LDA

$$p_k(x) = \frac{\pi_k \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{x-\mu_k}{\sigma}\right)^2\right)}{\underbrace{\text{const.}}}$$

$$\log(p_k(x)) = \boxed{\log(\pi_k) + \frac{1}{2}\frac{2\mu_k x}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2}} + \text{const}(\text{w.r.t. } k)$$

$$(\sigma^2 = \sigma_k^2)$$

$$\underline{\underline{\delta_k(x)}} = \frac{\mu_k}{\sigma^2} \cdot x - \frac{\mu_k^2}{2\sigma^2} + \log(\pi_k)$$

For example : $K = 2$, $\boxed{\pi_1 = \pi_2}$

Decision boundary $\delta_1(x) = \delta_2(x)$

$$x \cdot \frac{\mu_1}{\cancel{\sigma^2}} - \frac{\mu_1^2}{2\cancel{\sigma^2}} + \cancel{\log(0.5)}$$

$$= x \frac{\mu_2}{\cancel{\sigma^2}} - \frac{\mu_2^2}{2\cancel{\sigma^2}} + \cancel{\log(0.5)}$$

$$\Rightarrow x\cancel{(\mu_1 - \mu_2)} = \underbrace{\frac{\mu_1^2}{2} - \frac{\mu_2^2}{2}}_{\frac{1}{2}(\mu_1+\mu_2)\cancel{(\mu_1-\mu_2)}}$$

$$\Rightarrow \qquad \underbrace{x = \frac{\mu_1+\mu_2}{2}}_{\text{decision boundary}}$$

In general

$$\underbrace{\delta_1(x) = \delta_2(x)}$$

decision boundary:

$$x = \underbrace{\frac{\mu_1 + \mu_2}{2}} + \sigma^2 \frac{\overbrace{\log(\pi_2) - \log(\pi_1)}}{(\mu_1 - \mu_2)}$$

In practice, decision boundary is calculated with estimated values

$$x = \frac{\hat{\mu}_1 - \hat{\mu}_2}{2} + \hat{\sigma}^2 \frac{\log(\hat{\pi}_2) - \log(\hat{\pi}_1)}{(\hat{\mu}_1 - \hat{\mu}_2)}$$

$$\underline{(x_1 \;\; x_2)} \, \underline{\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}} \cdot \underline{\begin{pmatrix} -2 \\ -2 \end{pmatrix}}$$

$$-\frac{1}{2} (1 \;\; 1) \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \overbrace{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}^{\mu_A}$$

$$+\frac{1}{2}\begin{pmatrix}3 & 3\end{pmatrix}\begin{pmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{2}\end{pmatrix}\begin{pmatrix}3\\ 3\end{pmatrix} = 0$$

matrix/vector calculations

$$-x_1 - x_2 - \frac{1}{2} - \frac{9}{2} = 0$$

$$x_1 + x_2 = 4$$

$$\underline{\underline{x_2 = 4 - x_1}}$$