Hometask #6. Sequences and Induction.

1. Write the first four terms of sequences defined by the formulas in a-c ([ ] – integer part):

   a) $c_i = \frac{(-1)^i}{3^i}$, for all integers $i \geq 0$

   b) $e_n = \left[\frac{n}{2}\right] \cdot 2$, for all integers $n \geq 0$

   c) $f_n = \left[\frac{n}{4}\right] \cdot 4$, for all integers $n \geq 1$

2. Compute the summations:

   a) $\sum_{k=1}^{5} (k+1)$

   b) $\sum_{k=-1}^{1} (k^2+3)$

   c) $\sum_{m=0}^{3} \frac{1}{2^m}$

   d) $\sum_{n=1}^{10} \left(\frac{1}{n} - \frac{1}{n+1}\right)$

3. Compute the products

   a) $\prod_{k=2}^{4} k^2$

   b) $\prod_{j=0}^{4} (-1)^j$

   c) $\prod_{k=2}^{2} \left(1 - \frac{1}{k}\right)$

   d) $\prod_{i=2}^{5} \frac{i(i+2)}{(i-1)(i+1)}$

4. Write as single summation or product:

   a) $\sum_{i=1}^{k} i^3 + (k+1)^3$

   b) $\sum_{k=1}^{m} \frac{k}{k+1} + \frac{m+1}{m+2}$

   c) $\sum_{m=0}^{n} (m+1)2^m + (n+2)2^{n+1}$

   d) $2 \cdot \sum_{k=1}^{n} (3k^2+4) + 5 \cdot \sum_{k=1}^{n} (2k^2-1)$

   e) $\left(\prod_{k=1}^{n} \frac{k}{k+1}\right) \cdot \left(\prod_{k=1}^{n} \frac{k+1}{k+2}\right)$

5. Write, using summation or product notation:

   a) $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2$

   b) $(2^2-1)(3^2-1)(4^2-1)$

   c) $\frac{2}{3 \cdot 4} - \frac{3}{4 \cdot 5} + \frac{4}{5 \cdot 6} - \frac{5}{6 \cdot 7} + \frac{6}{7 \cdot 8}$

d) $(1-t)(1-t^2)(1-t^3)(1-t^4)$

6. Transform by making the change of variable $j = i - 1$

a) $\sum_{i=1}^{n-1} \frac{i}{(n-i)^2}$

b) $\prod_{i=n}^{2n} \frac{n-i+1}{n+i}$

7. Prove using mathematical induction

$4^3 + 4^4 + 4^5 + \ldots + 4^n = \frac{4(4^n - 16)}{3}$ for all $n \geq 3,\ n \in Z$

8. Prove using mathematical induction

$\prod_{i=0}^{n} \left(\frac{1}{2i+1} \cdot \frac{1}{2i+2} \right) = \frac{1}{(2n+2)!}$, for all integers $n \geq 1$