Event based ML fit for η' data

Methods

- `TTree::UnbinnedFit()`
- *RooFit* (WIP)
- event based fit using BAYESIAN inference
  - with and without fitting bkg contributions

each case: no measure of "goodness of fit" $\rightarrow$ toy MC necessary (as before)

Remembering PDF to be fitted...

$$
\begin{aligned}
-\ln\mathcal{L} = &\sum_{i=1}^{n} -\ln(p_{\text{prompt}}(\phi_i, p_{\gamma,i}, \Sigma, a_1 \ldots a_4, b_1 \ldots b_4)) + \\
&\sum_{j=1}^{m} -\ln\left(p_{\text{sideband}}(\phi_j, p_{\gamma,j}, \Sigma^{\text{bkg}}, a_1^{\text{bkg}} \ldots a_4^{\text{bkg}}, b_1^{\text{bkg}} \ldots b_4^{\text{bkg}})\right)
\end{aligned}
$$

where

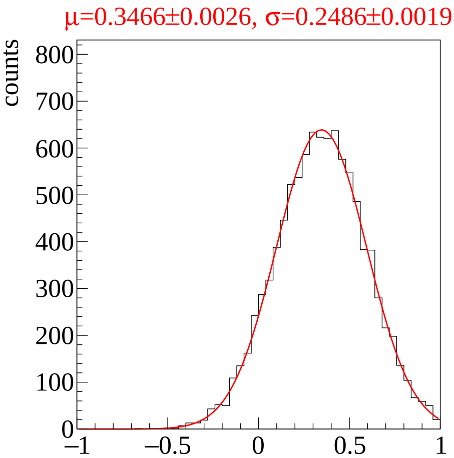
$$
\begin{aligned}
p_{\text{prompt}} &= f_{\text{sig}} \cdot \tilde{p}(\phi, p_\gamma, \Sigma, a_1 \ldots a_4, b_1 \ldots b_4) \\
&\quad + (1 - f_{\text{sig}}) \cdot \tilde{p}(\phi, p_\gamma, \Sigma^{\text{bkg}}, a_1^{\text{bkg}} \ldots a_4^{\text{bkg}}, b_1^{\text{bkg}} \ldots b_4^{\text{bkg}}) \\
p_{\text{sideband}} &= \tilde{p}(\phi, p_\gamma, \Sigma^{\text{bkg}}, a_1^{\text{bkg}} \ldots a_4^{\text{bkg}}, b_1^{\text{bkg}} \ldots b_4^{\text{bkg}})
\end{aligned}
$$

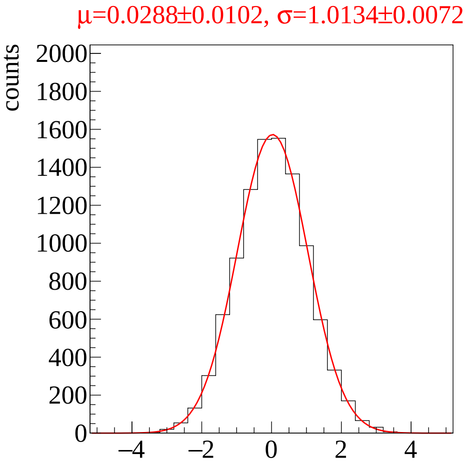
and

$$
\tilde{p}(\phi, \Sigma) = \frac{\left(1 + p_\gamma \Sigma \cos\left(2\left(\alpha^{\parallel} - \phi\right)\right)\right) \cdot \left(\sum_{k=0}^{4} a_k \sin(k\phi) + b_k \cos(k\phi)\right)}{1 - \frac{1}{2} a_2 p_\gamma \Sigma}
$$

Considering background contributions

- simulate η' statistics and bkg contributions in toy MC
- expectation: $\Sigma_{\text{meas}} = a \cdot \Sigma_{\text{true}} + b \cdot \Sigma_{\text{bkg}}$, where $a + b = 1$
- $\Sigma_{\text{true}} = 0.5, \Sigma_{\text{bkg}} = -0.3, a = 0.8 \rightarrow \Sigma_{\text{meas}} = 0.34$





Considering background contributions

- If background contribution is known, it can directly be included in the BAYESIAN fit by treating it as "missing data"
- $$\tilde{p}(\phi, \Sigma) = \frac{\left(1 + p_\gamma \Sigma \cos\left(2\left(\alpha^{\parallel} - \phi\right)\right)\right) \cdot \left(\sum_{k=0}^{4} a_k \sin(k\phi) + b_k \cos(k\phi)\right)}{1 - \frac{1}{2} a_2 p_\gamma \Sigma}$$

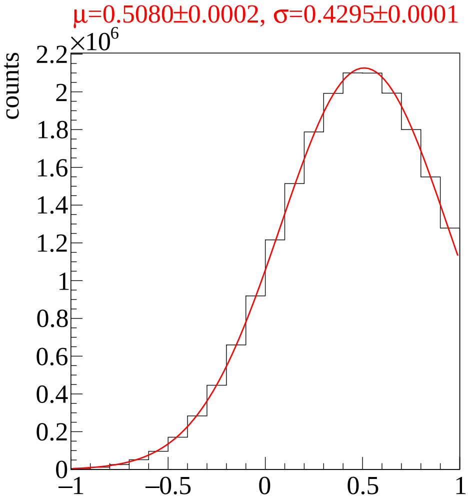
  with $\Sigma = (a \cdot \Sigma_{\text{true}} + b \cdot \Sigma_{\text{bkg}})$ and $a, b, \Sigma_{\text{bkg}}$ are read in as data
- statistical error of Σ_{bkg} can be accounted for by fitting an additional unknown parameter $\Sigma_{\text{bkg}}^{\text{true}}$, assuming

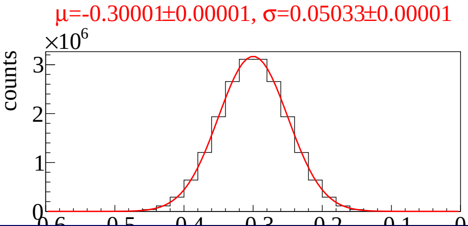
  $$\Sigma_{\text{bkg}}^{\text{true}} \sim \text{normal}(\Sigma_{\text{bkg}}^{\text{meas}}, \text{err}(\Sigma_{\text{bkg}}^{\text{meas}}))$$

  with uniform prior and then using $\Sigma_{\text{bkg}}^{\text{true}}$ as Σ_{bkg} in $p(\phi, \Sigma)$

Considering background contributions

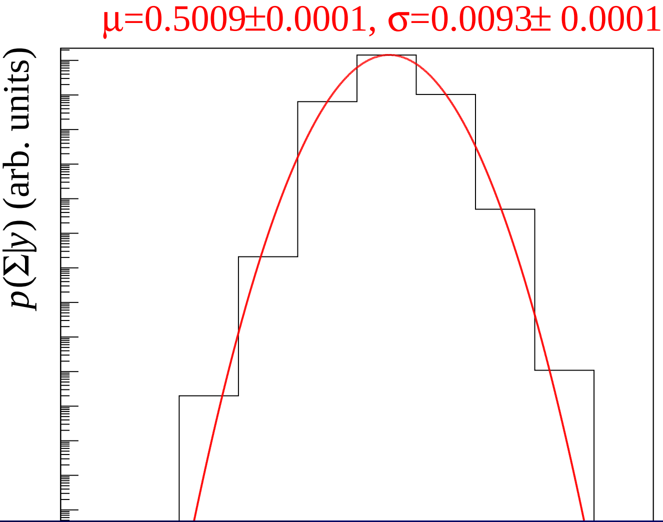
toy MC with same dataset as before (assuming $\mathrm{err}(\Sigma_{\mathrm{bkg}}^{\mathrm{meas}}) = 0.05$) reproduces input values very nicely



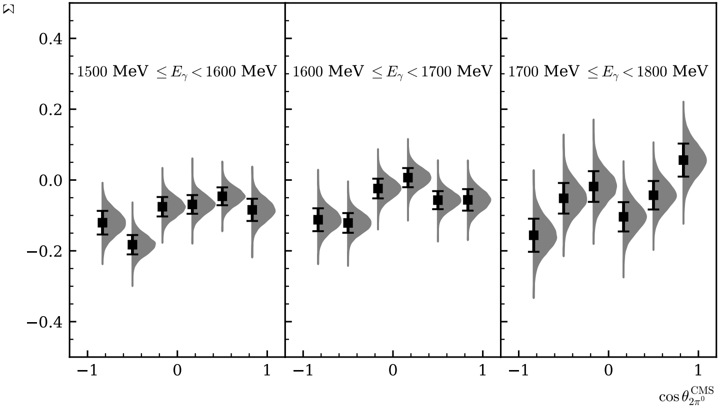


Considering background contributions

Also, likelihood pool combination of posteriors again works nicely

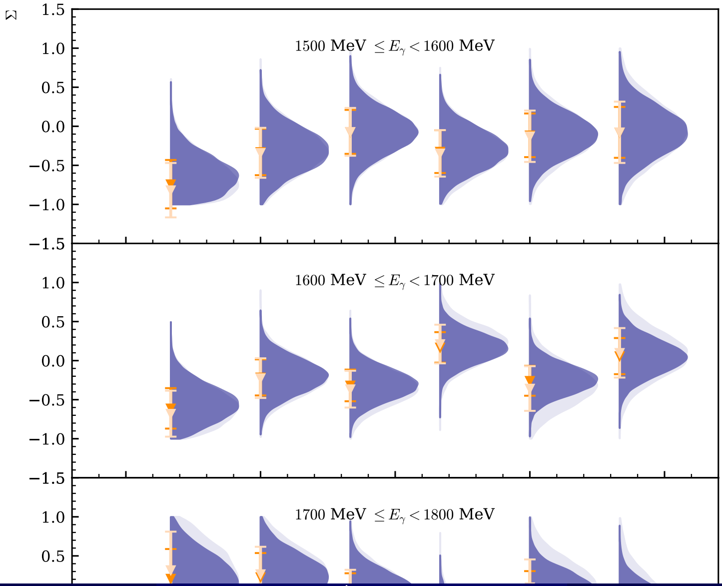


Results



Σ
0.4
0.2
0.0
−0.2
−0.4
1500 MeV ≤ E_γ < 1600 MeV
1600 MeV ≤ E_γ < 1700 MeV
1700 MeV ≤ E_γ < 1800 MeV
−1
0
1
$\cos\theta_{2\pi^0}^{\mathrm{CMS}}$

Results



Σ
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
1500 MeV ≤ E_γ < 1600 MeV
1600 MeV ≤ E_γ < 1700 MeV
1700 MeV ≤ E_γ < 1800 MeV