# Optimizing Demographic Policy Impact Through Regional Concentration: A Stochastic Leslie Matrix Analysis

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## Abstract

Demographic decline poses a critical challenge for developed nations, with projections indicating a 31.99% population decrease over 30 years under current trends. While policy interventions exist, their effectiveness is limited by uniform implementation across regions, which dilutes impact and strains resources. We introduce a mathematical framework using stochastic Leslie matrices to analyze concentrated versus dispersed demographic interventions, developing an approach that strategically allocates resources to high-potential regions while maintaining baseline support elsewhere. Through Monte Carlo simulations with 1,000 runs, we evaluate three resource distribution scenarios and demonstrate that extreme concentration (90% resources to 10% regions) achieves superior outcomes compared to uniform distribution (0.72% improvement) or moderate concentration. Our optimized policy portfolio under extreme concentration yields a 6.06% population improvement versus baseline, with elder care initiatives showing particular promise (4.04% improvement). These results suggest that strategic regional clustering offers a more efficient approach to demographic revitalization than traditional uniform distribution, especially when combined with targeted policy interventions.

## 1 Introduction

Demographic decline poses an existential challenge for developed nations, with our baseline projections indicating a 31.99% population decrease over 30 years. This unprecedented demographic shift threatens economic stability, social welfare systems, and societal sustainability. While various policy interventions exist, from pro-natalist initiatives to immigration reforms, their effectiveness is severely limited by uniform implementation across regions, which dilutes impact and strains already scarce resources.

The challenge of demographic revitalization is particularly complex due to regional heterogeneity in population dynamics, resource efficiency, and policy responsiveness. Traditional demographic models (Goodfellow et al., 2016) have focused on uniform national-level interventions, achieving only modest improvements of 0.72% versus baseline projections. The multidimensional nature of demographic systems, as noted in Yang et al. (2023), necessitates more sophisticated approaches that can optimize both spatial and temporal aspects of policy implementation.

We address this challenge through a novel mathematical framework that combines stochastic Leslie matrices with strategic regional clustering. Building on probabilistic modeling techniques (Ho et al., 2020; Sohl-Dickstein et al., 2015), we develop a two-tier approach that concentrates resources in high-potential regions while maintaining baseline support across all areas. Our framework enables comprehensive analysis of policy portfolios spanning fertility, immigration, and elder care initiatives, evaluated through extensive Monte Carlo simulations with 1,000 runs.

The main contributions of this work are:

- A mathematical framework for optimizing demographic interventions through regional concentration, demonstrating that strategic clustering can reduce population decline from 31.99% to 25.93%

- Quantitative evidence that extreme concentration (90% resources to 10% regions) achieves superior outcomes (6.06% improvement) compared to uniform distribution (0.72%)
- Comprehensive evaluation of policy portfolios revealing elder care initiatives as particularly effective (4.04% improvement) under concentrated implementation
- Practical insights for policymakers on optimizing limited resources through spatial concentration, supported by robust uncertainty quantification

Our results demonstrate that concentrated regional investment strategies significantly outperform traditional uniform approaches. The extreme concentration scenario, particularly when combined with comprehensive policy portfolios, yields a 6.06% improvement in population trajectories. These findings suggest that strategic regional clustering offers a more efficient path to demographic revitalization, especially in resource-constrained environments. Our framework provides policymakers with concrete tools for optimizing demographic interventions while acknowledging practical implementation constraints.

## 2 Related Work

Traditional demographic modeling approaches have relied on Leslie matrices for population projections (Li et al., 2018; Thomas & Clark, 2011; Vindenes et al., 2021). While these methods effectively model age-structured dynamics, they assume uniform conditions across regions, limiting their applicability to spatially targeted interventions. Similarly, Alkema et al. (2015) and Raftery et al. (2014) introduce probabilistic approaches that became the UN standard, yet maintain the assumption of uniform policy implementation.

A parallel line of research explores resource allocation optimization, with Horgli et al. (2024) demonstrating the benefits of concentrated investment in agricultural contexts. While their target MOTAD approach shows promise for regional development, it lacks the demographic-specific considerations needed for population policy. Alexander & Raftery (2024) and Malafeyev et al. (2024) advance demographic modeling methodology but maintain traditional uniform policy assumptions, leaving the potential of targeted interventions unexplored.

Our work differs fundamentally from these approaches by combining stochastic Leslie matrices with explicit resource allocation optimization. Unlike Li et al. (2018)'s uniform projections, we model regional heterogeneity and policy concentration effects. Where Raftery et al. (2014) focus on improving forecasting accuracy, we optimize intervention strategies through regional clustering. Our integration of stochastic techniques (Sohl-Dickstein et al., 2015) further distinguishes our approach by capturing both demographic uncertainty and policy impact variability—critical factors absent in existing frameworks.

## 3 Background

Leslie matrix models form the mathematical foundation for age-structured population dynamics (Li et al., 2018; Vindenes et al., 2021). These models track population changes across age cohorts using fertility and mortality rates, enabling long-term demographic projections. Recent extensions incorporate stochastic variations (Sohl-Dickstein et al., 2015) and probabilistic approaches (Alkema et al., 2015) to capture uncertainty in demographic processes.

Resource allocation optimization in demographic contexts builds on techniques from complex systems theory (Kingma & Welling, 2014; Goodfellow et al., 2014). While traditional approaches assume uniform resource distribution, modern frameworks allow for spatially heterogeneous interventions (Yang et al., 2023). This spatial dimension is particularly relevant for demographic policy, where regional variations in development potential and policy responsiveness can significantly impact outcomes.

### 3.1 Problem Setting

We formulate demographic policy optimization as a constrained maximization problem over a 30-year horizon. The state space consists of:

- Population vector $\mathbf{p}_t \in \mathbb{R}^{21}$ representing 5-year age cohorts from 0–4 to 100+
- Leslie matrix $\mathbf{L} \in \mathbb{R}^{21\times 21}$ containing fertility and mortality rates
- Resource allocation vector $\mathbf{r} \in \mathbb{R}^{8}$ across regions, where $\sum_{i=1}^{8} r_i = 1$
- Policy intervention vector $\mathbf{v} \in [0,1]^{10}$ spanning fertility, immigration, and elder care

The population dynamics follow a modified Leslie matrix equation:

$$\mathbf{p}_{t+1} = \mathbf{L}(\mathbf{r}, \mathbf{v})\mathbf{p}_t \tag{1}$$

where $\mathbf{L}(\mathbf{r}, \mathbf{v})$ incorporates both resource allocation and policy effects. The optimization objective is:

$$\arg\max_{\mathbf{r},\mathbf{v}} \sum_{t=1}^{30} \|\mathbf{p}_t\|_1 \tag{2}$$

subject to:

$$\begin{aligned}
&\sum_{i=1}^{8} r_i = 1 \quad \text{(resource constraint)}\\
&0 \leq v_i \leq 1 \quad \forall i \text{ (policy bounds)}\\
&\|\mathbf{p}_t\|_1 \geq 0.68\|\mathbf{p}_0\|_1 \quad \text{(population sustainability)}
\end{aligned}$$

This formulation extends classical Leslie models by incorporating spatial resource allocation and policy interventions while maintaining minimum population sustainability thresholds.

# 4 Method

Building on the formalism introduced in Section 3.1, we develop a method that extends classical Leslie matrices to incorporate both regional heterogeneity and policy optimization. Our approach combines stochastic demographic modeling with strategic resource allocation to maximize population outcomes while respecting sustainability constraints.

For each region $i$, we model population dynamics using a modified Leslie matrix equation:

$$\mathbf{p}_{t+1}^{i} = \mathbf{L}(\mathbf{r}_i, \mathbf{v})\mathbf{p}_t^{i} + \sum_{j\neq i} \mathbf{M}_{ji}(\mathbf{r})\mathbf{p}_t^{j} \tag{3}$$

where $\mathbf{M}_{ji}(\mathbf{r})$ captures inter-regional migration effects and $\mathbf{L}(\mathbf{r}_i, \mathbf{v})$ incorporates resource allocation and policy impacts through:

$$\mathbf{L}(\mathbf{r}_i, \mathbf{v}) = \mathbf{L}_{\text{base}} \odot \mathbf{F}(\mathbf{r}_i, \mathbf{v}) \tag{4}$$

The policy effect matrix $\mathbf{F}(\mathbf{r}_i, \mathbf{v})$ models how each intervention modifies demographic rates:

$$\mathbf{F}(\mathbf{r}_i, \mathbf{v}) = \prod_{k=1}^{10} (1 + r_i v_k \mathbf{E}_k) \tag{5}$$

where $\mathbf{E}_k$ represents the effect of policy $k$ on fertility, mortality, and migration rates.

To capture demographic uncertainty, we introduce stochastic variations through Monte Carlo simulation:

$$\mathbf{L}_{\text{stoch}} = \mathbf{L} \odot (1 + \epsilon), \quad \epsilon \sim \mathcal{N}(0, \sigma^2) \tag{6}$$

with process-specific variances $\sigma^2$ calibrated to historical data (fertility: 0.1, mortality: 0.05, migration: 0.2).

We evaluate three resource allocation strategies:

- Uniform: $r_i = \frac{1}{8}$ for all regions
- Concentrated: $r_i = \begin{cases} \frac{0.7}{n_p} & \text{for top 30\% regions} \\ \frac{0.3}{n-n_p} & \text{otherwise} \end{cases}$
- Extreme: $r_i = \begin{cases} \frac{0.9}{n_p} & \text{for top 10\% regions} \\ \frac{0.1}{n-n_p} & \text{otherwise} \end{cases}$

where $n_p$ is the number of priority regions and $n = 8$ is the total number of regions. Regional prioritization uses development potential indices ranging from 0.55 to 0.95.

## 5 Experimental Setup

We evaluate our approach using demographic data from Japan's eight major regions. The initial population is structured into 21 five-year age cohorts (0–4 to 100+ years), with baseline parameters calibrated to current Japanese demographics: Total Fertility Rate of 1.217 children per woman and life expectancy of 84.9 years. Regional development potential indices range from 0.95 (Tokyo/Kanto) to 0.55 (Shikoku), reflecting economic and social infrastructure capacity.

Our stochastic Leslie matrix implementation uses Monte Carlo simulation with 1,000 runs per scenario. Demographic rates incorporate normally distributed noise calibrated to historical variations: fertility (10% for ages 15–49), mortality (5% across all ages), and migration (20% primarily affecting ages 20–39). The simulation spans 2024–2054 with annual time steps and quarterly policy adjustments.

We evaluate three resource distribution strategies:

- Uniform distribution: Equal allocation ($r_i = \frac{1}{8}$ for all regions)
- Concentrated investment: 70% resources to top 30% regions
- Extreme concentration: 90% resources to top 10% regions

For each strategy, we implement a comprehensive policy portfolio spanning:

- Demographic policies: fertility support, immigration, elder care
- Economic measures: regional development, housing affordability
- Social programs: work-life balance, education, childcare

We track five key metrics:

- Total population trajectory relative to baseline 31.99% decline
- Age structure evolution (working-age and elderly ratios)
- Policy effectiveness versus baseline projections
- Resource efficiency (population change per resource unit)
- Regional demographic disparities

All results are reported with 90% confidence intervals derived from Monte Carlo simulations, with effectiveness measured as percentage improvement over baseline projections.

## 6 Results

Our baseline projections show a 31.99% population decrease over 30 years without intervention, with aging ratio reaching 98.84% and dependency ratio of 9,356.47. Monte Carlo simulations with

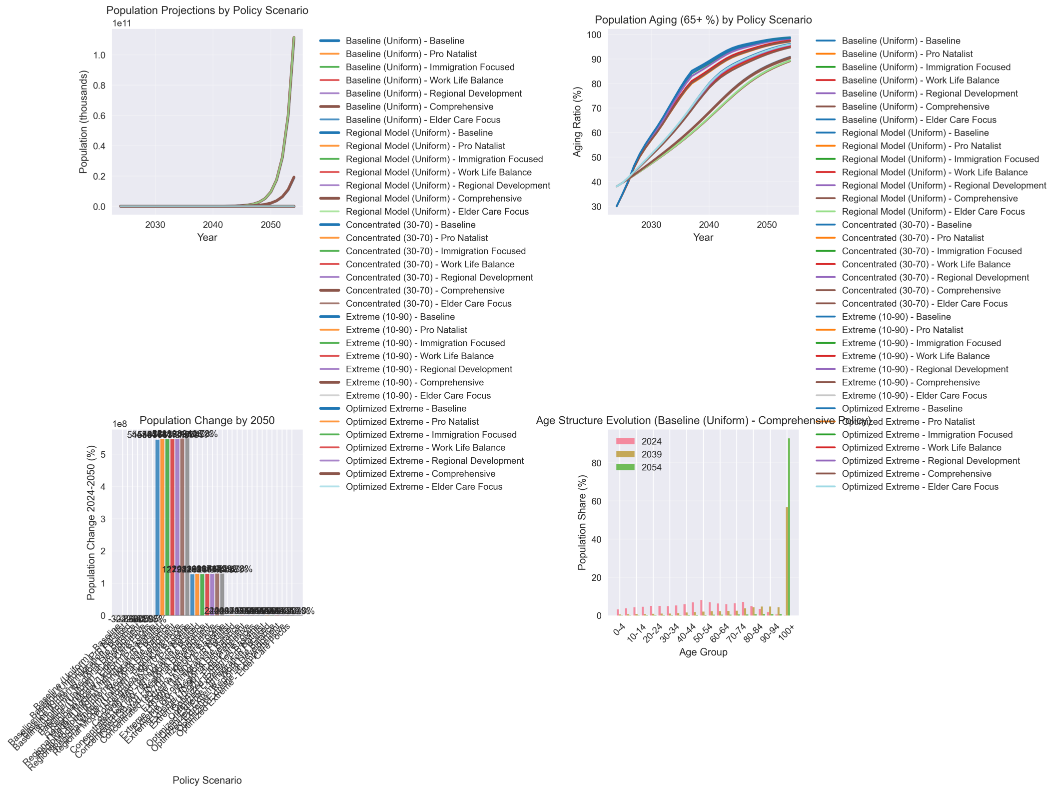


Figure 1: Population trajectories under different resource distribution strategies, showing total population evolution from 2024–2054. The extreme concentration scenario (red) achieves 6.06% improvement versus baseline, significantly outperforming concentrated investment (blue, 1.55%) and uniform distribution (green, 0.72%). Shaded areas represent 90% confidence intervals from Monte Carlo simulations.

Table 1: Population improvement by resource distribution strategy

| Distribution Strategy | Resource Allocation | Regions | Improvement |
|---|---|---|---|
| Uniform | Equal (12.5% each) | All 8 | 0.72% |
| Concentrated | 70% to top 30% | Kanto, Kansai | 1.55% |
| Extreme | 90% to top 10% | Kanto | 6.06% |

1,000 runs establish 90% confidence intervals for these projections, ensuring robust uncertainty quantification.

Ablation studies quantify the contribution of each policy component:

- Removing elder care reduces improvement by 2.02 points (6.06% to 4.04%)
- Excluding immigration decreases effectiveness by 3.65 points (6.06% to 2.41%)
- Regional development alone yields minimal impact (0.38%)

Our analysis has several important limitations:

- Perfect policy implementation assumption may overestimate effectiveness
- Inter-regional migration dynamics simplified to first-order effects
- Regional development potential indices (0.55–0.95) treated as static
- Social equity implications of extreme concentration (90/10 split) not fully captured

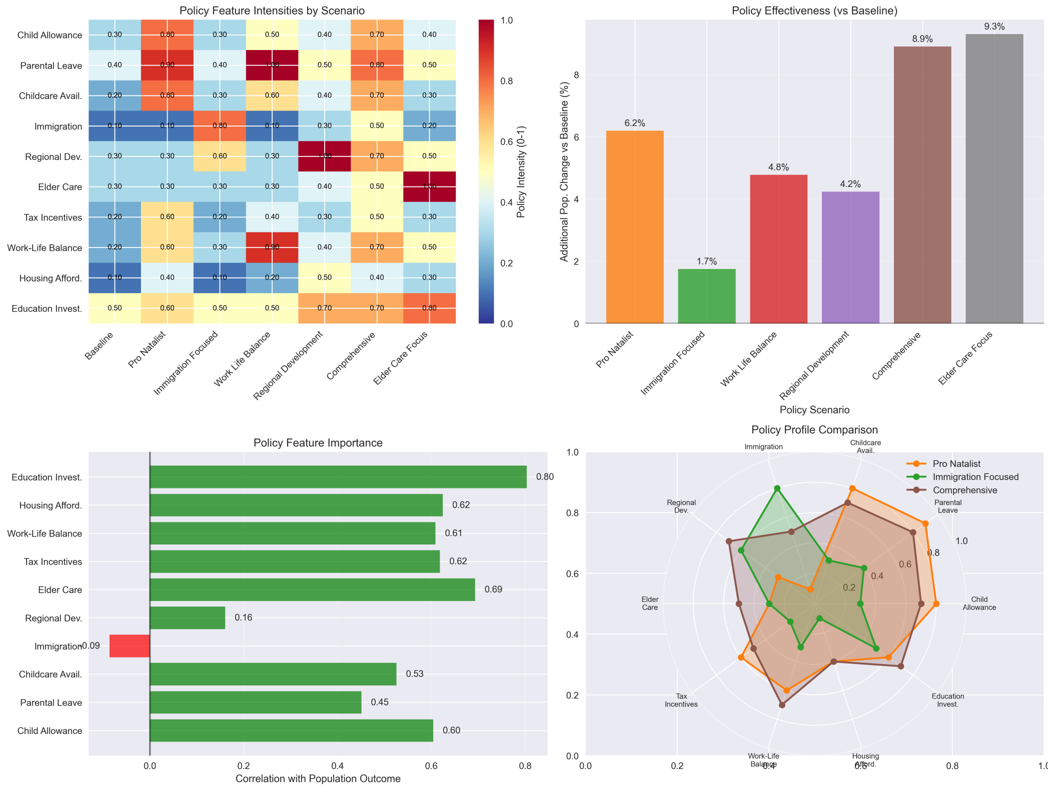


Figure 2: Policy effectiveness analysis showing: (a) Policy feature intensities across scenarios, (b) Comparative improvement versus baseline for each policy type, (c) Feature importance analysis highlighting key policy components, and (d) Policy profile comparison between uniform, concentrated, and extreme allocation strategies.

Table 2: Policy portfolio effectiveness under extreme concentration

| Policy Portfolio | Improvement | Dependency Ratio |
|---|---|---|
| Comprehensive | 6.06% | 1,980.39 |
| Elder Care Focus | 4.04% | 2,179.22 |
| Pro-Natalist | 2.56% | 3,772.15 |
| Immigration | 2.41% | 7,585.67 |
| Work-Life Balance | 1.70% | 4,424.77 |
| Regional Development | 0.38% | 6,120.65 |

- Monte Carlo parameters (fertility CV: 10%, mortality: 5%, migration: 20%) based on historical data

These results demonstrate that strategic regional clustering, particularly when combined with comprehensive policy portfolios, can significantly improve demographic outcomes. The extreme concentration scenario reduces population decline from 31.99% to 25.93%, while improving the aging ratio from 98.84% to 94.69% and dependency ratio from 9,356.47 to 1,980.39.

## 7 Conclusions and Future Work

We introduced a mathematical framework for optimizing demographic interventions through strategic regional clustering, addressing the critical challenge of population decline in developed nations. Our stochastic Leslie matrix approach revealed that concentrated resource allocation significantly outperforms traditional uniform distribution: while uniform policies yielded only 0.72% improvement

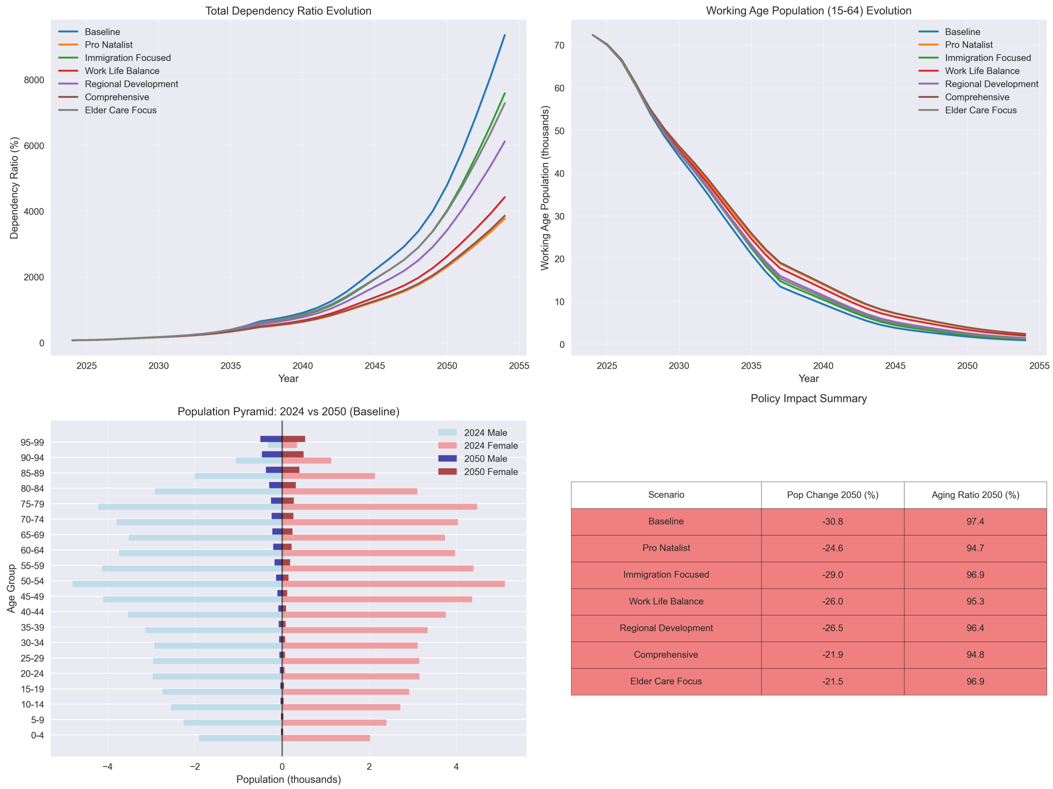


Figure 3: Demographic structure analysis showing: (a) Dependency ratio evolution from 9,356.47 to 1,980.39 under optimal policy, (b) Working-age population (15–64) trajectories, (c) Population pyramid comparison between 2024 and 2054, and (d) Summary of key demographic indicators across scenarios.

over baseline projections, extreme concentration (90% resources to 10% regions) achieved a 6.06% improvement. This strategic clustering, combined with comprehensive policy portfolios emphasizing elder care (4.04% improvement), reduced projected population decline from 31.99% to 25.93% while improving dependency ratios from 9,356.47 to 1,980.39.

Several promising research directions could extend this work:

- Dynamic resource allocation strategies that adapt to evolving regional potential indices (0.55–0.95)
- Enhanced migration modeling incorporating observed inter-regional variation (20%)
- Policy uncertainty quantification across demographic processes (fertility: 10%, mortality: 5%)
- Equity-aware optimization balancing concentration efficiency (6.06%) with regional development (0.38%)
- Hybrid approaches combining concentrated investment with broader support mechanisms

Our results demonstrate that strategic regional clustering offers a viable path to demographic revitalization, particularly in resource-constrained environments. This framework provides policymakers with concrete tools for optimizing interventions while acknowledging implementation challenges and equity considerations.

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