

Insertion Sort

Basics, Algorithm and Program

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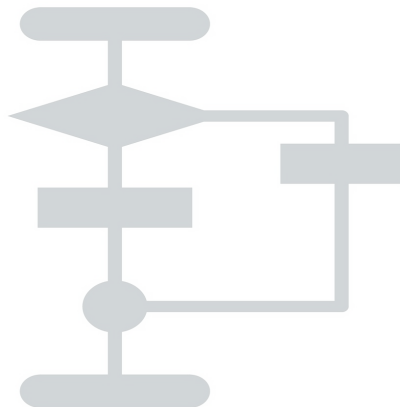
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Index

- 1 Introduction
- 2 Visual Representation
- 3 Implementation
- 4 Analysis

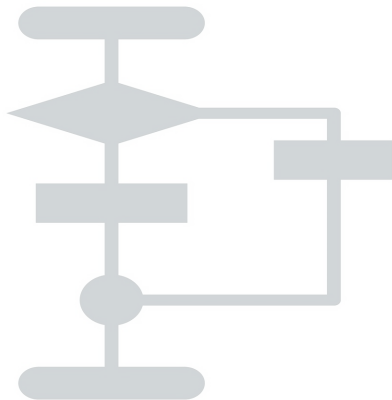


① Introduction

② Visual Representation

③ Implementation

④ Analysis



What is Insertion Sort?

Insertion Sort is a type of **sorting algorithm.**

Sorting Algorithm

A Sorting Algorithm is used to rearrange a given array or list of elements according to a comparison operator on the elements in computer language.

Example

- Selection Sort
- Insertion Sort
- Quick Sort
- Merge Sort

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Sorting: Rocket Science or Human Nature?



(a) Not Sorted



(b) Sorted

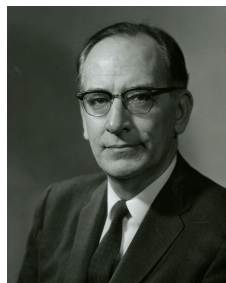
We have been using **sorting** for our own convenience since primitive times without really knowing or realizing any computer science documented algorithms!

Background

Historical Overview

Insertion Sort's real-life application ranges from sorting cards (see 5) to students' exam scripts. It might be hard to find the first person who came up with the idea behind insertion sort, because it is one of the basic ways humans would sort a list of items.

Knuth [1] writes that the variant of using binary insertion was mentioned by John Mauchly as early as 1946, in the first published discussion of computer sorting, being the algorithm's first documentation.



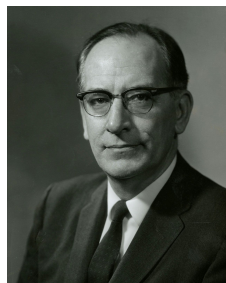
John Mauchly

Background

Historical Overview

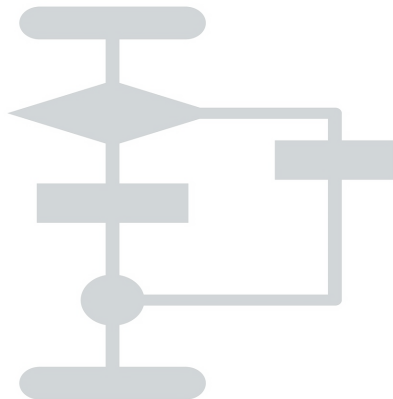
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John Mauchly

- 1 Introduction
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- 3 Implementation
- 4 Analysis



Visual Representation

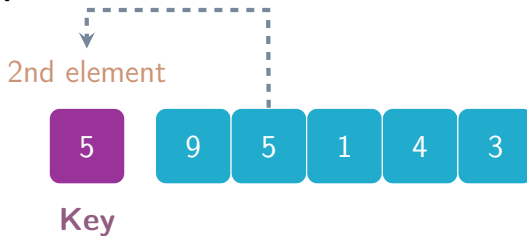
Steps in Illustration



Initial Array

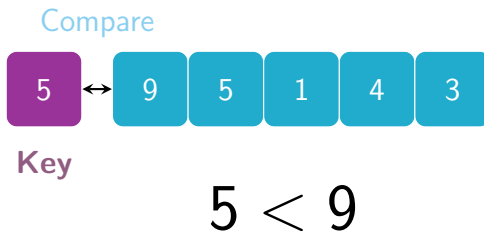
Steps in Illustration

Step 1:



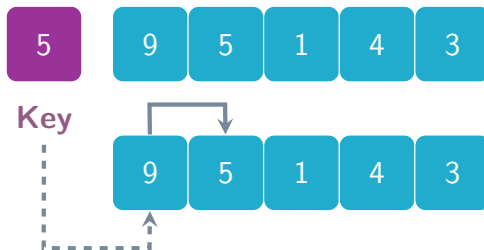
Steps in Illustration

Step 1:



Steps in Illustration

Step 1:



Steps in Illustration

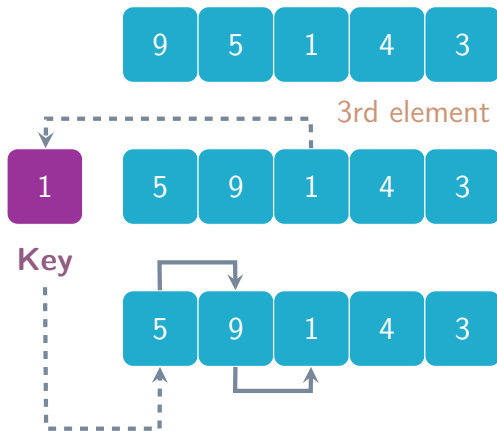
Step 1:



Places 5 at the beginning

Steps in Illustration

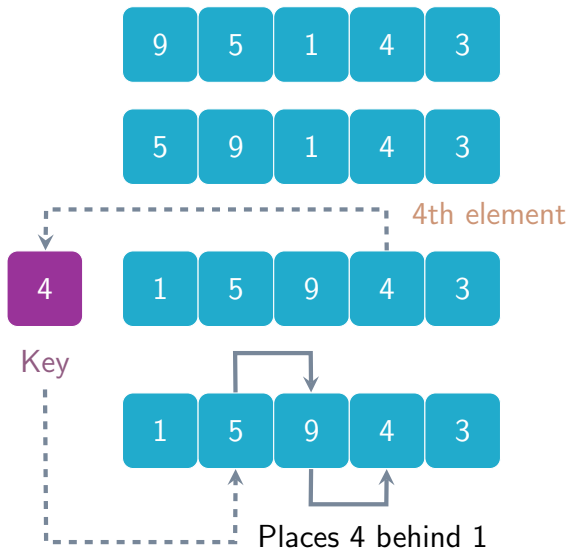
Step 2:



Places 1 at the beginning

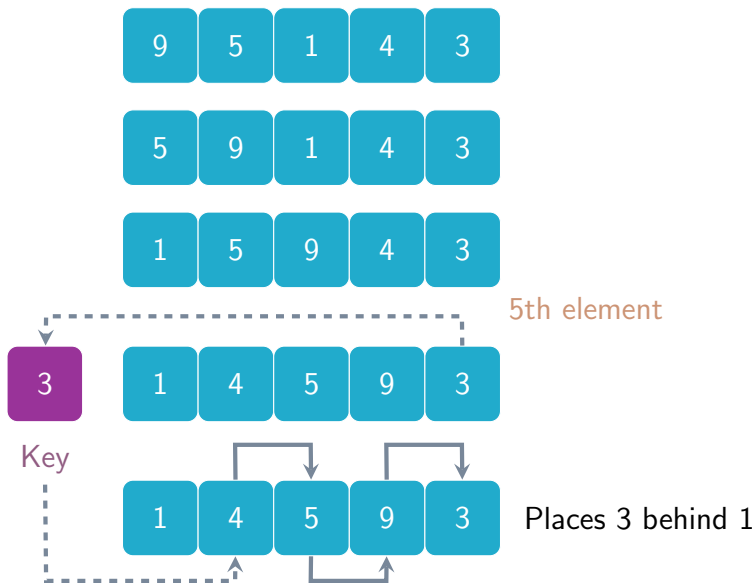
Steps in Illustration

Step 3:



Steps in Illustration

Step 4:



Steps in Illustration

9	5	1	4	3
---	---	---	---	---

Initial Array

5	9	1	4	3
---	---	---	---	---

step 1

1	5	9	4	3
---	---	---	---	---

step 2

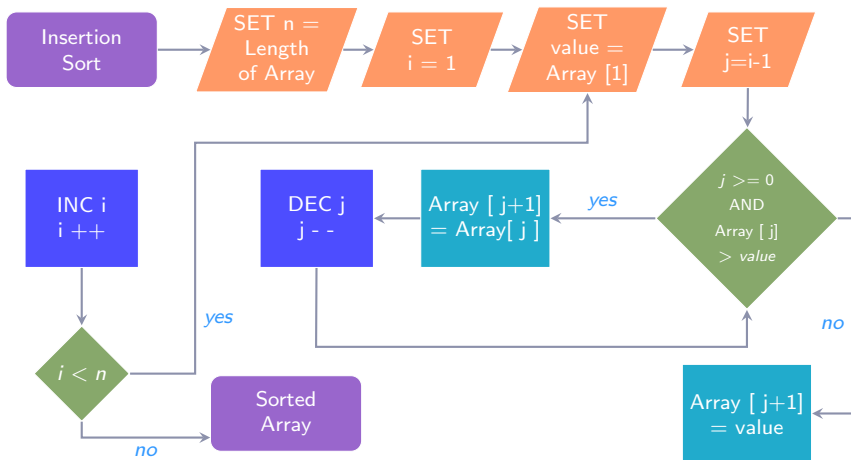
1	4	5	9	3
---	---	---	---	---

step 3

1	3	4	5	9
---	---	---	---	---

Sorted Array *step 4*

Flowchart



① Introduction

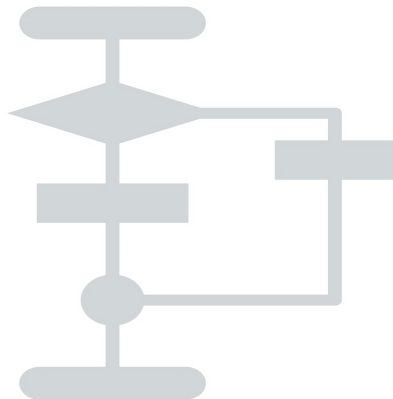
② Visual Representation

③ Implementation

Algorithm

Code

④ Analysis

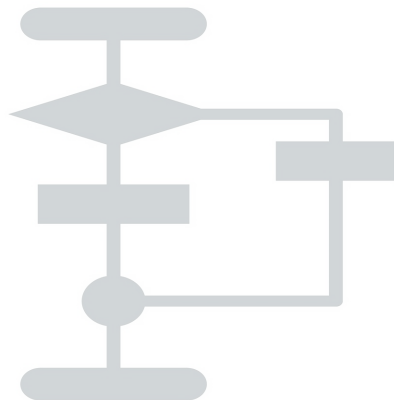


① Introduction

② Visual Representation

③ **Implementation**
Algorithm
Code

④ Analysis



Algorithm

Algorithm 1 Insertion algorithm

```
1: procedure INSERTIONSORT( $A : \text{array}$ )
2:    $n \leftarrow \text{length}(A)$ 
3:   for  $i := 1$  to  $n-1$  do
4:      $j \leftarrow i$ 
5:     while  $j > 0$  and  $A[j-1] > A[j]$  do
6:       swap( $A[j]$ ,  $A[j-1]$ )
7:        $j \leftarrow j - 1$ 
8:     end while                                ▷ inner loop end
9:   end for                                    ▷ outer loop end
10: end procedure
```

① Introduction

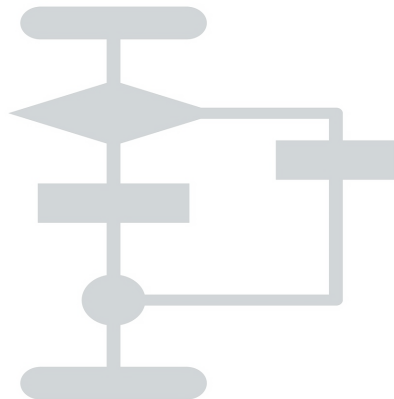
② Visual Representation

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Algorithm

Code

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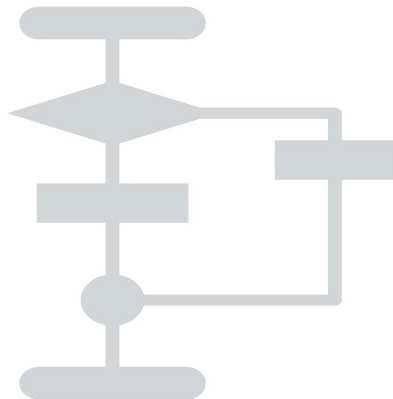


Complete Program in Python

```
1 # Insertion sort in Python
2 def insertionSort(array):
3
4     for step in range(1, len(array)):
5         key = array[step]
6         j = step - 1
7
8         # Compare key with each element on the left of it
9         # until an element smaller than it is found
10        while j >= 0 and key < array[j]:
11            array[j + 1] = array[j]
12            j = j - 1
13
14        # Place key after the element just smaller than it
15        array[j + 1] = key
16
17 data = [9, 5, 1, 4, 3]
18 insertionSort(data)
19 print('Sorted Array in Ascending Order:')
20 print(data)
```

See [2]

- ① Introduction
- ② Visual Representation
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- ④ Analysis



Best Case

The **best case** of Insertion Sort occurs **if the array is already sorted**. When $j = i - 1$, we always find the key $A[i]$ the first time.

Therefore, the running time of an algorithm equation-

$$\begin{aligned} T(n) = & c_1 n + c_2(n - 1) + c_4(n - 1) + c_5 \sum_{2 \leq j \leq n} (1) \\ & + c_6 \sum_{2 \leq n} (1 - 1) + c_7 \sum_{2 \leq n} (1 - 1) + c_8(n - 1) \end{aligned}$$

simplifies as,

$$T(n) = an + b = \mathcal{O}(n)$$

Worst Case

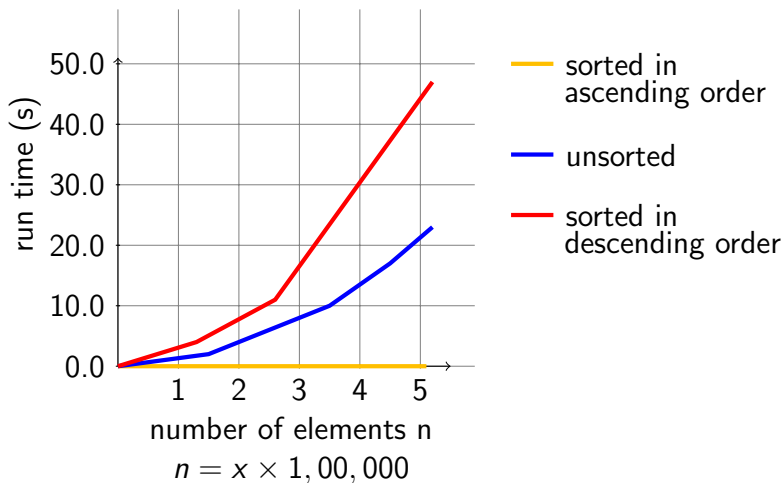
The **worst case** occurs **if the array is in reverse order of how we need to sort it**. So, we must compare each element $A[j]$ with element in the entire sorted subarray $A[i..j - 1]$.

The running term hence simplifies as,

$$T(n) = an^2 + bn + c = \mathcal{O}(n^2)$$

Since we focus more on worst case, the complexity of insertion sort is said to be n^2 .

Complexity Increase by Element



Complexity Overview

We can compare different sorting algorithm complexities with insertion sort from the given table.

Sorting Algorithm	Time Complexity			Space Complexity
	Best Case	Average Case	Worst Case	Worst Case
Bubble Sort	$\Omega(N)$	$\Theta(N^2)$	$O(N^2)$	$O(1)$
Selection Sort	$\Omega(N^2)$	$\Theta(N^2)$	$O(N^2)$	$O(1)$
Insertion Sort	$\Omega(N)$	$\Theta(N^2)$	$O(N^2)$	$O(1)$
Quick Sort	$\Omega(N \log_2 N)$	$\Theta(N \log_2 N)$	$O(N^2)$	$O(N)$
Merge Sort	$\Omega(N \log_2 N)$	$\Theta(N \log_2 N)$	$O(N \log_2 N)$	$O(N)$
Heap Sort	$\Omega(N \log_2 N)$	$\Theta(N \log_2 N)$	$O(N \log_2 N)$	$O(1)$

Time and Space Complexity of Sorting Algorithms

References

- [1] E Knuth Donald et al. “The art of computer programming”. In: *Sorting and searching* 3 (1999), pp. 426–458.
- [2] *Programiz*. URL: <https://www.programiz.com/dsa/insertion-sort>.