$$\operatorname{cov}(\underbrace{X}_{\begin{pmatrix} x_1 \\ \vdots \\ x_p \end{pmatrix}}) = E\left[(X-\mu)(X-\mu)^T\right]$$

$$= E\left[\begin{pmatrix} x_1-\mu_1 \\ \vdots \\ x_p-\mu_p \end{pmatrix} \cdot (x_1-\mu_1, \ldots, x_p-\mu_p)\right]$$

$$= E\begin{bmatrix} (x_1-\mu_1)\cdot(x_1-\mu_1) & \cdots & (x_p-\mu_p)(x_1-\mu_1) \\ (x_1-\mu_1)(x_2-\mu_2) & \ddots & \\ \vdots & & \ddots \\ (x_1-\mu_1)(x_p-\mu_p) & \cdots & (x_p-\mu_p)^2 \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_1^2 & \text{---} & \sigma_{p1} \\ \sigma_{12} & \diagdown & \vdots \\ \vdots & & \\ \sigma_{1p} & \sim & \sigma_p^2 \end{bmatrix}$$

$$\cdot\ \mu_z = E[Z] = E[CX] = E[\overset{d}{C}X \cdot \overset{d}{I}]$$

$$\underset{\text{slide 9}}{=} C\,E[X]\cdot I = C\cdot\mu_x$$

$$\cdot\ \mathrm{cov}(Z) = E\Big[(Z - \overset{\overset{C\cdot\mu_x}{b}}{\mu_z})(Z-\mu_z)^T\Big]$$

$$= E[((CX - C\cdot\mu_x)(CX - C\mu_x)^T]$$

$$= E[C\cdot(X-\mu_x)(X-\mu_x)^T\cdot C^T]$$

$$= C\ \underbrace{E[(X-\mu_x)(X-\mu_x)^T]}_{\mathrm{cov}(X) = \Sigma}\cdot C^T$$

$$\underline{C\cdot\Sigma\cdot C^T}$$

For univariate case: $Var(c \cdot X)$

$= c^2 \cdot Var(X)$

Aim: $\underset{3\times 1}{Y} = \begin{bmatrix} X_N - X_S \\ X_E + X_W \\ (X_E + X_W) - (X_N + X_S) \end{bmatrix}$

Find C such that

$C \cdot X = Y$

$Y = C \cdot X$

$C = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 1 & -1 & 1 \end{pmatrix}$

$cov(Y_1, Y_3)$?

$$\mathrm{cov}(Y) = \underbrace{C \cdot \mathrm{cov}(X) \cdot C^T}$$