$$E\left[(X-\mu)(X-\mu)^T\right]$$

$$= E\left[\begin{pmatrix} X_1-\mu_1 \\ X_2-\mu_2 \\ \vdots \\ X_p-\mu_p \end{pmatrix} (X_1-\mu_1, \dots, X_p-\mu_p)\right]$$

$$= \begin{bmatrix} E[(X_1-\mu_1)^2] & \cdots & E[(X_p-\mu_p)(X_1-\mu_1)] \\ E[(X_2-\mu_2)(X_1-\mu_1)] & & \\ \vdots & & \\ E[(X_p-\mu_p)(X_1-\mu_1)] & \cdots & E[(X_p-\mu_p)^2] \end{bmatrix}$$

Covariances

Variances

$$
= \begin{bmatrix} z_1^2 & z_{12} & \cdots & \\ z_{12} & z_2^2 & & \\ \vdots & & \ddots & \\ \vdots & & & z_p^2 \end{bmatrix}
$$

$$\mu_z = E[Z] = E[CX] = E[CXI]$$

$$= C\,E[X] \cdot I = C \cdot E[X] = C \cdot \mu$$

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$$\mathrm{Cov}(Z) = E\left[(Z-\mu_z)(Z-\mu_z)^T\right]$$

$$= E\left[(CX - C\cdot\mu)\,(CX - C\mu)^T\right]$$

$$= E\left[C(X-\mu)\cdot(X-\mu)^T \cdot C^T\right]$$

$$= C\, E[(X-\mu)(X-\mu)^T] \cdot C^T$$

$$= C \cdot \Sigma \cdot C^T$$

Need C such that

$$Y = \begin{bmatrix} X_1 - X_3 \\ X_2 + X_4 \\ (X_2+X_4)-(X_1+X_3) \end{bmatrix} = C \cdot \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix}$$

$$C \cdot X = Y$$

$$C = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 1 & -1 & 1 \end{pmatrix}$$

$$E[Y] = C \cdot E[X]$$

$$\mathrm{cov}(Y) = C \cdot \mathrm{cov}(X) \cdot C^T$$