Notation ~> See course book p. 9-12

p: # of predictors in our model:

Matrix

$$X_{n \times p} = \begin{bmatrix} x_{11} & \cdots & x_{1p} \\ x_{21} & & x_{2p} \\ \vdots & & \vdots \\ x_{n1} & \cdots & x_{np} \end{bmatrix}$$

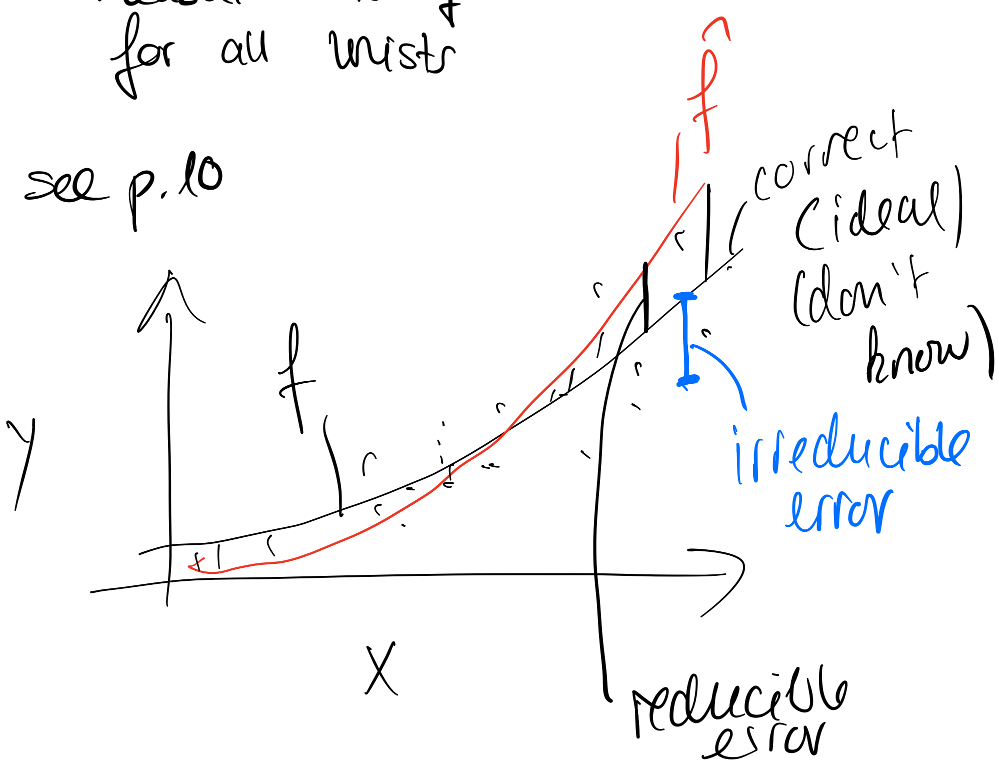
$\underbrace{x_{11}, \dots, x_{n1}}_{X_1}$ ... $\underbrace{x_{1p}, \dots, x_{np}}_{X_p}$

X_1^T all meas. for unit 1

n units

X_1 → measurements of variable 1 for all units

see p. 10



Reducible vs. irreducible error

Assume $\hat{f}$ and X are fixed

$$E\left[(\underbrace{Y-\hat{Y}}_{\hat{f}(X)})^2\right] = E\left[\left(f(X)+\varepsilon-\hat{f}(X)\right)^2\right]$$

$$= E\left[\left(f(X)-\hat{f}(X)\right)^2\right] + \overbrace{E\left[\varepsilon^2\right]}^{var(\varepsilon)}$$

$$+ \underbrace{2\,E\left[\left(\underbrace{f(X)-\hat{f}(X)}_{\text{fixed}}\right)\cdot\varepsilon\right]}_{=0 \quad (E(\varepsilon)=0)}$$

$$= \left(f(X)-\hat{f}(X)\right)^2 + var(\varepsilon)$$

Bias-variance trade-off

Expected test MSE at x_0

$$E\left[\left(\underbrace{y_0}_{f(x_0)+\varepsilon}-\hat{f}(x_0)\right)^2\right] = E\left[\left(f(x_0)+\varepsilon-\hat{f}(x_0)\right)^2\right]$$

$$= E\left[\left(f(x_0)\right)^2\right] + E\left[\varepsilon^2\right] + \overbrace{E\left[\left(\hat{f}(x_0)\right)^2\right]}^{Var(\hat{f}(x_0)) + E[\hat{f}(x_0)]^2}$$

$$-2E\left[f(x_0)\hat{f}(x_0)\right] + \underbrace{2E\left[\varepsilon \cdot f(x_0)\right]}_{=0} - \underbrace{2E\left[\varepsilon \cdot \hat{f}(x_0)\right]}_{=0}$$

$E(\varepsilon)=0$

$$= f(x_0)^2 + Var(\varepsilon) + Var(\hat{f}(x_0)) + E\left[\hat{f}(x_0)\right]^2$$

$$- 2E\left[f(x_0)\hat{f}(x_0)\right]$$

$$= \underbrace{Var(\varepsilon)}_{\text{irreducible error}} + \underbrace{Var(\hat{f}(x_0))}_{\text{Variance}} + \underbrace{\left(f(x_0) - E[\hat{f}(x_0)]\right)^2}_{\text{Squared bias}}$$