


# Determination of the beam asymmetry $\Sigma$ in $\eta$ - and $\eta'$ -photoproduction off the proton using Bayesian statistics

Master thesis for the CBELSA/TAPS collaboration

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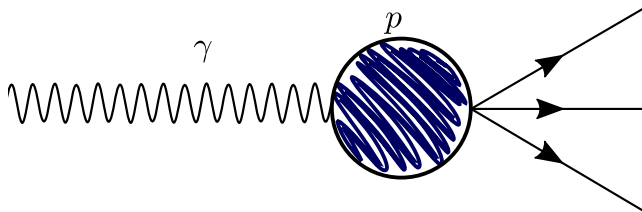
September 8/9 2022

# Setting the scene

## The Standard Model of Particle Physics

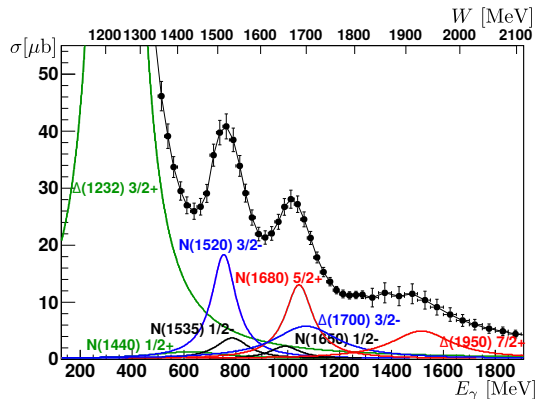
- ▶ matter consists of 12 (anti-) *fermions*
- ▶ quarks interact via *strong interaction*
- ▶ form bound states: mesons ( $q\bar{q}$ ) and baryons ( $qqq$ )

baryon spectroscopy (photoproduction) gives insight in strong interaction



# Setting the scene

Observe resonances  $N^*/\Delta^*$  in the cross sections  $\sigma(\gamma p \rightarrow pM)$



Total cross section  $\sigma(\gamma p \rightarrow p\pi^0)$  [Wunderlich et al. 2017]

→goal: (help to) identify contributing resonances as strong bound states!

1. Theoretical basics

2. Experimental Setup

3. Results

Determination of  $\Sigma_\eta$  using BAYESIAN statistics

Determination of  $\Sigma_{\eta'}$

4. Conclusion

## 1. Theoretical basics

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# Theoretical basics I

- ▶ resonances are broad, overlapping, require complicated partial-wave-analysis (PWA)
- ▶ constraints for the analysis can be derived from polarization observables
- ▶ ultimate goal: "complete experiment"; unambiguous, model-independent PWA solution  $\rightarrow$  several single and double polarization observables needed

## Beam-target polarization observables

photon	target polarization			
		$x$	$y$	$z$
unpolarized	$\sigma_0$	-	$T$	-
linearly polarized	$-\Sigma$	$H$	$-P$	$-G$
circularly polarized	-	$F$	-	$-E$

[Sandorfi et al. 2011]

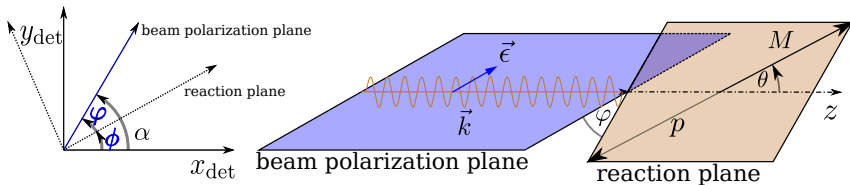
# Theoretical basics I

## Beam asymmetry $\Sigma$

$$\frac{d\sigma}{d\Omega}(E_\gamma, \cos\theta, \varphi) = \frac{d\sigma}{d\Omega_0}(E_\gamma, \cos\theta) \cdot \left[1 - p_\gamma^{\text{lin}} \Sigma \cos(2\varphi)\right]$$

polarization angle  $\varphi$ , polarization degree  $p_\gamma^{\text{lin}}$

[Sandorfi et al. 2011]



Definition of the polarization angle

## Theoretical basics II

- ▶ Polarization observables are input for further analysis
- ▶ Idea: increase amount of information gained from results using BAYESIAN inference

### BAYES' theorem

$$p(\theta|y) \propto p(y|\theta) \cdot p(\theta)$$
$$\textit{posterior} \propto \textit{likelihood} \cdot \textit{prior}$$

parameters  $\theta$  and data  $y$ .

[Gelman et al. 2014]



## Theoretical basics II

- ▶ Polarization observables are input for further analysis
- ▶ Idea: increase amount of information gained from results using BAYESIAN inference

### BAYES' theorem

parameters  $\theta$  and data  $y$

[Gelman et al. 2014]

## Theoretical basics II

$$p(\theta|y) \propto p(y|\theta) \cdot p(\theta)$$

- prior  $p(\theta)$  and likelihood  $p(y|\theta)$  can easily be specified  
→ gain *distributions*  $p(\theta|y)$  instead of point estimates with error bars

### BAYESIAN parameter inference

For each parameter  $\theta_n \in \theta$  we gain *marginal posteriors*

$$p(\theta_n|y) = \int d\theta_1 \cdots \int d\theta_{n-1} \int d\theta_{n+1} \cdots \int d\theta_N p(\theta_1 \dots \theta_N | y).$$

usually approximated using MARKOV-Chain Monte Carlo (MCMC) draws  $\theta^{(s)}$

[Sivia and Skilling 2005]

## Theoretical basics II

$$p(\theta|y) \propto p(y|\theta) \cdot p(\theta)$$

- prior  $p(\theta)$  and likelihood  $p(y|\theta)$  already specified  
→ gain *distributions*  $p(\theta|y)$  instead of point estimates with error bars

### BAYESIAN parameter inference

For each parameter  $\theta_n \in \Theta$  we want to find *distributions*

$$p(\theta_n|y) = \int d\theta_1 \dots \int d\theta_N p(\theta_1 \dots \theta_N | y).$$

usually approximated using Markov Chain Monte Carlo (MCMC) draws  $\theta^{(s)}$

[Sivia and Skilling 2005]

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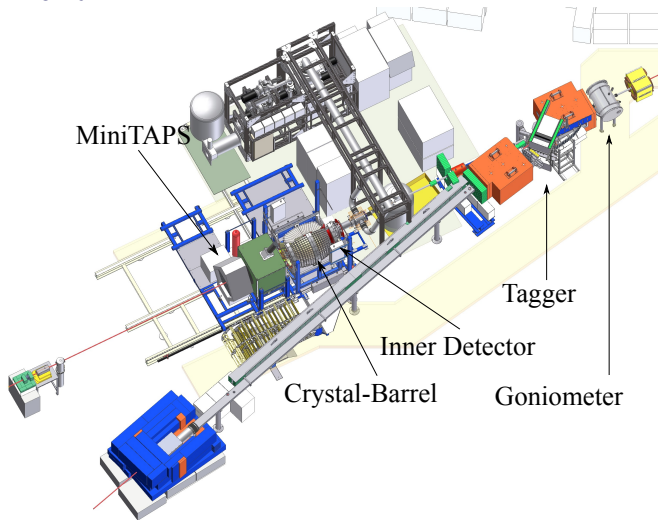
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# CBELSA/TAPS experiment

- ▶ generate photon beam from accelerated electrons via bremsstrahlung, with  $E_\gamma \leq 3.2 \text{ GeV}$
- ▶ photon beam impinges on liquid hydrogen target:  
 $\gamma p \rightarrow pM \rightarrow pX$
- ▶ measure decay products  $X$  of different final states:  
 $M = \pi^0/\eta/\eta'/\dots$
- ▶ data set:  
July-October 2013,  
1065 h beam time



Overview of the experimental area, adapted from [Walther 2021]

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# Confirming pre-published results of $\Sigma_\eta$

- ▶ Polarization observables are needed for different final states ( $\pi^0, \textcolor{red}{\eta}, \eta', \dots$ )
- ▶ high precision measurement of beam asymmetry for  $\eta$  production recently published [Afzal et al. 2020]
- ▶ goal: confirm results using BAYESIAN fitting methods

# Confirming pre-published results for $\Sigma_\eta$

## Event selection ( $\eta$ )

analysis performed in 11x12 bins of  $(E_\gamma, \cos \theta)$  by [Afzal et al. 2020]

## Method

- ▶ fit to event yield asymmetries using BAYESIAN inference
- ▶ BAYES' theorem:  $p(\theta|y) \propto \mathcal{L}(y|\theta) \cdot \pi(\theta)$ 
  - ▶ *marginal posteriors*:  $p(\theta_j|y) = \prod_{i \neq j} \int d\theta_i p(\theta|y)$
  - ▶ obtained using MARKOV-chain-Monte-Carlo (MCMC)

sampling algorithms: STAN

[Stan development team 2022]





# Confirming pre-published results for $\Sigma_\eta$

Event selection: ( )

analysis

Before

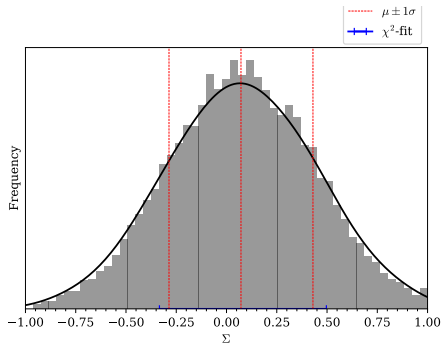
Point estimate:  $\Sigma_{\chi^2} \pm \Delta(\Sigma_{\chi^2})$

Now

marginal posteriors:  $p(\Sigma|y)$

Method

- f
- l

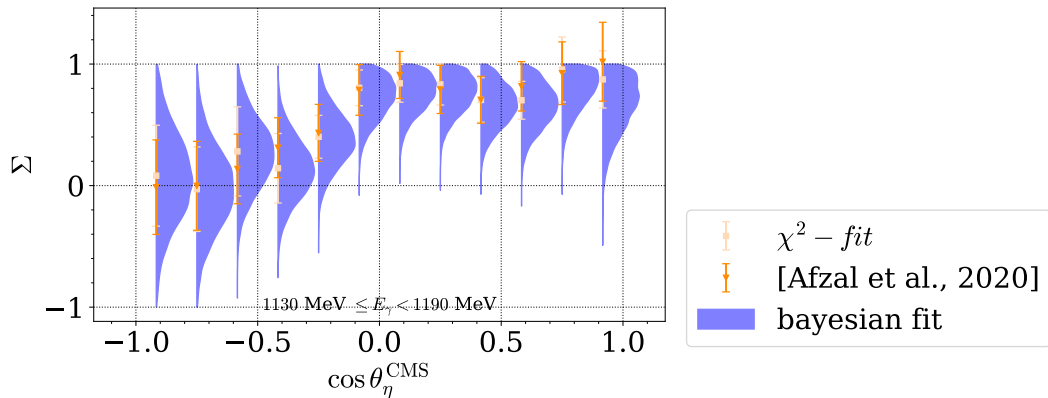


2022]

# Confirming pre-published results for $\Sigma_\eta$

Beam asymmetry  $\Sigma$  for all energy and angle bins

## Confirming pre-published results for $\Sigma_\eta$



Beam asymmetry  $\Sigma$  for one energy and all angle bins

Additional advantage: sample only in physically allowed parameter space

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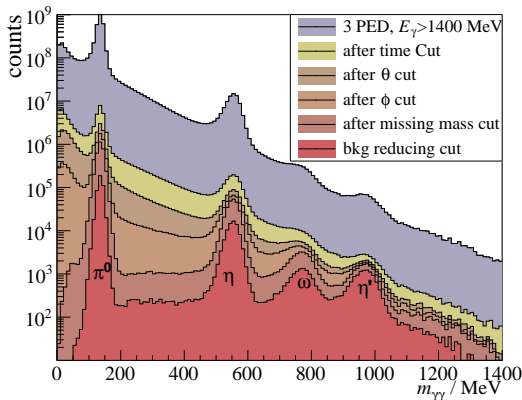
4. Conclusion

# Event selection of the $\eta' \rightarrow \gamma\gamma$ final state

Analysis performed in 3x6 bins of  $(E_\gamma, \cos \theta_{\eta'}^{\text{CMS}})$ ,  $E_\gamma \in [1500, 1800]$  MeV

- ▶ 3 detector hits, 2 uncharged, 1 charged
- ▶ coincident detector hits
- ▶ kinematic cuts derived from energy-momentum conservation
$$p_\gamma + p_p = p'_p + p_{\eta'}$$
- ▶ additional cuts to reduce background contributions

total:  $\sim 8000$   $\eta' \rightarrow \gamma\gamma$  events



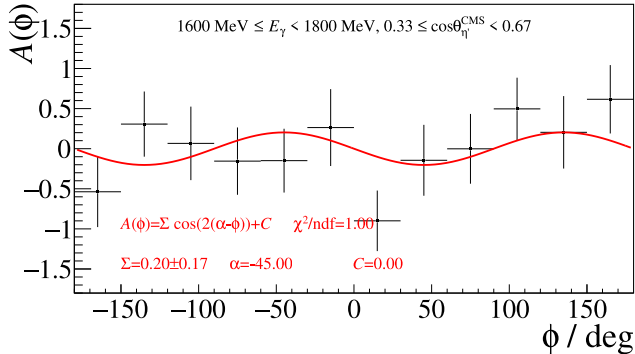
## Extraction method for $\Sigma_{\eta'}$

- ▶ measure in 2 distinct orthogonal polarization settings  $\perp, \parallel$

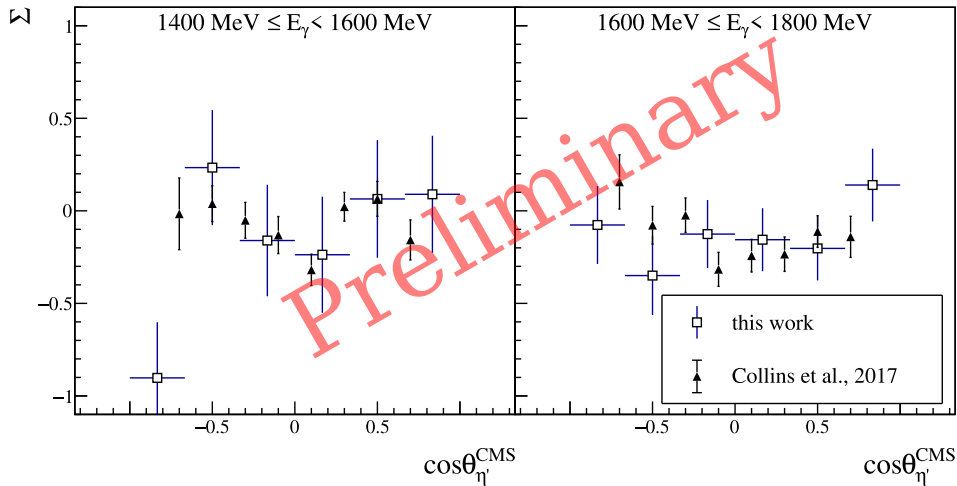
- ▶  $\chi^2$ -fit to event yield asymmetries

$$A(E_\gamma, \theta, \phi) = \frac{N^\perp(E_\gamma, \theta, \phi) - N^\parallel(E_\gamma, \theta, \phi)}{p_\gamma^\parallel N^\perp(E_\gamma, \theta, \phi) + p_\gamma^\perp N^\parallel(E_\gamma, \theta, \phi)} = \Sigma(E_\gamma, \theta) \cos(2(\alpha^\parallel - \phi))$$

- ▶ fit from  $\sim 800$   $\eta' \rightarrow \gamma\gamma$  events



# Preliminary results for $\Sigma_{\eta'}$



Beam asymmetry  $\Sigma_{\eta'}$  for all energy and angle bins, compared with results of [Mecking et al. 2003]

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# Conclusion

## Summary

- ▶  $\Sigma$  extracted for  $\eta$  and  $\eta'$  final state
- ▶  $\eta$  results obtained with BAYESIAN fit agree with previous results
- ▶  $\eta'$  results agree with previous results

## Outlook

- ▶ extract  $\Sigma$  using unbinned maximum likelihood fit for  $\eta/\eta'$
- ▶ apply BAYESIAN approach to above method
- ▶ consider bkg contaminations in results of  $\Sigma_{\eta'}$

# BACKUP & REFERENCES

## Additional theoretical basics

### Unpolarized differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}\rho \sum_{\text{spins}} |\langle f | \mathcal{F} | i \rangle|^2,$$

where

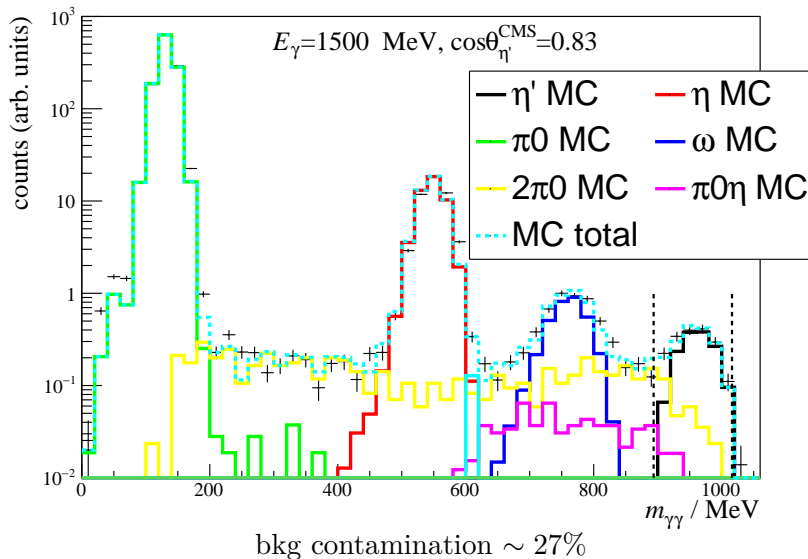
$$\mathcal{F} = i(\vec{\sigma} \cdot \vec{\epsilon})F_1 + (\vec{\sigma} \cdot \hat{q})(\vec{\sigma} \cdot (\hat{k} \times \vec{\epsilon}))F_2 + i(\vec{\sigma} \cdot \hat{k})(\hat{q} \cdot \vec{\epsilon})F_3 + i(\vec{\sigma} \cdot \hat{q})(\hat{q} \cdot \vec{\epsilon})F_4$$

$F_i$  : complex CGLN Amplitudes

[Chew et al. 1957]

$\frac{d\sigma}{d\Omega} \in \mathbb{R}$ , not sufficient to determine  $\mathcal{F}$  unambiguously  
→ Polarization Observables can be related to  $F_i$

# Background estimation using Monte-Carlo simulations

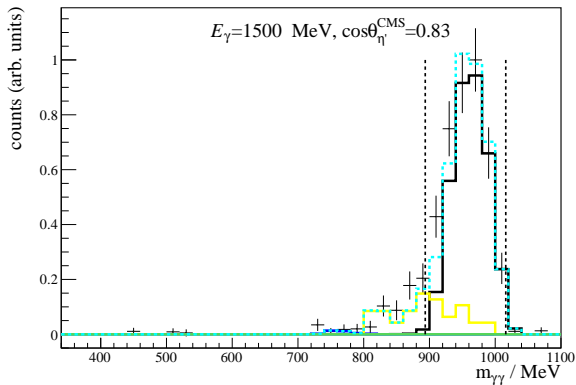


# Background estimation using Monte-Carlo simulations

$2\pi^0/\pi^0\eta$  events pass event selection, because  $E_{\gamma_i} \lesssim 20$  MeV, or  $\theta_{\gamma_i} \approx \theta_{\gamma_j}$

## Background reducing cuts

- ▶  $p$  in MT for  $E_\gamma < 1500$  MeV
- ▶  $E_{\gamma_i} < 1500$  MeV
- ▶ 1 PED/Cluster for  $\gamma_i$
- ▶  $\text{Clustersize}(p) < 6$
- ▶  $\text{Clustersize}(\gamma_i)$  in FW






bkg contamination  $\sim 13\%$




# Diagnostics of a BAYESIAN fit

- ▶  $\hat{R}$ : measure of convergence for chains
- ▶ Monte-Carlo-Standard-Error: measure for adequate sample size
- ▶ *posterior predictive checks*: "goodness of fit"

# References I




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