Homework list 4

1. A finite number of frogs sit on a points with integer coordiantes of a horizontal line so that no two frogs sit on a same point. At each move one frog jumps distance of 1 to the right, but only if it does not collide with another frog. We have calclulated the number of different ways in which these frogs can do n moves (for some initial configuration of frogs). Prove that if we change the rules of the game and allow them to jump to the left instead of jumping to the right, the number of different ways to make n moves will be the same!

2. 36 gangsters reside in Chicago and some of them are enemies. Each gangster belongs to some gangs and there are no two gangs with identical member rosters. Gangsters of the same gang are never enemies, but, if a gangster does not belong to some gang, then she always has an enemy in that gang. What is the maximum number of gangs in Chicago?

3. Two guards patrol around a circular wall and one of them walks twice as fast as the other one. The wall, which has a length of 1, also has some windows in it. A configuration of windows is considered *secure*, if at any moment at least one of the guards is at some window for some initial position of the guards.

a. If a secure system of windows consist of a single window, what is the minimum length that this sole window must have?

b. Prove that any secure system must have a total length of windows of more than $\frac{1}{2}$

c. Prove that for any $s > \frac{1}{2}$ there exists a secure configuration that has total length of windows less than s