DS502 - HW3
Vandana Anand
Kratika Agrawal

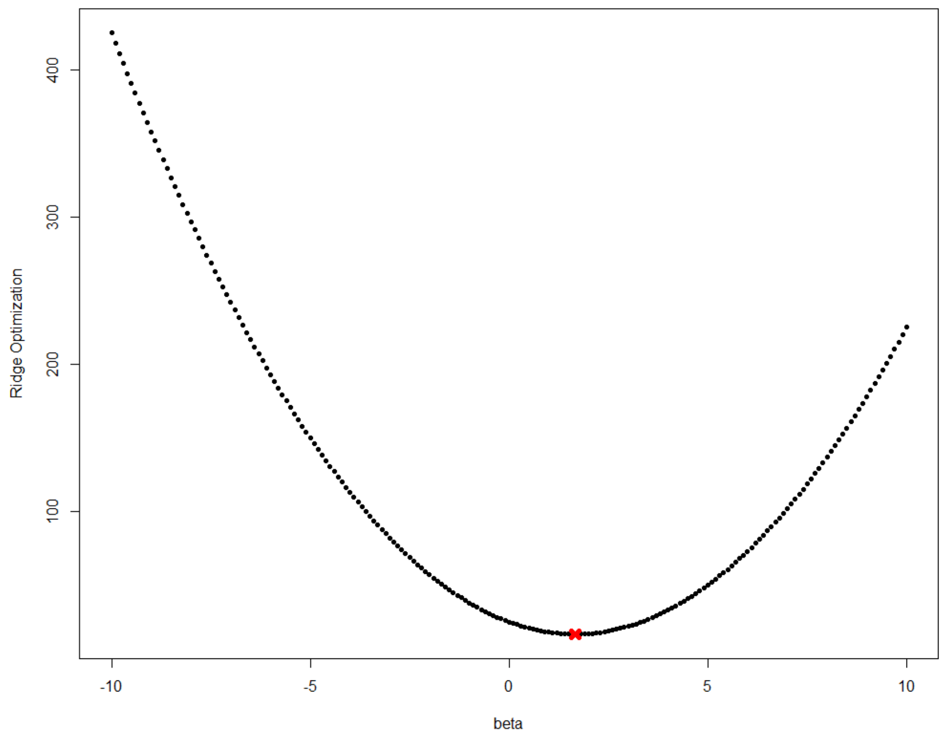
Question 1:

a) Best subset will have the smallest training RSS because it takes into account all the K predictors.
b) Cannot tell exactly. Best subset can have the smallest test RSS because it accounts for more models than forward or backward selection, but forward or backward selection can also pick the best one by chance.
c) i) True, the (k+1) model is achieved by adding an additional predictor to the model with k predictors
ii) True, the model with k predictors is achieved by removing a predictor from the model with (k+1) predictors
iii) False, there is no explicit relation between the forward and backward models
iv) False, same as above in which there is no explicit relation between forward and backward models
v) False, the model with (k+1) predictors is achieved by selecting from a list of possible (k+1) models which does not mean that all predictors will be used from the model with k predictors

Question 2:

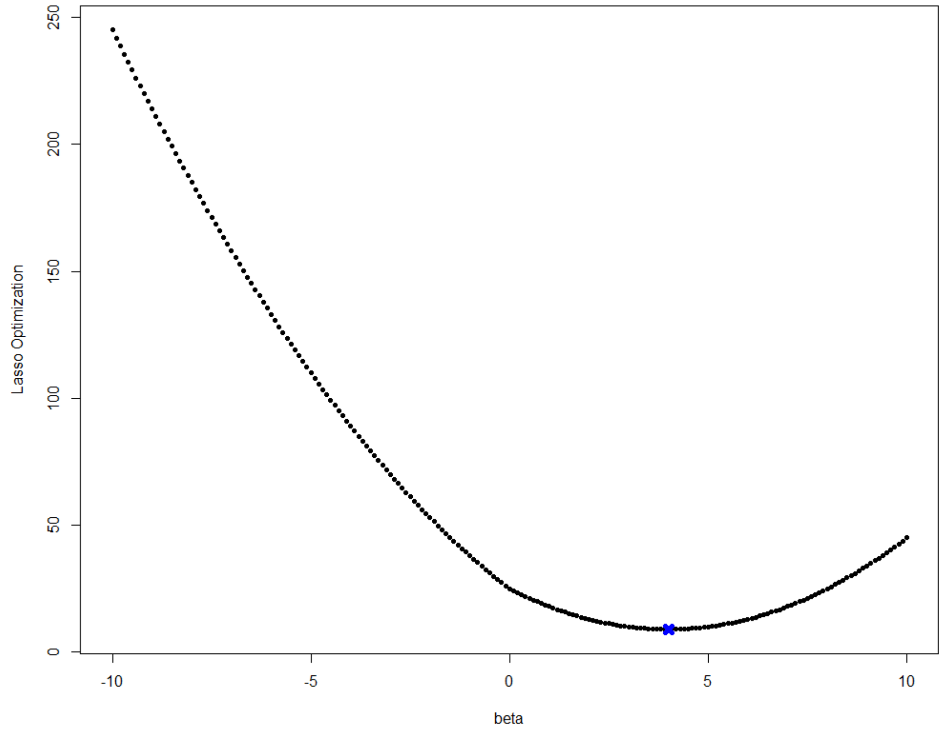
a) Ridge-Regression optimization plot for p=1. Minimum point is marked with red-cross

```
> #answer a
> y=5
> lambda=2
> beta=seq(from=-10,to=10,by=0.1)
> ridge = (y - beta)^2 + lambda * beta
> plot(beta, ridge, pch = 20, xlab = "beta", ylab = "Ridge Optimization")
> beta.est = y / (1 + lambda)
> points(beta.est, (y - beta.est)^2 + lambda * beta.est^2, col = "red", pch = 4, lwd = 5)
```



b) Lasso optimization plot for p=1. Minimum point is marked with blue-cross

```
> #answer b
> y1=5
> lambda1=2
> beta1=seq(from=-10,to=10,by=0.1)
> lasso = (y1 - beta1)^2 + lambda1 * abs(beta1)
> plot(beta1, lasso, pch = 20, xlab = "beta", ylab = "Lasso Optimization")
> beta.est1 = y1 - lambda1 / 2
> points(beta.est1, (y1 - beta.est1)^2 + lambda1 * abs(beta.est1), col = "blue", pch = 4, lwd = 5)
```



Question 3:

a)

```
> X = rnorm(100)
> print(X)
  [1]  1.572811625  0.780971848 -1.420947777  1.313922852  1.500466967  1.572601419  0.320873510
  [8]  0.266054665 -0.358781423 -1.455010644  0.008345388  0.915967690 -0.352883504  0.057068022
 [15] -0.371916790 -0.086373366  1.552282794  1.092933176 -0.336224420  0.746190244  0.412800629
 [22] -0.388417586 -0.202330250  2.643844846  0.486020881  0.546572608  0.296103739  0.211759239
 [29]  1.564742022 -0.648982423  0.018343735  0.331200438  0.469972889 -0.238855454  1.654410637
 [36]  0.273247011 -1.071088939 -0.848405608 -0.127589553 -0.617698635 -1.388631344  0.009807972
 [43] -0.179299439  2.310902142  0.964434329  0.911025079  0.539405024  0.396191029  1.194989355
 [50] -0.191333364  0.294093475  0.343087452  0.936413697 -1.111776497  0.917297671 -0.472013168
 [57]  1.139136757  0.447037643 -0.132862854  0.920285224  0.467461809  1.461597714 -0.765009390
 [64]  0.754972616  2.321174325 -0.802375570  0.668474997  0.246218868 -0.189907791 -0.389410318
 [71] -0.691435054 -0.869735489  0.578819965  0.693310427  1.202624968  0.276861500  0.216298711
 [78]  0.182300444 -0.038493102 -0.693806352  0.426144124 -0.047031392  0.481128044 -0.534493420
 [85]  0.432347342 -0.704107413  1.037099071 -0.735772798  1.013443704  0.073244111  1.086505438
 [92] -0.436003376 -0.634971143  0.521737335  0.678135522 -0.334026303 -0.663015093  1.081133166
 [99]  0.578612965 -0.828046011
> E = rnorm(100)
> print(E)
  [1] -0.291191968 -2.638856192 -0.641337403  1.270158583  0.731393983 -1.526897905 -1.047261087
  [8]  0.269386742  0.870344638  1.652005435 -0.239205373 -1.659932524 -0.229642418 -1.681208170
 [15]  1.805610377 -1.499714250 -0.750717640  2.230034278 -0.308857060  1.685940568  1.057690658
 [22] -0.859116952 -1.496276314  0.717054978  0.015507831  0.514452782 -0.817393772  0.850433080
 [29]  0.151039483  0.110196472  0.887531898 -3.139001113 -0.588382566 -1.197520957 -0.027254108
 [36] -0.112372991 -0.118370905 -0.068722638 -0.009291501  0.828773758  0.897427919 -0.100192455
 [43]  0.821919958 -1.639731500  0.095978863 -1.043410715 -0.947350283  0.041113225 -0.525870912
 [50]  3.158187532 -0.682065084 -1.164191983 -0.036394897 -1.221653419  1.713555298 -0.577989522
 [57] -1.332304516  0.373853174  0.280026949 -2.016144126  0.609746725  1.021660687  0.696618578
 [64]  0.839436304  0.095655024  0.629825305 -0.636449170  0.324710724  1.109422328  0.472699645
 [71]  0.440972833 -1.134264828  0.593960109 -0.742446841  0.368282707  0.566031099  0.595243755
 [78] -1.961764002 -0.501006504 -1.823720958  0.963744292 -0.585410127 -0.828554735 -0.227558098
 [85]  2.440124798  0.559880696 -0.526942037  0.082318847 -0.511886264  0.662005227  0.064772966
 [92] -0.180939483  0.194524969 -0.095734883  0.066424444 -1.354347279  0.951039771  0.089897617
 [99]  1.338292854  0.796837652
```

b)

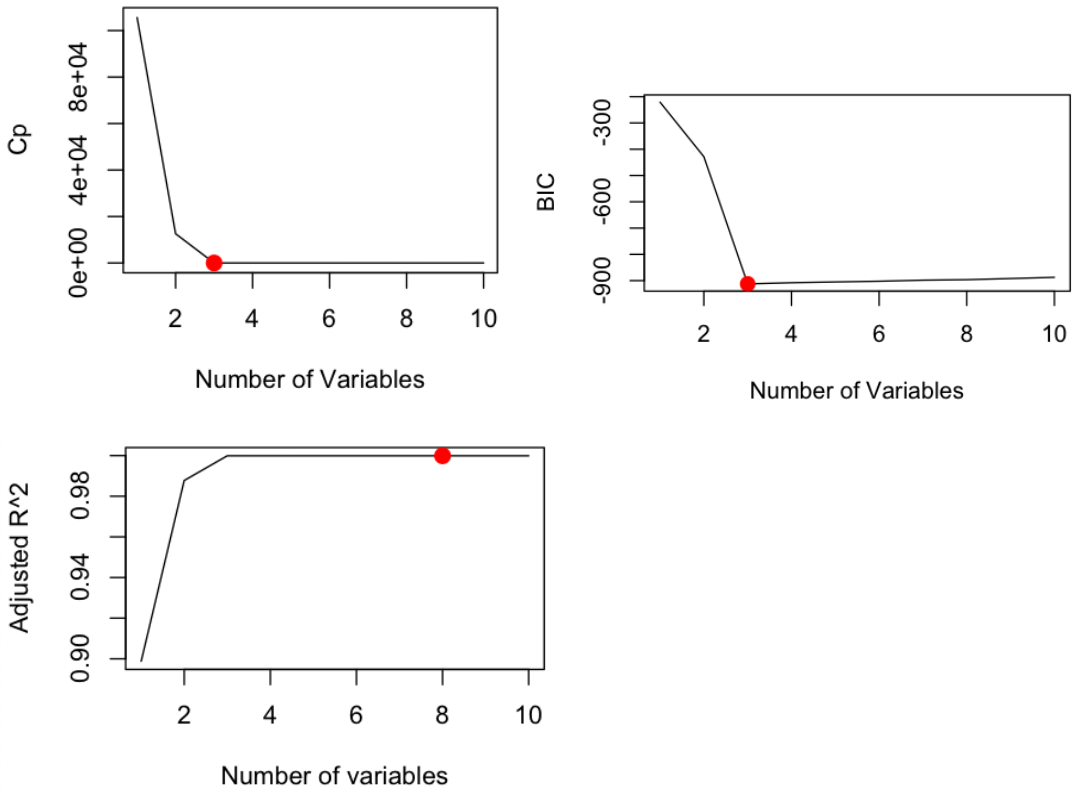
```
> Y = 3 + 5*X + 4*X^2 + 1*X^3 + E
> print(Y)
  [1] 24.6721501  9.9372536  1.1401586 18.6822370 22.8844033 24.6934981  5.0333949  4.5165812
  [9]  1.6802054  0.9336600  3.1949358 11.6902884  1.7885788  3.4017281  1.6452386  2.6881952
 [17] 24.1076222 14.3993634  1.5933002  9.3690480  5.7175668  1.6449421  2.2459965 62.6036447
 [25]  6.4334542  7.0391403  4.9170855  4.2445650 24.4347156  1.0440942  3.1511236  4.9955788
 [33]  6.2505458  2.0861037 26.7295770  4.7257564  1.0095364  1.0085213  2.4472225  1.0353361
 [41]  1.1528509  3.0780389  2.2018424 48.0143746 12.3352944 11.6088536  6.9491828  5.6619911
 [49] 16.5337417  1.9482560  4.6834387  5.2881887 12.0009792  1.0696424 11.8430433  1.4122298
 [57] 15.2808862  6.1554456  2.3536081 11.4338485  6.2342398 21.8950365  0.7687583  9.4028611
 [65] 48.6121087  1.0036843  8.4951871  4.4426273  2.2761267  1.6890510  1.1287273  1.0382720
 [73]  7.3744009  8.8853387 16.4509398  4.8366128  4.1861227  4.0902014  2.7443529  1.1455625
 [81]  5.7093421  2.7199172  6.3427651  1.3697124  6.0954730  1.1766829 13.7648439  1.0948876
 [89] 13.1135924  3.3990015 14.3148613  1.4760973  1.3360798  6.9702795  8.4383459  1.7155002
 [97]  1.0659152 14.4006120  7.4255756  1.1393767
```

c) Best Subset model

```
bestsubset = regsubsets(Y ~ X + I(X^2) + I(X^3) + I(X^4) + I(X^5) +
 I(X^6) + I(X^7) + I(X^8) + I(X^9) + I(X^10), data = dataX, nvmax =
 10)

bestsubsetSummary = summary(bestsubset)
```

```
          X   I(X^2) I(X^3) I(X^4) I(X^5) I(X^6) I(X^7) I(X^8) I(X^9) I(X^10)
1  ( 1 ) " " " "    "*"    " "    " "    " "    " "    " "    " "    " "
2  ( 1 ) "*" "*"    " "    " "    " "    " "    " "    " "    " "    " "
3  ( 1 ) "*" "*"    "*"    " "    " "    " "    " "    " "    " "    " "
4  ( 1 ) "*" "*"    "*"    "*"    " "    " "    " "    " "    " "    " "
5  ( 1 ) "*" "*"    "*"    " "    " "    " "    " "    "*"    " "    "*"
6  ( 1 ) "*" "*"    "*"    "*"    " "    "*"    "*"    " "    " "    " "
7  ( 1 ) "*" "*"    "*"    "*"    " "    "*"    " "    " "    "*"    "*"
8  ( 1 ) "*" "*"    "*"    " "    "*"    "*"    "*"    "*"    "*"    " "
9  ( 1 ) "*" "*"    "*"    "*"    "*"    "*"    " "    "*"    "*"    "*"
10  ( 1 ) "*" "*"   "*"    "*"    "*"    "*"    "*"    "*"    "*"    "*"
```

```
par(mfrow = c(2,2))

plot(bestsubsetSummary$cp, xlab = "Number of Variables", ylab = "Cp",
 type = "l")
points(which.min(bestsubsetSummary$cp),
 bestsubsetSummary$cp[which.min(bestsubsetSummary$cp)], col = "red",
 cex = 2, pch = 20)

plot(bestsubsetSummary$bic, xlab = "Number of Variables", ylab =
 "BIC", type = "l")
points(which.min(bestsubsetSummary$bic),
 bestsubsetSummary$bic[which.min(bestsubsetSummary$bic)], col = "red",
 cex = 2, pch = 20)

plot(bestsubsetSummary$adjr2, xlab = "Number of Variables", ylab =
 "Adjusted R^2", type = "l")
points(which.max(bestsubsetSummary$adjr2),
 bestsubsetSummary$adjr2[which.max(bestsubsetSummary$adjr2)], col =
 "red", cex = 2, pch = 20)
```

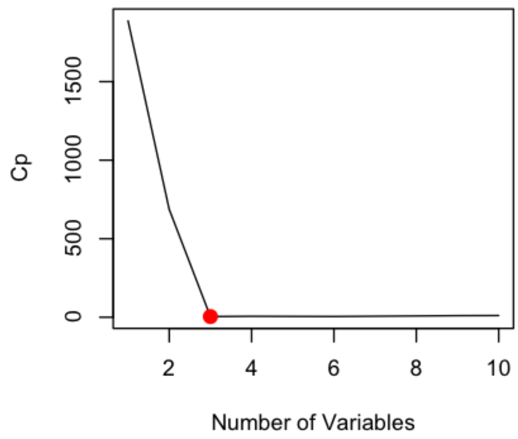


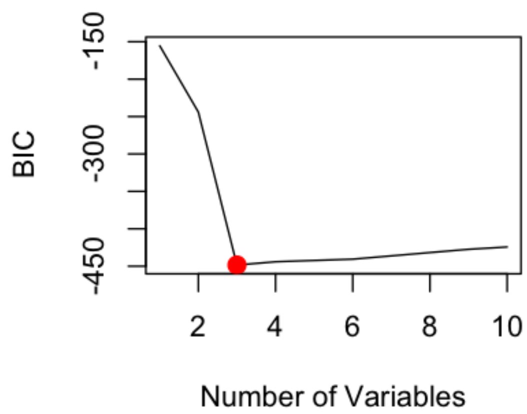
Cp - pick the 3-variables model, Bic - pick the 3 variables model, and
Adj R^2 - pick the 8 variables model

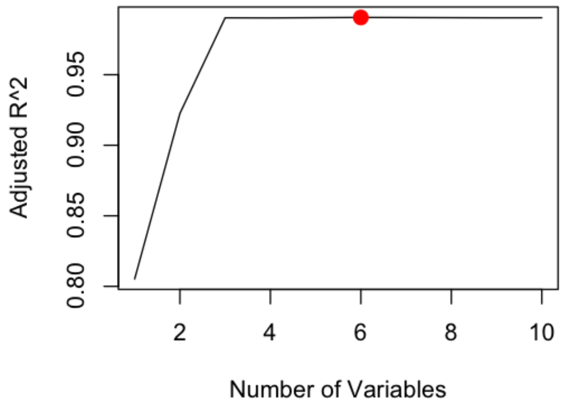
```
> coef(bestsubset, which.max(bestsubsetSummary$adjr2))
 (Intercept)            X      I(X^2)      I(X^3)      I(X^4)      I(X^8)     I(X^10)
 2.616881878  4.827080683  5.429391881  1.058400609 -0.723212109  0.037286705 -0.003569669
```

d) Forward model

```
plot(forwardSummary$cp, xlab = "Number of Variables", ylab = "Cp",
 type = "l")
points(which.min(forwardSummary$cp),
 forwardSummary$cp[which.min(forwardSummary$cp)], col = "red", cex =
 2, pch = 20)

plot(forwardSummary$bic, xlab = "Number of Variables", ylab = "BIC",
 type = "l")
points(which.min(forwardSummary$bic),
 forwardSummary$bic[which.min(forwardSummary$bic)], col = "red", cex =
 2, pch = 20)

plot(forwardSummary$adjr2, xlab = "Number of Variables", ylab =
 "Adjusted R^2", type = "l")
points(which.max(forwardSummary$adjr2),
 forwardSummary$adjr2[which.max(forwardSummary$adjr2)], col = "red",
 cex = 2, pch = 20)
```





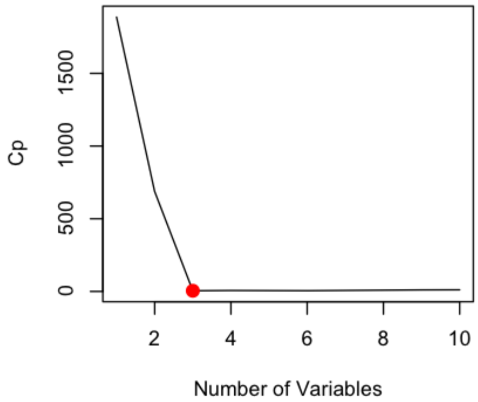


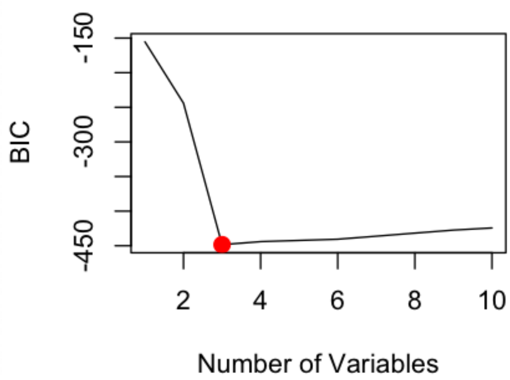
Cp - pick the 3 variables model, Bic - pick the 3 variables model, and Adj R^2 - pick the 6 variables model

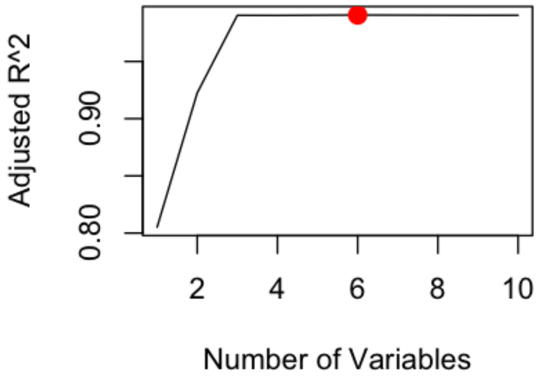
```
> coef(forward, which.max(forwardSummary$adjr2))
 (Intercept)           X      I(X^2)      I(X^3)      I(X^4)      I(X^6)     I(X^10)
 2.592789735 4.833287398 5.663984303 1.053787063 -1.094339766 0.191123540 -0.001228671
```

Backward model

```
plot(backwardSummary$cp, xlab = "Number of Variables", ylab = "Cp",
 type = "l")
points(which.min(backwardSummary$cp),
 backwardSummary$cp[which.min(backwardSummary$cp)], col = "red", cex =
 2, pch = 20)

plot(backwardSummary$bic, xlab = "Number of Variables", ylab = "BIC",
 type = "l")
points(which.min(backwardSummary$bic),
 backwardSummary$bic[which.min(backwardSummary$bic)], col = "red", cex
 = 2, pch = 20)

plot(backwardSummary$adjr2, xlab = "Number of Variables", ylab =
 "Adjusted R^2", type = "l")
points(which.max(backwardSummary$adjr2),
 backwardSummary$adjr2[which.max(backwardSummary$adjr2)], col = "red",
 cex = 2, pch = 20)
```





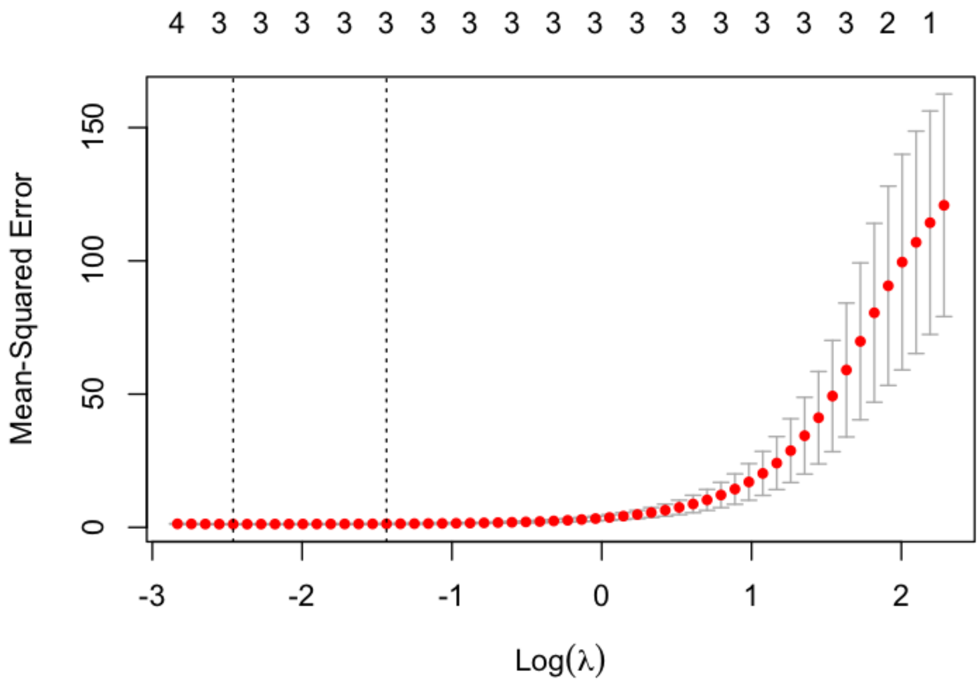


Cp - pick the 3 variables model, Bic - pick the 3 variables model, and Adj R^2 - pick the 6 variables model

```
> coef(backward, which.max(backwardSummary$adjr2))
(Intercept)           X      I(X^2)      I(X^5)      I(X^6)      I(X^7)      I(X^8)
 2.77591030  5.31679015  4.51613281  0.51143911 -0.12213077 -0.06768933  0.02116181
```

e) Lasso method

```
library(glmnet)

lasso = model.matrix(Y ~ X + I(X^2) + I(X^3) + I(X^4) + I(X^5) +
 I(X^6) + I(X^7) + I(X^8) + I(X^9) + I(X^10), data = dataX)[,-1]

cvLasso = cv.glmnet(lasso, Y, alpha=1)
plot(cvLasso)
```



The best lambda is:

```
> bestLambda = crossvalLasso$lambda.min
> print(bestLambda)
[1] 0.07092195
```

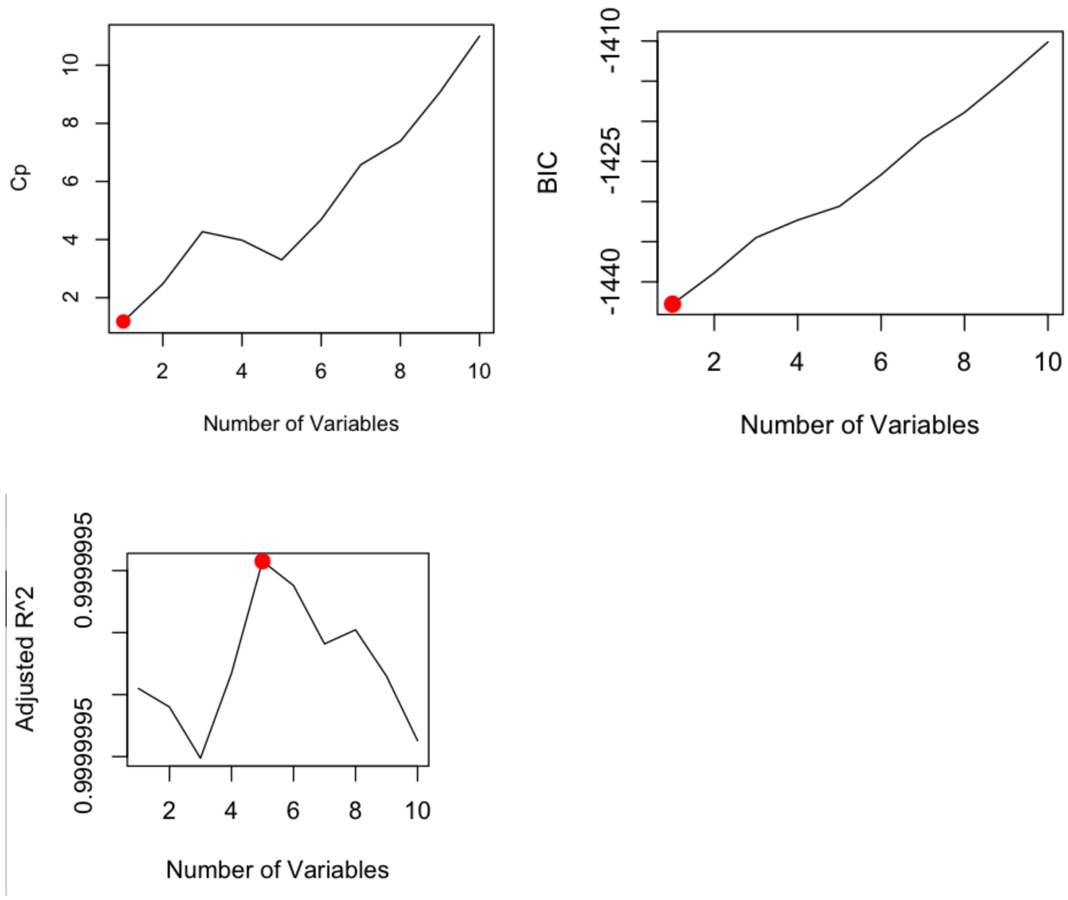
So we run the model using this value:

```
> lassoBestLambda = glmnet(lasso, Y, alpha = 1)
> predict(lassoBestLambda, s = bestLambda, type = "coefficients")[1:11,]
(Intercept)           X      I(X^2)      I(X^3)      I(X^4)      I(X^5)      I(X^6)      I(X^7)
   2.979571    4.753819    3.994324    1.049389    0.000000    0.000000    0.000000    0.000000
     I(X^8)      I(X^9)     I(X^10)
   0.000000    0.000000    0.000000
```

The lasso methods picks X, X^2, and X^3 as variables for the model.

f) Best subset model:

```
Y = 3 + 10*X^7 + E

datanewX = data.frame(X,Y)

bestsubset2 = regsubsets(Y ~ X + I(X^2) + I(X^3) + I(X^4) + I(X^5) + I(X^6) +
 I(X^7) + I(X^8) + I(X^9) + I(X^10), data = datanewX, nvmax = 10)

bestsubsetSummary2 = summary(bestsubset2)

par(mfrow = c(2,2))

plot(bestsubsetSummary2$cp, xlab = "Number of Variables", ylab = "Cp", type =
 "l")
points(which.min(bestsubsetSummary2$cp),
 bestsubsetSummary2$cp[which.min(bestsubsetSummary2$cp)], col = "red", cex =
 2, pch = 20)

plot(bestsubsetSummary2$bic, xlab = "Number of Variables", ylab = "BIC", type
 = "l")
points(which.min(bestsubsetSummary2$bic),
 bestsubsetSummary2$bic[which.min(bestsubsetSummary2$bic)], col = "red", cex
 = 2, pch = 20)

plot(bestsubsetSummary2$adjr2, xlab = "Number of Variables", ylab = "Adjusted
 R^2", type = "l")
points(which.max(bestsubsetSummary2$adjr2),
 bestsubsetSummary2$adjr2[which.max(bestsubsetSummary2$adjr2)], col = "red",
 cex = 2, pch = 20)
```



Cp - pick the 1 variable model, Bic - pick the 1 variable model, and Adj R^2 - pick the 5 variables model

```
> coef(bestsubset2, 1)
(Intercept)      I(X^7)
   2.955573   10.000653
> coef(bestsubset2, 2)
(Intercept)           X       I(X^7)
  2.9646268  -0.1021027  10.0009142
> coef(bestsubset2, 4)
(Intercept)      I(X^2)      I(X^4)      I(X^6)      I(X^7)
 2.73725660  0.78060829 -0.35892187  0.03864099  9.99875833
```

BIC picks the most accurate 1-variable model.

Lasso Method:

```
lasso2 = model.matrix(newY ~ X + I(X^2) + I(X^3) + I(X^4) + I(X^5) + I(X^6) +
 I(X^7) + I(X^8) + I(X^9) + I(X^10), data = datanewX)[,-1]

crossvalLasso2 = cv.glmnet(lasso2, newY, alpha=1)
> bestLambda2 = crossvalLasso$lambda.min
> print(bestLambda2)
[1] 0.07092195
```

```
> lassoBestLambda2 = glmnet(lasso2, newY, alpha = 1)
> predict(lassoBestLambda2, s = bestLambda2, type = "coefficients")[1:11,]
(Intercept)           X      I(X^2)      I(X^3)      I(X^4)      I(X^5)      I(X^6)      I(X^7) 
   8.079787    0.000000    0.000000    0.000000    0.000000    0.000000    0.000000    9.709129 
     I(X^8)      I(X^9)     I(X^10) 
   0.000000    0.000000    0.000000 
```

Lasso picks an accurate 1 variable model, but the intercept is off by about 5 units.

Question 4:

a) Split the College data into train and test in 7:3 ratio.

b)

```
Call:
lm(formula = Apps ~ ., data = train)

Coefficients:
(Intercept)   PrivateYes       Accept       Enroll    Top10perc    Top25perc  F.Undergrad  
 -638.64835   -596.53900      1.32983     -0.27428     52.60987    -17.61480      0.04737  
P.Undergrad     Outstate   Room.Board        Books     Personal          PhD     Terminal  
    0.04116     -0.07459      0.18067     -0.09611      0.02455    -10.13790     -5.64889  
  S.F.Ratio  perc.alumni       Expend    Grad.Rate  
   23.73129     -6.46646      0.12574     11.01750  
```

```
> err.lm = mean((test$Apps - pred.lm)^2)
> err.lm
[1] 1409723
```

Least Square Test Error on Linear Model is 1409723

c) Ridge Regression Best Lambda:

```
> ridge.lambda
[1] 28.48036
```

Ridge Regression minimum Test Error:

```
> err.ridge
[1] 1560730
```

d) Lasso Best Lambda:

```
> lasso.lambda
[1] 8.111308
```

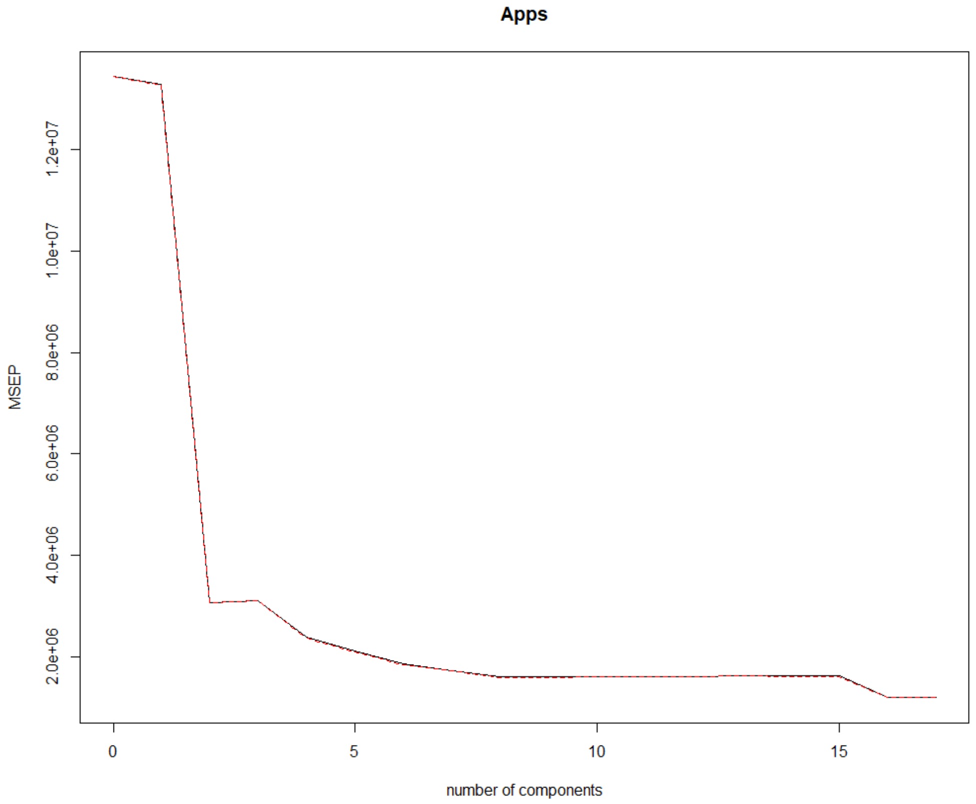
Lasso minimum Test Error:

```
> err.lasso
[1] 1448810
```

Lasso Non-zero coefficient estimates:

```
> coef.lasso[coef.lasso != 0]
  (Intercept)    PrivateYes        Accept     Top10perc     Top25perc   F.Undergrad   P.Undergrad 
-755.75876508 -582.61892808    1.28023352   46.91699381  -13.23745442    0.01776872    0.03662615 
     Outstate    Room.Board         Books      Personal           PhD      Terminal     S.F.Ratio 
  -0.06386837    0.16976881   -0.01678368    0.01062464   -8.59500784   -5.53121221   19.99611424 
  perc.alumni        Expend     Grad.Rate 
  -7.15656228    0.12107184    9.77722658 
```

e) PCR plot for value of M and MSEP:

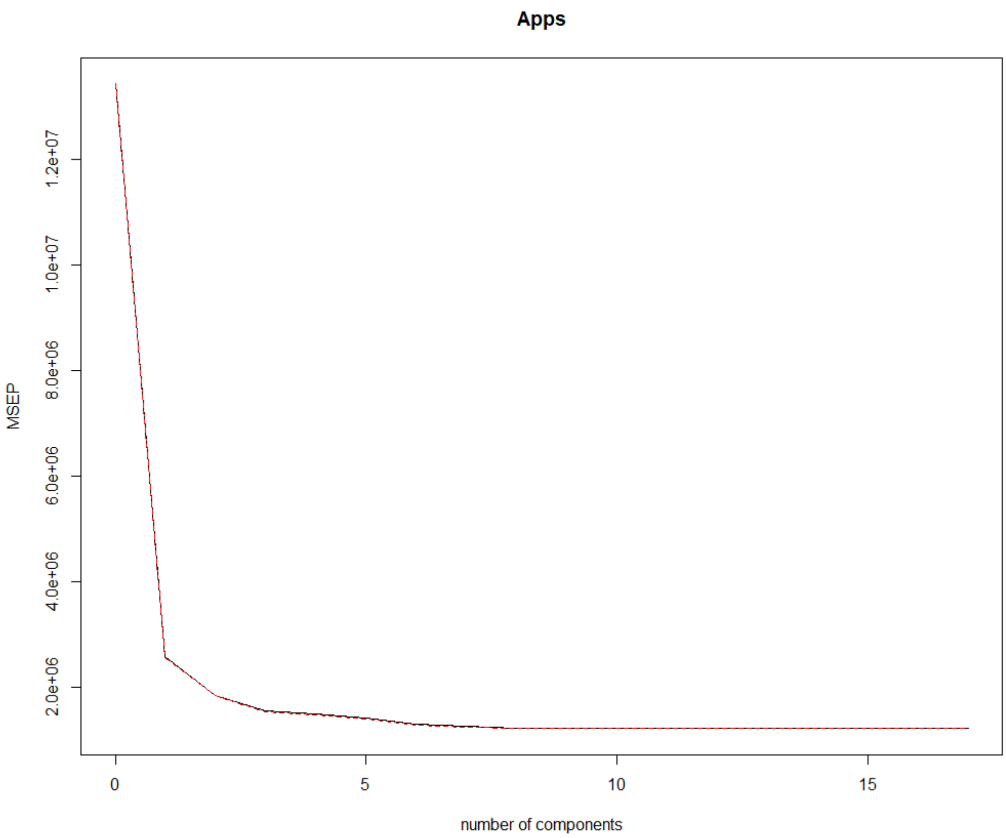


Visualizing the validation plot shows that MSEP is minimum for **M=16**.

Choosing M=16, yields PCR Test Error = 1542134:

```
> err.pcr
[1] 1542134
> summary(fit.pcr)
Data:   X dimension: 543 17
        Y dimension: 543 1
Fit method: svdpc
Number of components considered: 17

VALIDATION: RMSEP
Cross-validated using 10 random segments.
       (Intercept)  1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps  8 comps  9 comps
CV            3665     3643     1754     1764     1548     1454     1364     1313     1268     1268
adjCV         3665     3643     1753     1763     1542     1450     1362     1315     1265     1265
       10 comps  11 comps  12 comps  13 comps  14 comps  15 comps  16 comps  17 comps
CV         1270      1273      1273      1278      1277      1276      1098      1098
adjCV      1268      1271      1270      1274      1273      1273      1095      1094

TRAINING: % variance explained
      1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps  8 comps  9 comps  10 comps
X      32.539    58.51    65.54    70.98    76.18    80.88    84.60    88.00    91.00     93.36
Apps    1.865    77.55    77.67    83.12    85.15    86.79    87.78    88.66    88.66     88.79
      11 comps  12 comps  13 comps  14 comps  15 comps  16 comps  17 comps
X        95.35     97.03     98.13     98.99     99.48     99.84    100.00
Apps     88.80     89.00     89.04     89.06     89.24     91.98     92.16
```

f) PLS plot for value of M and MSEP:



Visualizing the validation plot shows that MSEP is minimum for **M=10**.

Choosing M=10, yields PCR Test Error = 1420562:

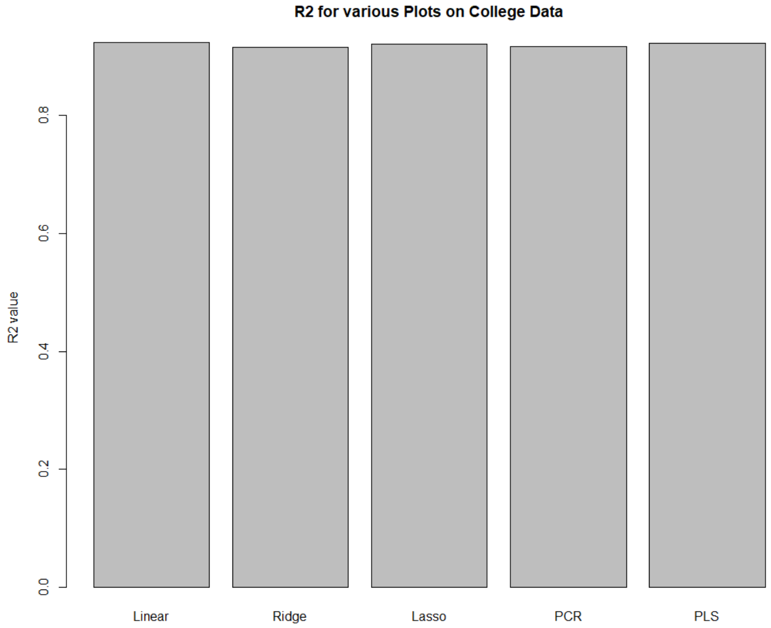
```
> err.plsr
[1] 1420562
> summary(fit.plsr)
Data:    X dimension: 543 17
         Y dimension: 543 1
Fit method: kernelpls
Number of components considered: 17

VALIDATION: RMSEP
Cross-validated using 10 random segments.
       (Intercept)  1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps  8 comps  9 comps
CV            3665     1606     1351     1237     1205     1148     1116     1101     1096     1094
adjCV         3665     1604     1354     1234     1200     1144     1112     1097     1092     1090
       10 comps  11 comps  12 comps  13 comps  14 comps  15 comps  16 comps  17 comps
CV         1092      1092      1092      1093      1093      1093      1093      1093
adjCV      1089      1088      1088      1089      1090      1090      1090      1090

TRAINING: % variance explained
      1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps  8 comps  9 comps  10 comps
X       26.10    42.12    63.55    67.17    70.56    74.76    78.01    81.33    84.04     87.16
Apps    81.44    87.06    89.38    90.32    91.37    91.90    92.07    92.11    92.13     92.14
      11 comps  12 comps  13 comps  14 comps  15 comps  16 comps  17 comps
X        89.65     92.05     93.58     94.52     96.94     98.36    100.00
Apps     92.15     92.15     92.15     92.16     92.16     92.16     92.16
```

g) R^2 value for various models is:

Linear Model, Ridge Regression, Lasso, PCR, PLS

```
> r2.all
[1] 0.9240041 0.9158635 0.9218970 0.9168660 0.9234197
```

The boxplot for these values is:



All the values and Boxplot shows that there isn't much difference in the accuracy of various models, and they perform almost the same. All models predict College applications with high accuracy of 91.5-92.4%.

Test Error value for various models:

```
> err.all
 Linear   Ridge   Lasso     PCR     PLS
1409723 1560730 1448810 1542134 1420562
```

Error is almost the same for all models but with maximum error for Ridge Regression(1560730) and minimum error for Linear Model(1409723).