ECE430.217 Data Structures

An Introduction to Hash Tables

Weiss Book Chapter 5

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Lecture slides were prepared based on Douglas W. Harder's notes (**dwharder@alumni.uwaterloo.ca).**

Outline

Discuss storing unrelated/unordered data

- IP addresses and domain names

Consider conversions between these two forms

Introduce the idea of hashing:

- Reducing $\mathbf{O}(\ln(n))$ operations to $\mathbf{O}(1)$

Consider some of the weaknesses

Problem: IP Addresses

Examples:

You want to map an IP address to a corresponding domain name

A 32-bit IP address are often written as four byte values from 0 to 255

- Consider IP Address
  - $10010011\ 00101110\ 01111001\ 00010110_2$
  - This can be written as `147.46.121.22`
- Suppose its domain name is
  - `compsec.snu.ac.kr`
- We use domain names because IP addresses are not human readable

Example: IP Addresses

Given an IP address, if we wanted to quickly find any associated domain name, we could create an array of size 2^{32} (4294967296) of strings:

```
int const MAX_IP_ADDRESSES = 4294967296;
string domain_name[MAX_IP_ADDRESSES];
```

For example, 147.46.121.22 can be translated

- As $147 * 256^3 + 46 * 256^2 + 121 * 256 + 22 = 2466293526$,

```
domain_name[2466293526] = “compsec.snu.ac.kr";
```

Can we use much less memory than this?

Goals and Requirements

Our goal:

Store data so that all operations are $\Theta(1)$ time

Requirement:

The memory requirement should be $\Theta(n)$

Q. Can we achieve this goal with data structures we covered before?

- Lists, stack, queue, trees, …

Goals and Requirements

In general, we would like to:

- Create an array of size M
- Store each of n objects in one of the M bins
- Have some means of determining the bin in which an object is stored

Idea: Grade Table Example

Let's try a simpler problem

- How do I store your examination grades so that I can access your grades in $\Theta(1)$ time?

Observation: SNU ID is an 9-digit number

- I can't create an array of size 10^9
- I can create an array of size 1000 though
- How could you convert an 9-digit number into a 3-digit number?
- First three digits might cause a problem
  - almost all students start with 2017, 2018, 2019, …
- The last four digits, however, are (somehow) random

Therefore, I could store the examination grade for SNU ID 201901011

```
grade[011] = 99;
```

Idea: Grade Table Example

Consequently, I have a function, mapping a student ID to a 3-digit number
- I can store the information in that location
- Storing it, accessing it, and erasing may take $\Theta(1)$
- Problem: two or more students may map to the same number:
  - Vayne has ID 200703456
  - Teemo has ID 200301456
  - Both would map to 456
  - ➔ So called “collision”

⋮	⋮
454	
455	
456	86
457	
458	
459	
460	
461	
462	
463	79
464	
465	
⋮	⋮

Probability of Collision

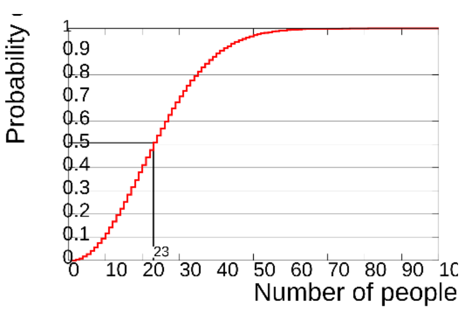
Question:

- What is the likelihood that in a class of size 100 that **no two students will have the same last three digits**?
- Not very high: i.e., a probability of having “no collision”

$$1 \cdot \frac{999}{1000} \cdot \frac{998}{1000} \cdot \frac{997}{1000} \cdot \quad \cdots \quad \cdot \frac{901}{1000} \approx 0.005959$$

- Probability of having collision(s): $1 - \ 0.005959 = 0.994041$
- Implication: If you insert 100 students to the table, there will be at least one collision at the probability of more than 99.4%
  - So highly likely there will be **a collision** if only using the last three digits

Check more:
https://en.wikipedia.org/wiki/Birthday_problem



The hashing problem

The process of mapping an object or a number onto an integer in a given range is called ***hashing***

Problem: multiple objects may hash to the same value

- Such an event is termed **a *collision***

Hash tables use a hash function together with a mechanism for dealing with collisions

The hash process

Object
(having a key/value pair)

$\downarrow$ **Hash function**

Map to an index $0, \ldots, M - 1$

$\downarrow$

Deal with collisions

Chained hash tables
Open addressing

- Linear probing
- Quadratic probing
- Double hashing

Definition: Hash Function

What is a hash of an object?

From Oxford Languages:

> *a dish of cooked meat cut into small pieces and cooked again, usually with potatoes.*

Our definition: map an arbitrary-size input to an fixed-size value (e.g., **`M`**)

`0, 1, 2, ..., M – 1`

https://cdn77-s3.lazycatkitchen.com/wp-content/uploads/2020/01/baked-vegan-hash-browns-plate-1000x1500.jpg

The hash process

Input key

32-bit integer

Map to an index $0, ..., M - 1$

Ideal properties of a hash function

A hash function is a function mapping an input key to a certain range (say 0 to M)

Necessary properties of such a hash function h are:

1a. Computation should be **fast**, ideally $\Theta(1)$

1b. The hash value must be ***deterministic***

- It must always return the same output: If $x = y \Rightarrow h(x) = h(y)$

1c. If two objects are randomly chosen, there should be only 1/M chance that they have the same hash value

(i.e., **uniform distribution**)

Types of hash functions

Hash functions for different types of input keys

- General class object
- Integer
- String

Hash Function for Class Object #1

#1. The easiest solution is to give each object **a unique number**

```
class Product {
    private:
        unsigned int hash_value;
        static unsigned int hash_count;
    public:
        Product();
        unsigned int hash() const;
};

unsigned int Product::hash_count = 0;
```

```
Product::Product() {
    hash_value = hash_count;
    ++hash_count;
}

unsigned int Product::hash() const {
    return hash_value;
}
```

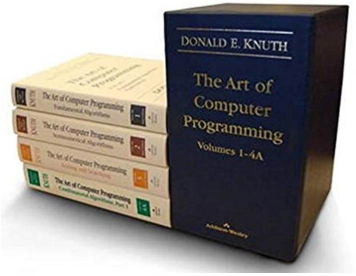
Hash Function for Class Object #2

#2. If we only need the hash value while the object exists in memory, you may use the **address**:

```
unsigned int Product::hash() const {
   return reinterpret_cast<unsigned int>( this );
}
```

Check more: https://en.cppreference.com/w/cpp/language/reinterpret_cast

Hash Function for Integer



- Knuth's Multiplicative Method

  hash(i) = i * 2654435761 mod 2^32

  - 2654435761 is the golden ratio of 2^32
  - 2654435761 and 2^32 have no common factors
    - So the multiplication produces a complete mapping of the key to hash result with no overlap
    - Having common factor "n" would only map to 1/n possible hashes
  - Issue: This preserves the divisibility. So for example, if your keys were even, their hashes are always even too.

  - Proof the claim:

    Consider h(x) = ax mod m.
    If a and m are coprime, $h(x_1) \,!= h(x_2)$ if x1 < x2 < m.

Reference: Integer hash function, Thomas Wang
https://gist.github.com/badboy/6267743

Hash Function for Integer

- Robert Jenkin's 32-bit integer hash function

```
uint32_t hash( uint32_t a)
{
   a = (a+0x7ed55d16) + (a<<12);
   a = (a^0xc761c23c) ^ (a>>19);
   a = (a+0x165667b1) + (a<<5);
   a = (a+0xd3a2646c) ^ (a<<9);
   a = (a+0xfd7046c5) + (a<<3);
   a = (a^0xb55a4f09) ^ (a>>16);
   return a;
}
```

Reference: Integer hash function, Thomas Wang
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Hash Function for Integer

- It’s difficult to tell if your hash function is good or bad
- It depends on the distribution of input keys
- What should be the goodness measure of hash function?
  - The hash function must be efficiently computed
  - The key mapping is closed to the uniform distribution

Hash Function for String

Two strings are equal if all the characters are equal and in the identical order

A string is simply an array of bytes:

- Each byte stores a value from 0 to 255

Any hash function must be a function of these bytes

Hash Function for String

We could, for example, just add the characters:

```
unsigned int hash( const string &str ) {
    unsigned int hash_value = 0;

    for ( int k = 0; k < str.length(); ++k ) {
        hash_value += str[k];
    }

    return hash_value;
}
```

Q. What’s the problem of this hash function?

Hash Function for String

Let the individual characters represent the coefficients of a polynomial in x:

$$p(x) = c_0 x^{n-1} + c_1 x^{n-2} + \cdots + c_{n-3} x^2 + c_{n-2} x + c_{n-1}$$

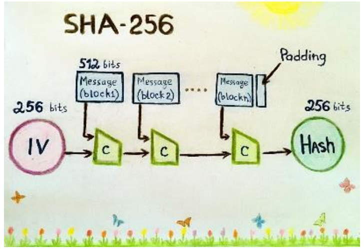
Use Horner's rule to evaluate this polynomial at a prime number, *e.g.*, $x = 12347$:

```
unsigned int hash( string const &str ) {
    unsigned int hash_value = 0;

    for ( int k = 0; k < str.length(); ++k ) {
        hash_value = 12347*hash_value + str[k];
    }
    return hash_value;
}
```

Note: Hash functions here != Cryptographic Hash

- All the hash functions discussed here are not cryptographic hash functions
  - MD5, SHA-1, SHA-512
  - https://en.wikipedia.org/wiki/Cryptographic_hash_function



https://en.bitcoinwiki.org/wiki/SHA-256

- Cryptographic hash functions have following security properties
  - Pre-image resistance
    - Given h, it is difficult to find m such that h = hash(m)
  - Second pre-image resistance
    - Given m_1, it is difficult to find m_2 such that hash(m_1) = hash (m_2)
  - Collision resistance
    - Difficult to find any m_1 and m_2 such that hash(m_1) = hash(m_2)
  - Be careful on the definition of being “difficult”

https://blog.bankofhodlers.com/the-value-of-proof-of-work/

Mapping down to 0, …, M-1

By far, we computed 32-bit hash values for different input keys

- Class object
- Integer
- String

Practically, we will require a hash value on the range $0, \ldots, M-1$:

- The modulus operator %

```
unsigned int hash_M( unsigned int n, unsigned int M ) {
       return n % M;
}
```

```
unsigned int hash_M( unsigned int n, unsigned int m ) {
    return n & ((1 << m) - 1); // M = 2^m
}
```

Summary

Hashing

- Discuss storing unordered data
- Discuss IP addresses and domain names
- Discussed the issues with collisions

Hash Function

- Ideal properties
- Hash functions for
  – Integer
  – String
  – Class object
- Map a 32-bit integer onto a smaller range $0, 1, ..., M - 1$

References

Wikipedia, http://en.wikipedia.org/wiki/Hash_table

[1] Cormen, Leiserson, and Rivest, *Introduction to Algorithms*, McGraw Hill, 1990.

[2] Weiss, Data Structures and Algorithm Analysis in C++, 3rd Ed., Addison Wesley.