Discrete Math
Homework 3

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1 Question

Write negations for each of the following statements:

a) John is 6 feet tall and he weights at least 200 pounds.

Negotiation:

This is not the case that John is 6 feet tall and he weights at least 200 pounds.

b) The bus was late or Toms watch was slow.

Negotiation:

This is not the case that the bus was late or Toms watch was slow.

2 Question

Construct truth tables for the statement forms:

a) $(\neg p \lor q) \to \neg q$

p	q	$\neg p$	$\neg p \lor q$	$\neg q$	$(\neg p \lor q) \to \neg q$
0	0	1	1	1	1
0	1	1	1	0	1
1	0	0	0	1	0
1	1	0	1	0	1

b) $(p \land \neg q) \to r$

p	q	r	$\neg q$	$p \land \neg q$	$(p \land \neg q) \to r$
0	0	0	1	0	1
0	0	1	1	0	0
0	1	0	0	0	1
0	1	1	0	0	0
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	0	0	1
1	1	1	0	0	0

3 Question

Let **p**, **q**, and **r** be the propositions:
p: Grizzly bears have been seen in the area.
q: Hiking is safe on the trail.
r: Berries are ripe along the trail.

Write these propositions using **p**, **q**, and **r** and logical connectives (including negations).

a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.

$r \wedge \neg p$

b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.

$\neg p \wedge q \wedge r$

c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

$r \rightarrow (q \leftrightarrow \neg p)$

4 Question

Find the bitwise OR, bitwise AND, and bitwise XOR of each of these pairs of bit strings.

a) 101 1110, 010 0001

a	1	0	1	1	1	1	0
b	0	1	0	0	0	0	1
|	1	1	1	1	1	1	1
&	0	0	0	0	0	0	0
$\wedge$	1	1	1	1	1	1	1

b) 1111 0000, 1010 1010

a	1	1	1	1	0	0	0	0
b	1	0	1	0	1	0	1	0
|	1	1	1	1	1	0	1	0
&	1	0	1	0	0	0	0	0
$\wedge$	0	1	0	1	1	0	1	0

5 Question

Given any statement form, is it possible to find a logically equivalent form that uses only $\neg$ and $\wedge$?

I think the answer is YES. Because disjuction we can represent as conjuction. With this three function we also can represent all another, for any statement form.

$$p \vee q \equiv \neg(\neg p \wedge \neg q)$$

6 Question

Prove the following:

a) Disjunctions is Commutative?

$(p \lor q)$, is equivalent to $(q \lor p)$?

p	q	$p \lor q$	$q \lor p$
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	1

This statements are logically equivalent, because their truth tables are same.

b) Disjunction is Associative?

$(p \lor q) \lor r$, is equivalent to $p \lor (q \lor r)$?

p	q	r	$p \lor q$	$q \lor r$	$(p \lor q) \lor r$	$p \lor (q \lor r)$
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	0	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

This statements are logically equivalent, because their truth tables are same.

7 Question

Prove that the statements given bellow are logically equivalent (or not). Justify your answers.

a) $\neg(p \land q)$ and $\neg p \land \neg q$?

p	q	$\neg(p \land q)$	$\neg p \land \neg q$
0	0	1	1
0	1	1	0
1	0	1	0
1	1	0	0

This statements are not logically equivalent, because their truth tables are different.

b) $\neg(p \wedge q)$ and $\neg p \vee \neg q$?

p	q	$\neg(p \wedge q)$	$\neg p \vee \neg q$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

This statements are logically equivalent, because their truth tables are same.

8 Question

Prove the statements:

a) Is implication logically equivalent to its contrapositive?
$(p \to q)$ and $(\neg p \to \neg q)$?

p	q	$(p \to q)$	$(\neg p \to \neg q)$
0	0	1	1
0	1	0	1
1	0	1	0
1	1	1	1

This statements are not logically equivalent, because their truth tables are different.

b) Is converse logically equivalent to the inverse?
$(q \to p)$ and $(\neg p \to \neg q)$?

p	q	$(q \to p)$	$(\neg p \to \neg q)$
0	0	1	1
0	1	1	1
1	0	0	0
1	1	1	1

This statements are logically equivalent, because their truth tables are same.

9 Question

Symbolically prove the following with explanation on each step on used law.

1. $(p \leftrightarrow q)$ equivalent to $(p \to q) \to (q \to p)$?

   Consider right side of statement:

   $(p \to q) \to (q \to p) \equiv (\neg p \vee q) \to (\neg q \vee p)$

   $(\neg p \vee q) \to (\neg q \vee p) \equiv \neg(\neg p \vee q) \vee (\neg q \vee p)$

   $\neg(\neg p \vee q) \vee (\neg q \vee p) \equiv (p \wedge \neg q) \vee (\neg q \vee p)$

2. $(p \wedge q)$ equivalent to $(p \vee \neg q) \wedge q$?

$(p \wedge q) \equiv (p \vee \neg q) \wedge q$ Initial statement

$(p \wedge q) \equiv q \wedge (p \vee \neg q)$ Commutative law

$(p \wedge q) \equiv (q \wedge p) \vee (q \wedge \neg q)$ Distributive law

$(p \wedge q) \equiv (q \wedge p) \vee C$ Identity law

$(p \wedge q) \equiv (q \wedge p)$ As desired