# Extracting the beam asymmetry $\Sigma$ from data (bins in $(E_\gamma, \cos\theta)$)

**Event yield asymmetries (binned in $\phi$)**

$$A(\phi) = \frac{N^\perp - N^\parallel}{p_\gamma^\parallel N^\perp + p_\gamma^\perp N^\parallel} = \Sigma \cos(2(\alpha^\parallel - \phi))$$

**Event based fit (unbinned in $\phi$)**

$$\begin{aligned} -\ln\mathcal{L} = &\sum_{i=1}^{n} -\ln(p_{\text{prompt}}(\phi_i, p_{\gamma,i}, \Sigma, a_1 \ldots a_4, b_1 \ldots b_4)) + \\ &\sum_{j=1}^{m} -\ln\left(p_{\text{sideband}}(\phi_j, p_{\gamma,j}, \Sigma^{\text{bkg}}, a_1^{\text{bkg}} \ldots a_4^{\text{bkg}}, b_1^{\text{bkg}} \ldots b_4^{\text{bkg}})\right) \end{aligned}$$

toy MC: generate events that follow pdfs $(\frac{d\sigma}{d\Omega})$ of $N^\parallel, N^\perp$ and $p_{\text{prompt}}, p_{\text{sideband}}$

# Investigating toy MC fit results

$\chi^2$ **fits**: investigate *normalized residuals* $\xi = \frac{\Sigma_{\text{estimated}} - \Sigma_{\text{true}}}{\text{err}(\Sigma_{\text{estimated}})}$,

$\rightarrow$ "good" fit yields $\xi \sim \mathcal{N}(0, 1)$

**bayesian fits:**

a. add up posteriors $p(\Sigma|y_i)$ to combined posterior $P(\Sigma|y_0, \ldots, y_{9999})$ (mixture model)
   $\rightarrow$ expect $P(\Sigma|y_0, \ldots, y_{9999}) \sim \mathcal{N}(\Sigma_{\text{true}}, \sigma)$

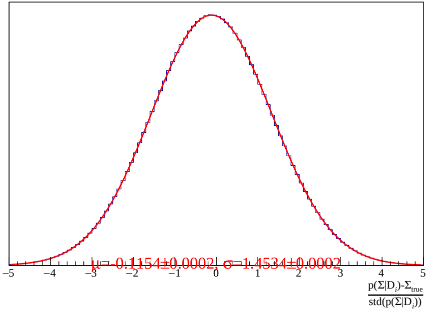
b. build "normalized residuals" $\Xi = \sum_{i=0} \frac{p(\Sigma|y_i) - \Sigma_{\text{true}}}{\text{std}(p(\Sigma|y_i))}$
   $\rightarrow$ expect $\Xi \sim \mathcal{N}(0, \mathcal{O}(1))$

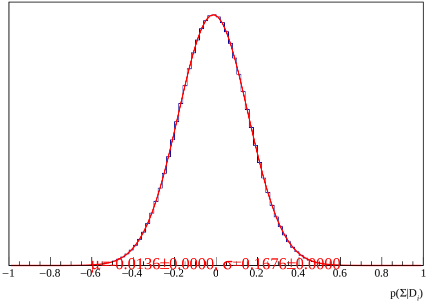
also check $\chi^2$ distribution and MCMC diagnostics

# Fit results (event yield asymmetries with Bayes)

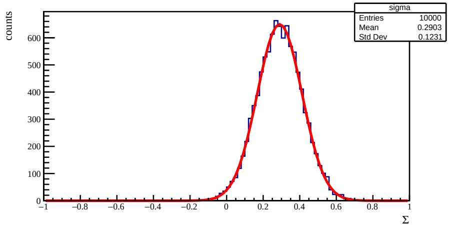
statistics of $\eta$ final state: (12 bins in $\phi$)

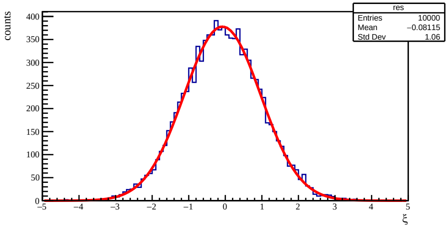
- 10000 toy bins
- $p_\gamma^{\parallel} = 0.25, p_\gamma^{\perp} = 0.3$, $\Sigma = 0.3$
- $n_{\text{events}}^{\parallel} \sim \text{Pois}(800)$, $n_{\text{events}}^{\perp} \sim \text{Pois}(1000)$

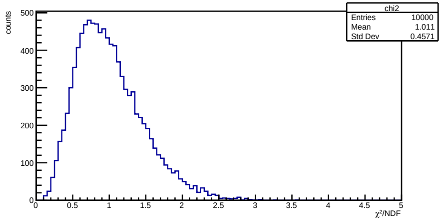


# Fit results (event yield asymmetries with $\chi^2$ fit)



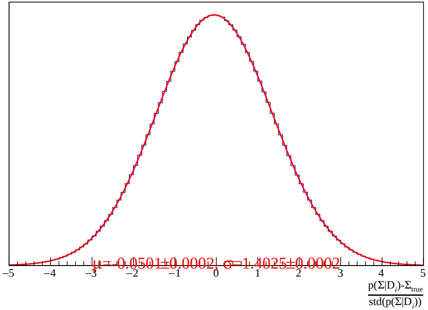



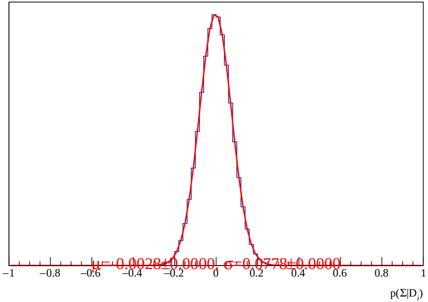



# Fit results (event yield asymmetries with Bayes)

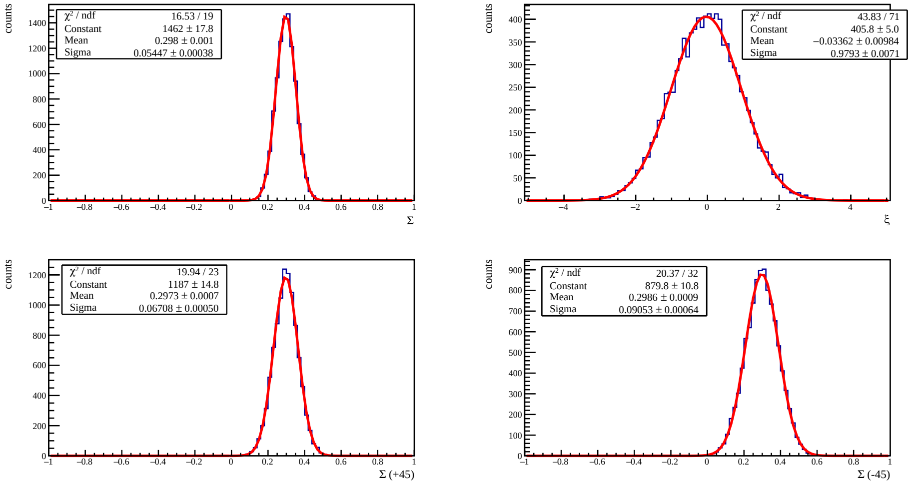
statistics of $\pi^0$ final state: (24 bins in $\phi$)

- 10000 toy bins
- $p_\gamma^{\parallel} = 0.25, p_\gamma^{\perp} = 0.3,\ \Sigma = 0.3$
- $n_{\text{events}}^{\parallel} \sim \text{Pois}(4000),\ n_{\text{events}}^{\perp} \sim \text{Pois}(5000)$



# Fit results (event yield asymmetries with $\chi^2$ fit)

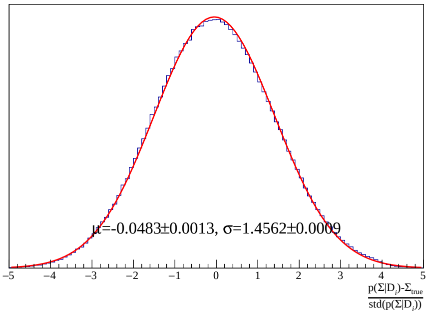


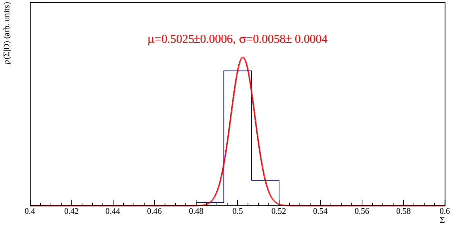

Event yield method okayish, but binning and statistics affect the fit siginificantly!

# Fit results (Event-based fit)

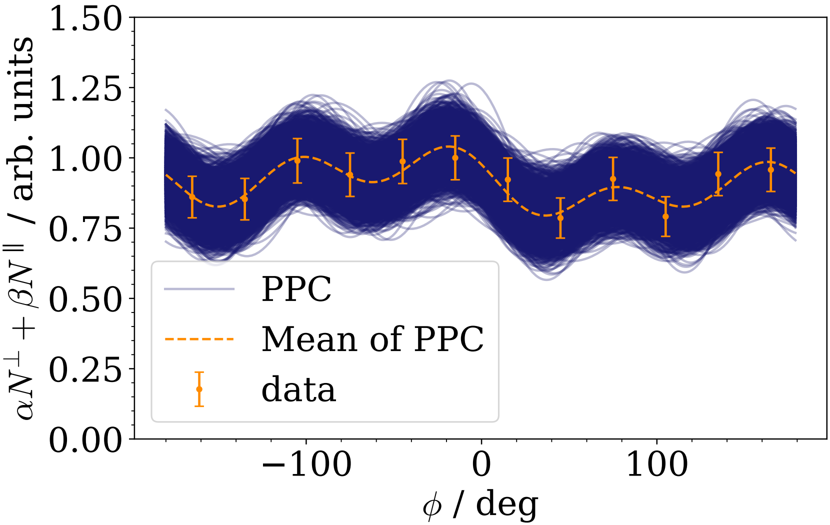
- 300 toy bins (fit takes very long)
- $p_\gamma^{\parallel} = 0.25, p_\gamma^{\perp} = 0.3$, $\Sigma = 0.3$, sideband events and weights similar to data, Eff. func: $f(\phi) = \frac{1}{10.5}(9.3 + 0.28\cos\phi + 0.24\sin 3\phi)$
- $n_{\text{events}}^{\parallel} \sim \text{Pois}(800)$, $n_{\text{events}}^{\perp} \sim \text{Pois}(1000)$
- either build "normalized residuals" $\Xi$ or "independent likelihood pool" $P(\Sigma|y_0 \ldots y_{300}) \propto \frac{\prod_i p(\Sigma|y_i)}{\pi(\Sigma)^{300-1}}$

# Fit results (Event-based fit)



# Fit results (Event-based fit)



# Fit results (Data), now "justified" :)