Hometask #7. Induction 2

1. Prove that $5^n + 9 < 6^n$, for all integers $n \geq 2$

2. On the outside rim of a circular disk, the integers from 1 through 30 are painted in random order. Show that no matter what this order is, there must be three successive integers whose sum is at least 45.

3. Any product of two or more integers is a result of successive multiplications of two integers at a time. For instance, here are a few of the ways in which $a_1 a_2 a_3 a_4$ might be computed: $(a_1 a_2)(a_3 a_4)$ or $((a_1 a_2) a_3) a_4)$ or $a_1((a_2 a_3) a_4)$. Use strong mathematical induction to prove that any product of two or more odd integers is odd.

4. Determine whether the given recursively defined sequence satisfies the explicit formula $a_n = (n - 1)^2$, for all integers $n \geq 1$.

   $a_k = 2a_{k-1} + k - 1$, for all integers $k \geq 2$

   $a_1 = 0$