Pigeonhole 1 - Problems

1. *[Hobbits]* 5 hobbits were frolicking on a meadow. There were 3 hobbit holes nearby. Suddenly an orcish war cry sounded from the forest and hobbits all ran and hid in hobbit holes. What can you say about number of hobbits in hobbit holes?

2. *[More hobbits]* What if there are 7 hobbits and 3 hobbit holes?

3. *[Even more hobbits]* What if there are n hobbits and m hobbit holes?

4. *[Football problem]* There are m football teams in Fromenistan, each has 11 players. All these players have gathered in an airport to go to tournament in Toland. Airplane has already made 10 flights each time bringing m footballers to Toland. One more footballer was so impatient that he went to Toland by bus (and has already arrived).

Prove that there is at least one full team in Toland already!

5. *[Amount of friends]* Prove that any company of 5 people has at least two persons that have the same amount of friends[1] within this group!

6. *[Unique differences]* We have 8 distinct natural numbers that do not exceed 15. Prove that there are at least three equal differences between these numbers!

7. *[Too many kings]* What is the maximum number of kings that can be placed on a chess board so that none of them can attack another [2]?

8. *[Too much money]* We randomly dropped 51 pointlike coins into a square with side length of 1 meter. Prove that it is always possible to cover at least three coins with a square paper sheet $20cm * 20cm$!

9. *[Existance of cliques]* Prove that between any 6 people it is always possible to find either three that know each other, or three that do not know each other!

10. *[Last digits of 3^x]* Prove that there exists such a natural n that 3^n ends with digits[3] 001 !

11. *[College problems]* In the college there are 11 students and 5 courses. Prove that such two students A and B exist that all courses attended by A is also attended by B!

12. *[Squares out of chaos]* Set A consists of 2018 distinct numbers. All prime factors of all these numbers are

a) less than 29

b) less than 30

Prove that A contains 4 distinct numbers a, b, c, d, such that $a \cdot b \cdot c \cdot d = n^2$ for some natural number n!

13. *[Bonus problem 1]* Come up with a generalization of Pidgeonhole principle - What can we know, if a hobbits all went into b hobbit-holes and it is known that maximum capacity of a hobbit hole is c?

14. *[Bonus problem 2]* Can you prove Pidgeonhole principle?

[1] herefrom we always consider friendships to be bi-directional
[2] standard chess board is $8 * 8$ squares, king attacks all nearby squares (including diagonals)
[3] always assume decimal, unless stated otherwise