Assignment #2 :

Q7 :

```
void scanInterval(int lo, int hi)
//@requires 0 <= lo && lo <= hi;   — L1
{
L0→int i;   ← L2  ← L3  ← L4
    for ( i = lo; i < hi; i++ )
    //@loop_invariant lo <= i && i <= hi; ← L5
    {
        //body
    }
    //@assert i==hi; ← L6
}
```

Termination :-

Theorem: Any strictly increasing int sequence bounded from above terminates (or, in other words finite)

① i is int by L0.
② i is strictly increasing by L4.
③ i is bounded from above by L3.

By ① + ② + ③ and the theorem the loop terminates.

Assertion proof (L6):

L6 is reached after the loop ends; hence the loop guard must be false.

LG is $i < hi$

LG false is $i \geq hi$. —①

Assuming LI at L5 is true we have $i \leq hi$. —②

①+② gives us $i = hi$ at L6.

Proof of LI (L5):

The loop invariant consists of two statements $lo \leq i$ and $i \leq hi$.

Proof of $lo \leq i$:

INIT step (Before the loop is executed)

① $i = lo$ by L2.

② $lo \leq lo$ fact.

③ $lo \leq i$ by ①+②.

PRES step (i.e the loop preserves the invariant).

① Assume that $lo \leq i$ before the loop is executed for any *arbitrary iteration* of the loop.

② Assume i changes to i' after execution of loop body once.

③ $i' = i + 1$ by 14

④ $lo + 1 \leq i' + 1$ by ①

⑤ $lo + 1 \leq i'$ by ③

⑥ $lo \leq lo + 1$ is a fact.

⑦ $lo \leq i'$ by ⑤ + ⑥.

This proves the LI: $lo \leq i$.

Proof of $i \leq hi$

Init step :

① $lo \leq hi$ by L1.

② $i = lo$ by L2.

③ $i \leq hi$ by ① + ②.

PRES step on next page :-

Let the value of i after loop be i'.

① Assume LI : $i \leq hi$ is true before the loop execution.

② As loop is executed; by L3 we have $i < hi$

$\Rightarrow i \leq hi - 1$. (fact).

③ add one on both sides of ②.

$i + 1 \leq hi$

④ $i' \leq hi$ using L4.

Hence proved.