DEPARTMENT OF INFORMATICS

TECHNICAL UNIVERSITY OF MUNICH

Master's Thesis in Informatics

Cloud Supercomputing with the ExaHyPE-Engine

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Wolken Hochleistungsrechnen mit der ExaHyPE-Engine

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I confirm that this master's thesis in informatics is my own work and I have documented all sources and material used.

Munich, January , 2019 Lukas Krenz

Acknowledgments

Quote!

(Booktitle)

Acknowledgements here.

Abstract

Abstract here!

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CHAPTER 1

Introduction

Introduction here.

CHAPTER 2

Methods

Methods here.

Organize into chapters!

2.1. Conservative Form

Standard (hyperbolic) form of conservation law

$$\frac{\partial}{\partial_t} Q + \nabla \cdot F(Q) = S(\mathbf{x}, t, Q) \tag{2.1}$$

extend to:

$$\frac{\partial}{\partial_t} Q + \nabla \cdot F(Q, \nabla Q) = S(\mathbf{x}, t, Q) \tag{2.2}$$

2.2. Compressible Euler

Vector of conserved quantities:

$$Q = (\rho, \rho \boldsymbol{v}, \rho E) \tag{2.3}$$

Flux:

$$F(Q) = \begin{pmatrix} \rho \boldsymbol{v} \\ \boldsymbol{v} \otimes \rho \boldsymbol{v} + p\mathbf{I} \\ \boldsymbol{v} \cdot (\mathbf{I}\rho E + \mathbf{I}p) \end{pmatrix} \tag{2.4}$$

$$p = (\gamma - 1)\left(\rho E - 0.5\left(\boldsymbol{v} \cdot \rho \boldsymbol{v}\right)\right) \tag{2.5}$$

2.3. Compressible Navier Stokes

Following [Dum10] by Dumbser.

Flux:

$$F(Q, \nabla Q) = \begin{pmatrix} \rho \boldsymbol{v} \\ \boldsymbol{v} \otimes \rho \boldsymbol{v} + \mathbf{I}p + \boldsymbol{\sigma}(Q, \nabla Q) \\ \boldsymbol{v} \cdot (\mathbf{I}\rho E + \mathbf{I}p + \boldsymbol{\sigma}(Q, \nabla Q)) - \kappa \nabla T \end{pmatrix} \tag{2.6}$$

With: Pressure p

$$p = (\gamma - 1)\left(\rho E - 0.5\left(\boldsymbol{v} \cdot \rho \boldsymbol{v}\right)\right) \tag{2.7}$$

Temperature T

$$\frac{p}{\rho} = RT \tag{2.8}$$

Heat conduction coefficient κ

$$\kappa = \frac{\mu\gamma}{Pr}\frac{1}{\gamma - 1}R \tag{2.9}$$

with R gas constant and Pr Prandtl number

Sutherland's viscosity law:

$$\mu(T) = \mu_0\left(\frac{T}{T_0}\right)^{\beta}\frac{T_0 + C}{T + C} \tag{2.10}$$

with $\beta = 1.5$, $C = \text{const.}$, reference temperature T_0 and reference viscosity $\mu_=$. Equal to

$$\lambda = \frac{\mu_0(T_0 + C)}{T_0^{\beta}} \tag{2.11}$$

$$\mu(T) = \lambda\frac{T^{\beta}}{T + C}, \tag{2.12}$$

where λ is constant for a given fluid.

stress tensor

$$\boldsymbol{\sigma}(Q, \nabla Q) = (2/3\mu\nabla \cdot \boldsymbol{v}) - \mu\left(\nabla(\boldsymbol{v}) + \nabla(\boldsymbol{v})^{\intercal}\right) \tag{2.13}$$

Max eigenvalue of convective part $|\lambda_c^{max}|$ i.e. of $(\partial F/\partial \mathbf{Q}) \cdot \mathbf{n}$, and viscous part $|\lambda_v^{max}|$ i.e. of $(\partial F/\partial\left(\nabla \mathbf{Q} \cdot \mathbf{n}\right)) \cdot \mathbf{n}$ is

$$|\lambda_c^{max}|| = \|\boldsymbol{v}\| + c \tag{2.14}$$

$$|\lambda_v^{max}|| = \max\left(\frac{4}{3}\frac{\mu}{\rho}, \frac{\gamma\mu}{Pr\,\rho}\right) \tag{2.15}$$

with speed of sound $c = \sqrt{\gamma RT}$

Maximum timestep (CFL)

$$\Delta t = \frac{CFL}{2N + 1}\frac{h}{|\lambda_c^{max}| + 2|\lambda_v^{max}|\frac{2N+1}{h}} \tag{2.16}$$

with N polynomial order and h characteristic length scale of elements

Numerical flux:

$$G(q_h^-, \nabla q_h^-; g_h^+, \nabla q_h^+) \cdot \mathbf{n} = \frac{1}{2}\left(F(q_h^+, \nabla q_h^+) + F(q_h^-, \nabla q_h^-)\right) - \frac{1}{2}s_{max}(q_h^+ - q_h^-) \tag{2.17}$$

with

$$s_{max} = \max\left(|\lambda_c(q_h^-)|, |\lambda_c(q_h^+)\right) + 2\eta \max\left(|\lambda_v(q_h^-)|, |\lambda_v(q_h^+)\right) \tag{2.18}$$

and

$$\eta = \frac{N+1}{h} \tag{2.19}$$

CHAPTER 3

Conclusion

Conclusion here.

APPENDIX A

Placeholder...

Bibliography

[Dum10] M. Dumbser. "Arbitrary high order PNPM schemes on unstructured meshes for the compressible Navier–Stokes equations." In: *Computers & Fluids* 39.1 (2010), pp. 60–76 (see p. 2).