

CSCI104 Written Homework #1

Problem 1: Runtime Analysis

a) $\theta(\log(\log n))$

i) i starts at $2 \rightarrow i^2 \rightarrow i_k = 2^{2^k}$

ii) Stops when $i_k \geq n = 2^{2^k} \geq n \rightarrow k \geq \log_2 \log_2 n$

iii) $T(n) = \sum_{k=0}^{\log_2 \log_2 n - 1} \theta(1)$

iv) $T(n) = \theta(\log(\log n))$

b) $\theta(n^{\frac{7}{2}})$

i) $T(n) = \sum_{i=1}^n (i \% \sqrt{n} = 0 \sum_{k=0}^{i^3-1} \theta(1))$

ii) $\sum_{j=1}^{\sqrt{n}} \sum_{k=0}^{(j\sqrt{n})^3-1} \theta(1) = \sum_{j=1}^{\sqrt{n}} (j\sqrt{n})^3 = \theta(n^{\frac{7}{2}})$

c) $\theta(n^2 \log n)$

i) $T(n) = \sum_{i=1}^n \sum_{k=1}^n 1 \sum_{m=1}^{\log_2 n} 1$

ii) $\sum_{i=1}^n \sum_{k=1}^n 1 = n^2 \rightarrow T(n) = n^2 \log n = \theta(n^2 \log n)$

d) $\theta(n)$

i) size = $10 \rightarrow 15 \rightarrow 22.5 (22) \rightarrow 33.75 (33) \rightarrow (49) \dots 10 \cdot \frac{3}{2}^x$

- ii) If statement runs for increments of $i < n$ divisible by size (which increases exponentially until reaching n) $\rightarrow 10(\frac{3}{2})^x \leq n \rightarrow x \leq \log_{\frac{3}{2}}(\frac{n}{10})$ (add +1 for $i=10$)

$$\text{iii) } T(n) = \sum_{i=0}^n \theta(1) + \sum_{k=0}^{\log_{\frac{3}{2}}(\frac{n}{10})} (1 + \sum_{j=0}^{10(\frac{3}{2})^i} 1) = \sum_{k=0}^{\log_{\frac{3}{2}}(\frac{n}{10})} 10(\frac{3}{2})^k \rightarrow \text{geometric series}$$

$$\text{iv) } T(n) = \theta(n) + \theta(10(\frac{3}{2})^{\log_{\frac{3}{2}}(\frac{n}{10})}) = \theta(n) + \theta(n)$$

Problem 2: Linked List Recursion Tracing

a) 1, 5, 2, 6, 3, 4

- i) Iteration 1: run llrec(1,5) \rightarrow 1,
- ii) Iteration 2 (1->next): run llrec(5,2) \rightarrow 1, 5
- iii) Iteration 3 (5->next): run llrec(2,6) \rightarrow 1, 5, 2
- iv) Iteration 4 (2->next): run llrec(6,3) \rightarrow 1, 5, 2, 6
- v) Iteration 5 (6->next): run llrec(3,null) \rightarrow 1, 5, 2, 6, 3
- vi) Iteration 6: returns 3 (in2 is null), 3's next is unmodified as 4 \rightarrow 1, 5, 2, 6, 3, 4

b) 2

- i) First if statement is triggered, with in1 being null. Since in2 is equal to 2, "2" is returned.