ExamPaper3-W16

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Example 11.11 Let us compare, on the basis of average response time, the performance of two identical servers each with its own separate queue, to the case when there is only a single queue to hold customers for both servers. The systems to compare are illustrated in Figure 11.17. We shall also check to see how these two possibilities compare to a single processor working twice as fast.

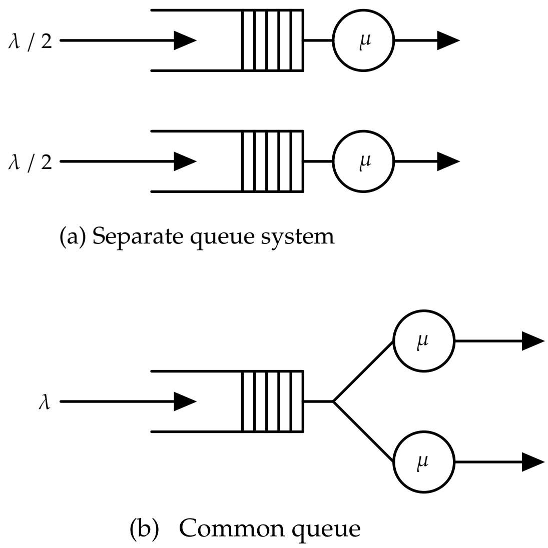


Figure 11.17. Two Configurations.

In the first case, we have two independent $M/M/1$ queues, each with arrival rate $\lambda/2$ and service rate μ. It follows that $\rho = (\lambda/2)/\mu = \lambda/(2\mu)$. The mean number in each $M/M/1$ queue is given by $\rho/(1-\rho)$ so that the mean number of customers in this first scenario is given as

$$L_1 = E[N_1] = 2 \times \frac{\rho}{1-\rho} = \frac{2\rho}{1-\rho}.$$

The average response time can now be found using Little's law. We have

$$E[R_1] = \frac{1}{\lambda}E[N_1] = \frac{1}{\lambda}\frac{2\rho}{(1-\rho)} = \frac{2}{2\mu-\lambda}.$$

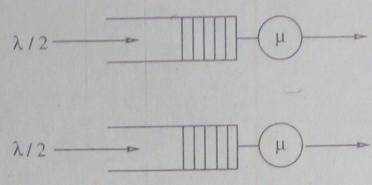
Now consider the second scenario in which the system may be represented as an $M/M/2$ queue. To find $E[R_2]$, we first must find $E[N_2]$ (= L_2). The mean number of customers in an $M/M/c$ queue with arrival rate λ and service rate μ per server is given by

$$E[N_2] = \frac{\lambda}{\mu} + \frac{(\lambda/\mu)^c\lambda\mu}{(c-1)!(c\mu-\lambda)^2}p_0 \quad \text{with} \quad \frac{\lambda}{c\mu} = \rho \quad \text{or} \quad \lambda/\mu = c\rho$$

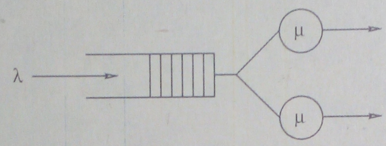
With c = 2, we obtain

$$\begin{aligned} L_2 = E[N_2] = \frac{\lambda}{\mu} + \frac{(\lambda/\mu)^2\lambda\mu}{(2\mu-\lambda)^2}p_0 &= \frac{\lambda}{\mu} + \frac{(\lambda/\mu)^2(\lambda/\mu)}{(1/\mu^2)(2\mu-\lambda)^2}p_0 \\ &= \frac{\lambda}{\mu} + \frac{(\lambda/\mu)^3}{(2-\lambda/\mu)^2}p_0 \\ &= 2\rho + \frac{(2\rho)^3}{(2-2\rho)^2}p_0. \end{aligned}$$

Example 11.11 Let us compare, on the basis of average response time, the performance of two identical servers each with its own separate queue, to the case when there is only a single queue to hold customers for both servers. The systems to compare are illustrated in Figure 11.17. We shall also check to see how these two possibilities compare to a single processor working twice as fast.



(a) Separate queue system



(b) Common queue

Figure 11.17. Two configurations.

In the first case, we have two independent $M/M/1$ queues, each with arrival rate $\lambda/2$ and service rate μ. It follows that $\rho = (\lambda/2)/\mu = \lambda/(2\mu)$. The mean number in each $M/M/1$ queue is given by $\rho/(1-\rho)$ so that the mean number of customers in this first scenario is given as

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$$E[N_2] = \frac{\lambda}{\mu} + \frac{(\lambda/\mu)^c\lambda\mu}{(c-1)!(c\mu-\lambda)^2}p_0 \quad \text{with} \quad \frac{\lambda}{c\mu} = \rho \quad \text{or} \quad \lambda/\mu = c\rho.$$

With $c = 2$, we obtain

$$\begin{aligned} L_2 = E[N_2] = \frac{\lambda}{\mu} + \frac{(\lambda/\mu)^2\lambda\mu}{(2\mu-\lambda)^2}p_0 &= \frac{\lambda}{\mu} + \frac{(\lambda/\mu)^2(\lambda/\mu)}{(1/\mu^2)(2\mu-\lambda)^2}p_0 \\ &= \frac{\lambda}{\mu} + \frac{(\lambda/\mu)^3}{(2-\lambda/\mu)^2}p_0 \\ &= 2\rho + \frac{(2\rho)^3}{(2-2\rho)^2}p_0. \end{aligned}$$

```
\documentclass{report}
\usepackage[utf8]{inputenc}
\usepackage{geometry}
\usepackage{amsmath}
\geometry{paper=a4paper,
left=32mm,
top=40mm,
right=30mm,
bottom=38mm,
}
\usepackage{fancyhdr,amssymb,txfonts,pxfonts}
\usepackage{pgf,tikz,pgfplots}
\pgfplotsset{compat=1.15}
\usepackage{mathrsfs}
\usepackage{caption}
\captionsetup[figure]{labelformat=empty}
\usetikzlibrary{arrows}
\fancyhf{}
\setcounter{page}{423}
\rhead{\small{\textbf{11.4 Multiserver Systems}\hspace{8mm}\thepage}}
\usepackage{graphicx}
\usepackage{listings}
\usepackage{microtype}
\hyphenpenalty=100
\pagestyle{fancy}
\title{ExamPaper3-W16}
\author{Kārlis Kreilis}
\date{May 2019}

\begin{document}

\maketitle
\setcounter{page}{423}


\noindent
\textbf{Example 11.11} Let us compare, on the basis of average response time, the performance
of two identical servers each with its own separate queue, to the case when there is only a
single queue to hold customers for both servers. The systems to compare are illustrated in
Figure 11.17. We shall also check to see how these two possibilities compare to a single
processor working twice as fast.

\begin{figure}[!h]
\begin{tikzpicture}[line cap=round,line join=round,>=triangle 45,x=1cm,y=1cm,
scale=0.8, transform shape]
\clip(-12,-0.55) rectangle (5.8,3.2);
\draw [->,line width=1pt] (-5,2) -- (-2.75,2);
\draw [line width=1pt] (-3.5,1.5)-- (-1,1.5);
\draw [line width=1pt] (-1,1.5)-- (-1,2.5);
\draw [line width=1pt] (-1,2.5)-- (-3.5,2.5);
\draw [line width=1pt] (-1.8,2.5)-- (-1.8,1.5);
\draw [line width=1pt] (-1.2,2.5)-- (-1.2,1.5);
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\draw [line width=1pt] (-1.6,2.5)-- (-1.6,1.5);

\draw [line width=1pt] (-1.4,2.5)-- (-1.4,1.5);
\draw [line width=1pt] (0,2) circle (0.5cm);
\draw [line width=1pt] (-1,2)-- (-0.5,2.);
\draw [->,line width=1pt] (0.5,2) -- (1.6,2);
\draw [line width=1pt] (-2,2.5)-- (-2,1.5);
\draw (-6,2.25) node[anchor=north west] {$ \lambda  /  2$};
\draw (-0.2,2.22) node[anchor=north west] {$\mu$};
\draw [->,line width=1pt] (-5.0,-0) -- (-2.75,-0);
\draw [line width=1pt] (-1,-0.5)-- (-1,0.5);
\draw [line width=1pt] (-1,0.5)-- (-3.5,0.5);
\draw [line width=1pt] (-0,-0) circle (0.5cm);
\draw [line width=1pt] (-1,-0)-- (-0.5,-0);
\draw [->,line width=1pt] (0.5,-0) -- (1.6,-0);
\draw [line width=1pt] (-1,-0.5)-- (-3.5,-0.5);
\draw [line width=1pt] (-1.2,0.5)-- (-1.2,-0.5);
\draw [line width=1pt] (-1.4,0.5)-- (-1.4,-0.5);
\draw [line width=1pt] (-1.6,0.5)-- (-1.6,-0.5);
\draw [line width=1pt] (-1.8,0.5)-- (-1.8,-0.5);
\draw [line width=1pt] (-2,0.5)-- (-2,-0.5);
\draw (-6,0.25) node[anchor=north west] {$ \lambda  /  2$};
\draw (-0.2,0.22) node[anchor=north west] {$\mu$};
\end{tikzpicture}
\caption{\small{(a) Separate queue system}}
\end{figure}
\begin{figure}[h!]
\begin{tikzpicture}[line cap=round,line join=round,>=triangle 45,x=1cm,y=1cm,
scale=0.8, transform shape]
\clip(-12,-0.55) rectangle (9,3.55);
\draw [->,line width=1pt] (-5,2) -- (-2.75,2);
\draw [line width=1pt] (-3.5,1.5)-- (-1,1.5);
\draw [line width=1pt] (-1,1.5)-- (-1,2.5);
\draw [line width=1pt] (-1,2.5)-- (-3.5,2.5);
\draw [line width=1pt] (-1.8,2.5)-- (-1.8,1.5);
\draw [line width=1pt] (-1.2,2.5)-- (-1.2,1.5);
\draw [line width=1pt] (-1.6,2.5)-- (-1.6,1.5);
\draw [line width=1pt] (-1.4,2.5)-- (-1.4,1.5);
\draw [line width=1pt] (1,3) circle (0.5cm);
\draw [line width=1pt] (-2,2.5)-- (-2,1.5);
\draw (-5.5,2.25) node[anchor=north west] {$ \lambda$};
\draw (0.75,3.22) node[anchor=north west] {$\mu$};
\draw (0.75,1.22) node[anchor=north west] {$\mu$};
\draw [line width=1pt] (1,1) circle (0.5cm);
\draw [line width=1pt] (-1,2)-- (-0.5,2);
\draw [line width=1pt] (-0.5,2)-- (0.5,1);
\draw [line width=1pt] (-0.5,2)-- (0.5,3);
\draw [->,line width=1pt] (1.5,3) -- (3,3);
\draw [->,line width=1pt] (1.5,1) -- (3,1);
\draw (-4,0.2) node[anchor=north west] {\large{(b)   Common queue}};
\end{tikzpicture}
\caption{Figure 11.17. Two Configurations.}
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\end{figure}

\hfill

In the first case, we have two independent $M/M/1$ queues, each with arrival rate $\lambda/2$
and service rate $\mu$. It follows that $\rho = (\lambda/2)/\mu = \lambda/(2\mu)$.
The mean number in each $M/M/1$ queue is given by $\rho/(1-\rho)$ so that the mean number of
customers in this first scenario is given as
\begin{equation*}
    L_{1}= E[N_1]= 2 \times \frac{\rho}{1-\rho}=\frac{2\rho}{1-\rho}.
\end{equation*}
\noindent
The average response time can now be found using Little's law. We have
\begin{equation*}
    E[R_1]= \frac{1}{\lambda}E[N_1]= \frac{1}{\lambda}
    \frac{2\rho}{(1-\rho)}=\frac{2}{2\mu-\lambda}.
\end{equation*}
\noindent
Now consider the second scenario in which the system may be represented as an $M/M/2$ queue.
To find $E[R_2]$, we first must find $E[N_2]\;(= L_2)$. The mean number of customers in
an $M/M/c$ queue with arrival rate $\lambda$ and service rate $\mu$ per server is given by
\begin{equation*}
    E[N_2]=\frac{\lambda}{\mu}+\frac{(\lambda/\mu)^c\lambda\mu}{(c-1)!(c\mu-\lambda)^2}p_0
    \hspace{3mm} \text{with} \hspace{3mm} \frac{\lambda}{c\mu}=\rho \hspace{3mm} \text{or}
    \hspace{3mm} \lambda/\mu=c\rho
\end{equation*}
\noindent
With c = 2, we obtain
\begin{align*}
L_2 = E[N_2] = \frac{\lambda}{\mu}+ \frac{(\lambda/\mu)^2\lambda\mu}{(2\mu-\lambda)^2}p_0
&= \frac{\lambda}{\mu}+ \frac{(\lambda/\mu)^2(\lambda/\mu)}{(1/\mu^2)(2\mu-\lambda)^2}p_0 \\
&= \frac{\lambda}{\mu}+
\frac{(\lambda/\mu)^3}{(2-\lambda/\mu)^2}p_0 \\
&= 2\rho+ \frac{(2\rho)^3}{(2-2\rho)^2}p_0.
\end{align*}
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