ECE430.217 Data Structures

Minimum Spanning Tree

Textbook: Weiss Chapter 9.5

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Lecture slides were prepared based on Douglas W. Harder's notes (**dwharder@alumni.uwaterloo.ca).**

Outline

In this topic, we will

- Define a spanning tree
- Define the weight of a spanning tree in a weighted graph
- Define a minimum spanning tree
- Consider applications
- List possible algorithms

Spanning trees

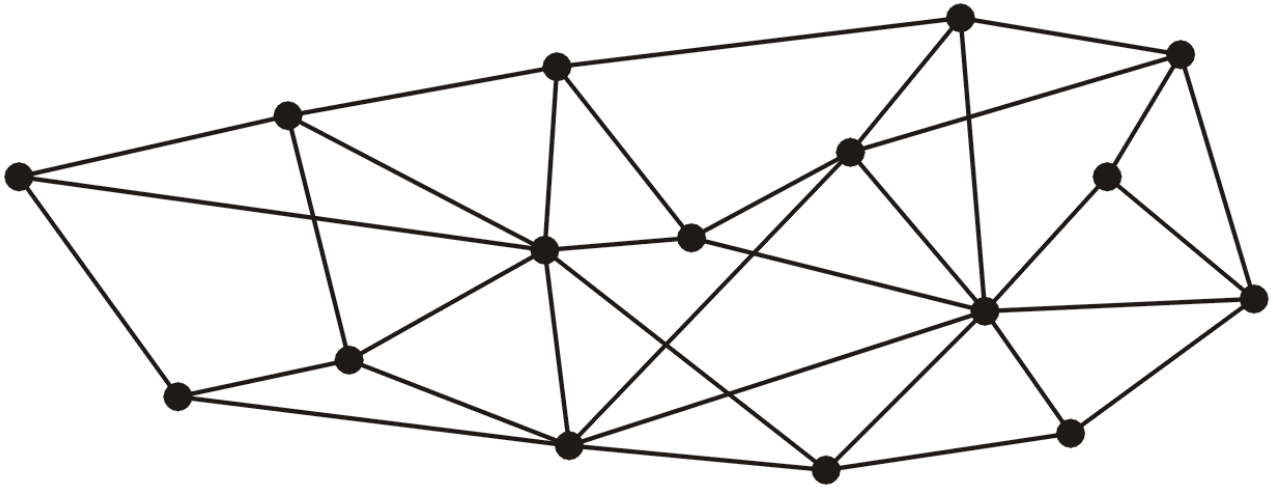
Given a connected graph with $|V| = n$ vertices, a spanning tree is defined a collection of $n - 1$ edges which connect all n vertices

- The n vertices and $n - 1$ edges define a connected sub-graph

A spanning tree is not necessarily unique

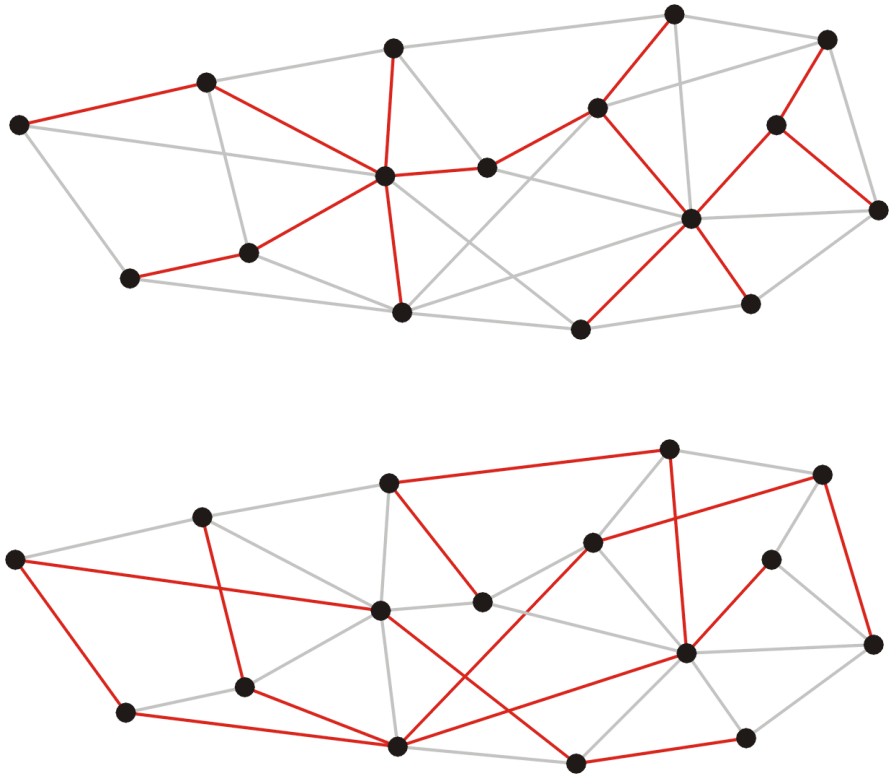
Spanning trees

This graph has 16 vertices and 35 edges



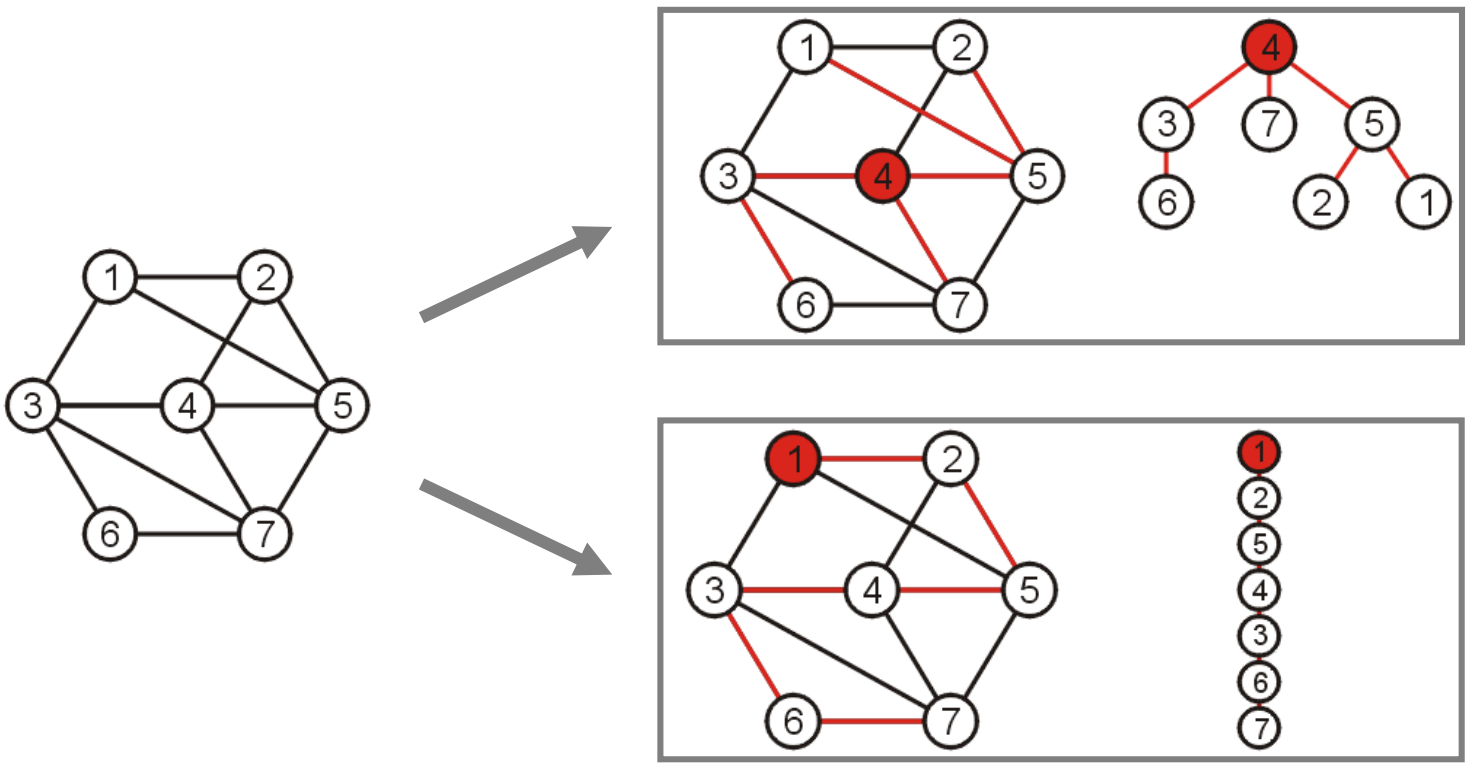
Spanning trees

These 15 edges form a minimum spanning tree



Spanning trees

Such a collection of edges is called a *tree* because if any vertex is taken to be the root, all vertices will be child/parent of others

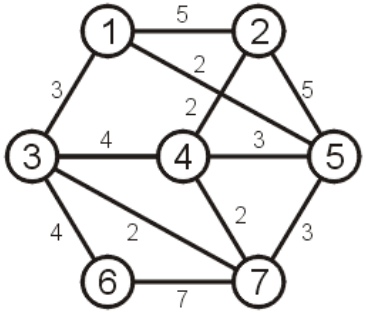


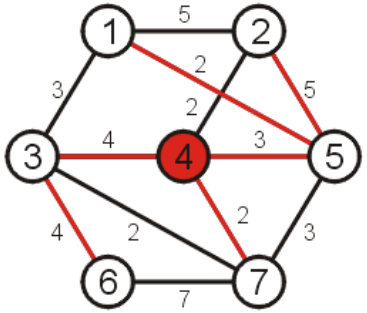
Spanning trees on weighted graphs

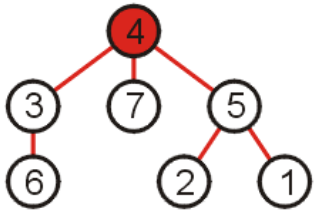
The weight of a spanning tree is the sum of the weights on all the edges which comprise the spanning tree

Which spanning tree minimizes the weight?

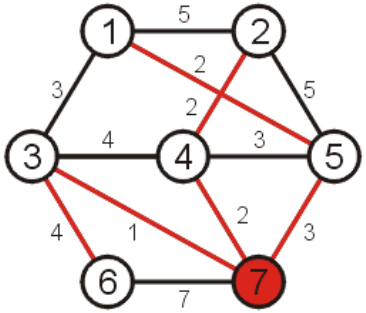
– Such a tree is termed ***a minimum spanning tree***

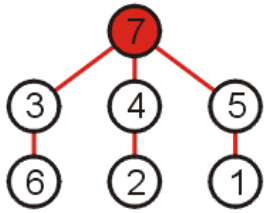






The weight is 20





The weight is 14

Unweighted graphs

Observation in unweighted graphs

- In an unweighted graph, we give each edge a weight of 1
- Consequently, all minimum spanning trees have weight $|V| - 1$

Application

- Supplying power to all circuit elements on a board
- Supplying power to all rooms in a building
- Flight costs with connection flights

A minimum spanning tree will give the lowest-cost solution

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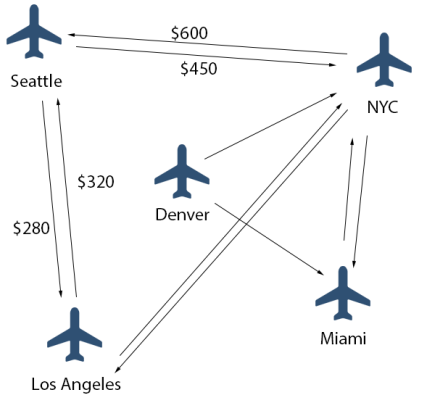

http://apleroy.com/posts/using-google-maps-to-visually-display-a-minimum-spanning-tree-post-1-of-4

Algorithms

Two common algorithms for finding minimum spanning trees are:

- Prim's algorithm
- Kruskal's algorithm

References

Wikipedia, http://en.wikipedia.org/wiki/Minimum_spanning_tree