

MEK3220 - Formulae sheet

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THE DISPLACEMENT GRADIENT

$$\nabla \mathbf{u} = \begin{pmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{pmatrix}$$

For $\nabla \mathbf{u} = (\nabla \mathbf{u})^T$ we have that $\nabla \mathbf{u} = \epsilon$, the *strain tensor*.

STRAIN TENSOR

$$\epsilon = \frac{1}{2}(\nabla \mathbf{u} + \nabla \mathbf{u}^T)$$

HOOKE'S LAW

$$\sigma = \lambda \text{tr}(\epsilon)I + 2\mu\epsilon$$

INVERTED HOOKE'S LAW

$$\epsilon = \frac{1+\nu}{E}\sigma - \frac{\nu}{E}\text{tr}(\sigma)I$$

CAUCHY'S EQUILIBRIUM EQUATION

$$\mathbf{f}_v + \nabla \cdot \sigma = 0$$

EQUATION OF CONTINUITY

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0$$

ISOTROPIC VISCOUS STRESS

$$\sigma = \text{pressure} + \text{shear stress} = (-p)I + \mu(\nabla \mathbf{v} + \nabla \mathbf{v}^T)$$

NAVIER-STOKES

$$\rho \frac{d\mathbf{v}}{dt} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \mathbf{g} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v}, \quad \nabla \cdot \mathbf{v} = 0$$

NAVIER'S EQUATION FOR ELASTIC MEDIA IN MOTION

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \mu \nabla^2 \mathbf{u} + (\lambda + \mu) \nabla(\nabla \cdot \mathbf{u}) + \mathbf{f}_v$$

LAMÈ COEFFICIENTS

$$E = \mu \frac{3\lambda + 2\mu}{\lambda + \mu}, \quad \nu = \frac{\lambda}{2(\lambda + \mu)}$$

$$\lambda = \frac{E\nu}{(1-2\nu)(1+\nu)}, \quad \mu = \frac{E}{2(1+\nu)}$$

ENERGY EQUATIONS

Kinetic energy(per volume) for fluid:

$$E_k = \frac{1}{2}\rho\mathbf{v}^2 = \frac{1}{2}\rho(u^2 + v^2 + w^2)$$

Work done by wall(at $y = h$ with length: L) from shear stress(σ_{xy}) on fluid:

$$W = U(h)L\sigma_{xy}(h)$$

Dissipation

$$\Delta = 2\mu\left[\frac{1}{2}(\nabla\mathbf{v} + \nabla\mathbf{v}^T)\right]^2$$

Heat transfer equation

$$\rho c \left(\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla)T \right) = k\nabla^2 T + h + \Delta$$

STRESS RELATIONS

Stress on surface (red)

$$\mathbf{t} = \sigma \cdot \mathbf{n}$$

Normal stress on surface (light blue)

$$\sigma_n = \mathbf{t} \cdot \mathbf{n}$$

$$\mathbf{t}_n = \sigma_n \mathbf{n}$$

Shear stress on surface (blue)

$$\mathbf{t}_t = \mathbf{t} - \mathbf{t}_n$$

