Algorithms and data structures 2: D&C

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Outline

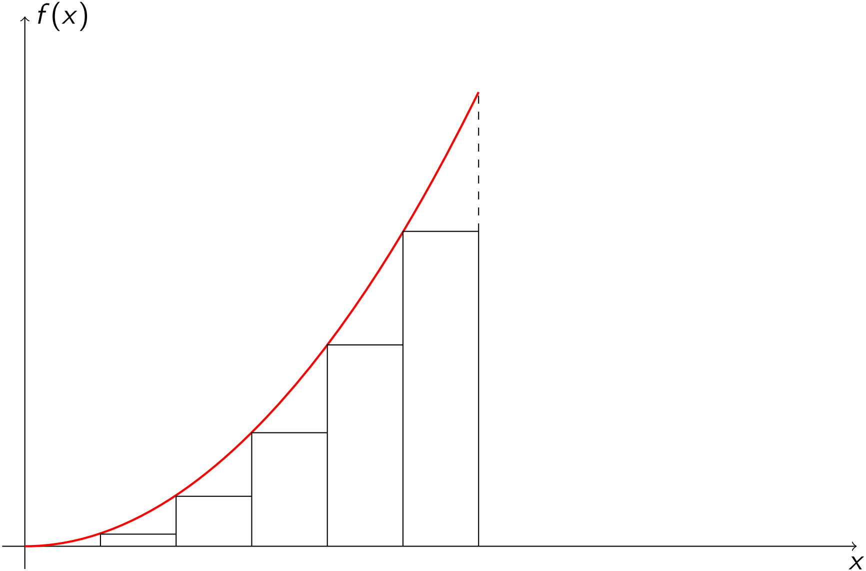
0. Revision
1. Sums
2. Recurrent relationss
3. Inversions
4. Karatsuba
5. Megainversions

Estimating sums: Abel transform

Th: $a_n, b_n,\ B_n = b_0 + \ldots + b_n$

$$\sum_{n=0}^{N} a_n b_n = a_n B_n - \sum_{n=0}^{N-1} B_n(a_{n+1} - a_n)$$

Estimating sums 2: integrals



Estimating sums 3: Stolz–Cesàro theorem

Th: a_n, b_n – int seq., b_n – strictly monotone, $\neg\exists b_0 \colon b_n \to b_0$.

$$\lim \frac{a_{n+1} - a_n}{b_{n+1} - b_n} = l$$

$$\Rightarrow \lim \frac{a_n}{b_n} = l$$

Generating functions

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Sug:

$$\sum x^n = \frac{1}{1-x}$$

Master theorem

Th: $a > 0, b > 1, d \geq 0$

$$T(n) = aT\left(\frac{n}{b}\right) + O(n^b)$$

$$T(n) = \begin{cases} O(n^d), & d > \log_b a \\ O(n^d \log n), & d = \log_b a \\ O(n^{\log_b a}), & d < \log_b a \end{cases}$$

Back to algos: inversions

Def: An inversion in array is pair of indexes i, j: $i < j$ and $A_i > A_j$.

Task: Given an array, find number of inversions in it.

Back to algos: multiplication

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- of $deg = 2$?

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Yes, devide!

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Def: one side inversion

EOF