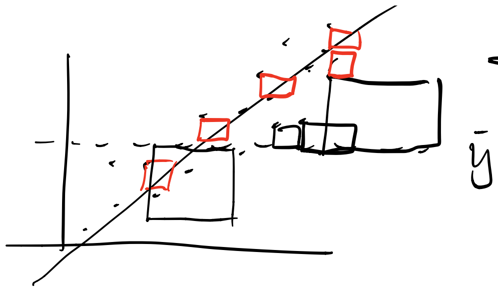

$\sum \square = TSS$

$$TSS = ESS + RSS$$

$$\sum_{i=1}^{n} (y_i - \bar{y})^2 = \sum_{i=1}^{n} (\hat{y} - \bar{y})^2 + \underbrace{\sum_{i=1}^{n} (y_i - \hat{y})}_{\text{residual}}$$

$$R = \frac{ESS}{TSS} = \frac{TSS - RSS}{TSS} = 1 - \frac{RSS}{TSS}$$

$$\frac{\partial}{\partial \beta} \underbrace{(Y - X\hat{\beta})^T (Y - X\hat{\beta})}_{LS(\beta)} = 0$$

$$LS(\hat{\beta}) = Y^T Y - Y^T X \beta - \underbrace{\beta^T X^T Y}_{= Y^T X \beta \text{ (scalar)}} + \beta^T X^T X \beta$$

$$= \cancel{Y^T Y} - \underbrace{2 \cdot Y^T X \beta} + \underbrace{\beta^T X^T X \beta}$$

$$\frac{\partial}{\partial \beta} (LS \beta) = \underbrace{-2 X^T Y}_{\frac{\partial}{\partial \beta} d^T \beta = d} + 2 \underbrace{X^T X}_{\text{because } (X^T X)^T = X^T X} \beta = 0$$

$$\Rightarrow \quad X^T Y = (X^T X)\hat{\beta}$$

$$\Rightarrow \quad (X^T X)^{-1} X^T Y = \underline{\underline{\hat{\beta}}}$$

$$e = \begin{pmatrix} e_1 \\ \vdots \\ e_n \end{pmatrix}$$

- $E(e) = E[Y - \hat{Y}] = E[Y] - E[\hat{Y}]$
  $= X\beta - X\beta = 0$

- $\underline{\underline{Cov(e)}} = Cov(Y - \underbrace{\hat{Y}}_{HY}) = Cov(Y - HY)$
  $= Cov(\underbrace{(I - H)}_{C} \cdot Y)$

$$= (I-H) \cdot \underbrace{Cov(Y)}_{\sigma^2 \cdot I} \cdot (I-H)^T$$

$$= \sigma^2 (I-H)(I-H)^T$$

$$= \sigma^2 (I - H - \underbrace{H^T}_{H} + \overbrace{H H^T}^{H})$$

$$= \sigma^2 (I - H - H + H) = \underline{\underline{\sigma^2 (I - H)}}$$