

## IMO SL list Misc - Problems

---

**1.** Given a positive integer  $k$  and other two integers  $b > w > 1$ . There are two strings of pearls, a string of  $b$  black pearls and a string of  $w$  white pearls. The length of a string is the number of pearls on it. One cuts these strings in some steps by the following rules. In each step:

- The strings are ordered by their lengths in a non-increasing order. If there are some strings of equal lengths, then the white ones precede the black ones. Then  $k$  first ones (if they consist of more than one pearl) are chosen; if there are less than  $k$  strings longer than 1, then one chooses all of them.
- Next, one cuts each chosen string into two parts differing in length by at most one.

(For instance, if there are strings of 5, 4, 4, 2 black pearls, strings of 8, 4, 3 white pearls and  $k = 4$ , then the strings of 8 white, 5 black, 4 white and 4 black pearls are cut into the parts (4, 4), (3, 2), (2, 2) and (2, 2), respectively.) The process stops immediately after the step when a first isolated white pearl appears. Prove that at this stage, there will still exist a string of at least two black pearls.

**2.** On a square table of 2011 by 2011 cells we place a finite number of napkins that each cover a square of 52 by 52 cells. In each cell we write the number of napkins covering it, and we record the maximal number  $k$  of cells that all contain the same nonzero number. Considering all possible napkin configurations, what is the largest value of  $k$ ?