DEPARTMENT OF INFORMATICS

TECHNICAL UNIVERSITY OF MUNICH

Master's Thesis in Informatics

Cloud Supercomputing with the ExaHyPE-Engine

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Wolken Hochleistungsrechnen mit der ExaHyPE-Engine

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I confirm that this master's thesis in informatics is my own work and I have documented all sources and material used.

Munich, January , 2019 Lukas Krenz

Acknowledgments

Quote!

(Booktitle)

Acknowledgements here.

Abstract

Abstract here!

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CHAPTER 1
Introduction

Introduction here.

CHAPTER 2

Methods

Methods here.

Organize into chapters!

2.1. Conservative Form

Standard (hyperbolic) form of conservation law

$$\frac{\partial}{\partial_t} Q + \nabla \cdot F(Q) = S(\boldsymbol{x}, t, Q) \tag{2.1}$$

extend to:

$$\frac{\partial}{\partial_t} Q + \nabla \cdot F(Q, \nabla Q) = S(\boldsymbol{x}, t, Q) \tag{2.2}$$

2.2. Compressible Euler

Vector of conserved quantities:

$$Q = (\rho, \rho\boldsymbol{v}, \rho E) \tag{2.3}$$

Flux:

$$F(Q) = \begin{pmatrix} \rho\boldsymbol{v} \\ \boldsymbol{v} \otimes \rho\boldsymbol{v} + p\mathbf{I} \\ \boldsymbol{v} \cdot (\mathbf{I}\rho E + \mathbf{I}p) \end{pmatrix} \tag{2.4}$$

$$p = (\gamma - 1)(\rho E - 0.5(\boldsymbol{v} \cdot \rho\boldsymbol{v})) \tag{2.5}$$

2.3. Compressible Navier Stokes

Following [Dum10] by Dumbser.

Flux:

$$F(Q, \nabla Q) = \begin{pmatrix} \rho\boldsymbol{v} \\ \boldsymbol{v} \otimes \rho\boldsymbol{v} + \mathbf{I}p + \boldsymbol{\sigma}(Q, \nabla Q) \\ \boldsymbol{v} \cdot (\mathbf{I}\rho E + \mathbf{I}p + \boldsymbol{\sigma}(Q, \nabla Q)) - \kappa\nabla T \end{pmatrix} \tag{2.6}$$

With: Pressure p

$$p = (\gamma - 1)\left(\rho E - 0.5\left(\boldsymbol{v} \cdot \rho \boldsymbol{v}\right)\right) \tag{2.7}$$

Temperature T

$$\frac{p}{\rho} = RT \tag{2.8}$$

Heat conduction coefficient κ

$$\kappa = \frac{\mu\gamma}{\mathrm{Pr}} \frac{1}{\gamma - 1} R \tag{2.9}$$

with R gas constant and Pr Prandtl number

Sutherland's viscosity law:

$$\mu(T) = \mu_0 \left(\frac{T}{T_0}\right)^{\beta} \frac{T_0 + C}{T + C} \tag{2.10}$$

with $\beta = 1.5$, $C = \text{const.}$, reference temperature T_0 and reference viscosity $\mu_=$. Equal to

$$\lambda = \frac{\mu_0 (T_0 + C)}{T_0^{\beta}} \tag{2.11}$$

$$\mu(T) = \lambda \frac{T^{\beta}}{T + C}, \tag{2.12}$$

where λ is constant for a given fluid.

stress tensor

$$\boldsymbol{\sigma}(Q, \nabla Q) = (2/3 \mu \nabla \cdot \boldsymbol{v}) - \mu\left(\nabla(\boldsymbol{v}) + \nabla(\boldsymbol{v})^{\intercal}\right) \tag{2.13}$$

Max eigenvalue of convective part $|\lambda_c^{max}|$ i.e. of $(\partial \mathbf{F}/\partial \mathbf{Q}) \cdot \boldsymbol{n}$, and viscous part $|\lambda_v^{max}|$ i.e. of $(\partial \mathbf{F}/\partial\left(\nabla \mathbf{Q} \cdot \boldsymbol{n}\right)) \cdot \boldsymbol{n}$ is

$$|\lambda_c^{max}\| = \|\boldsymbol{v}\| + c \tag{2.14}$$

$$|\lambda_v^{max}\| = \max\left(\frac{4}{3}\frac{\mu}{\rho}, \frac{\gamma\mu}{\mathrm{Pr}\,\rho}\right) \tag{2.15}$$

with speed of sound $c = \sqrt{\gamma R T}$

Maximum timestep (CFL)

$$\Delta t = \frac{\mathrm{CFL}}{2N + 1} \frac{h}{|\lambda_c^{max}| + 2|\lambda_v^{max}| \frac{2N+1}{h}} \tag{2.16}$$

with N polynomial order and h characteristic length scale of elements

Numerical flux:

$$G(q_h^-, \nabla q_h^-; g_h^+, \nabla q_h^+) \cdot \boldsymbol{n} = \frac{1}{2}\left(F(q_h^+, \nabla q_h^+) + F(q_h^-, \nabla q_h^-)\right) - \frac{1}{2} s_{\max}(q_h^+ - q_h^-) \tag{2.17}$$

with

$$s_{\max} = \max\left(|\lambda_c(q_h^-)|, |\lambda_c(q_h^+)\right) + 2\eta \max\left(|\lambda_\nu(q_h^-)|, |\lambda_\nu(q_h^+)\right) \tag{2.18}$$

and

$$\eta = \frac{N+1}{h} \tag{2.19}$$

CHAPTER 3 Conclusion

Conclusion here.

APPENDIX A

Placeholder…

Bibliography

[Dum10] M. Dumbser. "Arbitrary high order PNPM schemes on unstructured meshes for the compressible Navier–Stokes equations." In: *Computers & Fluids* 39.1 (2010), pp. 60–76 (see p. 2).