

DEPARTMENT OF INFORMATICS

TECHNICAL UNIVERSITY OF MUNICH

Master's Thesis in Informatics

**Cloud Supercomputing with the  
ExaHyPE-Engine**

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**Wolken Hochleistungsrechnen mit der  
ExaHyPE-Engine**

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I confirm that this master's thesis in informatics is my own work and I have documented all sources and material used.

Munich, January , 2019

Lukas Krenz

# Acknowledgments

Quote!

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*(Booktitle)*

Acknowledgements here.

# Abstract

Abstract here!

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CHAPTER 1

# Introduction

Introduction here.

Methods here.

Organize into chapters!

## 2.1. Conservative Form

Standard (hyperbolic) form of conservation law

$$\frac{\partial}{\partial t} Q + \nabla \cdot F(Q) = S(\mathbf{x}, t, Q) \quad (2.1)$$

extend to:

$$\frac{\partial}{\partial t} Q + \nabla \cdot F(Q, \nabla Q) = S(\mathbf{x}, t, Q) \quad (2.2)$$

## 2.2. Compressible Euler

Vector of conserved quantities:

$$Q = (\rho, \rho \mathbf{v}, \rho E) \quad (2.3)$$

Flux:

$$F(Q) = \begin{pmatrix} \rho \mathbf{v} \\ \mathbf{v} \otimes \rho \mathbf{v} + p \mathbf{I} \\ \mathbf{v} \cdot (\mathbf{I} \rho E + \mathbf{I} p) \end{pmatrix} \quad (2.4)$$

$$p = (\gamma - 1) (\rho E - 0.5 (\mathbf{v} \cdot \rho \mathbf{v})) \quad (2.5)$$

## 2.3. Compressible Navier Stokes

Following [Dum10] by Dumbser.

Flux:

$$F(Q, \nabla Q) = \begin{pmatrix} \rho \mathbf{v} \\ \mathbf{v} \otimes \rho \mathbf{v} + \mathbf{I} p + \boldsymbol{\sigma}(Q, \nabla Q) \\ \mathbf{v} \cdot (\mathbf{I} \rho E + \mathbf{I} p + \boldsymbol{\sigma}(Q, \nabla Q)) - \kappa \nabla T \end{pmatrix} \quad (2.6)$$



With: Pressure  $p$

$$p = (\gamma - 1) (\rho E - 0.5 (\mathbf{v} \cdot \rho \mathbf{v})) \quad (2.7)$$

Temperature  $T$

$$\frac{p}{\rho} = RT \quad (2.8)$$

Heat conduction coefficient  $\kappa$

$$\kappa = \frac{\mu \gamma}{Pr} \frac{1}{\gamma - 1} R \quad (2.9)$$

with  $R$  gas constant and  $Pr$  Prandtl number

Sutherland's viscosity law:

$$\mu(T) = \mu_0 \left( \frac{T}{T_0} \right)^\beta \frac{T_0 + C}{T + C} \quad (2.10)$$

with  $\beta = 1.5$ ,  $C = \text{const.}$ , reference temperature  $T_0$  and reference viscosity  $\mu_0$ . Equal to

$$\lambda = \frac{\mu_0 (T_0 + C)}{T_0^\beta} \quad (2.11)$$

$$\mu(T) = \lambda \frac{T^\beta}{T + C}, \quad (2.12)$$

where  $\lambda$  is constant for a given fluid.

stress tensor

$$\sigma(\mathbf{Q}, \nabla \mathbf{Q}) = (2/3 \mu \nabla \cdot \mathbf{v}) - \mu (\nabla(\mathbf{v}) + \nabla(\mathbf{v})^T) \quad (2.13)$$

Max eigenvalue of convective part  $|\lambda_c^{\max}|$  i.e. of  $(\partial \mathbf{F} / \partial \mathbf{Q}) \cdot \mathbf{n}$ , and viscous part  $|\lambda_v^{\max}|$  i.e. of  $(\partial \mathbf{F} / \partial (\nabla \mathbf{Q} \cdot \mathbf{n})) \cdot \mathbf{n}$  is

$$|\lambda_c^{\max}| = \|\mathbf{v}\| + c \quad (2.14)$$

$$|\lambda_v^{\max}| = \max \left( \frac{4}{3} \frac{\mu}{\rho}, \frac{\gamma \mu}{Pr \rho} \right) \quad (2.15)$$

with speed of sound  $c = \sqrt{\gamma R T}$

Maximum timestep (CFL)

$$\Delta t = \frac{CFL}{2N + 1} \frac{h}{|\lambda_c^{\max}| + 2|\lambda_v^{\max}| \frac{2N+1}{h}} \quad (2.16)$$

with  $N$  polynomial order and  $h$  characteristic length scale of elements

Numerical flux:

$$G(q_h^-, \nabla q_h^-; q_h^+, \nabla q_h^+) \cdot \mathbf{n} = \frac{1}{2} (F(q_h^+, \nabla q_h^+) + F(q_h^-, \nabla q_h^-)) - \frac{1}{2} s_{\max}(q_h^+ - q_h^-) \quad (2.17)$$

with

$$s_{\max} = \max(|\lambda_c(q_h^-)|, |\lambda_c(q_h^+)|) + 2\eta \max(|\lambda_v(q_h^-)|, |\lambda_v(q_h^+)|) \quad (2.18)$$

and

$$\eta = \frac{N+1}{h} \quad (2.19)$$

# CHAPTER 3

## Conclusion

Conclusion here.

Placeholder...

# Bibliography

- [Dum10] M. Dumbser. “Arbitrary high order PNPM schemes on unstructured meshes for the compressible Navier–Stokes equations.” In: *Computers & Fluids* 39.1 (2010), pp. 60–76 (see p. 2).