# Pascals triangle - Problems

**1.** *[Formula of a triangle]* Prove that $\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$

**2.** *[Construct of a triangle]* Construct a triangle from pointlike pins on a vertical plane so, that top row has one pin and each consequent row has one more pin than previous. A ball is dropped on a top pin so that, when it bounces off a pin, it has a equal chance to fall to left or right pin in next row downward (see Figure 1).

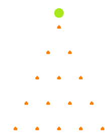

**Figure 1:** Ball and triangle of pins

Calculate the number of different paths that a ball can take to reach each pin! Is there any connection to result of problem *[Formula of a triangle]*?

**3.** *[Any number]* Does Pascals triangle contain number 2018?

**4.** *[Powers of mystery]* By what factor the sum of elements of 101*st* row of Pascals triangle is greater than sum of elements of 100*th* row?

**5.** *[Combinatorial proofs]* Prove combinatorically:

a. $\binom{2n}{2} = 2\binom{n}{2} + n^2$

b. $\binom{2n+2}{n+1} = \binom{2n}{n+1} + 2\binom{2n}{n} + \binom{2n}{n-1}$

**6.** *[Binomial theorem]* Prove the *Binomial Theorem*:

$$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \cdots + \binom{n}{n-1}xy^{n-1} + \binom{n}{n}y^n = \sum_{i=0}^{n}\binom{n}{i}x^{n-i}y^i$$

**7.** *[Some sums]* Calculate following sums:

a. $\binom{5}{0} + 2\binom{5}{1} + 2^2\binom{5}{2} + \cdots + 2^5\binom{5}{5}$

b. $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots + (-1)^n\binom{n}{n}$

c. $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n}$