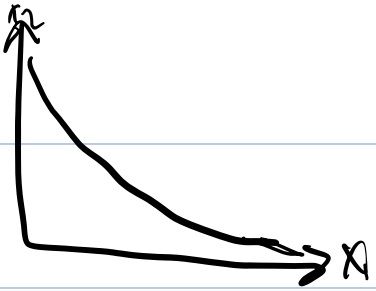
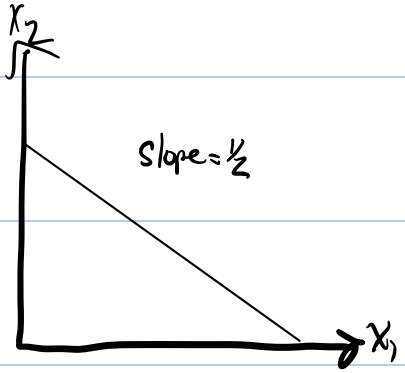
1) a. $y = f(x_1, x_2)$ (full time, part time)



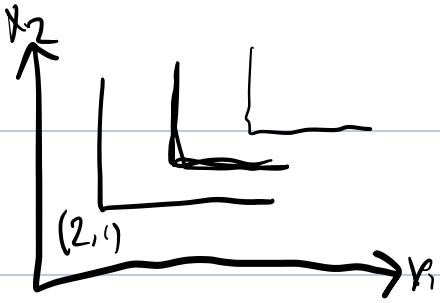
b) $y = f(x_1, x_2)$ (x_1: labor, x_2: capital)

$x_1 + x_2$



c) $y = f(x_1, x_2)$ (x_1: workers, x_2: machinery)

$\min\{x_1, 2x_2\}$



2) a. $f(x_1, x_2) = x_1^{1/2} x_2^{3/2}$

$\frac{\partial f}{\partial x_1} = \frac{1}{2} x_1^{-1/2} x_2^{3/2}$

b) $f(x_1, x_2) = x_1^{1/2} x_2^{3/2}$

$\frac{\partial f}{\partial x_2} = \frac{3}{2} x_1^{1/2} x_2^{1/2}$

c) $MRS = MP_1/MP_2 = \frac{\frac{1}{2} x_1^{-1/2} x_2^{3/2}}{\frac{3}{2} x_1^{1/2} x_2^{1/2}} = \frac{\frac{1}{2} x_2^{3/2}}{\frac{3}{2} x_1 x_2^{1/2}}$ $\boxed{\frac{1}{3}\frac{x_2}{x_1}}$

d) $\frac{1}{2} + \frac{3}{2} = 2 > 1$ → increasing returns to scale

3. $f(x_1, x_2) = A x_1^a x_2^b \quad a + b > 1$

a) $f(tx_1, tx_2) > tf(x_1, x_2)$

$f(tx_1, tx_2) = A(tx_1)^a(tx_2)^b = At^a x_1^a t^b x_2^b = \boxed{At^{a+b} x_1^a x_2^b}$

$tf(x_1, x_2) = tAx_1^a x_2^b = \boxed{t^{a+b} Ax_1^a x_2^b}$ $\quad a+b>1$, $t>0$

$f(tx_1, tx_2) > tf(x_1, x_2)$

b) CRS: $a+b=1$

DRS: $a+b<1$

4) $f(L,T) = L^{1/2}T^{1/2}$

a) Constant Returns to Scale bc $\frac{1}{2}+\frac{1}{2}=1$

b) $\frac{\partial f(L,T)}{\partial L} = \frac{1}{2}(1)^{-1/2}(1)^{1/2} = \boxed{\frac{1}{2} \text{ unit of output}}$

5) $f_c(x_1, x_2) = \min\{x_1, 2x_2\}$

a) $f_c(x_1, x_2) = x_1 = 20 \qquad f_c(x_1, x_2) = 2x_2 = 20 \quad x_2 = 10$

$x_1 = 20 \quad x_2 = 10$

b) $f_F(x_1, x_2) = 2x_1 = 20 \quad x_1 = 10$

$f_F(x_1, x_2) = x_2 = 20 \qquad x_2 = 20$

$x_1 = 10 \quad x_2 = 20$

1) $f(L) = 6L^{2/3}$ $w = 6$ $p = 3$

a) $-\frac{6}{3} = -2$

b) $\frac{\partial}{\partial L} = 4L^{-1/3} \cdot 3 = 12L^{-1/3}$

$12L^{-1/3} = 6$ $L^{-1/3} = \frac{1}{2}$

$L = \left(\frac{1}{2}\right)^{-3} = 8$ $\boxed{L = 8}$

c) $f(L) = 6(8)^{2/3} = \boxed{24}$

d) $TR = 3 \cdot 24 = 72$ $TC = 6 \cdot 8 = 48$

$72 - 48 = \boxed{24}$

2) $f(x) = 4\sqrt{x}$ input = 50 output = 100

a) $TR = 100 \cdot 4\sqrt{x}$ $TC = 50x$

$\boxed{400\sqrt{x} - 50x}$

b) $\frac{\partial \pi}{\partial x} = \frac{\partial}{\partial x}(400\sqrt{x} - 50x) = 0$ $\frac{400}{2\sqrt{x}} - 50 = 0$

$\frac{200}{\sqrt{x}} - 50 = 0$

$\frac{200}{\sqrt{x}} = 50$ $\sqrt{x} = \frac{200}{50} = 4$

$x = 4^2 = \boxed{16}$

c) $\pi(16) = 400\sqrt{16} - 50(16)$

$400 \cdot 4 - 50(16)$

$1600 - 800$

$\boxed{\$800}$

d) $(80 \cdot 4\sqrt{x}) - (40 \cdot x)$

$\frac{\partial}{\partial x}(320\sqrt{x} - 40x) = 0 = \frac{320}{2\sqrt{x}} - 40 = 0$

$\frac{320}{\sqrt{x}} = 40$

$\sqrt{x} = \frac{320}{40} = 8$

$\boxed{x = 64}$

e) $(80 \cdot 4\sqrt{64}) - (40 \cdot 64)$

$(80 \cdot 4 \cdot 8) - (40 \cdot 64)$

$2560 - 2560 = \boxed{0}$

3) $f(x_1, x_2) = \sqrt{2x_1} + \sqrt{4x_2}$ output $= 4$ $P_1 = \$2$ $P_2 = \$3$

a) $\frac{\partial}{\partial x_1}(4\sqrt{2x_1} + 3\sqrt{4x_2}) = \frac{4}{2\sqrt{2x_1}} = \frac{2}{\sqrt{2x_1}}$

$\frac{2}{\sqrt{2x_1}} = 2 \quad \sqrt{2x_1} = 1 \quad 2x_1 = 1 \quad \boxed{x_1 = \frac{1}{2}}$

b) $\frac{\partial}{\partial x_2}\left(4\sqrt{2x_1} + 3\sqrt{4x_2}\right) = \frac{3}{2\sqrt{4x_1}} = \frac{3}{4\sqrt{x_2}}$

$\frac{3}{4\sqrt{x_2}} = 3 \quad \frac{1}{\sqrt{x_2}} = 1$ $\boxed{k_2 = 1}$

c) $f\left(\frac{1}{2}, 1\right) = \sqrt{2\left(\frac{1}{2}\right)} + \sqrt{4(1)}$

$= \sqrt{1} + \sqrt{4}$

$= 1 + 2$

$= \boxed{3 \text{ units}}$