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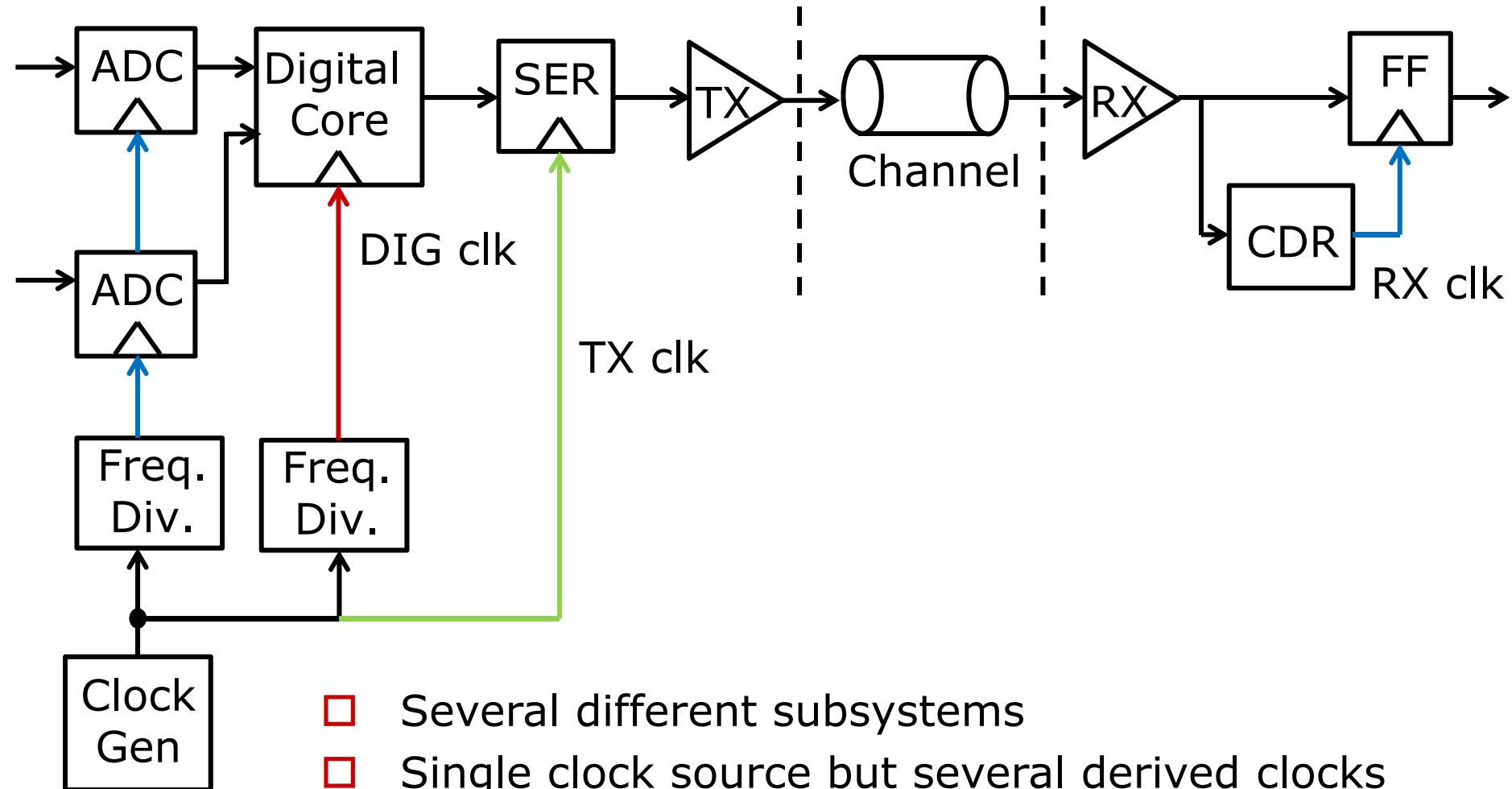
# ISSCC 2012 Tutorials

## **JITTER** **basic and advanced concepts,** **statistics and applications**

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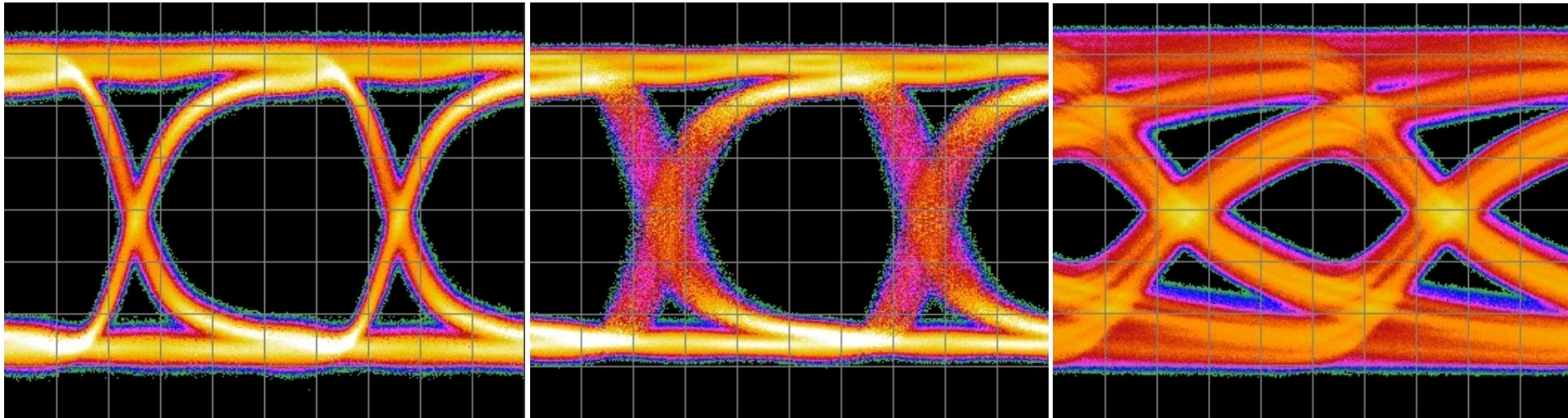
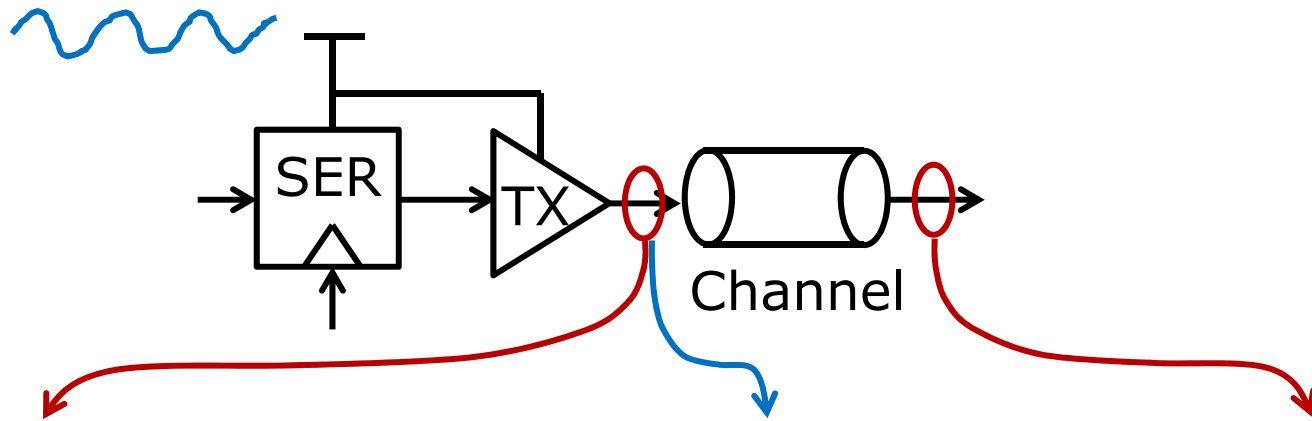
San Francisco, February 19th, 2012

# Motivation: a complex system



- ❑ Several different subsystems
- ❑ Single clock source but several derived clocks
- ❑ Each subsystem has own jitter requirements

# Motivation: jitter in serial high speed TX



□ Jitter can show different faces. How to define and quantify it?

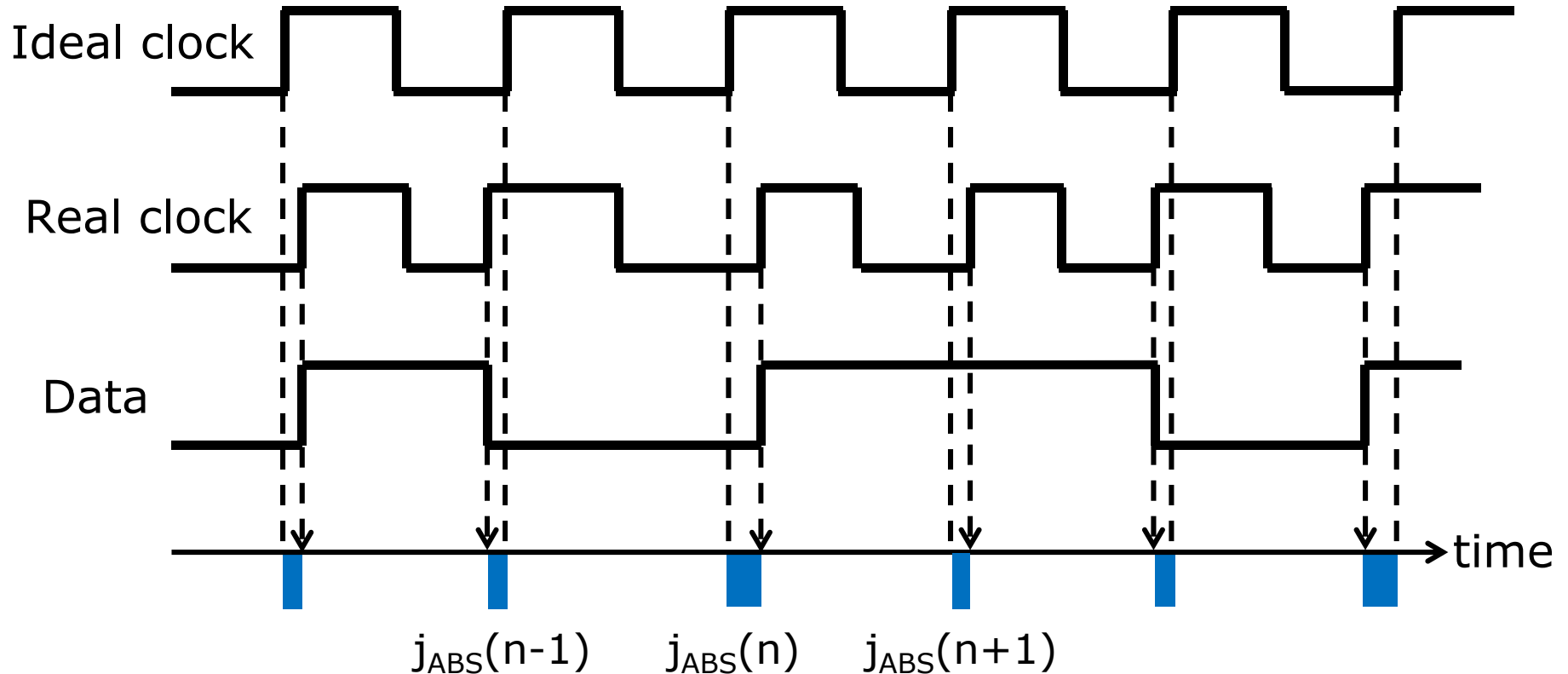
# Overview

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- ❑ Definition of absolute jitter
- ❑ Time and frequency domain measurements
- ❑ Phase noise and jitter
- ❑ Jitter in electronics systems and typical applications
- ❑ Self-referenced jitter and additional definitions
- ❑ Jitter statistics for gaussian and non-gaussian distributions
- ❑ Bathtub curves
- ❑ Application to high-speed serial data transmission
- ❑ Jitter and spectral spurious tones

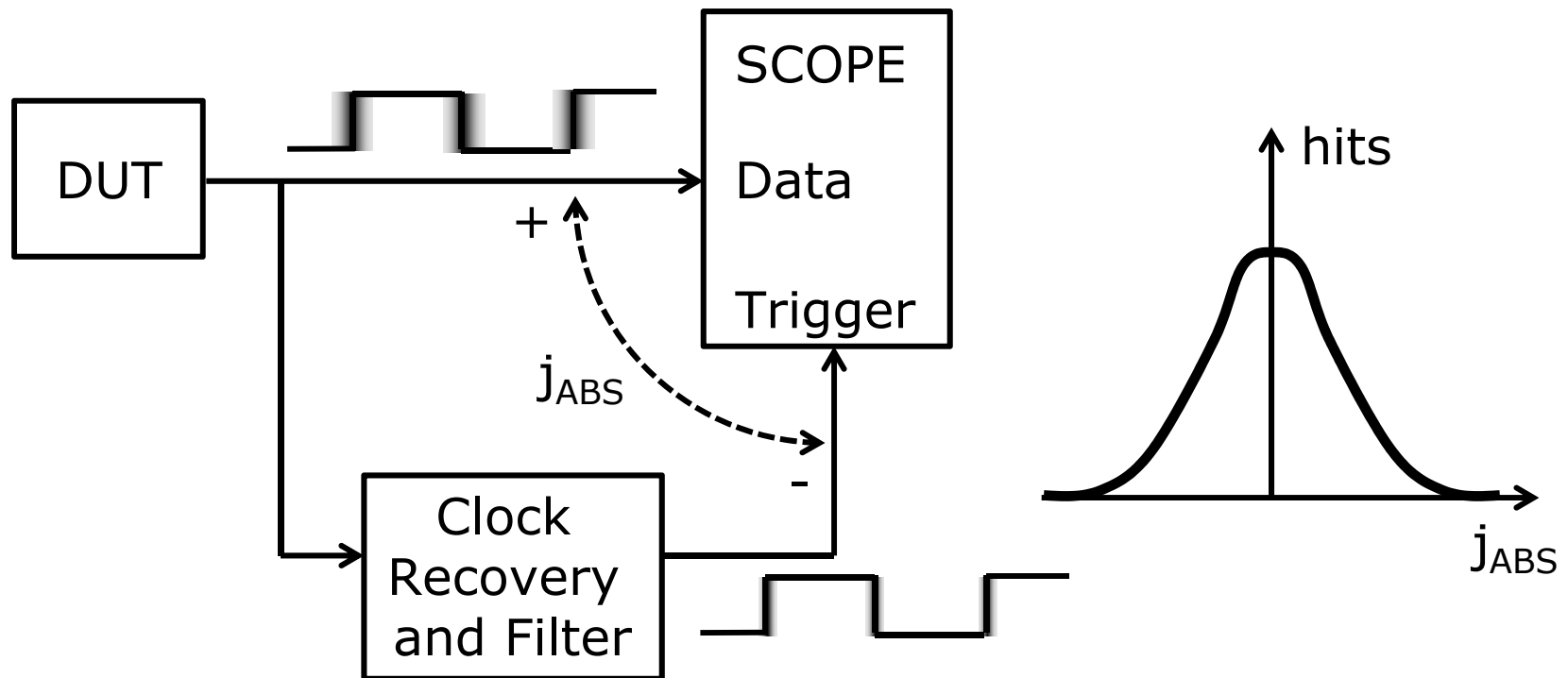
# Absolute Jitter : definition

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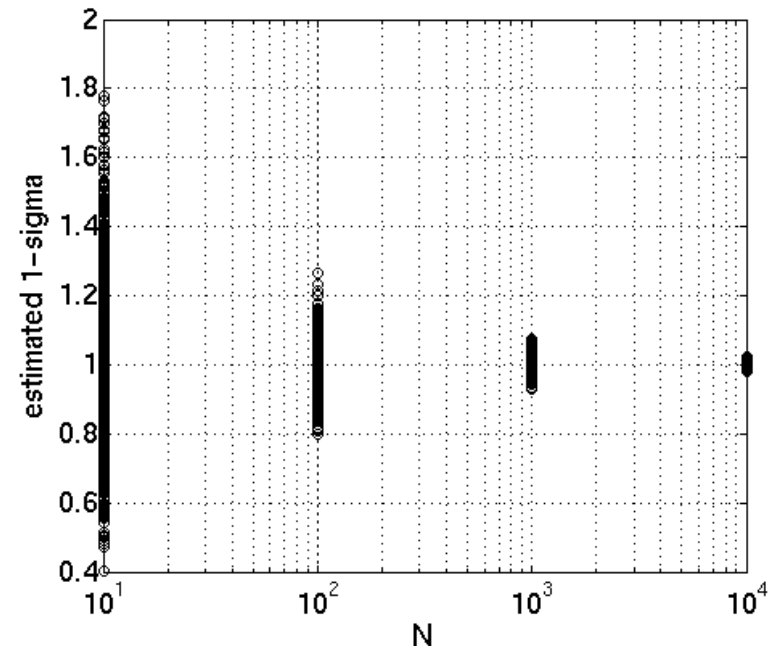
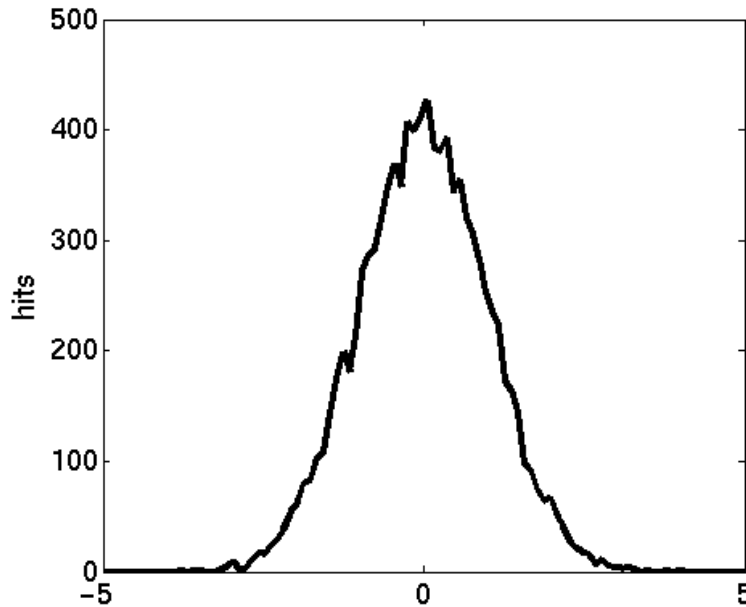
- **Absolute jitter** := Time difference between real and ideal edges

# Absolute Jitter : Time Domain Meas



- ❑ Clock recovery or PLL with clean up function (included in most high-end scopes)
- ❑ Scope measures the time difference from trigger to data
- ❑ Results are typically shown in form of histograms

# Estimation of RMS from Histogram (1/3)



- Estimated 1-sigma  $\hat{\sigma}_{ABS} = \sqrt{\frac{1}{N-1} \sum_{i=1}^N j_{ABS}^2(i)}$
- N = number of hits in histogram
- The larger N, the more accurate the estimation

# Estimation of RMS from Histogram (2/3)

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- ❑ A coffee company asks you to estimate the average size of coffee beans of a farm. **You'll have to pay a fine if you are wrong.**
- ❑ Take "N" coffee beans, measure, average = **11mm**
- ❑ You want to reserve a **margin for error** = **+/- 1mm** (this is called **Confidence Interval, CI**)
- ❑ **How sure are you?** You repeat the experiment lots of times (on different beans) and find out that in **90%** of the cases the average is indeed within 11mm +/- 1mm. This "90%" is called **Confidence Level, CL**.
- ❑ With larger N => CI can be tighter or CL improves

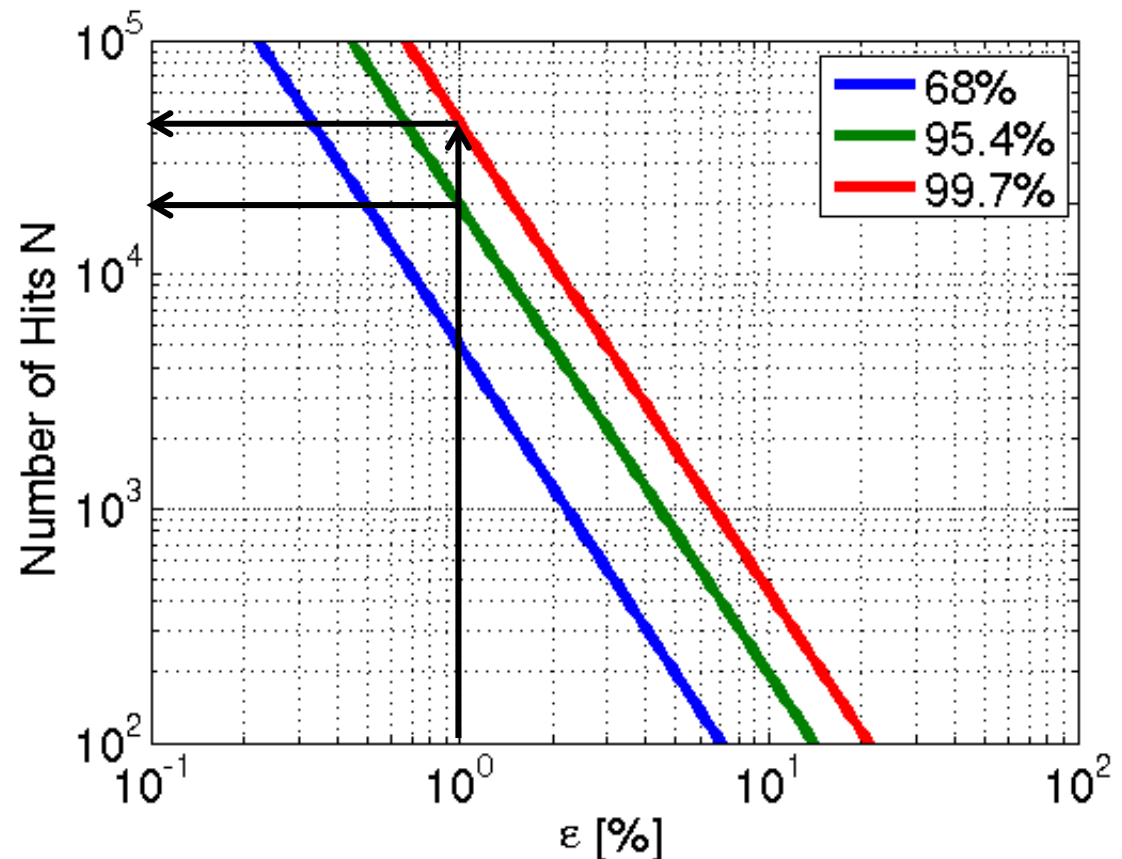


# Estimation of RMS from Histogram (3/3)

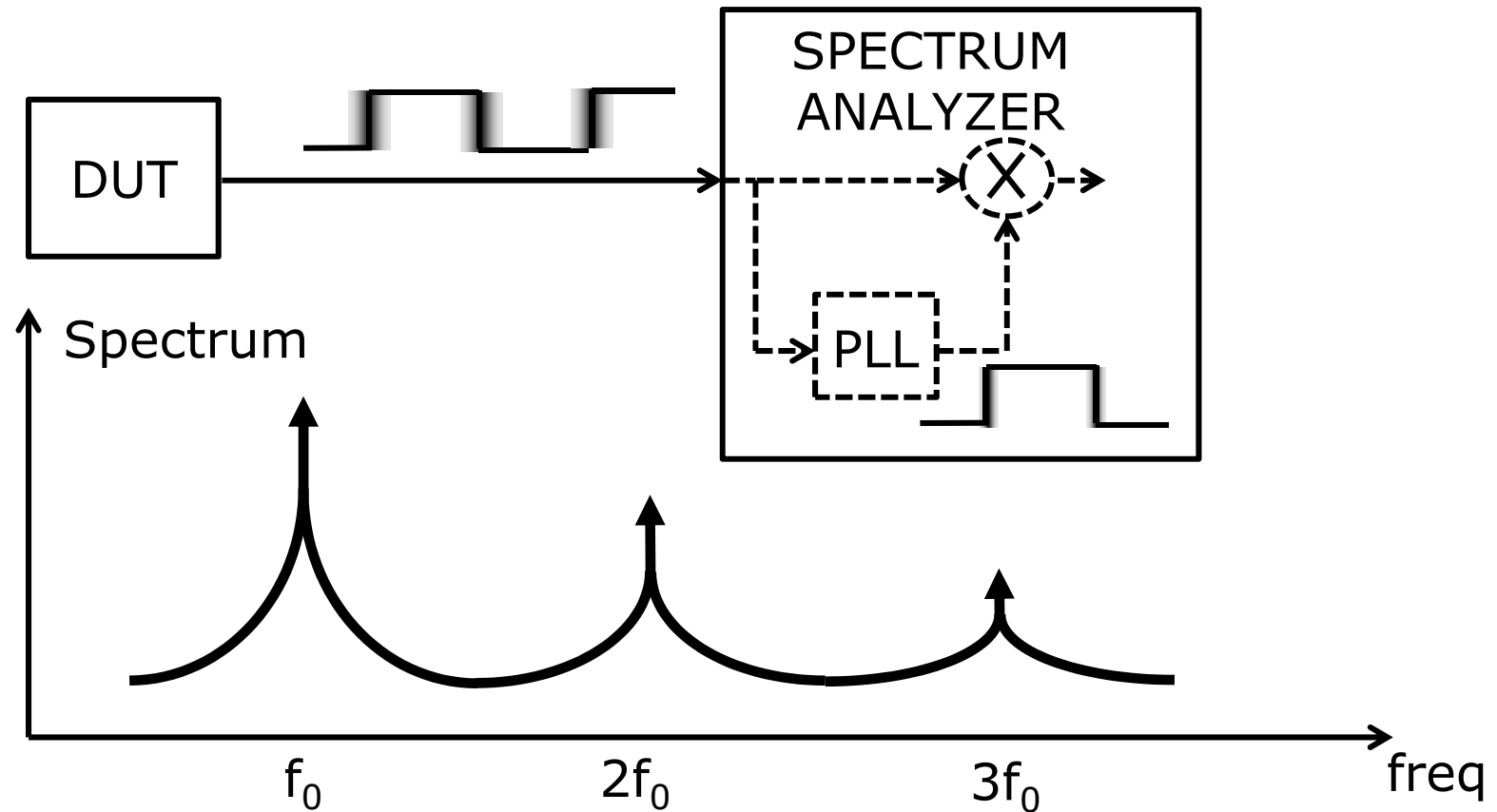
- Goal: Estimate the RMS within an error margin  $\varepsilon$  (confidence interval) and with a given confidence level
- Question: how large should N be?

$$N \geq \frac{1}{2} \left( \frac{Q}{\varepsilon} \right)^2$$

- Confidence Level:
  - 68% (Q=1)
  - 95% (Q=2)
  - 99.7% (Q=3)

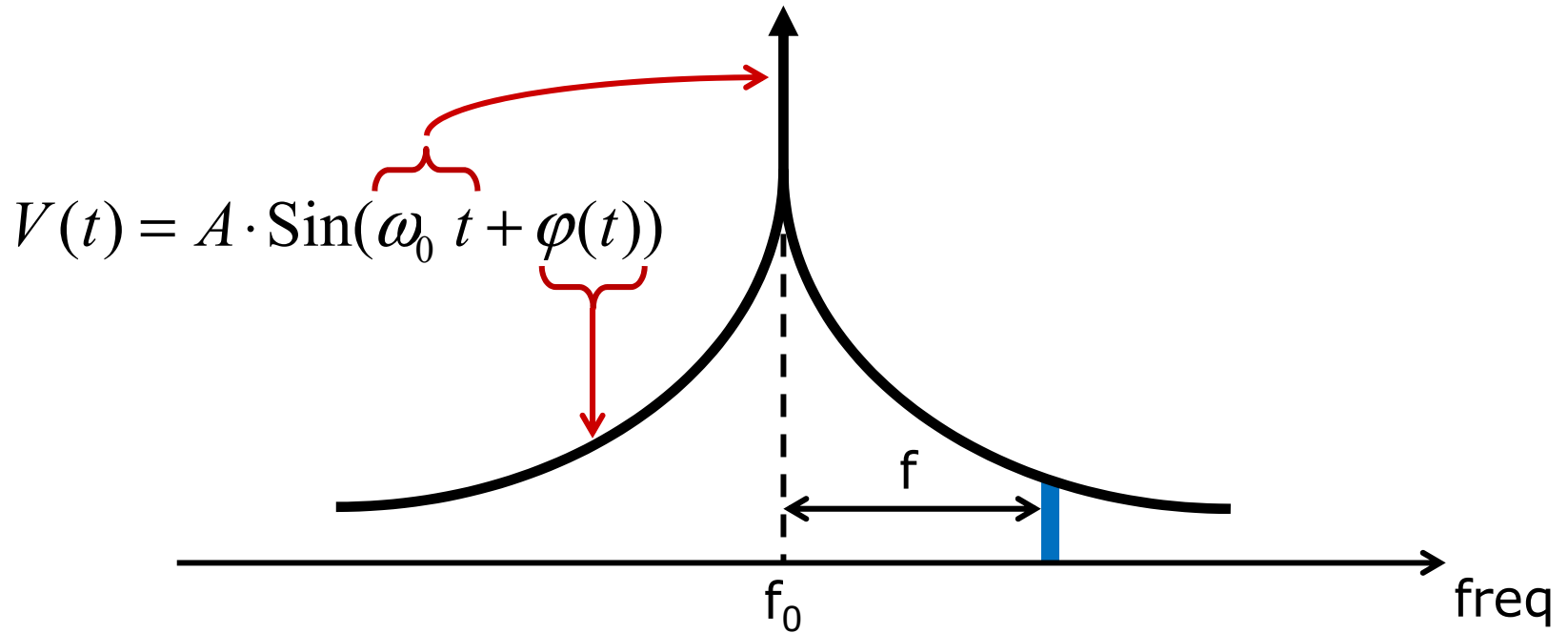


# Absolute Jitter : Freq Domain Meas



- ❑ Spectrum analyzer locks to ext clock and generates clean one
- ❑ Clean clock is used to mix ext clock and measure its power
- ❑ Result is shown in form of spectrum of voltage signal

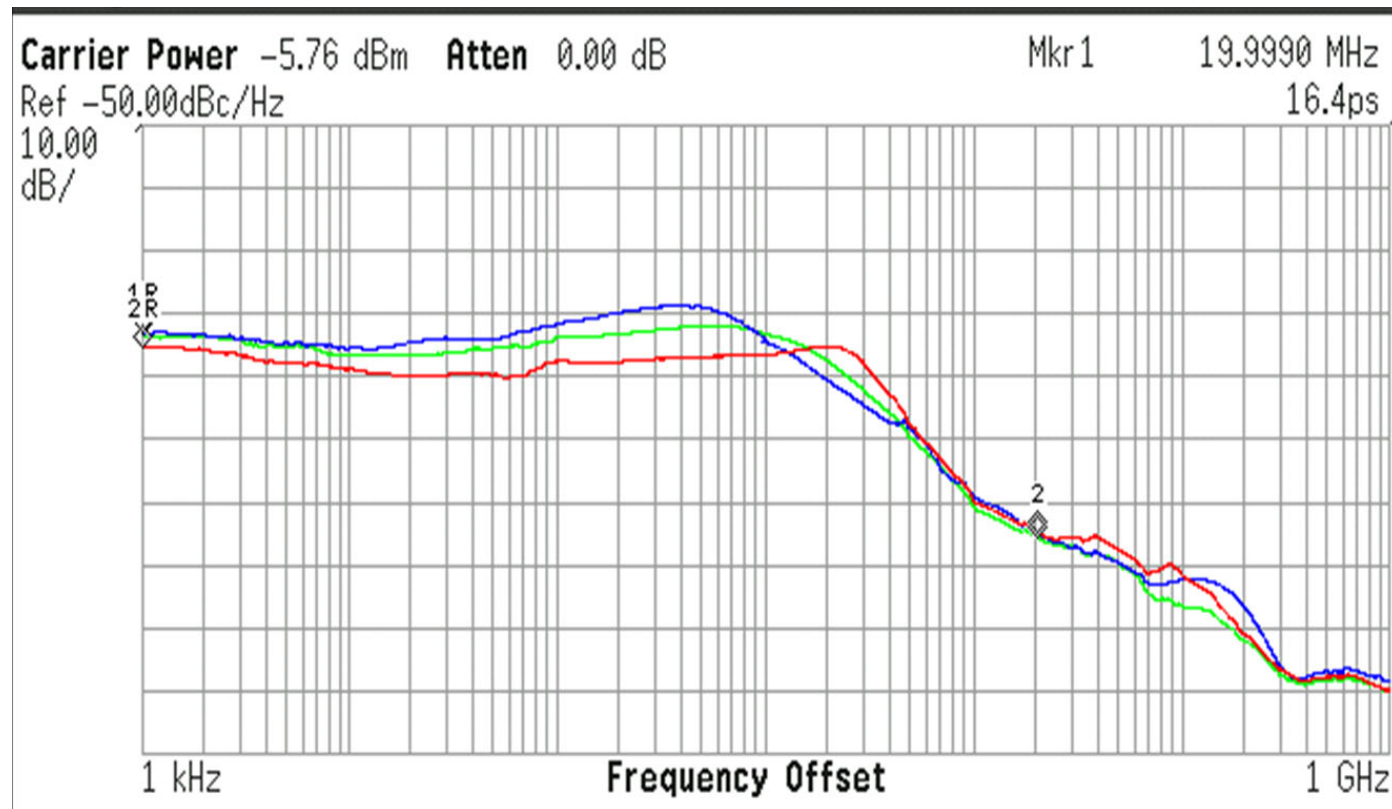
# Voltage Spectrum to Phase PSD



$$\frac{\text{Power of in 1 Hz BW @ } f \text{ from Carrier}}{\text{Power of Carrier}} \cong \text{PSD of } \varphi, S_{\varphi}(f)$$

- Assumptions: No AM, Small angle modulation
- PSD is Single Side Band

# Phase Noise



- ❑ PSD of  $\phi$  is called **Phase Noise,  $L(f)$**
- ❑ Shown in log-log scale vs. frequency from carrier
- ❑ Automatically computed by most spectrum analyzers

# Phase Noise to Absolute Jitter

$$V(t) = A \cdot \sin(\omega_0 t + \varphi(t)) = A \cdot \sin \left[ \omega_0 \left( t + \underbrace{\frac{\varphi(t)}{\omega_0}} \right) \right]$$

Time deviation wrt ideal clock,  
Absolute jitter

$$j_{ABS} = \frac{\varphi}{\omega_0}$$

PSD

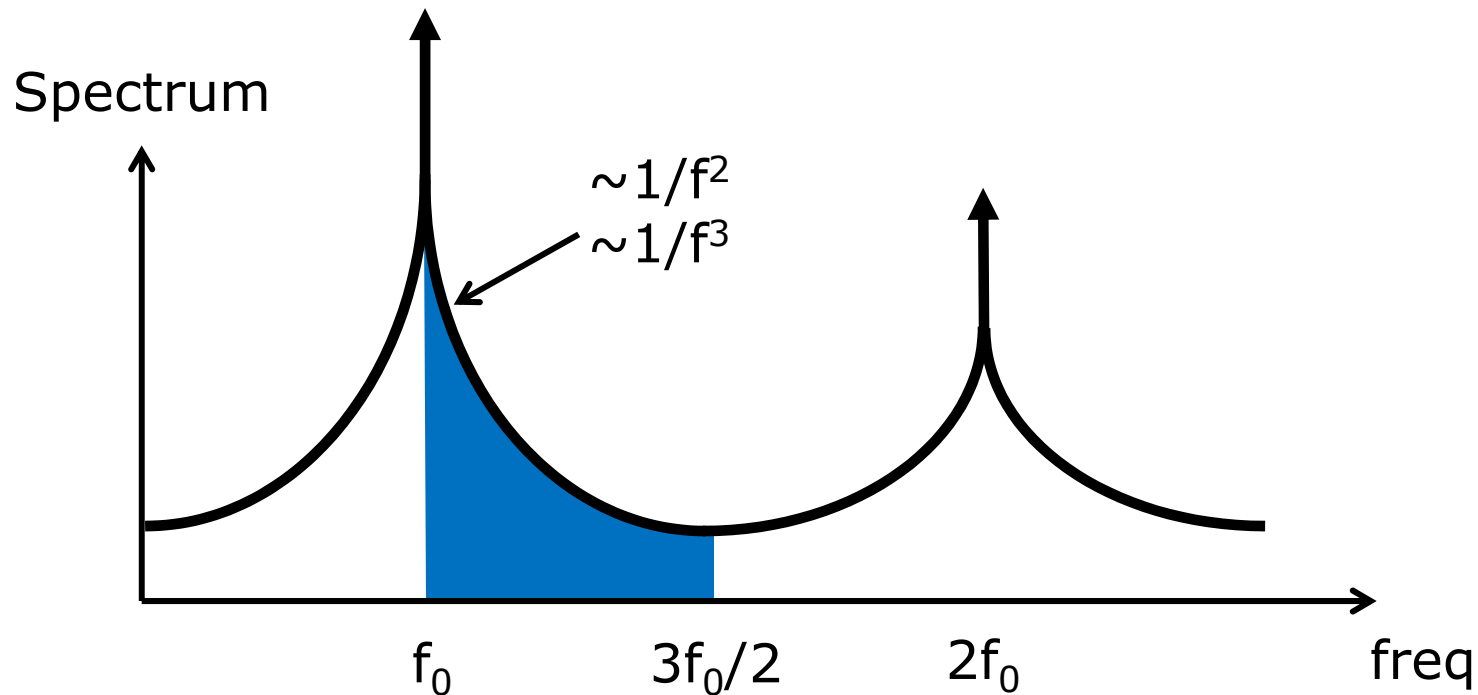
$$S_{j_{ABS}}(f) = \frac{L(f)}{\omega_0^2}$$

Wiener-Khinchin Theorem

$$\sigma_{ABS}^2 = \int_{-\infty}^{+\infty} S_{j_{ABS}}(f) df$$

$$\sigma_{ABS}^2 = \frac{2}{\omega_0^2} \int_0^{+\infty} L(f) df$$

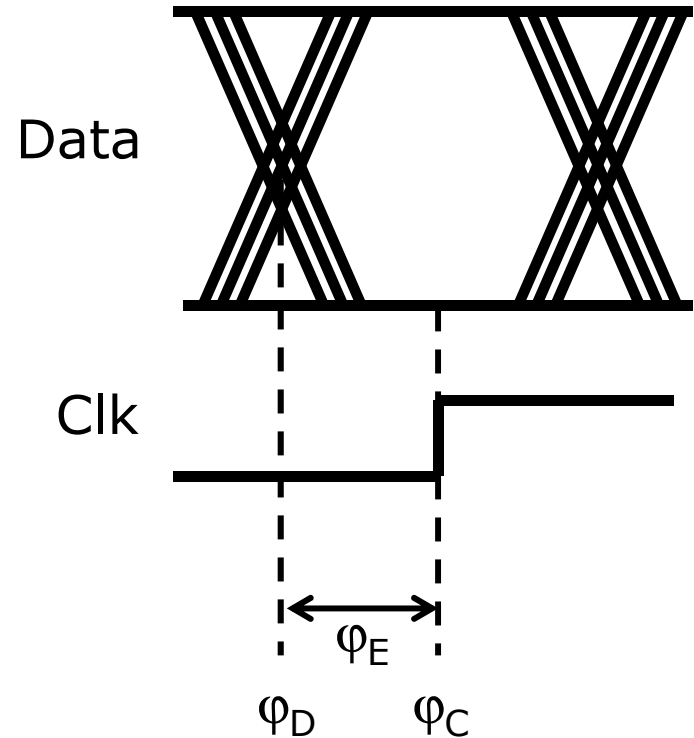
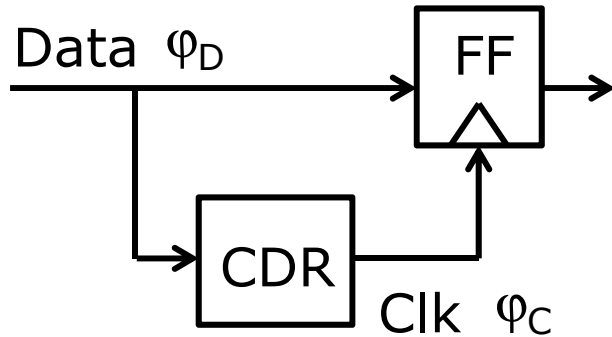
# Phase Noise integration limits



$$\sigma_{ABS}^2 = \frac{2}{\omega_0^2} \int_{f_{MIN}}^{f_0/2} L(f) df \quad \sigma_{ABS}^2 = \frac{2}{\omega_0^2} \int_0^{+\infty} \tilde{L}(f) |H_{SYS}(f)|^2 df$$

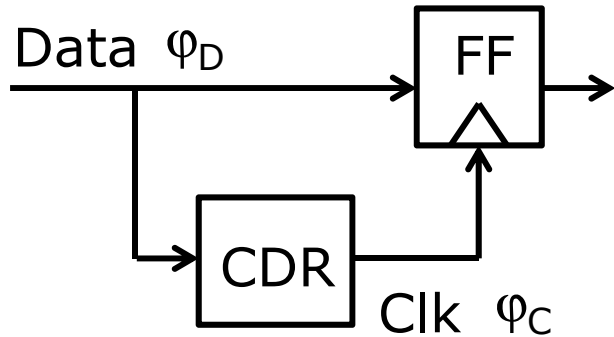
- Hard limits ( $f_{MIN}$  to  $f_0/2$ )
- Jitter transfer function of system under investigation

# Jitter filtering in CDR systems (1/3)



- ❑ CDR system is sensitive to  $\varphi_E$
- ❑ Low frequency jitter is tracked by CDR, does not hurt the BER
- ❑ High frequency jitter is not tracked by CDR, hurts the BER

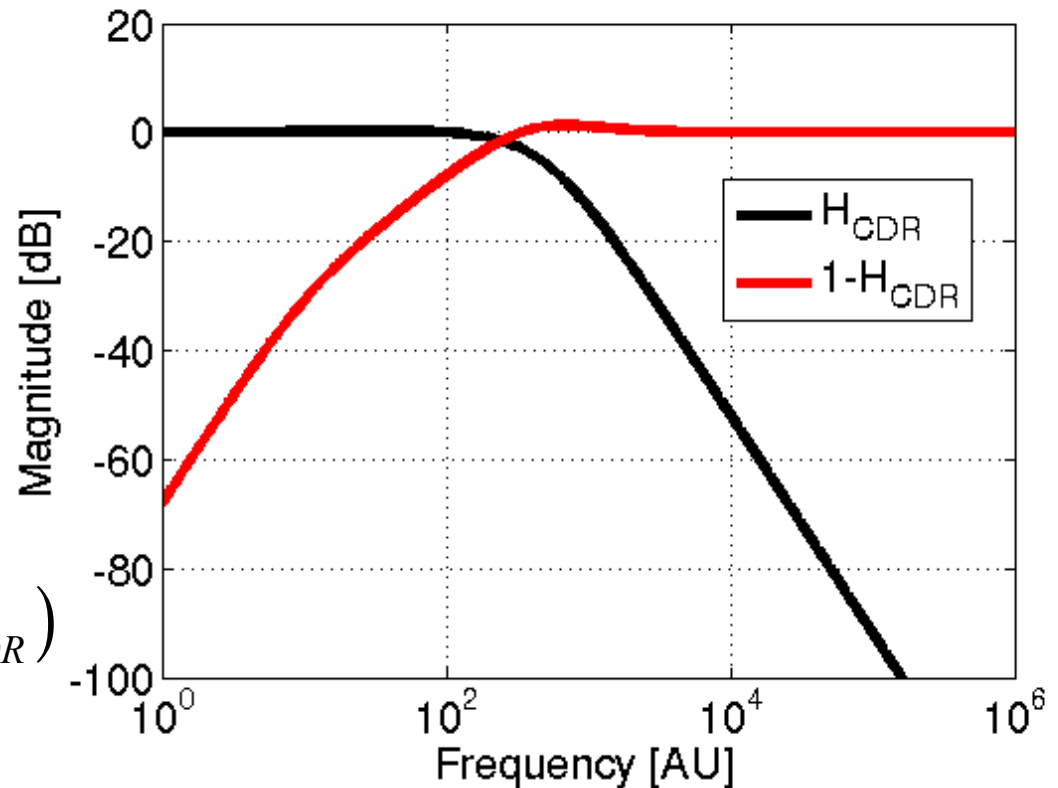
# Jitter filtering in CDR systems (2/3)



$$\varphi_C = H_{CDR} \cdot \varphi_D$$

$$\varphi_E = \varphi_D - \varphi_C = \varphi_D (1 - H_{CDR})$$

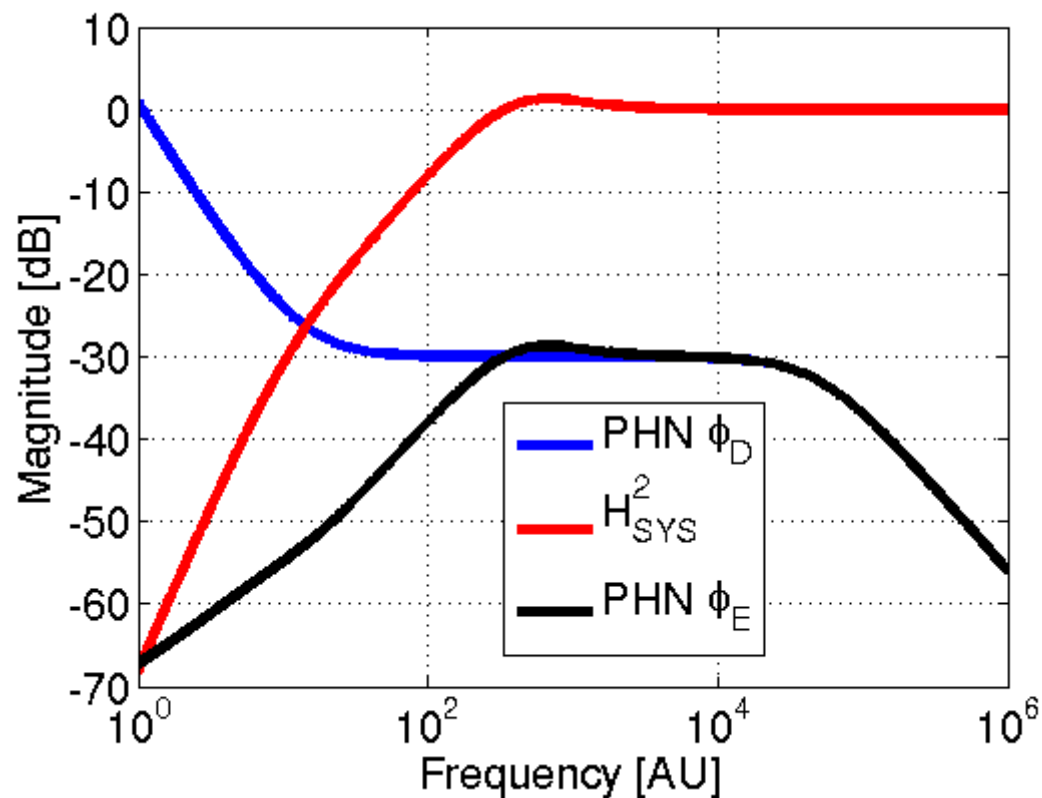
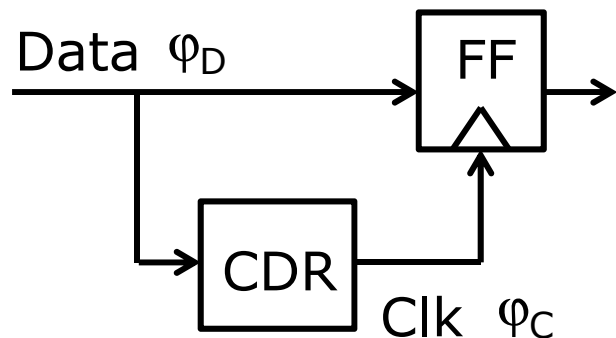
$$H_{SYS} = 1 - H_{CDR}$$



- Transfer function of CDR is a low-pass
- Transfer function of system ( $\varphi_D$  to  $\varphi_E$ ) is a high-pass



# Jitter filtering in CDR systems (3/3)



- ❑ Low-frequency jitter is filtered by the  $H_{\text{SYS}}$  before integration
- ❑ Integral from 0 to  $f_0/2$  of Phase Noise of  $\phi_E$  is finite

# Summary so far

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- ❑ Absolute jitter
- ❑ Measurement in time domain
- ❑ Estimation of 1-sigma value from histogram
- ❑ Measurement in frequency domain
- ❑ Phase Noise
- ❑ Relationship between Phase Noise and Absolute jitter
- ❑ Limitation of Phase Noise integration interval

## **What's next:**

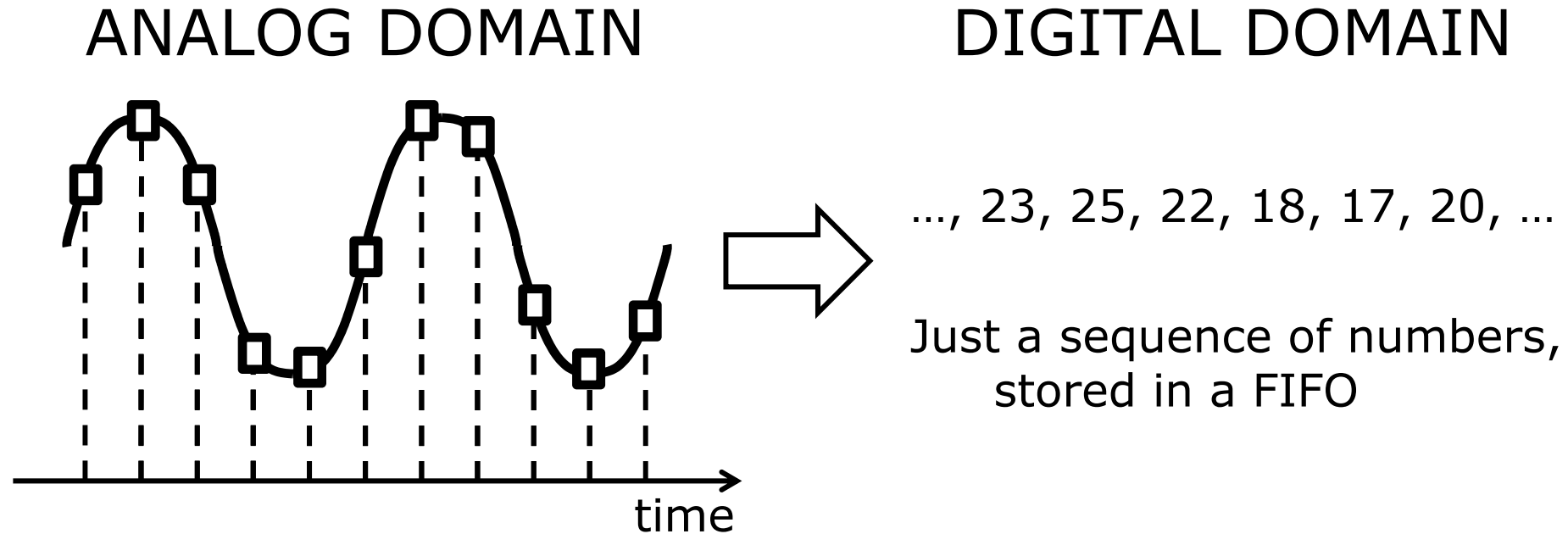
- ❑ Analyze typical applications and the type of jitter they are sensitive to
- ❑ New jitter types will be introduced

# Application Scope of Absolute Jitter

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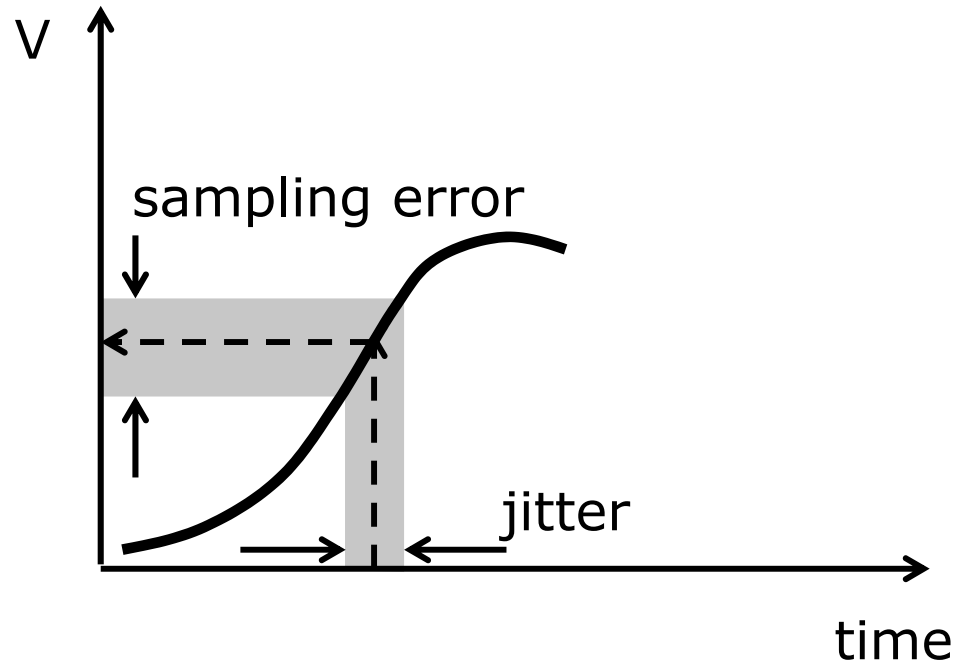
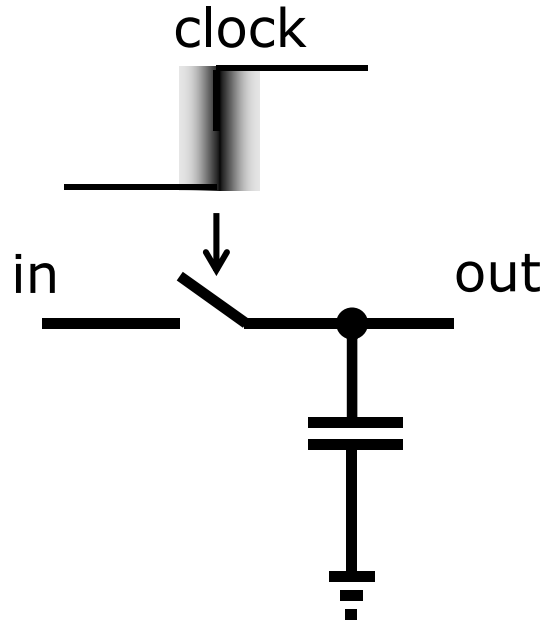
- ❑ RX for High Speed interfaces (see example of CDR before)
- ❑ Sampling of analog signals (i.e. in ADC)
- ❑ Generation of analog signal from digital (i.e. in DACs)
- ❑ **NOT** applicable to digital clocking

# Sampling of Analog Signals (1/2)



- ❑ In the digital domain the time information of the sampling instant is lost
- ❑ The implicit assumption is that the samples are taken on the edges of an ideal clock
- ❑ What counts is the absolute jitter

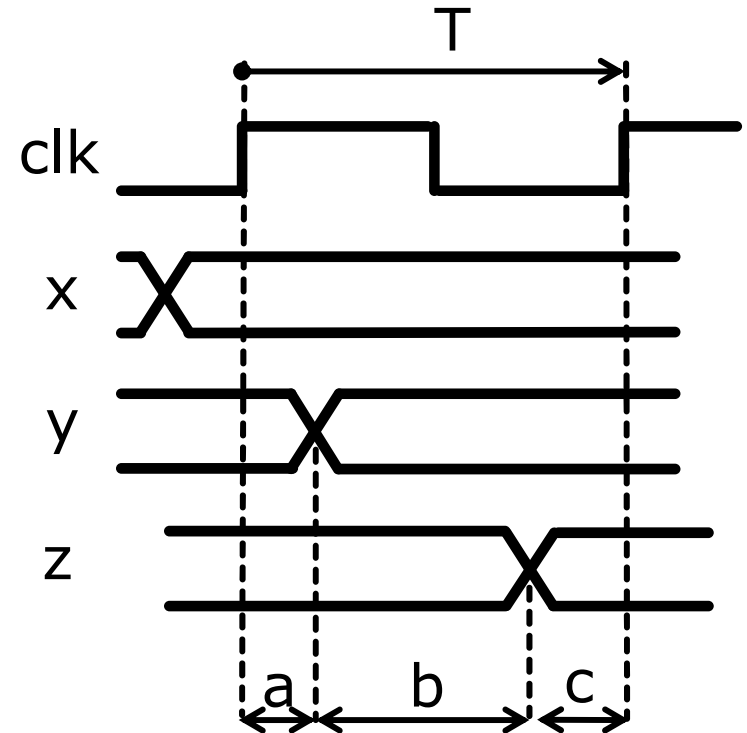
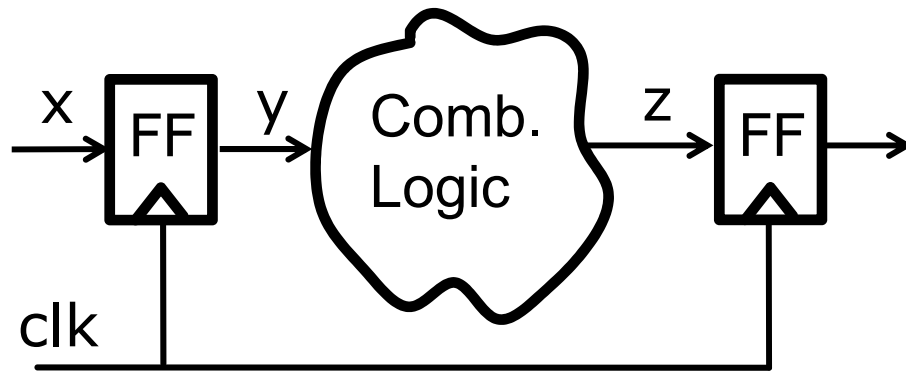
# Sampling of Analog Signals (2/2)



- ❑  $SNR = P_{\text{signal}} / P_{\text{error}}$
- ❑  $P_{\text{error}} = P_{\text{jitter}} * \langle \text{Slope}^2 \rangle$
- ❑  $P_{\text{jitter}} = \sigma_{\text{ABS}}^2$
- ❑  $SNR \sim 1 / \sigma_{\text{ABS}}^2$

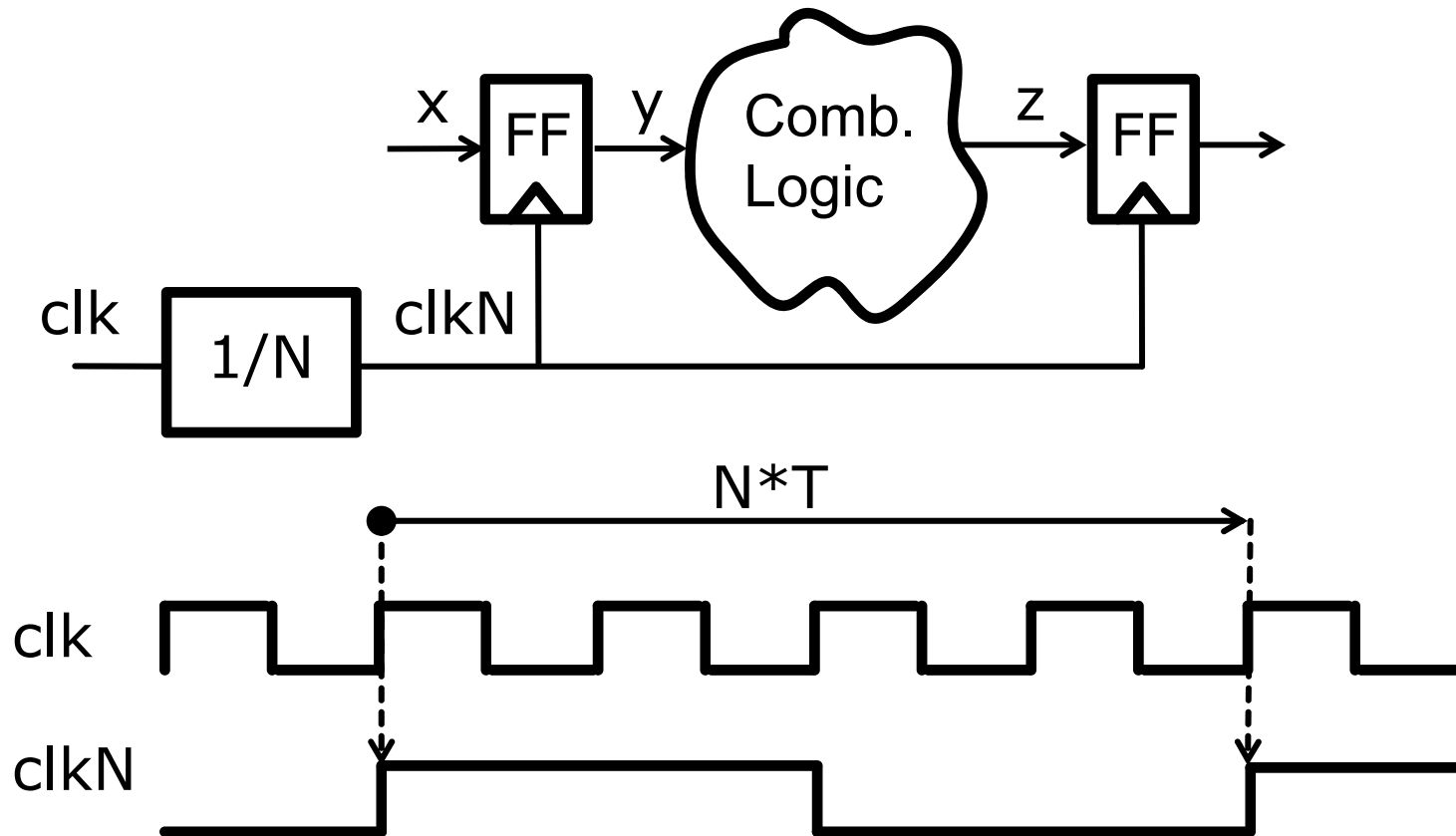
$$SNR = 20 \cdot \log_{10} \left( \frac{1}{2\pi f_{\text{ANA}} \sigma_{\text{ABS}}} \right) \text{dB}$$

# Digital Clocking (1/2)



- $a = clk2Q$
- $b = \text{delay of logic}$
- For correct operation:  $c > \text{FF setup time}$
- Constraint set on minimum duration of the clock period  $T$  (from edge to edge)

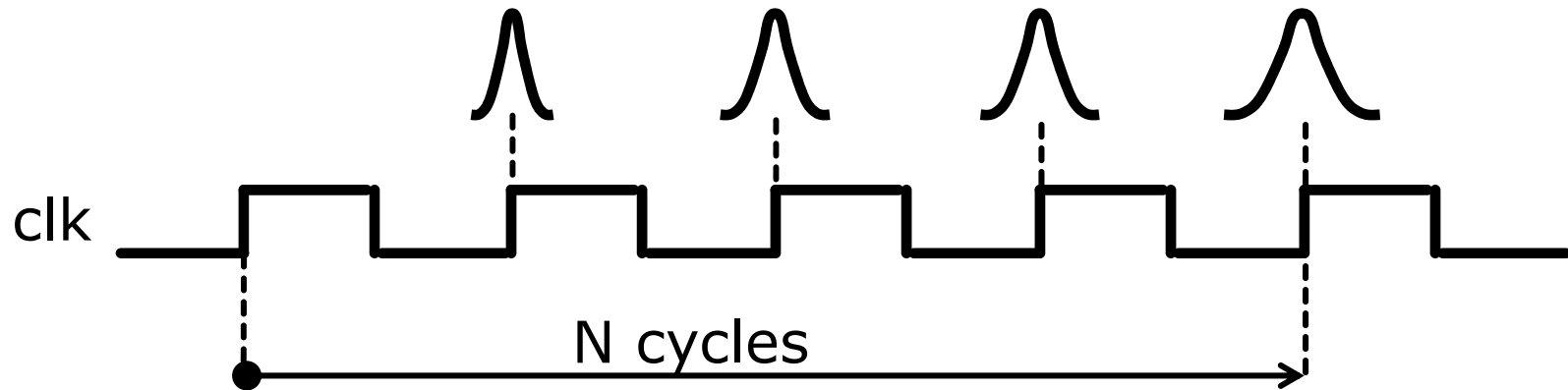
# Digital Clocking (2/2)



- Constraint set on minimum duration of  $N$  clock periods (from edge to the next  $N$ th edge)

# Accumulated Jitter ( $J_{ACC}$ )

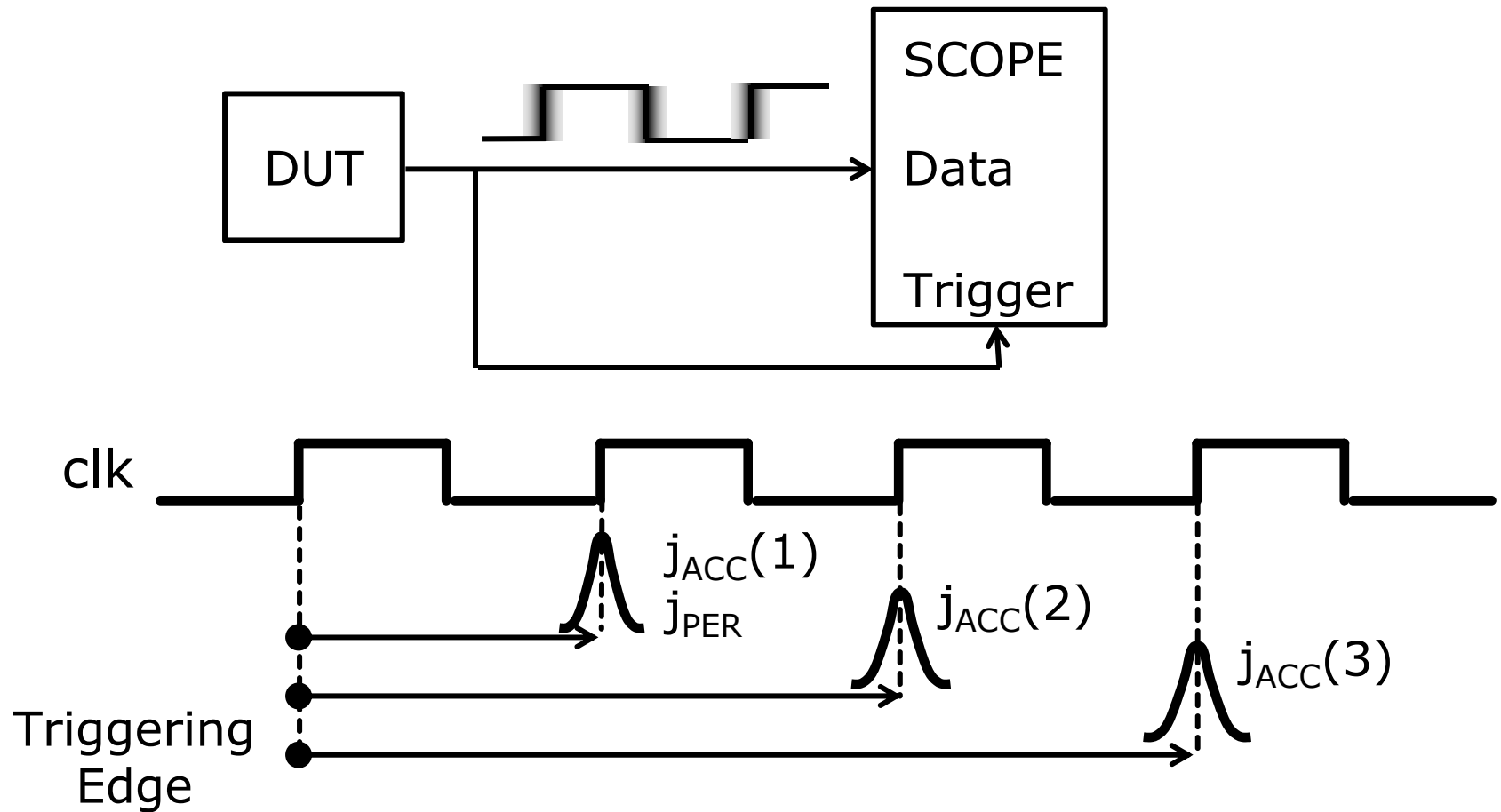
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- ❑ The jitter of one edge relative to the Nth previous edge is called **Accumulated Jitter on N Cycles,  $j_{ACC}(N)$**
- ❑ The Accumulated Jitter on 1 cycle is commonly called **Period Jitter,  $j_{PER}$**  (or Edge-to-Edge Jitter)



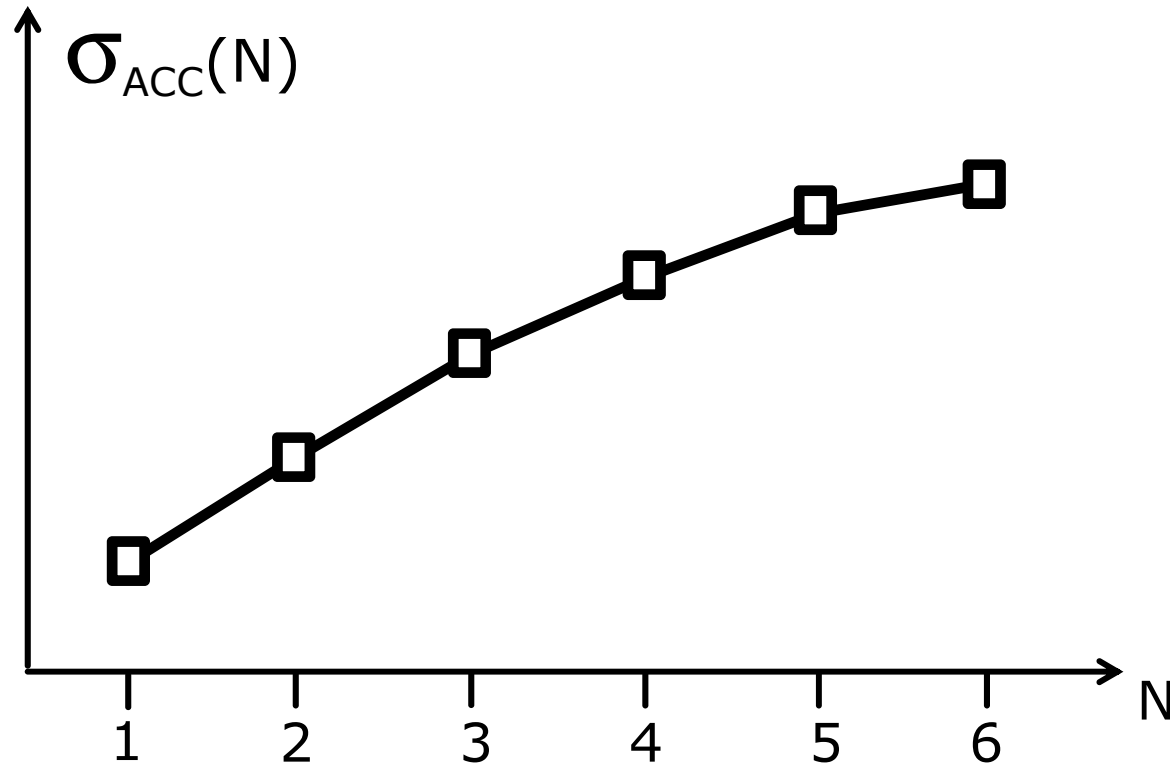
# $J_{ACC}$ : Time Domain Measurement (1/2)



□ Self-referenced measurement

# $J_{\text{ACC}}$ : Time Domain Measurement (2/2)

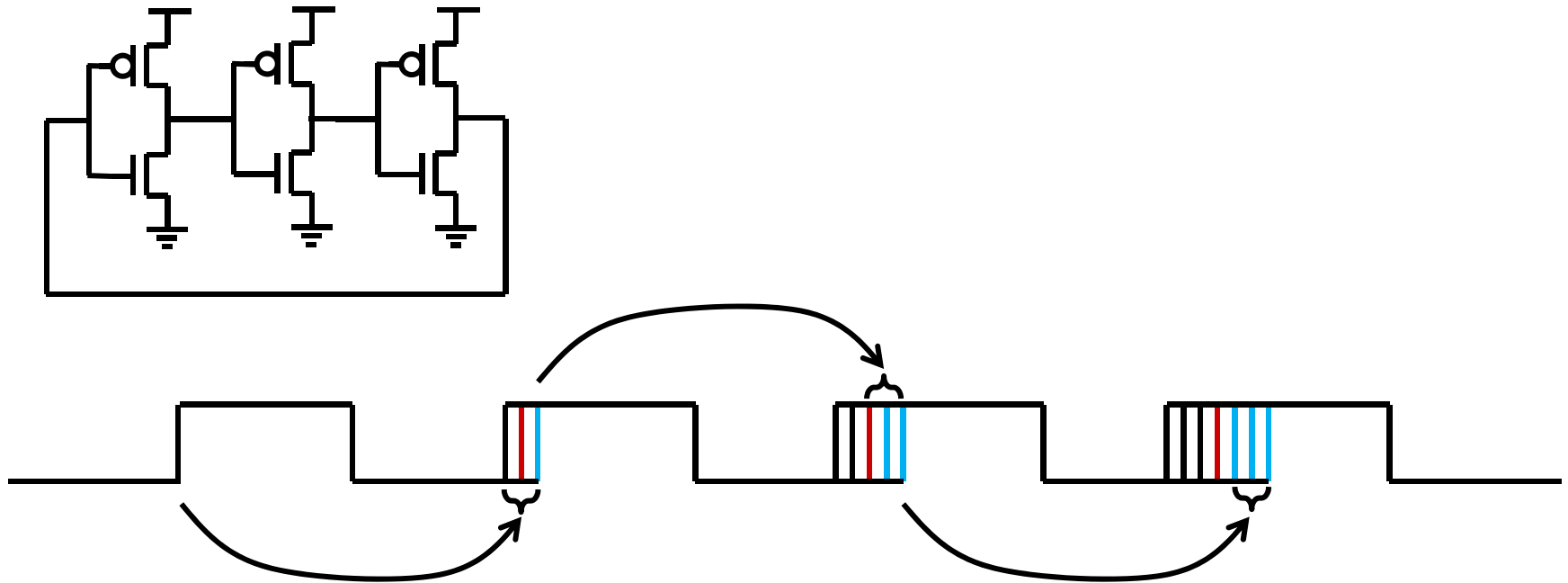
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- Function of number of cycles  $N$
- In real physical systems  $J_{\text{acc}}$  increases for low  $N$

# $J_{ACC}$ in Freerunning Oscillators (1/3)

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- ❑ Jitter on Edge (n) = Jitter on Edge (n-1) + Period Jitter
- ❑ Noise in each new cycle is uncorrelated
- ❑ Jitter accumulates over cycles
- ❑ No mechanism can counteract the accumulation

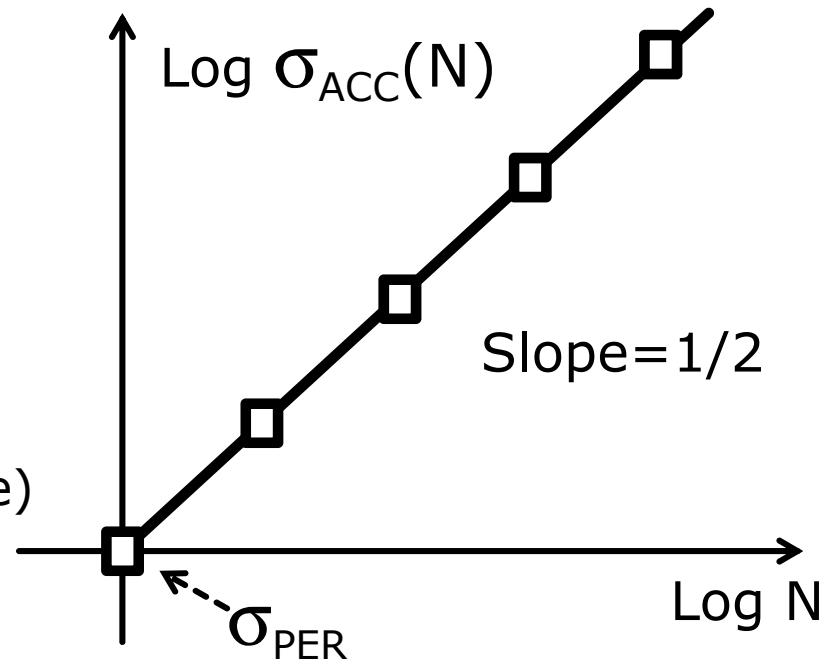
# $J_{ACC}$ in Freerunning Oscillators (2/3)

$$j_{ACC}(N) = j_{PER}(1) + j_{PER}(2) + \dots + j_{PER}(N)$$

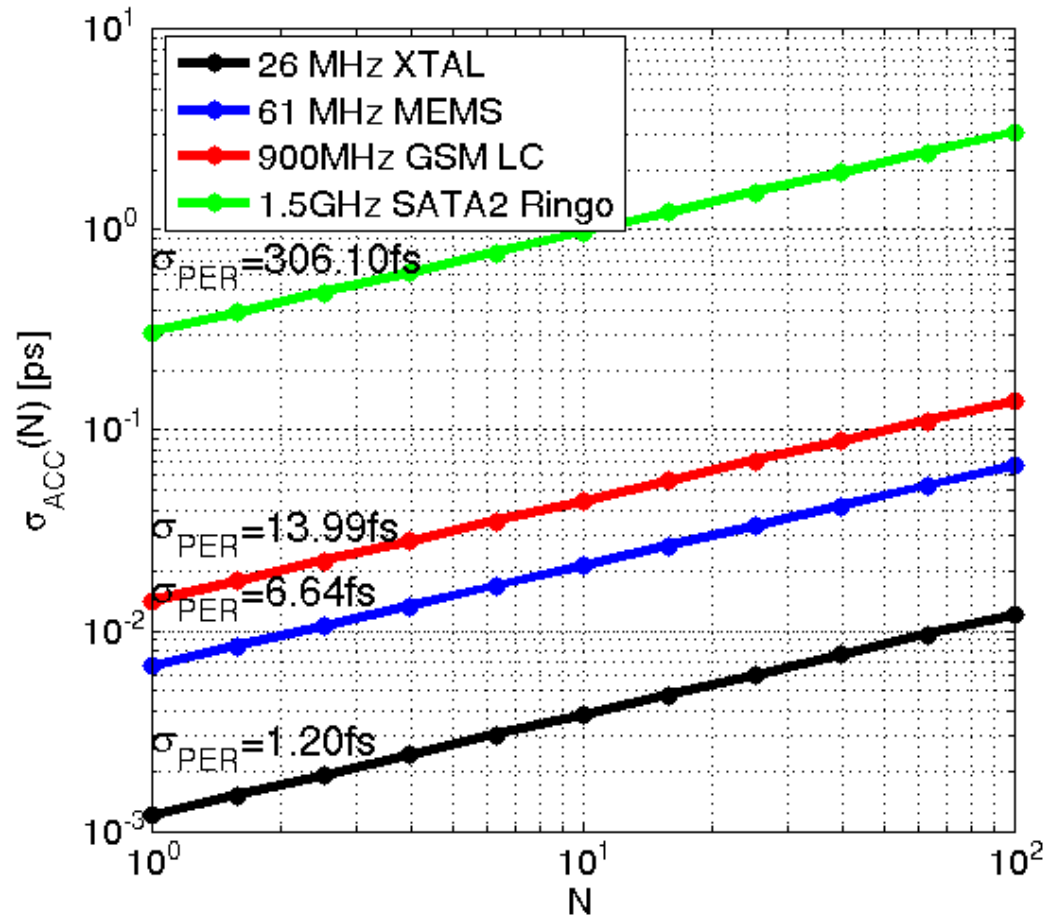
$$\sigma_{ACC}^2(N) = \sigma_{PER}^2(1) + \sigma_{PER}^2(2) + \dots + \sigma_{PER}^2(N)$$

$$\sigma_{ACC}(N) = \sqrt{N} \cdot \sigma_{PER}$$

- Assumptions
  - Jitter in different cycles is uncorrelated (e.g. white noise)
  - Jitter is time-independent (stationary random process)

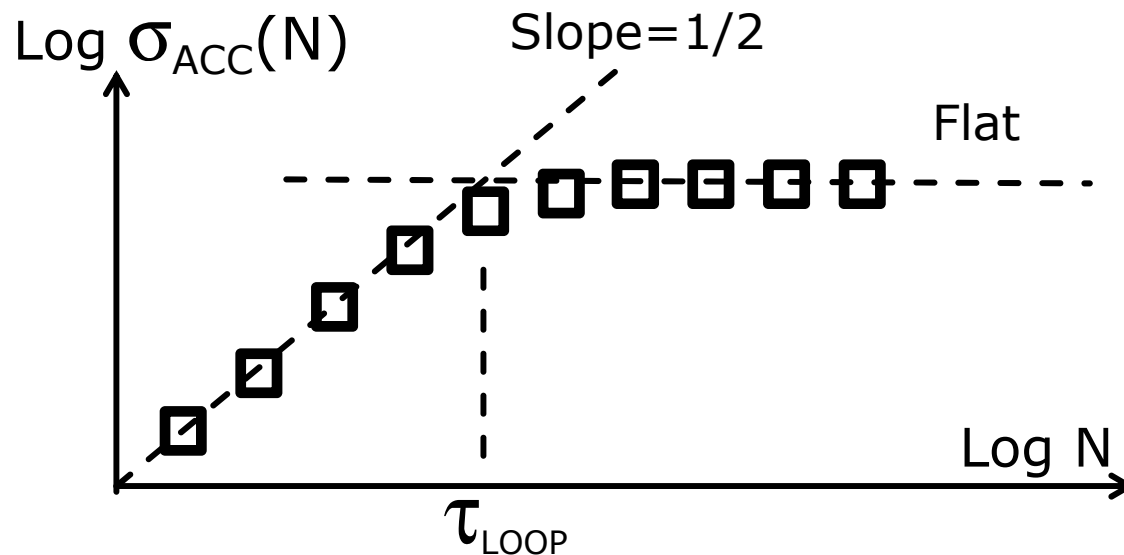
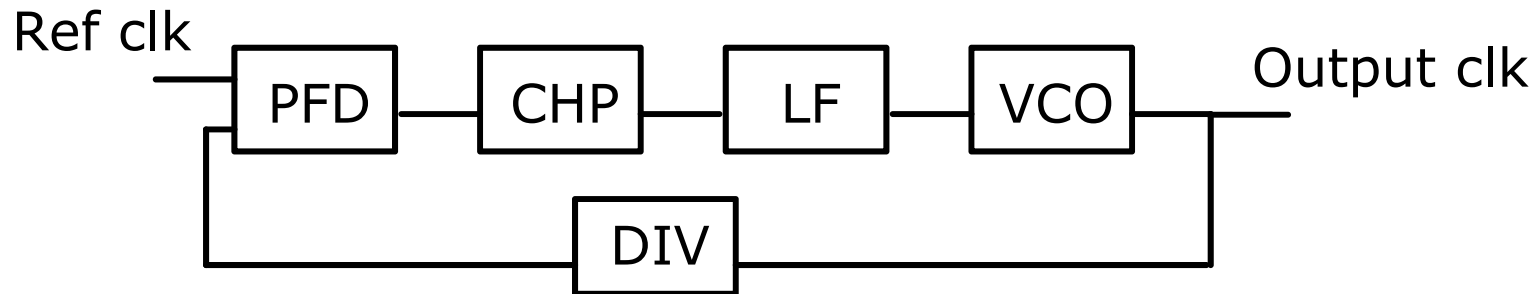


# $J_{ACC}$ in Freerunning Oscillators (3/3)



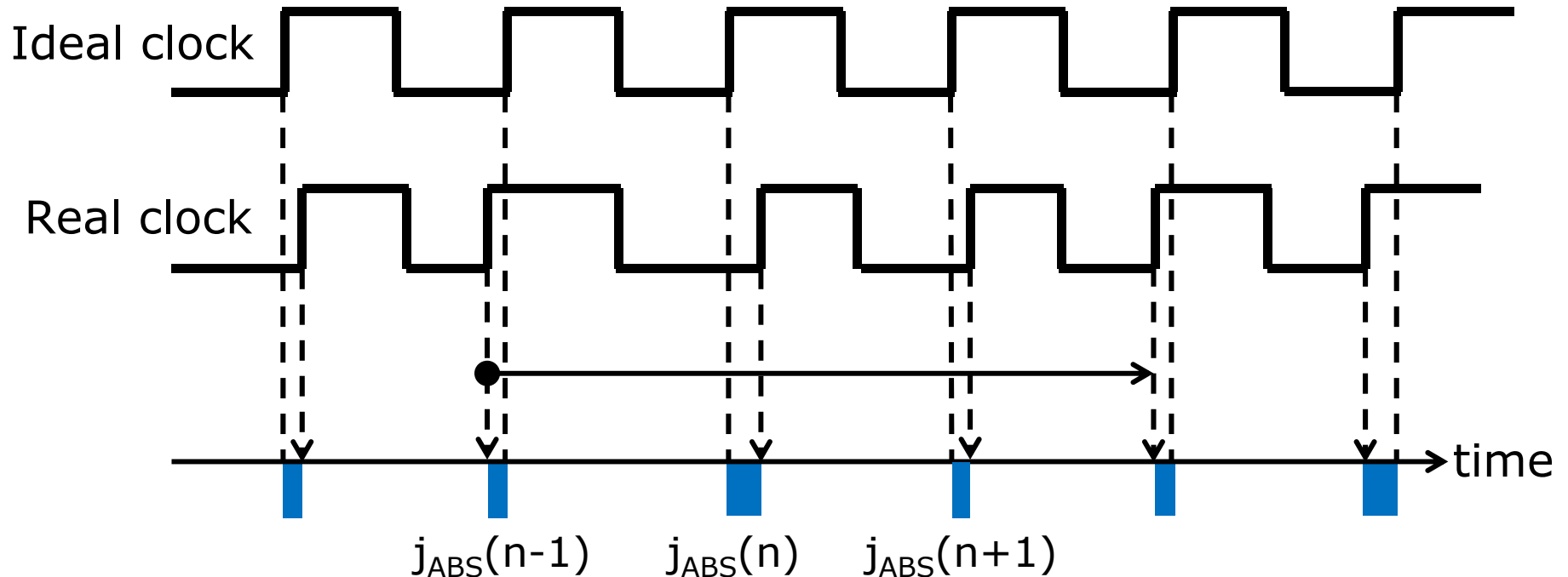
- 1.5G SATA2 RingO (5mA):
  - $-115\text{dBc/Hz}$  @  $10\text{e6}$
- 900MHz GSM LC (15mA):
  - $-138\text{dBc/Hz}$  @  $3\text{MHz}$
- 61MHz MEMS (1mA):
  - $-110\text{dBc/Hz}$  @  $1\text{kHz}$
- 26MHz XTAL (1mA):
  - $-136\text{dBc/Hz}$  @  $1\text{kHz}$
- Slope =  $1/2$  for all

# $J_{\text{ACC}}$ in PLLs



- Comparison with a clean reference limits jitter accumulation

# Accumulated vs Absolute Jitter



$$j_{ACC}(N) = j_{ABS}(n+N) - j_{ABS}(n)$$

$$j_{PER} = j_{ABS}(n+1) - j_{ABS}(n)$$

# Phase Noise to $J_{ACC}$

$$j_{ACC}(N) = j_{ABS}(n + N) - j_{ABS}(n) \longrightarrow \boxed{\sigma_{ACC}^2(N \gg 1) = 2 \sigma_{ABS}^2}$$



PSD

$$S_{j_{ACC}(N)}(f) = \left| 1 - e^{-i2\pi f N / f_0} \right|^2 \cdot S_{j_{ABS}}(f) = \frac{4}{\omega_0^2} \sin^2\left(\frac{\pi f N}{f_0}\right) L(f)$$

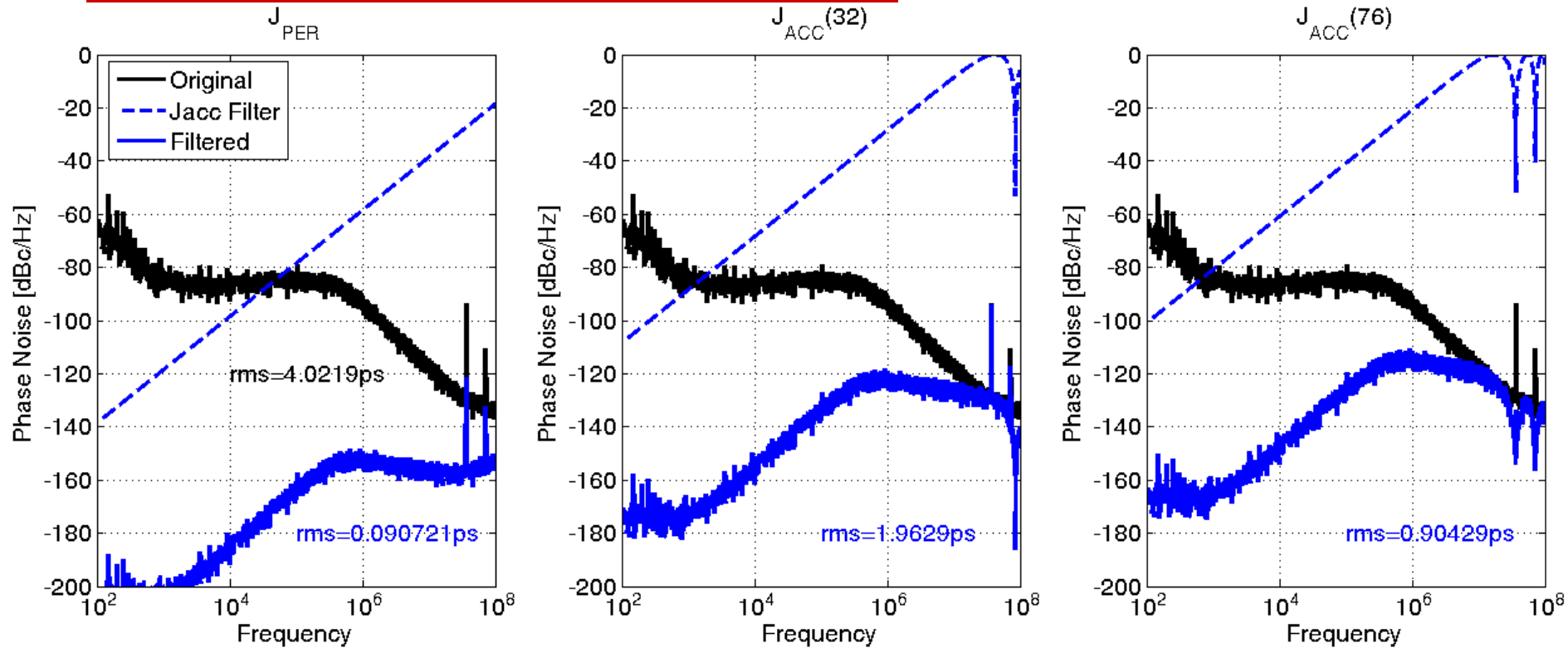


Wiener-Khinchin Theorem

$$\sigma_{ACC(N)}^2 = \frac{8}{\omega_0^2} \int_{f_{MIN}}^{f_0/2} \sin^2\left(\frac{\pi f N}{f_0}\right) L(f) df$$

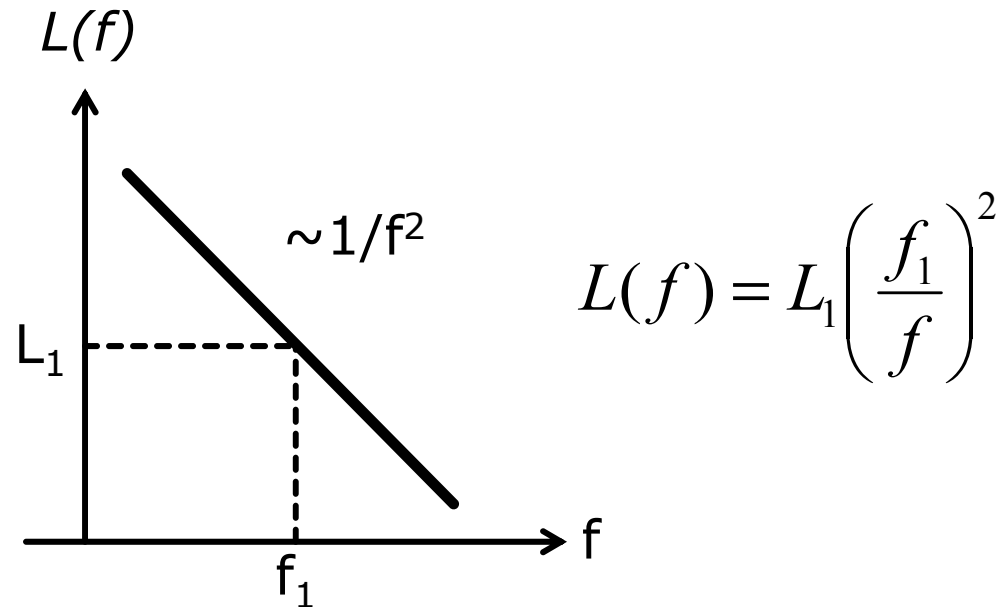


# Phase Noise to $J_{ACC}$ : Example



- PLL:  $F_{REF}=36MHz$ ,  $F_{OUT}=2736MHz$ ,  $DIV=76$
- $J_{PER}$  determined by high frequency components
- $J_{ACC}(N)$  determined by increasingly lower frequency components for increasing  $N$
- $J_{ACC}(N)$  can mask spurious tones

# Phase Noise to $J_{ACC}$ : Freerunning Oscillator

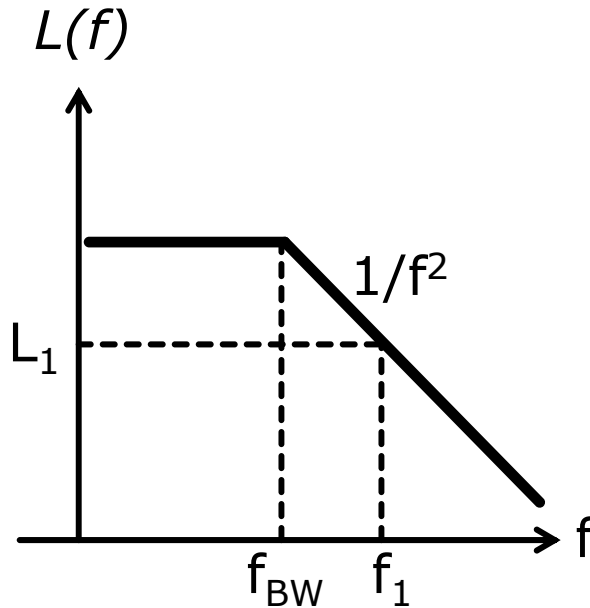


$$\sigma_{ACC(N)}^2 = \frac{L_1 f_1^2}{f_0^3} N$$

$$\sigma_{PER} = \sqrt{\frac{L_1 f_1^2}{f_0^3}}$$

□ RMS  $J_{acc}$  increases like  $\sqrt{N}$

# Phase Noise to $J_{ACC}$ : PLL



$$L(f) = \frac{L_1 f_1^2}{f_{BW}^2 + f^2}$$

$$\sigma_{ACC(N)}^2 = \frac{L_1 f_1^2}{f_0^3} \frac{f_0}{2\pi f_{BW}} \left(1 - e^{-2\pi f_{BW} N / f_0}\right)$$

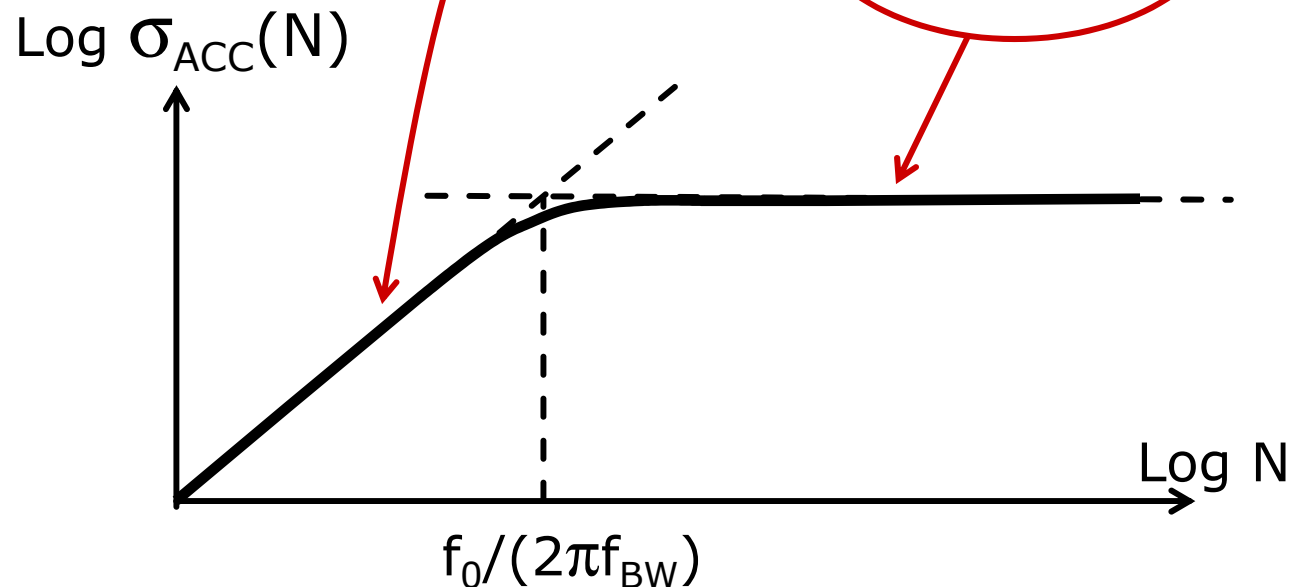
# Phase Noise to $J_{ACC}$ : PLL

□ For small N:

$$\sigma_{ACC(N)}^2 = \frac{L_1 f_1^2}{f_0^3} N = \sigma_{PER}^2 \cdot N$$

□ For large N:

$$\sigma_{ACC(N)}^2 = \sigma_{PER}^2 \frac{f_0}{2\pi f_{BW}}$$



# Summary so far

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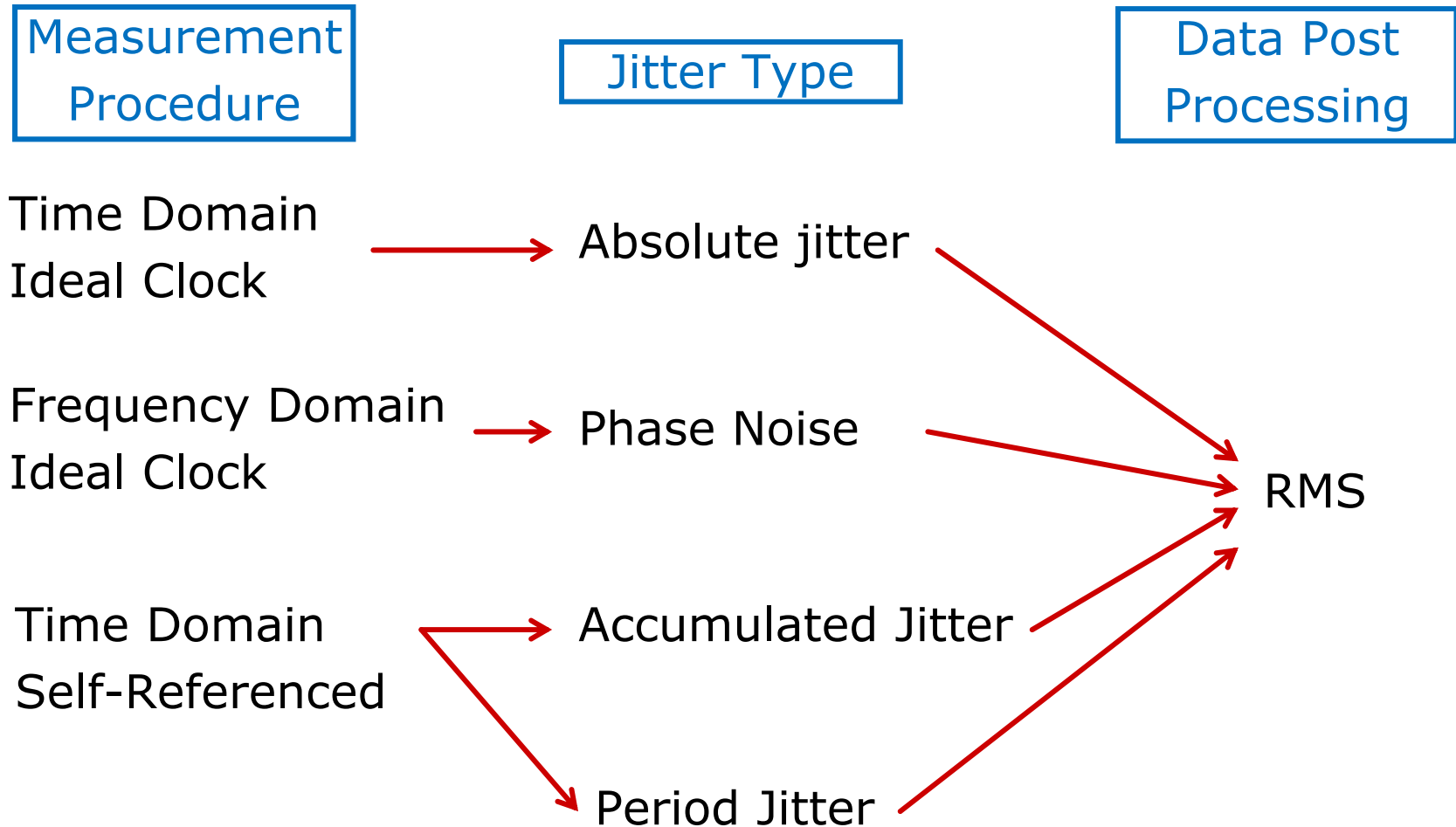
- ❑ Self-Referenced jitter (accumulated, period)
- ❑ Application to digital clocking
- ❑ Measurement in time domain
- ❑ Accumulated jitter for freerunning oscillators and PLLs
- ❑ Relation to absolute jitter
- ❑ How to compute  $J_{ACC}$  from Phase Noise

## **What's next:**

- ❑ Analyze the statistical properties of Jitter

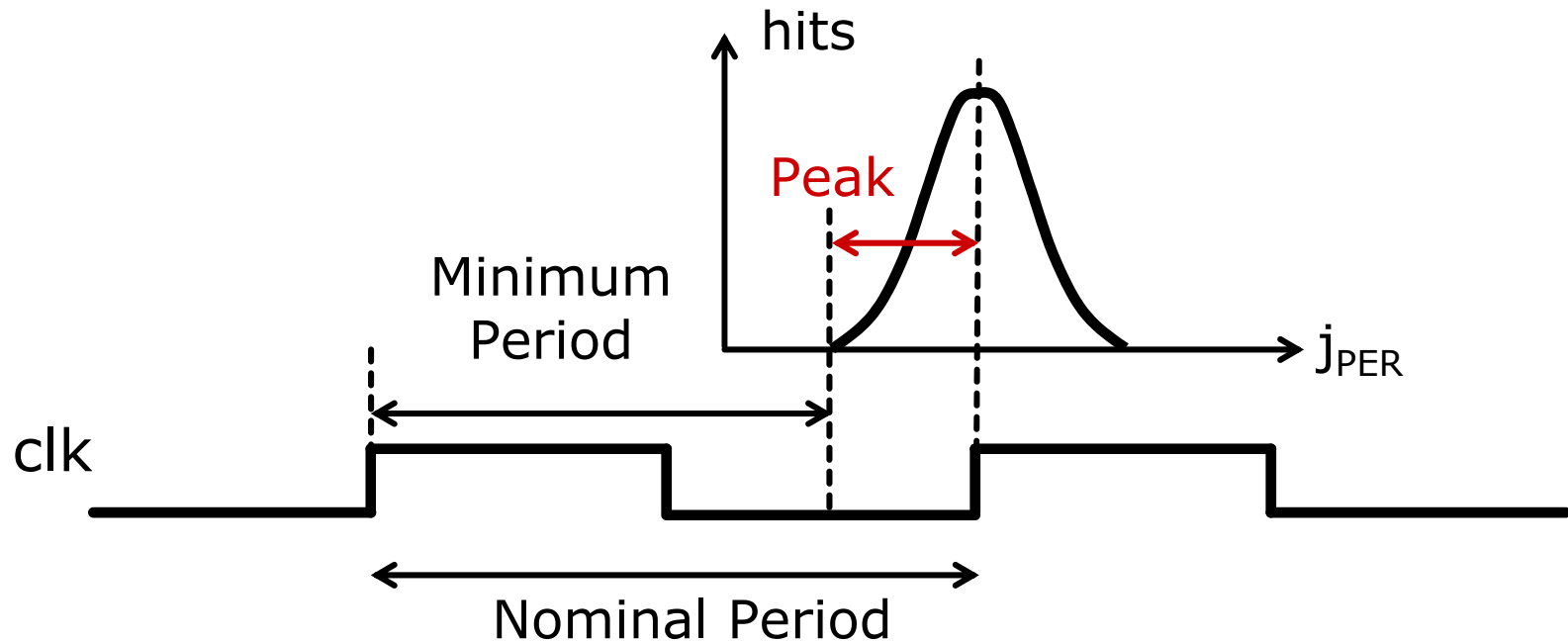
# Synoptics

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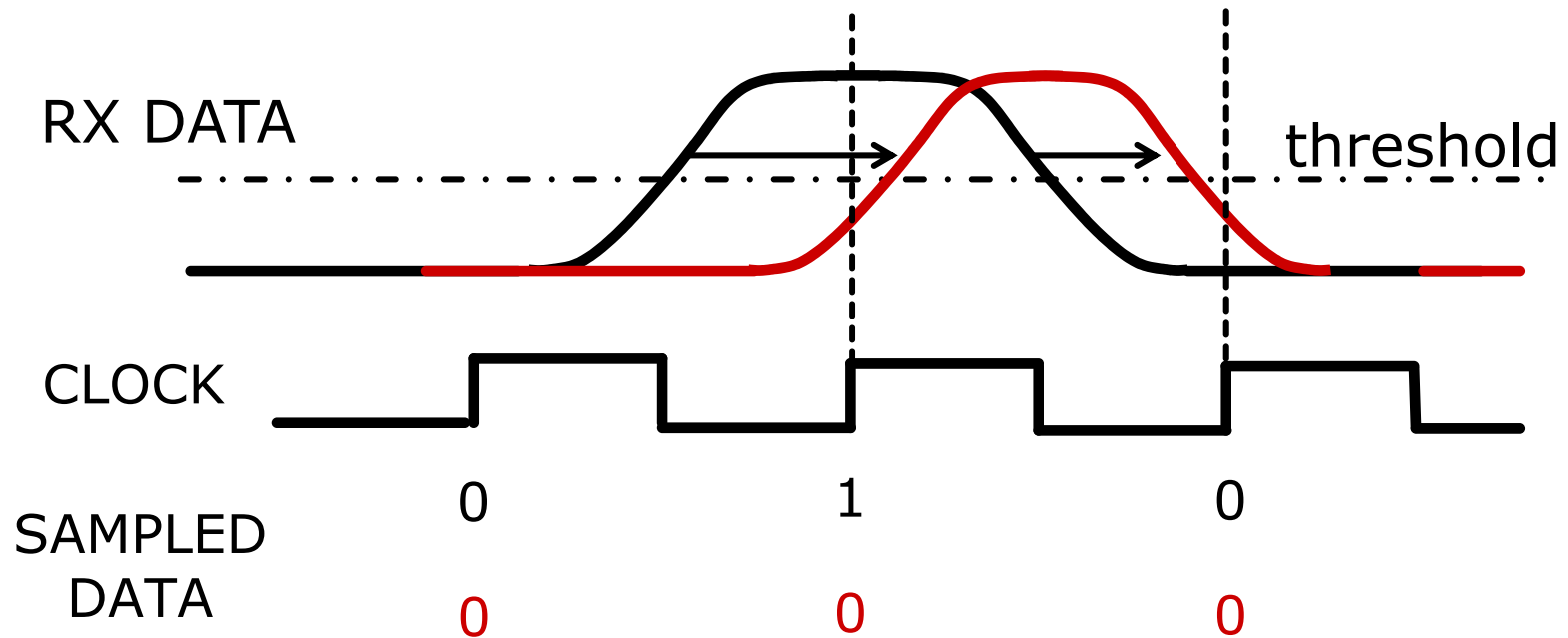
**How far can we go with the RMS value?**

# Digital time constraining



- ❑ Digital needs to satisfy setup-time for the digital block
- ❑ Looking for worst case minimum clock period
- ❑ The correct metrics is the peak, not the RMS!

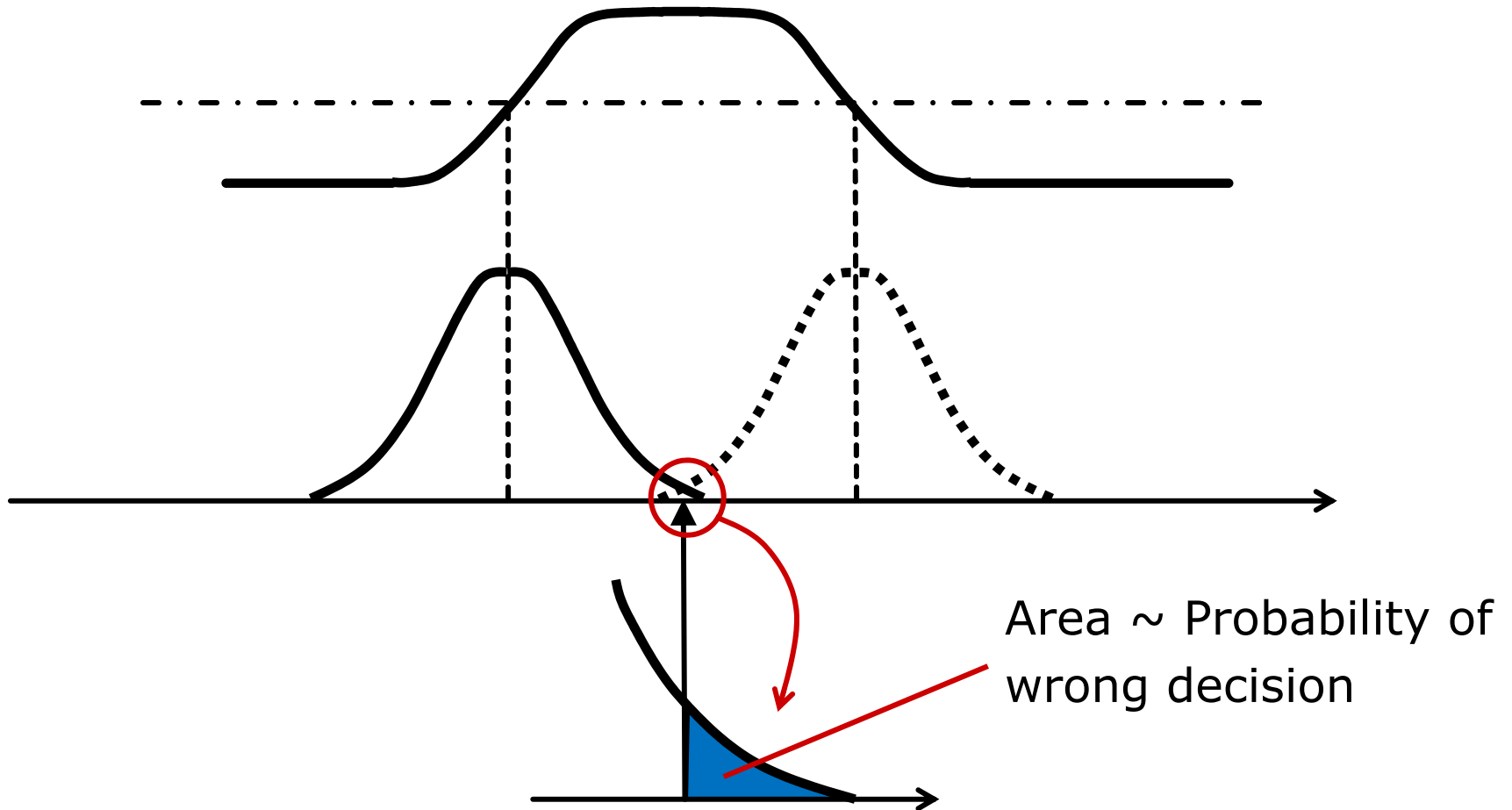
# High Speed Serial Data Sampling (1/2)



- ❑ Standards require Bit Error Rates  $\sim 1e-12$
- ❑ Jitter on RX data moves data transitions wrt sampling point
- ❑ If jitter on data edges is larger than 0.5 UI errors will occur



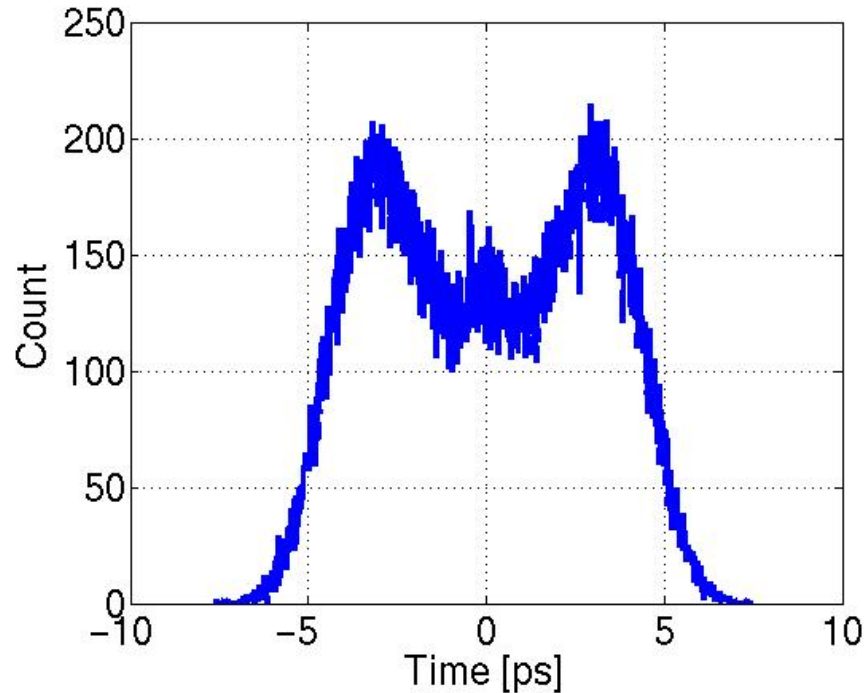
# High Speed Serial Data Sampling (2/2)



- ❑ BER determined by tails of the jitter distribution

# Jitter Histogram and Distribution

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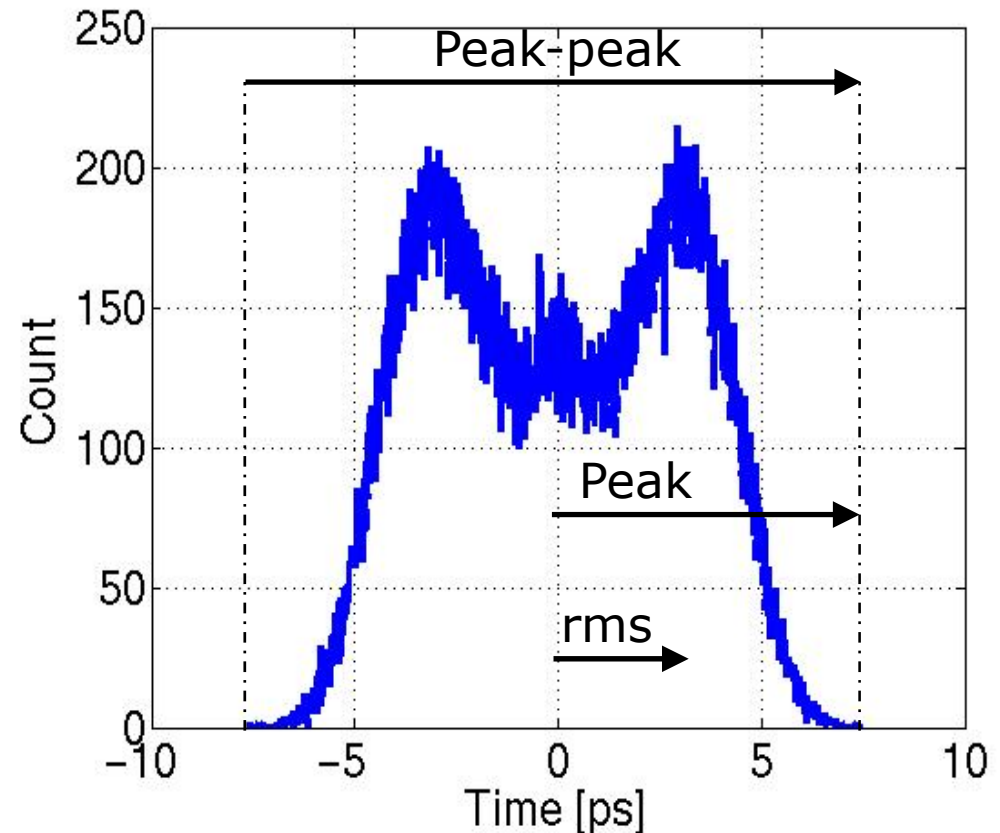


- ❑ Histogram is bounded (finite number of hits)
- ❑ Not necessarily gaussian
- ❑ Underlying jitter physical process might be unbounded

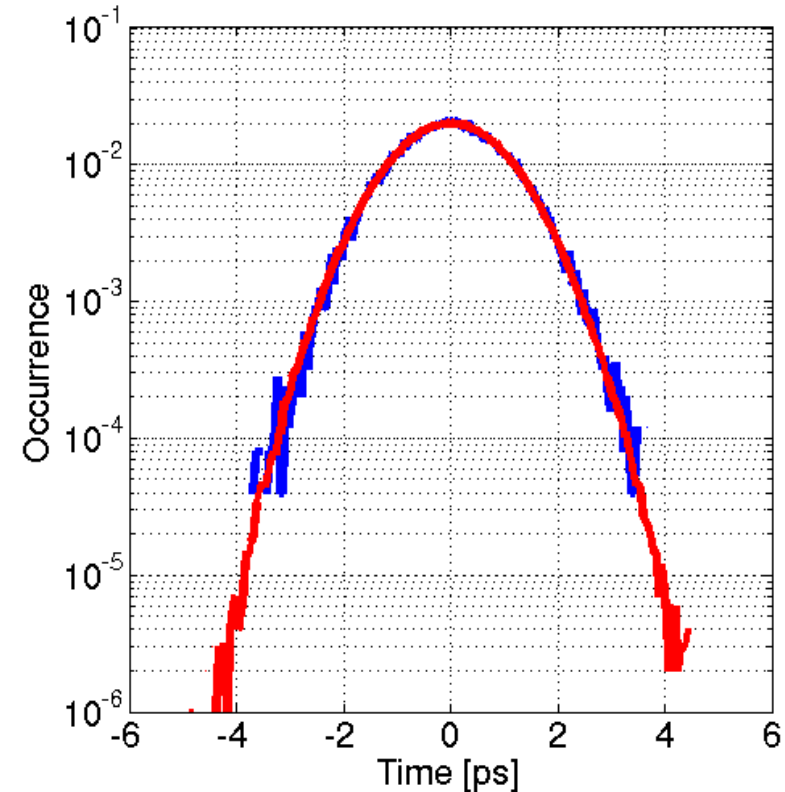
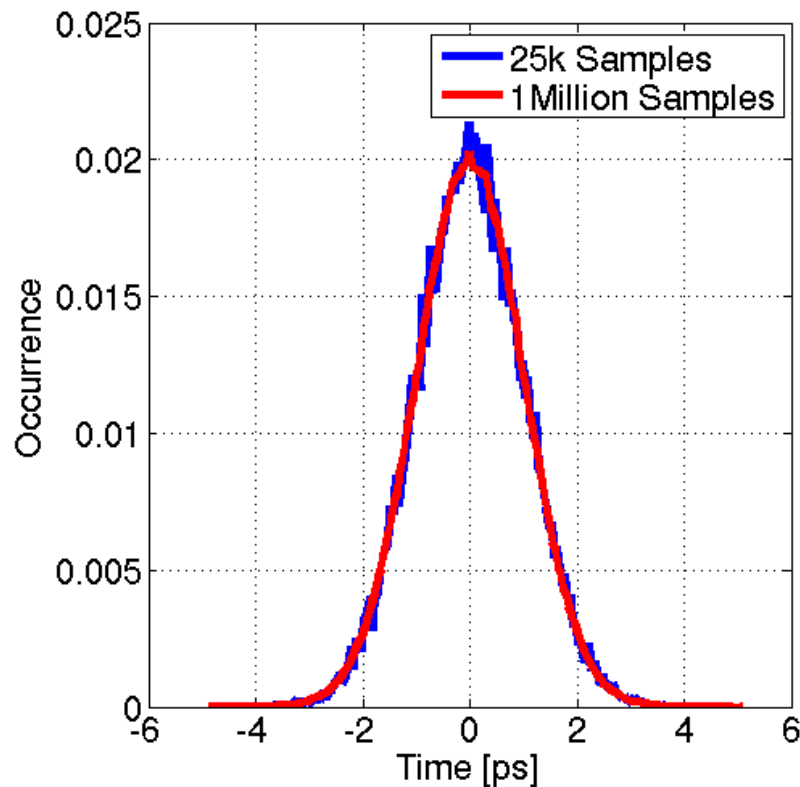
# Jitter statistics

- Out of the jitter distribution several different statistical values can be calculated

- Most used
  - rms, 1-sigma
  - peak
  - peak-peak
  - $3\text{-sigma} := 3 * \text{sigma}$



# The peak for unbounded jitter (1/4)



- ❑ In real circuits the thermal noise sources are gaussian
- ❑ Jitter distributions are therefore unbounded
- ❑ The peak-peak in the histogram depends on the number of hits

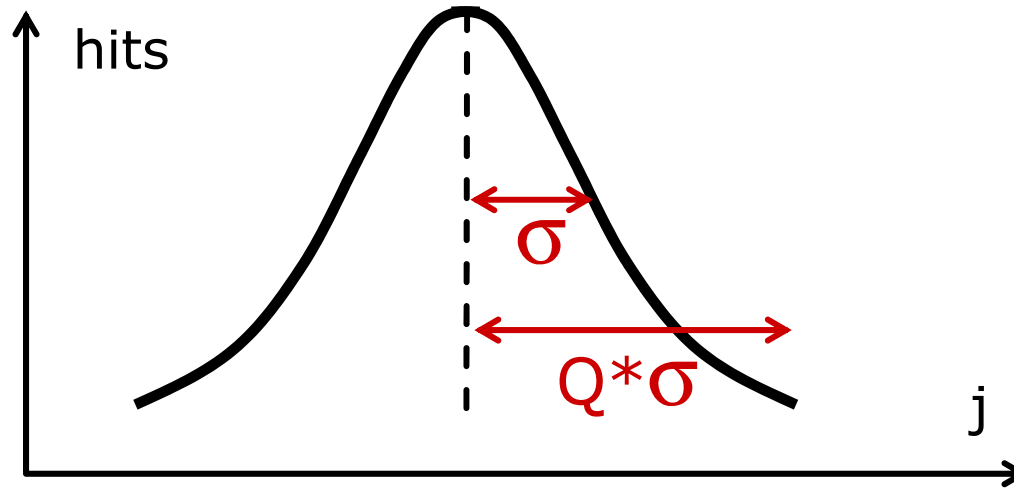
# The peak for unbounded jitter (2/4)

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- Calculate the “max–min” from the histogram
  - easy
  - result depends on number of hits
  - not very reliable, not insightful
  
- Calculate the rms and multiply it by a number  $Q$ 
  - easy
  - result does not depend on number of hits
  - applicable only to distributions which are close to gaussian

# The peak for unbounded jitter (3/4)

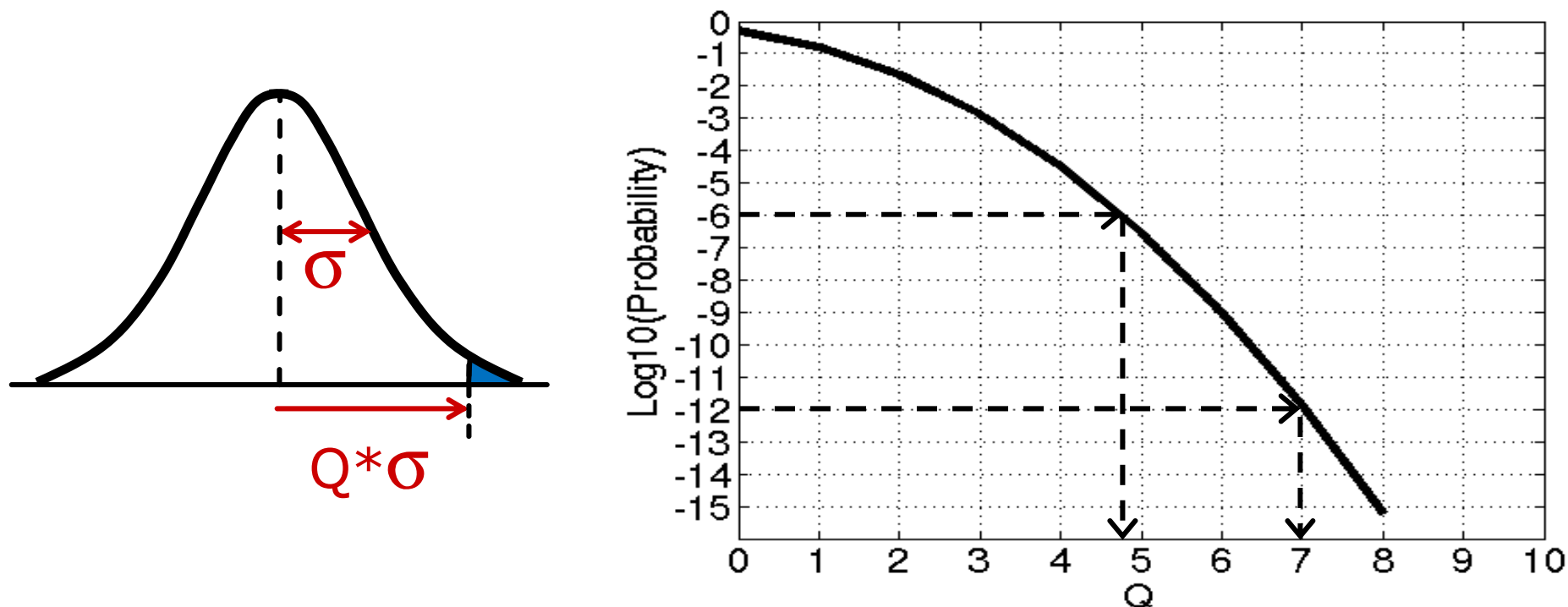
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$$\text{Peak} := Q*\sigma, \quad \text{Peak-peak} := 2*Q*\sigma$$

- ❑ The higher the  $Q$ , the lower the probability that a jitter event is beyond the peak or peak-peak (error event)
- ❑ Can be applied to any distributions, BUT
- ❑ Probability of error easy to know only for gaussian distributions

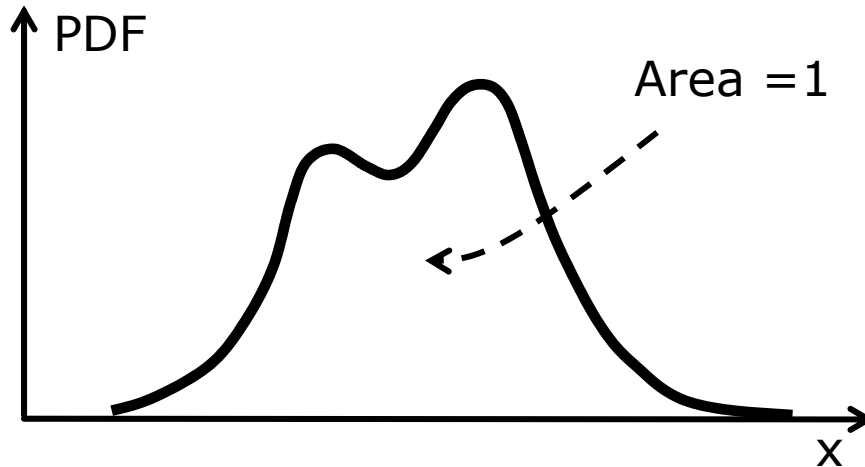
# The peak for unbounded jitter (4/4)



- Prob. of jitter event outside of specified peak depends on Q
- For gaussian jitter distributions easy to compute:
  - Prob <  $1e-3$   $\Rightarrow$   $Q=3.1$
  - Prob <  $1e-6$   $\Rightarrow$   $Q=4.8$
  - Prob <  $1e-12$   $\Rightarrow$   $Q=7.0$

$$\int_{Q \cdot \sigma}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} dx = \frac{1}{2} \text{erfc}\left(\frac{Q}{\sqrt{2}}\right)$$

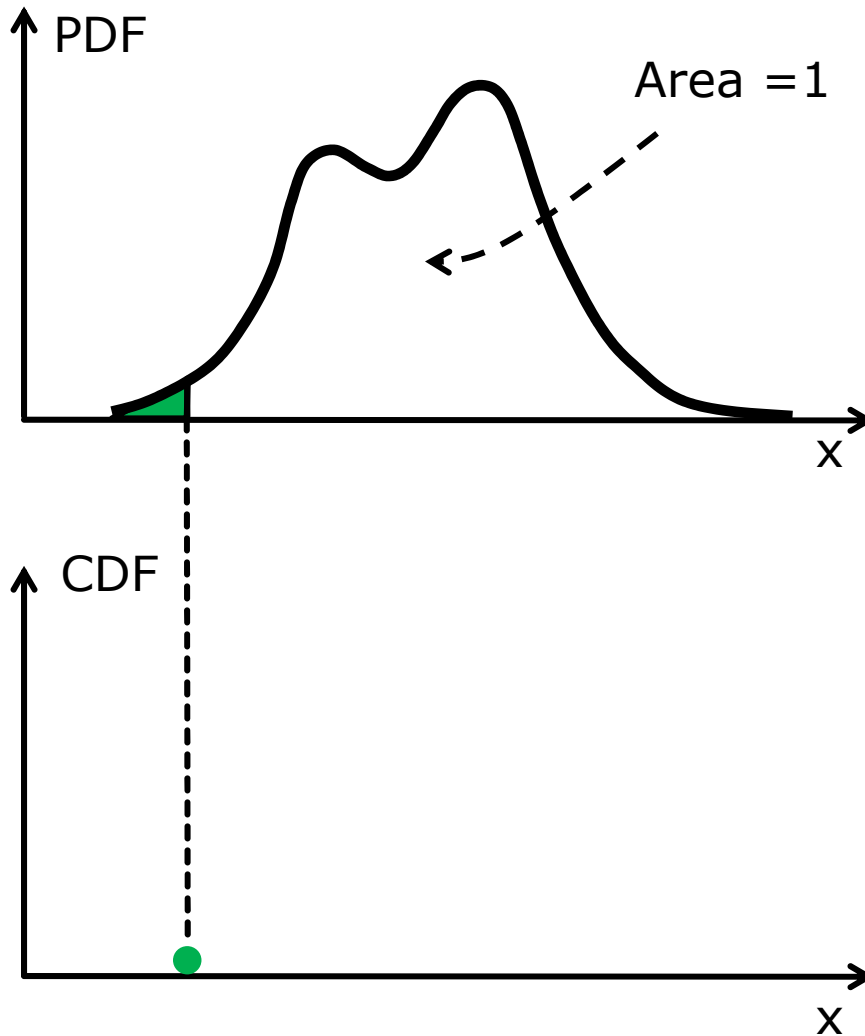
# PDF and CDF



- Random Variable "RV"
- Histogram of RV normalized to Area=1 is the **Probability Density Function (PDF)**
- The integral from  $-\infty$  to  $x$  of the PDF is the **Cumulative Distribution Function (CDF)**
- $\text{CDF}(x) = \text{Prob}[\text{RV} < x]$

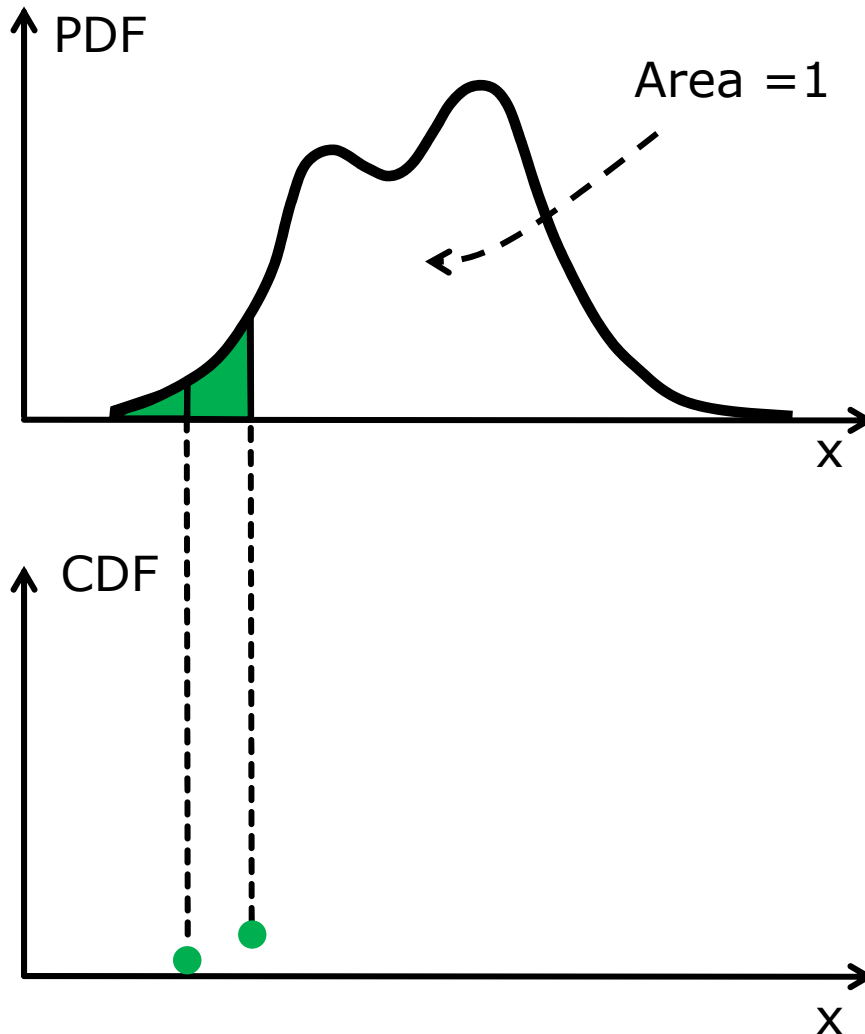


# PDF and CDF



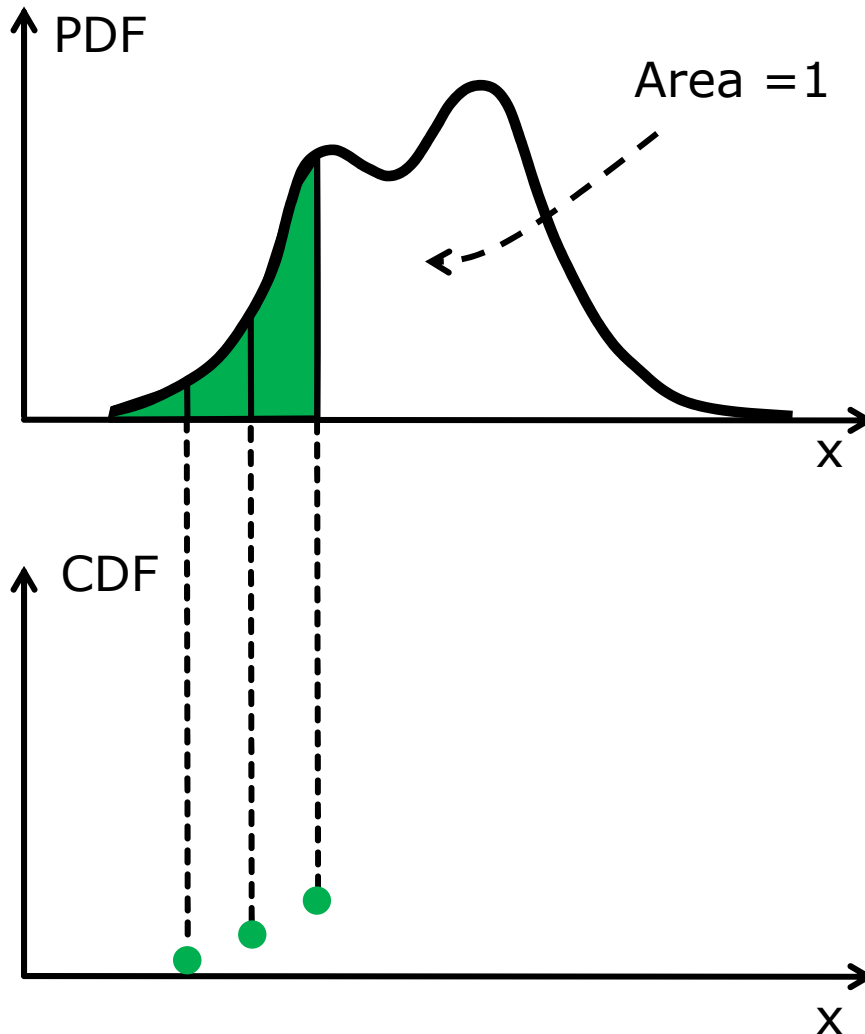
- ❑ Random Variable "RV"
- ❑ Histogram of RV normalized to Area=1 is the **Probability Density Function (PDF)**
- ❑ The integral from  $-\infty$  to  $x$  of the PDF is the **Cumulative Distribution Function (CDF)**
- ❑  $CDF(x) = \text{Prob}[RV < x]$

# PDF and CDF



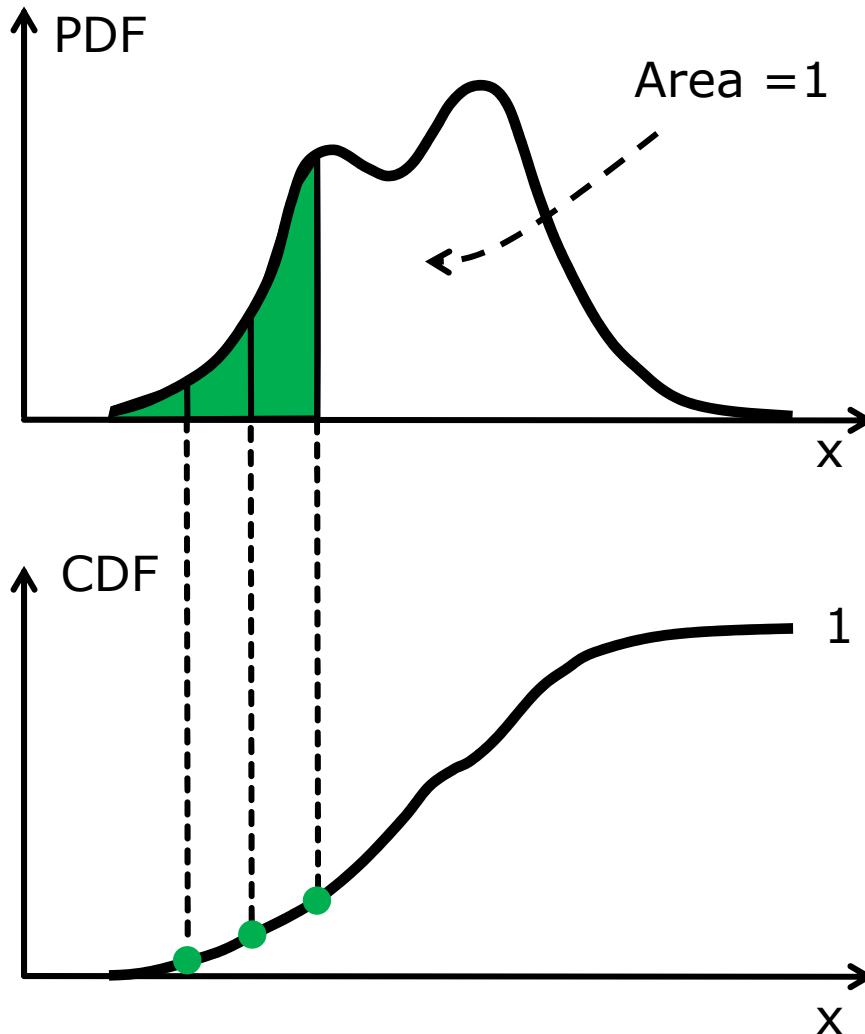
- Random Variable "RV"
- Histogram of RV normalized to Area=1 is the **Probability Density Function (PDF)**
- The integral from  $-\infty$  to  $x$  of the PDF is the **Cumulative Distribution Function (CDF)**
- $CDF(x) = \text{Prob}[RV < x]$

# PDF and CDF



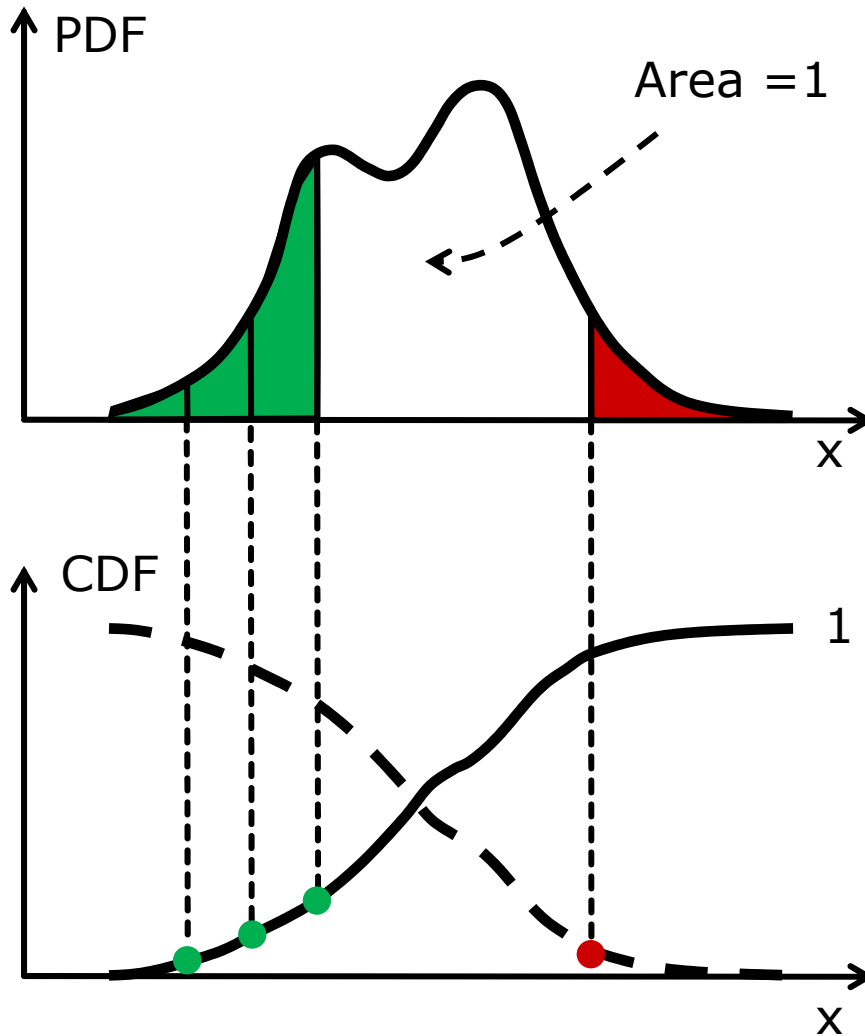
- Random Variable "RV"
- Histogram of RV normalized to Area=1 is the **Probability Density Function (PDF)**
- The integral from  $-\infty$  to  $x$  of the PDF is the **Cumulative Distribution Function (CDF)**
- $CDF(x) = \text{Prob}[RV < x]$

# PDF and CDF



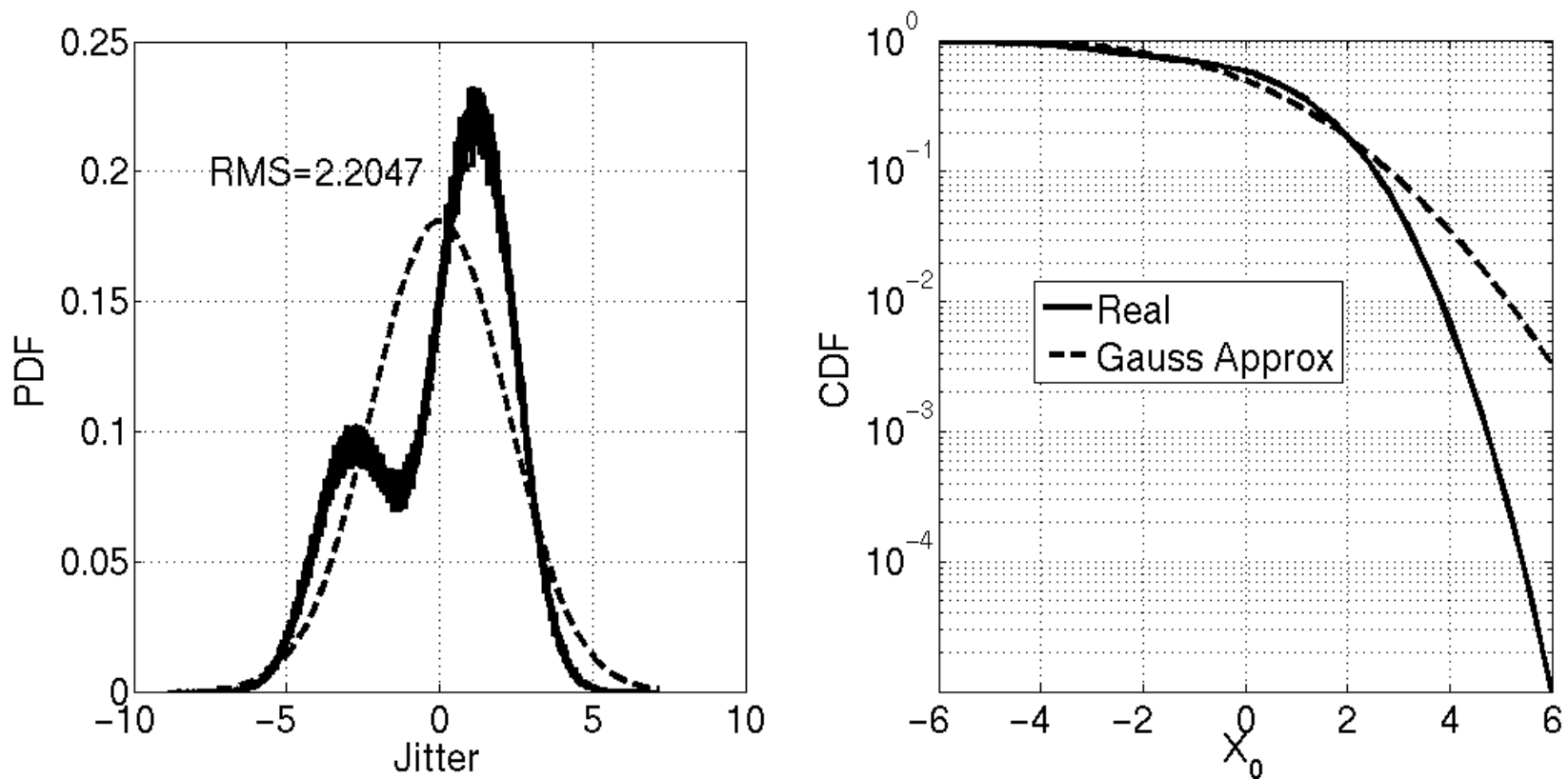
- Random Variable "RV"
- Histogram of RV normalized to Area=1 is the **Probability Density Function (PDF)**
- The integral from  $-\infty$  to  $x$  of the PDF is the **Cumulative Distribution Function (CDF)**
- $CDF(x) = \text{Prob}[RV < x]$

# PDF and CDF



- ❑ Random Variable "RV"
- ❑ Histogram of RV normalized to Area=1 is the **Probability Density Function (PDF)**
- ❑ The integral from  $-\infty$  to  $x$  of the PDF is the **Cumulative Distribution Function (CDF)**
- ❑  $\text{CDF}(x) = \text{Prob}[\text{RV} < x]$
- ❑  $\text{CDF}'(x) = \text{Prob}[\text{RV} > x]$

# What if jitter is not gaussian?



- $Q \cdot \sigma$  approach for non-gaussian distributions leads to wrong estimation of error probability

# Summary so far

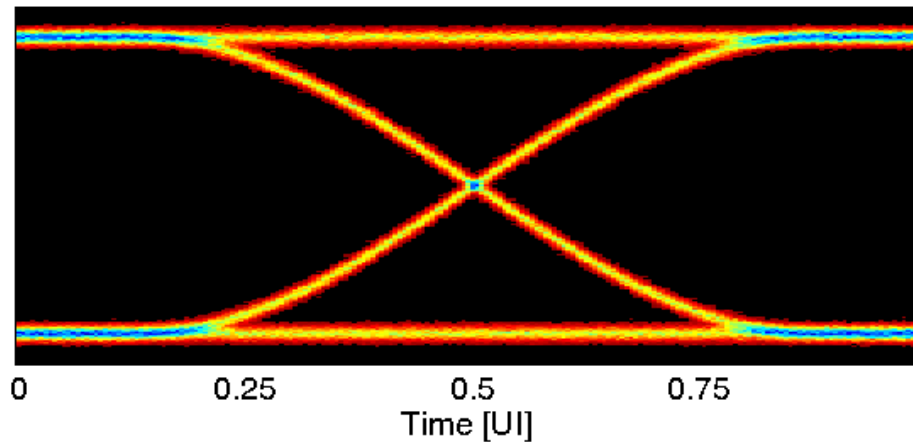
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- ❑ Lots of important applications are not affected by the RMS value of Jitter
- ❑ “Tail” behavior of jitter might be more important
- ❑ Statistical properties of gaussian jitter distributions
- ❑ Multiplying the rms jitter by factor  $Q$  leads to wrong results if jitter is not gaussian

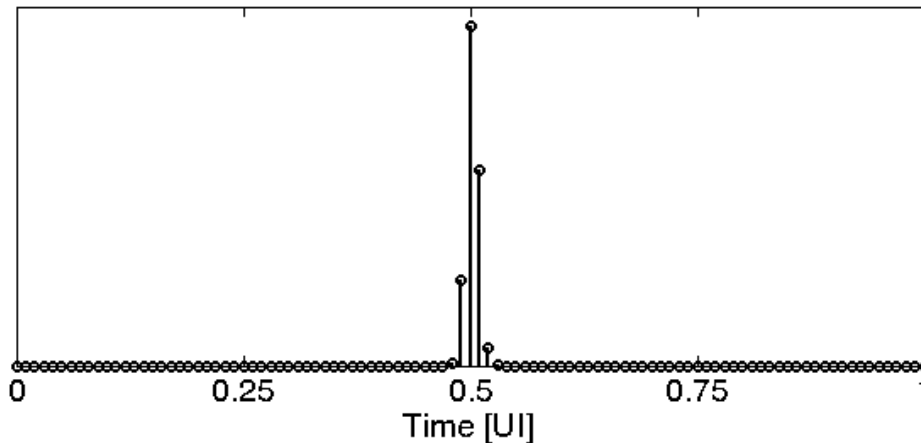
## **What's next:**

- ❑ Develop a method to characterize jitter distributions which are not gaussian

# Ideal Eye Diagram



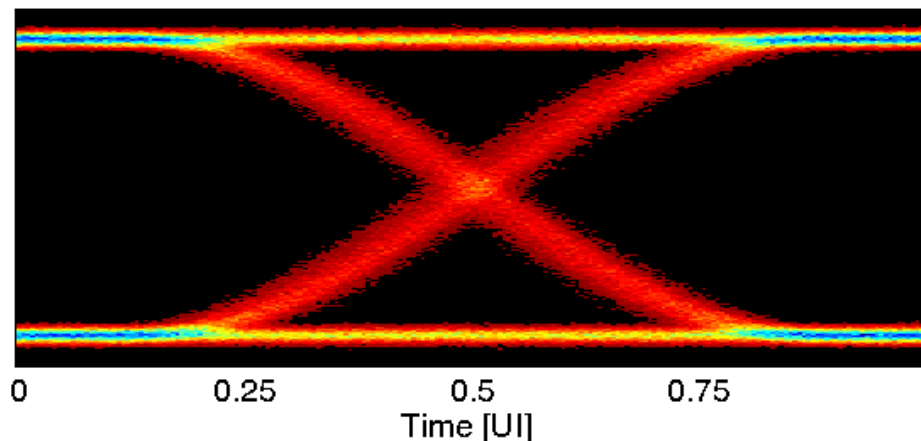
- Sharp Transition
- Histogram shows one peak



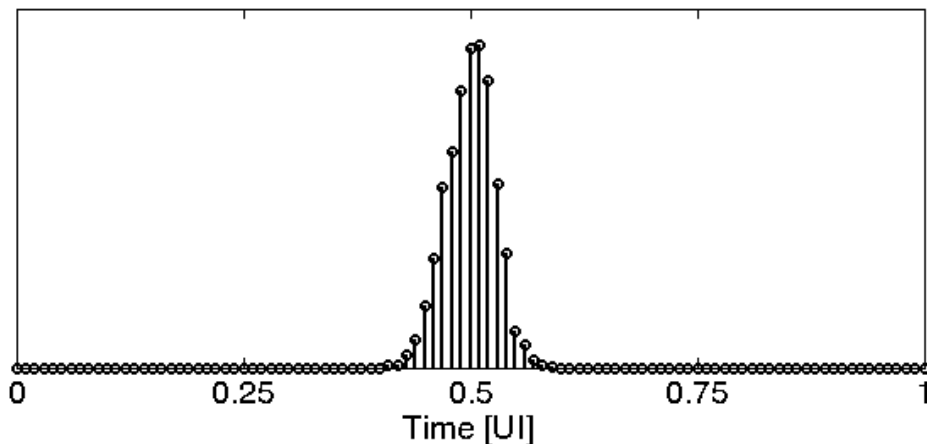


# Eye Diagram: Random Jitter (RJ)

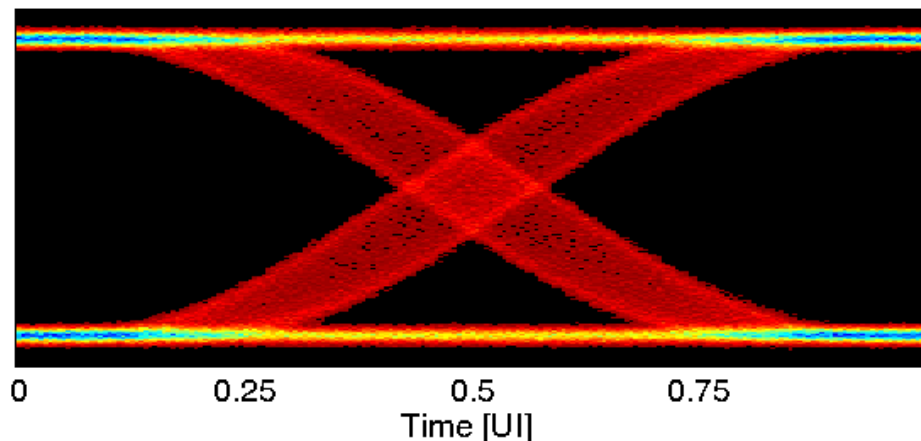
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- Transitions are blurred
- Histogram shows “gaussian” distribution
- Due to:
  - Device thermal noise
  - Device flicker noise
  - Composite effect of many uncorrelated noise sources



# Eye Diagram: Periodic Jitter (PJ,SJ)



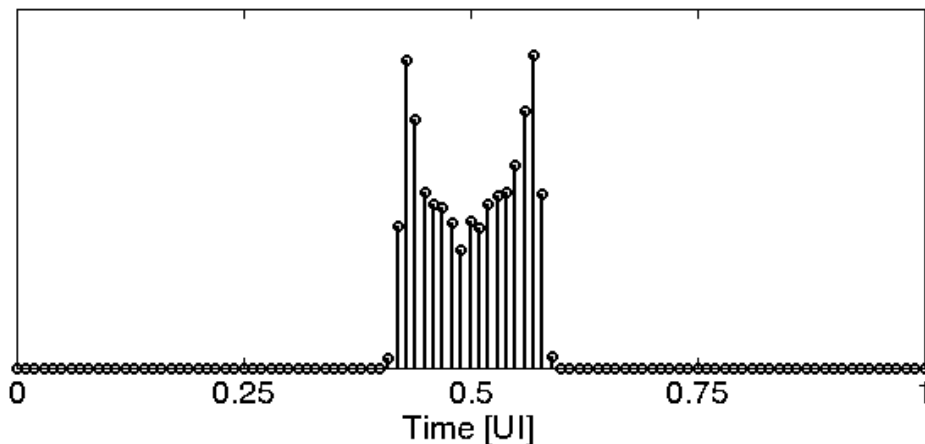
□ Transitions are spread

□ Histogram shows bounded distribution

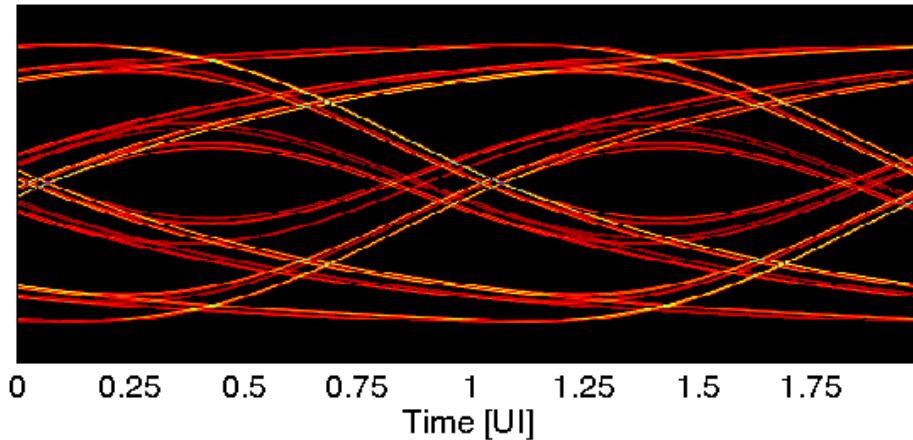
□ Figure shows example for case of sinusoidal jitter

□ Due to:

- Power supply noise
- Strong local RF carrier
- Spurious tones in PLL



# Eye Diagram: Data-Dep. Jitter (DDJ, ISI)

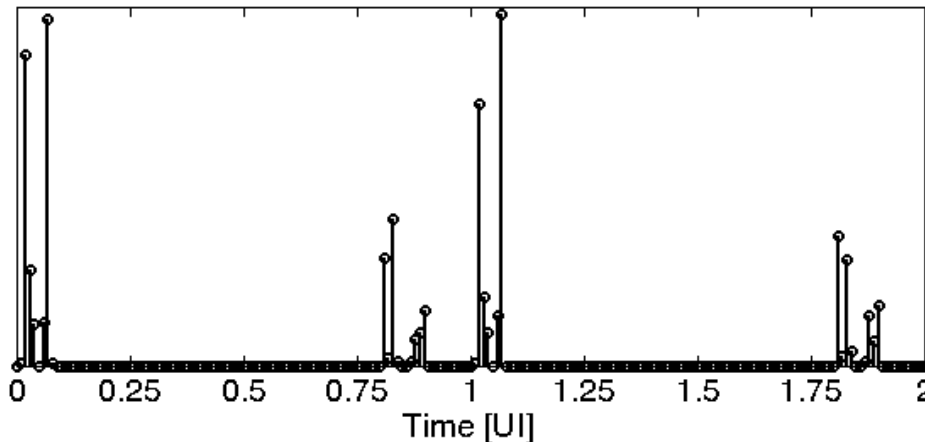


□ Transitions are “concentrated” in some “hot Spots”

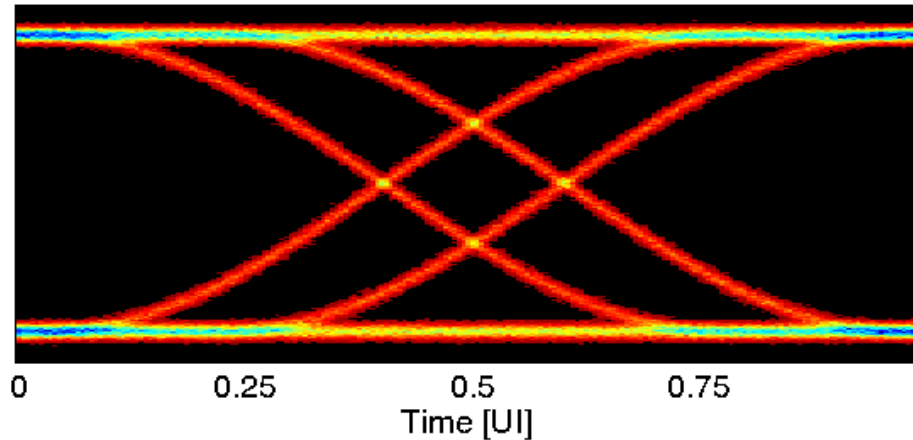
□ Histogram shows discrete peaks

□ Due to:

- Channel or TX bandwidth limitation (waveform does not reach full scale level unless there are several bits in a row with same polarity)

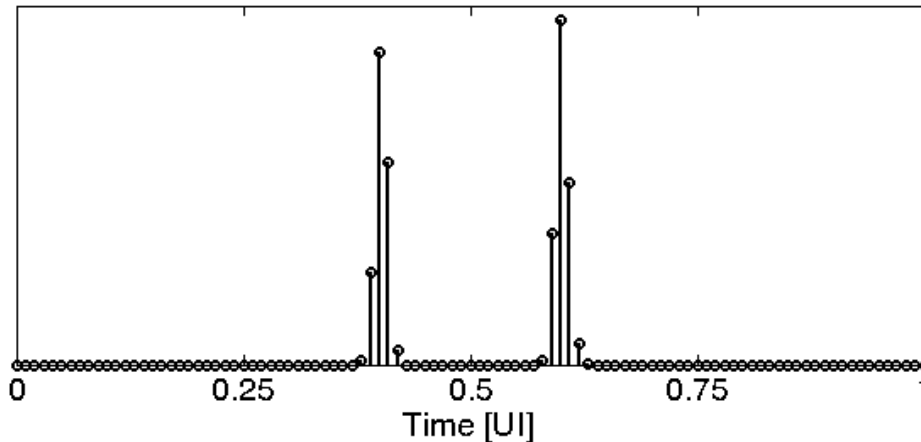


# Eye Diagram: Duty Cycle Distortion (DCD)



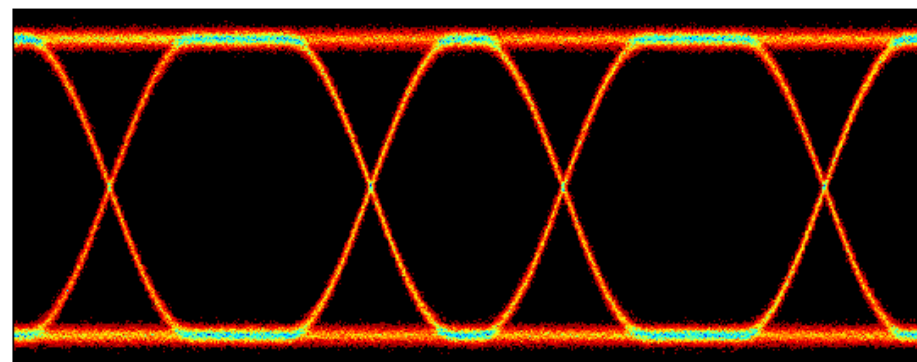
□ Transitions are concentrated in two hot spots

□ Histogram shows two equally large peaks



□ Due to:  
■ Duty Cycle Distortion (e.g. in DDR systems)

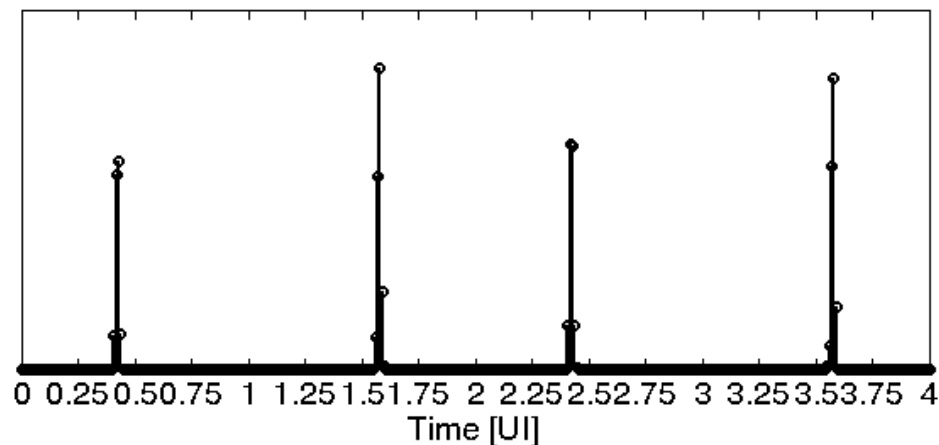
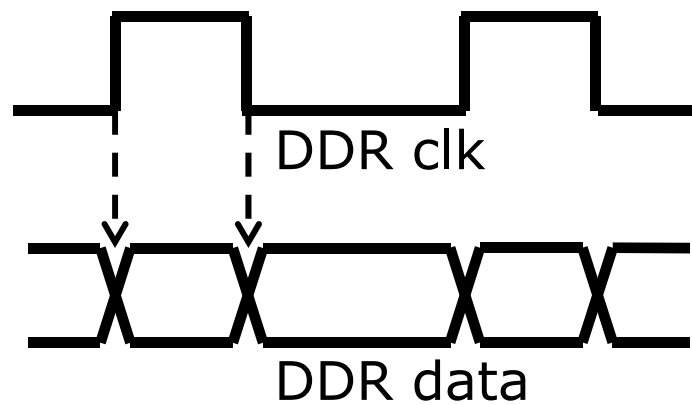
# Eye Diagram: Duty Cycle Distortion (DCD)



0 0.25 0.50 0.75 1 1.25 1.51.75 2 2.25 2.52.75 3 3.25 3.53.75  
Time [UI]

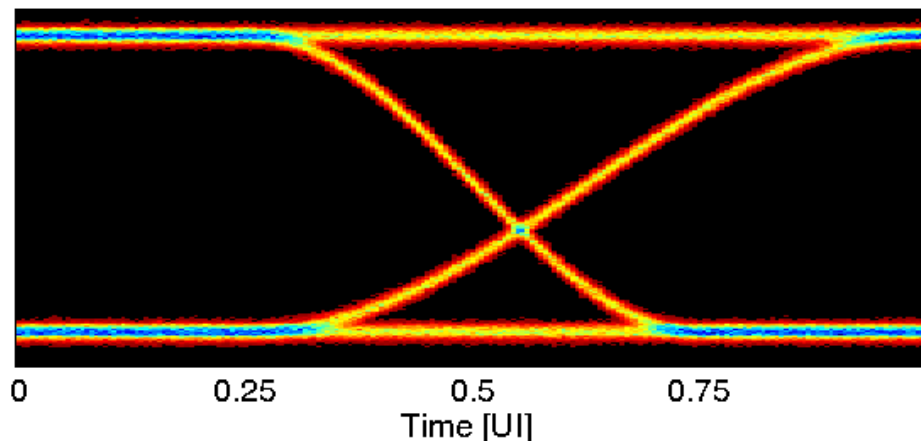
□ DCD is evident when the eye is unfolded over more UIs

□ Typical in DDR systems



0 0.25 0.50 0.75 1 1.25 1.51.75 2 2.25 2.52.75 3 3.25 3.53.75 4  
Time [UI]

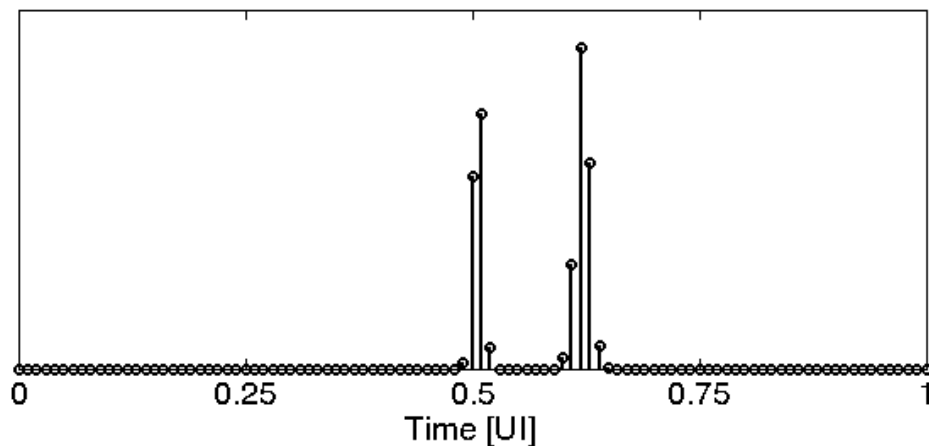
# Eye Diagram: Rise/Fall Time Asymm.



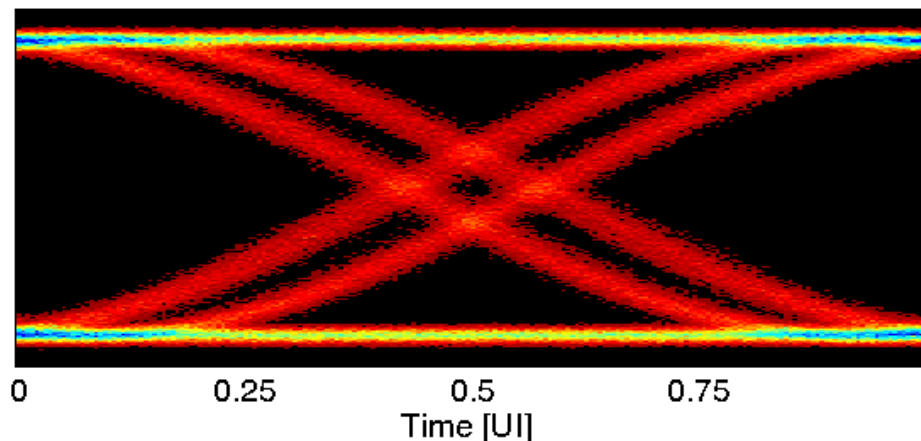
□ Similar Histogram as for DCD

□ Due to:

- Rise and Fall Time Asymmetry
- RX threshold not centered



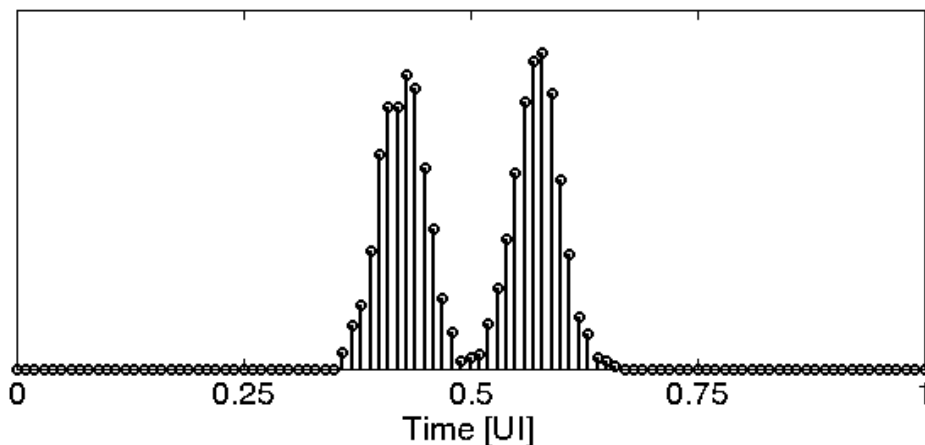
# Eye Diagram: Multiple Jitter Sources



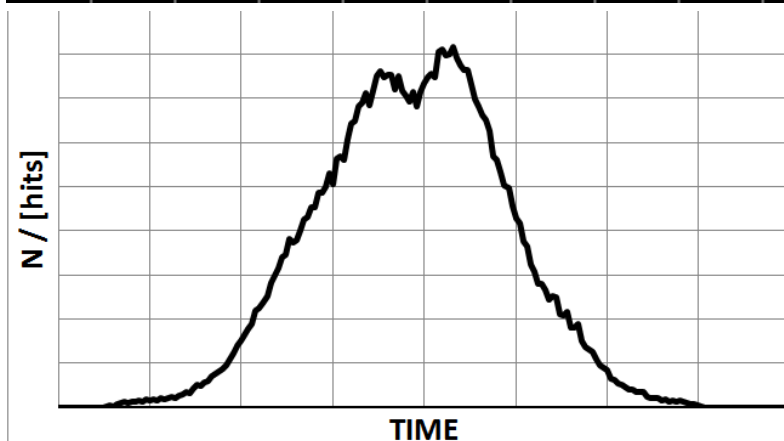
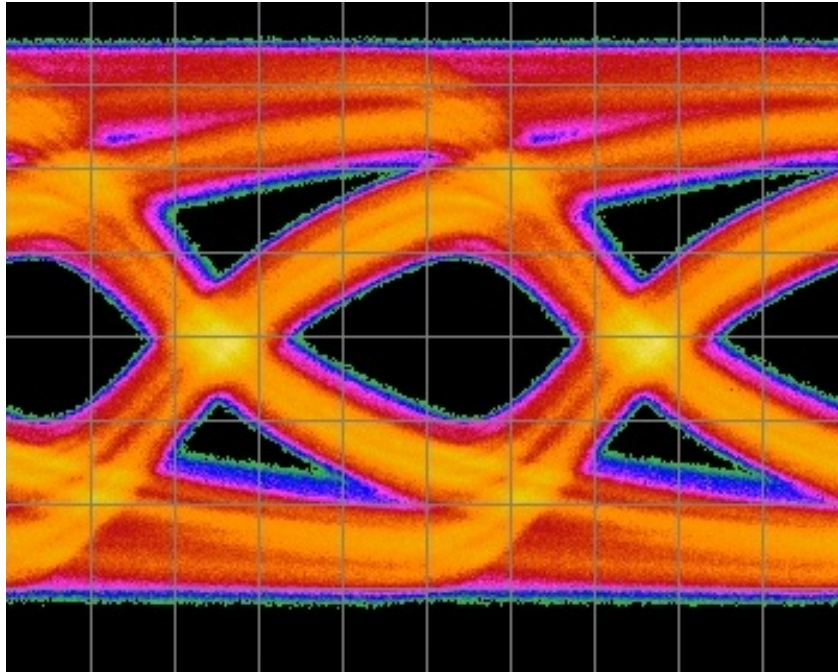
□ In real systems lots of jitter sources are present at the same time

□ In figure: example of DCD + RJ

□ **The resulting distribution is the convolution of the single distributions**



# Eye Diagram: Multiple Jitter Sources



- ❑ Measurement taken after 50 inches of FR4
- ❑ Data transitions show deterministic patterns plus blurred behavior
- ❑ Histogram shows an unbounded, clearly not gaussian distribution



# Jitter Decomposition (1/4)

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- Jitter is the result of:
  - Device Noise (flicker + thermal)
  - System non-idealities (nonlinearities, limited channel BW, spurious tones in PLL spectrum, duty-cycle, ...)
  - External disturbances (supply noise, switching noise, crosstalk, ...)
  
- Device noise contributors are gaussian, **unbounded**
  
- Other contributors generate **bounded** distributions

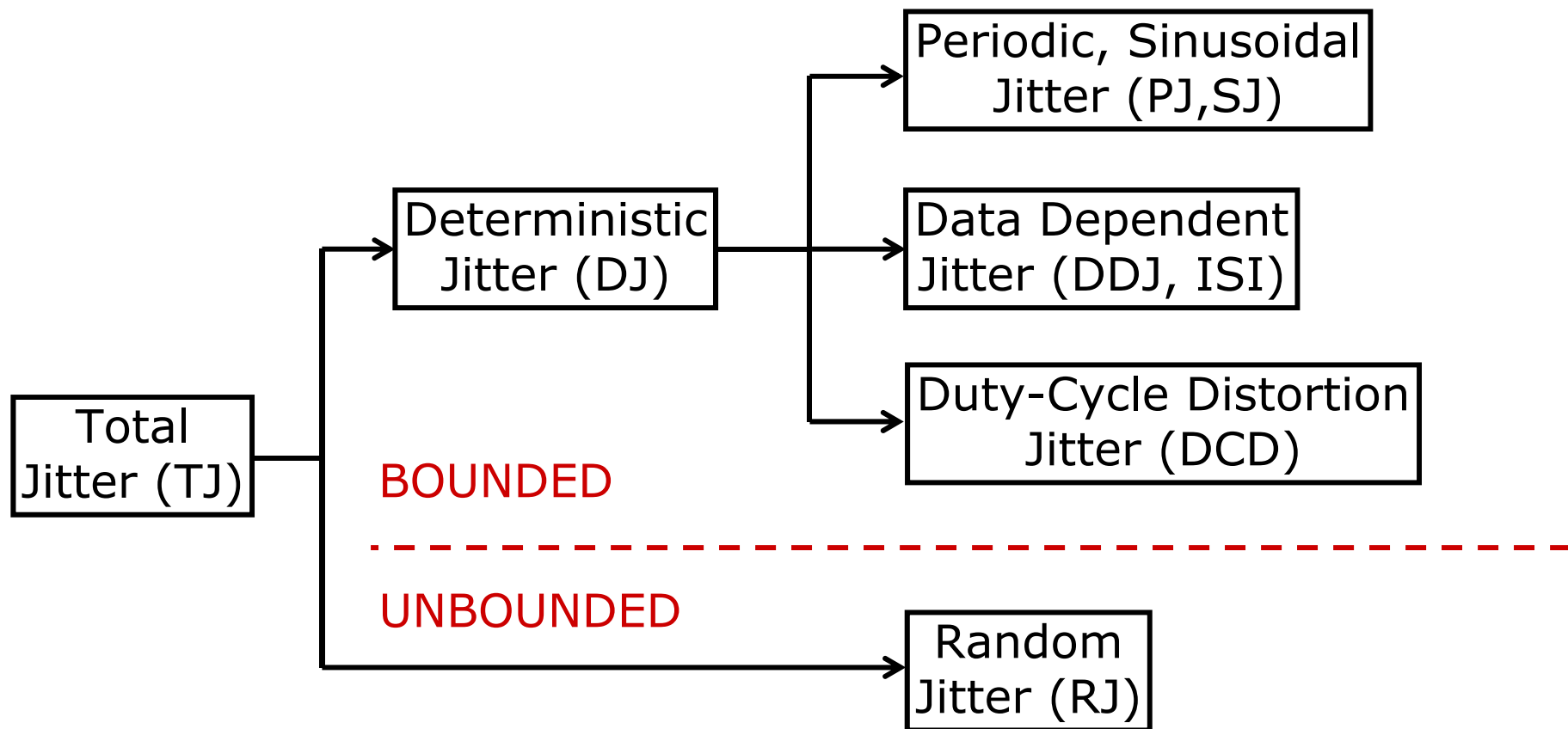
# Jitter Decomposition (2/4)

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- Idea: consider the total jitter distribution as the combination of:
  - deterministic components
  - random component
- **Deterministic Jitter (DJ)** := any jitter with **bounded** distribution
- **Random Jitter (RJ)** := any jitter with **unbounded** distribution (gaussian)
- The result of DJ and RJ is called **Total Jitter (TJ)**

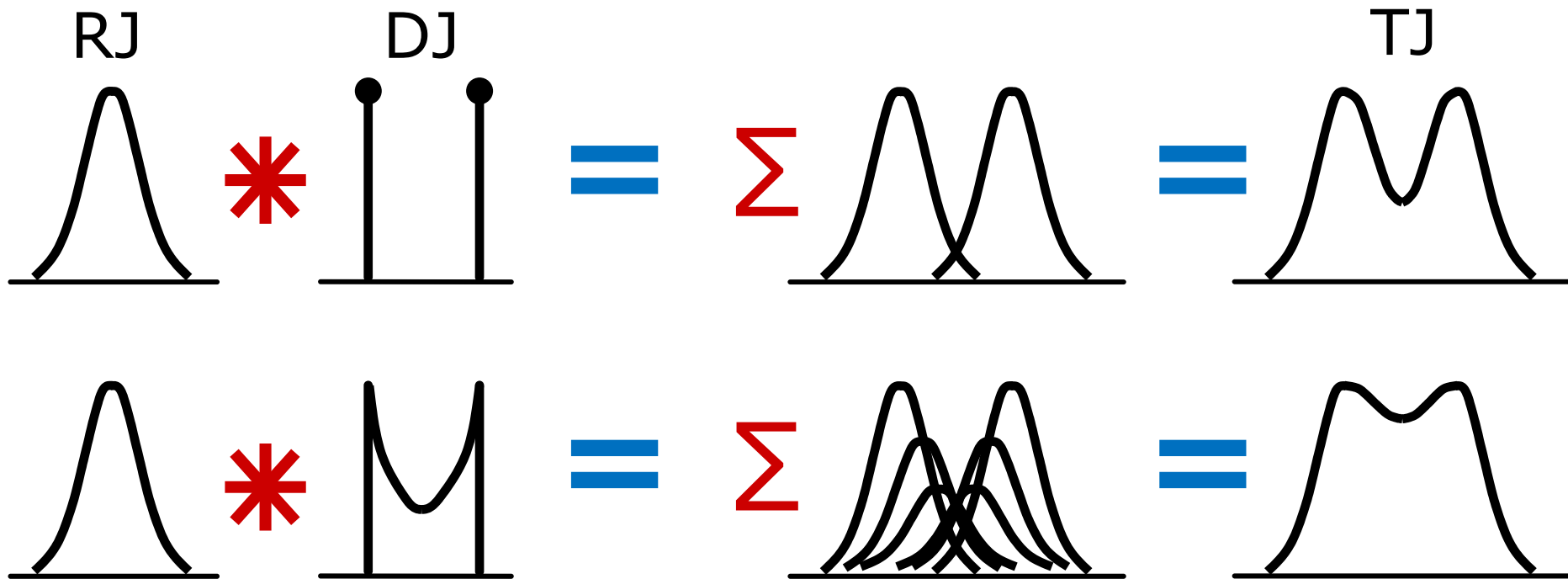
# Jitter Decomposition (3/4)

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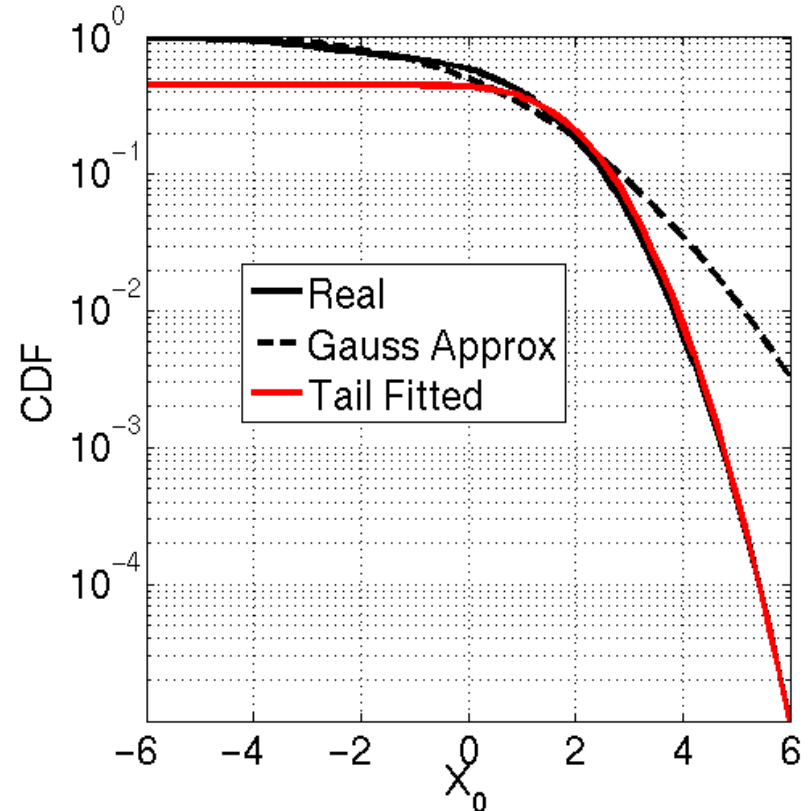
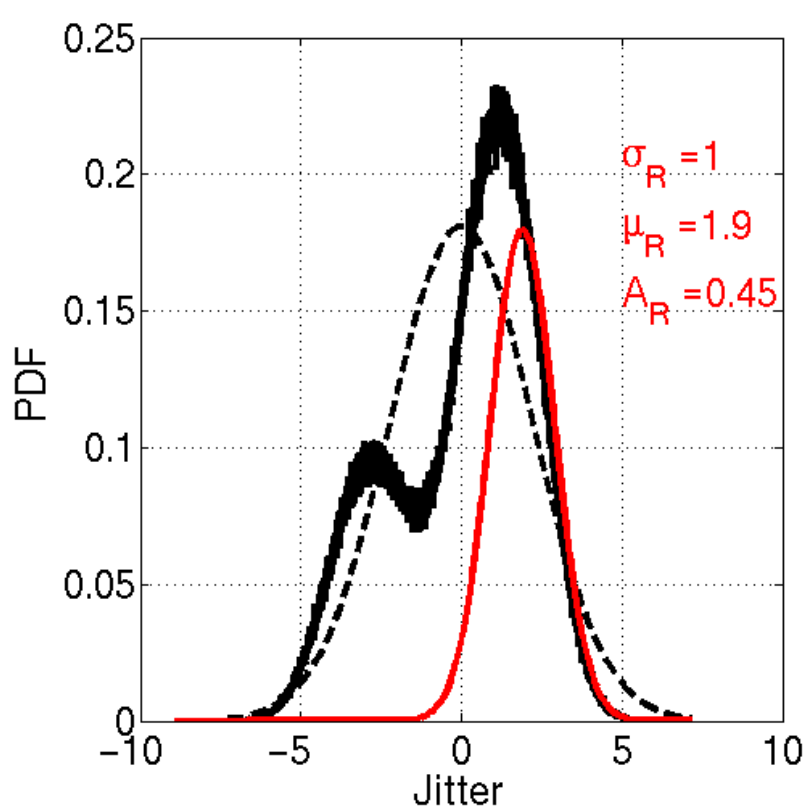


# Jitter Decomposition (4/4)

- From Probability Theory: the distribution of the sum of independent random variables is the **convolution** of the single distributions
- Hence: the distribution of TJ is the **convolution** of the distributions of RJ and DJ

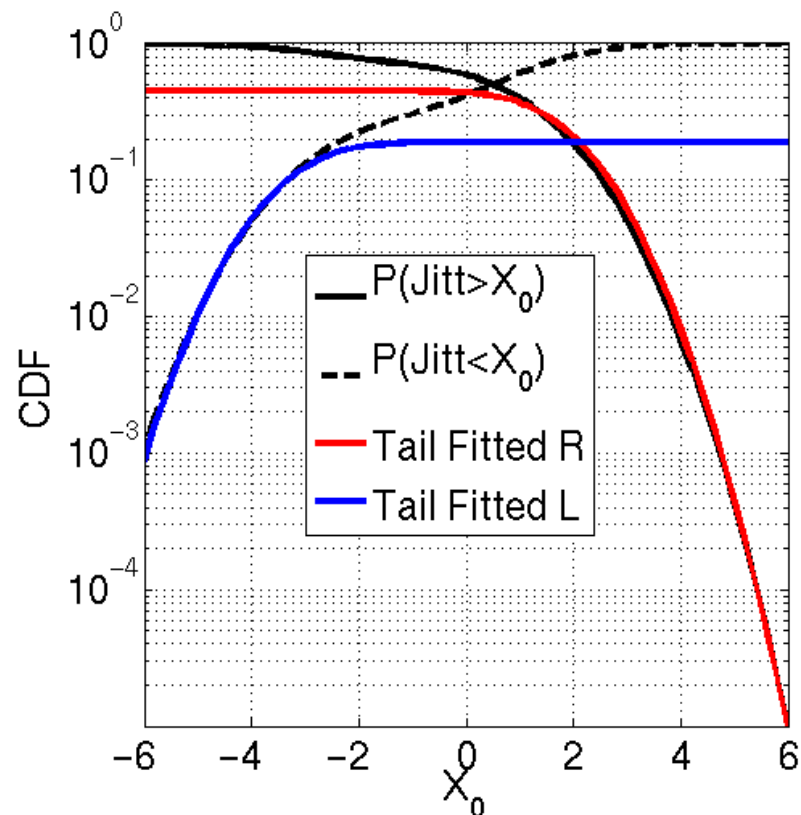
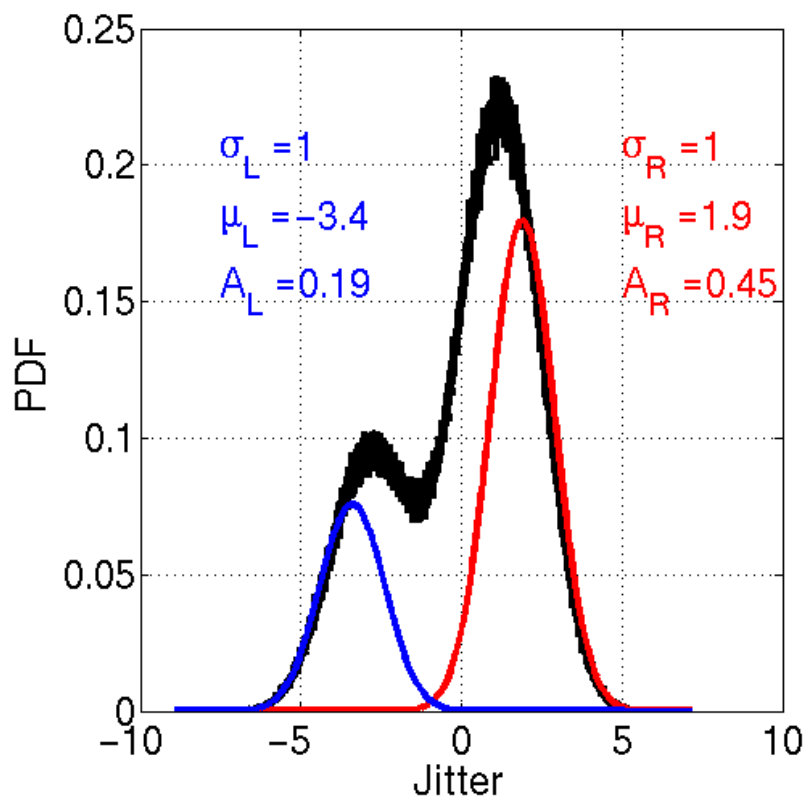


# Tail Fitting (1/3)



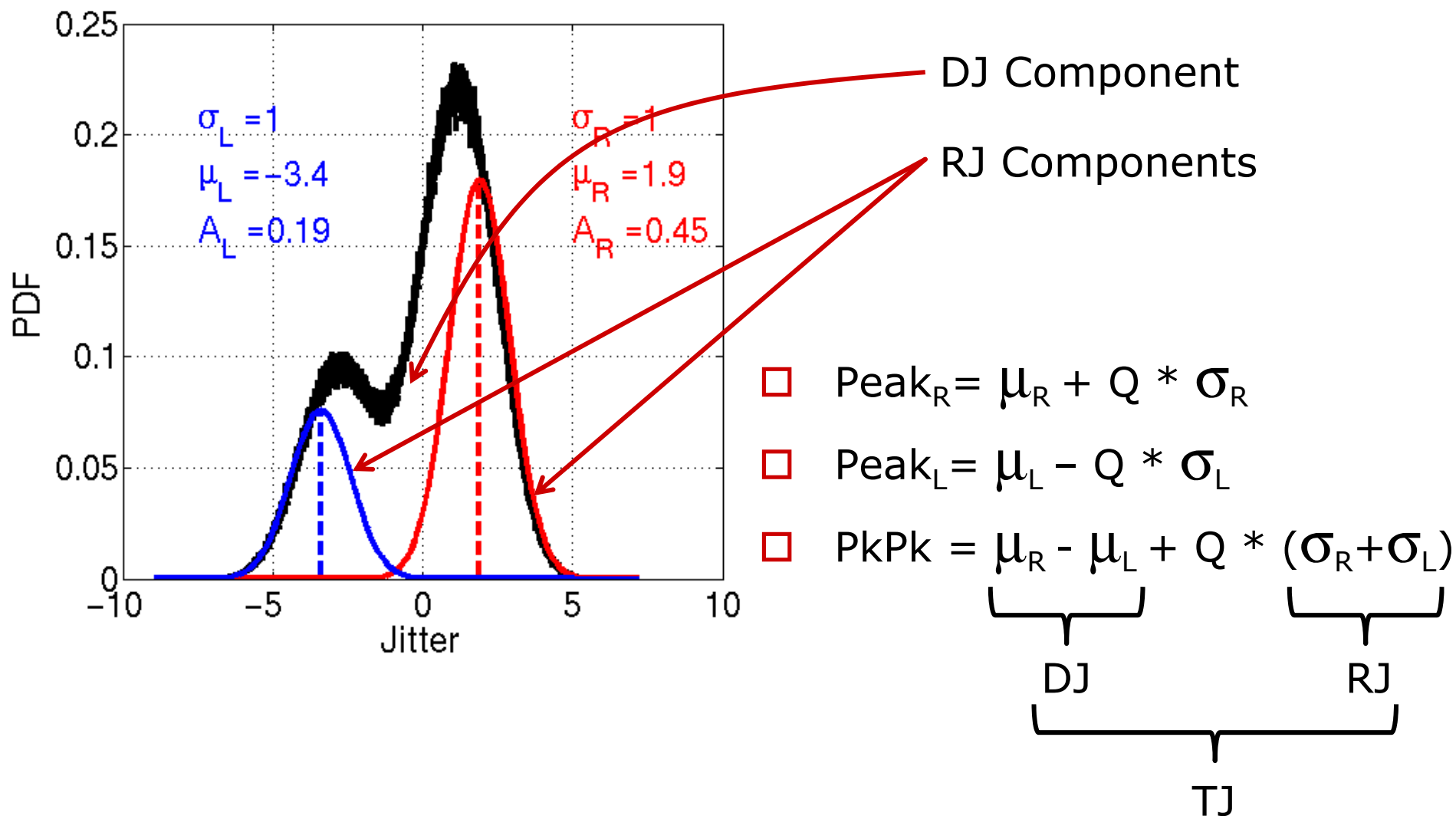
- ❑ Idea: fit a gaussian curve to the tail of the distribution
- ❑ Probability of error obtained using the fitted curve is matching the real one
- ❑ 3 Parameters:  $\sigma$ ,  $\mu$ ,  $A$

# Tail Fitting (2/3)

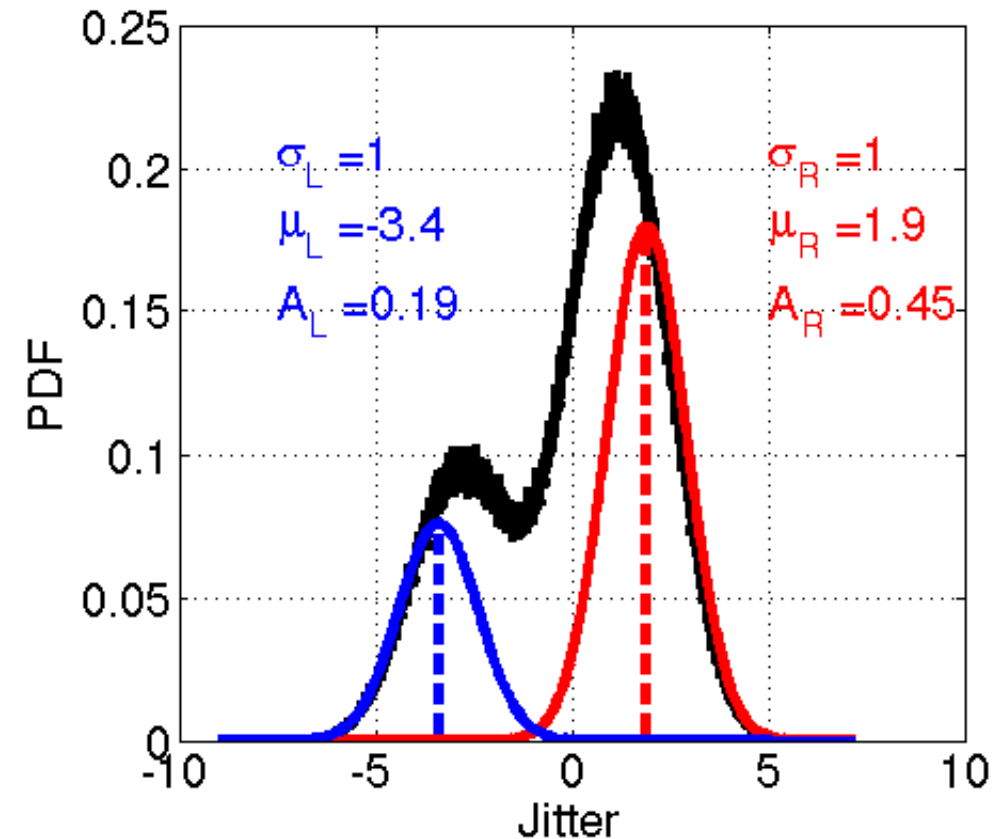


- Do the same for the left tail
- 6 Parameters:  $\sigma_R, \mu_R, A_R, \sigma_L, \mu_L, A_L$
- Probability of error left and right can be accurately described

# Tail Fitting (3/3)



# TJ and Probability



$$\square \quad TJ = \mu_R - \mu_L + Q * (\sigma_R + \sigma_L)$$

$$\square \quad \text{Usually } \sigma_R = \sigma_L = \sigma$$

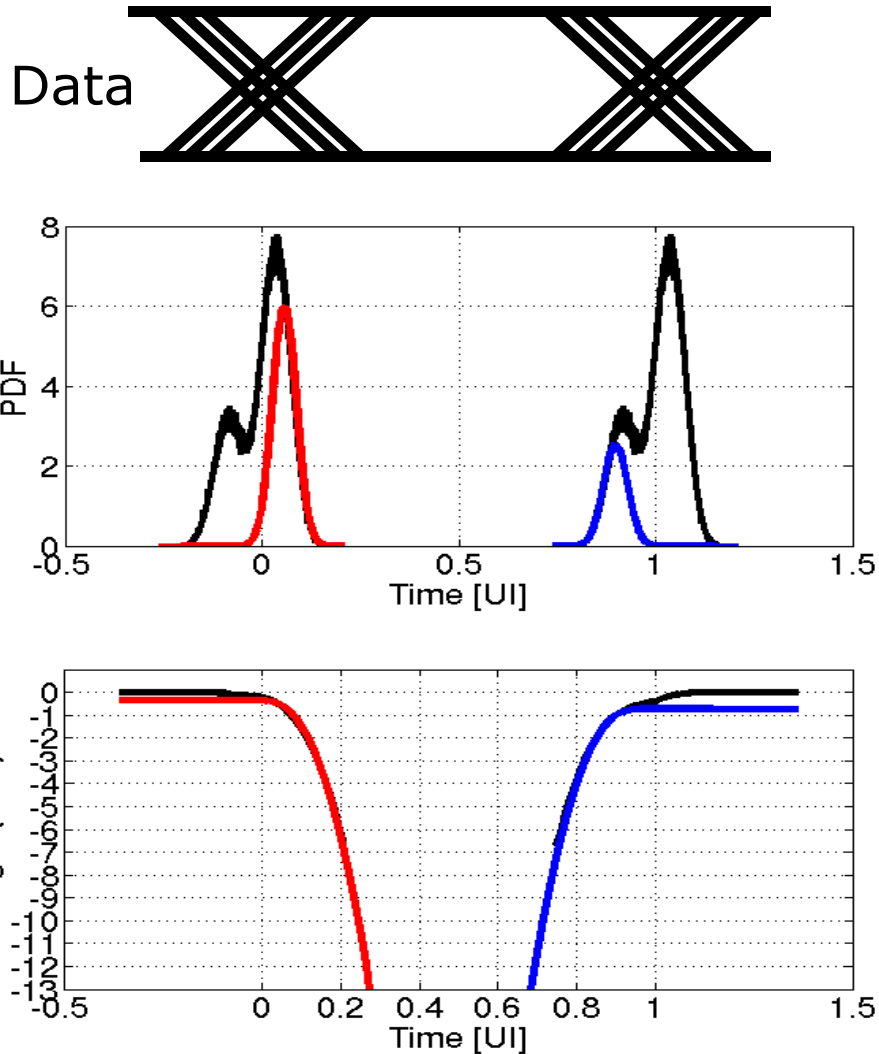


$$\square \quad TJ = DJ + 2 * Q * \sigma$$

$$\square \quad \text{Prob(Jitter outside TJ range)} = \frac{A_R + A_L}{2} \operatorname{erfc}\left(\frac{Q}{\sqrt{2}}\right) \cong \frac{1}{2} \operatorname{erfc}\left(\frac{Q}{\sqrt{2}}\right)$$

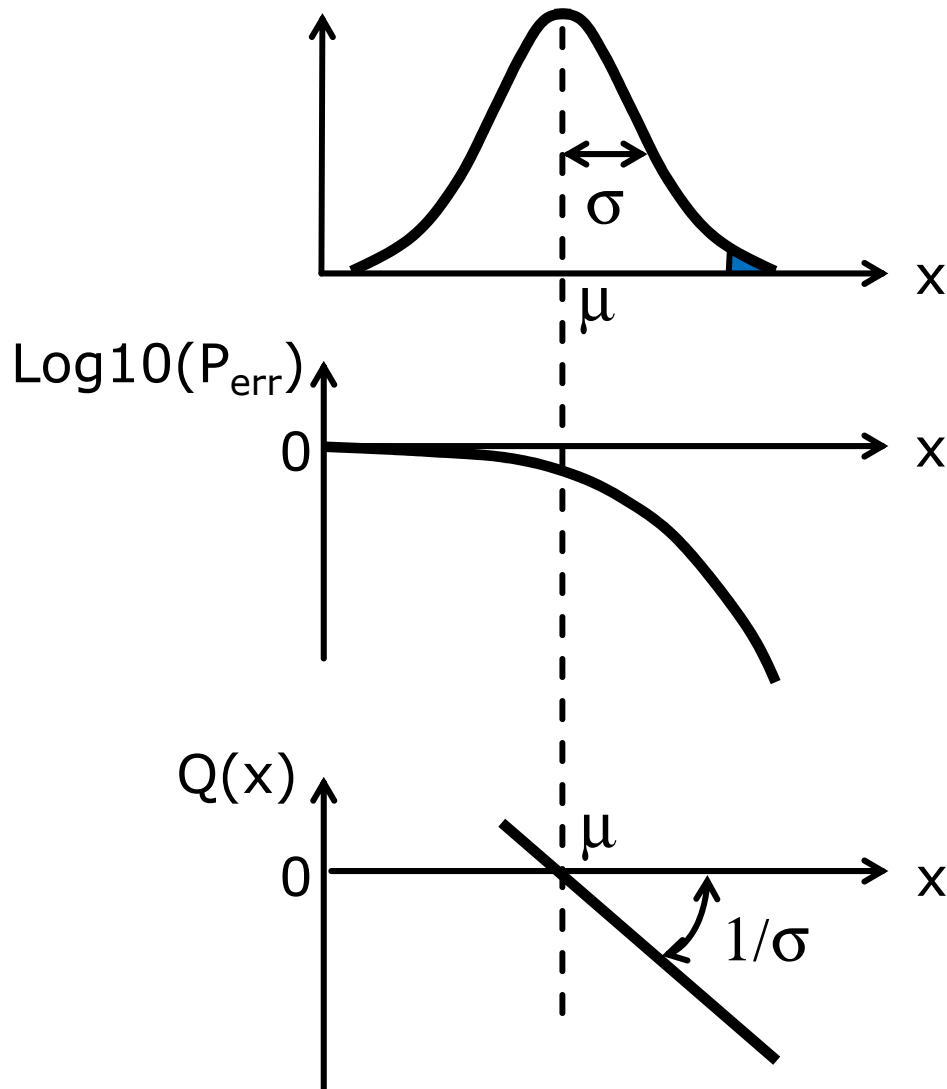


# Bathtub plot (1/3)



- The bathtub plot is obtained by building the CDFs of the jitter on the left and right data edges
- It shows the probability of error versus the sampling point
- Used to estimate the eye opening for very low BER levels

# Bathtub plot (2/3): Q-scale



- Q-scale used to linearize the bathtub plot for gaussian distributions

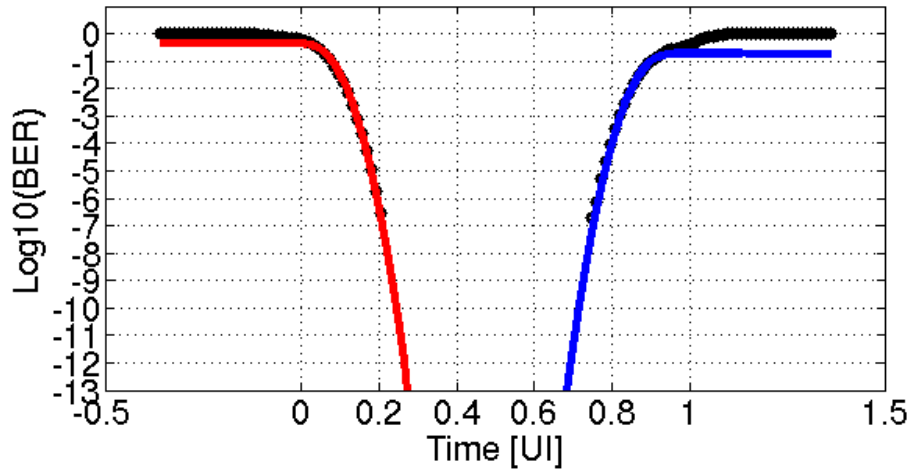
$$P_{\text{err}}(x) = \frac{1}{2} \text{erfc}\left(\frac{x - \mu}{\sqrt{2}\sigma}\right)$$

$$Q(x) := -\sqrt{2} \cdot \text{erfc}^{-1}(2 \cdot P_{\text{err}}(x))$$

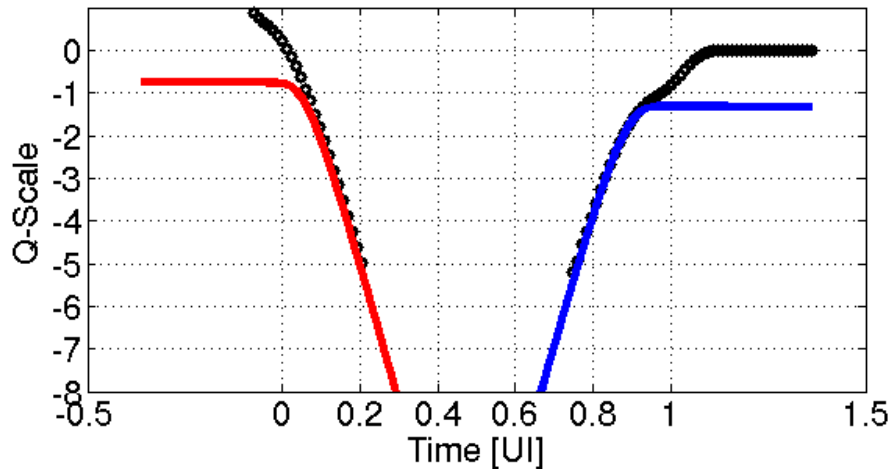
$$Q(x) = \frac{\mu - x}{\sigma}$$

- $P_{\text{err}} = 10^{-15} \rightarrow Q = -7.95$
- $P_{\text{err}} = 10^{-12} \rightarrow Q = -7.03$
- $P_{\text{err}} = 10^{-9} \rightarrow Q = -5.99$
- $P_{\text{err}} = 10^{-6} \rightarrow Q = -4.75$

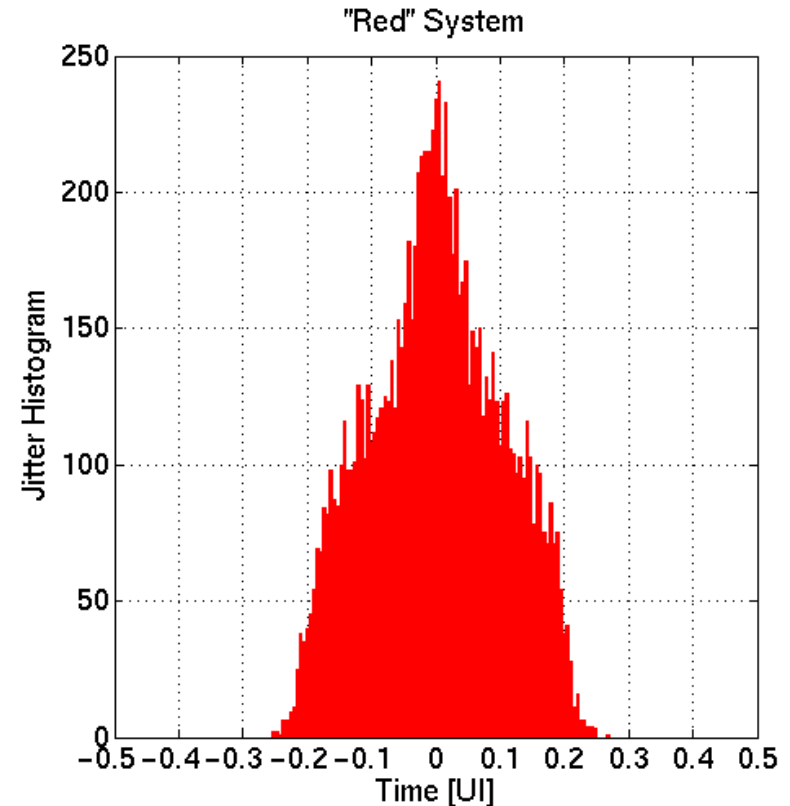
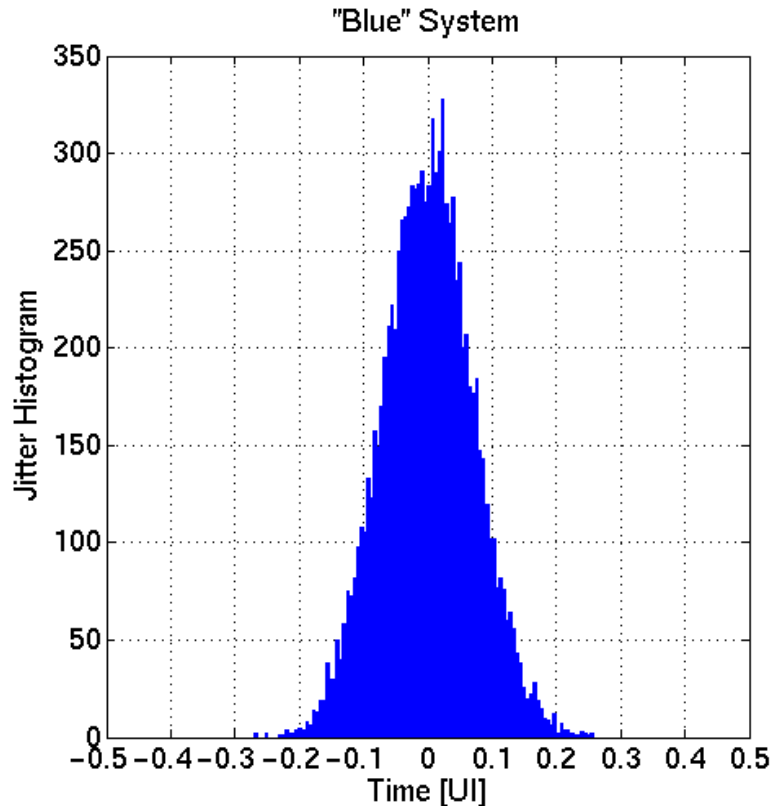
# Bathtub plot (3/3)



- Gaussian components in Q-scale are linear
- Linear interpolation to lower Q (BER)
- Used to understand when RJ contribution starts to dominate

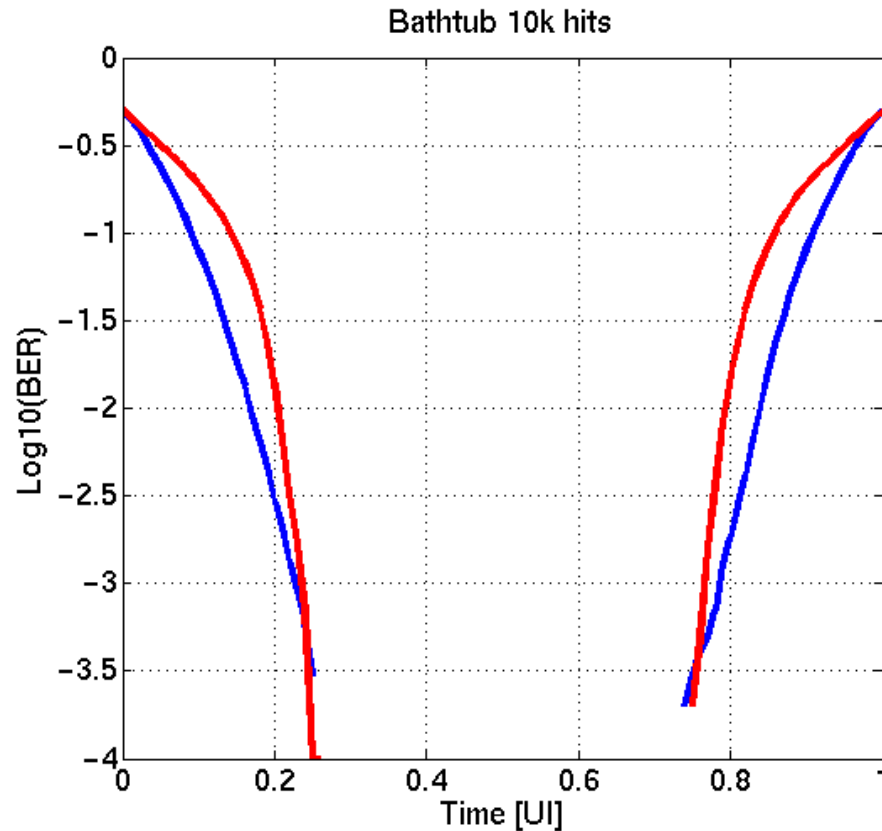


# Bathtub Example (1/4)



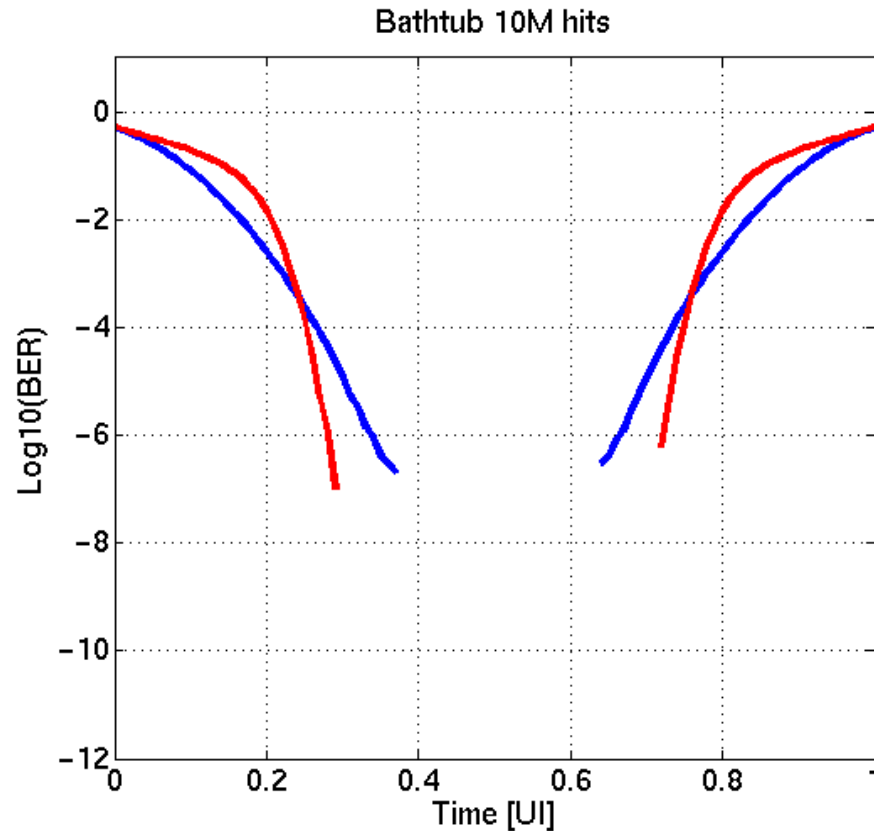
- Two systems with different jitter histograms over 10k hits
- Although the histograms are different the peak to peak jitter seems to be the same

# Bathtub Example (2/4)



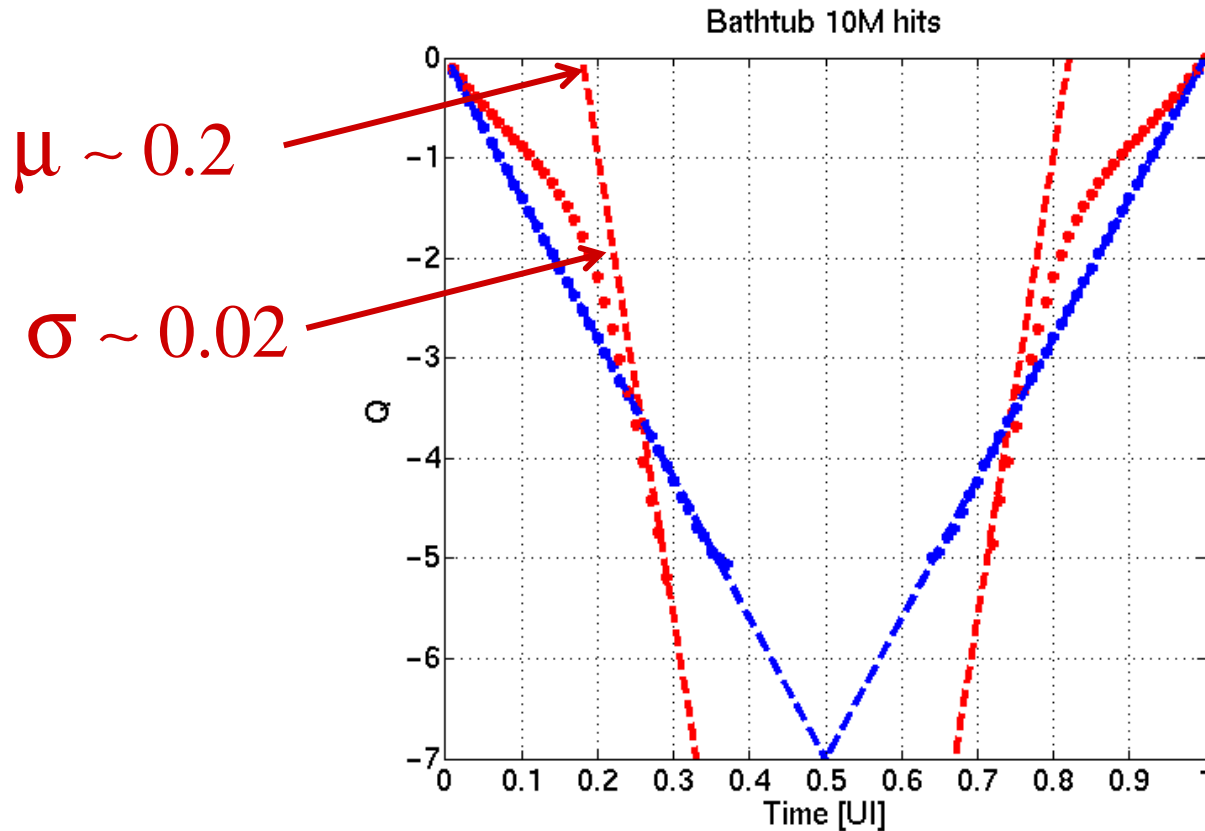
- ❑ Bathtub over 10k hits
- ❑ “Red” system has steeper slopes as “Blue” one
- ❑ For low BER, “Red” system seems to be better

# Bathtub Example (3/4)



- ❑ Bathtub over 10 Million hits
- ❑ “Red” system definitely has larger eye opening for low BER
- ❑ From this graph Eye Opening @  $1\text{e-}12$  not easy to predict

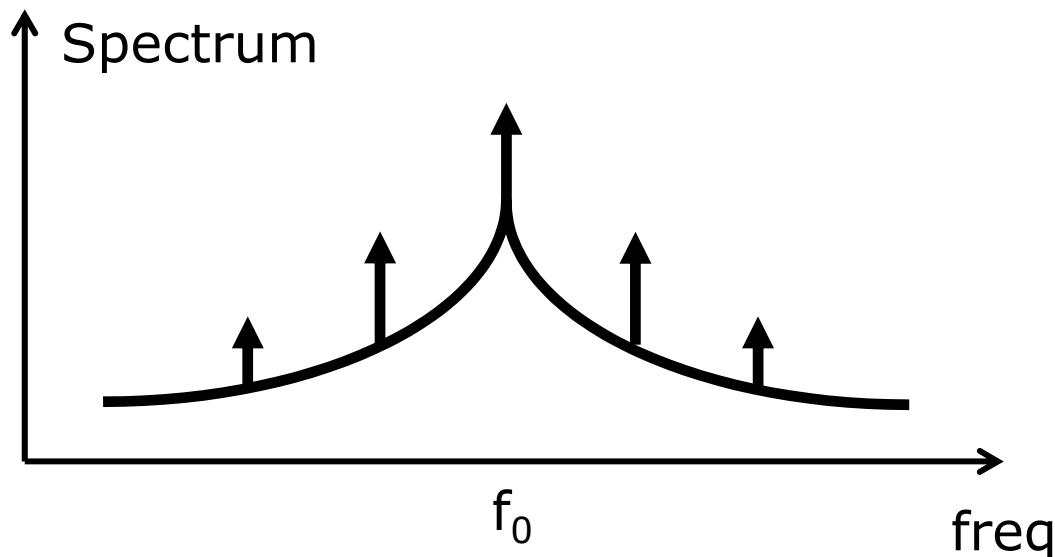
# Bathtub Example (4/4)



- ❑ Bathtub in Q-scale
- ❑ “Blue” has closed eye @  $1e-12$  (system not working)
- ❑ “Red” system has  $> 30\%$  eye opening @  $1e-12$

# Spurious Tones in Spectrum (1/4)

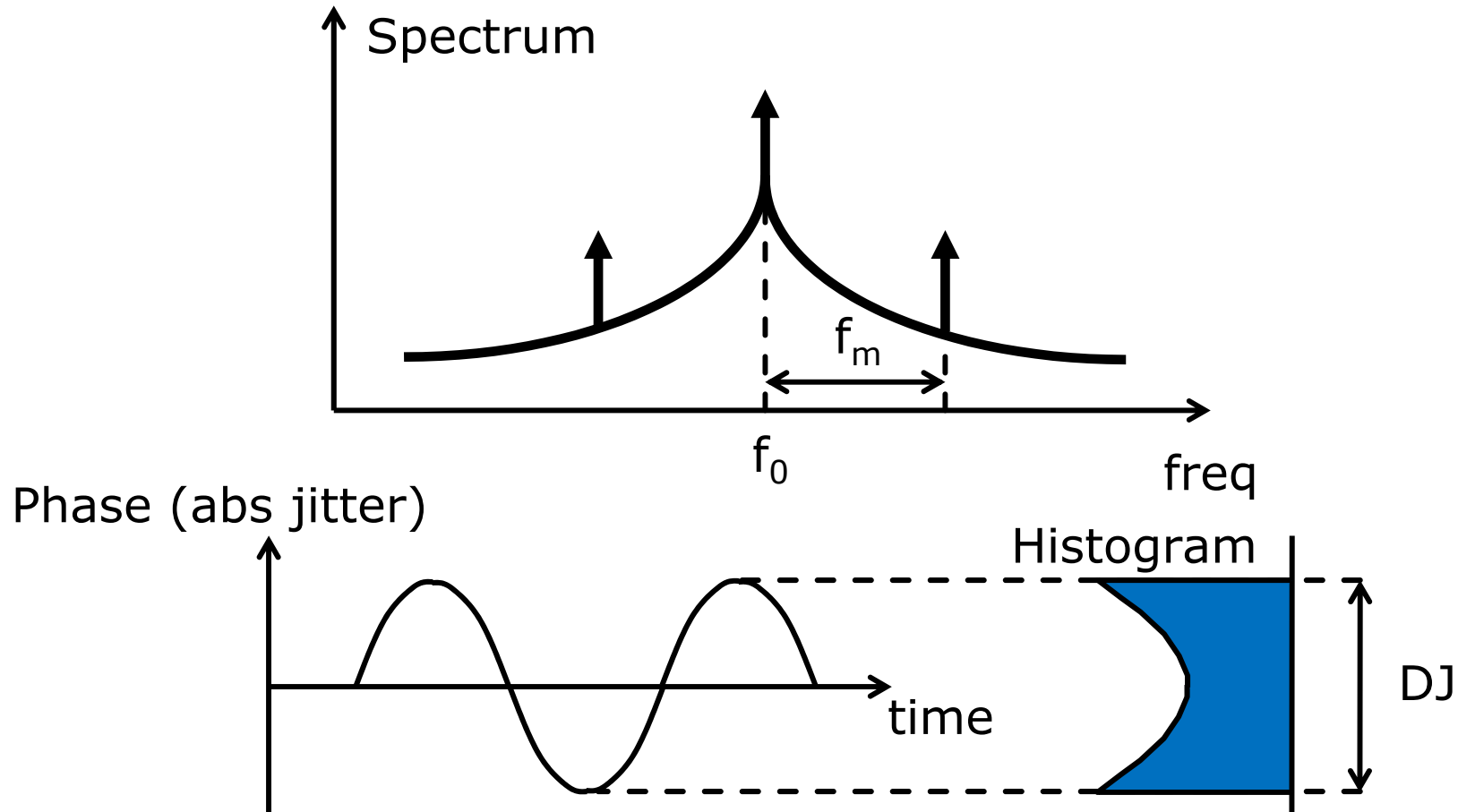
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- ❑ Spurious tones indicate periodical phase modulation
- ❑ Periodic signals can be decomposed in sinusoidal components
- ❑ Each sinusoidal component contributes to bounded jitter (DJ)

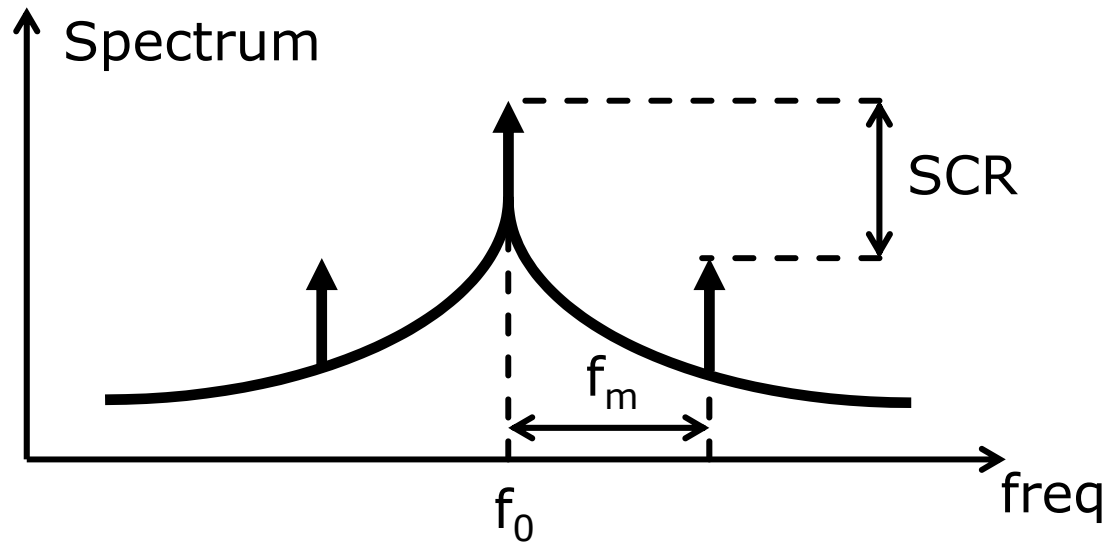


# Spurious Tones in Spectrum (2/4)



- Each sinusoidal component contributes to bounded jitter (DJ)

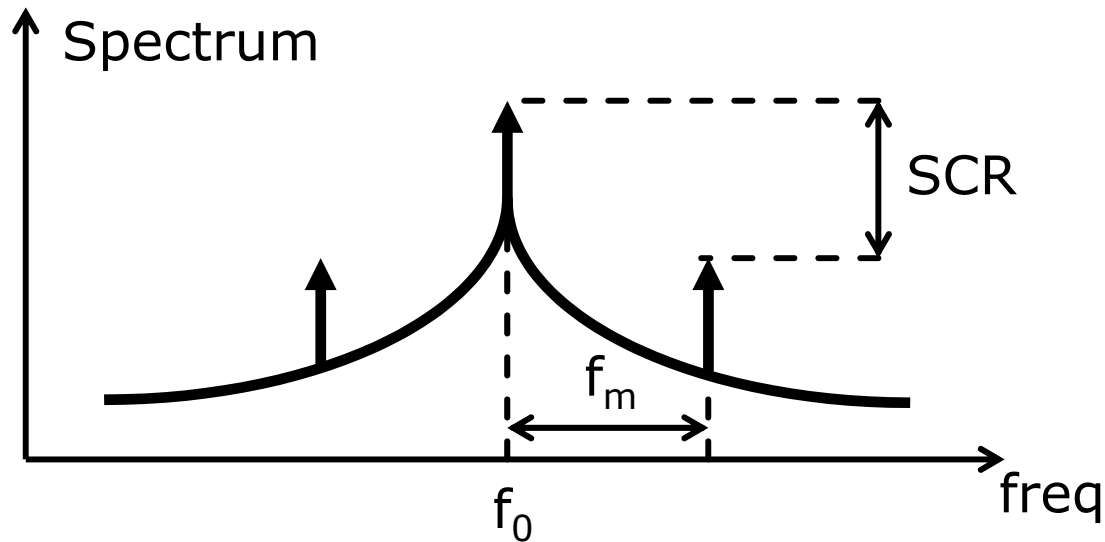
# Spurious Tones in Spectrum (3/4)



$$V(t) = V_0 \sin(\omega_0 t + A \cdot \sin(\omega_m t)) = V_0 \cdot \left[ J_0(A) \sin(\omega_0 t) - J_1(A) \cos((\omega_0 - \omega_m)t) + \dots \right. \\ \left. \dots + J_1(A) \cos((\omega_0 + \omega_m)t) + \dots \right]$$

$$\text{SCR} = 20 \log_{10} \left( \frac{J_1(A)}{J_0(A)} \right) = 20 \log_{10} \left( \frac{A}{2} \right) \quad \text{for small } A$$

# Spurious Tones in Spectrum (4/4)



$$DJ[s] = \frac{2A}{2\pi f_0} = \frac{2}{\pi f_0} 10^{SCR/20}$$

$$DJ[UI] = \frac{2}{\pi} 10^{SCR/20}$$

SCR [dBc]	DJ [UI]
-10	2.0E-01
-15	1.1E-01
-20	6.4E-02
-25	3.6E-02
-30	2.0E-02
-35	1.1E-02
-40	6.4E-03
-45	3.6E-03
-50	2.0E-03
-55	1.1E-03
-60	6.4E-04
-65	3.6E-04
-70	2.0E-04
-75	1.1E-04
-80	6.4E-05

□ DJ depends on SCR and  $f_0$ , not on  $f_m$

# Summary

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- ❑ Definition of several Jitter types (Absolute jitter, Period Jitter, Accumulated Jitter)
- ❑ Measurement in Time and Frequency Domain
- ❑ Phase Noise and relationship to Jitter
- ❑ Jitter in typical applications
- ❑ Analysis of statistical properties of Jitter for gaussian and non-gaussian distributions
- ❑ Jitter decomposition in RJ and DJ
- ❑ DJ in High Speed Data Communication (PJ/SJ,DDJ/ISI,DCD,...)
- ❑ Tail fitting
- ❑ Bathtub plots
- ❑ Extrapolation of eye opening @ very low BER

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