Combinatorial Geometry problems

1. *[Splitting a square]* Each of 9 lines divide a square into two quadrilaterals, whose areas have a proportion of $2:3$ to each other. Prove that there exists a point, where at least three of these lines meet!

2. [$Eulersformulaforpolyhedron$] Prove that, for any convex polyhedron $V+F-E=2$, where V - number of vertices, E - number of edges and F - number of faces of the polyhedron!

3. *[Splitting a circle]* What is the maximum number of areas that a circle is divided into by an inscribed polygon with n sides and all its diagonals?

4. [$BW1999PL10$] May the points of a disc of radius 1 (including its circumference) be partitioned into three subsets in such a way that no subset contains two points separated by a distance 1?

5. [$IMO2013PL2$] A configuration of 4027 points in the plane is called Colombian if it consists of 2013 red points and 2014 blue points, and no three of the points of the configuration are collinear. By drawing some lines, the plane is divided into several regions. An arrangement of lines is good for a Colombian configuration if the following two conditions are satisfied:

- no line passes through any point of the configuration;
- no region contains points of both colours.

Find the least value of k such that for any Colombian configuration of 4027 points, there is a good arrangement of k lines!

6. [$IMO2017PL3$] A hunter and an invisible rabbit play a game in the Euclidean plane. The rabbit's starting point, A_0, and the hunter's starting point, B_0, are the same. After $n-1$ rounds of the game, the rabbit is at point A_{n-1} and the hunter is at point B_{n-1}. In the nth round of the game, three things occur in order.

a. The rabbit moves invisibly to a point A_n such that the distance between A_{n-1} and A_n is exactly 1.

b. A tracking device reports a point P_n to the hunter. The only guarantee provided by the tracking device is that the distance between P_n and A_n is at most 1.

c. The hunter moves visibly to a point B_n such that the distance between B_{n-1} and B_n is exactly 1.

Is it always possible, no matter how the rabbit moves, and no matter what points are reported by the tracking device, for the hunter to choose her moves so that after 10^9 rounds she can ensure that the distance between her and the rabbit is at most 100?