

Tree Traversals

Weiss Book Chapter 4.1

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Outline

This topic will cover tree traversals:

- A means of visiting all the objects in a tree data structure
- We will look at
 - Breadth-first traversals
 - Depth-first traversals
- Applications
- General guidelines

Background

All the objects stored in an array or linked list can be accessed sequentially

Question: how can we iterate through all the objects in a tree in a predictable and efficient manner

- Requirements: $\Theta(n)$ run time and $o(n)$ memory

Types of Traversals

We have already seen one traversal:

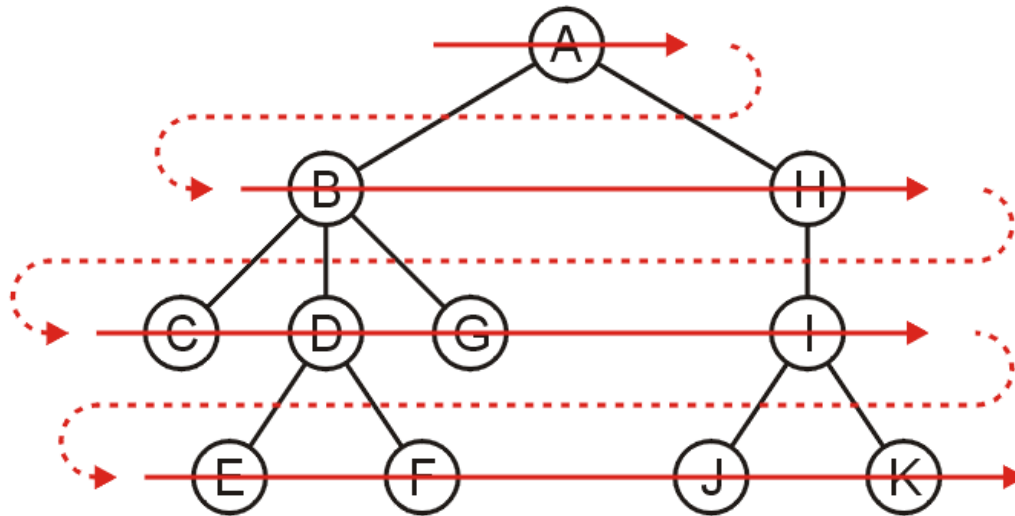
- The **breadth-first traversal** visits all nodes at depth k before proceeding onto depth $k + 1$
- Easy to implement using a queue

Another approach is to visit always go as deep as possible before visiting other siblings: ***depth-first traversals***

Breadth-First Traversal

Breadth-first traversals visit all nodes at a given depth

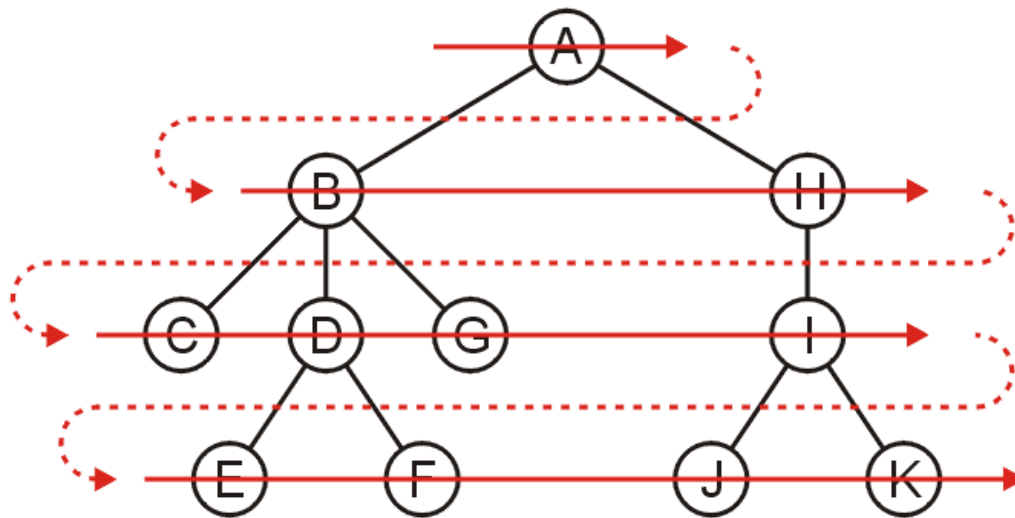
- Can be implemented using a queue
- Run time is $\Theta(n)$
- Memory is potentially expensive: maximum nodes at a given depth
- Order: A B H C D G I E F J K



Breadth-First Traversal

The implementation was already discussed:

- Create **a queue** and push the root node onto the queue
- While the queue is not empty:
 - Push all of its children of the front node onto the queue
 - Pop the front node

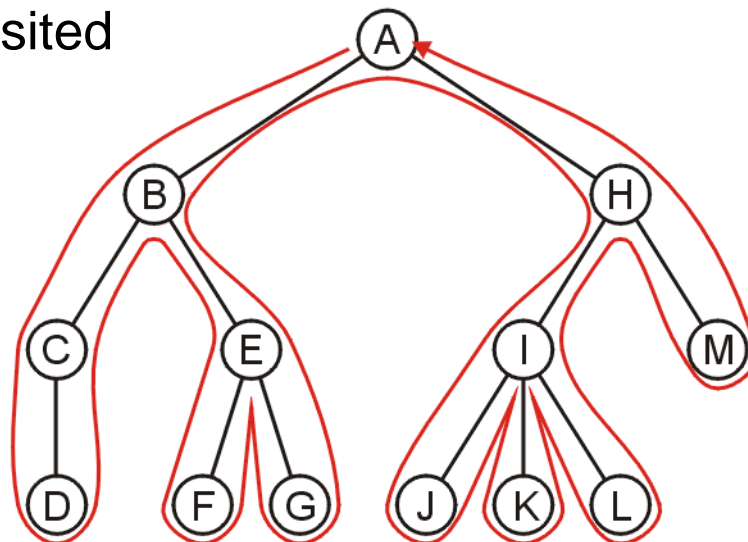


Backtracking

To discuss depth-first traversals, we will define **a backtracking algorithm** for stepping through a tree:

- At any node, we proceed to the first child that has not yet been visited
- Or, if we have visited all the children, we backtrack to the parent and repeat this decision making process

We end once all the children of the root are visited

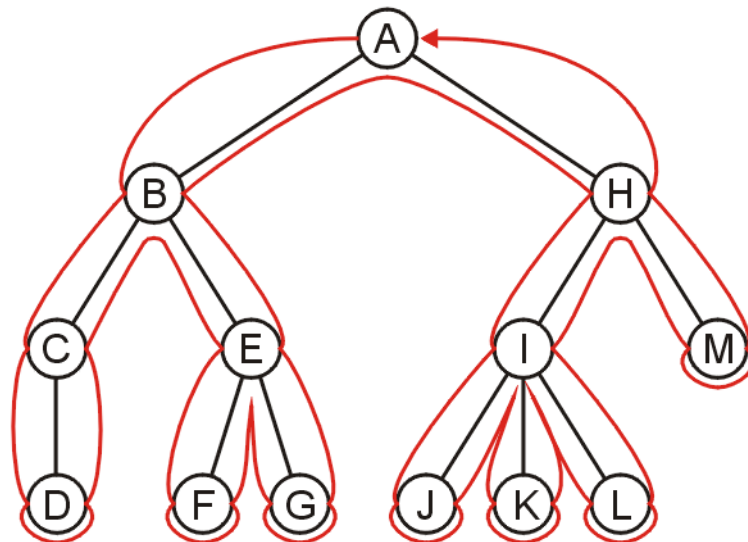


Depth-first Traversal

We define such a path as a *depth-first traversal*

We note that each node **could be visited twice** in such a scheme

- The first time the node is approached (before any children)
- The last time it is approached (after all children)



Implementing depth-first traversals

Depth-first traversals can be implemented with recursion:

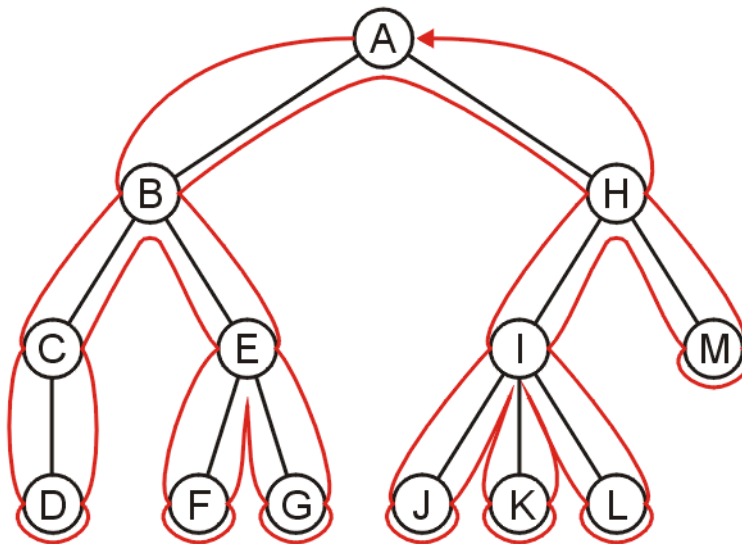
```
template <typename Type>
void print_dfs_tree(SimpleTree<Type> *tree, int depth=0) {
    // Perform pre-visit operations on the value
    std::cout << std::string(depth*4, ' ') << "<" << tree->get_value() << ">" << std::endl;

    // Perform a depth-first traversal on each of the children
    for (int i=0; i<tree->get_degree(); i++) {
        auto ctree = tree->get_child(i);
        print_dfs_tree(ctree, depth+1);
    }

    // Perform post-visit operations on the value
    std::cout << std::string(depth*4, ' ') << "</" << tree->get_value() << ">" << std::endl;
}
```

Implementing depth-first traversals

Performed on this tree, the output would be



```

<A>
  <B>
    <C>
      <D>
      </D>
    </C>
    <E>
      <F>
      </F>
      <G>
      </G>
    </E>
  </B>
  <H>
    <I>
      <J>
      </J>
      <K>
      </K>
      <L>
      </L>
    </I>
    <M>
    </M>
  </H>
</A>
  
```

Implementing depth-first traversals

Alternatively, we can use a stack:

- Create a stack and push the root node onto the stack
- While the stack is not empty:
 - Pop the top node
 - Push all of the children of that node to the top of the stack in reverse order
- Run time is $\Theta(n)$
- The objects on the stack are all unvisited siblings from the root to the current node
 - If each node has a maximum of two children, the memory required is $\Theta(h)$: the height of the tree

With the recursive implementation, the memory is $\Theta(h)$

Applications

Tree application: displaying information about directory structures and the files contained within

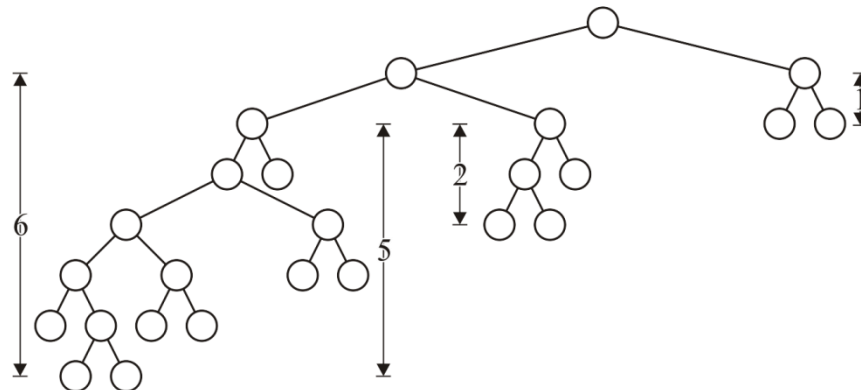
- Finding the height of a tree
- Printing a hierarchical structure

Height

The `int height()` const function is recursive in nature:

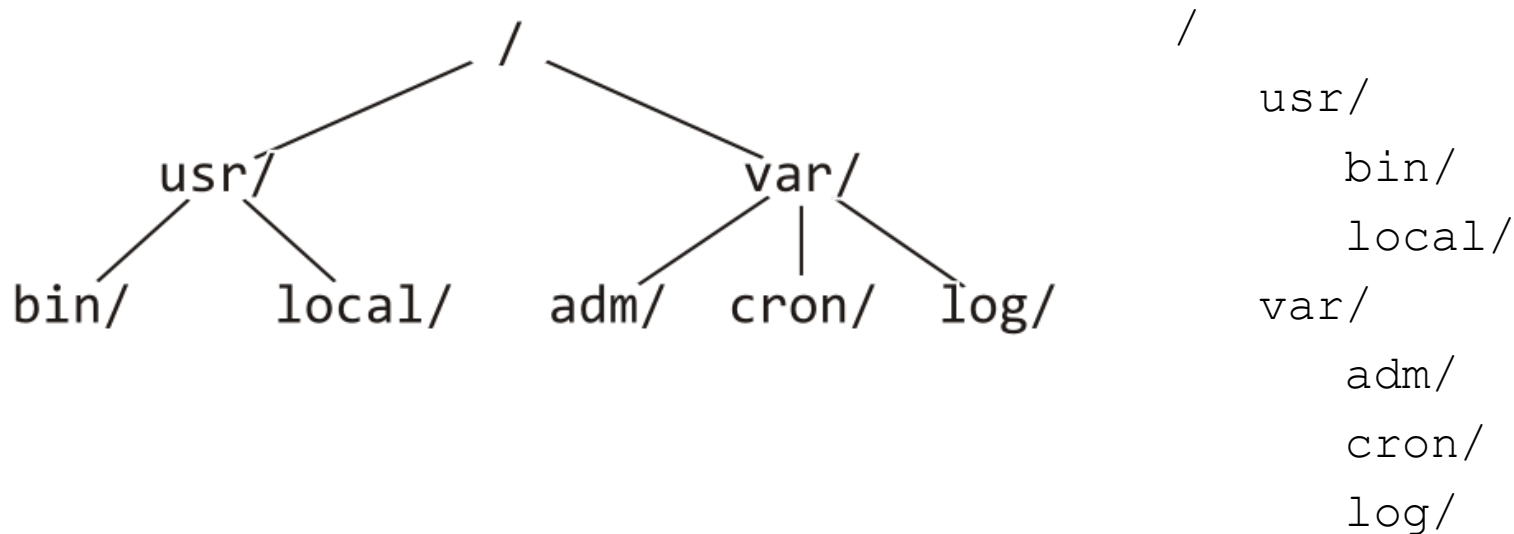
1. Before the children are traversed, we assume that the node has no children and we set the height to zero: $h_{\text{current}} = 0$
2. In recursively traversing the children, each child returns its height h and we update the height as $\max(1 + h, h_{\text{current}})$
3. Once all children have been traversed, we return h_{current}

When the root returns a value, that is the height of the tree



Printing a Hierarchy

Consider the directory structure presented on the left—how do we display this in the format on the right?



What do we do at each step?

Printing a Hierarchy

For a directory, we initialize a tab level at the root to 0

We then do:

1. Before the children are traversed, we must:
 - a) Indent an appropriate number of tabs, and
 - b) Print the name of the directory followed by a ' / '
2. In recursively traversing the children:
 - a) A value of one plus the current tab level must be passed to the children, and
 - b) No information are passed back
3. Once all children have been traversed, we are finished

Summary

This topic covered two types of traversals:

- Breadth-first traversals
- Depth-first traversals
- Applications
- Determination of how to structure a depth-first traversal