1. Prove, that the following statement is false:

   There exist an integer x>1 such that $\frac{x^8+x^4-2x^2+6}{x^4+2x^2+3}+2x^2-2$ is prime

   Solution:

   $$\frac{x^8+x^4-2x^2+6}{x^4+2x^2+3}+2x^2-2=\frac{\left(x^4-2x^2+2\right)\left(x^4+2x^2+3\right)}{x^4+2x^2+3}+2x^2-2=x^4-2x^2+2+2x^2-2=x^4$$

   $x^4 = x * x^3 = x^2*x^2$; $x, x^2, x^3 > 1$ and natural. QED

2. Find mistake in proof:
   Theorem: The difference between any odd integer and any even integer is odd.
   Proof: Suppose n is any odd integer, and m is any even integer. By definition of odd, $n = 2k+1$ 1 where k is an integer, and by definition of even, $m = 2k$ where k is an integer. Then $n - m = (2k + 1) - 2k = 1$. However, 1 is odd. Therefore, the difference between any odd integer and any even integer is odd.

   Solution
   The mistake in the "proof" is that the same symbol, k, is used to represent two different quantities. By setting m = 2k and n = 2k + 1, the proof implies that n = m + 1, and thus it deduces the conclusion only for this one situation. When m = 4 and n = 17, for instance, the computations in the proof indicate that n − m = 1, but actually n − m = 13. In other words, the proof does not deduce the conclusion for an arbitrarily chosen even integer m and odd integer n, and hence it is invalid.

3. Prove: For all integer n, $0.5+8\left(n^2\left(2n^2+3\right)+1\right)-\frac{\cos 2n}{2}+\frac{1}{1+\tan^2 n}$ is a perfect square

   Solution

   Giant statement can be composed to $\left(4n^2+3\right)^2$, which is a perfect square, because $4n^2+3$ is an integer.

4. Assume that a and b are both integers and that $b \neq 0$. Explain why $(10m+15n)/(4\mathrm{n})$ must be a rational number.

   Solution
   Because m and n are integers, $10m+15n$ and $4n$ are both integers (since differences and products of integers are integers). Also, by the zero product property, $4n^2 \neq 0$ because n is not a zero. Hence $(10m+15n)/(4\mathrm{n})$ Is a quotient of two integers with nonzero denominator, and so it is rational.

5. Prove that if one solution for a quadratic equation of the form $x^2 + bx + c = 0$ is rational (where b and c are rational), then the other solution is also rational.

   Solution
   Using the fact that if the solutions of the equation are r and s, then
   $x^2 + bx + c = (x - r)(x - s)$. After opening the brackets, we get $x^2-(s+r)x+sr$

So, $b=s+r$ and $c=sr$ rational, hence, if one of s and r is rational, than the other one also has to be rational, to get rational b and c

6. Prove that if a real number c satisfies a polynomial equation of the form
$r_3x^3+r_2x^2+r_1x+r_0=0$
where r0, r1, r2, and r3 are rational numbers, then c satisfies an equation of the form
$n_3x^3+n_2x^2+n_1x+n_0=0$
where n0, n1, n2, and n3 are integers

Solution
Suppose c is a real number such that $r_3x^3+r_2x^2+r_1x+r_0=0$, where r0,r1,r2, and r3 are rational numbers. By definition of rational, $r_0=\frac{a_0}{b_0}$ $r_1=\frac{a_1}{b_1}$, $r_2=\frac{a_2}{b_2}$, and $r_3=\frac{a_3}{b_3}$ for some integers, a0, a1, a2, a3, and nonzero integers b0, b1, b2, and b3. By substitution, $r_3c^3+r_2c^2+r_1c+r_0=\frac{a_3}{b_3}c^3+\frac{a_2}{b_2}c^2+\frac{a_1}{b_1}c+\frac{a_0}{b_0}=$
$\frac{b_0b_1b_2a_3}{b_0b_1b_2b_3}c^3+\frac{b_0b_1b_3a_2}{b_0b_1b_2b_3}c^2+\frac{b_0b_2b_3a_1}{b_0b_1b_2b_3}c+\frac{b_1b_2b_3a_0}{b_0b_1b_2b_3}=0$
Multiplying both sides by $b_0b_1b_2b_3$ gives $b_0b_1b_2a_3c^3+b_0b_1b_3a_2c^2+b_0b_2b_3a_1c+b_1b_2b_3a_0=0$. Let $n_3=b_0b_1b_2a_3$, $n_2=b_0b_1b_3a_2$, $n_1=b_0b_2b_3a_1$, and $n_0=b_1b_2b_3a_0$. Then n0, n1, n2, and n3 are all integers (being products of integers). Hence c satisfies the equation $n_3x^3+n_2x^2+n_1x+n_0=0$. where n0, n1, n2, and n3 are all integers. This is what was to be shown.

7. Suppose a, b, and c are integers and x, y, and z are nonzero real numbers that satisfy the following equations:
$\frac{xy}{x+y}=a$, $\frac{xz}{x+z}=b$ and $\frac{yz}{y+z}=c$
Is x rational? If so, express it as a ratio of two integers.

Solution
Let's rewrite equations in such form:
(1) $\frac{1}{x}+\frac{1}{y}=\frac{1}{a}$, (2) $\frac{1}{x}+\frac{1}{z}=\frac{1}{b}$ and (3) $\frac{1}{y}+\frac{1}{z}=\frac{1}{c}$

(1) – (3): (1*) $\frac{1}{x}-\frac{1}{z}=\frac{1}{a}-\frac{1}{c}$

(1*) + (2): (2*) $\frac{2}{x}=\frac{1}{b}+\frac{1}{a}-\frac{1}{c}$

Solving for x: $x=\frac{2abc}{ac+bc-ab}$

Since a,b and c are integers, then x is also integer, with condition $ac+bc\neq ab$

8. Prove that alphabet based on Hamming distance 3 is enough for code, fixing 1-error.

Solution

Let this statement is not true.

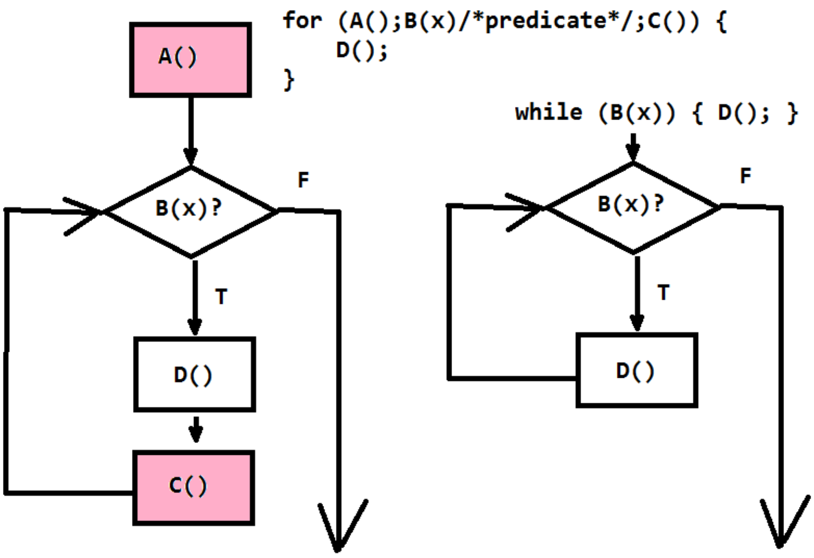
- $x_1 : d(x_1, err) == 1$, and moreover,
- there exist $x_2 \neq x_1 :$ $d(x_2, err) == 1$

According to triangle inequality

- 3 <= $d(x_1, x_2) \leq d(x_1, err) + d(x_2, err) = 2$.
- 3 <= 2. Contradiction

9. Prove, that every “while” loop can be transformed to “for (;;) {}” loop.

Solution by construction:



Consider activity diagrams of both loops. If take empty A() and C() we will get equal control flow graphs. This is equivalent to

```
for (; B(x); ) {
       D();
}
```