Event based maximum likelihood fit

$$
\begin{aligned}
-\ln\mathcal{L} = &\sum_{i=1}^{n} -\ln(p_{\text{prompt}}(\phi_i, p_{\gamma,i}, \Sigma, a_1 \ldots a_4, b_1 \ldots b_4)) + \\
&\sum_{j=1}^{m} -\ln\left(p_{\text{sideband}}(\phi_j, p_{\gamma,j}, \Sigma^{\text{bkg}}, a_1^{\text{bkg}} \ldots a_4^{\text{bkg}}, b_1^{\text{bkg}} \ldots b_4^{\text{bkg}})\right)
\end{aligned}
$$

where

$$
\begin{aligned}
p_{\text{prompt}} &= f_{\text{sig}} \cdot \tilde{p}(\phi, p_\gamma, \Sigma, a_1 \ldots a_4, b_1 \ldots b_4) \\
&\quad + (1 - f_{\text{sig}}) \cdot \tilde{p}(\phi, p_\gamma, \Sigma^{\text{bkg}}, a_1^{\text{bkg}} \ldots a_4^{\text{bkg}}, b_1^{\text{bkg}} \ldots b_4^{\text{bkg}}) \\
p_{\text{sideband}} &= \tilde{p}(\phi, p_\gamma, \Sigma^{\text{bkg}}, a_1^{\text{bkg}} \ldots a_4^{\text{bkg}}, b_1^{\text{bkg}} \ldots b_4^{\text{bkg}})
\end{aligned}
$$

and

$$
\tilde{p}(\phi, \Sigma) = \frac{\left(1 + p_\gamma \Sigma \cos\left(2\left(\alpha^{\parallel} - \phi\right)\right)\right) \cdot \left(\sum_{k=0}^{4} a_k \sin(k\phi) + b_k \cos(k\phi)\right)}{1 - \frac{1}{2} a_2 p_\gamma \Sigma}
$$

Event based maximum likelihood fit (in STAN)

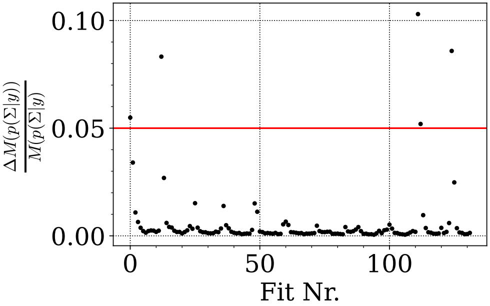
define priors and implement log-likelihood, truncate Σ to $[-1, 1]$

```
model{

//loop over prmpt peak events
for(k in 1:N){
    target+=log(f*mypdf(phi_prmpt[k],pol_prmpt[k],sigma,a,b)+(1-f)*mypdf(phi_prmpt[k],pol_prmpt[k],sigma_bkg,a_bkg,b_bkg));
}
//loop over sideband events
for(k in 1:M){
    target+=log(mypdf(phi_side[k],pol_side[k],sigma_bkg,a_bkg,b_bkg));
}

//priors for eff. coefficients, non-informative, broadly around 0
for(k in 1:4){
    a[k]~normal(0,0.1);
    b[k]~normal(0,0.1);
    a_bkg[k]~normal(0,0.1);
    b_bkg[k]~normal(0,0.1);
}
//priors for sigma
sigma ~ normal(0,1) T[-1,1];
sigma_bkg ~ normal(0,1) T[-1,1];
}
```

Fit diagnostics

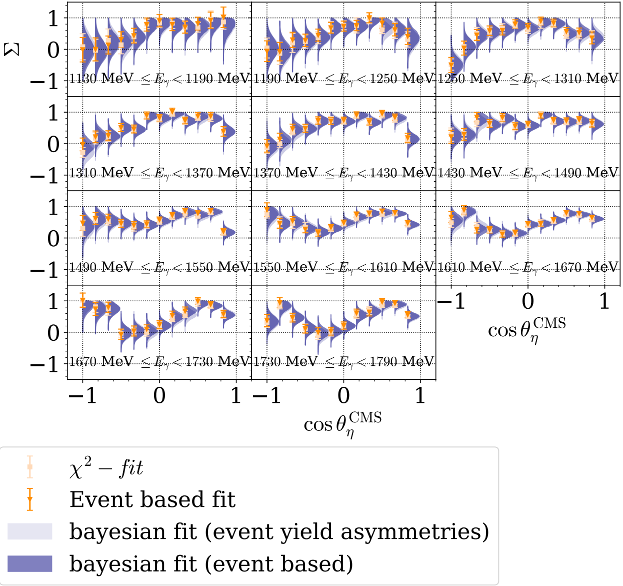
4 chains, 5000 samples, 1000 sample warmup

- ppd checks not possible (similar to "normal" event-based ML fit) $\to$ toy MC (?)
- $\hat{R} = 1$ for all parameters and all fits
- MCSE of median small: (nsamples chosen s.t. rel. error $\lesssim 0.05$)



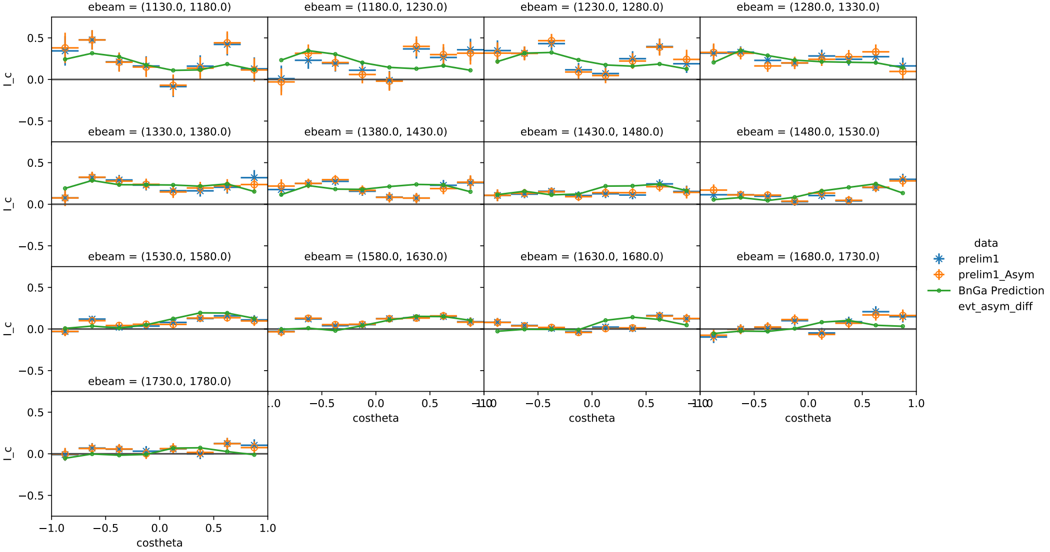
- check efficiency function (?)

Results

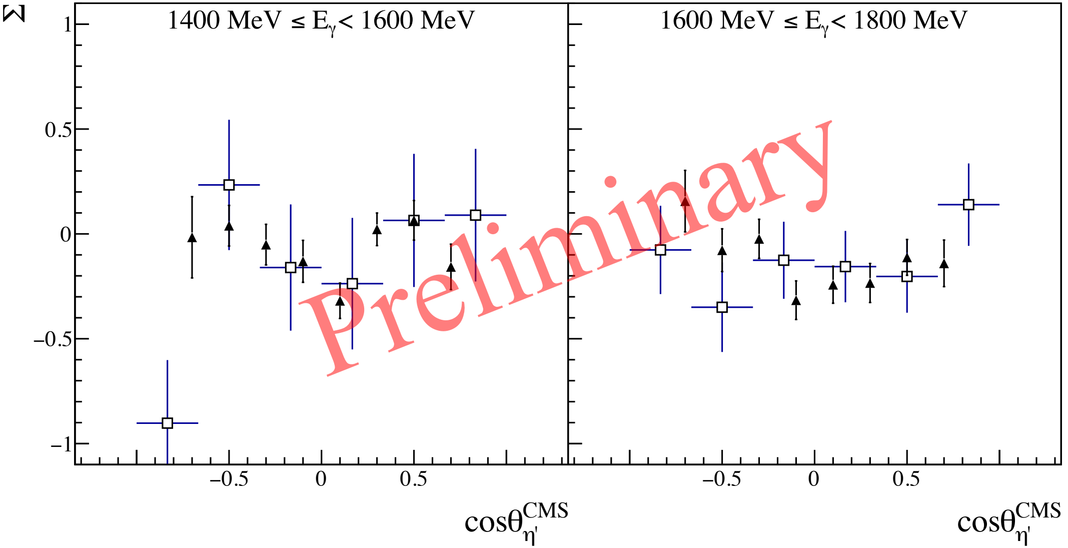


Σ
1
0
−1
1130 MeV $\leq E_\gamma <$ 1190 MeV
1190 MeV $\leq E_\gamma <$ 1250 MeV
1250 MeV $\leq E_\gamma <$ 1310 MeV
1310 MeV $\leq E_\gamma <$ 1370 MeV
1370 MeV $\leq E_\gamma <$ 1430 MeV
1430 MeV $\leq E_\gamma <$ 1490 MeV
1490 MeV $\leq E_\gamma <$ 1550 MeV
1550 MeV $\leq E_\gamma <$ 1610 MeV
1610 MeV $\leq E_\gamma <$ 1670 MeV
1670 MeV $\leq E_\gamma <$ 1730 MeV
1730 MeV $\leq E_\gamma <$ 1790 MeV
$\cos\theta_\eta^{\mathrm{CMS}}$
$\chi^2 - fit$
Event based fit
bayesian fit (event yield asymmetries)
bayesian fit (event based)

Σ in $\gamma p \to p 2\pi^0$



Σ in $\gamma p \to p\eta'$



To Do

- extract Σ using unbinned maximum likelihood fit for η'
- apply BAYESIAN approach
- consider bkg contaminations in results of $\Sigma_{\eta'}$, study toy MC