

Recurrences - Problems

1. *[Derangements of hats]* Suppose there are n people and each has one hat. They all throw hats in a pile and then each takes one hat out of pile.

How many possible ways are there to do this so that noone gets his own hat?

2. *[Guess - then prove method]* $b(0) = 0$; $b(1) = 1$; $b(n) = b(\lfloor \frac{n}{2} \rfloor) + b(\lceil \frac{n}{2} \rceil)$

3. *[How to guess - pattern spotting - differences]* $a(0) = 12$; $a(1) = 17$; $a(n) = a(\lfloor \frac{n}{2} \rfloor) + a(\lceil \frac{n}{2} \rceil) - 12$

4. *[How to guess - pattern spotting - 2nd order differences]* $a_0 = 7$; $a_1 = 12$; $a_n = a_{n-2} + 8n - 2$

5. *[How to guess - pattern spotting - n -th order differences]* $s_0 = 0$; $s_n = s_{n-1} + n^2$

6. *[How to guess - pattern spotting - ratios]* $g_0 = 2$; $g_1 = 6$; $g_n = g_{n-1} + 6g_{n-2}$

7. *[How to guess - pattern spotting - error terms]* $a_0 = 5$; $a_n = 2a_{n-1} + 1$

8. *[How to guess - pattern spotting in formulas - repeated substitutions]* $a_0 = 5$; $a_n = 3a_{n-1} + 2n$

9. *[Linear homogenous recurrences]** $a_0 = 1$; $a_1 = 1$; $a_n = a_{n-1} + a_{n-2}$

10. *[Linear non-homogenous recurrences]** $a_0 = 2$; $a_1 = 3$; $a_n = a_{n-1} + a_{n-2} + 3n + 1$

11. *[Tower of Hanoi]* Tower of Hanoi consists of 3 rods A , B and C and n discs of different sizes. Initially all discs are stacked on one rod so that no disc is placed on top of smaller one.

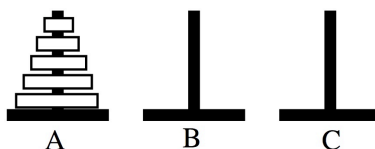


Figure 1: Tower of Hanoi

The objective is to move the entire stack to another rod, obeying the following simple rules:

- Only one disk can be moved at a time.
- Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack or on an empty rod.
- No disk may be placed on top of a smaller disk.

What is the minimal number of moves to move all discs from A to B ?

12. *[Pizza problem]* How many pieces of pizza is it possible to obtain using n straight cuts?

13. *[Josephus problem]* n people numbered 1 to n stand in a circle. Every second person starting from 1 is eliminated and leaves the circle until only one person remains.

For a given n what is the number of remaining person?