

MEK3220 - Formulae sheet

Krister Stræte Karlsen

November 22, 2015

THE DISPLACEMENT GRADIENT

$$\nabla \mathbf{u} = \begin{pmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{pmatrix}$$

Comment: If $\nabla \mathbf{u} = (\nabla \mathbf{u})^T$ we have that $\nabla \mathbf{u} = \epsilon$, the *strain tensor*.

STRAIN TENSOR

$$\epsilon = \frac{1}{2}((\nabla \mathbf{u})^T + \nabla \mathbf{u})$$

CAUCHY'S EQUILIBRIUM EQUATION

$$\mathbf{f}_v + \nabla \cdot \sigma^T = 0$$

HOOKE'S LAW

$$\sigma = \lambda I \sum_k \epsilon_{kk} + 2\mu \epsilon$$

LAMÈ COEFFICIENTS

$$E = \mu \frac{3\lambda + 2\mu}{\lambda + \mu}, \quad \nu = \frac{\lambda}{2(\lambda + \mu)}$$

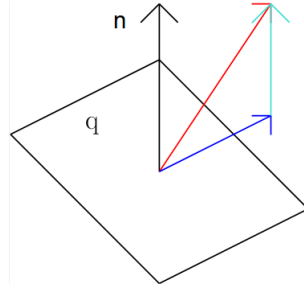
$$\lambda = \frac{E\nu}{(1 - 2\nu)(1 + \nu)}, \quad \mu = \frac{E}{2(1 + \nu)}$$

INVERTED HOOKE'S LAW

$$\epsilon = \frac{1+\nu}{E}\sigma - \frac{\nu}{E}I\sum_k \sigma_{kk}$$

STRESS RELATIONS

Let the stress be given by σ , a plane denoted q and its normal be \mathbf{n} . Then we have the following relations:



Stress on surface (red)

$$\vec{\sigma}_q = \sigma \cdot \mathbf{n}$$

Normal stress on surface (light blue)

$$\sigma_{q(n)} = \vec{\sigma}_q \cdot \mathbf{n}$$

Shear stress on surface (blue)

$$\vec{\sigma}_{q(s)} = \vec{\sigma}_q - \sigma_{q(n)}\mathbf{n}$$

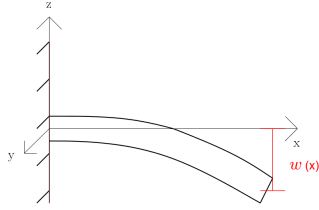
NAVIER'S EQUATION FOR ELASTIC MEDIA IN MOTION

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} = \mu \nabla^2 \mathbf{u} + (\lambda + \mu) \nabla (\nabla \cdot \mathbf{u}) + \mathbf{f}_v$$

Comment: For mechanical equilibrium set LHS. equal to zero.

BEAM EQUATIONS

Euler-Bernoulli and some additional important relations



$$\frac{d^2 w(x)}{dx^2} = -\frac{M_y(x)}{EI}, \quad \frac{dM_y(x)}{dx} = F_z, \quad \frac{dF_z}{dx} = -F_{ext}^{(z)}$$

EQUATION OF CONTINUITY

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0$$

NAVIER-STOKES

$$\frac{d\mathbf{v}}{dt} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \mathbf{g} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v}, \quad \nabla \cdot \mathbf{v} = 0$$

ISOTROPIC VISCOUS STRESS

$$\sigma = \text{pressure} + \text{shear stress} = (-p)I + \mu(\nabla \mathbf{v} + \nabla \mathbf{v}^T)$$

ENERGY EQUATIONS

Kinetic energy(per volume) for fluid:

$$E_k = \frac{1}{2} \rho \mathbf{v}^2 = \frac{1}{2} \rho (u^2 + v^2 + w^2)$$

Work done by wall(at $y = h$ with length: L) from shear stress(σ_{xy}) on fluid:

$$W = U(h)L\sigma_{xy}(h)$$

Dissipation

$$\Delta = 2\mu \left[\frac{1}{2} (\nabla \mathbf{v} + \nabla \mathbf{v}^T) \right]^2$$

Heat transfer equation

$$\rho c \left(\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla) T \right) = k \nabla^2 T + h + \Delta$$