Assignment - 2

Q.1

let & denote some notation for our purpose

G_i = Traffic light is Green at time = i
Y_i = Traffic light is Yellow at time = i
R_i = Traffic light is Red at time = i

S1

$$(G_i \leftrightarrow (\neg Y_i \wedge \neg R_i)) \wedge (Y_i \leftrightarrow (\neg G_i \wedge \neg R_i)) \wedge (R_i \leftrightarrow (\neg Y_i \wedge \neg G_i))$$

S2

$$(G_{i-1} \rightarrow (G_i \vee Y_i)) \wedge (Y_{i-1} \rightarrow (Y_i \vee R_i)) \wedge (R_{i-1} \rightarrow (R_i \vee G_i))$$

S3

$$(G_{i-3} \wedge G_{i-2} \wedge G_{i-1} \wedge \neg G_i) \wedge (Y_{i-3} \wedge Y_{i-2} \wedge Y_{i-1} \wedge \neg Y_i) \wedge (R_{i-3} \wedge R_{i-2} \wedge R_{i-1} \wedge \neg R_i)$$

Q.2

edge(n, m) : Node n is connected to node m.
color(n, x) : Node n has color x

S1

$$\forall n \forall m \forall x \, (\text{color}(n,x) \rightarrow \neg \exists y \, (y \neq x \wedge \text{color}(n,y)))$$

S2

$$\exists n \exists n' \, (\text{color}(n, \text{yellow}) \wedge \text{color}(n', \text{yellow}) \wedge n \neq n' \wedge \forall m \, (m \neq n \wedge m \neq n' \rightarrow \neg \text{color}(m, \text{yellow})))$$

S3

$\forall n \,(Color(n, red) \rightarrow \exists n_1 \,(edge(n, n_1) \wedge color(n_1, green))$

$\vee \;\exists n_1, n_2 \,(edge(n, n_1) \wedge edge(n_1, n_2) \wedge color(n_2, green)) \vee$

$\exists n_1, n_2, n_3 \,(edge(n, n_1) \wedge edge(n_1, n_2) \wedge edge(n_2, n_3) \wedge color(n_3, green)$

$\vee \exists n_1, n_2, n_3, n_4 \,(edge(n, n_1) \wedge edge(n_1, n_2) \wedge edge(n_2, n_3) \wedge edge(n_3, n_4) \wedge$
$color(n_4, green))))$

S4

$\forall x \,\exists n \; Color(n, x)$

S5

$(\forall x \,\exists n \; color(n, x)) \wedge (\forall n \,\exists x \; color(n, x)) \wedge$

$(\forall n \,\forall x \,(Color(n, x) \rightarrow \neg \exists y \,(y \neq x \wedge Color(n, y)))) \wedge$

$(edge(n, m) \vee \bigvee_{i=1}^{|N|} (\exists n_1 \ldots n_i \,(edge(n, n_1) \bigwedge_{j=1}^{i-1} edge(n_j, x_{j+1}) \wedge edge(n_i, m))))$

Q.3

let's define some proposition for broader purpose etc.

Read (x) = x can read

Lit (x) = x is literate

Dol (x) = x is a dolphin

Intelligent (x) = x is intelligen

1) $\forall x \,(Read(x) \rightarrow lit(x))$

2) $\forall x \,(Dol(x) \rightarrow \neg lit(x))$

3) $\exists x \,(Dol(x) \wedge Intelligent(x))$

4) $\exists x \,(Intelligent(x) \wedge \neg Read(x))$

⟨5⟩ $\forall y \exists x$ (Dol (x) $\land$ Intelligent (x) $\land$ Read (x) $\land$ [Dol (y) $\land$ Intelligent (y) $\land$ Read (y) $\rightarrow$ $\neg$ Literate (y)]

To prove 4 by resolution refutation

$\neg$ (Intelligent (x) $\land$ $\neg$ Read (x)) $\equiv$ $\neg$ Intelligent (x) $\lor$ Read (x)

($\neg$ Read(x) $\lor$ Lit(x))) ($\neg$ Dol (x) $\lor$ Lit (x))), Dol (x), Intelligent (x) Lit (x), ($\neg$ Intelligent (x) $\lor$ Read(x))

$\neg$ Lit(x

Read (x)

$\neg$ R(x)

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Since we are getting empty clause which is a contradiction

$\neg$ (Intelligent (x) $\land$ $\neg$ Read(x)) is False

$\Rightarrow$ Intelligent (x) $\land$ $\neg$ Read (x) is True

Since from ③ Dol(x) $\land$ Intelligent (x) is True

from ④ $\exists x$ $\neg$ Read(x) $\rightarrow$ $\exists x$ Read (x) is True

$\exists x$ (Dol(x) $\land$ Intelligent (x) $\land$ Read(x)) is True

By Resolution for universal Quantifier

(($\neg$ Dol(x) $\land$ Intelligent (x) $\land$ Read (x)) $\lor$ $\neg$ Lit (x))

Negation will be (Dol(x) $\land$ Intelligent (x) $\land$ Read (x)) $\lor$ $\neg$ Lit (x))

($\neg$ Read(x) $\lor$ Lit(x)), ($\neg$ Dol(x) $\lor$ $\neg$ Lit(x))), Dol(x), Intelligent(x), $\neg$ Read (x), Read (x), Lit (x)

$\neg$ Read (x), Literate $\lor$ Intelligent (x) $\neg$ Lit(x)

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The statement did not return empty clause

$\Rightarrow$ Statement remains unsatisfiable