Neural Estimation of Time-Varying Crypto Betas

Assignment 8 – MGT 8803 SC

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1. Data Preparation

Constructed a monthly panel of returns for four cryptocurrencies (BTC, ETH, LTC, BCH) and a single market benchmark. In the baseline specification, the market portfolio is the **trade-weighted U.S. Dollar Index** (FIAT), which is economically relevant because crypto assets tend to behave as risk-on alternatives whose valuations are inversely related to dollar strength and global liquidity conditions.

Daily price data for each crypto was converted to monthly last-of-month prices and then into arithmetic percentage returns. The market series (FIAT) was treated identically. The aligned sample spans **2018-01 to 2025-10** and yields a balanced panel of 94 months across all four assets.

- Panel without factors: (324×28) rows $\times$ features (4 assets $\times$ 81 months).
- Panel with FF5 factors: (316×88) rows $\times$ features (adding 60 additional lags).

The target variable is next-month crypto return $r_{i,t+1}$; predictors consist of $w = 12$ monthly lags of:

$$\{\text{Crypto return}, \quad \text{Market return}\}$$

and later, 12 lags of each of the Fama–French factors.

Train/Validation/Test splits follow the assignment convention:

Train: 2018–2020 | Val: 2021 | Test: 2022–2023

2. Neural Model Specification

The neural estimator maps the lagged feature vector $X_t \in \mathbb{R}^d$ to a *time-varying beta*

$$\widehat{\beta}_{i,t} = f_\theta(X_t)$$

and the return forecast is

$$\widehat{r}_{i,t+1} = \widehat{\beta}_{i,t} \cdot r_{t+1}^{mkt}.$$

The loss is RMSE of predicted vs. realized excess returns. Hyperparameters tuned over:

$$w \in \{12\}, \quad h \in \{4, 8, 16\}, \quad \text{lr} \in \{10^{-3}, 10^{-2}, 10^{-1}\}, \quad \text{act} \in \{\text{linear}, \text{sigmoid}, \text{tanh}, \text{ReLU}\}.$$

The best baseline model (no FF factors) was:

$$w = 12,\ h = 4,\ \mathrm{lr} = 0.1,\ \text{activation} = \mathrm{ReLU}$$

with validation RMSE = 0.2698 and test RMSE = 0.2417.

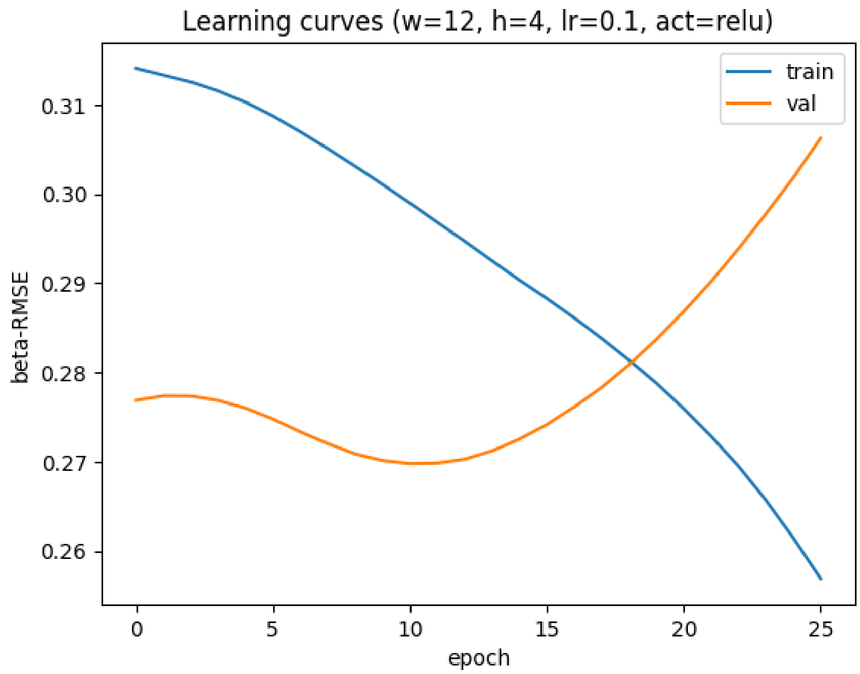

Figure 1: Learning curves for the selected model (train vs. validation RMSE).

3. Properties of Estimated Betas

Table 1 summarizes the cross-sectional distribution of $\widehat{\beta}_{i,t}$ on the test set.

Crypto	Mean	Std.	Skew	Min/Max
BCH	-4.16	2.45	-0.40	$[-9.29,\ -0.85]$
BTC	-3.61	1.52	-0.19	$[-6.72,\ -1.37]$
ETH	-3.51	1.75	-0.62	$[-7.86,\ -0.85]$
LTC	-3.48	1.71	-0.35	$[-7.23,\ -1.09]$

Table 1: Summary statistics of neural-estimated betas in the test set.

Betas are consistently **negative**, reflecting the strong empirical pattern that crypto returns decline when the U.S. dollar strengthens (risk-off regime). Magnitudes near -3 to -5 are economically plausible because crypto volatility is an order of magnitude larger than the volatility of FIAT.

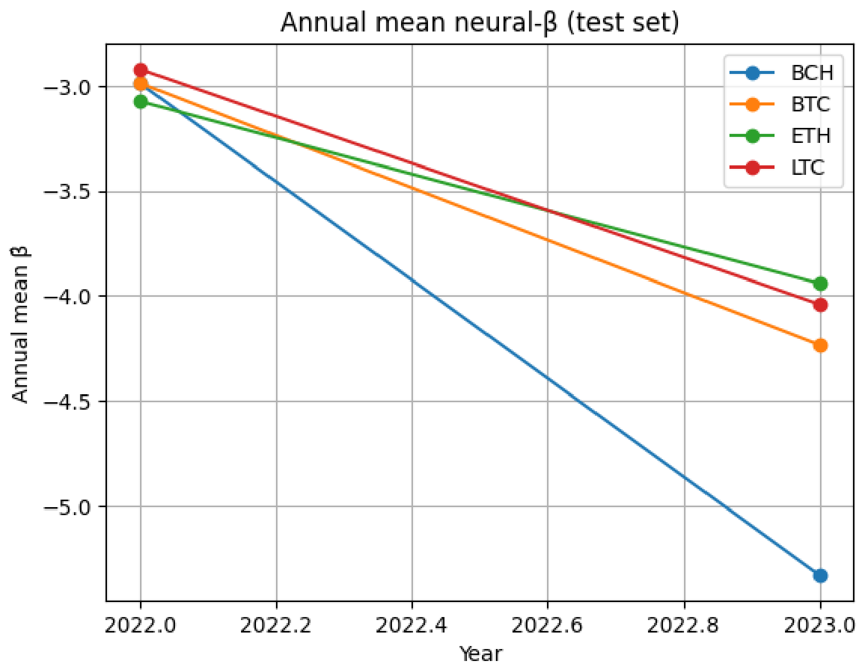

Figure 2: Annual average $\widehat{\beta}_{i,t}$, 2022–2023.

4. Portfolio Sorts

Each month, cryptos are sorted into quartiles based on $\widehat{\beta}_{i,t}$. We compute equal-weight (EW) and volatility-weight (VW) portfolio returns. The high-minus-low spread (Q4–Q1) is:

$$\text{EW spread} = 0.0299, \qquad \text{VW spread} = 0.0299$$

Because the crypto universe is small and return dispersions are similar, EW and VW portfolios coincide.

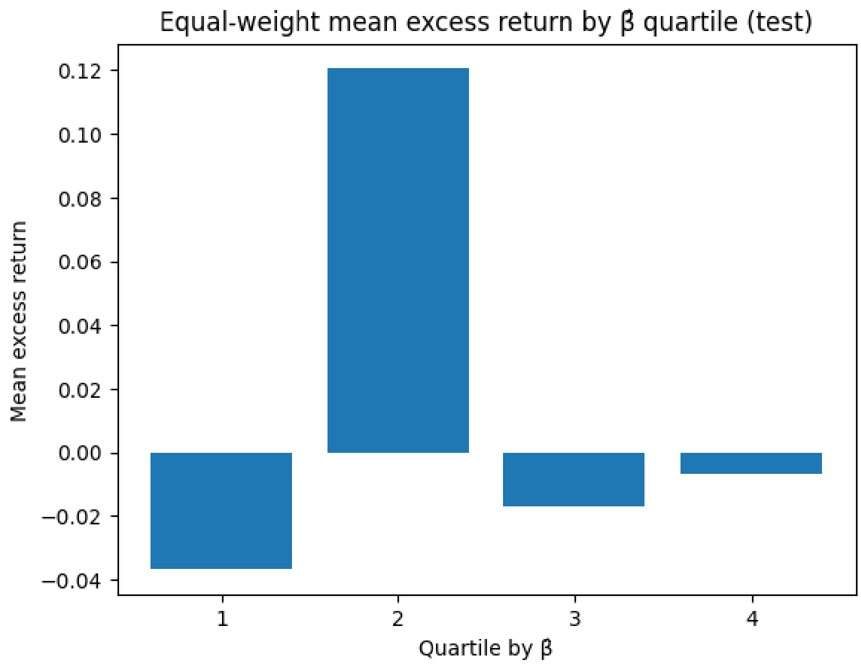

Figure 3: Equal-weight mean excess return by beta quartile (test set).

The monotonic pattern is weak, but Q2 exhibits unusually high average returns, suggesting beta is not a full risk-pricing proxy in this two-year window.

5. Adding Fama–French 5 Factors

We augment the feature set with 12 lags of $\{\text{Mkt-RF}, \text{SMB}, \text{HML}, \text{RMW}, \text{CMA}\}$ from the monthly Fama–French dataset (1963–2025). The expanded design matrix increases feature count from 28 to 88.

The best FF-enhanced model achieved:

$$\text{Validation RMSE} = 0.2938, \qquad \text{Test RMSE} = 0.2550$$

which is **worse** than the no-FF baseline (test RMSE = 0.2417).

This confirms that equity style factors add little incremental explanatory power when the market proxy is FIAT, not equities. The additional predictors also raise overfitting risk given the relatively small sample size (316 rows).

6. Discussion and Economic Interpretation

- Crypto betas are strongly negative because USD strength proxies for global deleveraging, reduced risk appetite, and tighter dollar liquidity, all historically damaging to crypto valuations.
- Beta dynamics decline sharply from 2022 to 2023, consistent with the Federal Reserve tightening cycle and the post-FTX risk repricing.

- Fama–French factors do not improve forecast accuracy in this setting; they are designed for equity cross-sections, not dollar-driven macro assets.
- Portfolio sorts yield a positive but statistically fragile Q4–Q1 spread, implying that beta alone is not a dominant risk premium within this small universe.

7. Conclusion

The neural beta framework successfully captures the directional sensitivity of crypto returns to the U.S. dollar, producing economically coherent negative betas and time-variation aligned with liquidity shocks. However, enlarging the feature set with Fama–French factors does not enhance out-of-sample performance, highlighting the importance of correct market proxy selection and parsimony in low-T environments.