INF5620 - EXERCISES

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7TH SEPTEMBER, 2015

Problem 1 - Vibrations and manufactured solutions

a)
Discretized:

$$\frac{u_{n+1} - 2u_n + u_{n-1}}{\Delta t^2} = \omega^2 u_n = f_n, \quad u_0 = I, \quad \frac{u_1 - u_{-1}}{2\Delta t} = V$$

Setting $n = 0$ and solving for u_1 gives:

$$u_1 = u_0 \Delta t + \frac{\Delta t^2}{2}(f_0 - \omega^2 u_0)$$

b)

$$u_e(0) = d = I, \quad \frac{du_e}{dt} = c = V$$

so

$$u_e = Vt + I.$$

Inserting u_e into the ODE gives the following source term:

$$f(t) = \omega^2(Vt + I).$$

Checking that $[D_t D_t t]^n = 0$:

$$[D_t D_t t]^n = \frac{t_{n+1} - 2t_n + t_{n-1}}{\Delta t^2} = \frac{n\Delta t + \Delta t - 2n\Delta t + n\Delta t - \Delta t}{\Delta t^2} = 0$$

Lets put u_e into the disceretized equation to see that everything is okay:

$$\begin{aligned} &[D_t D_t u_e]^n + \omega^2 u_n = f_n \\ &0 + \omega^2(Vn\Delta t + I) = \omega^2(Vn\Delta t + I) \quad OK! \end{aligned}$$

d)
By running the program we discover that a cubic polynomial gives a residual for the first step.

Exercise 21 - Simulating an elastic pendulum

c)
Looking at vertical motion only one ODE is of interest and it takes the simpler form:

$$y'' = -\frac{\beta}{1-\beta}\left(1 - \frac{\beta}{\sqrt{a-\beta}}\right) - \beta = -\frac{\beta}{1-\beta}(y - 1 + \beta) - \beta = -\frac{\beta}{1-\beta}y + C$$

Since the accelration is zero in equilimbrium, $y''(t) = 0$, so the constant C must be zero. We have now obtained a equation that can be written as a vibration equation:

$$y'' = -\frac{\beta}{1-\beta}y = -\omega^2 y, \quad \omega = \sqrt{\frac{\beta}{1-\beta}}$$

The exact solution to this ODE is: $y = Icos(\omega t)$.

d)
Running the simulation below

```
simulate_pendulum(0.99, 9, 0, 3, 600, plot=True)
```

gives the following output:

```
Equilimbrium-test passed
Vertical-test passed
```

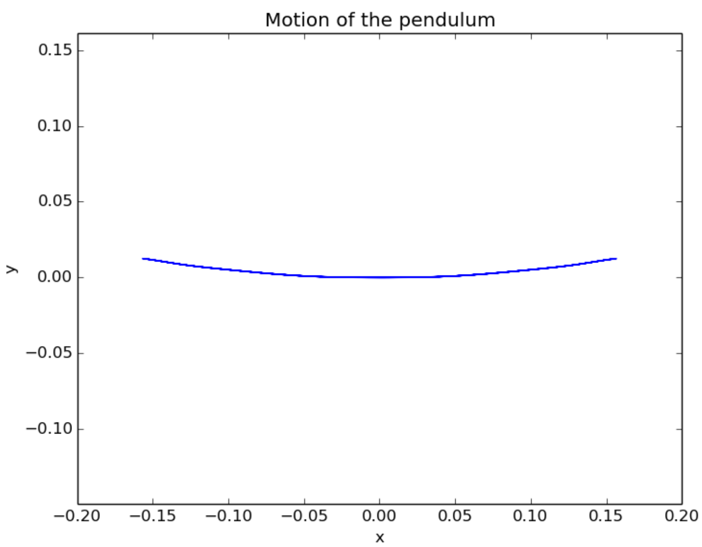

Figure 1. The real physical picture of the pendulum motion.

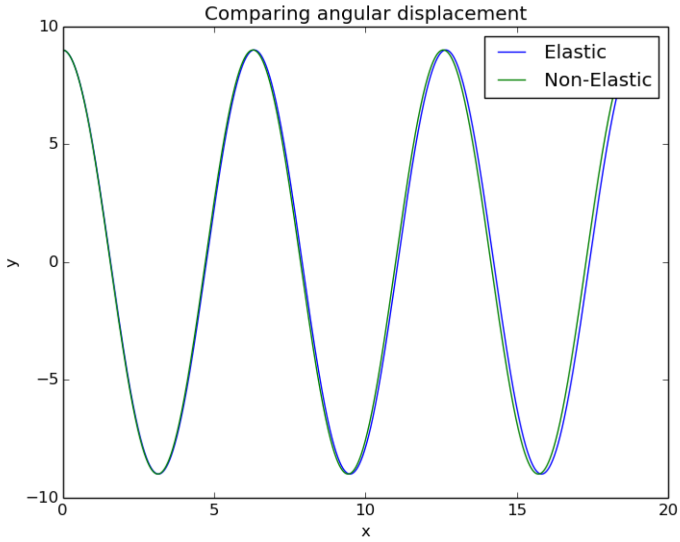

FIGURE 2. From comparing angular displacement one can observe that the difference grows with time.