

CS 411 F21 HW4

$$1a) T(n) = 4T(n/2) + n, T(1) = 1$$

$$a = 4, b = 2, f(n) = n, d = 1$$

$$a > b^d \Rightarrow T(n) \in \Theta(n^{\log_b a}) = \Theta(n^{\log_2 4}) = \Theta(n^2)$$

$$1b) T(n) = 4T(n/2) + n^2, T(1) = 1$$

$$a = 4, b = 2, f(n) = n^2, d = 2$$

$$a = b^d \Rightarrow T(n) \in \Theta(n^d \log n) = \Theta(n^2 \log n)$$

$$1c) T(n) = 4T(n/2) + n^3, T(1) = 1$$

$$a = 4, b = 2, f(n) = n^3, d = 3$$

$$a < b^d \Rightarrow T(n) \in \Theta(n^d) = \Theta(n^3)$$

2) Merge sort on average taking $n \log n$ comparisons and binary search taking on average $\log n$ compared to a sequential search taking on average $n/2$ we can set up an equation to solve.

x = smallest number of searches needed

$$n \log(n) + x \log(n) \leq x(n/2)$$

$$\frac{n \log(n)}{n/2 - \log(n)} \leq x$$

$$n = 1,000,000 \Rightarrow x \geq 40$$

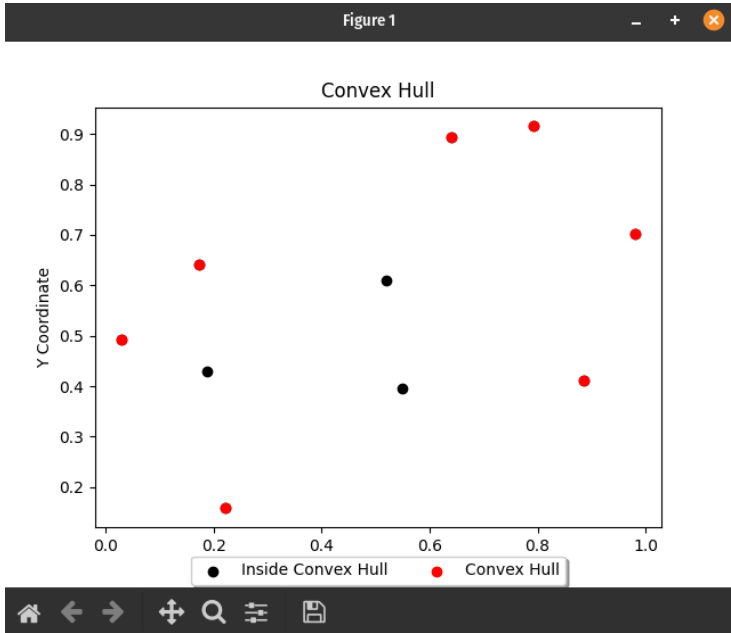
At least 40 searches will be needed to justify the time spent presorting an array of 1,000,000 elements.

3A)

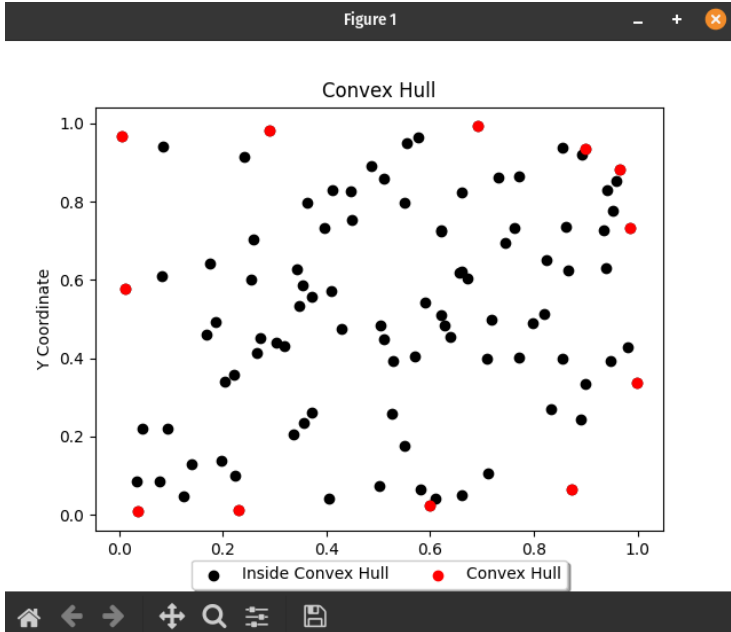
n (2D Randomly Distributed Points)	Run Time
10	0.00011444091796875 seconds

100	0.0009560585021972656 seconds
1000	0.023084402084350586 seconds
10000	0.28388452529907227 seconds

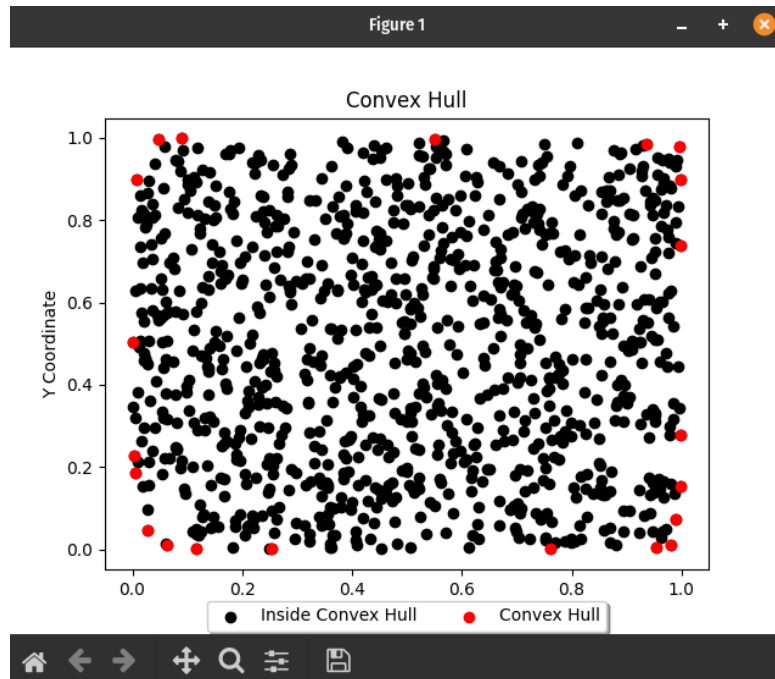
3B)
n = 10:



n = 100:



n = 1000:



n = 10000:

