

# Simulating blood flow in Zebrafish using Stokes' equations

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## Generating mesh from original image

From the original image a smaller part is extracted(see the green box in Figure 1) and then a mesh is created using *The Vascular Modeling Toolkit*(VMTK).

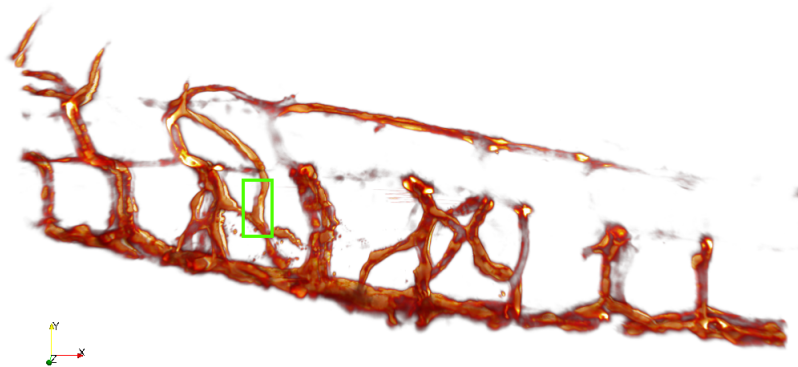


Figure 1: Original image of veins in Zebrafish.

To obtain a finite element mesh from the original image the following scripts/commands are executed:

- 1) Select a volume of interest(VOI)

```
vmtkimagevoiselector -ifile original.vti -ofile voi.vti
```

- 2) Segmentation

```
vmtklevelsetsegmentation -ifile voi.vti -ofile levelsets.vti
```

- 3) Create surface file

```
vtkmarchingcubes -ifile levelsets.vti -ofile surf.vtp
```

4) Smoothing of surface

```
vmtnksurfacesmoothing -ifile surf.vtp -passband 0.1 -iterations 30 -ofile  
sm_surf.vtp
```

5) Clip surface

```
vmtnksurfaceclipper -ifile sm_surf.vtp -ofile cl_surface.vtp
```

6) Generate mesh

```
vmtnkmeshgenerator -ifile cl_surface.vtp -ofile zebamsh.vtu -edgelenh  
1.0
```

7) Convert to dolfin-format

```
vmtnkmeshwriter -ifile zebamsh.vtu -entityidsarray CellEntityIds -ofile  
zebra_mesh.xml
```

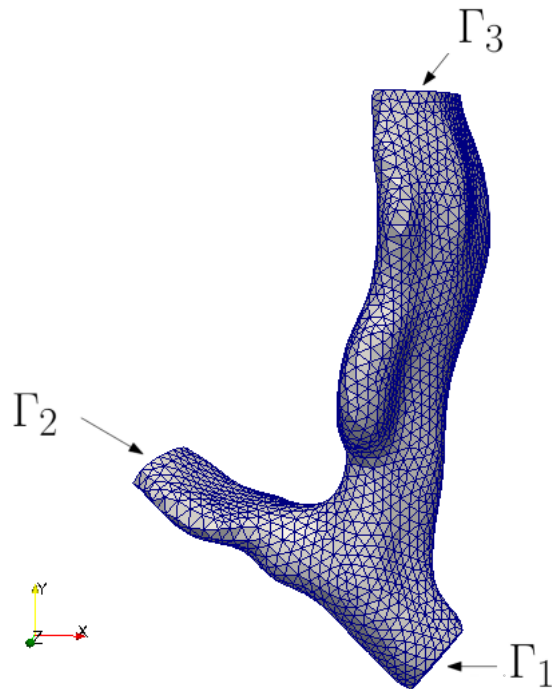


Figure 2: FEM mesh with marking of inlets/outlets.

## Mathematical problem formulation

Zebrafish veins are tiny and the blood velocities very low, thus using Stokes flow as an approximation is reasonable.

The governing equations are

$$\left. \begin{aligned} -\Delta u + \nabla p &= f \\ \nabla \cdot u &= 0 \end{aligned} \right\} \quad in \quad \Omega$$

with boundary conditions

$$\begin{aligned} p &= p_1 \quad on \quad \Gamma_1 \\ p &= p_2 \quad on \quad \Gamma_2 \\ p &= p_3 \quad on \quad \Gamma_3 \\ u &= 0 \quad on \quad \partial\Omega \setminus \{\Gamma_1, \Gamma_2, \Gamma_3\}. \end{aligned}$$

To obtain the results featured in Figure 3

$$p_1 > p_2 = p_3 = 0.$$

## Stabilized variational form of Stokes' equation

To get away with less degrees of freedom we use a stabilized  $P_1 - P_1$  formulation of Stokes' equations, i.e:

Find  $u, p \in W$ ,  $W = V \times Q$  such that

$$a((u, p), (v, q)) = L((v, q)) \quad \forall \quad v, q \in W$$

where

$$\begin{aligned} a((u, p), (v, q)) &= \int_{\Omega} \nabla u \cdot \nabla v - \nabla \cdot v \, p + \nabla \cdot u \, q + \epsilon \nabla q \cdot \nabla p \, dx, \\ L((v, q)) &= \int_{\Omega} f \cdot v \, dx + \epsilon \nabla q \cdot f \, ds. \end{aligned}$$

where  $\epsilon = \beta h^2$  and  $\beta$  is some number and  $h$  is the mesh cell size.

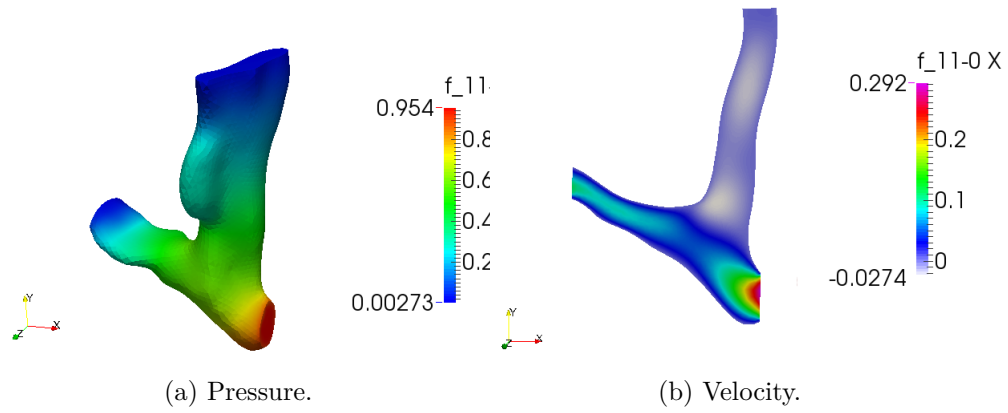


Figure 3: Velocity field of Stokes flow past a step computed using FEniCS.

## Results

Implementing and solving the problem above using *FEniCS* we get the results in Figure 3. For code see `p1p1_stokes.py` available at

<https://github.com/krikarls/SummerProjectMEK4250>