$$\log\left(p_k(x)\right)$$

$$= \log(\pi_k) + \frac{1}{2}\,\frac{2\mu_k x}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2}$$

$$+ \text{const } (\text{w.r.t. } k)$$

$$= \underbrace{x\,\frac{\mu_k}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2} + \log(\pi_k)}_{=:\,\delta_k(x)} + \text{const}$$

For example: $K=2$, $\pi_1 = \pi_2$

Decision boundary

$$\delta_1(x) = \delta_2(x)$$

$$x \cdot \frac{\mu_1}{\cancel{\sigma^2}} - \frac{\mu_1^2}{2\cancel{\sigma^2}} + \cancel{\log(0.5)}$$

$$= x \cdot \frac{\mu_2}{\cancel{\sigma^2}} - \frac{\mu_2^2}{2\cancel{\sigma^2}} + \cancel{\log(0.5)}$$

$$\Rightarrow x(\cancel{\mu_1-\mu_2}) = \underbrace{\frac{\mu_1^2}{2} - \frac{\mu_2^2}{2}}_{\frac{1}{2}(\mu_1+\mu_2)(\cancel{\mu_1-\mu_2})}$$

$$\Rightarrow x = \frac{\mu_1+\mu_2}{2}$$

$$\delta_1(x) = \delta_2(x)$$

$$x\frac{\hat{\mu}_1}{\hat{\sigma}^2} - \frac{\hat{\mu}_1^2}{2\hat{\sigma}^2} + \log(\hat{\pi}_1) = x\frac{\hat{\mu}_2}{\hat{\sigma}^2} - \frac{\hat{\mu}_2^2}{2\hat{\sigma}^2} + \log(\hat{\pi}_2)$$

$$\Rightarrow \quad x = \underbrace{\frac{\hat{\mu}_1+\hat{\mu}_2}{2}}_{} + \underbrace{\sigma^2\frac{\log(\hat{\pi}_2)-\log(\hat{\pi}_1)}{(\hat{\mu}_1-\hat{\mu}_2)}}_{}$$

reformulate