

# MEK3220 - Formulae sheet

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## THE DISPLACEMENT GRADIENT

$$\nabla \mathbf{u} = \begin{pmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{pmatrix}$$

Comment: If  $\nabla \mathbf{u} = (\nabla \mathbf{u})^T$  we have that  $\nabla \mathbf{u} = \epsilon$ , the *strain tensor*.

## STRAIN TENSOR

$$\epsilon = \frac{1}{2}((\nabla \mathbf{u})^T + \nabla \mathbf{u})$$

## CAUCHY'S EQUILIBRIUM EQUATION

$$\mathbf{f}_v + \nabla \cdot \sigma^T = 0$$

## HOOKE'S LAW

$$\sigma = \lambda(\nabla \cdot \mathbf{u})I + \mu\epsilon$$

or

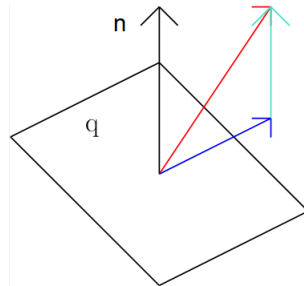
$$\begin{aligned} \sigma_{xx} &= (2\mu + \lambda)\epsilon_{xx} + \lambda(\epsilon_{yy} + \epsilon_{zz}), & \sigma_{yz} &= \sigma_{zy} = 2\mu\epsilon_{yz} \\ \sigma_{yy} &= (2\mu + \lambda)\epsilon_{yy} + \lambda(\epsilon_{xx} + \epsilon_{zz}), & \sigma_{xz} &= \sigma_{zx} = 2\mu\epsilon_{xz} \\ \sigma_{zz} &= (2\mu + \lambda)\epsilon_{zz} + \lambda(\epsilon_{yy} + \epsilon_{xx}), & \sigma_{xy} &= \sigma_{yx} = 2\mu\epsilon_{xy} \end{aligned}$$

## INVERTED HOOKE'S LAW

$$\epsilon = \frac{1+\nu}{E}\sigma - \frac{\nu}{E}I\sum_k \sigma_{kk}$$

## STRESS RELATIONS

Let the stress be given by  $\sigma$ , a plane denoted  $q$  and its normal be  $\mathbf{n}$ . Then we have the following relations:



Stress on surface (red)

$$\vec{\sigma}_q = \sigma \cdot \mathbf{n}$$

Normal stress on surface (light blue)

$$\sigma_{q(n)} = \vec{\sigma}_q \cdot \mathbf{n}$$

Shear stress on surface (blue)

$$\vec{\sigma}_{q(s)} = \vec{\sigma}_q - \sigma_{q(n)} \mathbf{n}$$

## NAVIER'S EQUATION FOR ELASTIC MEDIA IN MOTION

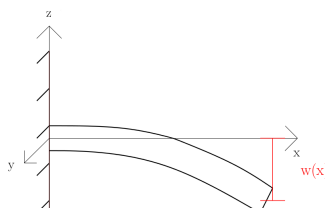
$$\frac{\partial^2 \mathbf{u}}{\partial t^2} = \mu \nabla^2 \mathbf{u} + (\lambda + \mu) \nabla \nabla \cdot \mathbf{u} + \mathbf{f}_v$$

Comment: For mechanical equilibrium set LHS. equal to zero.

## LAMÈ COEFFICIENTS

## BEAM EQUATIONS

*Euler-Bernoulli* and some additional important relations



$$\frac{d^2w(x)}{dx^2} = -\frac{M_y(x)}{EI}, \quad \frac{dM_y(x)}{dx} = F_z, \quad \frac{dF_z}{dx} = -F_{ext}^{(z)}$$

#### EQUATION OF CONTINUITY

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0$$

#### NAVIER-STOKES

$$\frac{d\mathbf{v}}{dt} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \mathbf{g} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v}, \quad \nabla \cdot \mathbf{v} = 0$$

#### ISOTROPIC VISCOUS STRESS

$$\sigma = \text{pressure} + \text{shear stress} = (-p)I + \mu(\nabla \mathbf{v} + \nabla \mathbf{v}^T)$$

#### ENERGY EQUATIONS

Kinetic energy(per volume) for fluid:

$$E_k = \frac{1}{2}\rho \mathbf{v}^2 = \frac{1}{2}\rho(u^2 + v^2 + w^2)$$

Work done by wall(at  $y = h$  with length:  $L$ ) from shear stress( $\sigma_{xy}$ ) on fluid:

$$W = U(h)L\sigma_{xy}(h)$$

Dissipation

$$\Delta = 2\mu[\frac{1}{2}(\nabla \mathbf{v} + \nabla \mathbf{v}^T)]^2$$

Heat transfer equation

$$\rho c \left( \frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla)T \right) = k \nabla^2 T + h + \Delta$$