Lecture 1

Thursday, January 27, 2022
10:11 AM

Permutation & combination.

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7

7 1 2 3

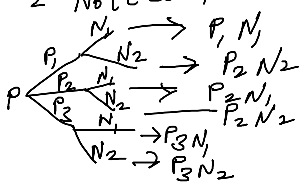
7 3 4 0

7 1 2 5

⋮ ⋮ ⋮ ⋮

ex. 3 Pens

2 Notebooks



$= 6$

$3 \times 2 = 6$ Pairs

ex. 2 bags, 3 Tiffin, 3 bottels

How many Pairs we can make

$2 \times 3 \times 3 = 18$.

✓ If an event can occur in m different ways, following which another event can occur in n different ways then total number of occurence of events are $= m \times n$.

ex. A, B, C, D, Find the number of 4 letter words, with or without meaning. Repeatition of letters is not allowed.

□ □ □ □

$4 \quad 3 \quad 2 \quad 1 \quad = 4 \times 3 \times 2 \times 1 = 24$

$4 \quad 4 \quad 4 \quad 4 \quad = 4^4 = 256$

(Repeatition is allowed)

ex. We have 4 flags of different colours & we want to make a signal taking two flags of different colours one below to another. How many different signals we can generate.

$\rightarrow 4$, $\rightarrow 3$ $= 4 \times 3 = 12$

Permutation.

A, B, C, D → ABCD
BCAD

A permutation is an arrangement in a definite order of number of objects taken some or all at a time.

ex. A B C D E F

How many 3 letter words we can make?

case - I

When all objects are distinct.

$$^nP_r = \frac{n!}{(n-r)!} \quad ; \quad 0 < r \le n.$$

i.e. Number of permutations of n different objects taken r at a time.

$$^6P_3 = \frac{6!}{(6-3)!} = \frac{6\times5\times4\times\cancel{3\times2\times1}}{\cancel{3\times2\times1}} = 120.$$

$n! = n\times(n-1)\times(n-2)\times\cdots\times1.$

$5! = 5\times4\times3\times2\times1 = 120$

$4! = 4\times3\times2\times1 = 24$

$0! = 1$

$12!$, $\lfloor 12$ } Factorial

ex. $\frac{12!}{10!\,2!} =$

$$\frac{12\times11\times\cancel{10!}}{\cancel{10!}\times2!} = 66.$$

ex. $\frac{1}{8!}+\frac{1}{9!} = \frac{x}{10!}$, $x = ?$

$$\frac{1}{8!}+\frac{1}{9\times8!} = \frac{x}{10!}$$

$$\frac{1}{8!}\left(1+\frac{1}{9}\right) = \frac{x}{10!}$$

$$\frac{1}{\cancel{8!}}\times\frac{10}{\cancel{9}} = \frac{x}{10\times\cancel{9\times8!}}$$

$$x = 100.$$

ex. $^nP_5 = 42\ ^nP_3$, $n > 4$.

$\because r \le n$ Find n., $n = 10$

case - 2 The number of permutations of n objects taken r at a time, repeation is allowed $= n^r$

ex. A, B, C, D, we want to make 4 letter words, repeation is allowed

$4^4 = 256.$

case - 3. The number of permutations of n objects, where p objects are of same kind $= \frac{n!}{p!}$

The number of permutations of n objects & p_1 objects are of same kind, p_2 objects are of same kind, ..., P_K objects are of same kind $= \dfrac{n!}{P_1!\, P_2! \cdots P_K!}$

ex. "ALLAHABAD"

$= \dfrac{9!}{4!\, 2!}$ | 4 A's, 2 L's

$= \dfrac{9\times8\times7\times6\times5\times\cancel{4!}}{\cancel{4!}\, 2!} = 7560$

ex. "INDEPENDENCE"

$= \dfrac{12!}{4!\, 3!\, 2!}$ | 4 E's, 3 N's, 2 D's

$= \boxed{1663200}$

ex $^5P_r = 2\, ^6P_{r-1}$, Find $r = ?$ | $r = 3, 10$

$\dfrac{5!}{(5-r)!} = 2 \times \dfrac{6!}{(6-r+1)!}$

$\dfrac{\cancel{5!}}{\cancel{(5-r)!}} = 2\times \dfrac{6\times\cancel{5!}}{(7-r)(6-r)\cancel{(5-r)!}}$

$(7-r)(6-r) = 12$

$r^2 - 13r + 42 = 12$

$r^2 - 13r + 30 = 0$

$(r-10)(r-3) = 0$

$r = 10, 3$

$= \underline{3}$

$ax^2 + bx + c = 0$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

(2) Combination.

The number of combinations of n different objects taken r at a time.

$$^nC_r = \frac{n!}{r!\,(n-r)!}, \quad 0 < r \le n$$

Relationship between permutation & combination

$$^nP_r = {}^nC_r \times r!$$

The main difference is in combination order doesn't matter, but in permutation order ~~no~~ matter.

ex $^5C_3 = \dfrac{5!}{3!\, 2!} = \dfrac{5\times4\times\cancel{3!}}{\cancel{3!}\times 2} = 10$

ex. $^nc_9 = {}^nc_8$, Find n

$$\frac{\cancel{n!}}{9!(n-9)!} = \frac{\cancel{n!}}{8!(n-8)!}$$

$$\frac{1}{9\times 8!(n-9)!} = \frac{1}{8!(n-8)(n-9)!}$$

$$n-8 = 9$$
$$= 17$$

ex. 52 Playing cards
4 cards out of 52

$$= {}^{52}c_4 = \frac{52!}{4!\,48!} = 270725$$

(i) Four cards are of same suit.

$$^{13}c_4 + {}^{13}c_4 + {}^{13}c_4 + {}^{13}c_4$$
$$= 4\,{}^{13}c_4 = 4\times\frac{13!}{4!\times 9!}$$
$$= 2860$$

(ii) cards are of same colour.

$$^{26}c_4 + {}^{26}c_4 = 2\times\frac{26!}{4!\times 22!}$$
$$= 29900$$

ex. $^nc_8 = {}^nc_2$, Find $^nc_2 = ?$

$$^nc_8 = {}^nc_2$$

$$\frac{\cancel{n!}}{8!(n-8)!} = \frac{\cancel{n!}}{2!(n-2)!}$$

$$2!(n-2)! = 8!(n-8)!$$

$$2!(n-2)(n-3)(n-4)(n-5)(n-6)(n-7)\cancel{(n-8)!} = 8!\cancel{(n-8)!}$$

$(n^6 + \cdots\cdots) = 4 \times 7!$

$^nc_r = {}^nc_{n-r}$ ✓

$^nc_8 = {}^nc_{n-8}$

$\boxed{^{10}c_8 = {}^{10}c_2}$ if $n = 10$

$n = 10 \Rightarrow {}^{10}c_2 = \dfrac{10!}{2!\,8!} = \dfrac{10 \times 9}{2} = 45$

Pascal's Triangle

Binomial expansion/ Theorem

$(a+b)^0 = 1$

$(a+b)^1 = a+b$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3.$

$(a+b)^4 =$

$(a+b)^{50} =$

$$(a+b)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n-1} b + {}^nC_2\, a^{n-2} b^2 + \cdots + {}^nC_n\, b^n$$

$$(a+b)^3 = {}^3C_0\, a^3 + {}^3C_1\, a^2 b + {}^3C_2\, a b^2 + {}^3C_3\, b^3 + \cancel{\boxed{{}^3C_4}}$$

$$= \frac{3!}{0!\,3!} a^3 + \frac{3!}{1!\,2!} a^2 b + \frac{3!}{2!\,1!} a b^2 + \frac{3!}{3!\,0!} b^3$$

$$= a^3 + 3a^2 b + 3ab^2 + b^3$$

$0! = 1$

Index	Coefficients
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1
6	1 6 15 20 15 6 1

$$(a+b)^5 = 1\cdot a^5 + 5a^4 b + 10a^3 b^2 + 10a^2 b^3 + 5ab^4 + 1\cdot b^5$$

$$(2x+5y)^6 = 1(2x)^6 + 6(2x)^5\, 5y + 15(2x)^4 (5y)^2 + 20(2x)^3 (5y)^3 + 15(2x)^2 (5y)^4 + 6(2x)(5y)^5 + 1(5y)^6$$

$$= 64x^6 + 960x^5 y + \cdots + 625 \times 25\, y^6$$

$$(2x - 5y)^3 = (\underbrace{2x}_{a} + \underbrace{(-5y)}_{b})^3$$

Groth of functions :

① **Big O, O( ).** Let $f(n)$ & $g(n)$ are two functions we say that $f(n) \in O(g(n))$, if $\exists$ a constant $c > 0$ & $n_0 \geq 1$ s.t. $f(n) \leq c\, g(n)$, $\forall\, n \geq n_0$

$\exists \rightarrow$ there exist

$\forall \rightarrow$ for all

ex $f(n) = 7n + 8$, $g(n) = n$

Is $f(n) \in O(g(n))$

$7n+8 \leq c \times n$ $\quad c = 1$

$7n+8 \leq n$, $n \geq 1$

$n = 100$

$708 \nleq 100$

$7n + 8 \leq 8n$

$7n + 8 \leq 8n$, $n \geq 8$

$c = 8$, $n_0 \geq 8$

$f(n) \in O(g(n))$ when $c = 8$, $n \geq 8$

$f(n) \notin O(g(n))$ when $c = 8$, $n < 8$

② **Little o, o( ).** Let $f(n)$ & $g(n)$ are two functions we say $f(n) \in o(g(n))$, $\exists\, c > 0$, $n_0 \geq 1$ s.t.

$f(n) < o(g(n))$, $\forall\, n \geq n_0$

ex. $f(n) = 7n + 8$, $g(n) = n$, is $f(n) \in O(g(n))$

$7n + 8 < c \cdot n$

$7n + 8 < 100n$ $\quad c = 100$

$f(n) \in O(g(n))$ if $c = 16$

$f(n) \notin O(g(n))$ if $c < 16$.

$7n + 8 < 16n$

$7n + 8 \not< 15n$

(3) Big Omega, $\Omega()$ $f(n)$ & $g(n)$ are two functions

we say $f(n) \in \Omega(g(n))$, if $\exists$ $c > 0$, $n_0 \geq 1$, s.t.

$f(n) \geq \Omega(g(n))$, $\forall$ $n \geq n_0$

(4) Little omega, $\omega()$ $f(n)$ & $g(n)$ are two functions

we say $f(n) \in \omega(g(n))$, if $\exists$ $c > 0$, $n_0 \geq 1$

s.t. $f(n) > \omega(g(n))$, $\forall$ $n \geq n_0$.

Module-III. Relations.

set. $A = \{a, b, \ldots\}$

Product A & B

$A = \{1, 2, 3\}$, $B = \{a, b, c\}$.

$$A \times B = \{(a,b) \in A \times B : a \in A \text{ and } b \in B\}.$$

$$R \subseteq A \times B = \{(1,a),(1,b),(1,c),(2,a),(2,b),(2,c),(3,a),(3,b),(3,c)\}.$$

Relation is a subset of product of sets.

If $(a,b) \in R$, then we say that a is related to b under the relation R.

A relation R in a set A is called

1. Reflexive, if $(a,a) \in R$, for every $a \in A$.
2. Symmetric, if $(a,b) \in R$ then $(b,a) \in R$, $\forall\, a,b \in A$
3. Transitive, if $(a,b) \in R$, $(b,c) \in R \Rightarrow (a,c) \in R$ $\forall\, a,b,c \in A$

A relation R in a set A is said to be an equivalence relation if it is reflexive, symmetric & transitive.

<u>ex.</u> T is the set of all triangles in a plane.
$R = \{(T_1, T_2) : T_1 \text{ is congruent to } T_2\}$
show that R is an equivalence Relation.

<u>Sol^n.</u> Every triangle is congruent to itself.

T_1 is Congruent to T_1

$(T_1, T_1) \in R$

(2) Symmetric

T_1 is congruent to $T_2 \Rightarrow T_2$ is congruent to T_1

R is symmetric.

(3) Transitive

T_1 is congruent to T_2

T_2 is congruent to T_3

$\Rightarrow T_1$ is congruent to T_3

R is an equivalance relation.

ex! Identity Relation.

$A = \{1, 2, 3\}$

$R = \{(1,1), (2,2), (3,3)\}$

$y = f(x) = x$

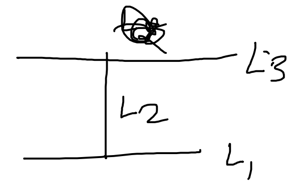
$x = 1$, $y = 1$ $\quad$ $x = 2$, $y = 2$

ex. L is set of all Lines in a plane.

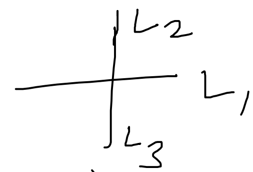
$R = \{(L_1, L_2) : L_1 \perp L_2\}$

Equivalance. ?

(1) Reflexive. A line l is not perpendicular to itself $(L_1, L_1) \notin R$



$L_1 \perp L_2 \Rightarrow L_2 \perp L_1$



Ex

$A = \{1, 2, 3\}$

$R = \{(1,1), (2,2), (3,3), \underline{(1,2), (2,3)}\}$

It is Reflexive, but not Symmetric nor Transitive $\underline{(1,3)\times}$