MEK3220 - Formulae sheet

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The displacement gradient

$$\nabla \mathbf{u} = \begin{pmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{pmatrix}$$

Comment: If $\nabla \mathbf{u} = (\nabla \mathbf{u})^T$ we have that $\nabla \mathbf{u} = \epsilon$, the *strain tensor*.

Strain tensor

$$\epsilon = \frac{1}{2}((\nabla \mathbf{u})^T + \nabla \mathbf{u})$$

Cauchy's equilibrium equation

$$\mathbf{f_v} + \nabla \cdot \sigma^T = 0$$

Hooke's law

$$\{\sigma\} = \lambda(\nabla \cdot \mathbf{u})I + \mu\epsilon$$

or

$$\begin{aligned} \sigma_{xx} &= (2\mu + \lambda)\epsilon_{xx} + \lambda(\epsilon_{yy} + \epsilon_{zz}), \quad \sigma_{yz} = \sigma_{zy} = 2\mu\epsilon_{yz} \\ \sigma_{yy} &= (2\mu + \lambda)\epsilon_{yy} + \lambda(\epsilon_{xx} + \epsilon_{zz}), \quad \sigma_{xz} = \sigma_{zx} = 2\mu\epsilon_{xz} \\ \sigma_{zz} &= (2\mu + \lambda)\epsilon_{zz} + \lambda(\epsilon_{yy} + \epsilon_{zz}), \quad \sigma_{xy} = \sigma_{yx} = 2\mu\epsilon_{xy} \end{aligned}$$

Inverted Hooke's law

Navier's equation for elastic media in motion

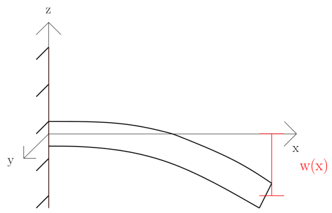
$$\frac{\partial^2 \mathbf{u}}{\partial t^2} = \mu \nabla^2 \mathbf{u} + (\lambda + \mu)\nabla\nabla \cdot \mathbf{u} + \mathbf{f_v}$$

Comment: For mechanical equilibrium set LHS. equal to zero.

Lamè coefficients

Beam equations

Euler-Bernoulli and some additional important relations



$$\frac{d^2 w(x)}{dx^2} = -\frac{M_y(x)}{EI}, \quad \frac{dM_y(x)}{dx} = F_z, \quad \frac{dF_z}{dx} = -F_{ext}^{(z)}$$

Equation of continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0$$

Navier-Stokes

$$\frac{d\mathbf{v}}{dt} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \mathbf{g} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v}, \quad \nabla \cdot \mathbf{v} = 0$$

Viscous shear stress

Energy equations

Kinetic energy(per volume) for fluid:

$$E_k = \frac{1}{2}\rho \mathbf{v}^2 = \frac{1}{2}\rho(u^2 + v^2 + w^2)$$

Work done by wall(at $y = h$ with length: L) from shear stress(σ_{xy}) on fluid:

$$W = U(h)L\sigma_{xy}(h)$$

[add figure]