



$$TSS = ESS + RSS$$

$$\sum_{i=1}^n (y_i - \bar{y})^2 = \sum_{i=1}^n (\hat{y}_i - \bar{y})^2 + \underbrace{\sum_{i=1}^n (y_i - \hat{y}_i)^2}_{\text{Residuals}}$$

$$R = \frac{ESS}{TSS} = \frac{TSS - RSS}{TSS} = 1 - \frac{RSS}{TSS}$$

$$\frac{\partial}{\partial \beta} \underbrace{(Y - X\hat{\beta})^T (Y - X\hat{\beta})}_{LS(\beta)} = 0$$

$$LS(\hat{\beta}) = Y^T Y - Y^T X \beta - \underbrace{\beta^T X^T Y}_{= Y^T X \beta \text{ (scalar)}} + \beta^T X^T X \beta$$

$$= \cancel{Y^T Y} - 2 \cdot Y^T X \beta + \beta^T X^T X \beta$$

$$\frac{\partial}{\partial \beta} (LS(\beta)) = \underbrace{-2 X^T Y}_{\frac{\partial}{\partial \beta} d^T \beta = d} + \underbrace{2 X^T X \beta}_{\text{because } (X^T X)^T = X^T X} = 0$$

$$\Rightarrow X^T Y = (X^T X) \hat{\beta}$$

$$\Rightarrow \underbrace{(X^T X)^{-1}}_{\substack{\uparrow \\ \uparrow}} \underbrace{X^T Y}_{\substack{\uparrow \\ \uparrow}} = \underline{\underline{\hat{\beta}}}$$