Determination of the beam asymmetry Σ in η- and η'-photoproduction off the proton using Bayesian statistics

Master thesis for the CBELSA/TAPS collaboration

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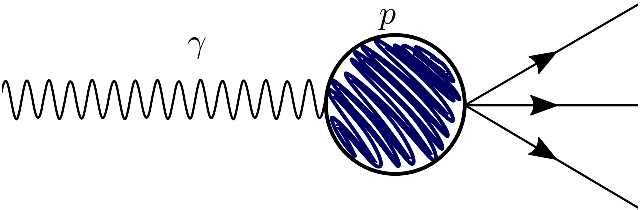
Setting the scene

The Standard Model of Particle Physics

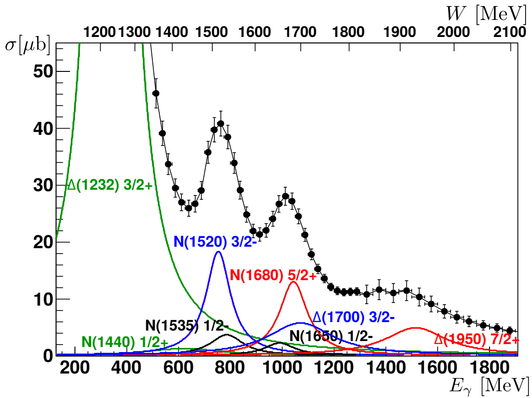
- matter consists of 12 (anti-)*fermions*
- quarks interact via *strong interaction*
- form bound states: mesons ($q\bar{q}$) and baryons (qqq)

baryon spectroscopy (photoproduction) gives insight in strong interaction



Setting the scene

Observe resonances N^*/Δ^* in the cross sections $\sigma(\gamma p \to pM)$



Total cross section $\sigma(\gamma p \to p\pi^0)$ [Wunderlich et al. 2017]

$\rightarrow$goal: (help to) identify contributing resonances as strong bound states!

1. Theoretical basics
2. Experimental Setup
3. Results
   - Determination of Σ_η using BAYESIAN statistics
   - Determination of $\Sigma_{\eta'}$
4. Conclusion

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Theoretical basics I

- resonances are broad, overlapping, require complicated partial-wave-analysis (PWA)
- constraints for the analysis can be derived from polarization observables
- ultimate goal: "complete experiment"; unambiguous, model-independent PWA solution $\rightarrow$ several single and double polarization observables needed

Beam-target polarization observables

		target polarization		
photon		x	y	z
unpolarized	σ_0	-	T	-
linearly polarized	$-\Sigma$	H	$-P$	$-G$
circularly polarized	-	F	-	$-E$

[Sandorfi et al. 2011]

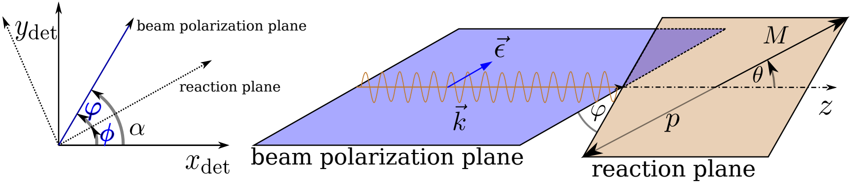
Theoretical basics I

Beam asymmetry $\boldsymbol{\Sigma}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(E_\gamma, \cos\theta, \varphi) = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}_0(E_\gamma, \cos\theta) \cdot \left[1 - p_\gamma^{\mathrm{lin}} \boldsymbol{\Sigma} \cos(2\varphi)\right]$$

polarization angle φ, polarization degree p_γ^{lin}

[Sandorfi et al. 2011]



Definition of the polarization angle

Theoretical basics II

- Polarization observables are input for further analysis
- Idea: increase amount of information gained from results using Bayesian inference

Bayes' theorem

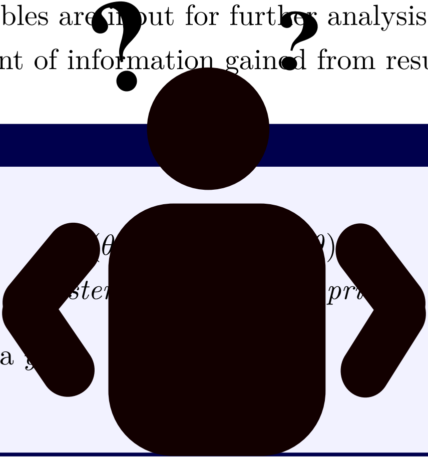
$$p(\theta|y) \propto p(y|\theta) \cdot p(\theta)$$
$$\textit{posterior} \propto \textit{likelihood} \cdot \textit{prior}$$

parameters θ and data y.

[Gelman et al. 2014]

Theoretical basics II

- Polarization observables are input for further analysis
- Idea: increase amount of information gained from results using BAYESIAN inference



BAYES' theorem

parameters θ and data

[Gelman et al. 2014]

Theoretical basics II

$$p(\theta|y) \propto p(y|\theta) \cdot p(\theta)$$

- prior $p(\theta)$ and likelihood $p(y|\theta)$ can easily be specified
  $\rightarrow$ gain *distributions* $p(\theta|y)$ instead of point estimates with error bars

Bayesian parameter inference

For each parameter $\theta_n \in \theta$ we gain *marginal posteriors*

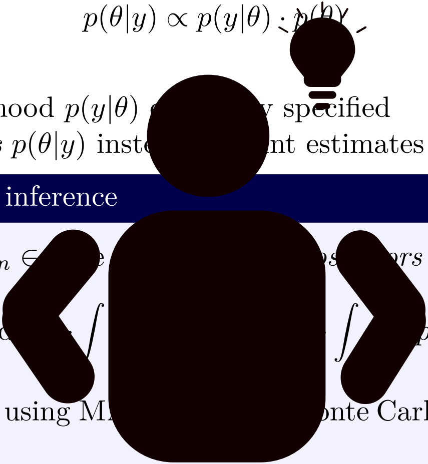
$$p(\theta_n|y) = \int \mathrm{d}\theta_1 \cdots \int \mathrm{d}\theta_{n-1} \int \mathrm{d}\theta_{n-1} \cdots \int \mathrm{d}\theta_N p(\theta_1 \ldots \theta_N|y).$$

usually approximated using Markov-Chain Monte Carlo (MCMC) draws $\theta^{(s)}$

[Sivia and Skilling 2005]

Theoretical basics II

$$p(\theta|y) \propto p(y|\theta) \cdot p(\theta)$$



- prior $p(\theta)$ and likelihood $p(y|\theta)$ [illegible] specified
  $\rightarrow$ gain *distributions* $p(\theta|y)$ inste[illegible]nt estimates with error bars

Bayesian parameter inference

For each parameter $\theta_n \in$ [illegible] *ors*

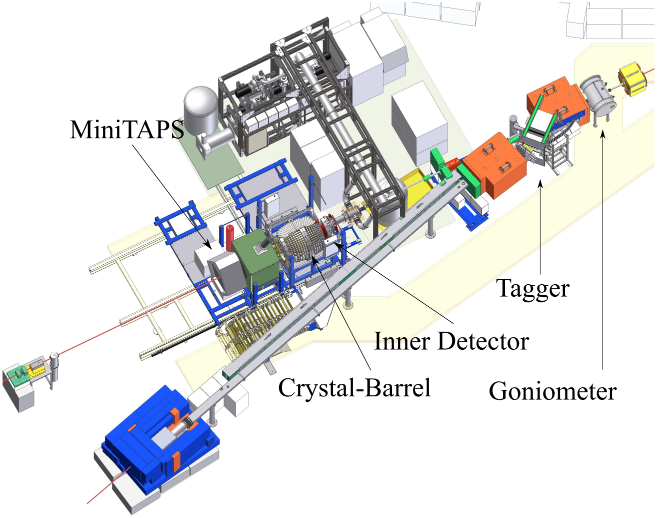
$$p(\theta_n|y) = \int d \ldots \int \ldots \int \ldots p(\theta_1 \ldots \theta_N|y).$$

usually approximated using M[illegible]onte Carlo (MCMC) draws $\theta^{(s)}$

[Sivia and Skilling 2005]

CBELSA/TAPS experiment

- generate photon beam from accelerated electrons via bremsstrahlung, with $E_\gamma \leq 3.2\,\mathrm{GeV}$
- photon beam impinges on liquid hydrogen target: $\gamma p \to pM \to pX$
- measure decay products X of different final states: $M = \pi^0/\eta/\eta'/\ldots$
- data set: July-October 2013, 1065 h beam time



Overview of the experimental area, adapted from [Walther 2021]

Confirming pre-published results of Σ_η

- Polarization observables are needed for different final states $(\pi^0, \eta, \eta', \dots)$
- high precision measurement of beam asymmetry for η production recently published [Afzal et al. 2020]
- goal: confirm results using Bayesian fitting methods

Confirming pre-published results for Σ_η

Event selection (η)

analysis performed in 11x12 bins of $(E_\gamma, \cos\theta)$ by [Afzal et al. 2020]

Methods

Remember: $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(E_\gamma, \cos\theta, \varphi) = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}_0(E_\gamma, \cos\theta) \cdot \left[1 - p_\gamma^{\mathrm{lin}} \boldsymbol{\Sigma} \cos(2\varphi)\right]$

Binned fit to event yield asymmetries

fit to event yield asymmetries

$$A(E_\gamma, \theta, \phi) = \frac{N^\perp(E_\gamma,\theta,\phi) - N^\parallel(E_\gamma,\theta,\phi)}{p_\gamma^\parallel N^\perp(E_\gamma,\theta,\phi) + p_\gamma^\perp N^\parallel(E_\gamma,\theta,\phi)} = \Sigma(E_\gamma, \theta) \cos\left(2\left(\alpha^\parallel - \phi\right)\right)$$

Confirming pre-published results for Σ_η

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analysis performed in 11x12 bins of $(E_\gamma, \cos\theta)$ by [Afzal et al. 2020]

Methods

Remember: $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(E_\gamma, \cos\theta, \varphi) = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}_0(E_\gamma, \cos\theta) \cdot \left[1 - p_\gamma^{\mathrm{lin}} \boldsymbol{\Sigma} \cos(2\varphi)\right]$

Unbinned maximum likelihood fit

Consider likelihood of each individual event

$$\tilde{p}(\phi, \Sigma) = \frac{\left(1 + p_\gamma \Sigma \cos\left(2\left(\alpha^{\parallel} - \phi\right)\right)\right) \cdot \epsilon\left(\phi\right)}{C}$$

Confirming pre-published results for Σ_η

Applying BAYESIAN approach to event yield asymmetries:

- assume GAUSSIAN errors, i.e.

$$A(\phi) = \Sigma \cos\left(2\left(\alpha^{\parallel} - \phi\right)\right) + \epsilon$$

  where $\epsilon \sim \mathcal{N}(0, \sigma)$
- likelihood $p(A|\Sigma)$ of each datapoint given by

$$y \sim \mathcal{N}\left(\Sigma \cos\left(2\left(\alpha^{\parallel} - \phi\right)\right), \sigma\right)$$

- prior:

$$p(\Sigma) \sim \mathcal{N}(0, 1)_{[-1,1]}$$

Sample from posterior $p(\Sigma|A) \propto p(A|\Sigma) \cdot p(\Sigma)$!

Confirming pre-published results for Σ_η

Applying BAYESIAN approach to unbinned fit:

- event based likelihood given by product of all single-event likelihoods
- assign priors for all fit parameters (18 in total)
- choose weakly informative, normal priors.
- truncate beam asymmetry to allowed region $[-1, 1]$
- perform toy Monte Carlo experiments

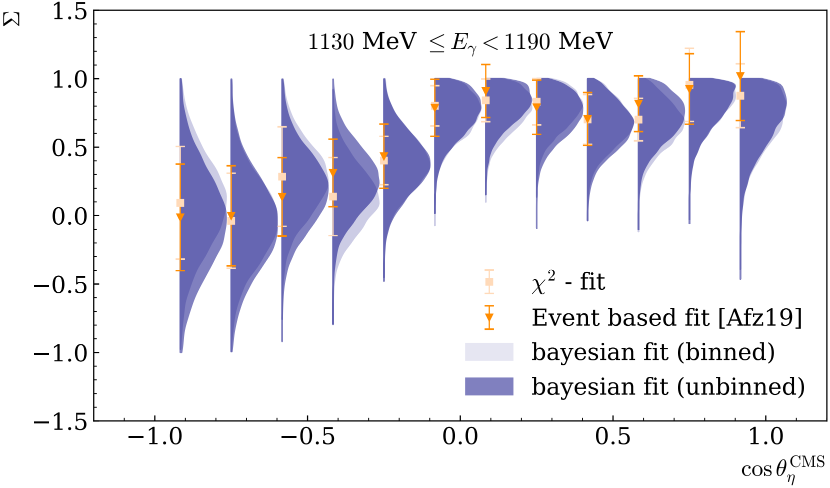
Sample from posterior !

Confirming pre-published results for Σ_η

- All Bayesian fits performed using the *Python* frontend of *Stan*
- MCMC-sampling: adaptive Hamiltonian Monte-Carlo (HMC), i.e. No-U-Turn-Sampling (NUTS)
  - generate samples $\theta^{(1)}, \theta^{(2)}, \ldots, \theta^{(S)}$ where each $\theta^{(t)}$ depends only on $\theta^{(t-1)}$
  - simulate draws from the posterior by updating at point t such that the posterior increases (importance sampling)
- diagnosing convergence of MCMC:
  - potential scale reduction statistic $1.00 \lesssim \widehat{R} \lesssim 1.01$
  - Monte Carlo standard error (MCSE) ‘small’

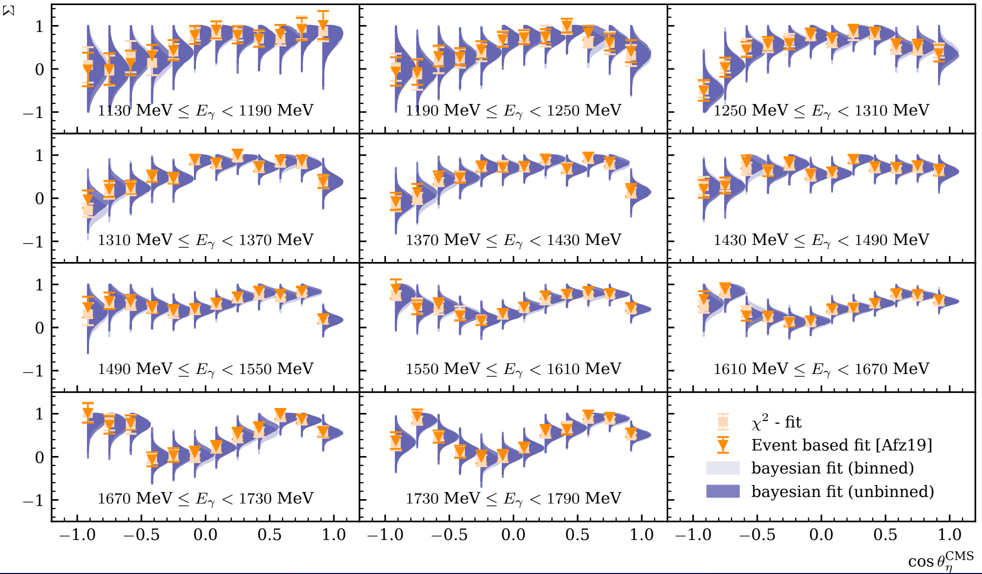
[Stan development team 2022; Hoffman and Gelman 2014]

Confirming pre-published results for Σ_η



Distinct advantage: sample only in physically allowed parameter space

Confirming pre-published results for Σ_η



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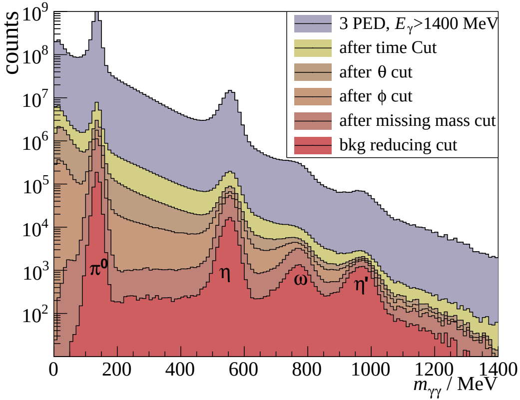
Event selection of the $\eta' \to \gamma\gamma$ final state

Analysis performed in 3x6 bins of $(E_\gamma, \cos\theta_{\eta'}^{\mathrm{CMS}})$, $E_\gamma \in [1500, 1800]$ MeV

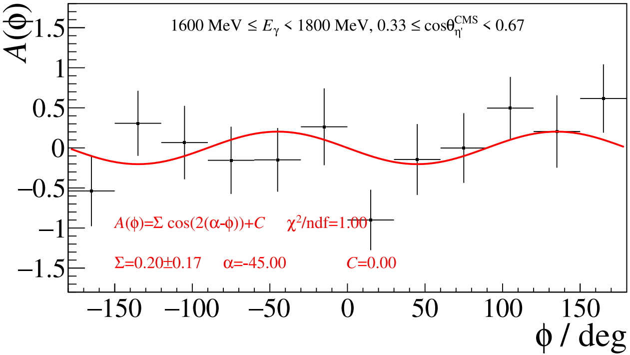
- 3 detector hits, 2 uncharged, 1 charged
- coincident detector hits
- kinematic cuts derived from energy-momentum conservation $p_\gamma + p_p = p'_p + p_{\eta'}$
- additional cuts to reduce background contributions

total: ~ 8000 $\eta' \to \gamma\gamma$ events

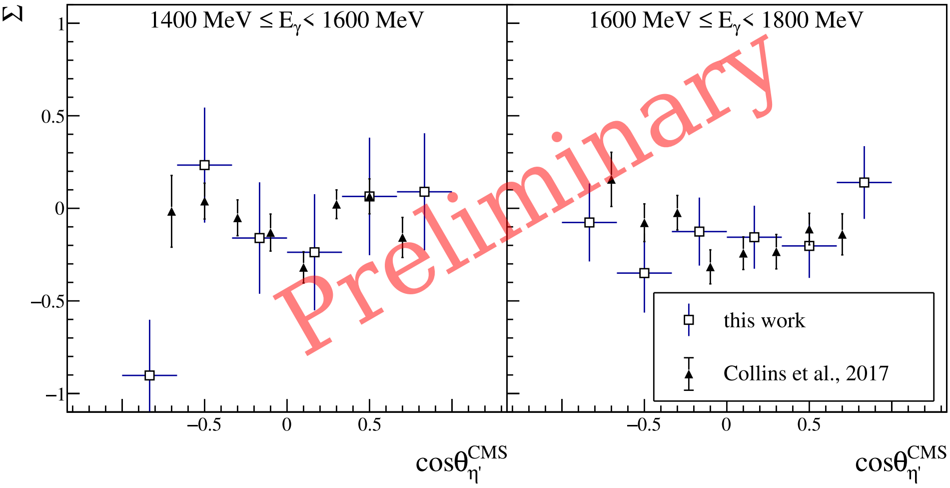


Extraction method for $\Sigma_{\eta'}$

- measure in 2 distinct orthogonal polarization settings $\perp, \parallel$
- χ^2-fit to event yield asymmetries
  $$A(E_\gamma, \theta, \phi) = \frac{N^\perp(E_\gamma,\theta,\phi) - N^\parallel(E_\gamma,\theta,\phi)}{p_\gamma^\parallel N^\perp(E_\gamma,\theta,\phi) + p_\gamma^\perp N^\parallel(E_\gamma,\theta,\phi)} = \Sigma(E_\gamma, \theta)\cos\left(2\left(\alpha^\parallel - \phi\right)\right)$$
- fit from ~ 800 $\eta' \to \gamma\gamma$ events



Preliminary results for $\Sigma_{\eta'}$



Beam asymmetry $\Sigma_{\eta'}$ for all energy and angle bins, compared with results of [Mecking et al. 2003]

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Conclusion

Summary

- Σ extracted for η and η' final state
- η results obtained with BAYESIAN fit agree with previous results
- η' results agree with previous results

Outlook

- extract Σ using unbinned maximum likelihood fit for η/η'
- apply BAYESIAN approach to above method
- consider bkg contaminations in results of $\Sigma_{\eta'}$

Backup & References

Full PDF for unbinned maximum likelihood fit

$$
\begin{aligned}
-\ln\mathcal{L} = &\sum_{i=1}^{n} -\ln(p_{\text{prompt}}(\phi_i, p_{\gamma,i}, \Sigma, a_1 \dots a_4, b_1 \dots b_4))+\\
&\sum_{j=1}^{m} -\ln\left(p_{\text{sideband}}(\phi_j, p_{\gamma,j}, \Sigma^{\text{bkg}}, a_1^{\text{bkg}} \dots a_4^{\text{bkg}}, b_1^{\text{bkg}} \dots b_4^{\text{bkg}})\right)
\end{aligned}
$$

where

$$
\begin{aligned}
p_{\text{prompt}} &= f_{\text{sig}} \cdot \tilde{p}(\phi, p_\gamma, \Sigma, a_1 \dots a_4, b_1 \dots b_4)\\
&\quad + (1 - f_{\text{sig}}) \cdot \tilde{p}(\phi, p_\gamma, \Sigma^{\text{bkg}}, a_1^{\text{bkg}} \dots a_4^{\text{bkg}}, b_1^{\text{bkg}} \dots b_4^{\text{bkg}})\\
p_{\text{sideband}} &= \tilde{p}(\phi, p_\gamma, \Sigma^{\text{bkg}}, a_1^{\text{bkg}} \dots a_4^{\text{bkg}}, b_1^{\text{bkg}} \dots b_4^{\text{bkg}})
\end{aligned}
$$

and

$$
\tilde{p}(\phi, \Sigma) = \frac{\left(1 + p_\gamma \Sigma \cos\left(2\left(\alpha^{\parallel} - \phi\right)\right)\right) \cdot \left(\sum_{k=0}^{4} a_k \sin(k\phi) + b_k \cos(k\phi)\right)}{1 - \frac{1}{2} a_2 p_\gamma \Sigma}
$$

Additional theoretical basics

Unpolarized differential cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{4}\rho \sum_{\text{spins}} |\langle f|\mathcal{F}|i\rangle|^2,$$

where

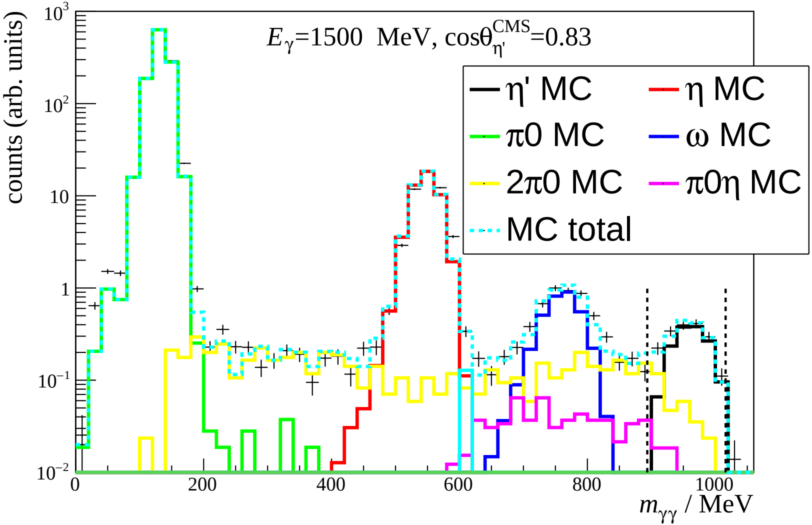
$$\mathcal{F} = i(\vec{\sigma}\cdot\vec{\epsilon})F_1 + (\vec{\sigma}\cdot\hat{q})(\vec{\sigma}\cdot(\hat{k}\times\vec{\epsilon}))F_2 + i(\vec{\sigma}\cdot\hat{k})(\hat{q}\cdot\vec{\epsilon})F_3 + i(\vec{\sigma}\cdot\hat{q})(\hat{q}\cdot\vec{\epsilon})F_4$$

F_i : complex CGLN Amplitudes

[Chew et al. 1957]

$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \in \mathbb{R}$, not sufficient do determine $\mathcal{F}$ unambiguously
$\rightarrow$ Polarization Observables can be related to F_i

Background estimation using Monte-Carlo simulations



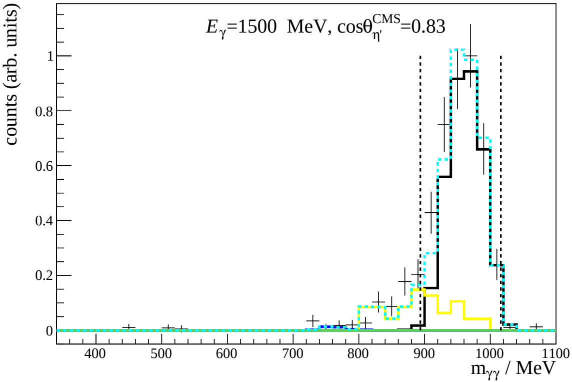
bkg contamination $\sim 27\%$

Background estimation using Monte-Carlo simulations

$2\pi^0/\pi^0\eta$ events pass event selection, because $E_{\gamma_i} \lesssim 20$ MeV, or $\theta_{\gamma_i} \approx \theta_{\gamma_j}$

Background reducing cuts

- p in MT for $E_\gamma < 1500$ MeV
- $E_{\gamma_i} < 1500$ MeV
- 1 PED/Cluster for γ_i
- Clustersize(p) < 6
- Clustersize(γ_i) in FW



bkg contamination $\sim 13\%$

Diagnostics of a BAYESIAN fit

- $\hat{R}$: measure of convergence for chains
- Monte-Carlo-Standard-Error: measure for adequate sample size
- *posterior predictive checks*: "goodness of fit"

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