TMA4268 Statistical Learning

Chapter 10: Unsupervised Learning

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The NCI60 data
The $\tt NCI60$ cancer cell line microarray data consists of 6,830 gene expression measurements on 64 cancer cenes.

library(ISLR)
nci.labs=NCI60\$labs
nci.data=NCI60\$data

The data has 64 rows and 6,830 columns.

```
dim(nci.data)
```

[1] 64 6830

Each cell line is labeled with a cancer type. We begin by examining the cancer types for the cell lines.

```
nci.labs[1:4]
```

```
## [1] "CNS" "CNS" "RENAL"
table(nci.labs)
```

```
## nci.labs
##
        BREAST
                         CNS
                                    COLON K562A-repro K562B-repro
                                                                       LEUKEMIA
                                                     1
                                                 NSCLC
                                MELANOMA
                                                            OVARIAN
                                                                       PROSTATE
## MCF7A-repro MCF7D-repro
##
                                        8
                                                     9
##
         RENAL
                    UNKNOWN
##
              9
                           1
```

PCA on the NCI60 Data

We first perform PCA on the data after scaling the variables (genes) to have standard deviation one, although one could reasonably argue that it is better not to scale the genes.

```
pr.out=prcomp(nci.data, scale=TRUE)
```

We now plot the first few principal component score vectors, in order to visualize the data. The observations (cell lines) corresponding to a given cancer type will be plotted in the same color, so that we can see to what extent the observations within a cancer type are similar to each other.

```
Cols=function(vec){
    cols=rainbow(length(unique(vec)))
    return(cols[as.numeric(as.factor(vec))])
}
par(mfrow=c(1,2))
plot(pr.out$x[,1:2], col=Cols(nci.labs), pch=19,xlab="Z1",ylab="Z2")
plot(pr.out$x[,c(1,3)], col=Cols(nci.labs), pch=19,xlab="Z1",ylab="Z3")
     20
                                                    4
                                                    20
      0
Z
                                              Z_3
     -20
                                                    0
     09-
                                                    -40
                           20
                                                                      0
                                                                         20
             -40
                       0
                                   60
                                                            -40
                                                                                  60
```

On the whole, cell lines corresponding to a single cancer type do tend to have similar values on the first few principal component score vectors. This indicates that cell lines from the same cancer type tend to have pretty similar gene expression levels.

Z1

summary(pr.out)

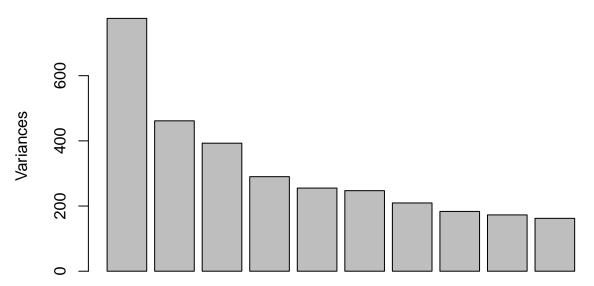
```
## Importance of components:
##
                               PC1
                                        PC2
                                                  PC3
                                                           PC4
                                                                     PC5
                           27.8535 21.48136 19.82046 17.03256 15.97181
## Standard deviation
                           0.1136
                                    0.06756
  Proportion of Variance
                                             0.05752
                                                       0.04248
                                                                0.03735
##
  Cumulative Proportion
                            0.1136
                                    0.18115
                                              0.23867
                                                       0.28115
                                                                 0.31850
##
                                PC6
                                         PC7
                                                   PC8
                                                            PC9
                                                                     PC10
## Standard deviation
                           15.72108 14.47145 13.54427 13.14400 12.73860
  Proportion of Variance
                            0.03619
                                     0.03066
                                              0.02686
                                                        0.02529
                                                                 0.02376
## Cumulative Proportion
                            0.35468
                                     0.38534
                                              0.41220
                                                        0.43750
                                                                 0.46126
##
                               PC11
                                        PC12
                                                  PC13
                                                           PC14
                                                                     PC15
## Standard deviation
                           12.68672 12.15769 11.83019 11.62554 11.43779
## Proportion of Variance
                                     0.02164
                                              0.02049
                            0.02357
                                                        0.01979
                                                                 0.01915
## Cumulative Proportion
                            0.48482
                                     0.50646
                                              0.52695
                                                        0.54674
```

Z1

```
PC17
                                                PC18
##
                              PC16
                                                        PC19
                                                                 PC20
## Standard deviation
                          11.00051 10.65666 10.48880 10.43518 10.3219
## Proportion of Variance 0.01772
                                   0.01663
                                            0.01611
                                                     0.01594
## Cumulative Proportion
                          0.58361
                                   0.60024
                                            0.61635
                                                     0.63229
                                                              0.6479
                              PC21
                                      PC22
                                             PC23
                                                     PC24
                                                              PC25
                                                                      PC26
## Standard deviation
                          10.14608 10.0544 9.90265 9.64766 9.50764 9.33253
## Proportion of Variance 0.01507
                                   0.0148 0.01436 0.01363 0.01324 0.01275
                          ## Cumulative Proportion
##
                             PC27
                                   PC28
                                            PC29
                                                   PC30
                                                            PC31
                                                                    PC32
                          9.27320 9.0900 8.98117 8.75003 8.59962 8.44738
## Standard deviation
## Proportion of Variance 0.01259 0.0121 0.01181 0.01121 0.01083 0.01045
## Cumulative Proportion 0.74433 0.7564 0.76824 0.77945 0.79027 0.80072
                            PC33
                                     PC34
                                            PC35
                                                    PC36
                                                             PC37
                                                                     PC38
## Standard deviation
                          8.37305 8.21579 8.15731 7.97465 7.90446 7.82127
## Proportion of Variance 0.01026 0.00988 0.00974 0.00931 0.00915 0.00896
## Cumulative Proportion 0.81099 0.82087 0.83061 0.83992 0.84907 0.85803
##
                                     PC40
                                             PC41
                                                   PC42
                             PC39
                                                            PC43
                                                                   PC44
## Standard deviation
                          7.72156 7.58603 7.45619 7.3444 7.10449 7.0131
## Proportion of Variance 0.00873 0.00843 0.00814 0.0079 0.00739 0.0072
## Cumulative Proportion 0.86676 0.87518 0.88332 0.8912 0.89861 0.9058
##
                            PC45
                                   PC46
                                            PC47
                                                   PC48
                                                            PC49
                                                                    PC50
## Standard deviation
                          6.95839 6.8663 6.80744 6.64763 6.61607 6.40793
## Proportion of Variance 0.00709 0.0069 0.00678 0.00647 0.00641 0.00601
## Cumulative Proportion 0.91290 0.9198 0.92659 0.93306 0.93947 0.94548
##
                            PC51
                                     PC52
                                             PC53
                                                    PC54
                                                             PC55
                                                                     PC56
## Standard deviation
                          6.21984 6.20326 6.06706 5.91805 5.91233 5.73539
## Proportion of Variance 0.00566 0.00563 0.00539 0.00513 0.00512 0.00482
## Cumulative Proportion 0.95114 0.95678 0.96216 0.96729 0.97241 0.97723
##
                            PC57
                                   PC58
                                            PC59
                                                   PC60
                                                            PC61
                                                                    PC62
## Standard deviation
                          5.47261 5.2921 5.02117 4.68398 4.17567 4.08212
## Proportion of Variance 0.00438 0.0041 0.00369 0.00321 0.00255 0.00244
## Cumulative Proportion 0.98161 0.9857 0.98940 0.99262 0.99517 0.99761
                            PC63
##
                                      PC64
## Standard deviation
                          4.04124 2.148e-14
## Proportion of Variance 0.00239 0.000e+00
## Cumulative Proportion 1.00000 1.000e+00
plot(pr.out)
```

3





Note that the height of each bar in the bar plot is given by squaring the corresponding element of pr.out\$sdev. However, it is more informative to plot the PVE of each principal component (i.e. a scree plot) and the cumulative PVE of each principal component. This can be done with just a little work. (Note that the elements of pve can also be computed directly from the summary, summary(pr.out)\$importance[2,], and the elements of cumsum(pve) are given by summary(pr.out)\$importance[3,].)

```
pve=100*pr.out$sdev^2/sum(pr.out$sdev^2)
par(mfrow=c(1,2))
plot(pve, type="o", ylab="PVE", xlab="Principal Component", col="blue")
plot(cumsum(pve), type="o", ylab="Cumulative PVE", xlab="Principal Component", col="brown3")
                                                      100
     10
                                                      80
                                                Cumulative PVE
     \infty
                                                      9
      9
                                                      40
     \alpha
                                                      20
     0
                                                                                 50
           0
               10
                       30
                                50
                                                           0
                                                               10
                                                                        30
              Principal Component
                                                               Principal Component
```

We see that together, the first seven principal components explain around 40% of the variance in the data. This is not a huge amount of the variance. However, looking at the scree plot, we see that while each of the

first seven principal components explain a substantial amount of variance, there is a marked decrease in the variance explained by further principal components. That is, there is an elbow in the plot after approximately the seventh principal component. This suggests that there may be little benefit to examining more than seven or so principal components (though even examining seven principal components may be difficult).

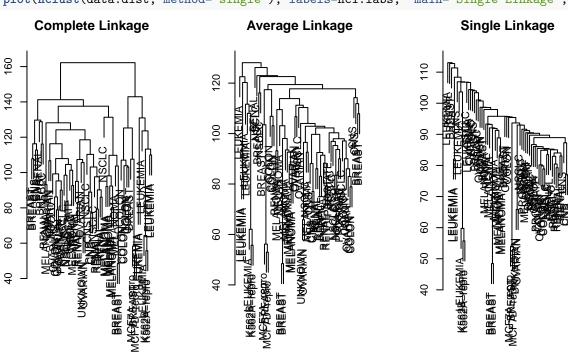
Clustering the Observations of the NCI60 Data

To begin, we standardize the variables to have mean zero and standard deviation one. As mentioned earlier, this step is optional and should be performed only if we want each gene to be on the same scale.

```
sd.data=scale(nci.data)
```

We now perform hierarchical clustering of the observations using complete, single, and average linkage. Euclidean distance is used as the dissimilarity measure.

```
par(mfrow=c(1,3))
data.dist=dist(sd.data)
plot(hclust(data.dist), labels=nci.labs, main="Complete Linkage", xlab="", sub="",ylab="")
plot(hclust(data.dist, method="average"), labels=nci.labs, main="Average Linkage", xlab="", sub="",ylab=
plot(hclust(data.dist, method="single"), labels=nci.labs, main="Single Linkage", xlab="", sub="",ylab=
```



Typically, single linkage will tend to yield trailing clusters: very large clusters onto which individual observations attach one-by-one. On the other hand, complete and average linkage tend to yield more balanced, attractive clusters. We will use complete linkage hierarchical clustering for the analysis that follows.

We can cut the dendrogram at the height that will yield a particular number of clusters, say four:

```
hc.out=hclust(dist(sd.data))
hc.clusters=cutree(hc.out,4)
table(hc.clusters,nci.labs)
```

```
nci.labs
## hc.clusters BREAST CNS COLON K562A-repro K562B-repro LEUKEMIA MCF7A-repro
                      2
                          3
                                2
                                              0
                                                           0
                                                                                  0
##
              1
                                                                     0
                          2
                                0
                                              0
                                                                     0
                                                                                  0
##
              2
                     3
```

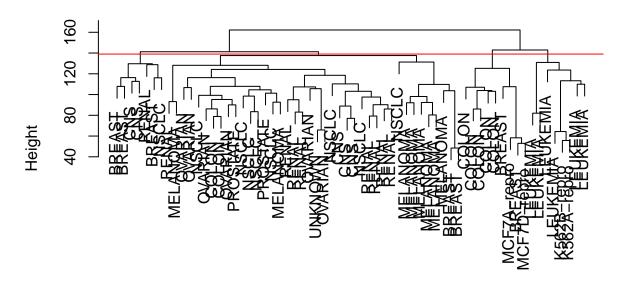
```
##
##
                       2
                            0
                                   5
##
                nci.labs
   hc.clusters MCF7D-repro MELANOMA NSCLC OVARIAN PROSTATE RENAL UNKNOWN
##
##
               1
                             0
                                       8
                                               8
                                                        6
                                                                  2
               2
                             0
                                       0
                                               1
                                                        0
                                                                  0
                                                                          1
                                                                                   0
##
##
               3
                             0
                                       0
                                               0
                                                        0
                                                                  0
                                                                          0
                                                                                   0
                             1
                                       0
                                               0
                                                        0
                                                                  0
                                                                                   0
##
                                                                          0
```

There are some clear patterns. All the leukemia cell lines fall in cluster 3, while the breast cancer cell lines are spread out over three different clusters.

We can plot the cut on the dendrogram that produces these four clusters:

```
par(mfrow=c(1,1))
plot(hc.out, labels=nci.labs)
abline(h=139, col="red")
```

Cluster Dendrogram



dist(sd.data) hclust (*, "complete")

Printing the output of hclust gives a useful brief summary of the object:

hc.out

```
##
## Call:
## hclust(d = dist(sd.data))
##
## Cluster method : complete
## Distance : euclidean
## Number of objects: 64
```

We claimed earlier that K-means clustering and hierarchical clustering with the dendrogram cut to obtain the same number of clusters can yield very different results. How do these NCI60 hierarchical clustering results

compare to what we get if we perform K-means clustering with K = 4?

```
set.seed(2)
km.out=kmeans(sd.data, 4, nstart=20)
km.clusters=km.out$cluster
table(km.clusters,hc.clusters)
```

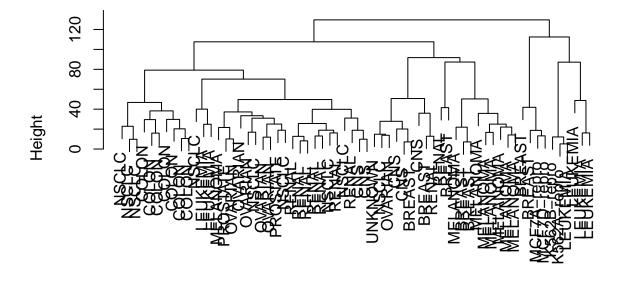
```
## hc.clusters
## km.clusters 1 2 3 4
## 1 11 0 0 9
## 2 20 7 0 0
## 3 9 0 0 0
## 4 0 0 8 0
```

We see that the four clusters obtained using hierarchical clustering and Kmeans clustering are somewhat different.

Rather than performing hierarchical clustering on the entire data matrix, we can simply perform hierarchical clustering on the first few principal component score vectors, as follows:

```
hc.out=hclust(dist(pr.out$x[,1:5]))
plot(hc.out, labels=nci.labs, main="Hier. Clust. on First Five Score Vectors")
```

Hier. Clust. on First Five Score Vectors



dist(pr.out\$x[, 1:5]) hclust (*, "complete")

```
table(cutree(hc.out,4), nci.labs)

## nci.labs

## BREAST CNS COLON K562A-repro K562B-repro LEUKEMIA MCF7A-repro
```

пπ		DIGLINDI	OIVD	COLON	MOOZA ICPIO	MOOZD ICPIO	LLUMLITA	nor the repre
##	1	0	2	7	0	0	2	0
##	2	5	3	0	0	0	0	0
##	3	0	0	0	1	1	4	0
##	Δ	2	Ο	0	0	0	0	1

##	nci	.labs						
##	MC	F7D-repro	MELANOMA	NSCLC	OVARIAN	PROSTATE	RENAL	UNKNOWN
##	1	0	1	8	5	2	7	0
##	2	0	7	1	1	0	2	1
##	3	0	0	0	0	0	0	0
##	4	1	0	0	0	0	0	0

Not surprisingly, these results are different from the ones that we obtained when we performed hierarchical clustering on the full data set.

Sometimes performing clustering on the first few principal component score vectors can give better results than performing clustering on the full data. In this situation, we might view the principal component step as one of denoising the data.