IMO SL list 2 geometric combinatorics - Problems

1. Let n points be given inside a rectangle R such that no two of them lie on a line parallel to one of the sides of R. The rectangle R is to be dissected into smaller rectangles with sides parallel to the sides of R in such a way that none of these rectangles contains any of the given points in its interior. Prove that we have to dissect R into at least $n-1$ smaller rectangles.

2. 2500 chess kings have to be placed on a 100×100 chessboard so that

- no king can capture any other one (i.e. no two kings are placed in two squares sharing a common vertex);
- each row and each column contains exactly 25 kings.

Find the number of such arrangements. (Two arrangements differing by rotation or symmetry are supposed to be different.)

3. Construct a tetromino by attaching two 2×1 dominoes along their longer sides such that the midpoint of the longer side of one domino is a corner of the other domino. This construction yields two kinds of tetrominoes with opposite orientations. Let us call them *S*- and *Z-tetrominoes*, respectively.

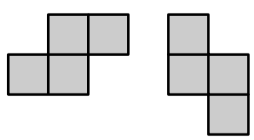

Figure 1: S-tetramino

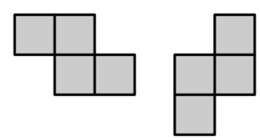

Figure 2: Z-tetramino

Assume that a lattice polygon P can be tiled with S-tetrominoes. Prove than no matter how we tile P using only S- and Z-tetrominoes, we always use an even number of Z-tetrominoes.

4. Consider $n \geq 3$ lines in the plane such that no two lines are parallel and no three have a common point. These lines divide the plane into polygonal regions; let F be the set of regions having finite area. Prove that it is possible to colour $\lceil \sqrt{n/2} \rceil$ of the lines blue in such a way that no region in F has a completely blue boundary. (For a real number x, $\lceil x \rceil$ denotes the least integer which is not smaller than x.)

5. On a square table of 2011 by 2011 cells we place a finite number of napkins that each cover a square of 52 by 52 cells. In each cell we write the number of napkins covering it, and we record the maximal number k of cells that all contain the same nonzero number. Considering all possible napkin configurations, what is the largest value of k?