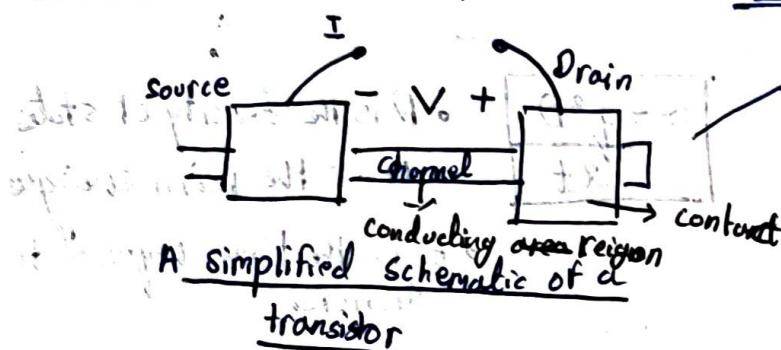


Unit 1: The new perspective. => L 1.1: Introduction.



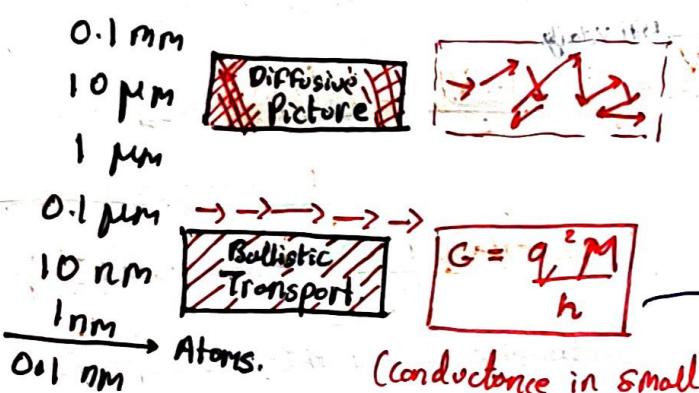
How do we think about small devices?

① How do we think about current flow in a device.

$$R = \frac{V}{I}$$

$$G = \frac{1}{R} = \frac{\sigma A}{L}$$

Material parameter.



② How do we think about conductivity.

$$\sigma = \frac{q^2 n T}{m}$$

Diffusion Current Picture / Perspective

Resistance of a very small conductor is now known.

$$G = \left(\frac{q^2}{h} \right) M$$

Fundamental constant \rightarrow Dimension of conductance $= \frac{1}{25 K\Omega}$

no. of moles

We would learn about this result in a course on Mesoscopic Physics.

Bottom up Approach.

$$G = \left[\left(\frac{q^2}{h} \right) M \right] \times \frac{\lambda}{\lambda + L}$$

Conductance of ANY device.

Ballistic conductance.

(G_B)

Length of device.

This factor.

$\lambda \rightarrow$ the distance an electron travels in a mean free time (before it hits something).

If $L \ll \lambda$ then $\frac{\lambda}{\lambda + L}$ is approximately 1.

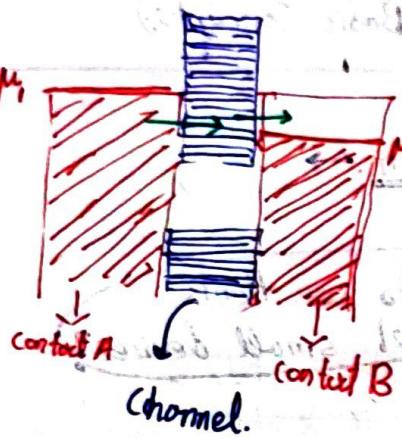
If $L \gg \lambda$ then $\frac{\lambda}{\lambda + L} \approx \frac{\lambda}{L}$ (A conductance which goes inversely as length)

So, I need to understand, for current flow, I need to understand the availability of Energy levels in the channel.

Footnotes : ① Transport in Large v/s Small devices. (conductance)

② A new perspective on conductance.

Each contact has states filled up to a certain level called the Fermi Level. (μ) or Electrochemical Potential.



A positive Potential lowers all energy levels by qV .

To bring eqm. \rightarrow contact A donates e^- so that the ECP of the channel matches the ECP of the contact.

The other contact wants to keep those states empty so it pulls them out.

[This is why current flows]

How far we need to fill up energy levels to accommodate all electrons.

$$G = \frac{q^2 D}{2t}$$

D is the density of states near the fermi energies
 t is the time required to transfer.

Drude Formula

Electrons move bcoz of an Electric Field!!!

diff velocity

$$\frac{m d\mathbf{v}_d}{dt} = -qE - \frac{m\mathbf{v}_d}{t} = 0 \quad (\text{At steady state})$$

no acc w.r.t. time.

$$I = A q n v_d = \left(A / \left(q^2 n t \right) \right) E \cdot v_d \cdot \left(q t / m \right) F$$

(Drude Conductance) $\rightarrow \sigma = \frac{q^2 n t}{m}$

$$\mu = \frac{q t}{m}$$

This picture does not work for small devices. (Drude Mobility)

If we think of electric field as driving the current. How do we explain

that. (unsetting, non intuitive) \rightarrow Why?
"Filled bonds do not conduct".

[n: "Free" electrons]

From our Theory

Both contacts want to fill up the filled up bonds thus they remain filled, end of story.

Conduction only takes place in a narrow range of Energies around a Fermi Energy.

\rightarrow The bands deep down do not take part in current flow.

- An expression for conductivity.

$$\sigma = q^2 \frac{D}{A L} \frac{D}{t}$$

$$t = \frac{L^2}{2D}$$

Now we will see how this comes about.

$$G = \frac{q^2 D}{2t}$$

And further how this can also be used to model diffusive current.

[Diffusion coefficient]

This form of conductivity is well known but it requires Advanced formalisms such as the Boltzmann formalism to derive.)

Elastic or inelastic $\lambda > 3$

Landauer Resistor.

This is much more general

When (e^-) goes from left to right, things get heated

(heat is dissipated.)

Footnotes: ① Movement of e^- /currents based on electrochemical potentials.

② Comparison with the Drude formula and why it's insufficient.

③ A more General Perspective on Conductivity.

Elastic or Landauer Resistor. \rightarrow In small devices, when an e^- goes from left to right, heat is dissipated.

However, the heating occurs in the contacts rather than the channel. \rightarrow (Whenever current flows, there is heat).

Thus, the medium provides the Resistance and the heating happens at the contacts.
[Mechanics: Force Drive]

[Thermodynamics: Entropy Driven]

In this case, both can be separated.

Boltzmann Transport:

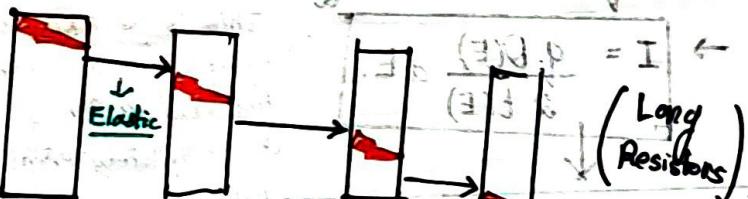
Equation:

Semi-classical

Transport theory.

Usually Study of Devices was ~~can't~~ tougher bcoz the two fields could not be separated. The BTE allows to combine the two.

We will deal with big devices by separating them into multiple smaller devices.

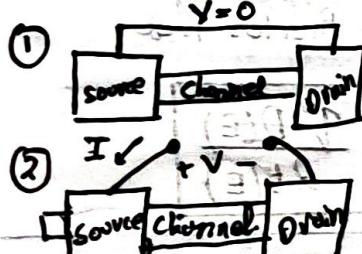


Elastic \neq Ballistic
[Please note.]

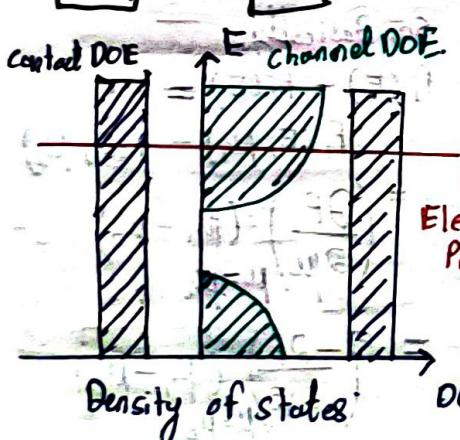
can be diffusive
Electron goes elastically from one point to another and then loses some energy and it's a series of this.

► Results Agree Exactly with BTE for low bias

L 1.2 : Two Key Concepts

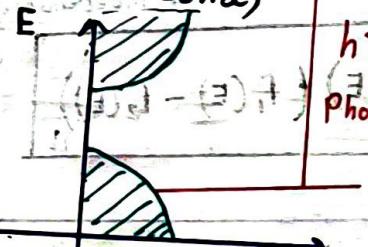


Equilibrium.



Current Flow.

In a solid,



Vacuum level.

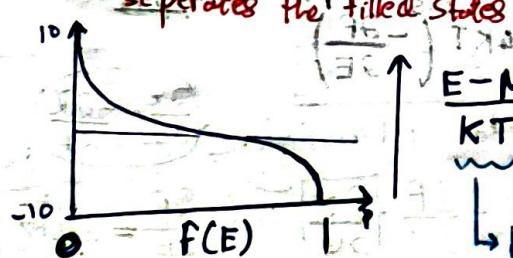
Photoemission.

To experimentally measure the density of states.

Electrons prefer to stay at lower energy level. need to provide light with sufficient energy.

* Note: Every state only holds 1 e^- but the states come in pairs. This is Pauli's exclusion principle.

Electrochemical Potential: μ separates states, one for upspin, one for down spin. Separates the filled states from the empty states.



$$F(E) = \frac{1}{1 + e^{(E-\mu)/kT}}$$

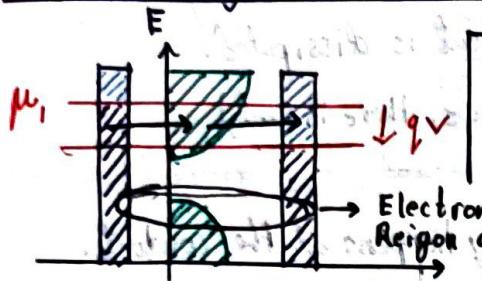
$kT \rightarrow [26 \text{ meV}]$

Footnotes: ① Bypassing the BTE by Separating the Classical and Thermodynamic Picture.

② Density of States

③ Fermi Energy Level

L 1.3 Why electrons Flow



Density of States (DE)
Source fills it up and the Drain
pulls e^- out.

in this Hard to understand
not flow if we say that
Both source
Brain wanna Electric Field
Fill. Stay's drives electrons
filled.

Approximately let's assume DOS is
constant in small window so,
 $n_{e^-} = (D \cdot qV)$ but divide by 2
bcz. half time filled, half time
empty logic.

$$\underline{\text{Actual}} : \frac{I}{q} * t(E) = \frac{Q(E) \cdot \Delta E}{2}$$

Time spent can also depend on the energy

$$\rightarrow I = \frac{q D(E)}{2 t(E)} dE.$$

This is the expression of filling states are completely full and emptying states.

$$I = \frac{1}{2} Q(E) (f_1(E) - f_2(E)) dE$$

~~E. Density of states | b. transfer time
are completely empty. (However they may be ~~completely~~ partially full and partially empty).~~

$$G \equiv \frac{I}{V} = \frac{q^2 D}{2c}$$

*H bond
wavy
Derivatives*

Current at a Single Energy

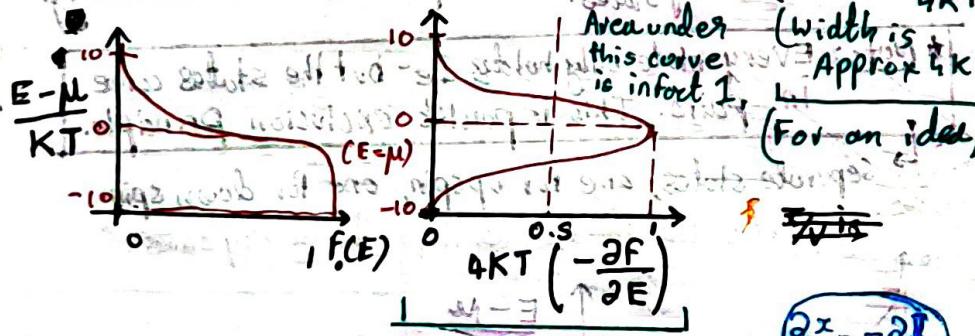
We're assuming e^- goes from left to right at the same energy.

Thus,

$$I = \frac{1}{q} \int_{-\infty}^{\infty} dE G(E) \{ f_1(E) - f_2(E) \}$$

$$\text{where } G(E) = \frac{q^2 R(E)}{2 t(E)}$$

L 1.4 : Conductance Formula



$$\text{# } P(E) = \underline{1}$$

卷之三

$$\frac{\partial F}{\partial \mu} = \frac{dF}{dx} \cdot \frac{\partial x}{\partial \mu} \quad \text{and} \quad \frac{\partial F}{\partial E} = \frac{dF}{dx} \cdot \frac{\partial x}{\partial E}$$

$$\begin{aligned}
 \text{So, Basically, } f_1(E) - f_2(E) &= \\
 f_1(E, \mu_1) - f(E, \mu_2) &= \\
 &= \left(\frac{\partial F}{\partial \mu} \right)_{\mu=\mu_0} (\mu_1 - \mu_2) \\
 &= \cancel{\left(\frac{-\partial F}{\partial E} \right)}_{\mu=\mu_0} \underbrace{(\mu_1 - \mu_2)}_{qV} \\
 &= qV \left(\frac{-\partial F}{\partial E} \right)_{\mu=0-}
 \end{aligned}$$

Footnotes: ① ~~Fig.~~ DOS Diagram for flow of current condition.

② General Current Formula Derivation: $I = \frac{q}{t} \int_0^\infty dE G(E) (F_1(E) - F_2(E))$

$$\textcircled{3} \text{ General conductance: } I/V = \int_{-\infty}^{\infty} dE \left(-\frac{\partial f_0}{\partial E} \right) G(E) \text{ (not PP) in eq 3}$$

$$\frac{I}{V} = \int_{-\infty}^{\infty} dE \left(-\frac{\partial f_0}{\partial E} \right) G(E)$$

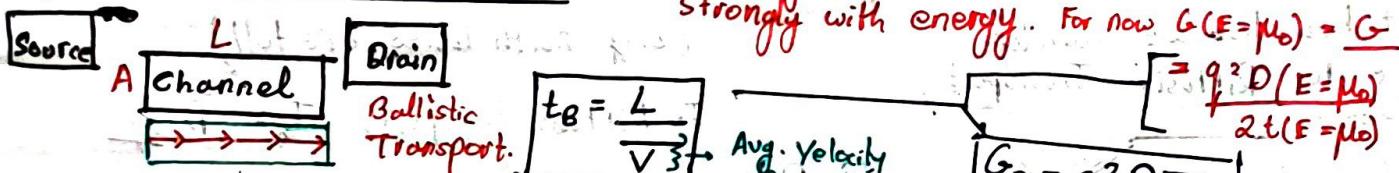
→ This is the measured conductance.
(It is like the Average of the conductance function $G(E)$).

→ The Observed conductance is largely determined by the value of the conductance function at ($\mu = \mu_0$). In fact at $T=0$, it is ~~that~~ exactly that.

As $T \rightarrow 0$, $\frac{\partial f}{\partial E}$ becomes a Delta Function. $\int_{-\infty}^{\infty} \left(-\frac{\partial f_0}{\partial E} \right) dE = f_0(-\infty) - f_0(+\infty) = 1 - 0 = 1$

* For this course, we will often use the Value of the conductance function at the Fermi energy rather than the observed conductance for ease of explanation.

L 1.5 s Ballistic Conductance. → The integral must be performed if $G(E)$ varies strongly with energy. For now $G(E=\mu_0) = G$

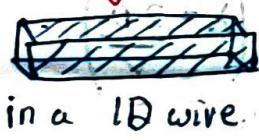


In Large Devices, ppl talk about the density of states per unit Area per unit volume. → [constant value for a particular material].

$$\text{Thus, } G_B \sim \frac{D}{L} \sim A$$

Point to note: The Ballistic conductance is independent of the length of the channel.

In Small Ballistic conductors rather than seeing something linear, you would see something in steps,



$$\frac{2t}{D} = h$$

in a 1D wire.

$$G_B = \frac{q^2}{h} = \frac{1}{25k\Omega}$$

This is the quantum of conductance.

A Device with a sharp Energy level

The uncertainty principle hints at this relation.

In Reality though, Energy levels are broader.

Such a Graph was observed experimentally around 1988 - Present.

[Shaped Modern Mesoscopic physics]. It's like a quantum of conductance times an integer.

Now, A Real conductor is lots of 1D conductors in parallel.

$$G_B = \frac{q^2}{h} \times M$$

M 1D conductors in parallel

However, this is normalized to $\frac{2q^2}{h}$ because energy levels come in pairs.

$$\nabla \equiv T \langle |V_z| \rangle \rightarrow e^-'s \text{ apparently only go from left to Right.}$$

Foot notes : ① Measured conductance v/s conductance function.

② General conductance to Ballistic conductance → Shorvin Resistance.

③ Quotients of conductance observed in small ballistic conductors.

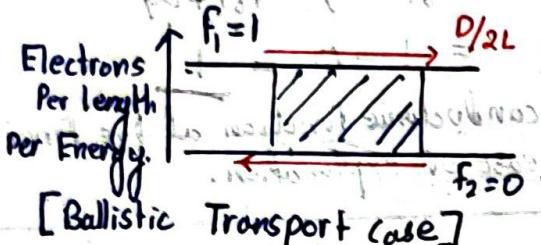
L 1.6 Diffusive Conductance $\div \left(G = q^2 D / 2t \right)$

$$b_D = \frac{L^2}{2D} \rightarrow G_D = \left(\frac{A}{L} \right) \left(q^2 \frac{D}{AL} \right)$$

σ : conductivity

Riffusive Transport.

Comes from the general theory of Random walks and Brownian motion.



$$g_n = I_n \times t$$

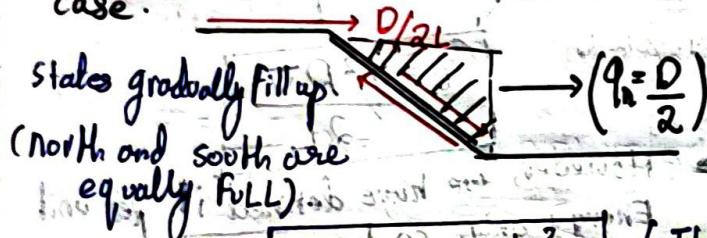
(# of e^- in channel) = (Electrons/m) \times (time in Channel) / second

"Half the states completely full (and half the states completely empty)"

$$\left(\frac{D}{2L} \right) \Rightarrow g_n = \frac{D}{2} \text{ and } t = \frac{L}{\sqrt{D}}$$

$$I_n = \frac{D \bar{V}}{2L}$$

- But how do we calculate time for Riffusive Transport case.



(only north lanes are full)
In Riffusive transport, $I_n = -D \frac{dn}{dz}$

$D/2L$ for both north bound and south bound lanes over a length of L

$$G = g_n / I_n = \frac{L^2}{2D}$$

This is a hand wavy way to get the diffusive time $\rightarrow \boxed{D = \langle V_z^2 T \rangle}$

L 1.7 Connecting Ballistic and Riffusive Conductance

write time as a polynomial expansion in L .

$$t = \frac{L}{\bar{V}} + \frac{L^2}{2D} = t_B + t_D = t_B \left(1 + \frac{L \bar{V}}{2D} \right)$$

Now, $t_D^2 = b_B \left(1 + \frac{L}{\lambda} \right)$ where λ is defined as $\left(\lambda = \frac{2D}{\bar{V}} \right)$

How would the picture look if transport is intermediated between ballistic and diffusive

(These Diagrams are fraction of e^- per unit length) over the channel length.

So what is the current in this case, $I_n = T(I_n)_B \Rightarrow t = \frac{g_n}{T(I_n)_B} = \frac{b_B}{T}$

This is in fact mean free path.

T : Fraction of e^- that make it from the source to the drain.

Footnotes: ① Riffusive time, conductance and conductivity.

② Electrons per length per Energy Picture : North bound and South bound lanes.

③ Combining Riffusive and Ballistic conductance.

Now $T = \frac{\lambda}{L + \lambda}$, use this as a definition for mean free path. [The channel length such that 50% of the e⁻ reach from source to Drain].

So, now, $t = t_B \left(1 + \frac{L}{\lambda}\right)$ and thus $\lambda = \frac{2D}{\sqrt{t}}$

$$G = \frac{q^2 D}{2t}$$

Thus, in general,

$$G = \frac{G_B}{1 + L/\lambda} = G_B \left(\frac{\lambda}{L + \lambda}\right) \text{ where } G_B = \frac{q^2 D}{2L}$$

Now, here's the interesting Result $G = \frac{G_B \lambda}{L + \lambda} \propto A$

when $L \gg \lambda \rightarrow G \propto \frac{A}{L}$

and when $L \ll \lambda, G \propto A$

$$\propto A = G_B \lambda$$

Thus, Rather than conductivity, Ballistic conductance is more fundamental to a material

L1.8 Angular Averaging.

At a particular energy, the speed of the e⁻ is a constant ($K.E. = \frac{1}{2}mv^2$).

$$1D \div \begin{array}{c} \xrightarrow{-v} \\ \xrightarrow{+v} \end{array} \text{Two possible velocities: } \begin{array}{l} +v \\ -v \end{array} \text{ so, } \langle |V_z| \rangle = \bar{v} = v$$

$$\text{And } \langle V_z^2 \tau \rangle = \bar{\tau} = v^2 \tau$$

$$\text{Thus, } \lambda = \frac{2\bar{\tau}}{\bar{v}} = 2\tau v \text{ [mean free time.]}$$

λ is mean free path for backscattering

when it collides

50% chance (Some direction) (Other direction)

Thus mean free time for back scattering is in fact $\frac{2\tau}{2} = \tau$. Thus $\lambda = 2v\tau$

$$2D \div \begin{array}{c} \xrightarrow{v} \\ \xrightarrow{\Delta\theta} \end{array} \bar{v} = \frac{\int_{-\pi}^{\pi} |V \cos \theta| d\theta}{2\pi} = \frac{\sqrt{1 \int_{-\pi}^{\pi} \cos^2 \theta d\theta}}{2\pi} = \frac{\sqrt{1 \cdot \pi}}{2\pi} = \frac{\sqrt{\pi}}{2\pi}$$

$$\text{And } \langle V_z^2 \tau \rangle = \bar{\tau} = \frac{\int_{-\pi}^{\pi} v^2 \cos^2 \theta d\theta}{2\pi} = \frac{v^2 \tau (\pi)}{2\pi} = \frac{v^2 \tau}{2}$$

$$\lambda = \left(\frac{\pi}{2}\right) v \tau \quad \text{we have a different const. for back scattering}$$

$$3D \div \begin{array}{c} \xrightarrow{v} \\ \xrightarrow{\phi} \end{array} \bar{v} = \frac{\int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta |V \cos \theta|}{2\pi \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta} = \frac{v}{2} \quad \bar{\tau} = \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta \frac{1}{2} v^2 \cos^2 \theta$$

$$\lambda = \left(\frac{1}{8}\right) v \tau$$

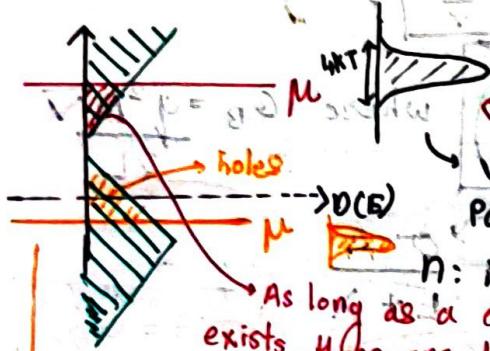
For 3D case.

Footnotes: ① Definition of mean free path, ② General Formula for conductance with conductivity defined.

③ Values of $\bar{v}, \bar{\tau}$ and λ for 1D, 2D and 3D cases. ④ Mean free time and path for backscattering.

$$\text{Overall: } \bar{V} = \frac{10}{V} \frac{2D}{2V} \frac{3D}{\frac{\pi}{2}} \quad \left| \begin{array}{l} \bar{D} = \sqrt{\tau} \cdot \frac{V^2 D}{2} \\ \bar{D} = \frac{3D}{8} \end{array} \right. \Rightarrow \lambda = 2\sqrt{\tau} \cdot \frac{(\pi)V^2}{2} = \frac{(4)}{3} V^2$$

L1.9 Drude Formula: In general, we know, $\sigma = q^2 n I / m$ from \vec{E} Perspective.



$$I/V = \int_{-\infty}^{\infty} dE \left(-\frac{\partial F_0}{\partial E} \right) G(E) \rightarrow G = \frac{q^2 D}{2t} \rightarrow \sigma = q^2 \frac{D}{AL}$$

We know that only electrons near the electrochemical potential participate in e^- flow. Thus even in the drude formula,

n : is free e^- .

As long as a clear gap exists, these are the free electrons.

Tough to come to terms with because in materials like graphene there is no clear band gap but even so we only count upto a particular limit.

What if this is μ ? In our conductivity formula, we only get the df/dE curve shifted down and consider the density of states there. BUT in the drude formula, you need to now consider the vacant spaces above the Electrochemical Potential i.e. the holes.

$$\text{Now, } \sigma = \frac{q^2 \tau}{m} (n) = \left(\int_{-\infty}^{\infty} F(E) \frac{D(E) dE}{AL} \right) \cdot \frac{q^2 \tau}{m} \quad \forall \sigma \quad \sigma = \left(\int_{-\infty}^{\infty} dE \left(-\frac{\partial F}{\partial E} \right) \frac{D(E)}{AL} \right) \cdot \frac{q^2 D}{AL}$$

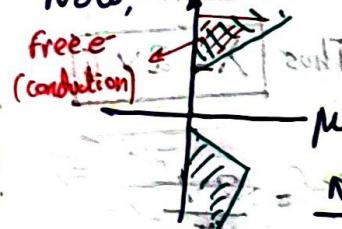
Drude Formula

These are two DIFFERENT functions.

what we obtained

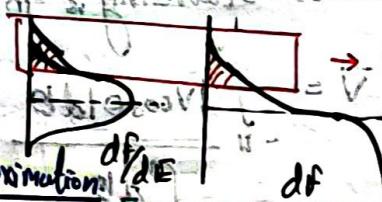
So, if the Electrochemical potential is somewhere in the Valence band / conduction band then that is a Degenerate Conductor. (Drude's formula will give an incorrect value of current due to the functions being different)

Now,



If the ECP is somewhere around here. Now this is the

Non-Degenerate Approximation



This part of both functions match, thus, we get the same (approx) value for σ .

$$* F_0(E) = \frac{1}{1 + e^{(E-\mu)/KT}} \propto e^{-(E-\mu)/KT}$$

[The tail of the two functions look very similar]

Maxwell Boltzmann Approximation

$$* -\frac{\partial F_0(E)}{2E} \approx \frac{f_0}{KT} \quad (\text{The tail of derivative is scaled by } k_F)$$

$$\text{Thus, } \frac{f_0}{2E} \cdot \mu = \frac{q \tau}{m}, \quad \mu = \frac{q D}{KT}$$

$$\textcircled{1} \quad \text{For holes, } 1 - f_0(E) \approx e^{(E-\mu)/KT}$$

Again, Maxwell Boltzmann Approximation.

$$\text{And } -\frac{\partial f_0(E)}{2E} \propto \frac{1-f_0}{KT}$$

(Fermi's Relation)

Only true for non-degenerate conductors, usually. There is an correction factor.

Makes the consis. of both expressions match.

Here μ is mobility.

Footnotes: ① How drude formula is applied to Band, meaning of n , ②: Solved

② Comparison b/w Drude formula and what we obtained

③ Non Degenerate Approximation and how what we obtained is general enough for degenerate conductors

L 1.10 Summing Up Unit 1 of Part A.

For each short section $\leftarrow qV \ll kT$

Current + I = $\frac{1}{q} \int_{-\infty}^{\infty} dE G(E) (f_1(E) - f_2(E))$. Now Assume Applied Voltage is small

BTE \rightarrow semi-classical Transport.

$$\lambda = \frac{2D}{V}$$

Ballistic

$$G_B = \frac{q^2 D V}{2L}$$

$$\frac{I}{V} = \int_{-\infty}^{\infty} dE \left(\frac{-\partial f_0}{\partial E} \right) G(E)$$

\downarrow In General This gives.

$$G = \frac{q^2 D}{2L} = \frac{G_B \lambda}{L + \lambda} \rightarrow \frac{\sigma A}{L + \lambda}$$

Unit 2: Energy Band Model + L 2.1 : Introduction

New Perspective:

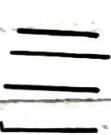
$$\sigma = \frac{G_B}{A} \lambda$$

Old Perspective:

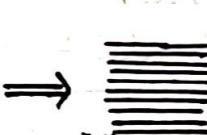
$$\sigma = \frac{q^2 n}{m} \tau$$

The two perspectives are related, one has λ (mean free path) whereas the other has τ (mean free time).

Discussing the Density of States (D):



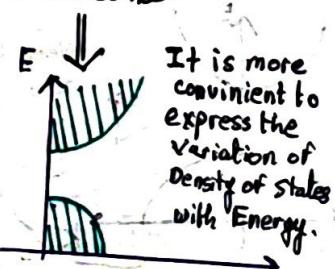
Singular atom like H-atom have good spacing between Energy levels



In larger things these energy levels squeeze more close together becoming more dense.

* Electrons in a so solid behave almost as if they were in vacuum except that they have a different E(p) or E(k) relation.

* Our objective will be to translate the E-K curve into the E-D(E) curve in this unit. Also further compare the Drude formula with the new perspective.



Gate (V_g) \rightarrow It is a third terminal of a nanotransistor, which is insulated from the channel which moves the Energy Levels of the device up and down.

($V_g > 0$) = everything pulled down and ($V_g < 0$) = everything pushed up. Thus the gate Potential is used to control the conductivity of the channel.

L 2.2 E(P) or E(k) relation:

while an electron is in vacuum:

$$E = \frac{p^2}{2m_0} + U$$

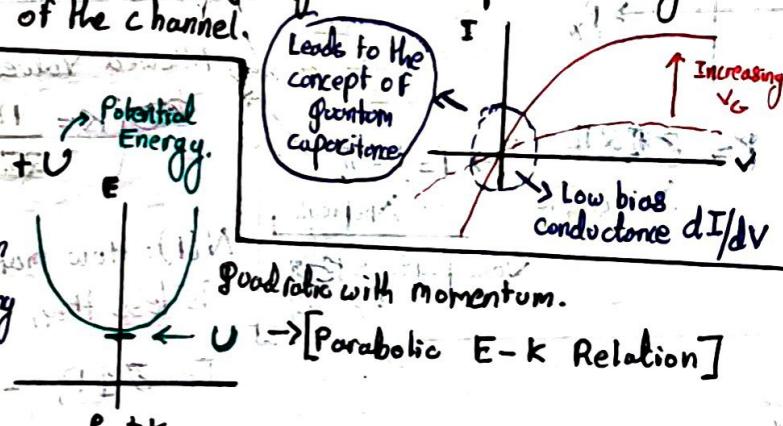
An e- sees a potential due to the atoms in a solid. Thus the PE is not const. It's changing periodically all over. \rightarrow Is there an E-k relation possible?

we can draw an Average Energy-Momentum curve in a perfectly crystalline solid where we can uniquely identify unit cells.

F \rightarrow can fit to the model via an effective mass, m_e .

$$E = \frac{p^2}{2m_e} + E_c$$

Electrons now have an effective mass.



Note: The energy levels only near the ECP matter thus, the small range of energies for which the Parabolic Approximation works is a good and acceptable Approximation.

* In case of parabolic bands, it is okay to say $P = mv$. o/w

P.T.O.

Footnotes: ① Summary of Unit 1 and Plan of Action for Unit 2.

② Discussion on DOS, E-K diagrams, and Variation of Gate Voltage + I-V Curves.

③ The E-K Relation in Vacuum v/s that in Solids.

^(Hamiltonian) General Form of laws of mechanics: $\frac{dz}{dt} = \frac{\partial E}{\partial P_z}$, $\frac{dP_z}{dt} = -\frac{\partial E}{\partial z}$

For a parabolic band, $v_z = \frac{\partial E}{\partial P_z} = \frac{P_z}{m}$

Velocity $\propto \frac{\partial E}{\partial P}$

Force \propto Gradient of Energy.

From Newton's Laws of Motion.

However, if the E-K graph was not parabolic, this relation wouldn't have worked out.
Retaristic momentum: $E^2 = (m_0 c^2)^2 + c^2 P_z^2$. \rightarrow not to worry about in a solid.

$$\rightarrow \frac{\partial E}{\partial P_z} = \frac{\partial c^2 P_z}{\partial P_z} \rightarrow v_z = c^2 P_z \rightarrow$$

In Freshman physics,

$$m = \frac{P_z}{v_z} = \frac{P_z}{c^2 P_z} = \frac{1}{c^2}$$

$$m = \frac{P_z}{v_z} = \frac{E}{c^2}$$

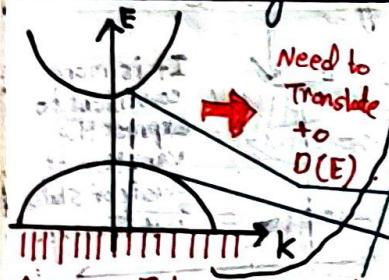
Doude Formula

\hookrightarrow "Effective mass".

[multiple bands and
-ve mass possible]

Thus in general in a solid, as opposed to vacuum, the Energy-momentum Relations, the mass can easily vary with Energy.

L 2.3 Counting States.



A e^- can take any momentum thus correspondingly it can have any energy.

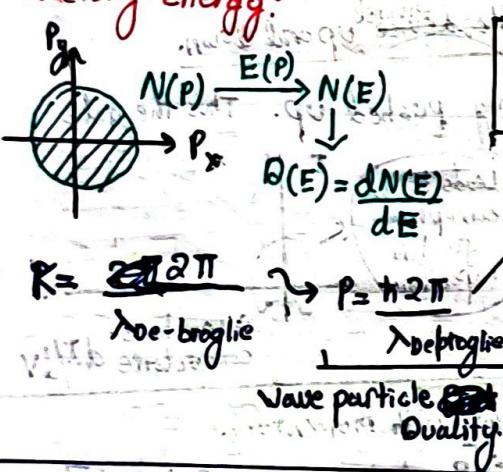
An e^- in a solid with a certain finite size, is $\not\equiv$ not allowed to take all values of momentum. Only certain momentum are allowed.

Discrete values of momentum and corresponding to each value of momentum there exists a value of Energy which is allowed \rightarrow Thus gives you on Energy level

Discrete momenta \iff Discrete Energy Levels

(How to count the discretization in momentum?)

Given a certain maximum value of momentum p , how many states are there whose momenta is less than that?



$$\lambda_{de\text{-broglie}} = \frac{h}{p}$$

$$L = n \left(\frac{h}{p} \right)$$

e^- has a certain wavelength, the length of the solid must be integer times that wavelength for the e^- to exist in it (fit in the box).

Allowed Values of k

$$k = \frac{n}{2\pi L}$$

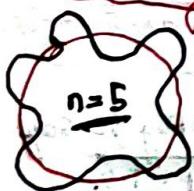
$$P_n = n \left(\frac{h}{L} \right)$$

Discrete values of $\frac{h}{L}$.

$N(P)$: How many momentum states allowed whose value is less than some value p .

Periodic Boundary Condition (PBC)

Length should be an integer no. times the wavelength h . Assumes solid is like a ring.



For larger solids, the conditions at the contacts don't matter.

PBC is mathematically easy.

$$In 1D, (-P) \leftrightarrow (P) \text{ then } N(P) = 2P/(h/l)$$

In Smaller Devices, PBC are not completely applicable

$$2D \div N(P) = \frac{\pi l^2}{(h/l)(h/w)}$$

$$N(P) = (P/h)^d \cdot C$$

$$3D \div N(P) = \frac{4\pi l^3}{(h/l)(h^2/A)}$$

\hookrightarrow some constant.

Footnotes: ① Hamiltonian Principles, and E-P relation in Relativistic mechanics \Rightarrow Solids.

② Deriving $N(P)$: no. of states $\leq P$ for 1D, 2D, and 3D via De Broglie condition.

③ Wave fitting and PBC.

L-2.4 Density of States $\div N(p) = \frac{K p^d}{\text{const.}} \xrightarrow{\substack{\text{Dimension} \\ \text{momentum}}} N(p)$ depends on dimension.

Now, $E(p) = E_c + B p^d$ \downarrow
 unimportant const. In general $d=2$



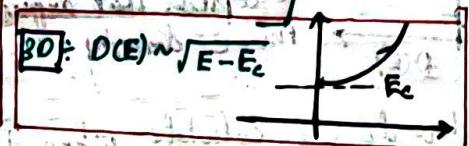
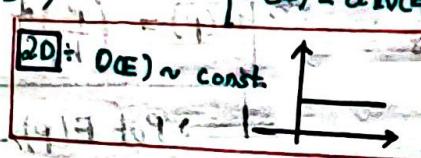
Assume $d=3, d=2$

$$p=\hbar k$$

$$\text{If } N(E) = K \left(\frac{E - E_c}{B} \right)^{\frac{d}{2}}$$

$$N(E) = K \left(\frac{E - E_c}{B} \right)^{d/2}$$

$$D(E) \sim \frac{(E - E_c)^{d/2-1}}{B} = \frac{d N(E)}{dE}$$



Thus the Density of States of any material is highly dependent on its E-p relation.

L-2.5 Number of Modes.

$$M = \frac{h D v}{2L} \{ 1, \frac{2}{\pi}, \frac{1}{2} \}$$

Now, here is a general Relation valid for all $E(p)$

$$D v p = N d$$

$$\text{We know, } N(p) = \left(\frac{p}{h} \right)^d \{ 2L, \pi WL, \frac{4}{3} \pi AL \} \rightarrow \frac{d N(p)}{d p} = d \left(\frac{p^{d-1}}{h^d} \right) \{ 2L, \pi WL, \frac{4}{3} \pi AL \}$$

$$\rightarrow P \times \frac{dN}{dE} \times \frac{dE}{dp} = d \frac{p^{d-1}}{h^d} \{ 2L, \pi WL, \frac{4}{3} \pi AL \} \times P \rightarrow D v p = N \cdot d$$

$$\text{So, } M = \frac{h}{2L} \left(\frac{N d}{P} \right) \{ 1, \frac{2}{\pi}, \frac{1}{2} \} \rightarrow M = \frac{1}{2L} \left(\frac{p}{h} \right)^{d-1} \{ 2L, 4WL, 2\pi AL \} \quad \begin{matrix} \text{General Relation valid for} \\ \text{all } E(p) \\ \text{multiplying} \\ 1 \text{ as well} \end{matrix}$$

$$M = \left(\frac{p}{h} \right)^{d-1} \{ 1, 2W, \pi A \}$$

no. of modes for all 3 dimensions.



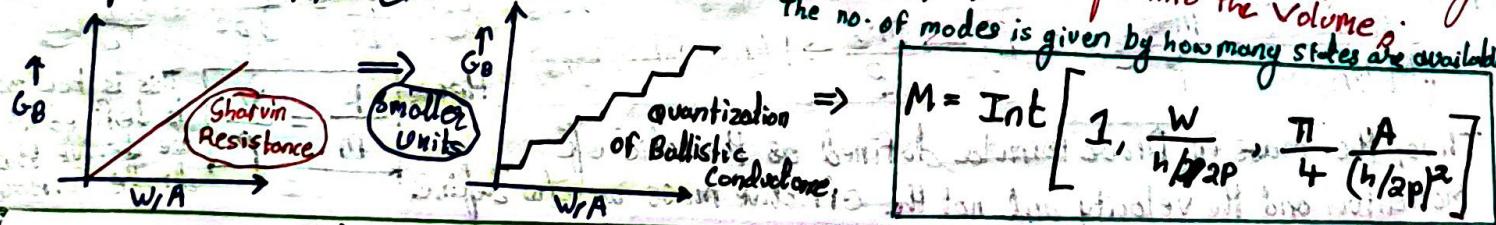
In 3D, it's the no. of half wavelengths that fit along the sides of the cross-section area.

The significance of no. of Modes is the no. of half wavelengths that fit in the width of the device.

Two Definitions of modes: ① $D v$ \div how many states you have \times how fast they move determine the amount of current you have

Similarly $N(p) = \left\{ \frac{L}{h/2p}, \frac{\pi}{4} \frac{WL}{(h/2p)^2}, \frac{\pi}{6} \frac{AL}{(h/2p)^3} \right\}$ $\xrightarrow{\text{Analogously,}}$ The no. of half-wavelengths that fit in the cross-sectional area.

No. of modes $\propto A, W \Rightarrow$ The no. of half-wavelengths that fit into the volume.



$$M = \text{Int} \left[1, \frac{W}{h/2p}, \frac{\pi}{4} \frac{A}{(h/2p)^2} \right]$$

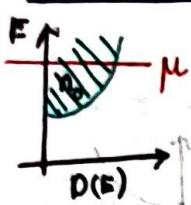
Footnotes: ① $N(p) \xrightarrow{E(p)} N(E) \rightarrow D(E) = \frac{dN(E)}{dE}$ for 1D, 2D and 3D.

② Derived. $D v p = N d$.

④ Defined no. of modes and $N(p)$ in terms of half-wavelengths.

The Linear Shorin Resistance is observed for a large no. of modes (1000, ---) whereas for a smaller no. of modes, quantization of Ballistic conductance is observed bcz the no. of modes is the no. of states which are available in that cross section, which can only be given by an Integer. Thus, M is [.] of what we've calculated.

L 2.6 Electron Density (n)

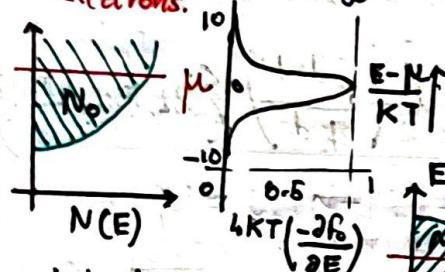


At zero temperature, all states below μ will be occupied. That defines the e-density: n_0 .

How do we calculate when $T \neq 0K$?

- So, the states are occupied according the Fermi function.

$$\text{no. of electrons} = N_0 = \int_{-\infty}^{\mu} dE D(E) f_0(E)$$



What about bands that run downwards?

"valence band".

Correspond to a above Energy E .

$$N(1-f_0) = 0 \text{ as } E \rightarrow -\infty, Nf_0 \rightarrow \infty \text{ as } E \rightarrow \infty \quad \text{Limits only vanish with } (1-f_0).$$

L 2.7 Conductivity v/s n .

$$\text{New Persp.} \div \sigma = \left(\frac{G_B}{A} \right) \lambda$$

$$\text{Drude Form.} \quad \sigma = \left(\frac{q^2 n}{m} \right) \frac{1}{T}$$

Taking the product of the two,

$$\left(\frac{G_B}{A} \right) (\lambda) = \frac{mv}{P} \left\{ 1, 1, 1 \right\} \rightarrow \frac{mv}{P} \rightarrow 1 \quad (\text{for parabolic band})$$

Thus, the mass in Drude formula defined as the ratio of momentum and the velocity and not the effective mass we often define.

- Footnotes:
- Reason for quantization of no. of modes and thus Ballistic Conductance.
 - Deriving an expression for electron density in a solid, general + measurable.
 - Comparing the ' σ ' of New perspective and Drude Formula.

$N(p)$: no. of states whose value of momentum $\leq p$.

(Total no. of states) $\rightarrow 1/L, 1/wL, 1/AL$ for e-density.

$$\text{So, } n(p) = \left(\frac{p}{h} \right)^d \left\{ 2, \pi, \frac{4\pi}{3} \right\} \quad \text{e-density at certain electrochemical Potential (ECP, μ)}$$

$$\rightarrow \text{Put } E(p) \rightarrow n(E) \quad \begin{matrix} E=\mu \\ T=0K \end{matrix} \quad n(\mu)$$

The e-density you would have if all states were filled upto μ . (which is the case at 0K)

$$\text{measurable e-density} \quad \int_{-\infty}^{\mu} dE \cdot \frac{dN}{dE} \cdot f_0(E) = \left[N f_0 \right]_{-\infty}^{\mu} + \int_{-\infty}^{\mu} dE \cdot N(E) \cdot \left(-\frac{\partial f_0}{\partial E} \right)$$

$$\text{Integration by parts. 3} \quad \text{where } n(E) \leq E(p) \quad n(p)$$

$$\Rightarrow N_0 = \int_{-\infty}^{\mu} dE n(E) \left(-\frac{\partial f_0}{\partial E} \right)$$

We need the no. of empty states now. (i.e., no. of holes)

$$\Rightarrow N_0 = \int_{-\infty}^{\mu} dE D(E) (1 - f_0(E)) \quad \left(-\frac{dN}{dE} \right) \quad \text{more states for holes as we go down.}$$

Thus $N(E)$ is increasing as you go down in energy and thus $D(E)$ is -ve of its derivative.

$$N_0 = \left[-N(1-f_0) \right]_{-\infty}^{\mu} + \int_{-\infty}^{\mu} dE N(E) \left(-\frac{\partial f_0}{\partial E} \right)$$

$$M = \left(\frac{P}{h} \right)^{d-1} \left\{ 1, 2w, \pi A \right\} \rightarrow \frac{G_B}{A} = \frac{q^2}{h} \left(\frac{P}{h} \right)^{d-1} \left\{ 1, 2, \pi \right\}$$

$$\therefore \frac{(G_B/A)}{(q^2 n/m)} = \frac{m}{P} \left\{ \frac{1}{2}, \frac{2}{\pi}, \frac{3}{4} \right\}$$

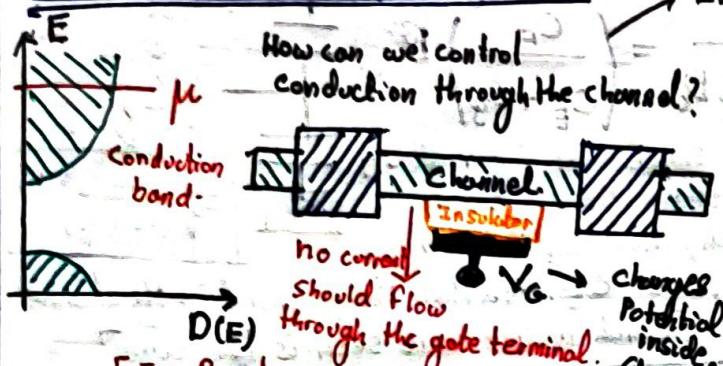
$$\therefore \frac{\lambda}{T} = \frac{V}{T} \left\{ \frac{2}{2}, \frac{\pi}{2}, \frac{4}{3} \right\}$$

$$\text{But, } \frac{1}{m} = \frac{d^2 E}{dP^2} = \frac{dV}{dP} \quad (X)$$

$$\text{Here, } \frac{1}{m} = \frac{V}{P} \quad \text{This is how mass is defined when we derive the drude formula.}$$

L2.8 Quantum Capacitance

Essence of a transistor:



$V_G < 0 \rightarrow$ Positive energy.
(ALL energy levels go up.)

[The ECP stays where it is bcoz that is controlled by the contacts]

In this case, DOS becomes smaller and the channel conducts less than before.

Let's say $V_G = U$.

[In practice, Insulators are quite thin and there is leakage current, which is undesirable.]

If μ goes below the band, we can use the non-degenerate approximation. $\rightarrow F_0(E) \rightarrow \frac{1}{e^{E-\mu/kT}}$

$$\propto \int_{-\infty}^{\infty} dE D(E) e^{-(E+U-\mu)/kT} \text{ (MB Approximation)}$$

$$\rightarrow \ln\left(\frac{N}{N_0}\right) = \frac{U}{kT} \quad \text{Now, } U = \beta(-qV_G) \rightarrow N \approx N_0 e^{-U/kT} \quad (\text{Initial is exponential})$$

$\beta < 1$ usually. $\rightarrow \sqrt{U} = \sqrt{\beta(-qV_G)} \rightarrow$ Ideally if V_G changes by 1V then the potential in the channel would change by only 1eV.

$$\text{Thus, } dV_G \propto \frac{1}{\beta} \cdot \frac{kT}{q} \cdot d\left(\ln\left(\frac{N}{N_0}\right)\right) \rightarrow \text{Thus, } \approx 60 \text{ mV/decade}$$

Assume $\beta=1$ 26 mV if we change e-density by a factor of 10.

But what if we can't apply the non-degenerate Approximation?

$$N = \int_{-\infty}^{\infty} dE D(E) F_0(E+U) \rightarrow \frac{dN}{dU} = \int_{-\infty}^{\infty} dE \cdot D(E) \frac{\partial F_0(E+U)}{\partial E} = -D_0 \rightarrow \frac{d(qN)}{d(U)} = q^2 D_0 = C_g$$

$$\boxed{\text{Quantum Capacitance}} = \frac{dN}{dU} = D_0 = \frac{C_g}{q^2}$$

As the no. of e- go up, it causes an increase in the potential energy.

(Self Potential Energy)

Quantum Capacitance \rightarrow Variation of no. of e- in channel with Gate Voltage

$$U = \beta(-qV_G) \Rightarrow \frac{dU}{d(-qV_G)} = \beta \quad \text{t.o.}(N-N_0)$$

$$\frac{dU}{d(-qV_G)} = \beta + U_0 \frac{dN}{d(-qV_G)} = \beta - U_0 D_0 \frac{dU}{d(-qV_G)}$$

$$U = \beta(-qV_G) + U_0(N - N_0)$$

Thus the band won't go down as much as you expect it to on application of gate Voltage.

When $N=N_0$, each -ve charge has a compensating +ve charge, now there is extra -ve charge.

Thus, $\frac{dU}{d(-qV_G)} = \frac{\beta}{1 + U_0 D_0}$

Avg. Density of States

Single e- charging energy.

extra e- adds Potential and make the band go up.

Footnotes: ① Introducing Gate Voltage and movement of Energy levels.

② Electron density and change of P.E in the channel with app. of V_G (non-degenerate approximation)

③ Non-degenerate Approx. and Quantum Capacitance, Electrostatic Potential.

Electrostatic Capacitance

$$U_0 = \frac{q^2}{CE}$$

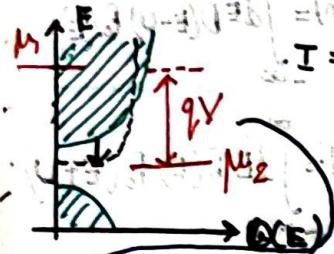
Due to Electrostatic Repulsion Energy.

$$\Rightarrow \frac{dU}{d(-qV_0)} = \frac{\beta}{1 + U_0 D_0} = \left(\frac{C_E}{C_E + C_g} \right) \beta (E_g)$$

(Electrostatic Potential in the channel)

When we were in non-degenerate region, DOS was small then C_g would be small and all the voltage would appear across C_g . However when we are in the Band, the DOS is high, thus C_g is high, this means that the entire voltage appears across C_E . And thus the potential in the channel is much less than the gate voltage we apply. [which as we have seen is the case].

L 2.9 The nano-transistor.

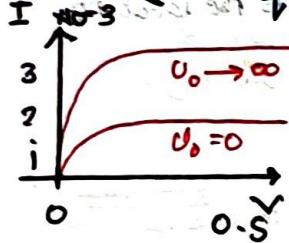


When we apply a voltage, not all of them are filled states are only half-filled leading to a lower total self-electrostatic energy. The Band moves down. [Shown in black]

Thus, we get more current than we would expect.

Again, $U = U_0(N - N_0) + \beta(-qV_0)$

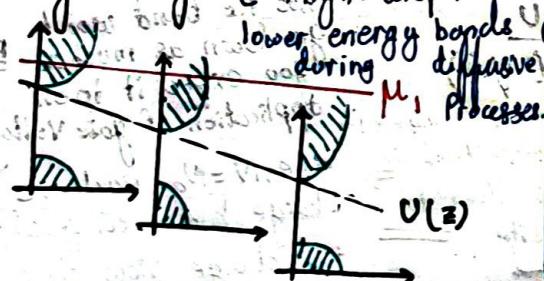
$I = \frac{t}{q} \int_{-\infty}^{\infty} dE G(E-U) (f_1(E) - f_2(E))$



A Good Transistor has current entirely controlled by Gate Voltage. [Perfect Saturation].

* Now to ensure that V_0 controls the nano-transistor better than V , we need the insulator to be really thin. i.e., if the transistor is 100 atoms across, we need the insulator to be about 10 atoms across. This is a major engineering challenge.

In Practically, V is not a single number but varies across the channel.



we are now interested in the entire I-V characteristic rather than just around the origin. Integrate this to get exact I-V.

current flows b/w μ_1 and μ_2 but once N_2 goes below the upper DOS, there are no new states and the current saturates.

Numerical Techniques to Solve

solved using numerical methods, And if we solve them self-consistently, we get a current larger than before.

But this is not the case as the potential in the channel is also affected by

Drain Voltage.
Drain induced Barrier Lowering



$$U = U_0(N - N_0) + \beta(-qV_0) + d(-qV_D)$$

As $V(T) = U(T)$ and the band moves downwards increasing the amount of current.

A Good nano-transistor has a small effect of V . The smaller the device, the larger the effect of V .

V depends on the length of device.

V_0 depends on thickness of insulator.

- ① Capacitive model of Potential in a transistor (Quantum and Electrostatic Capacitance)
- ② Introducing electrostatic effects in derivation of the I-V Current.
- ③ Effect of Drain Voltage on channel potential.
- ④ Continuity of channel Potential.

* Actual Device modelling [Continuity] $G = \frac{G_B \lambda}{L + \lambda} \rightarrow \frac{\sigma A}{L + \lambda} \sim R = R_B \left(\frac{L + \lambda}{\lambda} \right) = \frac{R_B L}{\lambda} + R_B$

uses, $I = -\sigma(z) \cdot d\mu / dz, dI = 0$.

[Drift-Diffusion Equation]

And, $\frac{d}{dz} \left(\epsilon \frac{dU}{dz} \right) = q^2 (n - n_0)$

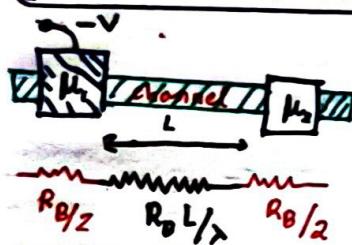
[Poisson Equation]

what does our new perspective bring to the plate with the old equations working.

$R_{B/2}$ $R_{BL/\lambda}$ $R_{B/2}$ [A Ballistic Resistance at the ends.]

This is essentially the fundamental limit to how much we can reduce the contact resistance.

Unit 3: What and where is the Voltage?

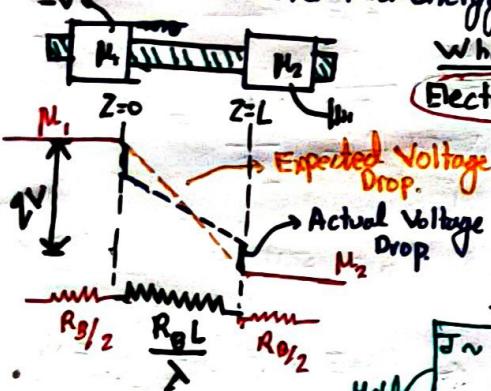


$$I = G_0 V$$

$$G_0 = G_B \left(\frac{\lambda}{L + \lambda} \right)$$

$$R_0 = \frac{1}{G_0} = R_B + \frac{R_B L}{\lambda}$$

If I punch a hole in middle of the then the Resistance WILL go up. But that doesn't mean the heating associated with that Resistance will happen in the channel. Rather, it will still happen at the contacts as the e- need some time to collide and transfer their energy to as heat.



What is Voltage? Electrochemical Potential

Resistance is associated with heating: $I^2 R$. [Joule Heating]

• But in small devices the heating is not in the channel at all, most of the heating is in the contacts.

• The channel is an Elastic or London Resistor.

→ Thus "Where the Heat is" is NOT a good indicator of where the Resistance is.

④ The Voltage Drop is a good criteria.

→ The areas with larger voltage drops have higher Resistances.

The common Answer is Electrostatic potential

$$J \sim \sigma_0 E = \sigma_0 \cdot \frac{dV}{dz} \quad (\times \text{Wrong.})$$

$$J \sim \sigma_0 \frac{d\mu}{dz} \quad (\text{Current depends on the slope of the Electrochemical Potential.})$$

In Homogeneous conductors, ECP Potential also falls with the current. Is there a current in this situation?

However, it is possible to have an In Homogeneous conductor with more e- on the Right than at the Left. The 'μ' remains const. throughout. And even as ECP Potential falls, no current flows. This is at Equilibrium. $e \rightarrow [Drift]$ $\rightarrow [Diffusion]$

Traditionally, Electron Current, $J = -\sigma_0 \frac{dV}{dz} - i q \frac{dn}{dz} \Rightarrow -\sigma_0 \frac{d}{dz} (\mu - V)$

(However, μ (ECP) is difficult to define under non-eqm conditions.) → Thus people don't talk about it.

Out of Eq M we don't have a single ECP, rather the right and left moving carriers have different ECP's.

These are [Quasi-Fermi Levels]

This is the Average of the two QFL's. μ_{up} μ_{down} μ_{avg}

(Variation of e-density) Another way to see Diffusion Current that solves the Discrepancy. \rightarrow Topological Insulators

This is spin-momentum locking, i.e., e- moving right have up spin and vice versa moving left have down spin.

Footnotes: ① Ballistic Resistance, what traditional device modelling misses out.

② What determines the location of Resistance. ③ Voltage or Electrochemical Potential.

What is Mesoscopic Physics.

It is a subdiscipline of condensed matter physics

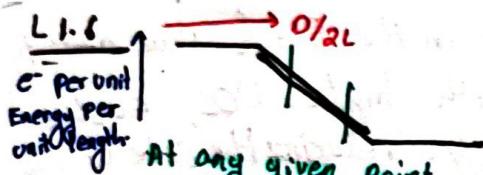
which deals with materials of intermediate size.

These materials range b/w the nanoscale for a quantity of atoms and of materials measuring micrometers.

→ Learn Boltzmann's Transport Equation (It's good to know classical transport).

$$\text{④ } t_0 = \frac{L^2}{2D}$$

→ Revise how this comes from the General theory of Random walks and Brownian motion.



At any given point, e^- going from left → right = e^- going from right → left. (no current should flow)

I still find the derivation for I_n even in terms of north bound and south bound lanes to be sketchy.

How is $\frac{dn}{dz} = (D/L^2)$ → Doesn't make sense.

Unit 2 → why does the shape of E-p relation change for electrons in a solid. How do we define momentum if not a product of mass and velocity?

→ So in Newtonian mechanics, there is no "non-linearity" thus $p=mv$ works.
The equation, $\frac{dE}{dt} = \frac{\partial E}{\partial p_z}$ comes from Hamiltonian Mechanics.

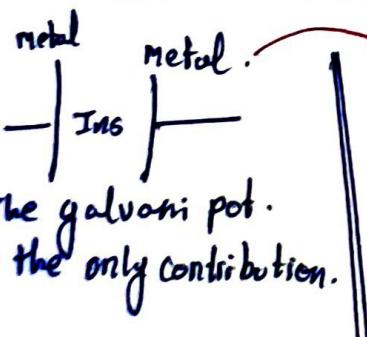
Q.) What exactly is the energy associated with (Take a course on Classical Mechanics)

Ans) [Source: Wikipedia] → ① Quantum capacitance is what exactly is quantum capacitance?

② When a voltmeter is used to measure on an electronic device, it does not quite measure the pure electric potential but instead it measures the Electrochemical potential, which is the (total free energy difference per e-)

(Also called "Fermi level difference") which includes not only its electric potential energy but all other forces and influences on the e^- (Such as kinetic energy of the wave-function).

So when we use $\phi = CV$ → we can divide the potential in Galvanic (electric) Potential and everything else.



The galvanic pot. is the only contribution.

If this is a semiconductor. → the Galvanic potential is not necessarily the only contribution.

As the capacitor charge increases, the negative plate fills up with e^- which occupy higher energy states in the band structure. While the positive plate loses e^- , leaving behind e^- with lower energy states in the band structure.