Survey Computation and Adjustment

EG 2104 GE

Unit-5 Theory of Measurement Errors and Adjustments

Public Namuna Secondary School

Lecture by

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Error Concept

Error

- It is the difference between measured value and true value of a quantity.

Error = True value – Measured value

- The true value of a quantity is never known and hence the true error too. However, when a number of measurements are to fulfil a known condition, true error can be obtained.

For example, the theoretical sum of exterior included angles in a traverse of N sides is (2N+4)*90. After traversing when you add all your angles you will get the sum of observed angle. If both don’t matches you will know you have got error.

Sources of Error

There are three sources of error. They are:

1. Personal

- Due to limitations in human sight in observing and touch in manipulating instruments, carelessness, fatigue etc.
- For example, an error may occur in taking the level reading or reading and angle on the circle of a theodolite. Such errors are known as personal errors.
- It includes centering error, levelling error, reading error, recording error, missing observation, focusing error etc.

2. Instrumental

- Surveying errors which arise due to imperfection or faulty instrument adjustment with which measurement is being taken.
- For example, a tape may be too long, or an angle measuring instrument may be out of adjustment.

Sources of Error

There are three sources of error. They are:

3. Natural

- Error in surveying may also be due to variations in natural phenomena such as temperature, humidity, gravity, atmospheric pressure, refraction and magnetic declination.
- If they are not properly observed while taking measurements, the results will be incorrect.
- For example, a tape may be 20 meters at 20°C, but its length will change if the field temperature is different.
- It includes error due to wind vibration, unequal expansion in various parts of instrument due to temperature difference, etc.

Types of Error

There are mainly three types of error. They are:

1. Mistakes (gross errors)

- Mistakes arise from inattention, inexperience, carelessness and poor judgment or confusion in the observer's mind.
- They do not follow any mathematical rule (law of probability) and can’t be handled even statistically.
- In the theory of errors, mistakes are also known as gross errors, and they cannot be measured.
- Detection: Mistakes can be detected by repeated observations of the same quantity.
- Remedy: The measurements must be observed and recorded carefully and in a controlled way so as to minimize them.

2. Systematic errors (Cumulative Errors or biases)

- A systematic or cumulative error is an error that occurs from well-understood causes and follows some definite mathematical or physical law, and correction can be determined and applied.
- Causes: They are caused by the surveying equipment, observation methods and certain environmental factors.
- Remedy: Keeping equipment in proper working order and following established surveying procedures, many of the systematic errors can be eliminated.

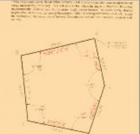
Types of Error

There are mainly four types of error. They are:

3. Random errors (Accidental error or Probable error)

- Random errors occur due to the combination of causes that are unpredictable and beyond the control of the surveyor.
- Their occurrence, magnitude and direction (positive & negative) cannot be predicted.
- Since accidental error follows the law of probability it is also called probable error.
- Remedy: It can be minimized by taking redundant observations and adjusting by method of least square. the process is referred to as the “Adjustment of observation” or “Adjustment computation”

Error Propagation



Error Propagation:

- In surveying we have quantities measured and quantities required. If there are errors in the measured quantities, then automatically there will be errors in the required quantities.
- Error propagation is the evaluation of the errors in the required quantities as functions of the errors in the measurements.
- Usually we combine measurements together to compute other quantities. Errors in those measurements affect the accuracy of the resulting computation. This is what's meant by error propagation.
- For example, Surveyors may measure angles and lengths of lines between points directly and use these measurements to compute station coordinates. During this procedure, the errors that were present in the original direct observations (angles & lengths) are propagated (distributed) by the computational process into the indirect values. Thus, the indirect measurements (computed station coordinates) contain errors that are functions of the original errors. This distribution of errors is known as error propagation.

Three of the more common error propagations are:

1. Error of a Sum
2. Error of a Series
3. Error of a Product.

Error Analysis

Error Analysis:

- Error analysis is an important aspect of surveying that involves identifying and quantifying the errors associated with the measurements made during the survey.
- Error analysis helps to identify the magnitude and sources of these errors and provides guidance on how to minimize their impact on surveying results.
- To perform error analysis in surveying, it is essential to use appropriate statistical methods such as standard deviation, mean, and variance to evaluate the quality of the survey data.

The process of error analysis in surveying involves several steps, including:

1. Measurement of errors:

- The first step is to measure the errors in the survey measurements.
- This can be done by taking multiple measurements and comparing the results to identify any differences.

2. Classification of errors:

- The next step is to classify the errors into random errors and systematic errors.
- Random errors are those that occur due to chance, whereas systematic errors are caused by a constant bias in the measurement process.

Error Analysis

The process of error analysis in surveying involves several steps, including:

3. Quantification of errors:

- Once the errors have been classified, they can be quantified.
- This involves determining the magnitude of the errors and expressing them in terms of their standard deviation.

4. Mitigation of errors:

- Finally, the errors can be mitigated by taking appropriate corrective measures.
- These may include adjusting the equipment, using more accurate instruments, and improving measurement techniques.

Accuracy vs Precision

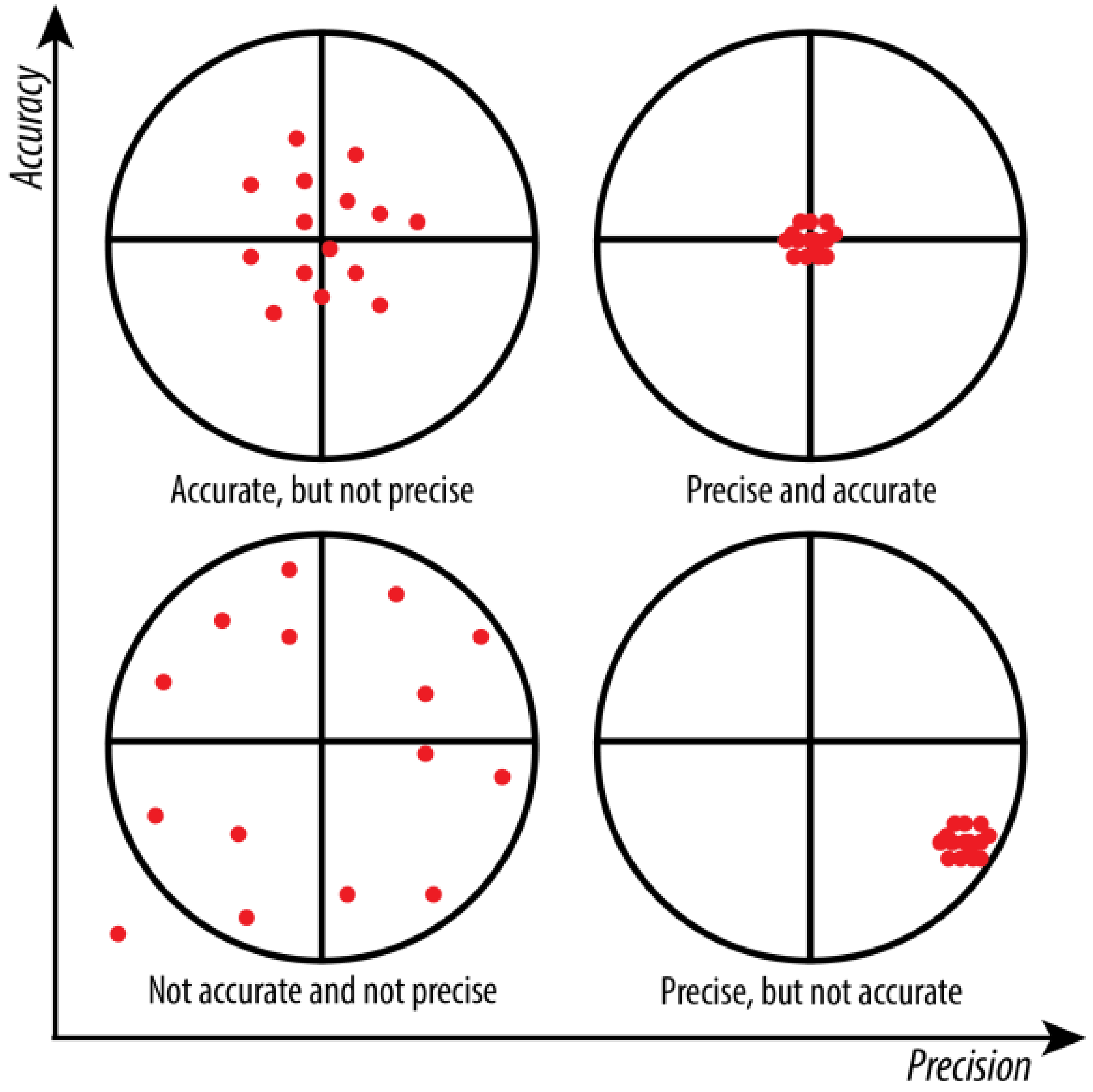
Accuracy:

- Accuracy refers how close a measurement or observation is to the "true value”.
- Since the true value of a quantity can never be determined, accuracy is always an unknown.
- The level of accuracy required for particular applications varies greatly.
- Highly accurate data can be very difficult and costly to produce and compile.

Precision:

- Precision refers how close measure values are to each other.
- The level of precision required for particular applications varies greatly. Engineering projects such as road and utility construction require very precise information.
- The degree of precision attainable is dependent on the stability of the environment during the time of measurement, the quality of the equipment used to make the observations, and the observer’s skill with the equipment and observational procedures.

Accuracy vs Precision



Source: [Precision Vs. Accuracy – Information Technology (stolaf.edu)]

Basic Terms:

1. Measurement

- It is the observation made to determine the unknown quantity.
- It is of two types: (i) Direct measurement (ii) Indirect measurement

1.1 Direct measurement

- It is the observation made in the field by using an instrument to determine the unknown quantity.

1.2 Indirect measurement

- It is the observation made to determine the unknown quantity from its mathematical relationship to direct measurements.
- Indirect measurements are obtained when it is not possible or practical to make direct measurements.
- Surveyors may, for example, measure angles and lengths of lines between points directly and use these measurements to compute station coordinates.

Basic Terms:

2. Value of a quantity

- The surveying measurements are made under different conditions causing variations in the measurements made.
- Therefore, the value of a measurement made may be true value, observed value or the probable value.

2.1 True value

- It is the value which is absolutely free from all the errors.

2.1.1 True error (Absolute error)

- It is the difference between true value and measured value of a quantity.

$$\text{True error} = \text{True value} - \text{Measured value}$$

- True error of a measurement can never be found since the true value of a measurement can not be determined.

Basic Terms:

2.2 Measured value

- It refers to the value obtained from direct measurements using surveying instruments, such as total stations, GPS receivers, or leveling equipment.
- These measurements are typically taken in the field by surveyors and are based on the physical observations made during the surveying process.

2.3 Observed value

- It refers to the value obtained after applying corrections, adjustments, or computations to the measured values that may have errors in the measurements due to instrumental errors, atmospheric conditions, curvature, refraction of the Earth, and other systematic or random errors associated with the survey.
- It considered to be more accurate and refined than measured values as it is applied with corrections.

3. Most Probable value (MPV):

- When many observations are made for same quantity, most probable value of a quantity can be defined as that value which is most likely to be the true value.
- Since in surveying, the true value of a measurement is never know for the purpose of calculating errors, the arithmetic mean is taken to be the true value or MPV.

Basic Terms:

3.1 Determination of most probable value

On basis of measurement direct or indirect, MPV can be obtained.

3.1.1 MPV of directly observed independent quantities.

- Quantities of equal weight
- Quantities of unequal weight

3.1.2 MPV of indirectly observed independent quantities.

Basic Terms:

Quantities of equal weight:

- The MPV of a quantity measured by direct observations of equal weight is given by the arithmetic mean of the measured values.
- If the measured values of a quantity x be $x_{1,}$ $x_{2,}$ ……x_n with all measured values of equal weight. The arithmetic mean will be the MPV of the quantity and is given by,

$$\mu_x = \sum_{i=1}^{n} \frac{\sum x_i}{n}$$

Quantities of unequal weight:

- The MPV of a quantity measured by direct observations of unequal weight is given by multiplying each results by its weight and dividing the sum of the products by the sum of the weights.
- If the measured values of a quantity x be $x_{1,}$ $x_{2,}$ ……x_n with all measured values of unequal weights $w_{1,}$ w_2, ,…….w_n . The arithmetic mean will be the MPV of the quantity and is given by,

$$\mu_x = \sum_{i=1}^{n} \frac{\sum w_i x_i}{\sum w_i}$$

Numericals: MPV

Example 3.1 An angle A was observed by the method of repitition by three different observers and the values were as below.

Observer no.	$\angle A$	Number of measurements
1	40°20′	2
2	40°30′	3
3	40°45′	4

Find the most probable value of the angle.

Ans: $40°34'26.67''$

Example 3.17 An angle was measured by different persons and the following values were observed. Find the most probable value of the angle.

Angle	Number of measurements
54°30′20″	2
54°29′40″	2
54°30′10″	2

Ans: $54°30'3.33''$

Numericals: MPV

Example 3.17 An angle was measured by different persons and the following values were observed. Find the most probable value of the angle.

Angle	Number of measurements
54°30′20″	2
54°29′40″	2
54°30′10″	2

Solution Since the angles are measured with equal weights, the most probable value of the angle will be the arithmetic mean of the observations. Hence, the most probable value

$$= \frac{54°30'20'' + 54°29'40'' + 54°30'10''}{3} = 54°30'3.33''$$

Example 3.18 An angle is measured by different observers with the following values. Find the MPV of the angle.

Angle	Number of measurements
54°30′20″	2
54°29′40″	3
54°30′10″	4

Solution The MPV will be the weighted arithmetic mean. Hence, the most probable value

$$= \frac{(54°30'20'' \times 2) + (54°29'40'' \times 3) + (54°30'10'' \times 4)}{2+3+4} = 54°30'2.2''$$

Basic Terms:

Residual (Residual error):

- It is the difference between the MPV and the measured value (direct or indirect measurement) of a quantity.

Basic Terms:

Weight

- The weight of an observation is a measure of its relative trustworthiness, precision and confidence enjoyed by the observer.
- To derive the value of the quantity measured, it is not enough to simply take the arithmetic mean, but the quality of each measurement also must be taken into account. The quality of each measurement result is evaluated by means of an index number known as weight.
- Weight describes the reliability of measured quantity.

For example if a distance is measured three times with an EDM and the measured values are 185.67, 185.67, and 185.68 then the weights for 185.67 is 2 and 185.68 is 1.

Basic Terms:

Discrepancy:

- It is the difference between two measurements of a same quantity.
- Large discrepancy indicates a mistakes in the observation.
- If the mistake is significant and unpredictable, leading to a large disprepancy it is called a blunder.

Improper levelling of the surveying instrument, incorrect entering of the control point number in the data collector or misplacing the decimal point are some of the examples of blunders.

Redundant observation:

- If the measurement is made in excess of the required number that are needed to determine the unknowns then it is called redundant observation.
- For example, two measurements of the length of a line yield one redundant observation. The first observation would be sufficient to determine the unknown length, and the second is redundant.

Here, the second observation is very useful in two ways:

i. First by calculating discrepancy we can know the size of error and if needed again can take measurement.

ii. Second, the redundant observation permits an adjustment to be made to obtain a final value for the unknown line length, and that final adjusted value will be more precise statistically than either of the individual observations.

Basic Terms:

Redundant observation:

- Surveyors always make redundant observations in their work, for the two important reasons :

1. to make it possible to assess errors and make decisions regarding acceptance or rejection of observations,
2. to make possible an adjustment whereby final values with higher precisions are determined for the unknowns.

Error Analysis

Standard error

- Standard error describes precision of the measurements, or how close the measurements are to their mean value.
- Standard error of a set of repetitive measurements is given by,

$$E_s = \pm\sqrt{\frac{\sum v^2}{n-1}}$$

Where,

v= residual = MPV - measured value

n= no. of observations

- Standard error also called standard deviation or root mean square error.

NOTE: Variance is the square of standard deviation and is another way of defining precision.

Standard error of the mean

- Since the mean is computed from individual observed values, each of which contains an error = σ or E_s , the error of the mean is found by the relation:

$$\bar{\sigma} = \frac{\sigma}{\sqrt{n}}$$

Where,

n= no. of observations

Error Analysis: Standard Error

Numerical:

Q.1 Line AB was measured 6 times as: 81.240; 81.245; 81.243; 81.242;

81.241; and 81.246 m. Calculate the mean , residual of each measurement, standard error of each measurement , and standard error of the mean.

Ans: Mean = = 81.243, standard error of the mean= ±0.001 m

Error Analysis: Standard Error

Numerical: Solution

length	Residual (V)	v^2
81.240	-0.003	9E-6
81.245	0.002	4E-6
81.243	0.000	0.000
81.242	-0.001	1E-6
81.241	-0.002	4E-6
81.246	0.003	9E-6
487.458		27E-6

$$\bar{M} = \frac{\sum M}{n} = \frac{487.458}{6} = 81.243$$

$$\sigma = \pm\sqrt{\frac{\sum v^2}{n-1}} = \sqrt{\frac{27E-6}{6-1}} = 2.32\text{E-3}$$

$$= \pm 0.002 \text{ m}$$

$$\bar{\sigma} = \frac{\sigma}{\sqrt{n}} = \frac{0.002}{\sqrt{6}} = \pm 0.001 \text{ m}$$

Error Analysis: Standard Error

Numerical:

Q.2

Example 2.1 An angle was measured six times, the observed values being 49°23′00″, 49°23′20″, 49°22′40″, 49°22′20″, 49°23′40″ and 49°24′00″. Calculate the most probable value of the angle and the standard error of the measurement.

Ans: ±37.41’’

Error Analysis: Standard Error

Numerical:

Q.2 Ans:

Angle	Residual v	v^2
49°23′00″	− 10″	100
49°23′20″	+ 10″	100
49°22′40″	− 30″	900
49°22′20″	− 50″	2500
49°23′40″	+ 30″	900
49°24′00″	+ 50″	2500
$\Sigma\ 6 \times 49° + 139'$	Mean = 49°23′10″	$\Sigma v^2 = 7000$

$$\text{Standard deviation } \sigma = \pm\sqrt{\frac{v^2}{n-1}}$$

$$= \pm\sqrt{\frac{7000}{6-1}}$$

$$= \pm\ 37.41''$$

Error Analysis

Laws of Accidental (Random) errors:

- Accidental error follows the law of probability and is so called probable error.
- This law defines the occurrence of errors and can be expressed in the form of equation which is used to compute the probable value or the probable precision of a quantity.
- Accidental errors always written after the observation quantity with the plus and minus sign.
- The residuals between the true value and measurements made due to random error (error remaining after removing systematic errors and mistakes) when plotted takes the shape of curve called the probability curve of error and is given by

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-v^2/2\sigma^2}$$

where y is the frequency of residual error, v is the size of the residual (deviation from the mean), and σ is the standard error (a measure of precision).

Laws of Accidental (Random) errors:

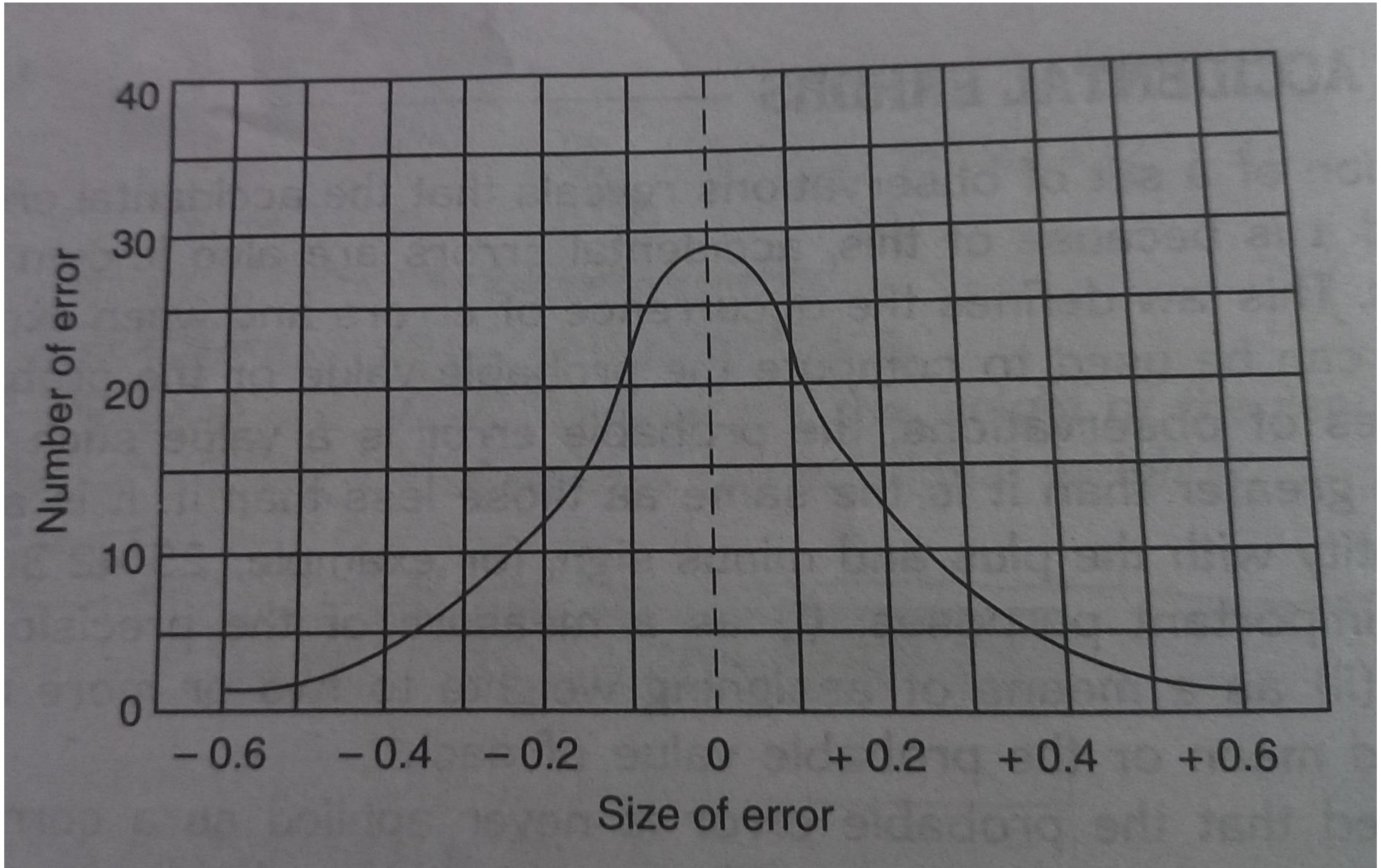


Fig. 3.1 *Probability curve*

Source: S.K Duggal, Vol-II

Error Analysis

Laws of Accidental (Random) errors:

The following point may be noted in the normal probabilities curve:

1. Positive and negative errors are equal in size and frequency, as the curve is symmetrical.
2. Small error are more frequent than large error.
3. Very large error seldom occur and are impossible.:

Error Analysis

Probable error:

- It is the error remaining after removing systematic error and mistakes. Since it follows the law of probability it is called probable error.
- Probable error is measurement calculated from the probability curve of error.

From the probability curve of error,

1. Probable error of the single measurement is given by

- $E_s = \pm 0.6745 \sqrt{\Sigma v^2/n - 1}$
- Es = probable error of the single observation
- v = difference between any single observation and mean of the series.
- n = number of the observation in the series

Probable error of the mean:

- The arithmetic mean or the average in terms of probable error for a repetitive measurement is given by,

$$Em \pm Es / \sqrt{n}$$

Em = probable error of the mean

Determination of Probable error:

The determination of probable error for various cases are:

i. Direct observation of equal weight

ii. Direct observation of unequal weight

iii. Indirect observation of independent quantities

iv. Indirect observation involving equation of condition

Error Analysis: Determination of Probable error:

i. Direct observation of equal weight

- ***Probable error of a single observation is given by,***

1. Probable error of the single measurement is given by

- Es = ± 0.6745 $\sqrt{\Sigma v^2/n-1}$
- Es = probable error of the single observation
- v = difference between any single observation and mean of the series.
- n = number of the observation in the series

- ***Probable error of an arithmetic mean (or average) is***

Em $\pm$Es / $\sqrt{n}$

Em = probable error of the mean

Error Analysis: Determination of Probable error:

i. Direct observation of equal weight

Probable error of a sum of measurements,

$$E_{sm} = \sqrt{E_1^2 + E_2^2 + E_3^2 + \ldots + E_n^2}$$

where n = number of observations in the series

v = difference between any single observation and the mean of the series

$E_1, E_2, \ldots, E_n$ = probable errors of several observations which sum up in a measurement

Source: S.K Duggal, Vol-II

Error Analysis: Determination of Probable error:

ii. Direct observation of unequal weight

Probable error of a single observation of unit weight,

$$E_s = 0.6745 \sqrt{\frac{\Sigma(wv^2)}{n-1}}$$

Probable error of an observation of weight w,

$$E_{su} = \frac{E_{s}}{\sqrt{w}}$$

Probable error of weighted arithmetic mean,

$$E_{smw} = \frac{E_s}{\sqrt{\Sigma w}}$$

where w = weight of an observation

Source: S.K Duggal, Vol-II

NUMERICALS:

Example 3.9 Following observations were recorded for horizontal angles of a triangle:

$$\angle A = 20°10' \pm 0.2$$

$$\angle B = 100°40' \pm 0.1$$

$$\angle C = 59°10' \pm 0.2$$

Determine the probable error of its summation.

Ans: ± 0.3

Source: S.K Duggal, Vol-II

Error Analysis: Probable Error

NUMERICALS:

Solution Sum of angles, $S = \angle A + \angle B + \angle C$

Let e_A, e_B, and e_C be the probable errors of angles A, B, and C.

$$\text{Then, } e_S = \sqrt{e_A^2 + e_B^2 + e_C^2}$$

$$= \sqrt{0.2^2 + 0.1^2 + 0.2^2} = \pm\, 0.3$$

The probable error of summation is ± 0.3

Source: S.K Duggal, Vol-II

Error Analysis: Numerical

Determination of Probable error:

i. Direct observation of equal weight

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Example 3.4 The following observations were recorded for an angle under identical conditions:

162°20′00″	162°21′20″	162°21′40″
162°20′40″	162°19′40″	162°21′20″

Calculate (a) the probable error of a single observation, (b) the probable error of the mean, and (c) the probable value of the angles.

Source: S.K Duggal, Vol-II

Error Analysis: Numerical

Determination of Probable error:

Direct observation of equal weight

Example 3.4 The following observations were recorded for an angle under identical conditions:

162°20′00″	162°21′20″	162°21′40″
162°20′40″	162°19′40″	162°21′20″

Calculate (a) the probable error of a single observation, (b) the probable error of the mean, and (c) the most probable value of the angles.

Solution

Value	*Mean*	v	v^2
162°20′00″		–46.67″	36′18.08″
162°21′20″		+33.34″	18′31.56″
162°21′40″	162°20′46.66″	+53.33″	47′24.08″
162°20′40″		–6.67″	44.49″
162°19′40″		–1′6.67″	74′4.88″
162°21′20″		+33.34″	18′31.56″
		$\Sigma v = 0$	$\Sigma v^2 = 3°15′34.57″$

$$E_s = \pm\, 0.6745 \times \sqrt{3°15′34.57″/(6-1)} = 32.676′$$

Probable error of the mean $E_m = E_s/\sqrt{n} = 32.676′/\sqrt{6} = 13.34′$

Most probable value of the angle = 162°20′46.66″ ± 13.34′ = 162°34′ 7.06″ or 162°7′26.26″.

Source: S.K Duggal, Vol-II

Error Analysis: Probable Error

NUMERICAL:Direct observation of unequal weight

Example 2.12 The following are the observed values of an angle:

Angle	Weight
40°20′20″	2
40°20′18″	2
40°20′19″	3

Find

(i) Probable error of single observation of unit weight.
(ii) Probable error of weighted arithmetic mean.

Ans: ± 0.9538'', ± 0.3605''

Source: S.K Duggal, Vol-II

Here, Weighted arithmetic mean = MPV = $\dfrac{2*40^{o}20'20'' + 2*40^{o}\,20'18'' + 3*40^{o}\,20'19''}{2+2+3} = 40^{o}\,20'19''$

Error Analysis: Probable Error

NUMERICAL:Direct observation of unequal weight

Ans:

Then

v	v^2	pv^2
+1	1	2
−1	1	2
0	0	0
		$\Sigma pv^2 = 4$

Probable error of a single measurement $= \pm 0.6745 \sqrt{\frac{\Sigma pv^2}{n-1}}$

$= \pm 0.6745 \sqrt{\frac{4}{2}}$

$= \pm 0.9538''$

Probable error of weighted mean

$= \pm 0.6745 \sqrt{\frac{\Sigma pv^2}{\Sigma p(n-1)}}$

$= \pm 0.6745 \sqrt{\frac{4}{7(2)}}$

$= \pm 0.3605$

Source: S.K Duggal, Vol-II

Error adjustment

Error adjustment

- Error adjustment in surveying involves the process of minimizing or correcting errors that may have occurred during the surveying process.

The common technique of error adjustment are:

i. Least squares method

ii. Adjustment by observations

Error adjustment

Least square methods

- It is an adjustment where the sum of the weighted squares of the residuals is at a minimum.
- It is used to find the MPV of a quantity, which has been measured for several times.

Least square methods

Proof:

From the equation of probability curve for normal distribution, we have

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{v^2}{2\sigma^2}}$$

Replacing, $\frac{1}{2\sigma^2}$ by h^2 in the above equation

$$y = \frac{he^{-h^2v^2}}{\sqrt{\pi}}$$

Least square methods

Proof:

where h is the index of precision and y the probability of occurrence of an error

Let $\frac{1}{\sqrt{\pi}} = C$, then

$$y = Che^{-h^2v^2}$$

Differentiating partially the above equation with respect to h and putting it equal to zero for maximum y gives

$$\frac{\partial y}{\partial h} = 1 \cdot e^{-h^2v^2 + h\left(-2hv^2e^{-h^2v^2}\right)}$$

$$\Rightarrow \quad e^{-h^2v^2}\left(1-2h^2v^2\right) = 0$$

$$v^2 = \frac{1}{2h^2}$$

Considering errors,

$$v_1^2 + v_2^2 + \ldots + v_n^2 = \frac{1}{2h_1^2} + \frac{1}{2h_2^2} + \ldots + \frac{1}{2h_n^2}$$

Then,

$$\sum v^2 = \sum \frac{1}{2h^2}$$

As h increases, $1/2h^2$ decreases, thus maximum accuracy is achieved when

$$v^2 = 1/\left(2h^2\right) \text{ is minimum}$$

According to the principle of least squares, the MPV of a quantity x is the one for which the sum of the weighted errors squared or weighted residuals squared is minimum.

$$\sum wv^2 = \text{minimum.}$$

where, w is the weight of the observation, v is the residual error $(x - \mu_x)$, and μ_x is the mean of observations of quantity x.

Source: S.K Duggal, Vol-II

References

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Survey Computation and Adjustment

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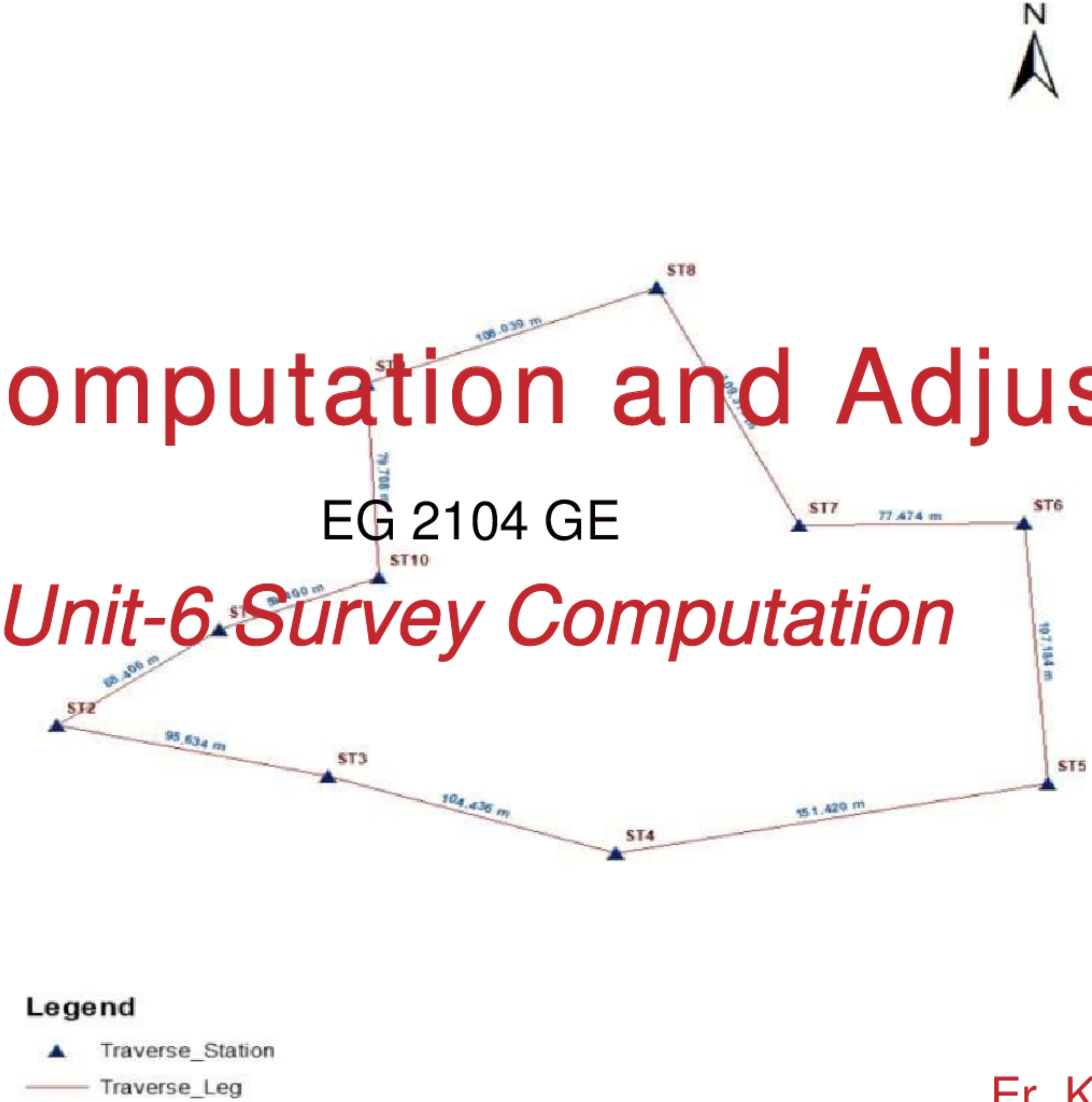
Unit-6 Survey Computation



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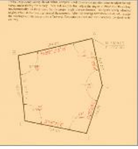


Traverse Computation

Traversing

- It is that type of survey in which a number of connected survey lines form the framework and the directions and lengths of the survey lines are measured with the help of an angle measuring instrument and tape or chain respectively.
- A traverse consists of a series of connected straight lines of known length, each joining two points on the ground and related to each other by known angles between the lines.
- It is one of the method for establishing horizontal control point.
- Points defining the ends of traverse lines are called traverse stations or traverse points.

Traversing

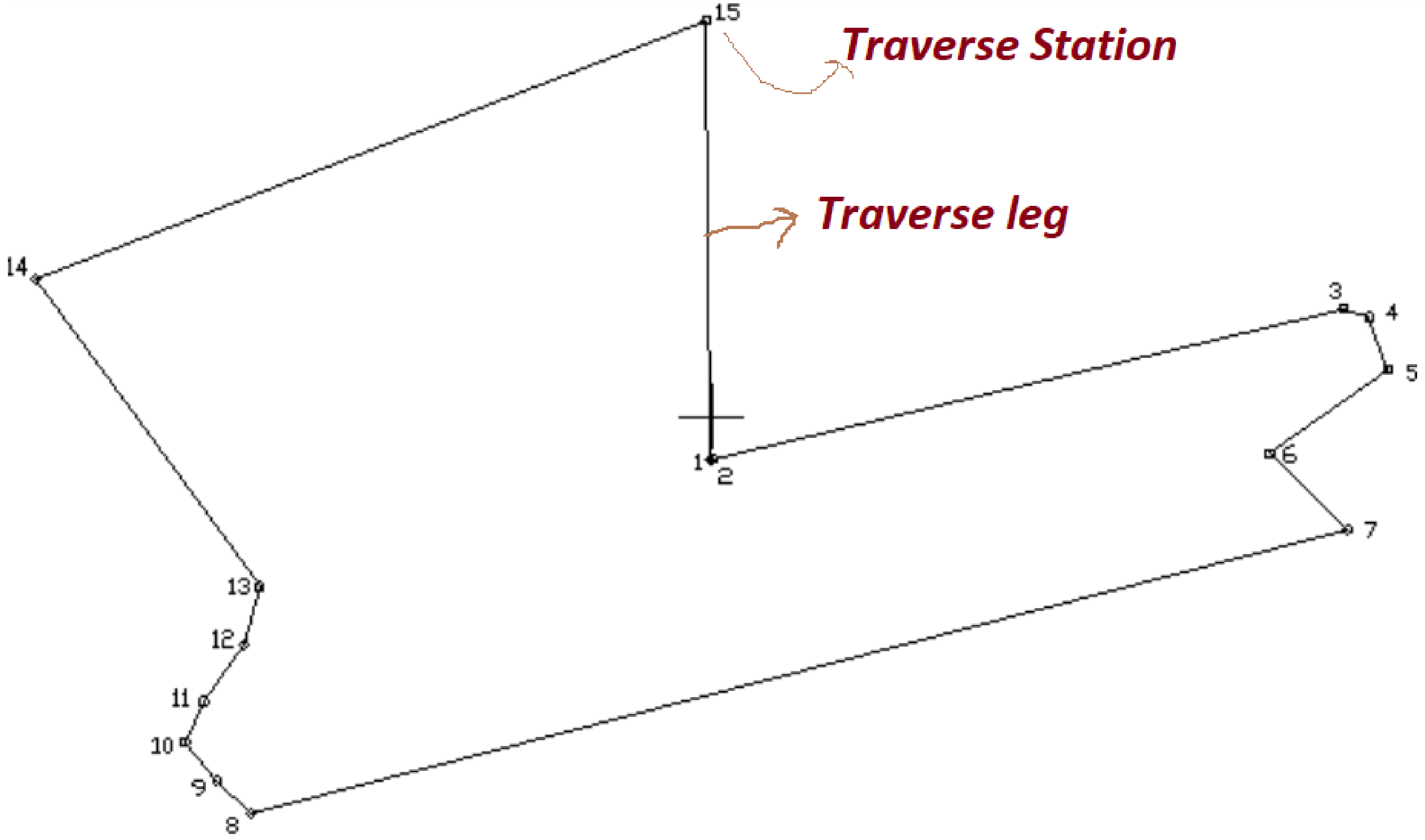


Fig: showing a close traverse

Meridian

- Meridian is a fixed direction from which, the bearings of survey lines are measured.

There are three types of meridians.

1) True meridian

2) Magnetic meridian

3) Arbitrary meridian

True meridian

- It is a line of intersection of earth's surface formed by a plane passing through north and south poles and the given place.

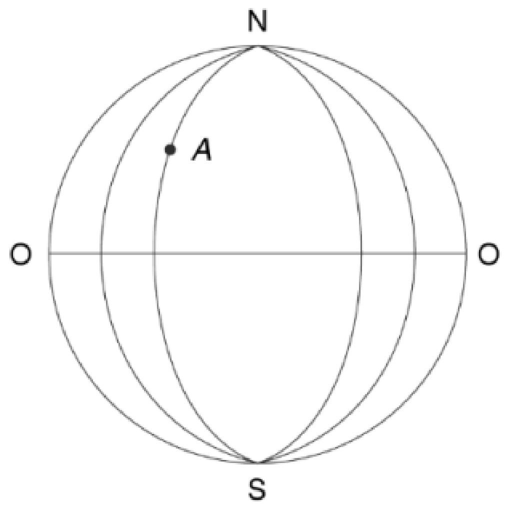


Fig: showing true meridian; source: SK Duggal Vol-I

Traverse Computation

Meridian

Magnetic meridian

- It is the direction indicted by a freely suspended magnetic needle.

Arbitary meridian

- It is any convenient direction assumed as meridian for measuring bearings of survey lines.

Traverse Computation

Bearing

- It is a horizontal angle made by the survey line with reference to the meridian.

Based on the meridian the bearings are three types.

1) True bearing
2) Magnetic bearing
3) Arbitrary bearing

True bearing

- It is the angle made by a survey line with reference to true meridian (true or geographical north) in clockwise direction.
- Since, true meridian does not changes with time; true bearing remains constant.

Traverse Computation

Bearing

Magnetic bearing

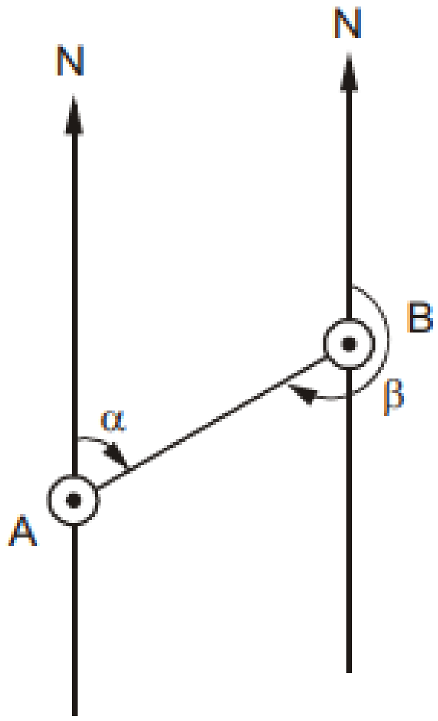
- The angle made by a survey line with reference to magnetic meridian is called magnetic bearing.
- Since, magnetic meridian changes with time; magnetic bearing also changes.

Arbitrary bearing:

- The angle made by a survey line with reference to arbitrary meridian is called arbitrary bearing.

Traverse Computation

Bearing



$\angle NAB = \alpha$ = Fore Bearing
$\angle NBA = \beta$ = Back Bearing
$\therefore$ Fore Bearing – Back Bearing = 180°

Fig: fore bearing and back bearing

- For line BA: forebearing = β; back bearing =α
- For line AB: forebearing = α; back bearing = β

Traverse Computation

Bearing

Fore Bearing (FB)

- It is the bearing of a line in the direction of progress of the survey.
- In the below figure, if the bearing of line AB is measured from A towards B, it is known as forward bearing or fore bearing.

Back Bearing (BB)

- It is the bearing of a line in the opposite direction of progress of the survey.
- In the below figure, if the bearing of same line AB is measured from B towards A, it is known as backward bearing or back bearing.

Traverse Computation

Designation of Bearing

There are commonly two systems used to express bearings. They are:

i. Whole circle bearing system (WCBS)

ii. Quadrantal bearing system (QBS)

Whole circle bearing system

- In this system, bearing of a line is measured from north point of the reference meridian towards the line right around the circle in clockwise direction.
- Here, $\emptyset_1$, $\emptyset_2$, are whole circle bearing for lines OA and OB.

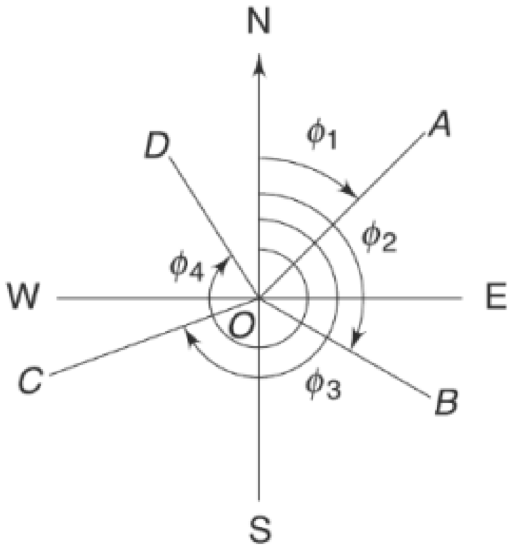


Fig: showing whole circle bearing;
source: SK Duggal Vol-I

Traverse Computation

Relation between fore bearing and back bearing

1. In WBC system

- If the fore bearing of a line is known then,

$$\text{Back bearing} = \text{Fore bearing} \pm 180^\circ$$

- *NOTE: (+) sign is used if fore bearing is less than 180^0 and (–) sign is used if it is more than 180^0.*

Calculation of included angles from bearings [Traverse in anticlockwise]

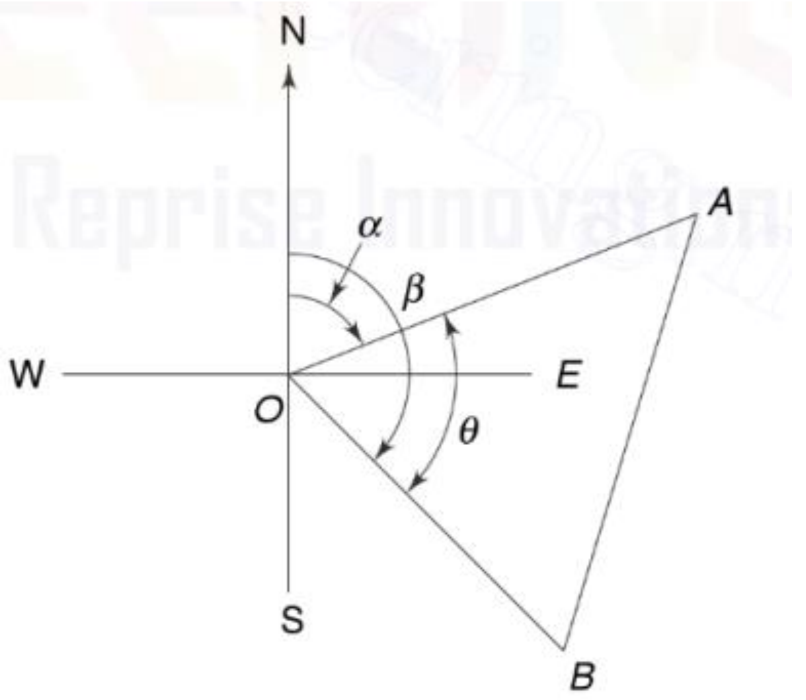
In case the traversing is done in anticlockwise direction, say *OBAO*, then *OB* is the forward line and *AO* is the previous line.

The included angle $\angle AOB = \theta$

= F.B. of the forward line – B.B. of the previous line

$= \beta - \alpha$

= a positive value (interior included angle)



- NOTE: When traversing is done is clockwise direction included angle is exterior and interior for anticlockwise direction.

Calculation of included angles from bearings [Traverse in clockwise]

In a clockwise close traverse *OABO*, *OA* is the forward line (next line) and *BO* is the previous line at station *O*.

Let W.C.B. of the line $OA = \alpha$

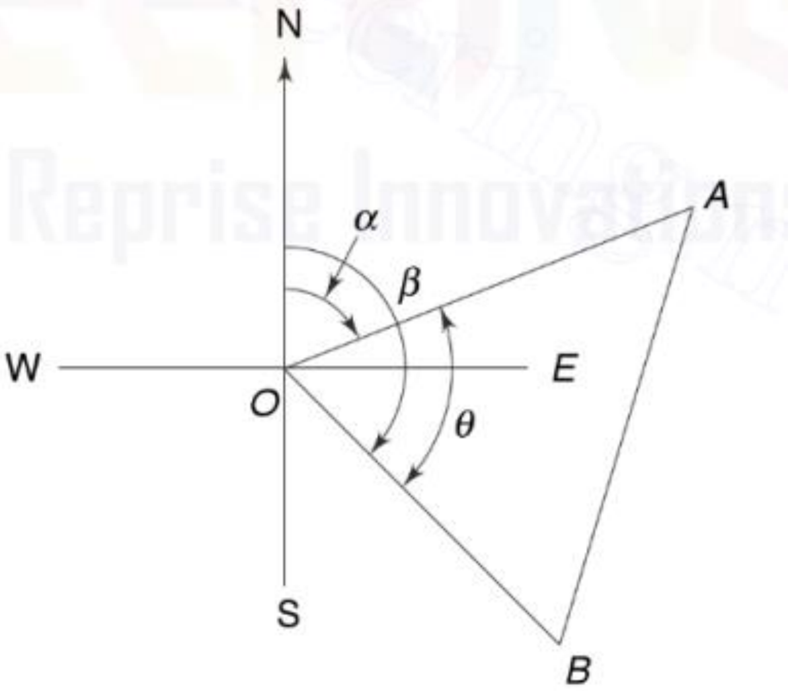
and W.C.B. of the line $OB = \beta$

The included angle $\angle AOB = \theta$

= F.B. of the forward line – B.B. of the previous line

$= \alpha - \beta$

= a negative value



Note *In the process of calculating the included angle, if the value is a negative one (as above), add 360° to get the actual included angle which will be the exterior included angle.*

Bearing

Q.N 1

Determine the value of included angles in a closed compass traverse *ABCDA* (Fig. 3.12) conducted in clockwise direction, given the following fore bearings of the respective lines.

Line	F.B.
AB	40°
BC	70°
CD	210°
DA	280°

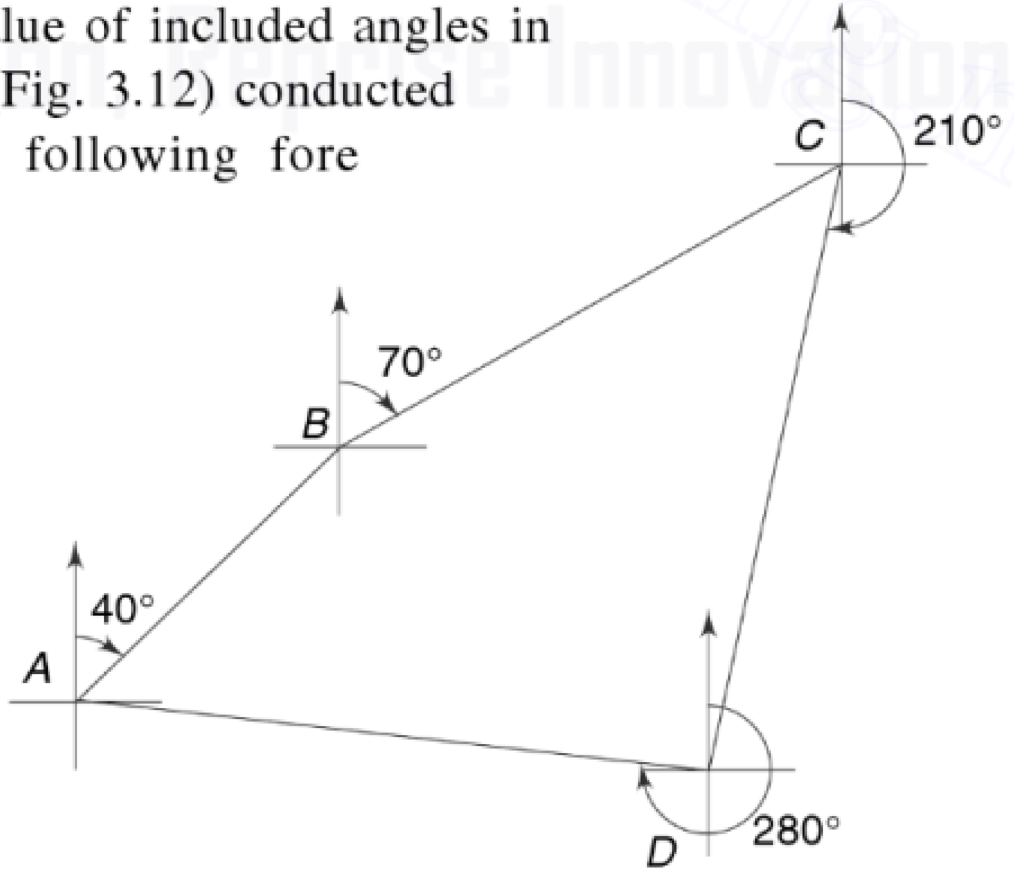


Fig: fore bearing and back bearing

Q.N 1: Ans

Solution As the traversing is done in clockwise direction, the included angles will be the exterior angles.

Included angle = F.B. of next line – B.B. of the previous line.

$\angle A$ = F.B. of AB – B.B. of DA

$= 40° - (280° - 180°) + 360° = 300°$

$\angle B$ = F.B. of BC – B.B. of AB

$= 70° - (40° + 180°) + 360° = 210°$

$\angle C$ = F.B. of CD – B.B. of BC

$= 210° - (180° + 70°) + 360° = 320°$

$\angle D$ = F.B. of DA – B.B. of CD

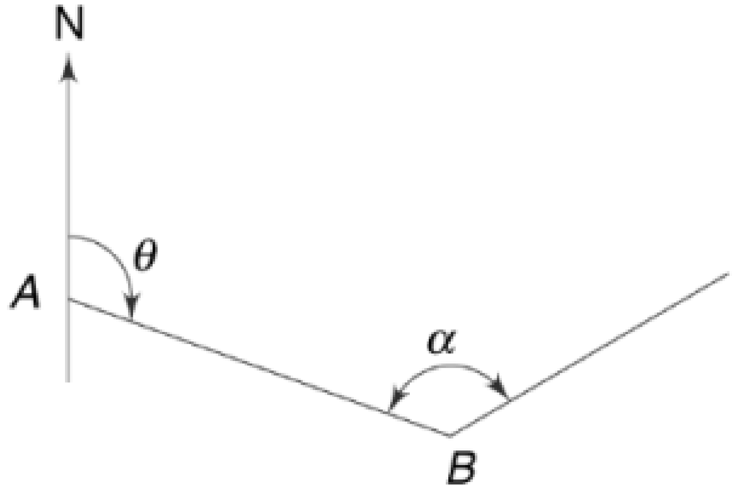
$= 280° - (210° - 180°) = 250°$

Traverse Computation

Calculation of bearings from included angles

Knowing the bearing of a line and the various included angles of a traverse, the bearings of other lines may be calculated as below:

Let the observed fore bearing of a line AB be θ and the included angle with the next adjacent line be α (Fig. 3.14). The fore bearing of the next line is equal to the F.B. of the previous line plus included angle.



Bearing from included angles

- FB of next line = FB of previous line + included angle

Calculation of bearings from included angles

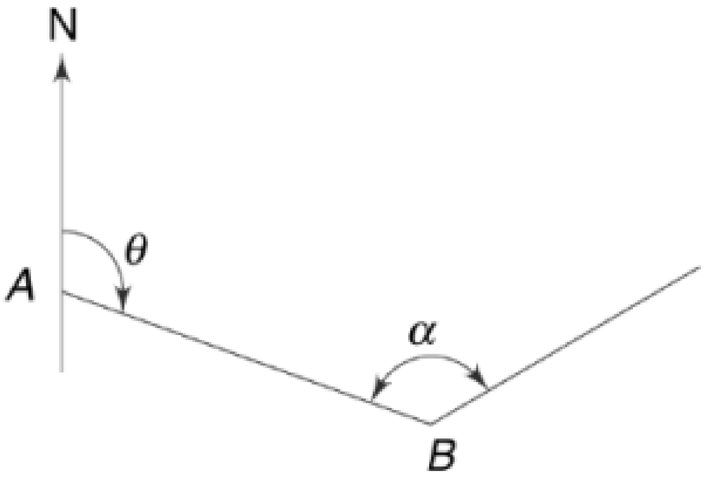
- FB of next line = FB of previous line + included angle

NOTE:

i. If sum is more than 180° , deduct 180°

ii. If sum is more than 540° , deduct 540°

iii. If sum is less than 180° , add 180°



Bearing from included angles

NOTE: This formula is only for case of closed traverse not for tri

And other cases.

Note

(i) *In a closed traverse running in anticlockwise direction, the observed included angles are interior angles.*

(ii) *In a closed traverse running in clockwise direction, the observed included angles are exterior angles.*

(iii) *Included angles are measured clockwise from the preceding line to the forward line.*

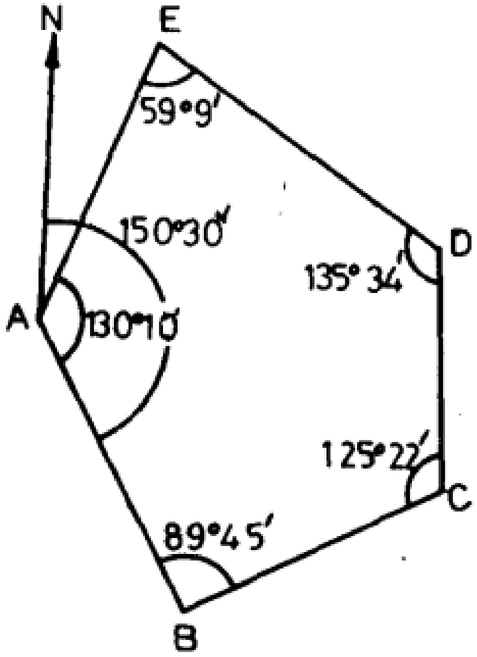
Bearing Calculation: Numerical

In a closed traverse ABCDE, the bearings of the line AB was measured as $150^{0}30'$. The included angles were measured as under: $\angle A = 130^{0}10'$, $\angle B = 89^{0}45'$, $\angle C = 125^{0}22'$, $\angle D = 135^{0}34'$, $\angle E = 59^{0}9'$.

Calculate the bearings of all other lines.

Solution

See following figure.



Bearing Calculation: Numerical

Ans: Since included angles given are interior so traverse runs in anti-clockwise direction.

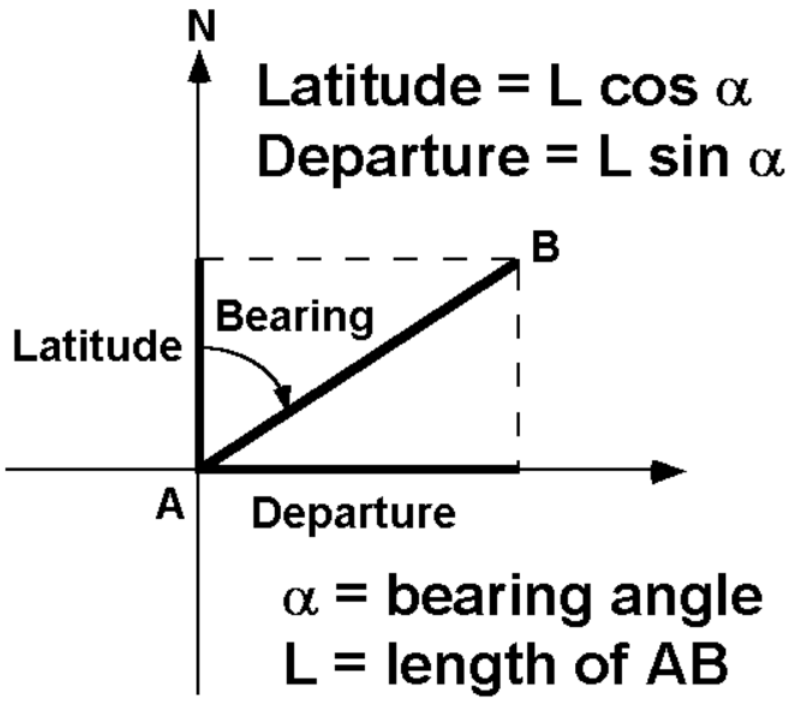
Bearing of BC = Bearing of A + angle ABC = $(150^{0}30' + 180^{0}) + 89^{0}45' = 420^{0}15' = 60^{0}15'$

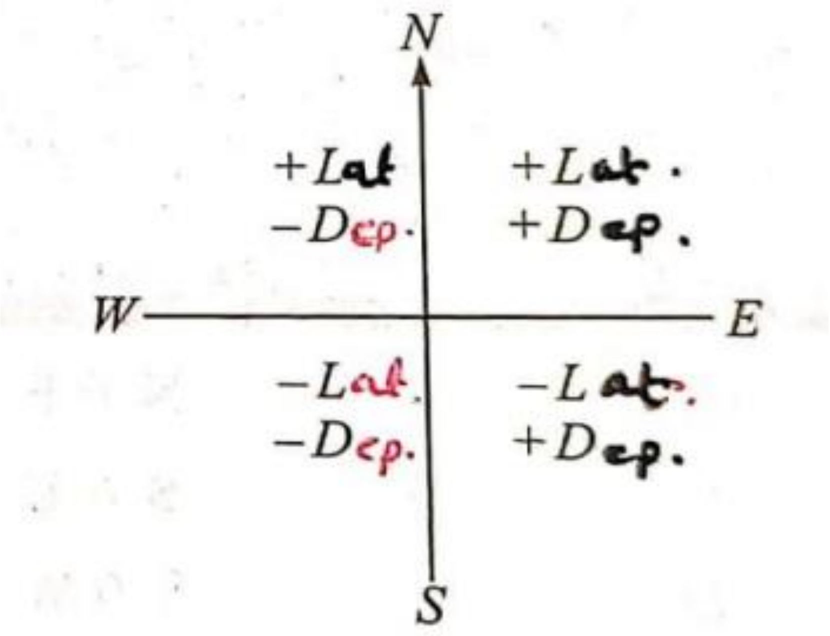
Bearing of CD = Bearing of CB + angle BCD = $(60^{0}15' + 180^{0}) + 125^{0}22' = 365^{0}37' = 5^{0}37'$

Bearing of DE = Bearing of DC + angle CDE = $(5^{0}37' + 180^{0}) + 135^{0}34' = 321^{0}11'$

Bearing of EA = Bearing of ED + angle DEA = $(321^{0}11' - 180^{0}) + 59^{0}9' = 200^{0}20'$

Latitude and Departure





Source: http://gis.washington.edu/phurvitz/courses/esrm304/lectures/2009/Hurvitz/procedures/latitudes_and_departures.html#:~:text=The%20departure%20of%20a%20line,(also%20known%20as%20easting).

Fig: showing concept of latitude and departure

Source: http://www.ce.memphis.edu/1112/notes/project_3/traverse/Surveying_traverse.pdf

Traverse Computation: Latitude and Departure

Latitude

- The latitude of a line is its projection on the north-south meridian and is equal to the length of the line times the cosine of its bearing.
- The latitude is the y component of the line (also known as northing).

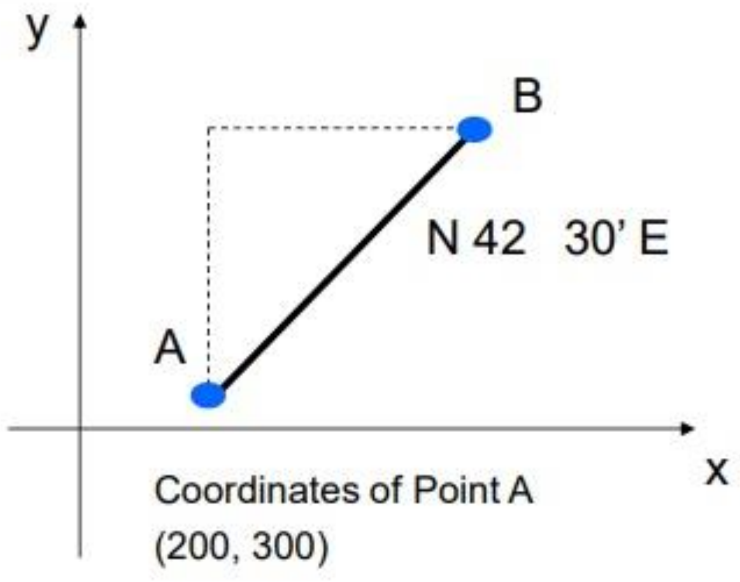
Departure

- The departure of a line is its projection on the east-west meridian and is equal to the length of the line times the sine of its bearing.
- It is the x component of the line (also known as easting).

Coordinate Calculation from latitude and departure

1.

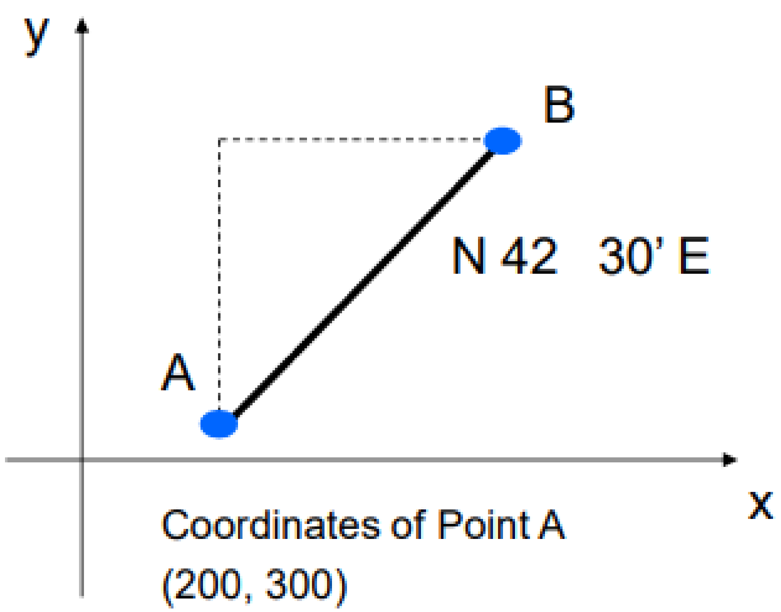
In this example, the length of AB is 300 ft. and bearing is shown in the figure below. Determine the coordinates of point B



Coordinate Calculation from latitude and departure

1.

In this example, the length of AB is 300 ft. and bearing is shown in the figure below. Determine the coordinates of point B



$$\text{Latitude}_{AB} = 300 \text{ ft. } \cos(42\ \ 30') = 221.183 \text{ ft.}$$

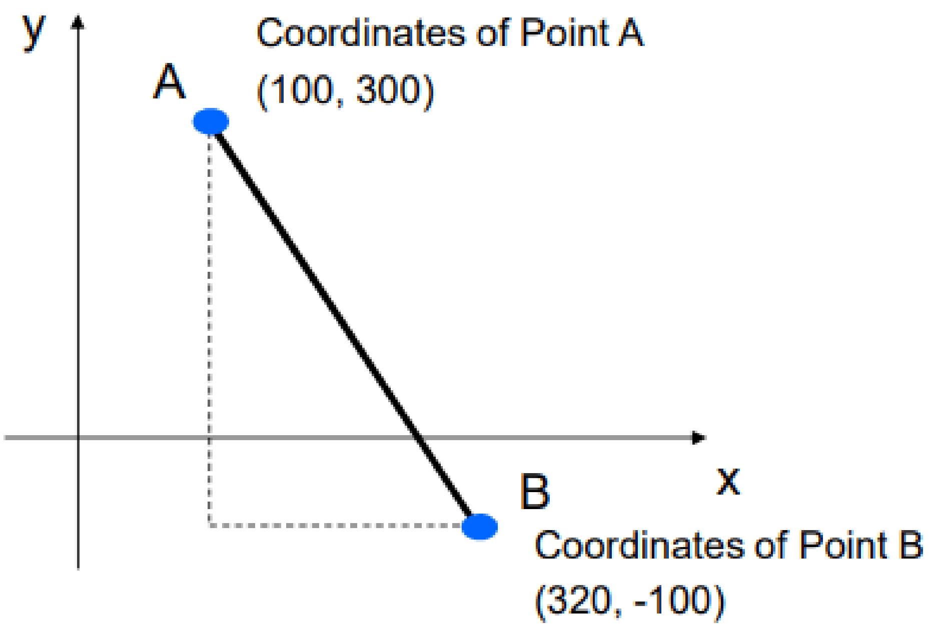
$$\text{Departure}_{AB} = 300 \text{ ft. } \sin(42\ \ 30') = 202.677 \text{ ft.}$$

$$x_B = 200 + 202.667 = 402.667 \text{ ft.}$$

$$y_B = 300 + 221.183 = 521.183 \text{ ft.}$$

2.

In this example, it is assumed that the coordinates of points A and B are know and we want to calculate the latitude and departure for line AB



NOTE:

i. Latitude and departure can be positive or negative depending on value of bearing and according line will lie on respective quadrant.

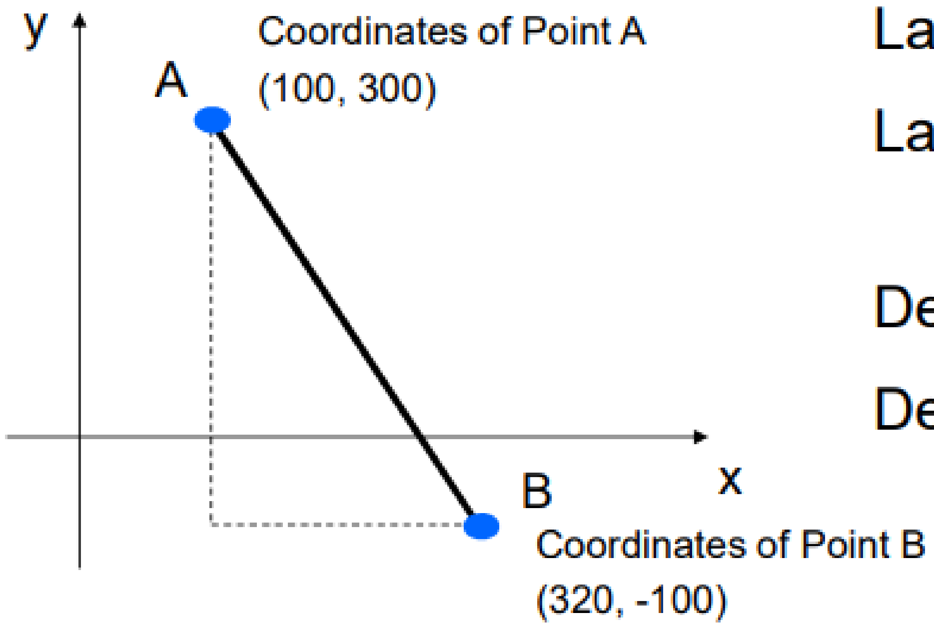
ii. We can estimate bearing value by (+/- sign) of latitude and departure.

iii. Don't assume quadrant according to sign of coordinates.

Coordinate Calculation from latitude and departure

2.

In this example, it is assumed that the coordinates of points A and B are know and we want to calculate the latitude and departure for line AB



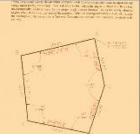
$$\text{Latitude}_{AB} = y_B - y_A$$

$$\text{Latitude}_{AB} = -400 \text{ ft.}$$

$$\text{Departure}_{AB} = x_B - x_A$$

$$\text{Departure}_{AB} = 220 \text{ ft.}$$

Coordinate Calculation from latitude and departure



3. If the coordinate of station A is (436948.1347, 3050431.68) and bearing of line AB and length are 272º 56’ 44’’ and 47.14 respectively. Calculate the coordinate of station B. [Ans: 436901.0570, 3050434.10238322]

4. Two points have the following coordinates:

Point	Easting (m)	Northing (m)
A	32255.751	49076.286
B	12231.864	36939.667

Calculate the bearing A to B and plan distance AB.

Coordinate Calculation from latitude and departure

Ans:

Now find the ΔE and ΔN:

$\Delta E = E_B - E_A = 12231.864 - 32255.751 = -20023.887$ m

$\Delta N = N_B - N_A = 36939.667 - 49076.286 = -12136.619$ m

Note: To find the bearing of A to B we take B coordinates minus A

Using Pythagoras's Theorem to solve d_{AB}:

$$d_{AB}^2 = \Delta E^2 + \Delta N^2 = -20023.887^2 + -12136.619^2$$

$$d_{AB} = \sqrt{-20023.887^2 + -12136.619^2}$$

$\boxed{d_{AB} = 23\,414.815 \text{ m}}$

Coordinate Calculation from latitude and departure

Ans:

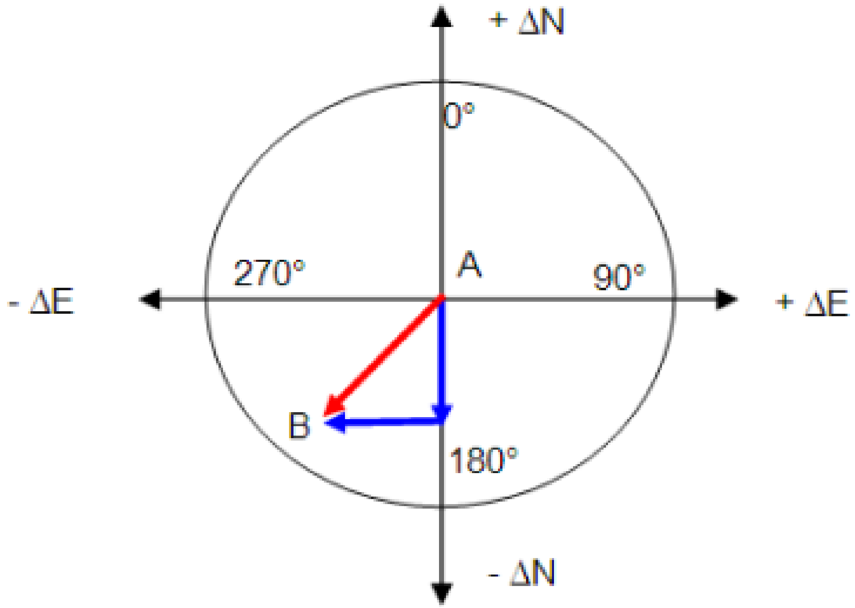
Now for the Bearing β: $\beta = \tan^{-1}\left(\frac{\Delta E}{\Delta N}\right)$

The signs of ΔE and ΔN will determine the quadrant of the bearing. The diagram shows that our bearing will be in the 3rd quadrant, between 180° and 270°

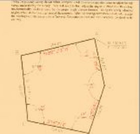
$$\beta = \tan^{-1}\left(\frac{-20023.887}{-12136.619}\right) = 58.77965125°$$

For WCB, we have to add 180°

58.77965125° + 180° = 238.77965125°



Coordinate Calculation from latitude and departure



5.

Given $N_A = 106.132$ $\quad$ $E_A = -11.063$

$N_B = -110.296$ $\quad$ $E_B = 103.000$

Calculate, (a) L_{AB} and (b) θ_{AB}

Ans: [244.646, 152 12'35"]

Consecutive Coordinates Calculation

Consecutive Coordinates/ Dependent coordinate

- The latitude & departure of a point calculated with reference to the preceding point for what are called consecutive coordinates.
- Consecutive coordinates may be positive or negative, depending upon the quadrant in which they lie.

Traverse Computation: Closing Error

Angular error

Angular error of closure is defined as the difference between the sum of the measured angles and the theoretical sum of the angles of a closed traverse.

- The sum of angles of a regular polygon is given by:
  Sum= $(2n \pm 4) \times 90°$
  - '+'ve if the traverse is done in clockwise direction (exterior angles)
  - '-'ve if the traverse is done in anticlockwise direction (interior angles)
  - Where, n = total number of stations

- The angular error of a closure is distributed equally among all angles, if measured with equal precision. .

Traverse Computation: Closing Error

Distribution of angular error/ Horizontal angle adjustment

Q.1 .

A interior angle in a closed traverse is as follows

$A= 84^{0}58'$, $B=157^{0}38'$, $C=24^{0}37'$

$D= 153^{0}\ 14'$, $E=103^{0}\ 54'$, $F= 139'\ 06'$ $G= 236^{0}\ 49'$

Compute the error of closure and adjust the interior angle.

Traverse Computation: Closing Error

Distribution of angular error / Horizontal angle adjustment

Q.1 .

Solution:

Station	Observed Interior angle	Correction	Adjusted interior Angle
A	84^0 58'	-0^0 2'	$84^0$56'
B	$157^0$38'	$-0\ ^0$2'	$157^0$36'
C	24^0 37'	$-0\ ^0$2'	24 35'
D	153^0 14'	$-0\ ^0$2'	153 12'
D	103 ^{0}54'	$-0\ ^0$2'	103 52'
F	139 ^{0}06'	$-0\ ^0$2'	139 04'
G	236^0 47'	$-0\ ^0$2'	236 45'
Sum	900^0 14' 00"		$900^0$00'00'
$(n-2)180^0$	900^0 00 ' 00"		
Error of closure	0^0 14'00"		

Traverse Computation: Closing Error

Closing error/ Linear error of closure

- The distance by which a traverse fails to close is known as closing error or error of closure.
- The finishing point may not coincide with the starting point of a closed traverse due to the errors in the field measurements of angles & lengths.
- In fig. the traverse $ABCDA_1$ fails to close by a distance AA1 which is the closing error of this traverse.

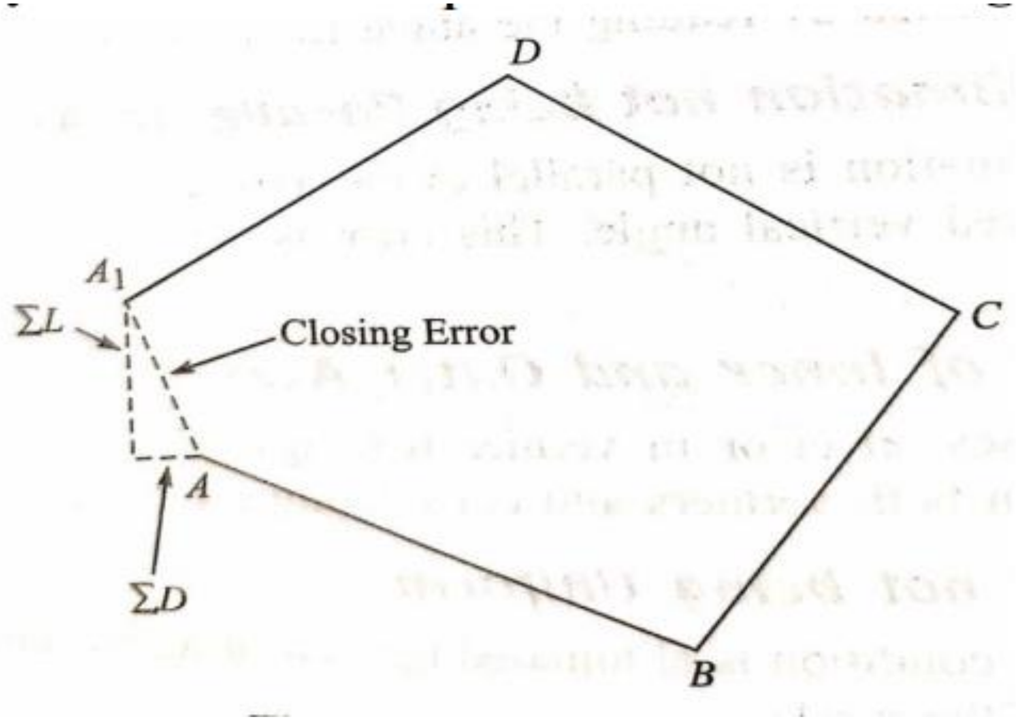


Fig: showing a traverse being short of being closed by a distance AA_1

Traverse Computation: Closing Error

Closing error / Linear error of closure

In a closed traverse if the work is correct, the algebraic sum of the latitudes (L) should be equal to zero, i.e., $\Sigma L = 0$, and the algebraic sum of the departures (D) should also be equal to zero, i.e., $\Sigma D = 0$.

The two components OO_1 and O_1O_2 (Fig. 5.20) of the closing error OO_2 may be obtained by calculating ΣL and ΣD and then the closing error can be computed:

$$\text{Closing error, } e = \sqrt{(OO_1)^2 + (O_1O_2)^2}$$

$$= \sqrt{\Sigma L^2 + \Sigma D^2}$$

The direction of the closing error is determined from $\tan\ \theta = \Sigma D/\Sigma L$.

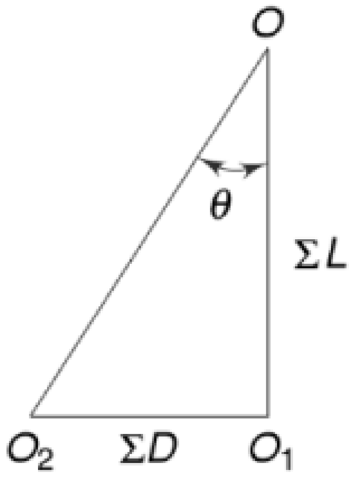


Fig. *Closing error*

Traverse Computation: Closing Error

Balancing the traverse

- It is the correction applied to the latitude and departure of a traverse.

In a closed traverse the following conditions must be satisfied:

$$\Sigma \text{ Departure} = \Sigma D = 0$$

$$\Sigma \text{ Latitude} = \Sigma L = 0 \qquad \ldots(4.5)$$

- It is the operation of adjusting the closing error in a closed traverse by applying corrections to departures and latitudes to satisfy the conditions given by the Eq 4.5

Methods of balancing a traverse:

i. Bowditch rule

- .

ii. Transit rule

Traverse Computation: Balancing the traverse

Methods of balancing a traverse

Bowditch rule

- It is used when the angular and linear measurements are equally precise.
- In this rule, the total error in latitude and that in departure is distributed in proportion to the lengths of the sides.

$$\text{Correction to latitude (or departure of any side)} = \text{total error in latitude (or departure)} \times \frac{\text{length of that side}}{\text{perimeter of traverse}}$$

Transit rule

- This rule is used to balance the traverse when the angular measurement are more precise than the linear measurements.

Correction in latitude $= \text{total error in latitude} \times \left(\frac{\text{latitude of that side}}{\text{arithmetical sum of all latitudes}}\right)$

Correction in departure $= \text{total error in departure} \times \frac{\text{departure of that side}}{\text{arithmetical sum of all departures}}$

Traverse Computation: Questions to THInk

1. When should we use Bowditch rule and transit rule ?

Traverse Computation: Assignment

1.

The following observations were made for a closed traverse round an obstacle. Due to obstructions, lengths of lines *DE* and *EA* could not be measured. Find out the missing lengths.

Line	Length (m)	Bearing
AB	500	98°30′
BC	620	30°20′
CD	468	298°30′
DE	?	230°00′
EA	?	150°10′

Ans: 695.24, 273.97

Traverse Computation: Numericals

2. A clockwise traverse ABCDEA was surveyed with the following results:

AB = 101.01 m, <BAE = 128°10'20" < DCB = 84°18'10“, BC = 140.24 m, CD = 99.27 m, < CBA = 102°04'30" <EDC = 121°30'30"

The angle < AED and the sides DE and EA could not be measured directly. Assuming no error in survey, find the missing lengths and their bearings if AB is due North.

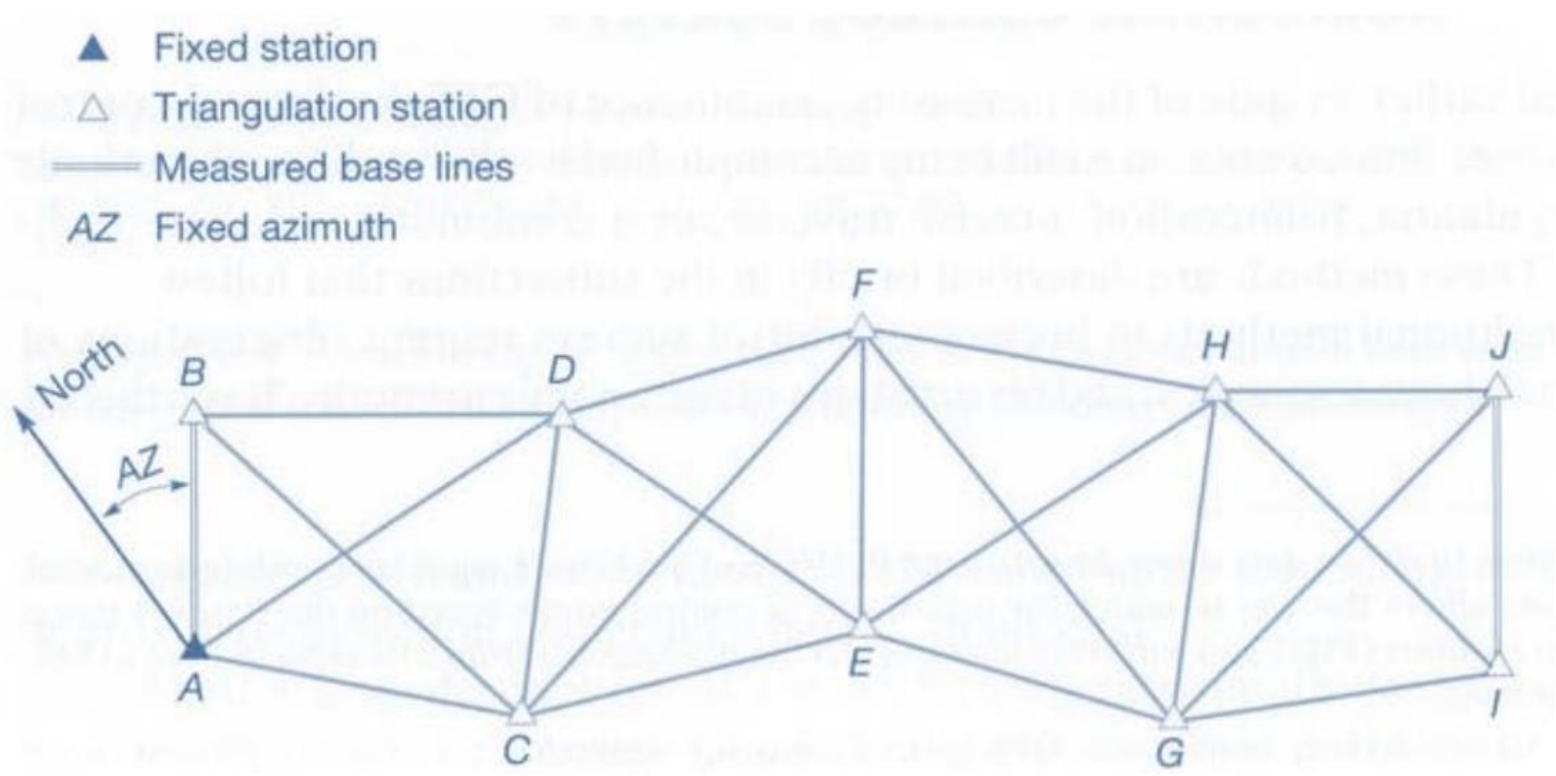
Ans: 120.01m, 232°06'50", 67.99 m, 128°10'20"

6.2. Triangulation and Trilateration Computation

Triangulation

- It is a kind of surveying based on the trigonometric theory that if one side and two angles of a triangle are known, the remaining sides and angle can be computed.
- It is a method used to transfer horizontal control.
- A triangulation system consists of a series of joined or overlapping triangles in which an occasional side called as base line, is measured and remaining sides are calculated from the angles measured at the vertices of the triangles. The vertices of the triangles are known as triangulation stations.

Fig: showing triangulation concept.



Source: Department of Geodetic Engineering Training Center for Applied Geodesy and Photogrammetryy; GE-10

6.2. Triangulation and Trilateration Computation

Triangulation figures or systems

- It is a system consisting of triangulation stations connected by a chain of triangles.
- The most common type of figures used in a triangulation system are triangles, quadrilaterals and polygons.

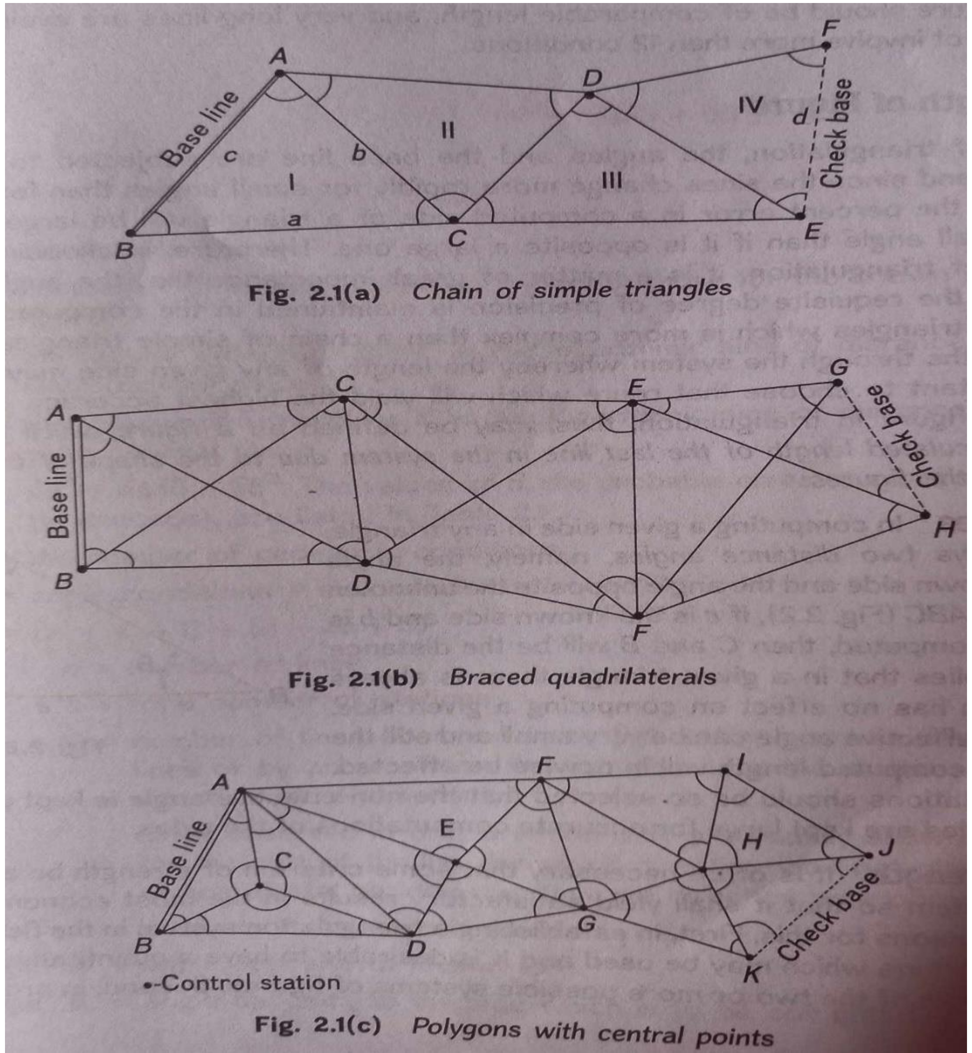


Fig: showing triangulation figure

Source: SK Duggal, Vol-I

6.2. Triangulation Adjustment

Triangulation Adjustment

- It is an error adjustment done in triangulation stations by distributing the observational errors.
- The different methods to adjust the triangulation network are:

i. Least square method

ii. Approximate method / Angle Adjustment (method of adjusting angles)

a. Station adjustment

b. Figure adjustment

- It includes the adjustment of following geometric figures:

a. Adjustment of a triangle

b. Adjustment of two connected triangles

c. Adjustment of chain of triangles

d. Adjustment of geodetic quadrilateral

6.2. Triangulation Adjustment

Triangulation Adjustment

Least Squares Method

- Uses the angle and side condition equations as inputs to the adjustment process.
- Rigorous, complex and requires lengthy computations.

6.2. Triangulation Adjustment

Triangulation Adjustment

Approximate method (method of adjusting angles)

- After completion of field work of measurements of angles, it is necessary to adjust the angles.
- Generally the angles of a triangle and chain of triangles are adjusted under two heads.

(i) Station Adjustment

(ii) Figure Adjustment

- In case of adjustment of quadrilateral, trigonometric/side equation is also calculated along with station and figure adjustment.

Triangulation Adjustment

Station Adjustment

- It consists of determining the MPV_s of two or more angles measured at a station to satisfy the geometric conditions involved.
- The various conditions can be

i. Sum of all the angles should be 360^o (closing of horizon).

ii. Measuring the angles with equal or unequal weights

iii. Measuring different angles at a station individually (directly) and in combination (indirect observation), for example using two angles to calculate third one with horizon not being closed.

- In the first case; if there is discrepancy the correction is distributed equally to all measured angles.
- In second case, the error is distributed inversely as the respective weights for unequal weights.
- In last case, normal equations are formed and solved simulataneously.

Triangulation Adjustment

Station Adjustment

- For a given triangle station adjustment can be done as

➢ Sum of angles about a station (S_m) = 360°

➢ Correction per station = 360° – S_m

➢ Correction per measured angle = (360° – S_m)/n

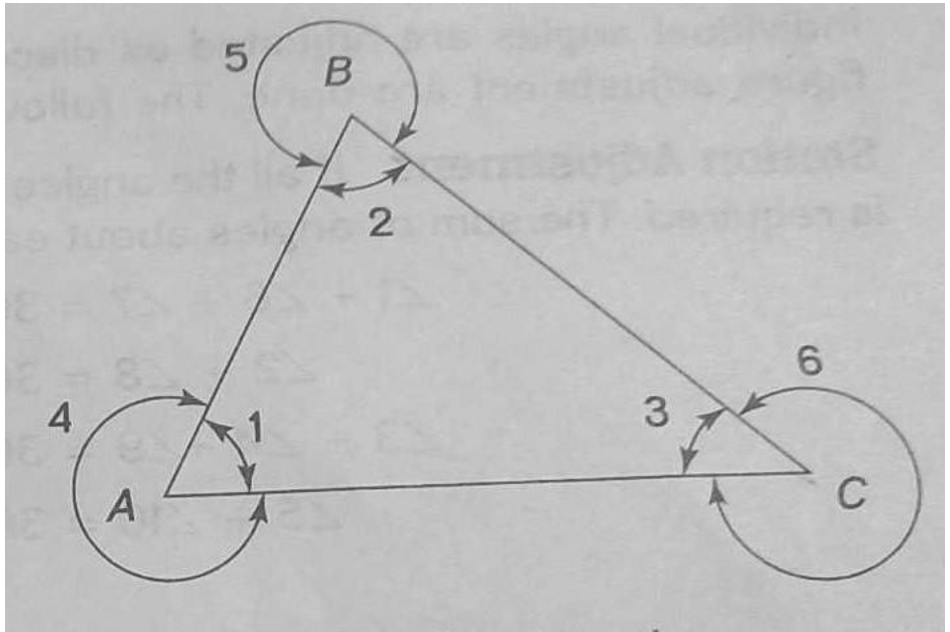


Fig. 3.3 Triangle

6.2. Triangulation Adjustment

***Figure Adjustment*:**

- The determination of most probable values of angles involved in any geometrical figure so as to fulfill the geometrical conditions is called the figure adjustment.
- It is performed after station adjustment.
- It uses adjusted values from previous adjustment.
- All cases of figure adjustment necessarily involve one or more conditional equations. The geometrical figures used in a triangulation system are:

(a) Triangles.

(b) Quadrilaterals.

(c) Polygons with central stations.

6.2. Triangulation Adjustment

Figure Adjustment

For triangles,

- Sum of Interior angles or a triangle = S_{int}
- Correction per interior angle = $(180^o – S_{int})/3$

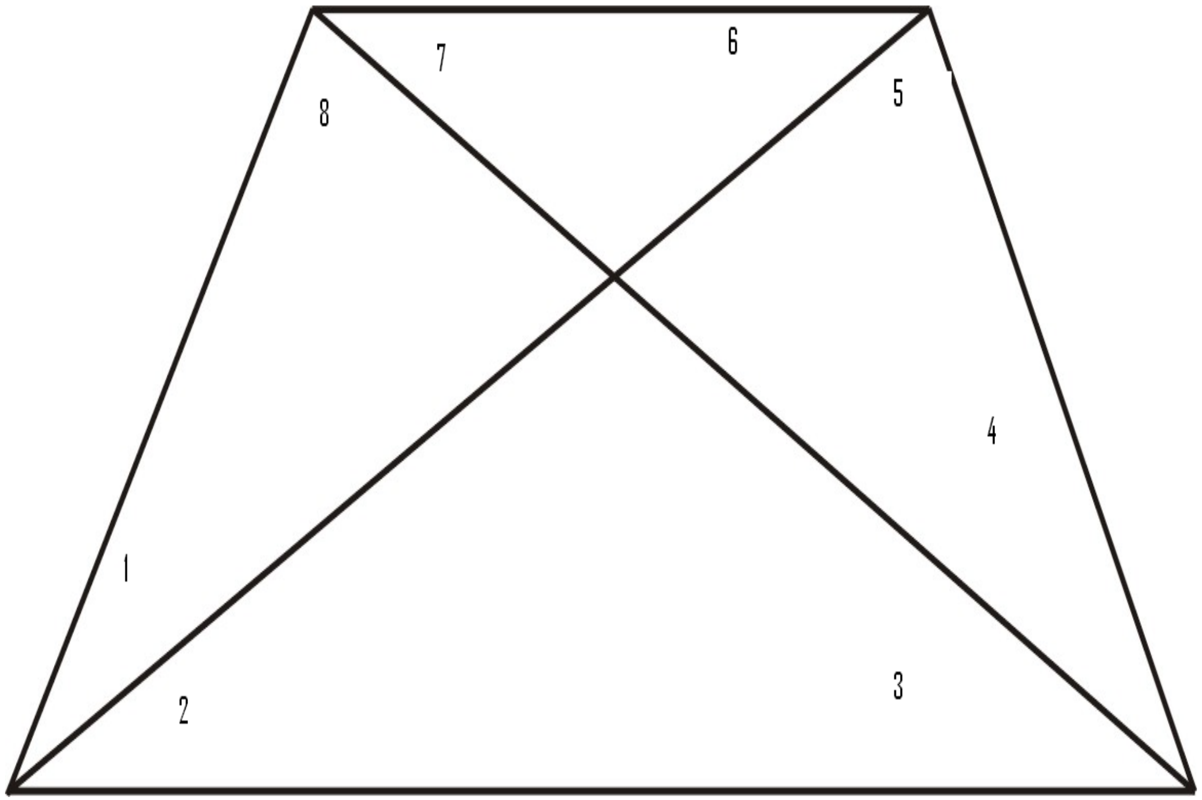
For quadrilaterals,

(a) Sum of all the angles should be equal to 360^o.

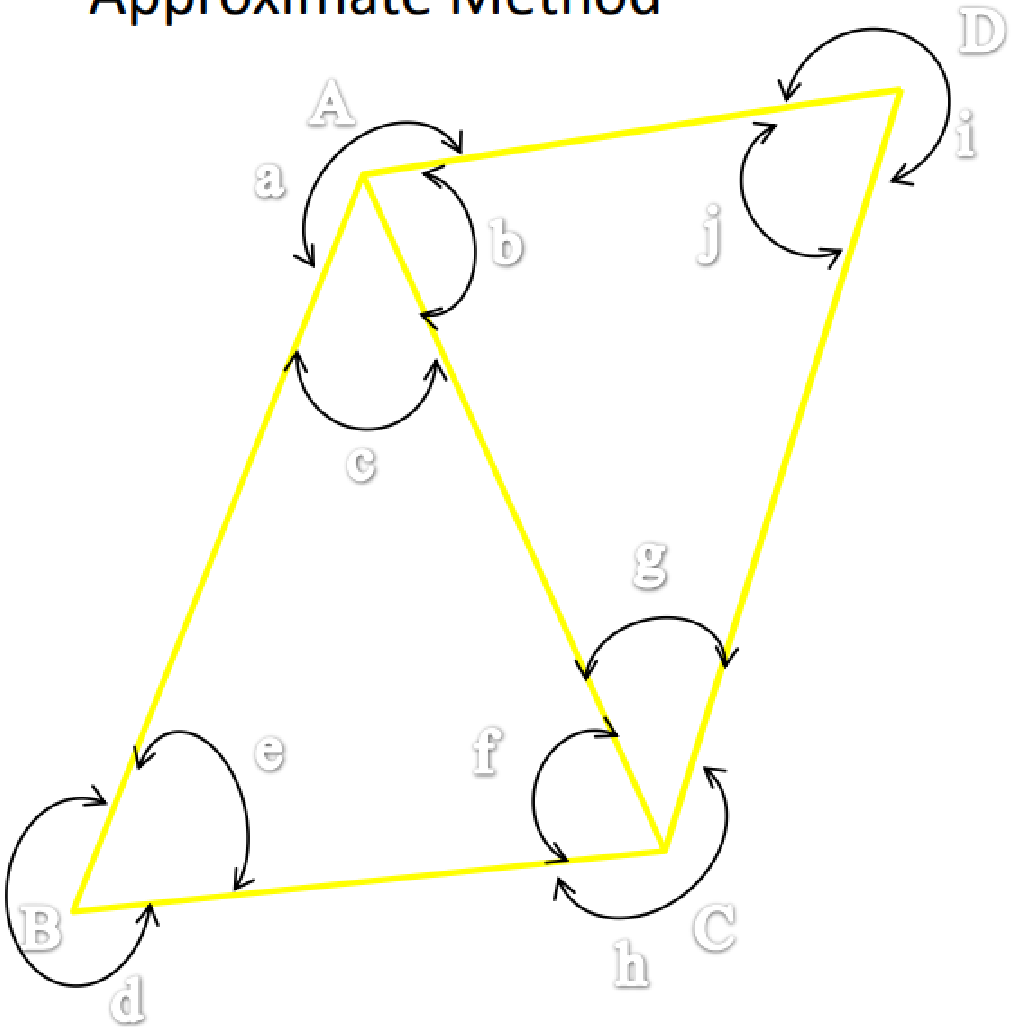
(b) Sum of equal pair of angle should be equal.

$< 2 + < 3 = < 6 + < 7$

$< 1 + < 8 = < 4 + < 5$



1. Adjust the measured angles of the given chain of triangles using the Approximate Method

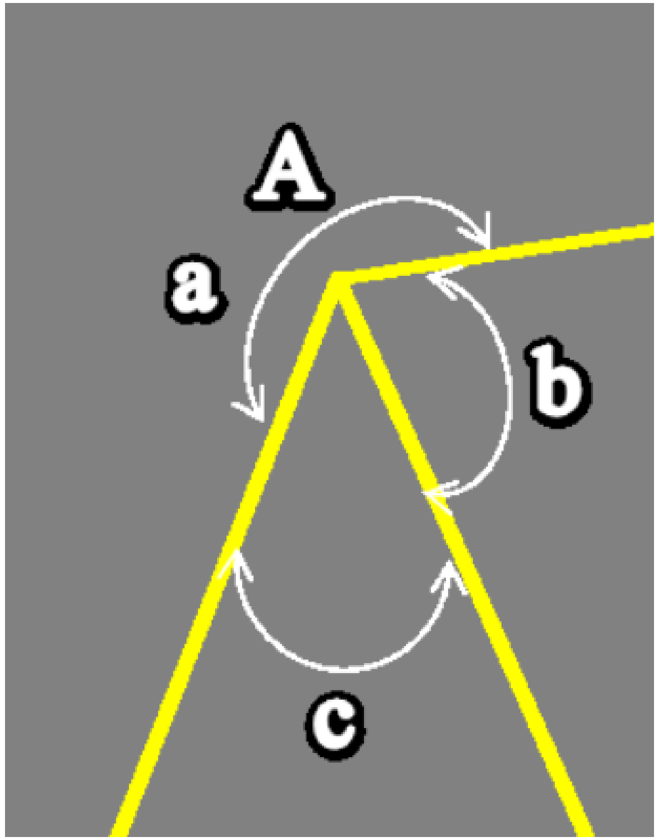


Angle	Observed Value
a	240-21-00
b	60-29-10
c	59-10-05
d	301-34-49
e	58-25-15
f	62-25-10
g	59-25-10
h	238-09-31
i	299-54-54
j	60-05-10

Station Adjustment: $\Sigma_m = 360^0$

- About station A:
  - Σ_m = a + b + c = 360-00-15
  - correction = -15”
  - Correction per angle (cor)= -15”/3 = -5”
  - Adjusted angles:
    - ✓a’= a + cor = 240-20-55
    - ✓b‘= b + cor = 60-29-05
    - ✓c‘ = c + cor = <u>59-10-00</u>

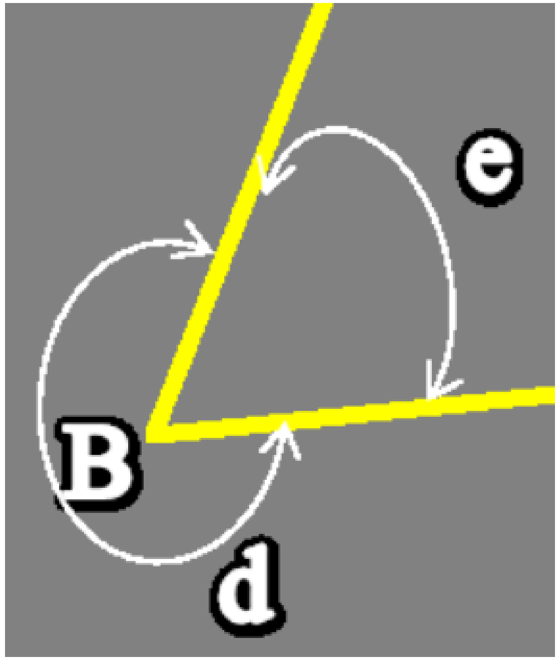
      Sum check: 360-00-00



Station Adjustment: $\Sigma_m = 360^0$

- About station B:
  - ➢ Σ_m = d + e = 360-00-04
  - ➢ correction = -04”
  - ➢ correction per angle (cor)= -04”/2 = -2”
  - ➢ Adjusted angles:
    - ✓ d’= d + cor = 301-34-47
    - ✓ e‘ = e + cor = <u>58-25-13</u>

    Sum check: 360-00-00

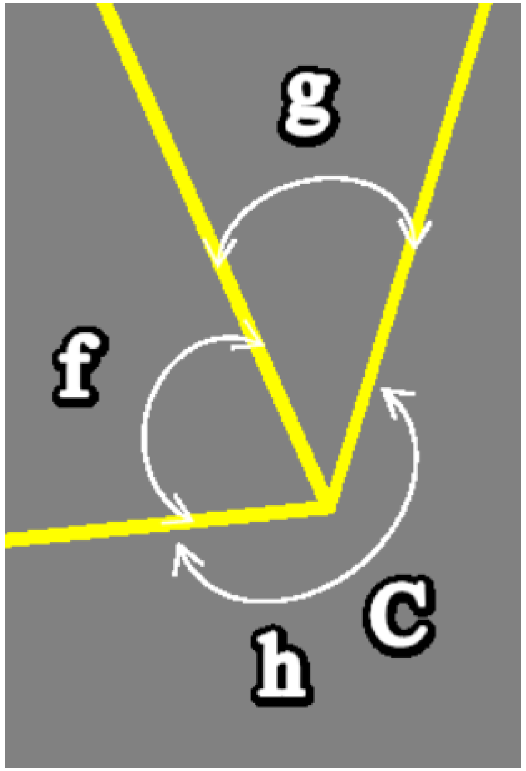


6.2. Triangulation Adjustment: Approximate method

Station Adjustment: $\Sigma_m = 360^0$

- About station C:
  - ➢ Σ_m = f + g + h= 359-59-51
  - ➢ correction = +09”
  - ➢ correction per angle (cor)= +09”/3 = +3”
  - ➢ Adjusted angles:
    - ✓ f’= f + cor = 62-25-13
    - ✓ g‘= g +corr = 59-25-13
    - ✓ h‘ = h + cor = <u>238-09-34</u>

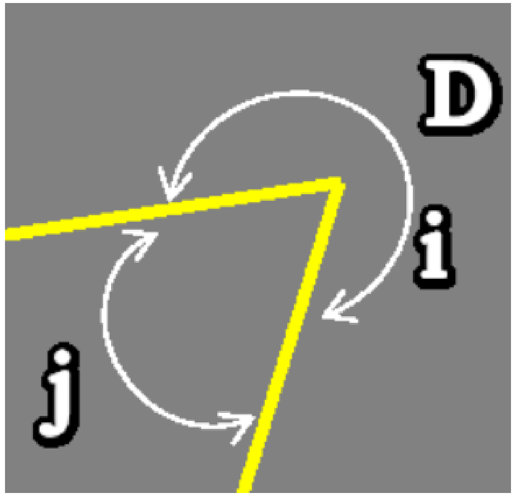
      Sum check: 360-00-00



Station Adjustment: $\Sigma_m = 360^0$

- About station D:
  - ➢ Σ_m = i + j = 360-00-04
  - ➢ correction = -04”
  - ➢ correction per angle (cor)= -04”/2 = -2”
  - ➢ Adjusted angles:
    - ✓ i’= i + cor = 299-54-52
    - ✓ j‘ = j + cor = 60-05-08

      Sum check: 360-00-00



6.2. Triangulation Adjustment: Approximate method

Angle	Observed Value	Adjusted Angle	Adjusted Value (after Station Adjustment)
a	240-21-00	a‘	240-20-55
b	60-29-10	b’	60-29-05
c	59-10-05	c‘	59-10-00
d	301-34-49	d’	301-34-47
e	58-25-15	e’	58-25-13
f	62-25-10	f’	62-25-13
g	59-25-10	g’	59-25-13
h	238-09-31	h’	238-09-34
i	299-54-54	i’	299-54-52
j	60-05-10	j’	60-05-08

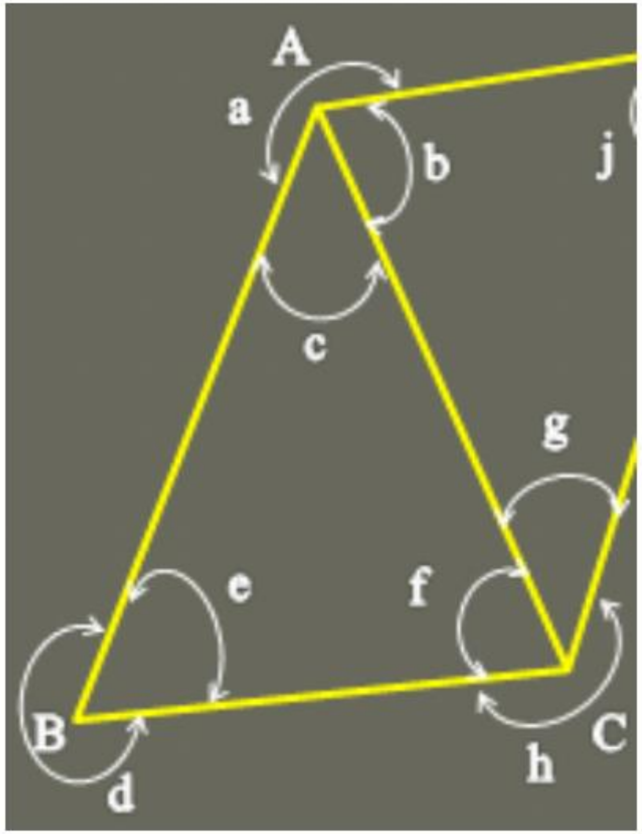
6.2. Triangulation Adjustment: Approximate method

Figure Adjustment: $\Sigma_{int} = 180^0$

- Triangle ABC:
  - ➢ Σ_{int} = c’ + e’ + f’ = 180-00-26
  - ➢ correction = -26”
  - ➢ correction per angle:

    = -26”/3 = -8.67” say -9”
  - ➢ Adjusted angles:
    - ✓ c’’= c’ + (-9”) = 59-09-51
    - ✓ e’’ = e’ + (-9”) = 58-25-04
    - ✓ f’’ = f’ + (-8”) = 62-25-05

      Sum check: 180-00-00

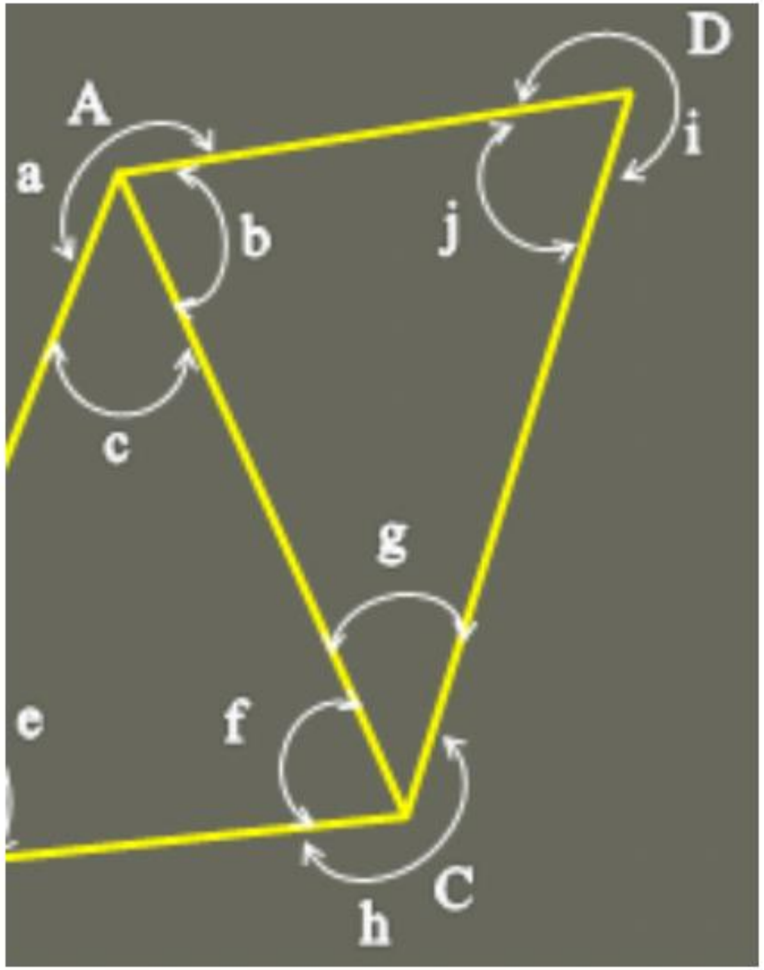


6.2. Triangulation Adjustment: Approximate method

Figure Adjustment: $\Sigma_{int} = 180^0$

- Triangle ACD:
  - Σ_{int} = b' + g' + j' = 179-59-26
  - correction = +34"
  - correction per angle:
    = +34"/3 = +11.33" say +11"
  - Adjusted angles:
    - ✓b''= b' + 11" = 60-29-16
    - ✓g'' = g' + 11" = 59-25-24
    - ✓j'' = j' + 12" = 60-05-20

    Sum check: 180-00-00



6.2. Triangulation Adjustment: Approximate method

Triangle	Angle	Adjusted Value (after Station Adjustment)	Corrections by Figure Adjustment	Adjusted Value (after Figure Adjustment)
ABC	c	59-10-00	-9”	59-09-51
	e	58-25-13	-9”	58-25-04
	f	62-25-13	-8”	62-25-05
ACD	b	60-29-05	+11”	60-29-16
	g	59-25-13	+11”	59-25-24
	j	60-05-08	+12”	62-05-20

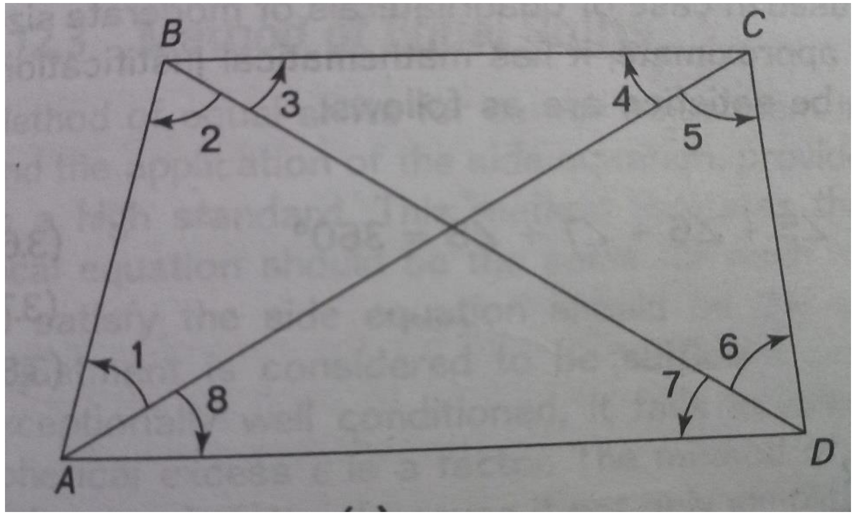
Adjustment of geodetic quadrilateral

- There are three methods for adjusting quadrilateral. They are:

a. Method of least square

b. Approximate method

c. Method of equal shift



NOTE: For geodetic quadrilateral, angles should be adjusted for spherical excess before applying above methods.

6.2. Triangulation Adjustment: Approximate method

Adjustment of Quadrilateral

The adjustment of quadrilateral using approximate method involves:

a. Angle equation

Step 1: Station adjustment (if exterior angles given)

- Sum of each angles at a station is equal to 360^o .

Step 2: Figure adjustment

b. Trigonometric Condition/ Side equation

6.2. Triangulation Adjustment: Approximate method

Adjustment of Braced Quadrilateral

Figure Adjustment

(a) Sum of all the angles should be equal to 360°.

(b) Sum of equal pair of angle should be equal.

$\angle 2 + \angle 3 = \angle 6 + \angle 7$

$\angle 1 + \angle 8 = \angle 4 + \angle 5$

Suppose L.H.S > R.H.S by 12''

Divide this 12 seconds by 4; correction = 12''/4 = 3''

Add 3'' to the angles of R.H.S and subtract 3'' from the angles of L.H.S.

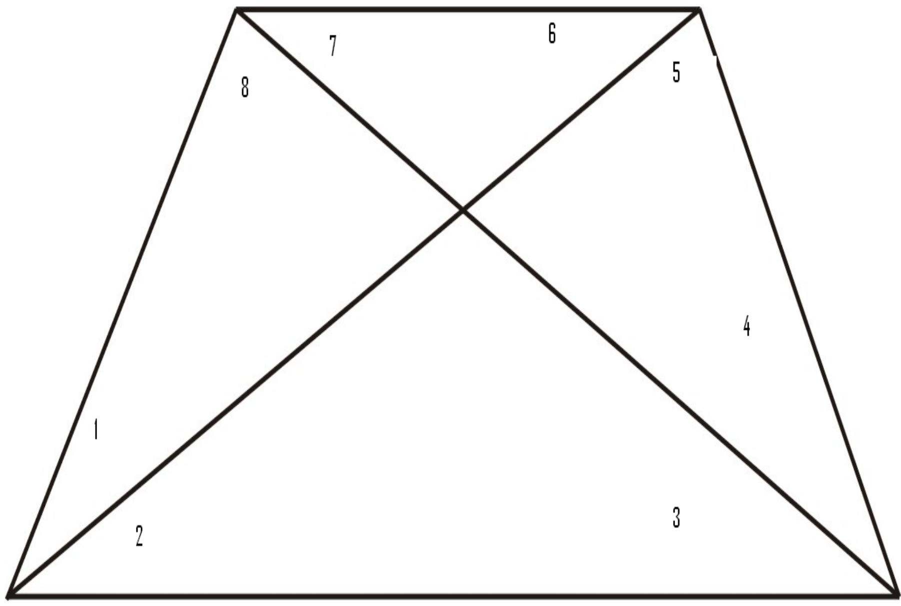


Fig: showing geodetic quadrilateral

6.2. Triangulation Adjustment: Approximate method

Adjustment of Braced Quadrilateral

Trigonometric Condition/ Side equation

$$\log(\sin 1) + \log(\sin 3) + \log(\sin 5) + \log(\sin 7) = \log(\sin 2) + \log(\sin 4) + \log(\sin 6) + \log(\sin 8)$$

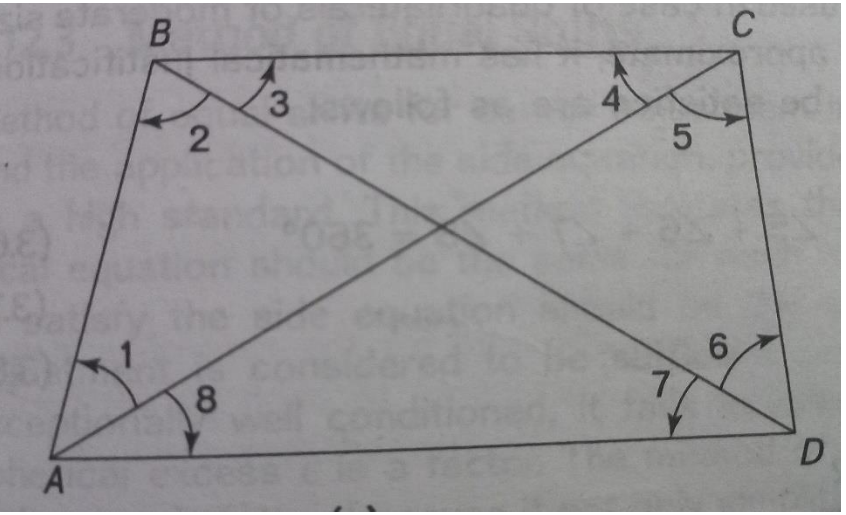
Here < 1, < 7, < 5 and < 3 are called left hand angles and < 8, < 6, < 4 and < 2 are right hand angles while standing at the intersection of the two diagonals and looking towards the sides.

For this adjustment following procedure is adopted :

6.2. Triangulation Adjustment: Approximate method

Adjustment of Braced Quadrilateral

Trigonometric Condition/ Side equation



The steps below may be followed.

1. Sum of all the angles observed is found, and if it does not equal 360°, the discrepancy d is obtained. Each of the angle measured is corrected by distributing the discrepancy (by $d/8$) so as to satisfy Eq. (3.6).
2. Find the difference between angles ($\angle 1 + \angle 2$) and ($\angle 5 + \angle 6$). The discrepancy, if any, is distributed equally in all the four angles to satisfy Eq. (3.7).
3. Find the difference between angles ($\angle 3 + \angle 4$) and ($\angle 7 + \angle 8$). The discrepancy, if any, is equally distributed in all the four angles to satisfy Eq. (3.8).
4. Once all the angles are satisfied by the station adjustment, these are tested for side equation Eq. (3.9).
   $\sum \log \sin R - \sum \log \sin L = d'$
   In case of any discrepancy d' the side equation Eq. (3.9) needs to be satisfied.
5. Let $d_1, d_2, \ldots, d_8$ be the tabular differences 1″ for log sin ∠1, log sin ∠2, ..., log sin ∠8. Determine $\sum d^2$ by

$$\sum d^2 = d_1^2 + d_2^2 + \ldots + d_8^2$$

6. Determine corrections to be applied to the angles 1 to 8, by

$$\text{Correction to } \angle 1 = \frac{d_1}{\sum d^2} d'$$

$$\text{Correction to } \angle 2 = \frac{d_2}{\sum d^2} d'$$

....

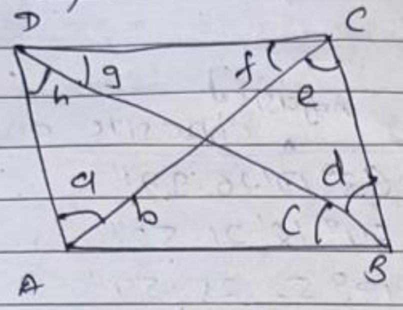
$$\text{Correction to } \angle 8 = \frac{d_8}{\sum d^2} d'$$

Adjustment of Braced Quadrilateral

Derivation of Trigonometric Condition/ Side equation

3. Side condition:-

proof:



Let, a quadrilateral ABCDE, whose all 8 angles a, b, c, d, e, f, g, h are measured independently.

Use sine law in ΔABC'

$$\frac{BC}{\sin b} = \frac{AB}{\sin e}$$

$$BC = AB \cdot \frac{\sin b}{\sin e} \quad \text{— (1)}$$

Use sine law in ΔBCD,

$$\frac{BC}{\sin g} = \frac{CD}{\sin d}$$

$$CD = BC \times \frac{\sin d}{\sin g}$$

$$CD = AB \cdot \frac{\sin b \sin d}{\sin e \sin g} \quad \text{— (2)} \quad (\because \text{using eq}^n\ 1)$$

Source: Er. Arjun Kandel

Adjustment of Braced Quadrilateral

Derivation of Trigonometric Condition/ Side equation

Again, using sine law in, ΔDAC,

$$\frac{AD}{\sin f} = \frac{CD}{\sin a}$$

$$AD = AB \cdot \frac{\sin b \sin d \sin f}{\sin a \cdot \sin e \sin g} \quad \text{--- (3)} \quad (\because \text{using eqn (2)})$$

Also,

using sine law in ΔABD:

$$\frac{AB}{\sin h} = \frac{AD}{\sin c}$$

$$AB = AD \cdot \frac{\sin h}{\sin c}$$

$$\Rightarrow \cancel{AB} = \cancel{AB} \cdot \frac{\sin b . \sin d . \sin f . \sin h}{\sin a . \sin c . \sin e . \sin g} \quad \text{--- (4)} \quad (\because \text{using eqn 3})$$

$$\Rightarrow 1 = \frac{\sin b . \sin d . \sin f . \sin h}{\sin a . \sin c . \sin e . \sin g} \quad \text{--- (5)}$$

Source: Er. Arjun Kandel

Adjustment of Braced Quadrilateral

Derivation of Trigonometric Condition/ Side equation

So, To adjust the above quadrilateral above ~~following~~ equation must be satisfied. (Ideally)

Let's, Rearrange the equation above;

we get,

$$\Rightarrow \sin a . \sin c . \sin e . \sin g = \sin b . \sin d . \sin f . \sin h.$$

Taking log on both sides, we get

$$\Rightarrow \log(\sin a . \sin c . \sin e . \sin g) = \log(\sin b . \sin d . \sin f . \sin h)$$

using the log product property

$$\log(\sin a) + \log(\sin c) + \log(\sin e) + \log(\sin g) = \log(\sin b) + \log(\sin d) + \log(\sin f) + \log(\sin h).$$

which also can be written as;

$$\text{i.e. } \Sigma \log(\sin(a, c, e, g)) = \Sigma \log(\sin(b, d, f, h))$$

Source: Er. Arjun Kandel

Adjustment of Braced Quadrilateral

Q: Adjust the angles of given quadrilateral by figure adjustment.

$a = 63°17'26.18''$

$b = 84°18'22.69''$

$c = 17°52'26.72''$

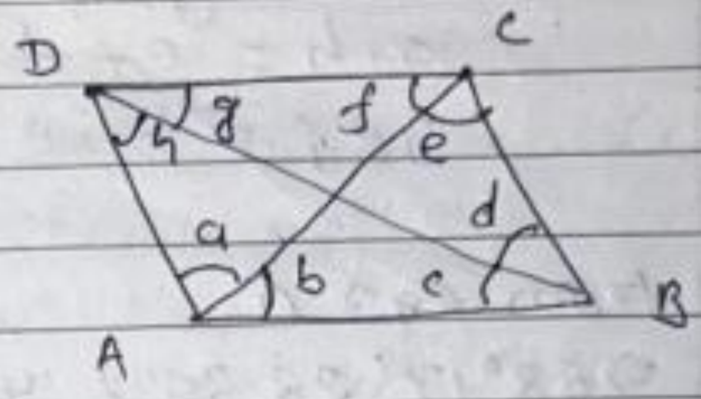
$d = 30°41'18.49''$

$e = 47°07'55.18''$

$f = 66°35'54.84''$

$g = 35°34'49.01''$

$h = 14°31'45.18''$



Source: Er. Arjun Kandel

Adjustment of Braced Quadrilateral

sum = 359°59'58.29"

Error = 360° − 359°59'58.29"
= 0°0'1.71"

Correction = $\frac{\text{Error}}{8}$ = 0°0'0.21"

Now, add the above correction each 8 angles above.

* Angles adjusted by 360° method:

unadjusted angles	360° adjusted angles.
63°17'26.18"	63°17'26.39"
84°18'22.69"	84°18'22.91"
17°52'26.72"	17°52'26.93"
30°41'18.49"	30°41'18.71"
47°07'55.18"	47°07'55.39"
66°35'54.84"	66°35'55.06"
35°34'49.01"	35°34'49.22"
14°31'45.18"	14 31'45.39"
	= 360°

Compiled by Er. Arjun Kandel

Source: Er. Arjun Kandel

Adjustment of Braced Quadrilateral

Also,

Angle adjustment by summation of opposite angles (i.e. using angle eq^ns).

⇒ The angles equations are,

$a+h = e+d$ --- (i)

$g+f = b+c$ --- (ii)

From eq^n (i)

$\Rightarrow 63°17'26.39'' + 14°31'45.39''$

$= 47°07'55.39'' + 30°41'18.71''$

$\Rightarrow 77°49'11.78'' = 77°49'14.1''$

Difference $= 0°0'2.32''$

Correction $= \dfrac{0°0'2.32''}{4}$

$= 0°0'0.58''$

so, the correction will be positive on angles 'a' and 'h', and it will be negative on angles e and d, because summation of angles on right hand sides is greater than left hand sides.

Source: Er. Arjun Kandel

Adjustment of Braced Quadrilateral

Again from eqn (2)

$35°34'49.22'' + 66°35'55.06''$

$= 84°18'22.91'' + 17°52'26.93''$

$102°10'44.28'' = 102°10'49.84''$

Difference $= 5.56''$

Correction $= \dfrac{5.56''}{4}$

$= 0°0'1.39''$

Source: Er. Arjun Kandel

Adjustment of Braced Quadrilateral

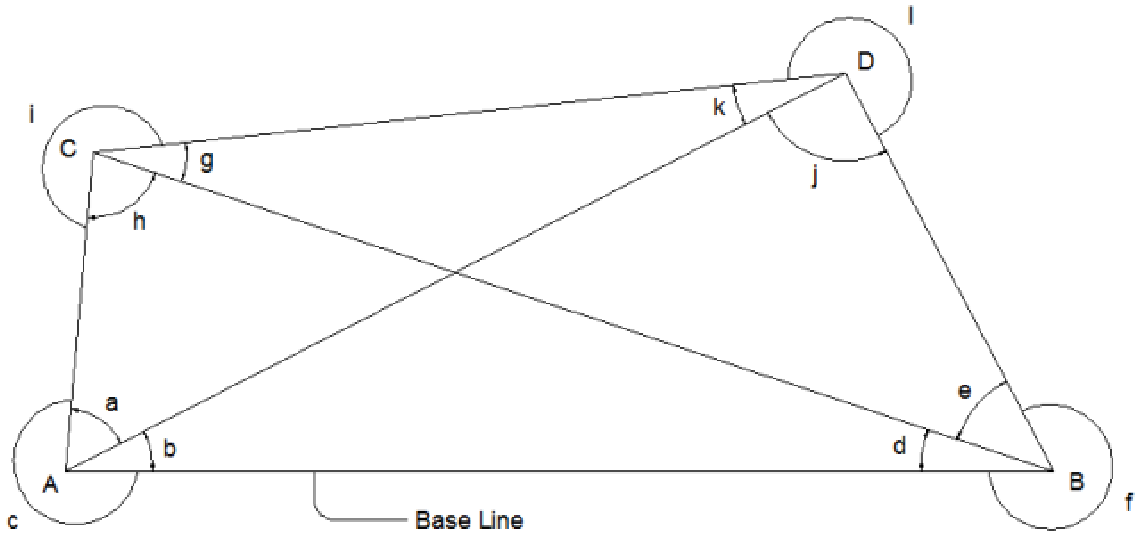
So, the correction will be positive on angles g and f, and it will be negative on b and c.

360° adjusted angles	Adjusted opposite angles.
63° 17' 26.39''	63° 17' 26.97''
84° 18' 22.91''	84° 18' 21.52''
17° 52' 26.93''	17° 52' 25.54''
30° 41' 18.71''	30° 41' 18.13''
47° 07' 55.39''	47° 07' 54.81''
66° 35' 55.06''	66° 35' 56.45''
35° 34' 49.22''	35° 34' 50.61''
14° 31' 45.39''	14° 31'' 45.97''

Source: Er. Arjun Kandel

Quadrilateral

.. ADJUSTMENT OF QUADRILATERAL. The observed angles of a quadrilateral are given in the accompanying tabulation and sketch. Adjust the angles about each station by the approximate method.



ANGLE	OBSERVED VALUE	ANGLE	OBSERVED VALUE
a	57° 21'10"	g	30° 01'55"
b	31° 37'05"	h	71° 15'05"
c	271° 01'30"	i	258° 42'45"
d	22° 22'00"	j	68° 00'00"
e	57° 31'25"	k	21° 50'30"
f	280° 06'50"	l	270° 09'30"

6.2. Triangulation Adjustment: Approximate method

Quadrilateral

I. STATION ADJUSTMENT:

About station A:

Sum = a + b + c

Sum = 57° 21'10" + 31° 37'05"
+ 271° 01'30"
= 359° 59'45"

Error = 360° - 359° 59'45"
= 0° 0'15"

Corr = 15"/3
= 5"

Adjusted Angles:

a' = 57° 21'10" + 5" = 57° 21'15"
b' = 31° 37'05" + 5" = 31° 37'10"
c' = 271° 01'30" + 5"= 271° 01'35"

About station B:

Sum = d + e + f
= 22° 22'00" + 57° 31'25"
+ 280° 06'50"
= 360° 00' 15"

Error = 360° - 360° 00' 15"
= -15"

Corr = -15"/3
= -5"

Adjusted Angles:

d' = 22° 22'00" – 5" = 22° 21'55"
e' = 57° 31'25" – 5" = 57° 31'20"
f' = 280° 06'50" – 5"= 280° 06'45"

6.2. Triangulation Adjustment: Approximate method

Quadrilateral

About station C:

Sum = g + h + i
= 30° 01'55" + 71° 15'05"
+ 258° 42'45"
= 359° 59'45"

Error = 360° - 359° 59'45"
= 15"

Corr = 15"/3
= 5"

Adjusted Angles:
g' = 30° 01'55" + 5" = 30° 02'00"
h' = 71° 15'05" + 5" = 71° 15'10"
i' = 258° 42'45" + 5" = 258° 42'50"

About station D:

Sum = j + k + l
= 68° 00'00" + 21° 50'30" + 270° 09'30"
= 360° 00'00"

(No correction to be applied.)

6.2. Triangulation Adjustment: Approximate method

Quadrilateral

About station C:

Sum = g + h + i
= 30° 01'55" + 71° 15'05"
+ 258° 42'45"
= 359° 59'45"

Error = 360° - 359° 59'45"
= 15"

Corr = 15"/3
= 5"

Adjusted Angles:

g'= 30° 01'55"+ 5" = 30° 02'00"
h'= 71° 15'05"+ 5" = 71° 15'10"
i' = 258° 42'45" + 5" = 258° 42'50"

About station D:

Sum = j + k + l
= 68° 00'00" + 21° 50'30" + 270° 09'30"
= 360° 00'00"

(No correction to be applied.)

NOTE: Also do rest of the figure and trigonometric/side equation adjustment

6.2. Triangulation Adjustment: Approximate method

Braced Quadrilateral Adjustment:

Example 3.31 Following are the values of the eight angles of a geodetic quadrilateral (Fig. 3.9(a)) after being adjusted for spherical excess. The angles may be assumed to be of equal weights. Adjust the quadrilateral using approximate method.

(Angles Only)

Angle	Value	Angle	Value
1	29°25′34″	5	30°29′25″
2	58°41′20″	6	57°37′33″
3	69°36′20″	7	50°35′44″
4	22°17′02″	8	41°17′26

Solution Angle equations

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360° \quad \text{(i)}$$

$$\angle 1 + \angle 2 = \angle 5 + \angle 6 \quad \text{(ii)}$$

$$\angle 3 + \angle 4 = \angle 7 + \angle 8 \quad \text{(iii)}$$

Sum of the angles designated as 1, 2, 3, 4, 5, 6, 7, 8 is

$$\sum_{n=1}^{8} n = 360°0'24''$$

$$E_1 = 360° - 360°0'24'' = -24''$$

Distributing this equally to all the eight angles, the correction to each angle $= -\frac{1}{8} \times 24'' = -03''$.

Hence, the corrected angles are

$$\angle 1 = 29°25'34'' - 03'' = 29°25'31''$$
$$\angle 2 = 58°41'20'' - 03'' = 58°41'17''$$
$$\angle 3 = 69°36'20'' - 03'' = 69°36'17''$$
$$\angle 4 = 22°17'02'' - 03'' = 22°16'59''$$
$$\angle 5 = 30°29'25'' - 03'' = 30°29'22''$$
$$\angle 6 = 57°37'33'' - 03'' = 57°37'30''$$
$$\angle 7 = 50°35'44'' - 03'' = 50°35'41''$$
$$\angle 8 = 41°17'26'' - 03'' = 41°17'23''$$
$$\text{Sum} = 360°0'0''$$

Now,

$$\angle 1 + \angle 2 = 29°25'31'' + 58°41'17'' = 88°06'48''$$
$$\angle 5 + \angle 6 = 30°29'22'' + 57°37'30'' = 88°06'52''$$
$$E_2 = 88°06'48'' - 88°06'52'' = -04''$$

Distributing this equally to all four angles, the correction to each angle $= \frac{1}{4} \times 04'' = 01''$. The correction will be positive to ∠1 and ∠2 and negative to ∠5 and ∠6.

Hence, the corrected angles are

$$\angle 1 = 29°25'31'' + 01'' = 29°2532''$$
$$\angle 2 = 58°41'17'' + 01'' = 58°41'18''$$

6.2. Triangulation Adjustment: Approximate method

Braced Quadrilateral Adjustment:

$$\angle 5 = 30°29'22'' - 01'' = 30°29'21''$$

$$\angle 6 = 57°37'30'' - 01'' = 57°37'29''$$

Now,

$$\angle 3 + \angle 4 = 69°36'17'' + 22°16'59'' = 91°53'16''$$

$$\angle 7 + \angle 8 = 50°35'41'' + 41°17'23'' = 91°53'04''$$

$$E_3 = 91°53'16'' - 91°53'04'' = 12''$$

Distributing this equally to all four angles, the correction to each angle $= \frac{1}{4} \times 12'' = 03''$. The correction will be negative to $\angle 3$ and $\angle 4$ and positive to $\angle 7$ and $\angle 8$.

Hence, the corrected angles are

$$\angle 3 = 69°36'17'' - 03'' = 69°36'14''$$

$$\angle 4 = 22°16'59'' - 03'' = 22°16'56''$$

$$\angle 7 = 50°35'41'' + 03'' = 50°35'44''$$

$$\angle 8 = 41°17'23'' + 03'' = 41°17'26''$$

NOTE: Also do rest of the trigonometric/side equation adjustment

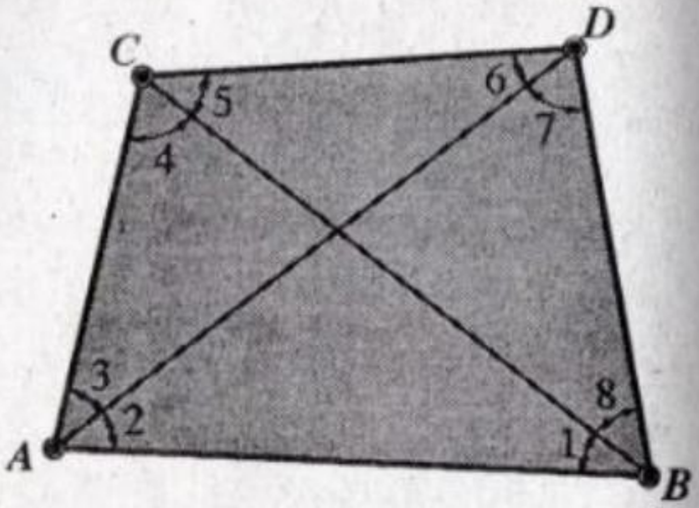
Braced Quadrilateral Adjustment: ASSIGNMENT

17. Problem:

Given the quadrilateral shown which has been adjusted using angle condition. It is required to adjust the angles using the side condition.

① Compute the adjusted angle 3.
② Compute the adjusted angle 5.
③ Compute the adjusted angle 8.

$\angle 1 = 39°37'49''$
$\angle 2 = 26°25'52''$
$\angle 3 = 75°12'13''$
$\angle 4 = 38°44'06''$
$\angle 5 = 23°44'35''$
$\angle 6 = 42°19'06''$
$\angle 7 = 44°52'00''$
$\angle 8 = 69°04'19''$
$\text{Sum} = 360°00'00''$



6.2. Triangulation Adjustment: Angle Adjustment

Angle Adjustment:

- Angle adjustment consists of determining the MPV of each individual angles and their weights from a set of repeated measurements or observations of each of them.
- The measured angles can be of equal or unequal weights.
- If all the measurements are made under precisely similar conditions and by same surveyor, they can be considered to be of same precision or same weight.
- If the conditions are different or different surveyors are involved in making the observations, they are assigned different weights.

6.2. Triangulation Adjustment: Angle Adjustment

Rule 1 For angles of equal weight, the discrepancy d is distributed equally among all the three angles:

$$c_A = c_B = c_C = \frac{1}{3} d$$

Rule 2 For angles of unequal weight, the discrepancy is distributed in inverse proportions to the weights:

$$c_A : c_B : c_C = \frac{1}{w_A} : \frac{1}{w_B} : \frac{1}{w_C}$$

$$c_A = \frac{\dfrac{1}{w_A}}{\dfrac{1}{w_A} + \dfrac{1}{w_B} + \dfrac{1}{w_C}} d, \quad c_B = \frac{\dfrac{1}{w_B}}{\dfrac{1}{w_A} + \dfrac{1}{w_B} + \dfrac{1}{w_C}} d, \quad c_C = \frac{\dfrac{1}{w_C}}{\dfrac{1}{w_A} + \dfrac{1}{w_B} + \dfrac{1}{w_C}} d$$

Rule 3 If instead of weights, number of observations is given, then the discrepancy is distributed in inverse proportions to the number of observations:

$$c_A = \frac{\dfrac{1}{n_A}}{\dfrac{1}{n_A} + \dfrac{1}{n_B} + \dfrac{1}{n_C}} d, \quad c_B = \frac{\dfrac{1}{n_B}}{\dfrac{1}{n_A} + \dfrac{1}{n_B} + \dfrac{1}{n_C}} d, \quad c_C = \frac{\dfrac{1}{n_C}}{\dfrac{1}{n_A} + \dfrac{1}{n_B} + \dfrac{1}{n_C}} d$$

where,

Source: SK Duggal, Vol-II

c = correction to observed angles, $c_{A,}$ $c_{B,}$ c_c are respective correction to angles A, B, C

w = weight of the angle, $w_{A,}$ w_B , w_C are respective weights of angles A, B, C

6.2. Triangulation Adjustment: Angle Adjustment

Rule 5 The corrections are proportional to the square of the probable errors:

$$c_A = \frac{E_A^2}{E_A^2 + E_B^2 + E_C^2} d, \quad c_B = \frac{E_B^2}{E_A^2 + E_B^2 + E_C^2} d, \quad c_C = \frac{E_C^2}{E_A^2 + E_B^2 + E_C^2} d$$

Rule 6 When the weights of the observations are not given directly, then if v is the difference between the mean observed value and the observed value of an angle, the weight of the angle is given by

$$w = \frac{\frac{1}{2} n^2}{\Sigma v^2}$$

Thus,

$$\frac{1}{w_A} = \frac{\Sigma v_A^2}{\frac{1}{2} n_A^2}, \quad \frac{1}{w_B} = \frac{\Sigma v_B^2}{\frac{1}{2} n_B^2}, \quad \frac{1}{w_C} = \frac{\Sigma v_C^2}{\frac{1}{2} n_C^2}$$

$$c_A = \frac{\frac{1}{w_A}}{\frac{1}{w_A} + \frac{1}{w_B} + \frac{1}{w_C}} d, \quad c_B = \frac{\frac{1}{w_B}}{\frac{1}{w_A} + \frac{1}{w_B} + \frac{1}{w_C}} d, \quad c_C = \frac{\frac{1}{w_C}}{\frac{1}{w_A} + \frac{1}{w_B} + \frac{1}{w_C}} d$$

Source: SK Duggal, Vol-II

NUMERICALS

Q.1

Example 3.26 Following angles were measured at a station. Find their most probable values:

Angle	*Value*	*Weight*
AOB	126°30′20″	2
BOC	74°20′10″	2
COA	159°12′20″	2

Solution Refer to Fig. 3.3. The sum of the observed angles

$$= 126°30'20'' + 74°20'10'' + 159°12'20'' = 360°2'50''$$

$$\text{Discrepancy} = 360°2'50'' - 360° = 2'50''$$

$$\therefore \quad \text{Correction} = -\,2'50''$$

$$\text{Correction to each angle} = \frac{-2'50''}{3} = -\,56.66''$$

Most probable values of angles are

$$\angle AOB = 126°30'20'' - 56.66''$$
$$= 126°29'23.34''$$
$$\angle BOC = 74°20'10'' - 56.66''$$
$$= 74°19'13.34''$$
$$\angle COA = 159°12'20'' - 56.66''$$
$$= 159°11'23.34''$$

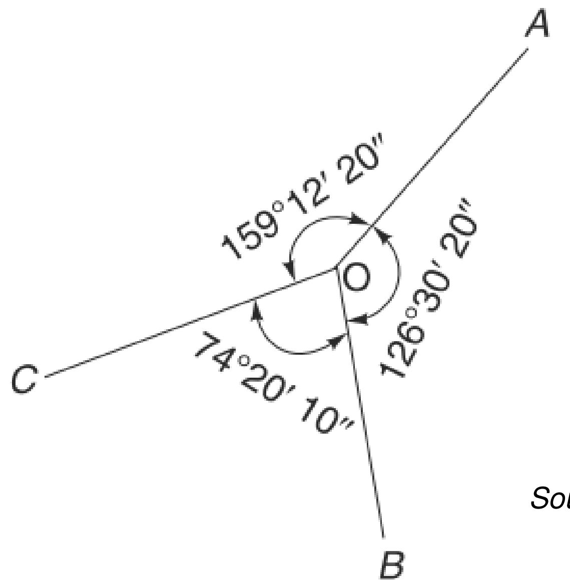


Fig. 3.3

Source: SK Duggal, Vol-II

NOTE: Finding MPVs also means adjusting angles.

NUMERICALS

Q.2

Example 2.10 The following are the observed values of angles in a triangle of a triangulation survey. Adjust the angles.

$$A = 87°35'11.1'' \quad \text{weight } 1$$
$$B = 43°15'17.0'' \quad \text{weight } 2$$
$$C = 49°09'34.1'' \quad \text{weight } 3$$

[AMIE A.S. Winter 1985]

Solution The angles of a plane triangle should sum 180°. Here

$$A + B + C = 180°00'2.2''$$

Hence there is a total error of 2.2″ and correction – 2.2″

As per rule 4 of Sec. 2.12 corrections are to be distributed inversely to weights of observations.

Therefore $C_A : C_B : C_C :: \frac{1}{1} : \frac{1}{2} : \frac{1}{3} :: 6 : 3 : 2$

Hence
$$C_A = -1.2''$$
$$C_B = -0.6''$$
$$C_C = -0.4''$$

NOTE: Also find final adjusted angles.

Source: *Fundamental of Surveying, S.K Roy, Prentice –Hall of India, New Delhi*

NUMERICALS

Q.3

Example 3.16 Neglecting spherical excess, adjust the angles of a triangle of which the observed values are:

∠A	48°18′22″	Weight = 3
∠B	76°32′47.2″	Weight = 1
∠C	55°08′53.8″	Weight = 3

Ans: < A = 48^{o} 18’21.4’’, < B = 76^{o} 32’45.4’’ , < C = 55^{o} 08’ 53.2’’

NOTE: Adjusting angles means find MPVs.

Source: SK Duggal, Vol-II

NUMERICALS

Q.4

Example 3.28 Adjust the following angles of a triangle:

∠A	∠B	∠C
34°22′13″	69°32′48″	76°03 ′18″
12″	44″	22″
16″	45″	21″
17″	49″	17″
11″	46″	
9″		

c_A = 42.66'' , c_B = 22.94'' c_C = 35.49''

Hint: Use rule 5

NOTE: Do Adjustment for mean of each angles.

Source: SK Duggal, Vol-II

NUMERICALS

Q.5

Example 3.18 On a station O, three horizontal angles, closing the horizon, were measured as follows:

$$\angle A = 34°10'20'' \pm 3'', \angle B = 176°40'32'' \pm 4'', \angle C = 149°09'04'' \pm 5''$$

Calculate the corrected angles.

Ans: < A = $34^{o}10'20.72''$, < B = $176^{o}40'33.28''$, < C = $149^{o}09'06''$

NOTE: Closing the horizon means sum of each angles is equal to 360^{o}.

Source: SK Duggal, Vol-II

Numericals

1. **Example 2.13** Directions are observed from a satellite station *S*, 62.18 m from station *C*. Following were the results:

$$\angle A = 0°0'00'', \angle B = 71°54'32'', \angle C = 296°12'02''$$

The approximate lengths of *AC* and *BC* were 8240.60 m and 10 863.60 m. Calculate the angle *ACB*.

Solution From triangle *BSC* in Fig. Ex. 2.13,

$$\angle CSB = 360° - (296°12'02'' - 71°54'32'') = 135°42'30''$$

Ans: 71° 44' 58.5''

NOTE: Eccentric distance means that distance from satellite station to the closet triangulation station.

6.2.1. Solution of triangle using Sine and Cosine Law

Numericals

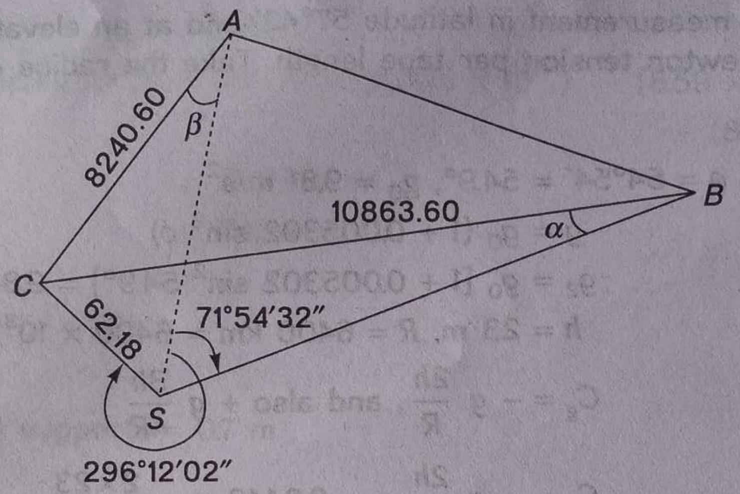


Fig. Ex. 2.13

$$\frac{62.18}{\sin\alpha} = \frac{10863.60}{\sin 135°42'30''}$$

or $$\sin\,\alpha = \frac{62.18 \times \sin 135°42'30''}{10863.60}$$

or $$\alpha = 0.00399 \times 206265 = 822.99''$$

From triangle *CSA*,

$$\angle CSA = 360° - 296°12'02'' = 63°47'58''$$

$$\frac{62.18}{\sin\beta} = \frac{8240.60}{\sin 63°47'85''}$$

or $$\sin\,\beta = \frac{62.18}{8240.6}\ \sin\ 63°47'58''$$

or $$\beta = 0.00677 \times 206265 = 1396.47''$$

$\therefore$ $$\angle ACB = 71°54'32'' - 1396.47'' + 822.99'' = 71°44'58.5''$$

6.2. Triangulation and Trilateration Computation

Trilateration

- Trilateration is a kind of surveying in which the lengths of the sides of a series of joined or overlapping triangles are measured (usually with the EDM equipment) and the angles are computed from the lengths.
- It is a method used to transfer horizontal control.

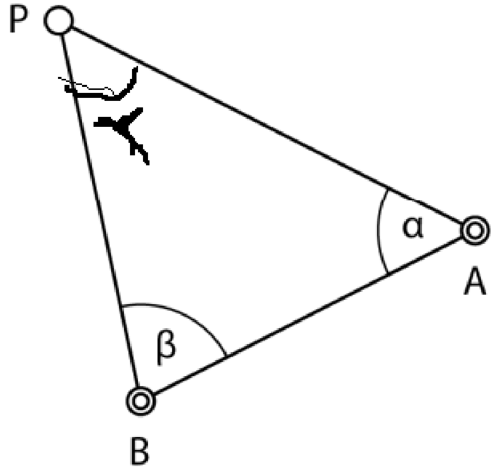
Source: https://lecengineers.files.wordpress.com/2016/03/8-0-triangulation-and-trilteration-notes.pdf

6.3. Intersection and Resection computation

Intersection

- Intersection is a surveying technique used to determine the position of an unknown point using at least two known points through certain direction (angle or bearing) or distance measurements from known points to unknown points.
- It is commonly used in land surveying, construction, and mapping to establish accurate locations of features or boundaries.

1. Intersection using inner angles



Source: https://edu.epito.bme.hu/local/coursepublicity/mod/resource/view.php?id=70161

6.3. Intersection computation

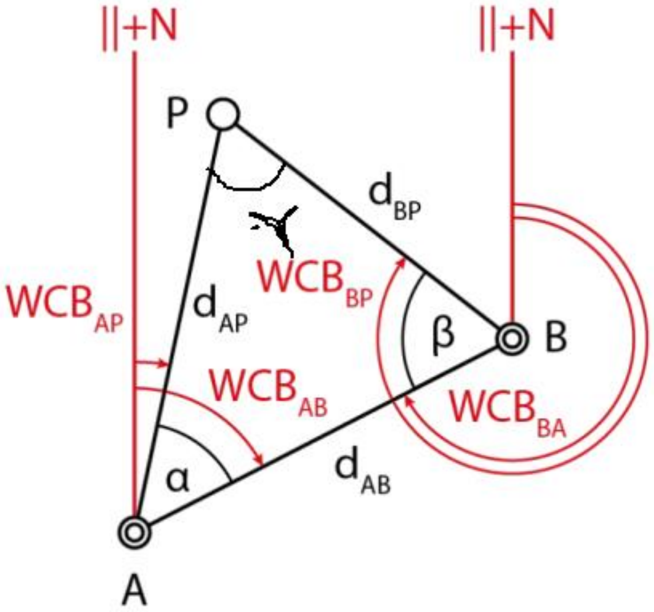
1. Intersection using inner angles

Let A and B be the known coordinates and P be the unknown point. Also < PAB = α and < PBA = β be the interior angles of a triangle APB considering in clockwise direction.

Using the measured interior angles, we compute the WCBs between the control points and the unknown point WCB_{AP} and WCB_{BP}.

According to the figure:

$$WCB_{AP} = WCB_{AB} - \alpha$$
$$WCB_{BP} = WCB_{BA} + \beta$$



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6.3. Intersection computation

1. Intersection using inner angles

Using sine theorem, we compute the distances between control points and unknown point P.

$$\frac{\sin(\alpha)}{\sin(\alpha+\beta)} = \frac{d_{BP}}{d_{AB}} \Rightarrow d_{BP} = \frac{\sin(\alpha)}{\sin(\alpha+\beta)} \cdot d_{AB}$$

$$\frac{\sin(\beta)}{\sin(\alpha+\beta)} = \frac{d_{AP}}{d_{AB}} \Rightarrow d_{AP} = \frac{\sin(\beta)}{\sin(\alpha+\beta)} \cdot d_{AB}$$

From A and B coordinates of point P can be calculated as:

$$E_P = E_A + d_{AP} \cdot \sin(\mathrm{WCB}_{AP})$$
$$N_P = N_A + d_{AP} \cdot \cos(\mathrm{WCB}_{AP})$$

or

$$E_P = E_B + d_{BP} \cdot \sin(\mathrm{WCB}_{BP})$$
$$N_P = N_B + d_{BP} \cdot \cos(\mathrm{WCB}_{BP})$$

NOTE:

$\Upsilon = 180 - (\alpha + \beta)$

or, $\sin \Upsilon = \sin\{180 - (\alpha + \beta)\}$

or, $\sin \Upsilon = \sin(\alpha + \beta)$

6.3. Intersection computation

1. Intersection using inner angles

Since, coordinate of point P can be obtained from both point A and B which can be later used to cross check the final results.

NOTE:

1. There must be a line of sight between stations to use this method.

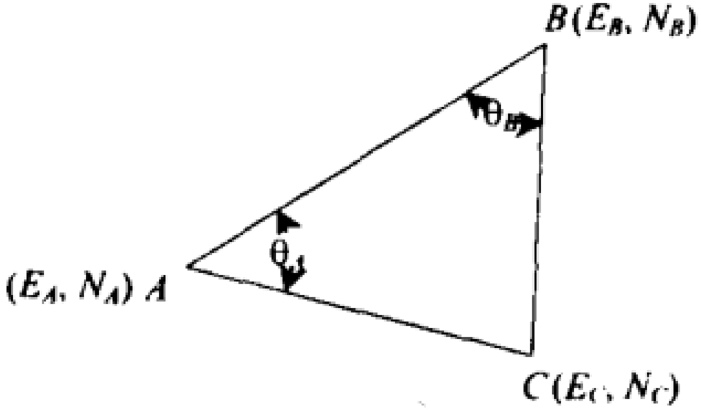
2. You will need bearing and angles.

But if you can have got only angles you can use the following method.

Considering known points A, B and unknown point C and in clockwise direction.

$$E_C = \frac{E_A \cot \theta_B + E_B \cot \theta_A + (N_B - N_A)}{\cot \theta_A + \cot \theta_B}$$

$$N_C = \frac{N_A \cot \theta_B + N_B \cot \theta_A - (N_B - N_A)}{\cot \theta_A + \cot \theta_B}$$



6.3. Intersection computation: Numericals

Q.N 1:

Coordinates of the control points:

Point	E [m]	N [m]
A	658 077.70	247 431.38
B	657 310.23	247 123.54

Measured angles:

$$\alpha = 81\text{-}34\text{-}45$$

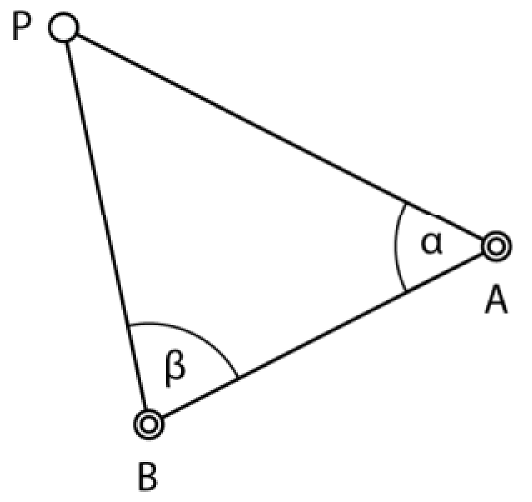
$$\beta = 66\text{-}45\text{-}57$$

The clockwise order of the points is B, P, A.

6.3. Intersection computation: Numericals

Q.N 1: Ans:

According to the coordinates and the clockwise order, we can create the following figure:



Distance and WCB between the control points

$$d_{AB} = 826.907 \text{ m}$$

$$\text{WCB}_{AB} = 248\text{-}08\text{-}38$$

$$\text{WCB}_{BA} = 68\text{-}08\text{-}38$$

The WCBs from the control points to point P:

$$\text{WCB}_{AP} = \text{WCB}_{AB} + \alpha = 329\text{-}43\text{-}23$$

$$\text{WCB}_{BP} = \text{WCB}_{BA} - \beta = 1\text{-}22\text{-}41$$

The distances between the control points and point P:

$$d_{AP} = \frac{\sin(\beta)}{\sin(\alpha+\beta)} \cdot d_{AB} = 1447.866 \text{ m}$$

$$d_{BP} = \frac{\sin(\alpha)}{\sin(\alpha+\beta)} \cdot d_{AB} = 1558.655 \text{ m}$$

6.3. Intersection computation: Numericals

Q.N 1: Ans:

According to the coordinates and the clockwise order, we can create the following figure:

The coordinates of P computed from point A:

$$E_P = E_A + d_{AP} \cdot \sin(\mathrm{WCB_{AP}}) = 657\,347.714 \text{ m} \approx 657\,347.71 \text{ m}$$

$$N_p = N_A + d_{AP} \cdot \cos(\mathrm{WCB_{AP}}) = 248\,681.754 \text{ m} \approx 248\,681.75 \text{ m}$$

The coordinates of P computed from point B:

$$E_P = E_B + d_{BP} \cdot \sin(\mathrm{WCB_{BP}}) = 657\,347.714 \text{ m} \approx 657\,347.71 \text{ m}$$

$$N_p = N_B + d_{BP} \cdot \cos(\mathrm{WCB_{BP}}) = 248\,681.754 \text{ m} \approx 248\,681.75 \text{ m}$$

The two sets of coordinates match at least up to centimeter precision.

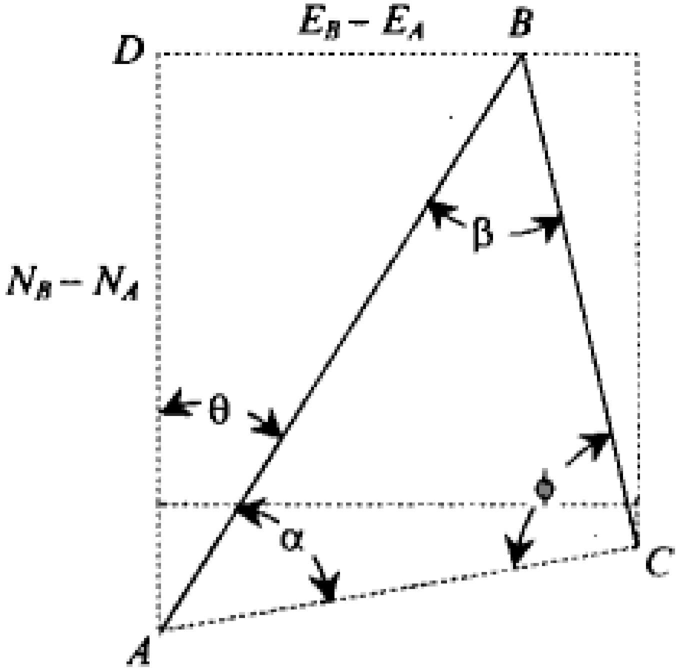
Assignment

Q.N 1:

Example 9.2. From two triangulation stations *A* and *B* the clockwise horizontal angles to a station *C* were measured as $\angle BAC = 50°05'26^2$ and $\angle ABC = 321°55'44^2$. Determine the coordinates of *C* given those of *A* and *B* are

A	*E* 1000.00 m	*N* 1350.00 m
B	*E* 1133.50 m	*N* 1450.00 m.

Ans: E_C = 1100.16 m, N_C = 1326.41



Assignment

Q.N 1:

$$\alpha = 50°05'26''$$

$$\beta = 360° - 321°55'44^2 = 38°04'16''.$$

If the coordinates of A, B, and C are (E_A, N_A), (E_B, N_B), and (E_C, N_C), respectively, then

$$BD = E_B - E_A = 1133.50 - 1000.00 = 133.50 \text{ m}$$

$$AD = N_B - N_A = 1450.00 - 1350.00 = 100.00 \text{ m}.$$

Therefore bearing of AB

$$\theta = \tan^{-1}\frac{BD}{AD}.$$

$$= \tan^{-1}\frac{133.50}{100.0} = 53°09'52''.$$

$$\text{Bearing of } AC = \theta_{AC} = \text{Bearing of } AB + \alpha$$

$$= 53°09'52'' + 50°05'26''$$

$$= 103°15'18''.$$

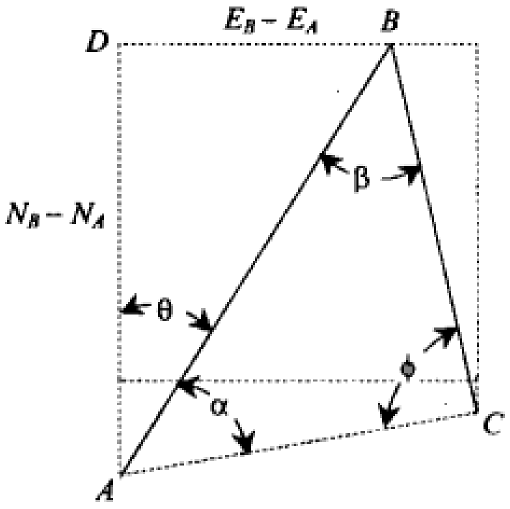


Fig. 9.8

Assignment

Q.N 1:

$$\text{Bearing of } BA = \theta_{BC} = \text{Bearing of } AB + 180°$$

$$= 53°09'52'' + 180° = 233°09'52''.$$

$$\text{Bearing of } BC = \theta_{BC} = \text{Bearing of } BA - b$$

$$= 233°09'52'' - 38°04'16'' = 195°05'36''.$$

In ΔABC, we have

$$\angle ACB = 180° - (\alpha + \beta)$$

$$= 180° - (50°05'26'' + 38°04'16'') = 91°50'18''.$$

$$AB = \sqrt{BD^2 + AD^2}$$

$$= \sqrt{133.50^2 + 100.00^2} = 166.80 \text{ m.}$$

$$AC = \frac{AB \sin\beta}{\sin\phi}$$

$$= \frac{166.80 \times \sin 38°04'16''}{\sin 91°50'18''} = 102.90 \text{ m}$$

$$BC = \frac{AB \sin\alpha}{\sin\phi}$$

$$= \frac{166.80 \times \sin 50°05'26''}{\sin 91°50'18''} = 128.01 \text{ m.}$$

6.3. Intersection computation: Numericals

Assignment

Q.N 1:

Let the latitude and departure of C, considering the line AC and BC, be respectively L_{AC}, D_{AC}, and L_{BC}, D_{BC}.

$$L_{AC} = AC \cos \theta_{AC} = 102.90 \times \cos 103°15'18'' = -23.59 \text{ m}$$

$$D_{AC} = AC \sin \theta_{AC} = 102.90 \times \sin 103°15'18'' = 100.16 \text{ m}$$

$$L_{BC} = BC \cos \theta_{BC} = 128.01 \times \cos 195°05'36'' = -123.59 \text{ m}$$

$$D_{BC} = BC \sin \theta_{BC} = 128.01 \times \sin 195°05'36'' = -33.33 \text{ m.}$$

Coordinates of C

$$E_C = E_A + D_{AC} = 1000.00 + 100.16 = 1100.16 \text{ m}$$

$$= E_B + D_{BC} = 1133.50 - 33.33 = 1100.17 \text{ m} \quad (Okay)$$

$$N_C = N_A + L_{AC} = 1350.00 - 23.59 = 1326.41 \text{ m}$$

$$= N_B + L_{BC} = 1450.00 - 123.59 = 1326.41 \text{ m} \quad (Okay).$$

Thus, the coordinates of C are ***E*** **1100.17 m** and ***N*** **1326.41 m**.

2. Intersection using Arc section method

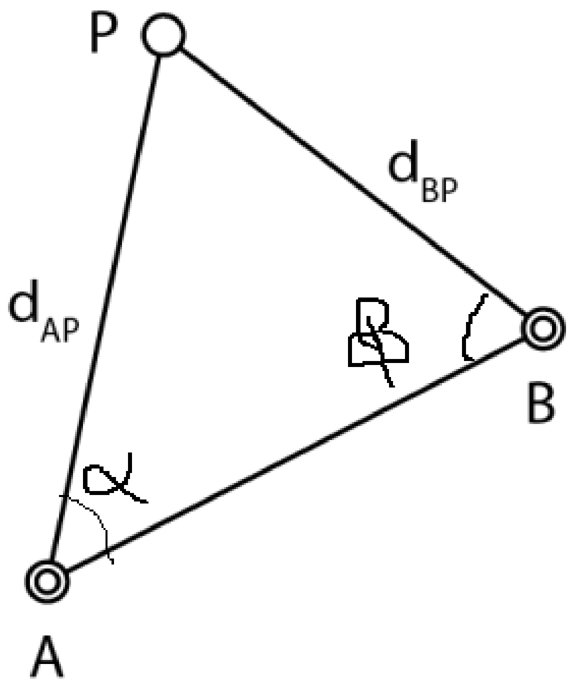
- In case of the arc section, instead of angles, we measure distances between the control points and the unknown point.

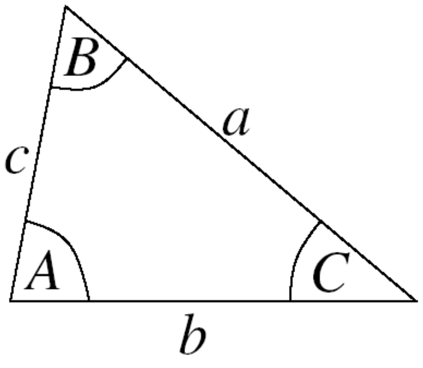
Let us consider two known points A and B with unknown point P. Also let points A, P and B are in clockwise order. Let d_{AP} , d_{BP} and d_{AB} be the known distance.

From here rest everything is same as intersection

Using inner angles except required interior angles

Are calculated using cosine rule.





<u>Sine Rule</u>

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \quad \text{or} \quad \frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

(for finding sides) (for finding angles)

<u>Cosine Rule</u>

$$a^2 = b^2 + c^2 - 2bc\cos(A) \quad \text{or} \quad \boxed{\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}}$$

(for finding sides) (for finding angles)

6.3. Intersection computation: Numericals

Q.N 2: Assignment

Coordinates of the control points:

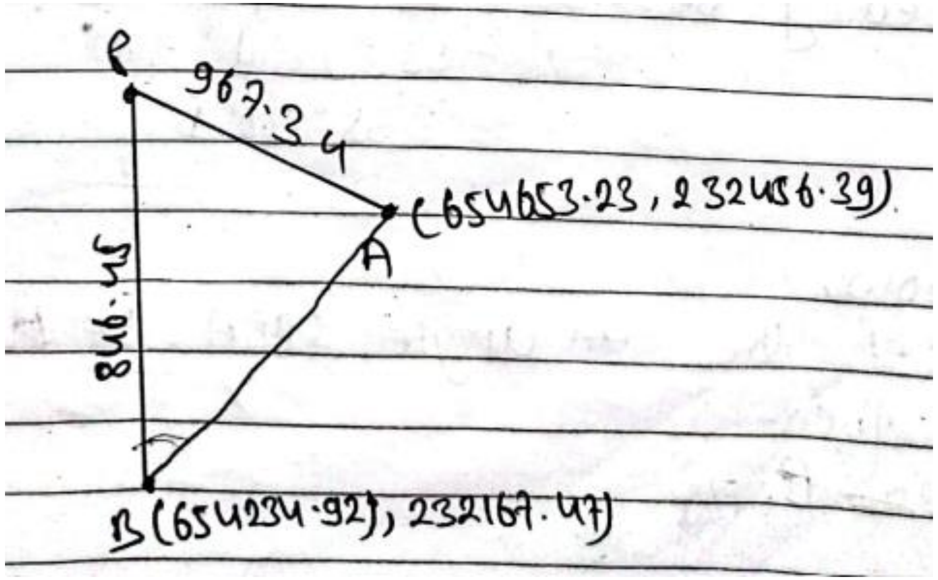
Point	E [m]	N [m]
A	654 653.23	232 456.39
B	654 234.92	232 167.47

Measured distances:

$$d_{AP} = 967.34 \text{ m}$$

$$d_{BP} = 846.45 \text{ m}$$

The clockwise order of the points is B, P, A.



Calculate coordinate of point P.

Ans: E_P = 653786.09, N_P = 232885.12

6.3. Intersection and Resection computation

Resection

- Intersection is a surveying technique used to determine the position of an unknown point with the help of at least three known stations.
- It is mainly done by measuring the angles subtended by lines of sight from unknown point to the three known points.
- There are several methods of solving the three point resection problem, e.g.

✓ Collins method

✓ Tangent method (Blunt's method)

✓ Cassini method

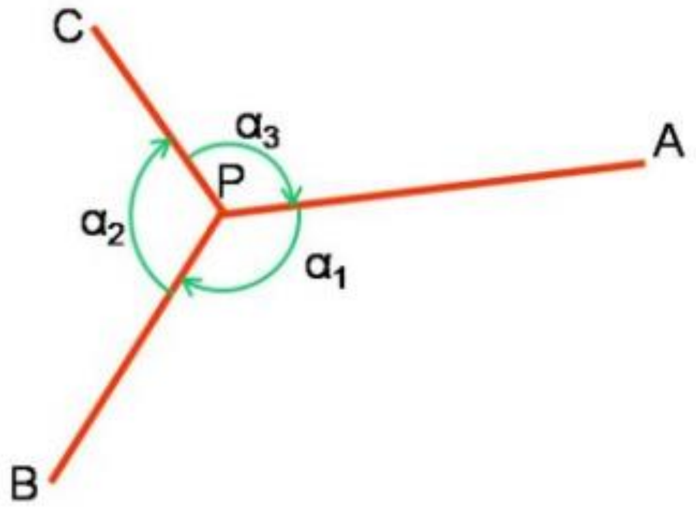
✓ Tienstra's Barycentric method

✓ Willerdings formula

✓ Solver tool in Microsoft Excel

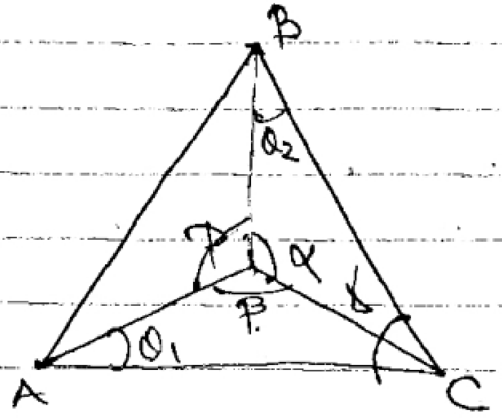
✓ Semigraphic method

✓ Least Squares



Resection

Three point problem (Collins method).



$$\alpha+\beta+\theta_1+\theta_2+\gamma = 360^\circ$$

$$\theta_1+\theta_2 = 360^\circ - (\alpha+\beta+\gamma) \quad \text{(i)}$$
$$= \phi$$

From Sine law:

or, $\dfrac{AC}{\sin\beta} = \dfrac{CP}{\sin\theta_1}$ & $\dfrac{CP}{\sin\theta_2} = \dfrac{CB}{\sin\alpha}$

or, $CP = \dfrac{AC}{\sin\beta}\sin\theta_1$ or, $\dfrac{CB}{\sin\alpha}\sin\theta_2$

Source: Er. Kiran Neupane,, WRC BGE 070

6.3. Resection computation

Resection

or, $$\frac{\sin\theta_1}{\sin\theta_2} = \frac{CB}{AC}\frac{\sin\beta}{\sin\alpha} = K \quad \text{--(ii)}$$

from (i) & (ii)

or, $$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin(\phi - \theta_2)}{\sin\theta_2} = K$$

or, $$\frac{\sin\phi\cos\theta_2}{\sin\theta_2} - \frac{\cos\phi\sin\theta_2}{\sin\theta_2} = K$$

or, $$\sin\phi\cot\theta_2 - \cos\phi = K$$

or, $$\sin\phi\cot\theta_2 - \cos\phi = K$$

or, $$\sin\phi\cot\theta_2 = K + \cos\phi$$

or, $$\theta_2 = \cot^{-1}\left(\frac{K+\cos\phi}{\sin\phi}\right)$$

or, $$\boxed{\theta_2 = \tan^{-1}\left(\frac{\sin\phi}{K+\cos\phi}\right)}$$

Resection

$$\text{or, } \frac{\sin\theta_1}{\sin\theta_2} = \frac{CB}{AC}\frac{\sin\beta}{\sin\alpha} = K \quad \text{---(ii)}$$

from (i) & (ii)

$$\text{or, } \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin(\phi-\theta_2)}{\sin\theta_2} = K$$

$$\text{or, } \frac{\sin\phi\cos\theta_2}{\sin\theta_2} - \frac{\cos\phi\sin\theta_2}{\sin\theta_2} = K$$

$$\text{or, } \sin\phi\cot\theta_2 - \cos\phi = K$$

$$\text{or, } \sin\phi\cot\theta_2 - \cos\phi = K$$

$$\text{or, } \sin\phi\cot\theta_2 = K + \cos\phi$$

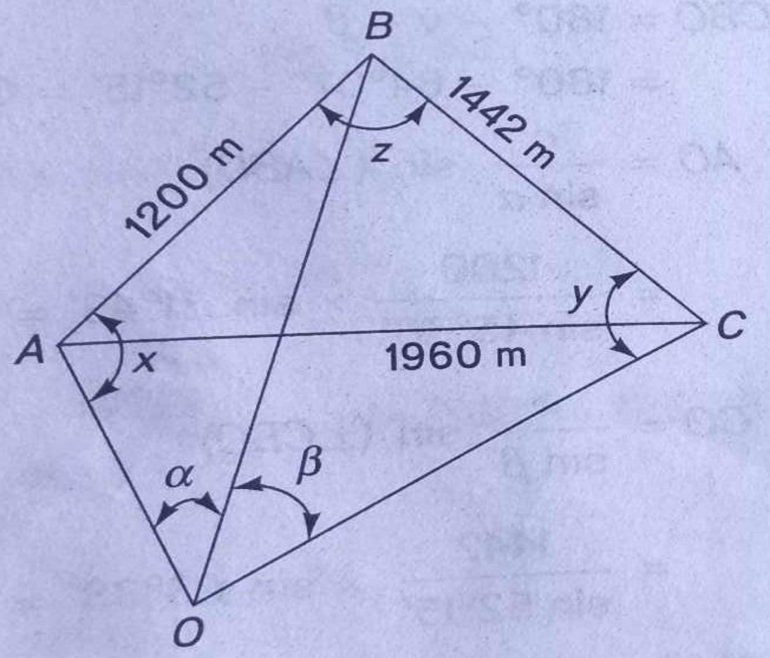
$$\text{or, } \theta_2 = \cot^{-1}\left(\frac{K+\cos\phi}{\sin\phi}\right)$$

$$\Rightarrow$$

$$\text{or, } \boxed{\theta_2 = \tan^{-1}\left(\frac{\sin\phi}{K+\cos\phi}\right)}$$

6.3. Resection computation: Numerical

Q. N 1: A, B and C are three visible stations in a hydrographic survey. The computed sides of the triangle ABC are AB = 1200 m, BC = 1442 m and CA = 1960 m. A station O is established outside the triangle and its position is to be determined by resection on A, B and C, the angle AOB and BOC being respectively 45°30′ and 52°15′. Determine distances of OA and OC, if O and B are on the opposite sides of line AC.



Source: SK Duggal, Vol- II

Ans: OA = 884.49 m , OC = 1634 m

6.3. Resection computation: Numerical

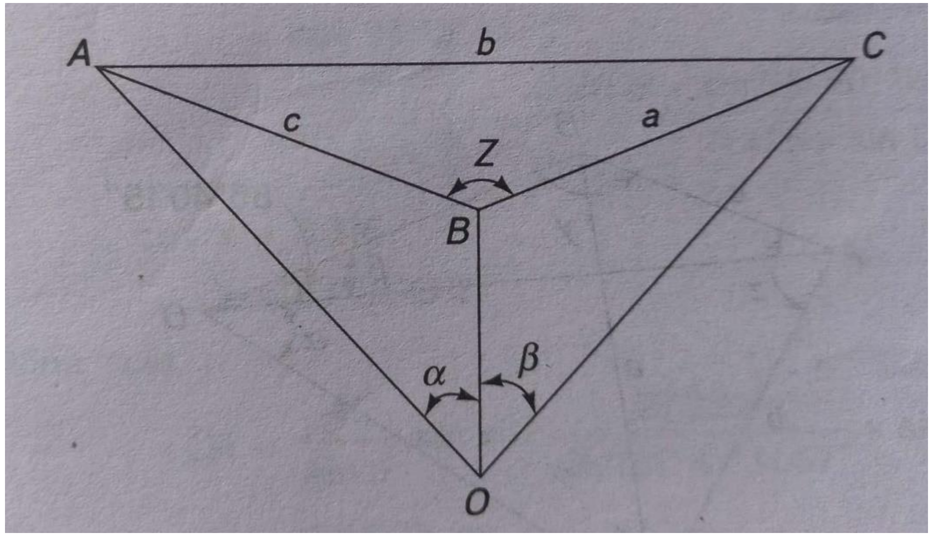
Q. N 2:

The following observations were made on three stations A, B and C from a boat at O with the help of a sextant. Station B and O being on the same side of AC.

$\angle AOB = 30°25'$, $\angle BOC = 45°25'$, $\angle ABC = 130°10'$

$AB = 4000$ m, $BC = 4995$ m

Calculate the distances of the boat from the three stations.



Source: SK Duggal, Vol- II

Ans: OA = 6537.159 m , OB = 3391.068 m, OC = 6752.60 m

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