Determination of the beam asymmetry Σ in η- and η'-photoproduction using Bayesian statistics

Master thesis for the CBELSA/TAPS collaboration

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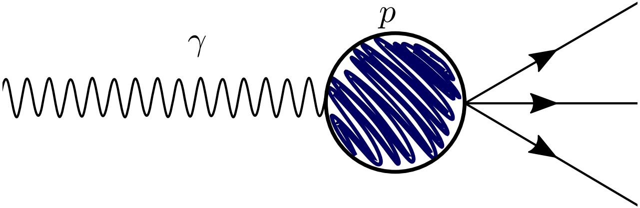
Setting the scene

The Standard Model of Particle Physics

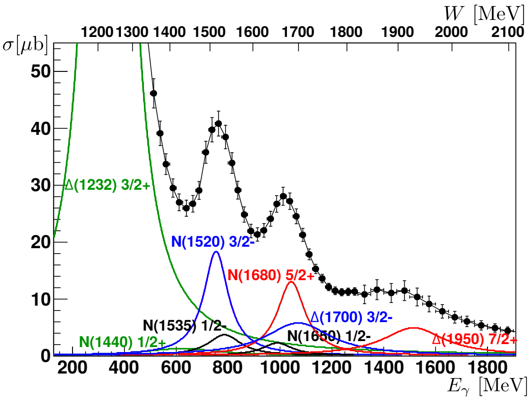
- matter consists of 12 (anti-)*fermions*
- quarks interact via *strong interaction*
- form bound states: mesons (e.g. $q\bar{q}$) and baryons (e.g. qqq)

baryon spectroscopy (photoproduction) gives insight in strong interaction



Setting the scene

observe resonances N^*/Δ^* in the cross sections $\sigma(\gamma p \to pM)$



Total cross section $\sigma(\gamma p \to p\pi^0)$ [Wunderlich et al. 2017]

$\rightarrow$goal: (help to) identify contributing resonances as strong bound states!

Theoretical basics I

- resonances are broad, overlapping, require complicated partial-wave-analysis (PWA)
- constraints for the analysis can be derived from polarization observables
- ultimate goal: "complete experiment"; unambiguous, model-independent PWA solution $\rightarrow$ several single and double polarization observables needed

Beam-target polarization observables

			target polarization	
photon		x	y	z
unpolarized	σ_0	-	T	-
linearly polarized	$-\Sigma$	H	$-P$	$-G$
circularly polarized	-	F	-	$-E$

[Sandorfi et al. 2011]

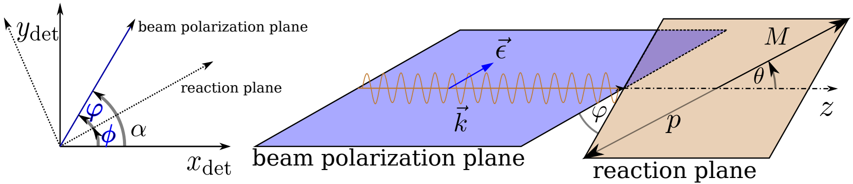
Theoretical basics I

Beam asymmetry $\boldsymbol{\Sigma}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(E_\gamma, \cos\theta, \varphi) = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}_0(E_\gamma, \cos\theta) \cdot \left[1 - p_\gamma^{\mathrm{lin}} \boldsymbol{\Sigma} \cos(2\varphi)\right]$$

polarization angle $\varphi = \alpha - \phi$ with $\alpha = \alpha^{\perp/\parallel} = \pm 45°$, polarization degree p_γ^{lin}

[Sandorfi et al. 2011]



Definition of the polarization angle

Theoretical basics II

- polarization observables are input for further analysis
- idea: increase amount of information gained from results using Bayesian inference

Bayes' theorem

$$p(\theta|y) \propto p(y|\theta) \cdot p(\theta)$$
$$\mathit{posterior} \propto \mathit{likelihood} \cdot \mathit{prior}$$

parameters θ and data y.

[Gelman et al. 2014]

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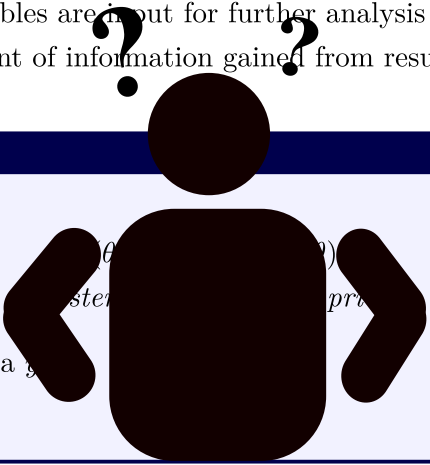

Bayes' theorem

$(\theta$ … $\theta)$

ster … *pri*

parameters θ and data y

[Gelman et al. 2014]

Theoretical basics II

$$p(\theta|y) \propto p(y|\theta) \cdot p(\theta)$$

- prior $p(\theta)$ and likelihood $p(y|\theta)$ can be specified once the model is known $\rightarrow$ gain *distributions* $p(\theta|y)$ instead of point estimates with error bars

Bayesian parameter inference

For each parameter $\theta_n \in \theta$ we gain *marginal posteriors*

$$p(\theta_n|y) = \int \mathrm{d}\theta_1 \cdots \int \mathrm{d}\theta_{n-1} \int \mathrm{d}\theta_{n-1} \cdots \int \mathrm{d}\theta_N p(\theta_1 \ldots \theta_N|y).$$

usually approximated using Markov-Chain Monte Carlo (MCMC) draws $\theta^{(s)}$

[Sivia and Skilling 2005]

Theoretical basics II

- all BAYESIAN fits performed using the *Python* frontend of *Stan*

[Stan development team 2022; Hoffman and Gelman 2014]

Theoretical basics II

- all Bayesian fits performed using the *Python* frontend of *Stan*
- MCMC-sampling: adaptive Hamiltonian Monte-Carlo (HMC), i.e. No-U-Turn-Sampling (NUTS)
  - generate samples $\theta^{(1)}, \theta^{(2)}, \dots, \theta^{(S)}$ where each $\theta^{(t)}$ depends only on $\theta^{(t-1)}$
  - simulate draws from the posterior by updating at point t such that the posterior increases (importance sampling)

[Stan development team 2022; Hoffman and Gelman 2014]

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- diagnosing convergence of MCMC:
  - potential scale reduction statistic $1.00 \lesssim \widehat{R} \lesssim 1.01$
  - Monte Carlo standard error (MCSE) 'small', e.g. for mean μ $\mathrm{MCSE} = \frac{\mathrm{std}(\{\theta^{(s)}\})}{S}$

[Stan development team 2022; Hoffman and Gelman 2014]

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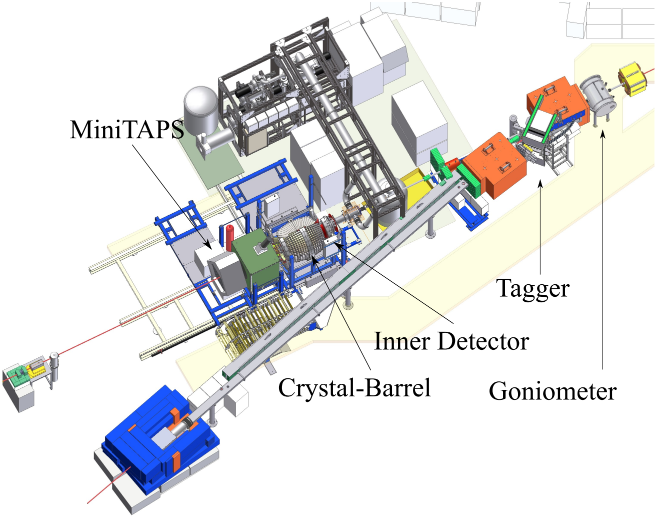
  - Monte Carlo standard error (MCSE) 'small', e.g. for mean μ $\mathrm{MCSE} = \frac{\mathrm{std}(\{\theta^{(s)}\})}{S}$
- goodness of fit: posterior predictive checks (PPC)
  - $p(y^{\mathrm{rep}}|y) = \int \mathrm{d}\theta p(y^{\mathrm{rep}}|\theta)p(\theta|y)$
  - in practice: $y^{\mathrm{rep},(s)} \sim p(y|\theta^{(s)})$

[Stan development team 2022; Hoffman and Gelman 2014]

CBELSA/TAPS experiment

- generate photon beam from accelerated electrons via bremsstrahlung, with $E_\gamma \leq 3.2\,\mathrm{GeV}$
- photon beam impinges on liquid hydrogen target: $\gamma p \to pX \to pM$
- $M = \pi^0/\eta/\eta'/\ldots$
- data set: July-October 2013, 1065 h beam time



Overview of the experimental area, adapted from [Walther 2021]

1. Theoretical basics

2. Experimental Setup

3. Results
   Determination of Σ_η using Bayesian statistics
   Determination of $\Sigma_{\eta'}$

4. Conclusion

Confirming pre-published results of Σ_η

- polarization observables are needed for different final states $(\pi^0, \eta, \eta', \dots)$
- high precision measurement of beam asymmetry for η production recently published [F. Afzal et al. 2020]
- goal: confirm results using BAYESIAN fitting methods

Confirming pre-published results for Σ_η

Event selection (η)

analysis performed in 11x12 bins of $(E_\gamma, \cos\theta)$ for $\gamma p \to p\eta \to p\gamma\gamma$ by [F. Afzal et al. 2020]

Methods

$$\text{Remember: } \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(E_\gamma, \cos\theta, \varphi) = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}_0(E_\gamma, \cos\theta) \cdot \left[1 - p_\gamma^{\text{lin}} \boldsymbol{\Sigma} \cos(2\varphi)\right]$$

binned fit to event yield asymmetries:

$$A(E_\gamma, \theta, \phi) = \frac{N^\perp(E_\gamma, \theta, \phi) - N^\parallel(E_\gamma, \theta, \phi)}{p_\gamma^\parallel N^\perp(E_\gamma, \theta, \phi) + p_\gamma^\perp N^\parallel(E_\gamma, \theta, \phi)} = \Sigma(E_\gamma, \theta) \cos\left(2\left(\alpha^\parallel - \phi\right)\right)$$

Confirming pre-published results for Σ_η

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unbinned maximum likelihood fit:

$$\tilde{p}(\phi, \Sigma) = \frac{\left(1 + p_\gamma \Sigma \cos\left(2\left(\alpha^{\parallel} - \phi\right)\right)\right) \cdot \epsilon\left(\phi\right)}{C}$$

Confirming pre-published results for Σ_η

applying Bayesian approach to event yield asymmetries:

- assume Gaussian errors, i.e.

$$A(\phi) = \Sigma \cos\left(2\left(\alpha^{\parallel} - \phi\right)\right) + \epsilon$$

where $\epsilon \sim \mathcal{N}(0, \sigma)$

- likelihood $p(A|\Sigma)$ of given by

$$y_i \sim \mathcal{N}\left(\Sigma \cos\left(2\left(\alpha^{\parallel} - \phi_i\right)\right), \sigma_i\right) \qquad \mathcal{L} = \prod_i p(y_i|\theta)$$

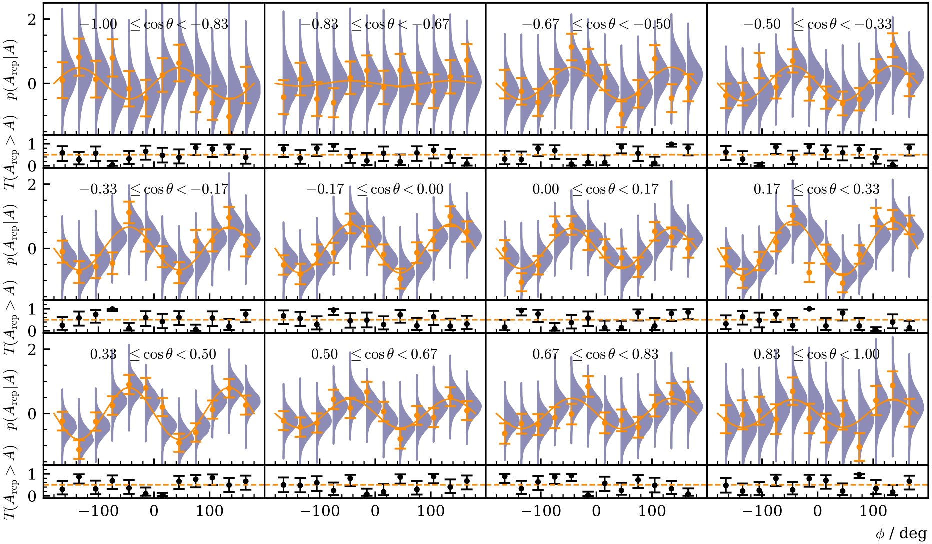
- prior:

$$p(\Sigma) \sim \mathcal{N}(0, 1)_{[-1,1]}$$

Sample from posterior $p(\Sigma|A) \propto p(A|\Sigma) \cdot p(\Sigma)$!

Confirming pre-published results for Σ_η

PPC for *binned* Bayesian fit:



Confirming pre-published results for Σ_η

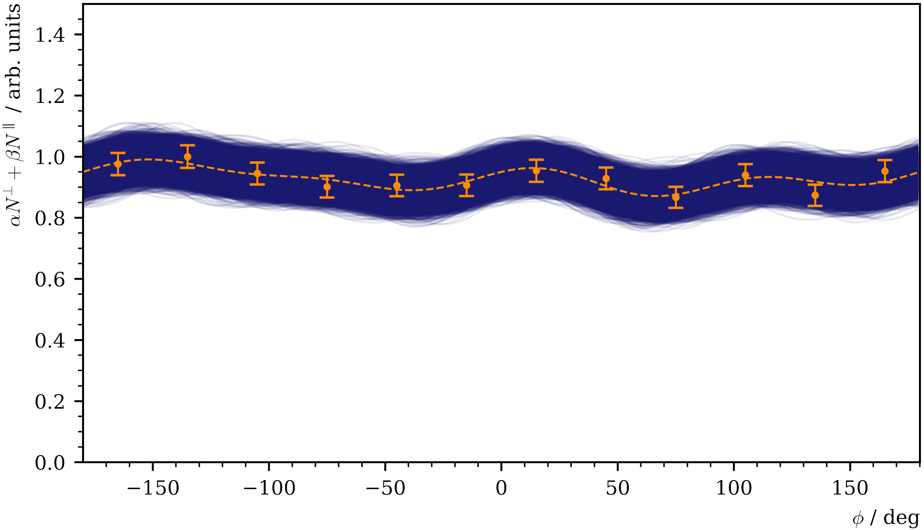
applying Bayesian approach to unbinned fit:

- event based likelihood given by product of all single-event likelihoods
- assign priors for all fit parameters (18 in total)
- truncate beam asymmetry to allowed region $[-1, 1]$
- perform toy Monte Carlo experiments

Sample from posterior !

Confirming pre-published results for Σ_η

PPC for *unbinned* Bayesian fit:



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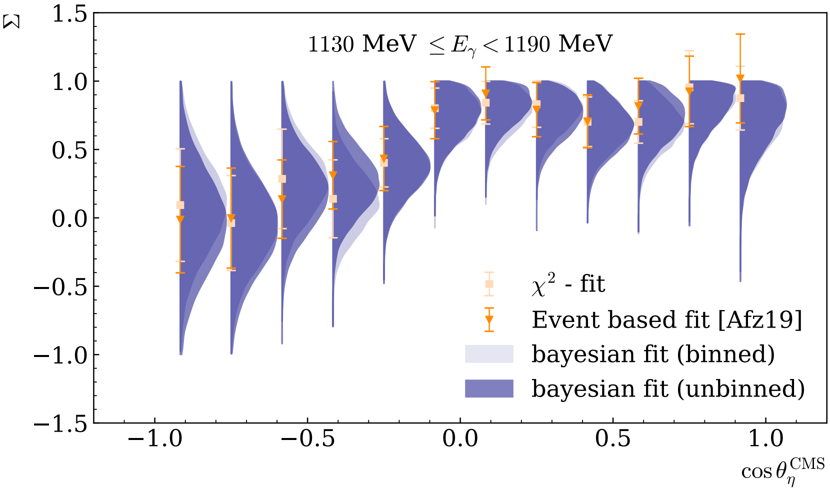
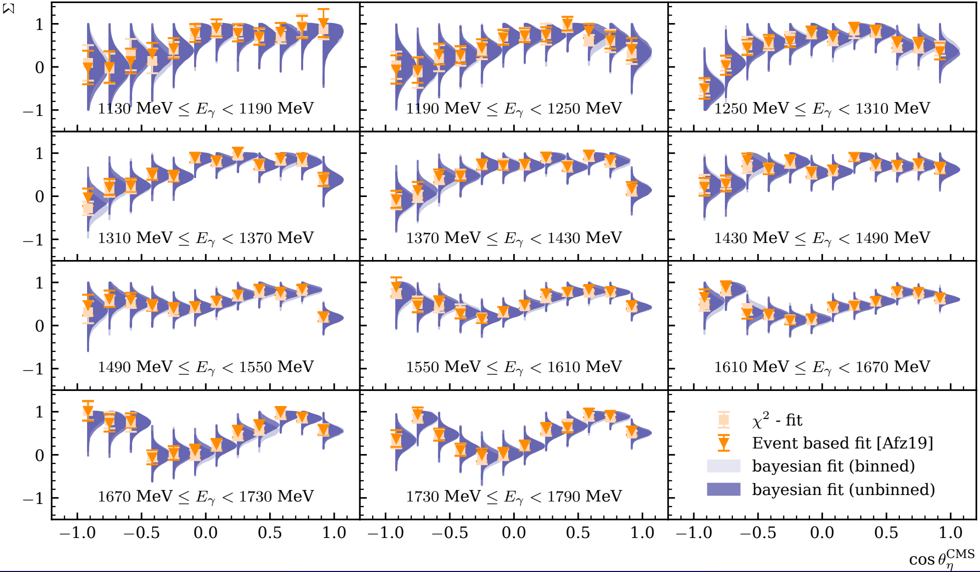


Distinct advantage: sample only in physically allowed parameter space

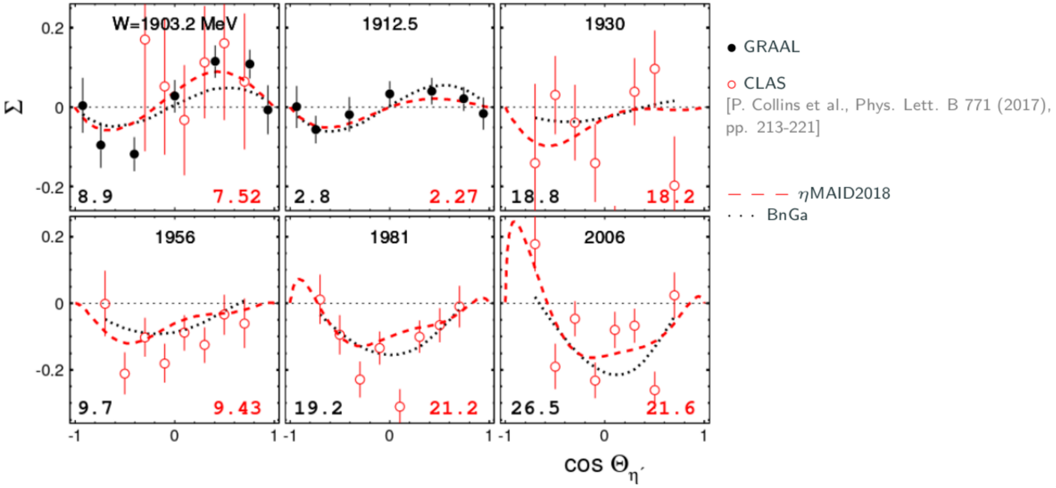
Confirming pre-published results for Σ_η



Determining the beam asymmetry in η' photoproduction

existing database:

ηMAID2018: L. Tiator et al., Eur. Phys. J. A 54.12 (2018)



Determining the beam asymmetry in η' photoproduction

First, perform event selection regarding η' photoproduction

η'	
Decay mode	Branching ratio
$\pi^+\pi^-\eta$	42.6%
$\rho^0\gamma(\to \pi^+\pi^-\gamma)$	28.9% (28.9%)
$\pi^0\pi^0\eta(\to 6\gamma)$	22.8% (8.8%)
$\omega\gamma(\to \pi^+\pi^-\pi^0\gamma/\pi^0\gamma\gamma)$	2.52% (2.2%/0.21%)
$\gamma\gamma$	2.3%

[Workman et al. 2022]

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[Workman et al. 2022]

Determining the beam asymmetry in η' photoproduction

event selection:

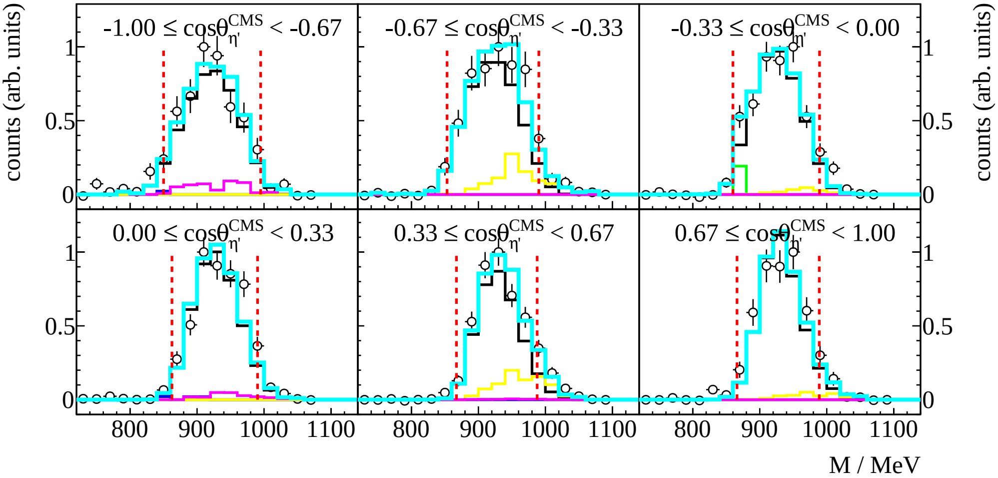
analysis performed in 3x6 bins of $(E_\gamma, \cos\theta)$ for $\gamma p \to p\eta' \to p\gamma\gamma$, $E_\gamma \in [1500, 1800]$ MeV

$$p_\gamma + p_p = p_{\eta'} + p_{\text{recoil}} = \underbrace{p_{\gamma_1} + p_{\gamma_2}}_{=p_{\eta'}} + p_{\text{recoil}}$$

$$p_\gamma + p_p = p_{\eta'} + p_X = \underbrace{p_{\gamma_1} + p_{\gamma_2}}_{=p_{\eta'}} + p_X$$

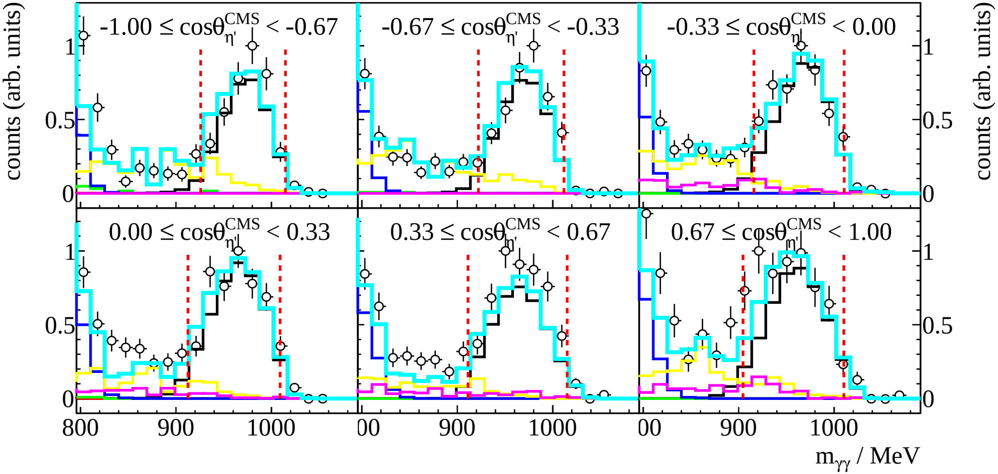
- one charged, two uncharged detector hits in coincidence with beam γ
- Coplanarity $\Delta\phi = \phi_{\eta'} - \phi_{\text{recoil}} \stackrel{!}{=} 180°$
- Polar angle $\theta_X \stackrel{!}{=} \theta_{\text{recoil}}$
- Missing mass $m_X \stackrel{!}{=} m_p$
- Invariant mass $m_{\gamma\gamma} \stackrel{!}{=} m_{\eta'}$

Determining the beam asymmetry in η' photoproduction



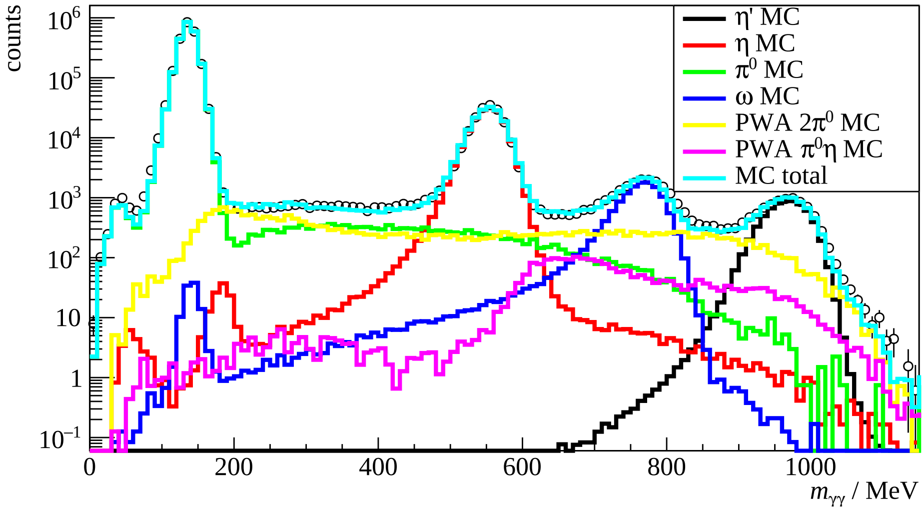
data points: m_X, MC histograms: total, $p\eta'$, $p2\pi^0$, $p\pi^0\eta$

Determining the beam asymmetry in η' photoproduction



data points: $m_{\gamma\gamma}$, data points: m_X, MC histograms: total, $p\eta'$, $p2\pi^0$, $p\pi^0\eta$, $p\omega$

Determining the beam asymmetry in η' photoproduction



Determining the beam asymmetry in η' photoproduction

why these contributions from 4γ final states $2\pi^0$ (and $\pi^0\eta$)??

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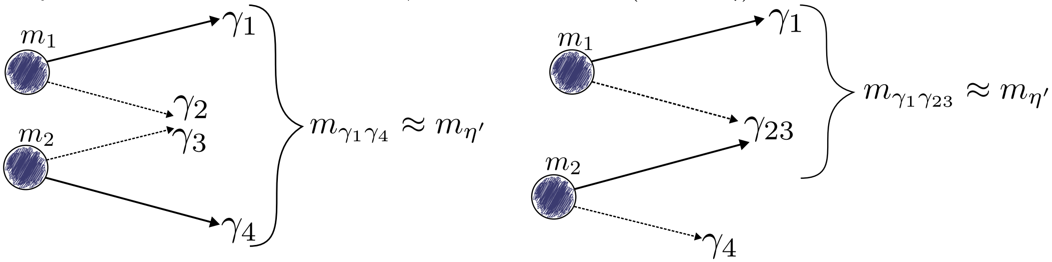

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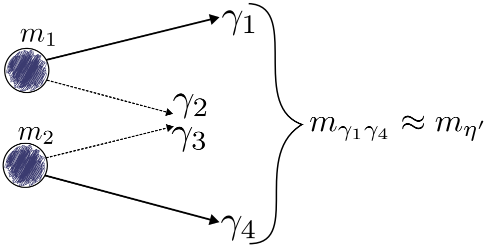
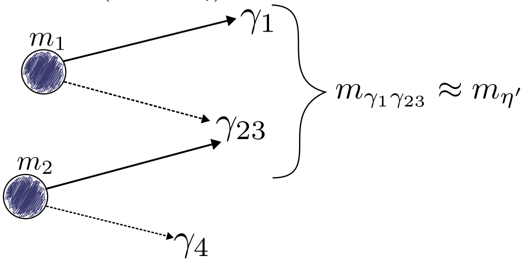




acceptance for $2\pi^0$ events is almost vanishing $A(E_\gamma, \cos\theta) < 2 \cdot 10^{-3}$, yet

$$R = \frac{\sigma_{2\pi^0} \cdot \mathrm{BR}_{2\pi^0 \to \gamma\gamma} \cdot \tilde{A}_{2\pi^0}}{\sigma_{\eta'} \cdot \mathrm{BR}_{\eta' \to \gamma\gamma} \cdot \tilde{A}_{\eta'}} = \frac{5\,\mu\mathrm{b} \cdot 0.9765 \cdot 2 \cdot 10^{-3}}{1\,\mu\mathrm{b} \cdot 0.023 \cdot 0.61} \approx 0.7!$$

explains high background contributions of up to 45% (background to total)

[Workman et al. 2022; Crede et al. 2009; Dieterle et al. 2020]

Determining the beam asymmetry in η' photoproduction

How do we get rid of background contributions?

- real photons mimic a two-photon final state, no sensible additional cuts have been found

Determining the beam asymmetry in η' photoproduction

How do we get rid of background contributions?

- real photons mimic a two-photon final state, no sensible additional cuts have been found
- main background from $2\pi^0$ photoproduction
- beam asymmetry for this reaction determined by [Mahlberg 2022]
- correct estimates for Σ according to the amount of background

$$\Sigma^{\text{meas}} = (1-\delta)\cdot\Sigma_{\eta'} + \delta\Sigma_{2\pi^0}$$

Determining the beam asymmetry in η' photoproduction

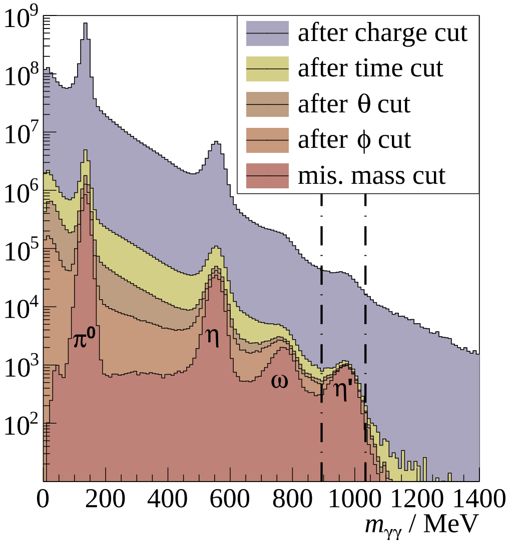
total: $\sim 8000\ \eta' \to \gamma\gamma$ events

- perform unbinned fit as maximum likelihood fit and BAYESIAN fit
- BAYESIAN fit: modify likelihood

$$\tilde{p}\,(\phi, \Sigma) \to \tilde{p}\left(\phi, (1-\delta)\Sigma_1 + \delta\Sigma_2^{\text{true}}\right)$$

$$\Sigma_2^{\text{true}} \sim \mathcal{N}(\Sigma_2^{\text{meas}}, \tau)$$

- unbinned maximum likelihood fit: shift point estimates *after fit*



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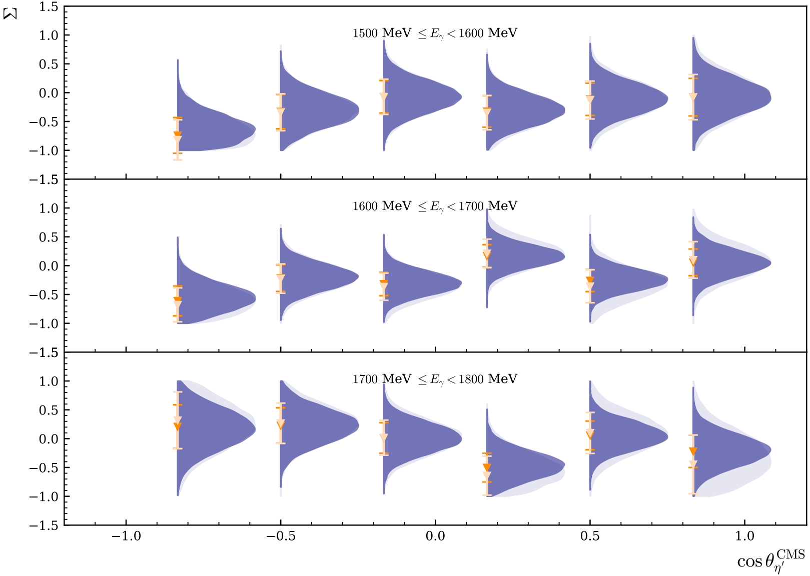


Determining the beam asymmetry in η' photoproduction

Systematic error: $\Delta\Sigma^{\text{sys}}_{\eta'} = \sqrt{\left(\frac{\Delta p_\gamma}{p_\gamma}\Sigma_{\eta'}\right)^2 + \left(\Delta\Sigma_{\eta'}\right)^2}.$

- polarization degree

$$\frac{\Delta p_\gamma}{p_\gamma} = \begin{cases} 0.05 & E_\gamma < 1600\,\text{MeV}, \\ 0.08 & \text{otherwise} \end{cases}$$

- background contributions

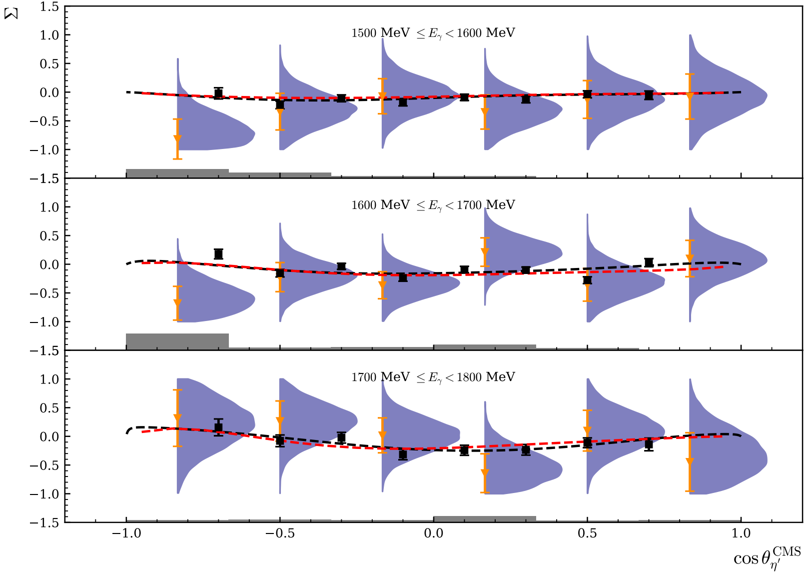
$$\Sigma^{\text{meas}} = (1 - \delta_1 - \delta_2) \cdot \Sigma_{\eta'} + \delta_1 \Sigma_{2\pi^0} + \delta_2 \Sigma^{\text{r bkg}},$$

thus:

$$\Delta\Sigma_{\eta'} = \max\left[\left|\frac{\Sigma^{\text{meas}} - \delta_1 \cdot \Sigma_{2\pi^0} \pm \delta_2 \cdot 1}{1 - \delta_1 - \delta_2} - \frac{\Sigma^{\text{meas}} - \delta \cdot \Sigma_{2\pi^0}}{1 - \delta}\right|\right]$$

[F. N. Afzal 2019; Eberhardt 2012]

Determining the beam asymmetry in η' photoproduction



■ measurement at CLAS [Collins et al. 2017] ,

- - - etaMAID PWA [Tiator et al. 2018],

- - - BnGa PWA [Anisovich et al. 2018]

Conclusion

Summary

- Σ extracted for η and η' final state
- η results obtained with BAYESIAN fit agree with previous results
- η' results agree with previous measurements

Outlook

- use posterior *distributions* for PWA calculations
- increase precision of existing data , i.e. investigate $\eta' \rightarrow \pi^0\pi^0\eta$
- measurement of observables near η' production threshold

Backup & References

Additional theoretical basics

Unpolarized differential cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{4}\rho \sum_{\text{spins}} |\langle f|\mathcal{F}|i\rangle|^2,$$

where

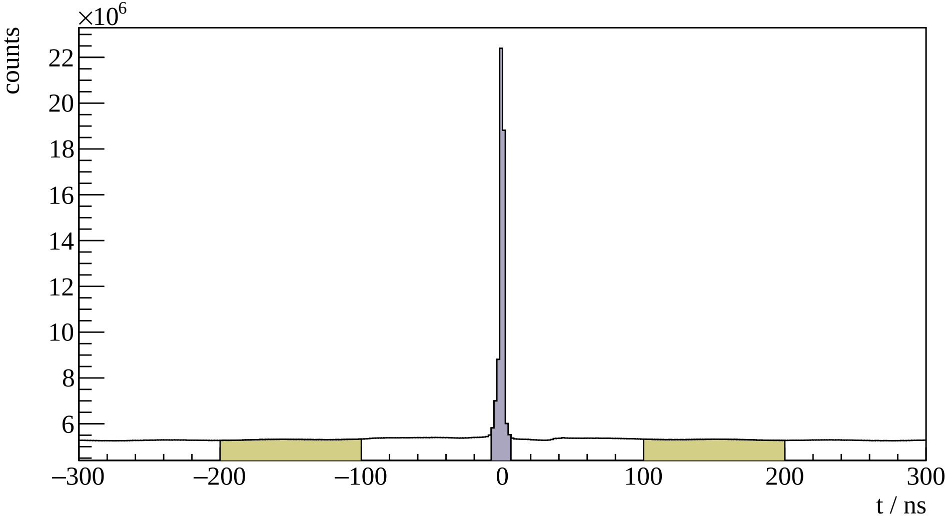
$$\mathcal{F} = i(\vec{\sigma}\cdot\vec{\epsilon})F_1 + (\vec{\sigma}\cdot\hat{q})(\vec{\sigma}\cdot(\hat{k}\times\vec{\epsilon}))F_2 + i(\vec{\sigma}\cdot\hat{k})(\hat{q}\cdot\vec{\epsilon})F_3 + i(\vec{\sigma}\cdot\hat{q})(\hat{q}\cdot\vec{\epsilon})F_4$$

F_i : complex CGLN Amplitudes

[Chew et al. 1957]

$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \in \mathbb{R}$, not sufficient do determine $\mathcal{F}$ unambiguously
$\rightarrow$ Polarization Observables can be related to F_i

Time cut



$\times10^6$
counts
22
20
18
16
14
12
10
8
6
–300
–200
–100
0
100
200
300
t / ns

Full PDF for unbinned maximum likelihood fit

$$
\begin{aligned}
-\ln\mathcal{L} = &\sum_{i=1}^{n} -\ln(p_{\text{prompt}}(\phi_i, p_{\gamma,i}, \Sigma, a_1 \dots a_4, b_1 \dots b_4)) + \\
&\sum_{j=1}^{m} -\ln\left(p_{\text{sideband}}(\phi_j, p_{\gamma,j}, \Sigma^{\text{bkg}}, a_1^{\text{bkg}} \dots a_4^{\text{bkg}}, b_1^{\text{bkg}} \dots b_4^{\text{bkg}})\right)
\end{aligned}
$$

where

$$
\begin{aligned}
p_{\text{prompt}} &= f_{\text{sig}} \cdot \tilde{p}(\phi, p_\gamma, \Sigma, a_1 \dots a_4, b_1 \dots b_4) \\
&\quad + (1 - f_{\text{sig}}) \cdot \tilde{p}(\phi, p_\gamma, \Sigma^{\text{bkg}}, a_1^{\text{bkg}} \dots a_4^{\text{bkg}}, b_1^{\text{bkg}} \dots b_4^{\text{bkg}}) \\
p_{\text{sideband}} &= \tilde{p}(\phi, p_\gamma, \Sigma^{\text{bkg}}, a_1^{\text{bkg}} \dots a_4^{\text{bkg}}, b_1^{\text{bkg}} \dots b_4^{\text{bkg}})
\end{aligned}
$$

and

$$
\tilde{p}(\phi, \Sigma) = \frac{\left(1 + p_\gamma \Sigma \cos\left(2\left(\alpha^{\parallel} - \phi\right)\right)\right) \cdot \left(\sum_{k=0}^{4} a_k \sin(k\phi) + b_k \cos(k\phi)\right)}{1 - \frac{1}{2} a_2 p_\gamma \Sigma}
$$

Hamiltonian Monte Carlo

Introduce artificial "momentum" $\boldsymbol{\phi}$

$$\begin{aligned} & p(\boldsymbol{\theta},\boldsymbol{\phi}|y) = p(\boldsymbol{\theta}|y)\cdot p(\boldsymbol{\phi}|\boldsymbol{\theta},y) = p(\boldsymbol{\theta}|y)\cdot p(\boldsymbol{\phi}) \\ \Leftrightarrow & -\log p(\boldsymbol{\theta},\boldsymbol{\phi}|y) = -\log p(\boldsymbol{\theta}|y) - \log p(\boldsymbol{\phi}) \\ \overset{\text{Def.}}{\Leftrightarrow} & \; H(\boldsymbol{\theta},\boldsymbol{\phi}) := V(\boldsymbol{\theta}) + T(\boldsymbol{\phi}), \end{aligned}$$

Applying the well known equations of motion to this Hamiltonian one finds

$$\frac{\mathrm{d}\theta_i}{\mathrm{d}t} = \frac{\partial H}{\partial \phi_i} = \frac{\partial T}{\partial \phi_i} \qquad \frac{\mathrm{d}\phi_i}{\mathrm{d}t} = -\frac{\partial H}{\partial \theta_i} = -\frac{\partial V}{\partial \theta_i}.$$

Integrate to arrive at proposals $\boldsymbol{\theta}^*, \boldsymbol{\phi}^*$ fed into Metropolis-Hastings accept-reject step

Diagnostics of a BAYESIAN fit I

The $\widehat{R}$-statistic is defined for a set of M MARKOV chains $\boldsymbol{\theta}_m$ with N samples $\boldsymbol{\theta}_m^{(n)}$. The *between-chain-variance* B is estimated as

$$B = \frac{N}{M-1} \sum_{m=1}^{M} \left(\bar{\boldsymbol{\theta}}_m^{(\bullet)} - \bar{\boldsymbol{\theta}}_\bullet^{(\bullet)} \right)^2 ,$$

where

$$\bar{\boldsymbol{\theta}}_m^{(\bullet)} = \frac{1}{N} \sum_{n=1}^{N} \boldsymbol{\theta}_m^{(n)}, \qquad \bar{\boldsymbol{\theta}}_\bullet^{(\bullet)} = \frac{1}{M} \sum_{m=1}^{M} \bar{\boldsymbol{\theta}}_m^{(\bullet)}.$$

Further the *within-chain variance* W is averaged over all chains

$$W = \frac{1}{M} \sum_{m=1}^{M} s_m^2,$$

Diagnostics of a BAYESIAN fit II

where

$$s_m^2 = \frac{1}{N-1} \sum_{n=1}^{N} \left(\boldsymbol{\theta}_m^{(n)} - \bar{\boldsymbol{\theta}}_m^{(\bullet)} \right)^2 .$$

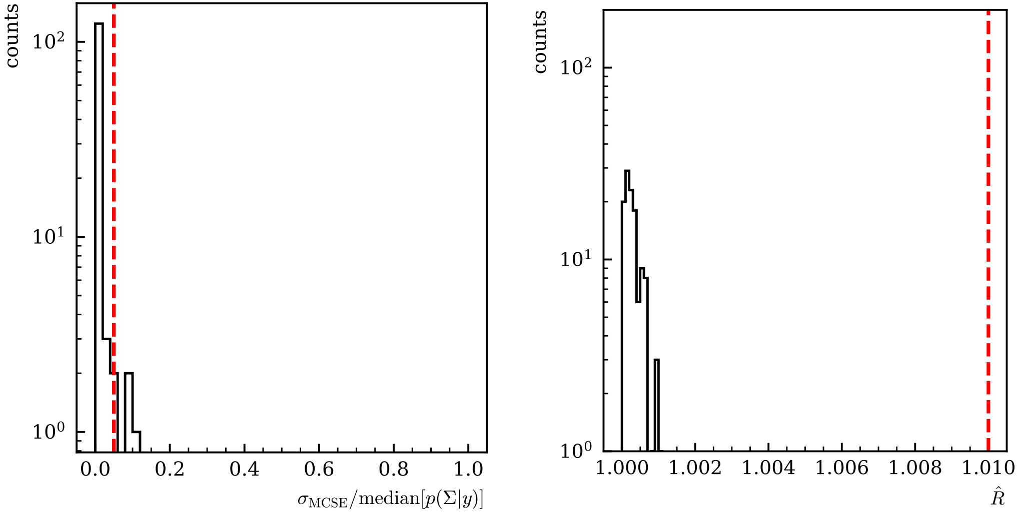
Combining the between and within chain variances into the *variance estimator*

$$\widehat{\mathrm{var}}^{+}(\boldsymbol{\theta}|y) = \frac{N-1}{N} W + \frac{1}{N} B$$

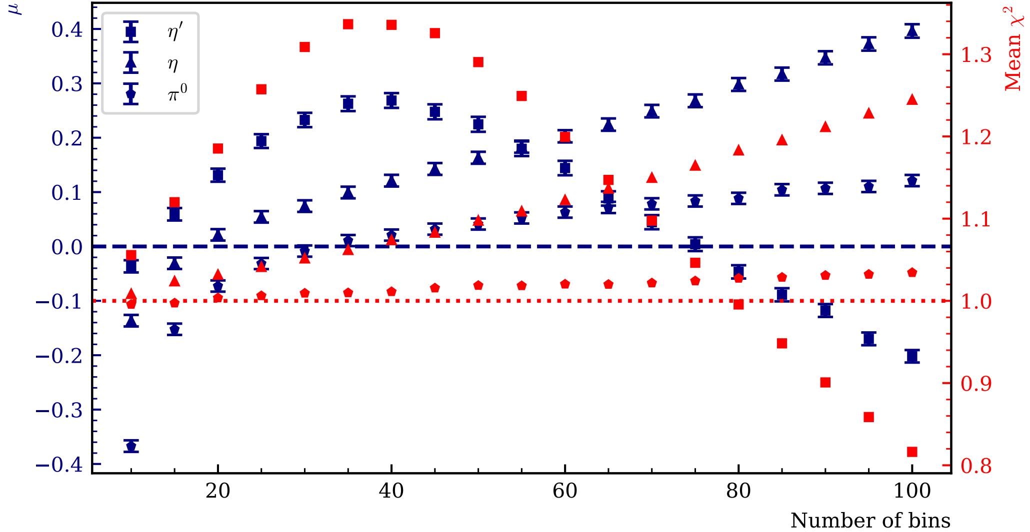
one finally arrives at the *potential scale reduction statistic*

$$\widehat{R} = \sqrt{\frac{\widehat{\mathrm{var}}^{+}(\boldsymbol{\theta}|y)}{W}}.$$

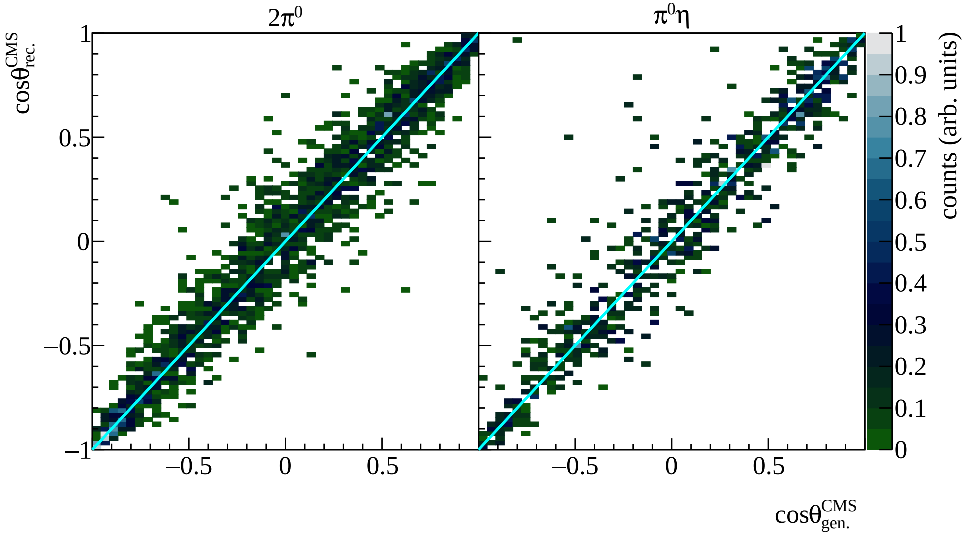
Diagnosis of the unbinned BAYESIAN fit



Systematic effects of binned fitting



$\cos\theta(4\gamma)$ vs. $\cos\theta(2\gamma)$



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