MEK3220 - Formulae sheet

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November 21, 2015

The displacement gradient

$$\nabla \mathbf{u} = \begin{pmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{pmatrix}$$

Comment: If $\nabla \mathbf{u} = (\nabla \mathbf{u})^T$ we have that $\nabla \mathbf{u} = \epsilon$, the *strain tensor*.

Strain tensor

$$\epsilon = \frac{1}{2}((\nabla \mathbf{u})^T + \nabla \mathbf{u})$$

Cauchy's equilibrium equation

$$\mathbf{f_v} + \nabla \cdot \sigma^T = 0$$

Hooke's law

$$\sigma = \lambda I \sum_k \epsilon_{kk} + 2\mu\epsilon$$

or

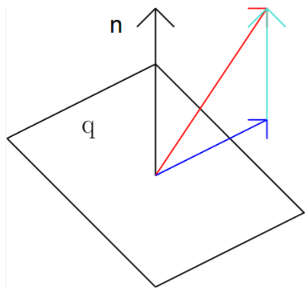
$$\begin{aligned} \sigma_{xx} &= (2\mu + \lambda)\epsilon_{xx} + \lambda(\epsilon_{yy} + \epsilon_{zz}), \quad & \sigma_{yz} = \sigma_{zy} = 2\mu\epsilon_{yz} \\ \sigma_{yy} &= (2\mu + \lambda)\epsilon_{yy} + \lambda(\epsilon_{xx} + \epsilon_{zz}), \quad & \sigma_{xz} = \sigma_{zx} = 2\mu\epsilon_{xz} \\ \sigma_{zz} &= (2\mu + \lambda)\epsilon_{zz} + \lambda(\epsilon_{yy} + \epsilon_{zz}), \quad & \sigma_{xy} = \sigma_{yx} = 2\mu\epsilon_{xy} \end{aligned}$$

Inverted Hooke's law

$$\epsilon = \frac{1+\nu}{E}\sigma - \frac{\nu}{E} I \sum_k \sigma_{kk}$$

Stress relations

Let the stress be given by σ, a plane denoted q and its normal be $\mathbf{n}$. Then we have the following relations:



Stress on surface (red)

$$\vec{\sigma}_q = \sigma \cdot \mathbf{n}$$

Normal stress on surface (light blue)

$$\sigma_{q(n)} = \vec{\sigma}_q \cdot \mathbf{n}$$

Shear stress on surface (blue)

$$\vec{\sigma}_{q(s)} = \vec{\sigma}_q - \sigma_{q(n)}\mathbf{n}$$

Navier's equation for elastic media in motion

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} = \mu \nabla^2 \mathbf{u} + (\lambda + \mu)\nabla\nabla \cdot \mathbf{u} + \mathbf{f_v}$$

Comment: For mechanical equilibrium set LHS. equal to zero.

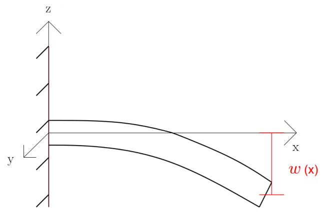
Lamè coefficients

$$E = \mu \frac{3\lambda + 2\mu}{\lambda + \mu}, \quad \nu = \frac{\lambda}{2(\lambda + \mu)}$$

$$\lambda = \frac{E\nu}{(1 - 2\nu)(1 + \nu)}, \quad \mu = \frac{E}{2(1 + \nu)}$$

Beam equations

Euler-Bernoulli and some additional important relations



$$\frac{d^2w(x)}{dx^2} = -\frac{M_y(x)}{EI}, \quad \frac{dM_y(x)}{dx} = F_z, \quad \frac{dF_z}{dx} = -F^{(z)}_{ext}$$

Equation of continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0$$

Navier-Stokes

$$\frac{d\mathbf{v}}{dt} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \mathbf{g} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v}, \quad \nabla \cdot \mathbf{v} = 0$$

Isotropic viscous stress

$$\sigma = pressure + shear\ stress = (-p)I + \mu(\nabla \mathbf{v} + \nabla \mathbf{v}^T)$$

Energy equations

Kinetic energy(per volume) for fluid:

$$E_k = \frac{1}{2}\rho \mathbf{v}^2 = \frac{1}{2}\rho(u^2 + v^2 + w^2)$$

Work done by wall(at $y = h$ with length: L) from shear stress(σ_{xy}) on fluid:

$$W = U(h)L\sigma_{xy}(h)$$

Dissipation

$$\Delta = 2\mu[\frac{1}{2}(\nabla \mathbf{v} + \nabla \mathbf{v}^T)]^2$$

Heat transfer equation

$$\rho c \left(\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla)T \right) = k\nabla^2 T + h + \Delta$$