Hometask #4. Predicate logic

1. Let P(x), Q(x), R(x), and S(x) be the statements “x is a baby,” “x is logical,” “x is able to manage a crocodile,” and “x is despised,” respectively. Suppose that the domain consists of all people. Express each of these statements using quantifiers; logical connectives; and P(x), Q(x), R(x), and S(x).
   a) Babies are illogical.
   b) Nobody is despised who can manage a crocodile.
   c) Illogical persons are despised.
   d) Babies cannot manage crocodiles.
   ∗e) Does (d) follow from (a), (b), and (c)? If not, is there a correct conclusion?

2. Let the domain of x be the set of geometric figures in the plane, and let Ell(x) be “x is an ellipse” and Circ(x) be “x is a circle.” Translate into English, and say whether statement is true of false.
   a. $\exists x$ such that Ell(x) ∧ Circ(x).
   b. $\exists x$ such that Circ (x)∧ ∼Ell(x).
   c. $\forall x$, Circ(x) →Ell(x).

3. Which of the following is a negation for “All innopolis university students are blonde”? More than one answer may be correct.
   a. There is an Innopolis university student who is a not blonde.
   b. All Innopolis university students are not a blonde.
   c. There is a not blonde person who is an Innopolis university student.
   d. No Innopolis university students are blonde.
   e. Some Innopolis university students are not blonde.
   f. No blonde people are Innopolis university students.

4. Let D = {−42, −14, −6, 0, 3, 5, 18, 25, 28, 32, 48}. Determine which of the following statements are true and which are false. Provide counterexamples for those statements that are false.
   a. ∀ x ∈ D, if x is odd then x > 0.
   b. ∀ x ∈ D, if x is less than 0 then x is even.
   c. ∀ x ∈ D, if x is even then x ≤ 0.
   d. ∀ x ∈ D, if the ones digit of x is 2, then the tens digit is 3 or 4.
   e. ∀ x ∈ D, if the ones digit of x is 6, then the tens digit is 1 or 2.

5. Write a negation: ∀ real numbers x , if $x^2 \geq 4$ then $x > 2$.

6. Let G(x, y) be “$\cos(x) - x^2 \sin x + tg(x) > y + 1 / y - \ln y^2$.” Indicate which of the following statements are true and which are false.
   a. G(1,1)
   b. G(1.25,10/4)
   c. G(20,20)
   d. G(0.5,0.3)

7. Let S be the set of students at Innopolis University, let M be the set of books that have ever been written, and let B(s,m) be “student *s* has read book *m*.” Rewrite each of the following statements without using the symbol ∀ , the symbol ∃ , or variables.
   a. ∃ s ∈ S such that B(s, LOTR).
   b. ∀ s ∈ S, B(s, 50 shades of Gray).
   c. ∀ s ∈ S, ∃ m ∈ M such that B(s,m).

d. ∃ m ∈ M such that ∀ s ∈ S, B(s,m).
e. ∃ s ∈ S, ∃ t ∈ S, and ∃ m ∈ M such that $s \neq t$ and B(s,m) ∧ B(t,m).
f. ∃ s ∈ S and ∃ t ∈ S such that $s \neq t$ and ∀ m ∈ M, B(s,m) → B(t,m).

8. Indicate which of the following statements are true and which are false. Justify your answers as best you can.
   a. ∀ x ∈ Z+, ∃y ∈ Z+ such that x = y + 1.
   b. ∀ x ∈ Z, ∃y ∈ Z such that x = y + 1.
   c. ∃ x ∈ R such that ∀ y ∈ R, x = y + 1.
   d. ∀ x ∈ R+, ∃y ∈ R+ such that xy = 1.
   e. ∀ x ∈ R, ∃y ∈ R such that xy = 1.
   f. ∀ x ∈ Z+ and ∀ y ∈ Z+, ∃z ∈ Z+ such that z = x − y.
   g. ∀ x ∈ Z and ∀ y ∈ Z, ∃z ∈ Z such that z = x − y.
   h. ∃ u ∈ R+ such that ∀ v ∈ R+, uv < v.

9. Indicate whether the argument is valid or invalid. Support your answer by drawing diagram.

   All discrete mathematics students can tell a valid argument from an invalid one.
   All thoughtful people can tell a valid argument from an invalid one.
   ∴ All discrete mathematics students are thoughtful.

10. What rules do you use to prove or disprove next arguments
    a. All cats purr.
       John does not purr.
       ∴ John is not a cat.

    b. All teachers handsome
       Arthur is a teacher
       ∴ Arthur handsome

    c. All Russians are happy
       Gérard Depardieu is Russian
       ∴ Gérard Depardieu is happy