Hometask-3

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1. Proove some basic facts about matricies:

a. Matrix power is defined as:

$$A^k = \underbrace{A \cdot A \cdots A}_{k}$$

Proove, that: $A = diag(d_1, d_2, d_3, \ldots, d_n) \Rightarrow A^k = diag(d_1^k, d_2^k, d_3^k, \ldots, d_n^k)$

b.

$$J(2,\lambda) = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$$

Matrix $J(2,\lambda)$ is called Jordan block of size 2. Proove that:

$$J(2,\lambda)^k = \begin{pmatrix} \lambda^k & k\lambda^{k-1} \\ 0 & \lambda^k \end{pmatrix}$$

c. «In my paper the fact that XY was not equal to YX was very disagreable to me. I felt this was the only point of difficulty in the whole scheme.» *(Werner von Heisenberg)*

«Now when Heisenberg noticed that, he was *really* scared.» *(Paul Dirac)*

Let $X, Y \in M(n, \mathbb{R})$.

- Show that if both X and Y are diagonal, $XY = YX$.
- Show that Jordan blocks commute: $J(n,\lambda)J(n,\mu) = J(n,\mu)J(n,\lambda)$

d. We are used to think that 0 has no divisors. But when it comes to matrix multiplications, this is not true. Give exmaple of two matricies A and B such that $A \neq 0$, $B \neq 0$, $AB = 0$.

2. Let's apply matrix multiplication to some discrete math:

a. *(We have already solved this at first lection)* Remember Fibonacci numbers: $F_{n+2} = F_{n+1} + F_n$, $F_0 = 0$, $F_1 = 1$. We can rewrite it in matrix form:

$$\begin{pmatrix} F_{n+2} \\ F_{n+1} \end{pmatrix} = A \begin{pmatrix} F_{n+1} \\ F_n \end{pmatrix}$$

Find out, what matrix A looks like. How can we compute F_n?

3. **Def**: A bipartite graph is a graph $G = (V, E)$ whose whose verticies can be partitioned into two sets ($V = V_1 \cup V_2$ and $V_1 \cap V_2 =$) such that there are no edges between verticies in the same set (for instance, if $u, v \in V_1$, there is no $e \in E$, $e = \{u, v\}$).

a. Give a linear-time algorithm to determine whether an undirected graph is bi[partite.

b. Proove that undirected graph is bipartite, iff it contains no cycles of odd length.

4. For each vertex in undirected graph compute sum of it's neighbors degrees in $O(E)$ time. Graph is given as adjacency list.