Phys 121 Lab **Fall 2012**

Hints and Help

November 12, 2012 Giordon Stark

Contents

1 Introduction and Review

This week's lab is back to rotational motion. We'll be extending our study of Newton's Second Law

$$\sum F = ma$$

into rotational dynamics.

1.1 Lab Notebooks / Write ups

Make sure the following are covered in your write ups

- numbers are reported with units where possible (except for fractionals)
- measurements are reported with standard deviation ($g = 9.8 \pm 0.01\ m/s^2$)
- tables of numbers are labeled, the columns are appropriately labeled and boxed to separate it from the rest of the scribbles you might have on the page
- plots are titled, axes labeled with variable and units
- all fits performed on plots must report the results of said fits along with the plots
- sections are divided clearly, non-obvious calculations are shown by specifying what you're doing to calculate a number (e.g. "Take these values $\cdots$ and plug into equation $\cdots$")
- for those using carbon-copy, make sure your writing is clearly visible on the carbon-copy before turning it in and fix it if it's not (e.g. write harder)

2 Last Week's Lab

Last week's lab was a discussion on elastic collisions, inelastic collisions, and explosions. Let's review what we should have learned:

2.1 Explosions

Kinetic Energy is not conserved. Momentum is conserved. This is evident from initial and final states:

$$E_i = 0 \qquad E_f = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

and momentum

$$p_i = 0 \qquad p_f = m_1 v_1 - m_2 v_2$$

Where does the energy come from? It's the energy stored in the rubber band! You should have seen that momentum is conserved meaning that $p_i = p_f$. It should be quite clear, since energy is a scalar, that kinetic energy is not conserved (something positive plus something positive can not equal zero).

2.2 Elastic Collisions

Kinetic Energy is conserved. Momentum is conserved. We had one mass originally at rest. Our initial and final states are:

$$E_i = \frac{1}{2}m_1 v_i^2 \qquad E_f = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

and momentum

$$p_i = m_1 v_i \qquad p_f = m_1 v_1 + m_2 v_2$$

This is not as obvious. One can (and should) show that these quantities are conserved by measuring initial and final states and showing that these equal.

2.3 Inelastic Collision

Kinetic Energy is not conserved. Momentum is conserved. We had one mass originally at rest, and the two masses collide and stick and move together. Our initial and final states are

$$E_i = \frac{1}{2}m_1 v_i^2 \qquad E_f = \frac{1}{2}(m_1 + m_2) v_f^2$$

and momentum

$$p_i = m_1 v_i \qquad p_f = (m_1 + m_2) v_f$$

Where does our extra energy go here? It goes into sticking the two masses together. How much energy did we lose?

$$E_{loss} = E_i - E_f = \frac{1}{2}m_1 v_i^2 - \frac{1}{2}(m_1 + m_2) v_f^2$$

From conservation of momentum, we know that $v_f = \frac{m_1}{m_1+m_2} v_i$. You can substitute this in and simplify the E_{loss} in terms of the masses m_1, m_2 and the initial velocity v_i.

3 Rotational Dynamics

Instead of doing a section-by-section summary. We'll have a brief discussion about different fundamental ideas to impart from this lab. I assume, if you're reading this, that you're somewhat familiar with the lab manual's write up here.

3.1 Tension approximation

We can show the tension formula in equation (20) by discussing Newton's 2nd Laws rather briefly. Realize that we need to make some assumptions here. The first is $a = \alpha r$. We make this assumption because of the string connecting the acceleration of the small mass to the rotation of the disc. We also assume that the rope/string does not slip as it is accelerating / rotating. Therefore, from a free-body diagram on the hanging mass:

$$\sum F = ma = mg - T$$

where the acceleration a is in the direction of gravity (or pointing away from the spinning disc). Applying the law again on the rotational dynamics of the spinning disc:

$$\sum \tau = I\alpha = Tr$$

where I is the moment of inertia of the disc of radius R which is $\frac{1}{2}MR^2$ (M is the mass of the disc). Combine these equations together to show:

$$m\alpha r = mg - T \qquad\qquad I\alpha = Tr$$

$$\rightarrow T = mg\frac{I}{I + mr^2} \qquad\qquad \alpha = mg\frac{r}{I + mr^2}$$

We can make the approximation $T \approxeq mg$ as long as $I = \frac{1}{2}MR^2 \gg mr^2$. We can judge how strong our approximation is by computing

$$\%\text{strength} = 1 - \frac{mr^2}{I} = 1 - \frac{2mr^2}{MR^2}$$

3.2 Angles, Rotation, and Distances

There is a very awesome formula to calculate the arclength of an arc radius r subtended through an angle θ:

$$s = r\theta$$

Convince yourself that if $\theta = 2\pi$ (or a circle), that we have the formula for the circumference of a circle.

3.3 Conservation of Energy

As m falls to the floor, the potential energy it starts with is converted into kinetic energy of the mass and rotational energy of the disc:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

To show that this holds (or not), we simply need to show that each side equals each other. You know the initial state here mgh since you measure that, so you know $E_i = mgh$. Now, how do we get E_f? We know that the velocity of the mass is related to ωr, and we know ω from the computer measurement, and r from our measurement. So we can write the final energy (at the point which the block hits the ground):

$$E_i = mgh$$

$$\begin{aligned} E_f &= \frac{1}{2}m(\omega r)^2 + \frac{1}{2}(\frac{1}{2}MR^2)\omega^2 \\ &= \frac{1}{2}\left(\frac{1}{2}MR^2 + mr^2\right)\omega^2 \end{aligned}$$

We know M, m, R, r, g, h, ω so we can show that our calculated E_i equals our calculated E_f.