Section - A

Q-1

Sol 1 (i) $y^2p - xyq = x(z-2y)$

Given equation is of form $Pp + Qq = R$

$\Rightarrow P = y^2$

$\Rightarrow Q = -xy$

$\Rightarrow R = x(z-2y)$

The Lagrange's subsidiary equation are $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

i.e., $\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x(z-2y)}$

$\Rightarrow$ step I $\Rightarrow$ $\frac{dx}{y^2} = \frac{dy}{-xy}$

$\frac{dx}{y} = \frac{dy}{-x}$

$x\,dx = -y\,dy$

$\int x\,dx = -\int y\,dy$

$\frac{x^2}{2} = -\frac{y^2}{2} + c1/2$

$x^2 + y^2 = c1$

$u = x^2+y^2$

$\Rightarrow$ step II $\Rightarrow$ $\frac{dy}{-xy} = \frac{dz}{x(z-2y)}$

$\frac{dy}{-y} = \frac{dz}{(z-2y)}$

$(z-2y)\,dy = -y\,dz$

$z\,dy - 2y\,dy = (-y\,dz)$

$y\,dz + z\,dy = 2y\,dy$

$\int d(yz) = \int 2y\,dy$

$\int d(yz) = \int 2y\,dy$

$yz = y^2 + c2$

$v = yz - y^2$

So, solution for given eq^n is

$\boxed{f(x^2+y^2,\ yz-y^2) = 0}$

Sol 1 (ii) $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = 1$

According to Langranges Theorem.

$$\frac{\partial z}{\partial x} = P \quad , \quad \frac{\partial z}{\partial y} = q$$

$$(P)^2 + (Q)^2 = 1$$

It is of type I, equation whose solution is

$$z = ax + by + c$$

where,

$$a^2 + b^2 = 1$$

$$b = \sqrt{1-a^2}$$

$$z = ax + \sqrt{1-a^2}\, y + c.$$

Q-2.

Sol 2 Inverse fourier sine transformation $f(x)$, if function is $\frac{e^{-as}}{s}$

$\Rightarrow$ throughout $s = \alpha$

$$f_s(\alpha) = \frac{e^{-a\alpha}}{\alpha} \quad , \quad x > 0$$

By. diff. of inverse fourier sine transformation

$$f_s^{-1}\{f_s(\alpha)\} = f(x) = \frac{2}{\pi}\int_0^\infty f_s(\alpha)\sin(\alpha x)\,dx$$

$$f(x) = \frac{2}{\pi}\int_0^\infty \frac{1}{\alpha} e^{-ax} \sin(\alpha x)\,d\alpha \quad — \quad \text{I}$$

Differentiating undo integral sign with respect to x

$$\frac{dI}{dx} = \frac{2}{\pi}\int_0^\infty \frac{d}{dx}\left(\frac{1}{\alpha} e^{-a\alpha} \sin(\alpha x)\right] d\alpha$$

$$= \frac{2}{\pi}\int_0^\infty \frac{1}{\alpha} e^{-a\alpha} \cos(\alpha x) . \alpha . d\alpha$$

$$\boxed{\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)}$$

Hence $x = \alpha$, $a = -a$, $b = x$

$$\frac{dI}{dx} = 2/\pi\left[\frac{e^{-a\alpha}}{(-a)^2+x^2}\left[-a\cos\alpha x + x\sin\alpha x\right]\right]_0^\infty$$

$$= 2/\pi\left[\cancel{\frac{e^{-\infty}}{a^2+x^2}\left[-a\cos\infty + x\sin\infty\right]}\right] - \left[\frac{e^0}{a^2+x^2}\left(-a\cos 0 + \cancel{x\sin 0}\right)\right]$$

$$\frac{dI}{dx} = \frac{2}{\pi}\left[\frac{a}{a^2+x^2}\right]$$ which is the Diff. eq^n of 1^{st} order 1^{st} degree

$$dI = \frac{2}{\pi}\frac{a}{a^2+x^2}dx$$

on integration

$$I = \frac{2}{\pi}\int\frac{a}{a^2+x^2}dx$$

$$f(x) = \frac{2}{\pi}\int_0^\infty \frac{1}{\alpha}e^{-\alpha a}\sin(\alpha x)\,dx$$

$$= \frac{2}{\pi}\tan^{-1}(x/a) + c$$

To find c let $x = 0$

$$\frac{2}{\pi}\int_0^\infty \frac{1}{\alpha}e^{-\alpha a}\cdot\sin 0\,dx = \frac{2}{\pi}\tan^{-1}0 + c$$

$$0 = 0 + c$$

$$\boxed{c = 0}$$

$$\boxed{\therefore f(x) = \frac{2}{\pi}\tan^{-1}(x/a)}$$

Q-3
Sol 3(i)

$$f(t) = \sin(2t)\cos(t)$$

$$f(t) = 2\sin t \cos t \cos t$$

$$= 2\sin t \cos^2 t$$

$$= 2\sin t\,[1-\sin^2 t]$$

$$= 2\sin t - 2\sin^3 t$$

$$F(s) = \int_0^\infty e^{-st} f(t)\,dt$$

$$= -\int_0^\infty e^{-st}[2\sin t - 2\sin^3 t]\,dt$$

$$\int_0^\infty e^{-st} 2\sin t - 2\int_0^\infty e^{-st}\sin^3 t\,dt$$

$$\therefore \sin^3 t = \frac{1}{4}[3\sin t - \sin 3t]$$

$$f(s) = 2\left[\int_0^\infty e^{-st}\sin t\,dt - \int_0^\infty e^{-st}\frac{1}{4}[3\sin t - \sin 3t]dt\right.$$

$$= \frac{1}{2}\int_0^\infty e^{-st}\sin t\,dt + \frac{1}{2}\int_0^\infty e^{-st}\sin 3t\,dt$$

$$= -\frac{1}{2}\left[\frac{1}{s^2+1}\right] + \frac{1}{2}\left[\frac{3}{s^2+9}\right]$$

$$= \frac{1}{2}\left[\frac{+s^2+9+3s^2 \pm 3}{(s^2+1)(s^2+9)}\right] = \boxed{\frac{[2s^2+6]}{(s^2+1)(s^2+9)}}$$

Q-3 Sol 3(ii) $F(t) = tu(t-3)$, u is unit step function

$$\Rightarrow \; = tu(t-3)$$

$$= (t-3+3)\,u(t-3)$$

$$= \underbrace{(t-3)u(t-3)}_{(i)} + \underbrace{3u(t-3)}_{(ii)}$$

Solving eq (i)

Comparing, $(t-3)\ u(t-3)$ with $F(t-a)u(t-a)$

$F(t) = t$

$L\,F(t) = \frac{1}{s^2}$

By second shift Property

$L\{(t-3)\ u(t-3)\} = \frac{e^{-3s}}{s^2}$

Solving eqn (ii)

$= 3u\ (t-3)$

$\therefore\ L(u\ (t-a)) = \frac{e^{-as}}{s}$

So $3L\ (u(t-3))$

$= 3 \times \frac{e^{-3s}}{s}$

Put values of eqn (i) & eqn (ii)

$$= \boxed{\frac{e^{-3s}}{s^2} + \frac{3e^{-3s}}{s}}$$

Q-4
Ans 4(i) Solve: $(D^2 + D'^2)z = \sin 2x$

Complementary functions (C.F) is obtaining by putting

$D^2 = M$, $D'^2 = 1$

$(D^2+1) = 0 \rightarrow D = \sqrt{-1},\ D = \pm i$

$\therefore C.F = C_1 \cos x + C_2 \sin x$

Particular Integral, $P.I. = \frac{\sin 2x}{(D^2+D'^2)} = \frac{\sin 2x}{(-1^2+D'^2)} = \frac{\sin 2x}{(D'^2-1^2)}$

[one writing -1^2 for D^2, y is not present so we taking 0

$PI = \frac{\sin 2x}{-1} = -\sin 2x$

$\boxed{\text{General Solution} = C_1\cos x + C_2 \sin x - \sin 2x}$

Q4

Sol 4(b)

Obtain fourier series for the function

$F(x) = x + x^e, \ -\pi < x < \pi$

→ due carefully commute the fourier co-efficients

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} (x + x^2)\,dx$$

$$= \frac{1}{\pi}\left(2\int_0^{\pi} x^2\,dx + 0\right)$$

$$= \left[\frac{2}{\pi} \cdot \frac{x^3}{3}\right]_0^{\pi}$$

$$= \frac{2\pi^2}{3}$$

For $n \geq 1$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (x + x^2)\cos(nx)\,dx$$

$$= 2/\pi \int_0^{\pi} x^2 \cos(nx)\,dx, \text{ vid even/odd}$$

$$= 2/\pi \left(x^2 \cdot \frac{\sin(nx)}{x} - 2x \cdot \frac{-\cos(nx)}{n^e} + 2 \cdot \frac{-\sin(nx)}{n^3}\right)_0^{\pi}$$

$$= \frac{4(-1)^n}{n^2}$$

Similarly,

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (x + x^2)\sin(nx)\,dx$$

$$= 2/\pi \int_0^{\pi} x\sin(nx)\,dx, \text{ vid even/odd.}$$

$$= 2/\pi \left(x \cdot \frac{-\cos(nx)}{n} + 1 \cdot \frac{-\sin(nx)}{n^2}\right]_0^{\pi}$$

$$= \frac{2(-1)^{n+1}}{n}$$

Hence the fourier series for $F(x) = x + x^2$ on $(-\pi, \pi)$ is given by

$$\boxed{\frac{1}{2} \cdot \frac{2\pi^2}{3} + \sum_{n=1}^{\infty} \left(\frac{4(-1)^n}{n^2}\cos(nx) + \frac{2(-1)^{x+1}}{n}\sin(nx)\right)}$$

Q-5

Sol5 → Minimize

$$U = -3x + y - 2z$$

Subject to

$$x + 3y + z \le 5$$
$$2x - y + z \ge 2$$
$$4x + 3y - 2z = 5$$
$$x, y, z \ge 0$$

linear max = - linear min

⇒ Maximize $U = 3x - y + 2z$

with subject equations

Converting inequalities

⇒ Max $0x + 0y + 0z + 0x_4 + 0x_5 + (-A_1) + (-A_2)$

$$\Rightarrow x + 3y + z + x_4 = 5$$
$$2x - y + z - x_5 + A_1 = 2$$
$$4x + 3y - 2z + A_2 = 5$$

Table 1

C_B	C_J Basic Var	0 x	0 y	0 z	0 x_4	0 x_5	-1 A_1	-1 A_2	Values
0	x_4	1	3	1	1	0	0	0	5
-1	A_1	2	-1	1	0	-1	1	0	2
-1	A_2	4	3	-2	0	0	0	1	5
$Z_j - C_j$		-6	-2	1	0	1	0	0	

Key Column = x_1 Column

min = 1

Key role = A_1 row

Pivot element = 2

A_1 departs and x_3 enters

Table 2

C_B	C_J / Basic Var.	x_1 (0)	y_2 (0)	z (0)	x_4 (0)	x_5 (0)	A_2 (−1)	Values $b=y$
0	x_4	0	7/2	1/2	1	1/2	0	4
0	x	1	−1/2	1/2	0	−1/2	0	1
−1	A_2	0	5	−4	0	2	1	1
$z_j - c_j$		0	−5	4	0	−2	0	

Table 3

C_B	C_J / Basic Var.	x (3)	y (−1)	z (2)	x_4 (0)	x_5 (0)	Solution Values $b = x_B$
0	x_4	0	0	33/10	1	−9/10	
3	x	1	0	1/10	0	−3/10	
−1	y	0	1	−4/5	0	2/5	
$z_j - c_j$		0	0	−9/10	0	−13/10	

~~Table 4~~

C_B	C_j / Basic Var.	x (3)	y (−1)	z (2)	x_4 (0)	x_5 (0)	Values $b = (x_B)$
0	x_4	0	9/4	3/2	2/3	0	5/2
3	x	1	3/4	0	−1/3	0	5/2
0	x_5	0	5/2	0	4/3	1	1/2
$Z_j - C_j$		0	13/4	0	7/3	0	

~~Ans optical policy is $x_1 = 5/2$, $y = 0$~~

~~$z = 5/2$~~

~~Optimal value $= M = 25/2$~~

Table 4

CB	C_j / Basic Var.	3 x	-1 y	2 z	0 x_4	0 x_5	Values $b = (x_B)$
0	x_4	0	9/4	3/2	1	0	13/4
3	x	1	3/4	-1/2	0	0	5/4
0	x_5	0	5/2	-2	0	1	1/2
$Z_j - C_j$		0	13/4	-7/2	0	0	

Table 5

CB	C_j / Basic Var.	3 x	-1 y	2 z	0 x_4	0 x_5	Values $b = (x_B)$
2	z	0	3/2	1	2/3	0	5/2
3	x	1	3/2	0	1/3	0	5/2
0	x_5	0	1/2	0	4/3	1	1/2
$Z_j - C_j$		0	17/2	0	7/3	0	

Ans optimal policy is $x = 5/2$ $y = 0$

$z = 5/2$

optimal value $= u = \underline{\underline{25/2}}$