Hometask #5. Methods of proof

1. Prove, that the following statement is false:

   There exist an integer x>1 such that $\frac{x^8 + x^4 - 2x^2 + 6}{x^4 + 2x^2 + 3} + 2x^2 - 2$ is prime

2. Find mistake in proof:
   Theorem: The difference between any odd integer and any even integer is odd.
   Proof: Suppose n is any odd integer, and m is any even integer. By definition of odd, $n = 2k + 1$ 1 where k is an integer, and by definition of even, $m = 2k$ where k is an integer. Then $n - m = (2k + 1) - 2k = 1$. However, 1 is odd. Therefore, the difference between any odd integer and any even integer is odd.

3. Prove: For all integer n, $0.5 + 8\left(n^2\left(2n^2 + 3\right) + 1\right) - \frac{\cos 2n}{2} + \frac{1}{1 + \tan^2 n}$ is a perfect square.

4. Assume that *m* and *n* are both integers and that *n ≠ 0*. Explain why $(10m + 15n)/(4\mathrm{n})$ must be a rational number.

5. Prove that if one solution for a quadratic equation of the form $x^2 + bx + c = 0$ is rational (where b and c are rational), then the other solution is also rational.

6. Prove that if a real number ***x=c*** satisfies a polynomial equation of the form
   $r_3x^3 + r_2x^2 + r_1x + r_0 = 0$
   where r_0, r_1, r_2, and r_3 are rational numbers, then ***x=c*** satisfies an equation of the form
   $n_3x^3 + n_2x^2 + n_1x + n_0 = 0$
   where n_0, n_1, n_2, and n_3 are integers.

7. Suppose *a, b,* and *c* are integers and *x, y,* and *z* are nonzero real numbers that satisfy the following equations:
   $\frac{xy}{x+y} = a$, $\frac{xz}{x+z} = b$ and $\frac{yz}{y+z} = c$
   Is *x* rational? If so, express it as a ratio of two integers.

8. Prove that alphabet based on Hamming distance 3 is enough for code, fixing 1-error.

9. Prove, that every “`while`” loop can be transformed to “`for (;;) {}`” loop.