Krikun G. BS3 # 2

Integers

Prove or disprove:

1) if $a*b = a$ then $b = 1$.

lets multiply both sides by a^{-1}

$a^{-1}*a*b = a^{-1}*a$, it follows from commutativity rules and..

$b = 1$ As desired

2) The difference of any two odd integers is even

odd integers we can represent as: $(2k \pm 1)$ and $(2l \pm 1)$, then

$(2k \pm 1)-(2l \pm 1) = m$, so

$m = 2(k-l) \pm 1 \mp 1$

this part can be: $\{0; 2; -2\}$,

then $m = 2(k-l)$, or

$m = 2(k-l \pm 1)$

Prime numbers

1) Is n^k-1 prime for any integers n and k?

Not, it isn't! for example

$n=3$, $k=2$ $\quad 3^2-1 = 8 \leftarrow$ composite

2) Is expression n^2-n+41 a prime number? Not, it isn't!

firstly, lets take n as common factor: $n\cdot\left(n-1+\frac{41}{n}\right)$

As we can see, this expression is multiplication of n and $\left(n-1+\frac{41}{n}\right)$, (or 41 and $\left(\frac{n^2}{41}-\frac{n}{41}+1\right)$), so when 41 divides n then its divide n^2-n+41, and part $\left(n-1+\frac{41}{n}\right)$ or $\left(\frac{n^2}{41}-\frac{n}{41}+1\right)$ is integer, so expression result is composite number, when 41 divides n.

Divisibility

1) Prove that sum of $2n+1$ consecutive numbers is divisible $2n+1$

$$sum = \sum_{i=1}^{2n+1}(k+i) \Rightarrow sum = (2n+1)k + \sum_{i=1}^{2n+1} i \leftarrow \text{arithmetic progression}$$

where $k \in \mathbb{Z}$

$sum = \frac{n(n+1)}{2}$ can prove if you want...

$$sum = (2n+1)\cdot k + \frac{(2n+1)(2n+2)}{2}$$

this can be divided by $(2n+1)$ $\Leftarrow$ $sum = (2n+1)\cdot(k+n+1)$

2) Find quotient and divisor of

a. n^3+2n-1 divided by n — $n\cdot\left(n^2+2-\frac{1}{n}\right)$ easy!

b. $12n^5+10n^4+2$ divided by $2n+1$

$$
\begin{array}{l|l}
12n^5+10n^4+2 & 2n+1 \\
\hline
12n^5+6n^4 & 6n^4+2n^3-n^2+\frac{n}{2}-\frac{1}{4} \\
\quad 4n^4+2 & \text{quotient} \\
\quad 4n^4+2n^3 & \\
\quad\quad -2n^3+2 & \\
\quad\quad -2n^3-n^2 & \\
\quad\quad\quad n^2+2 & \\
\quad\quad\quad n^2+\frac{n}{2} & \\
\quad\quad\quad\quad -\frac{n}{2}+2 & \\
\quad\quad\quad\quad -\frac{n}{2}-\frac{1}{4} &
\end{array}
$$

the remainder $\rightarrow 2\frac{1}{4}$

so $12n^5+10n^4+2 = (2n+1)\left(6n^4+2n^3-n^2+\frac{n}{2}-\frac{1}{4}\right)+2\frac{1}{4}$

Rational numbers and Real numbers

Write each rational number as a ratio of two integers:

$x = 0{,}46271(6271)\ldots$ let $y = x\cdot 10 = 4{,}(6271)\ldots$, then

$10^4y - y = 46271{,}(6271)\ldots - 4{,}(6271)\ldots$

$(10^4-1)y = 46267 \Rightarrow y = \frac{46267}{9999}$, then $x = \frac{y}{10}$

$x = \frac{46267}{99990}$

$x = 12{,}(112)\ldots$, then $10^3x - x = 12112{,}(112)\ldots - 12{,}(112)\ldots$

$(10^3-1)x = 12100$

$x = \frac{12100}{999}$

Prove or disprove:

1) if r is any rational number, then $3r^2-2r+4$ is rational

r - rational - so we can represent $r = \frac{m}{n}$ where $m \in \mathbb{Z}$, $n \in \mathbb{N}$, so

$3\left(\frac{m}{n}\right)^2 - 2\left(\frac{m}{n}\right) + 4 = \frac{3m^2-2m\cdot n+4n^2}{n^2}$, in this ratio the numerator

$3m^2 \in \mathbb{Z}$, $-2mn \in \mathbb{Z}$, $4n^2 \in \mathbb{N}$, so sum $\in \mathbb{Z}$

and the denominator $n^2 \in \mathbb{N}$ — is ~~the~~ rational, as desired

2) Product and sum of two rational numbers is rational

Product:

$\left(\frac{m}{n}\right)\cdot\left(\frac{k}{l}\right) = \frac{m\cdot k}{n\cdot l}$ as soon as $m, k \in \mathbb{Z}$, $n, l \in \mathbb{N}$ $\Rightarrow$ $m\cdot k \in \mathbb{Z}$, $n\cdot l \in \mathbb{N}$

$\left(r_1 = \frac{m}{n},\ r_2 = \frac{k}{l}\right)$ so $\frac{m\cdot k}{n\cdot l}$ - rational, as desired

Sum:

$\left(\frac{m}{n}\right)+\left(\frac{k}{l}\right) = \frac{ml+nk}{nl}$ as soon as $m, k \in \mathbb{Z}$, $n, l \in \mathbb{N}$ $\Rightarrow$ $ml, n\cdot k \in \mathbb{Z}$, $n\cdot l \in \mathbb{N}$

$\left(r_1 = \frac{m}{n}\right)\left(r_2 = \frac{k}{l}\right)$ so $\frac{ml+nk}{nl}$ $\left(\frac{m}{n}+\frac{k}{l}\right)$ - rational, as desired