MEK3220 - Formulae sheet

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The displacement gradient

$$\nabla \mathbf{u} = \begin{pmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{pmatrix}$$

For $\nabla \mathbf{u} = (\nabla \mathbf{u})^T$ we have that $\nabla \mathbf{u} = \epsilon$, the *strain tensor*.

Strain tensor

$$\epsilon = \frac{1}{2}(\nabla \mathbf{u} + \nabla \mathbf{u}^T)$$

Hooke's law

$$\sigma = \lambda tr(\epsilon) I + 2\mu\epsilon = \lambda(\nabla \cdot \mathbf{u}) I + 2\mu\epsilon$$

Inverted Hooke's law

$$\epsilon = \frac{1+\nu}{E}\sigma - \frac{\nu}{E} tr(\sigma) I$$

Cauchy's equilibrium equation

$$\mathbf{f_v} + \nabla \cdot \sigma = 0$$

Equation of continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0$$

Isotropic viscous stress

$$\sigma = pressure + shear\ stress(\tau) = (-p)I + \mu(\nabla \mathbf{v} + \nabla \mathbf{v}^T)$$

Navier-Stokes

$$\frac{d\mathbf{v}}{dt} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \mathbf{g} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v}, \quad \nabla \cdot \mathbf{v} = 0$$

Navier's equation for elastic media in motion

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \mu \nabla^2 \mathbf{u} + (\lambda + \mu)\nabla(\nabla \cdot \mathbf{u}) + \mathbf{f_v}$$

Lamè coefficients

$$E = \mu \frac{3\lambda + 2\mu}{\lambda + \mu}, \quad \nu = \frac{\lambda}{2(\lambda + \mu)}$$

$$\lambda = \frac{E\nu}{(1-2\nu)(1+\nu)}, \quad \mu = \frac{E}{2(1+\nu)}$$

Energy equations

Heat transfer equation

$$\rho c \left(\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla) T \right) = k \nabla^2 T + \Delta$$

Kinetic energy(per volume) for fluid:

$$E_k = \frac{1}{2} \rho \mathbf{v}^2 = \frac{1}{2} \rho (u^2 + v^2 + w^2)$$

Work done by wall(at $y = h$ with length: L) from shear stress(σ_{xy}) on fluid:

$$W = U(h) L \sigma_{xy}(h)$$

Dissipation

$$\Delta = 2\mu [\frac{1}{2} (\nabla \mathbf{v} + \nabla \mathbf{v}^T)]^2 = \tau : \nabla \mathbf{v}$$

Stress relations

Stress on surface (red)

$$\mathbf{t} = \sigma \cdot \mathbf{n}$$

Normal stress on surface (light blue)

$$\sigma_n = \mathbf{t} \cdot \mathbf{n}$$
$$\mathbf{t}_n = \sigma_n \mathbf{n}$$

Shear stress on surface (blue)

$$\mathbf{t}_t = \mathbf{t} - \mathbf{t}_n$$