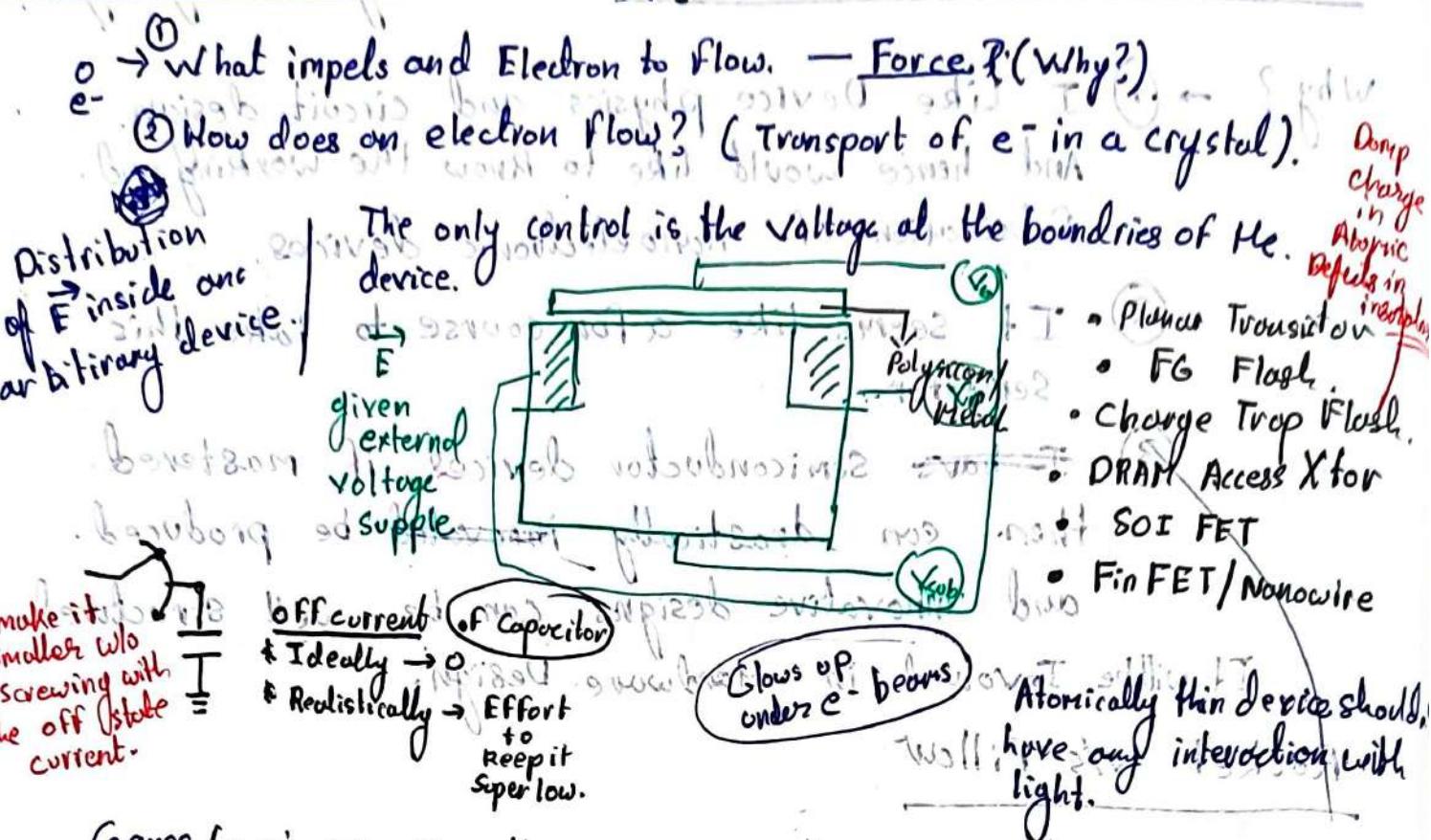


Lecture - 2 8/13/01/24



Gauss Law: The Flux through a closed surface depends on the charge enclosed inside it.

Stokes Law: The voltage drop across a loop is zero. (Field lines)

$\nabla \cdot B = 0 \Rightarrow$ The magnetic flux through a closed surface is always 0.

Amperes Law: The line integral of the magnetic field around an area depends on the current through it.

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

=> Charge creates Electrical Field

what is a free e?

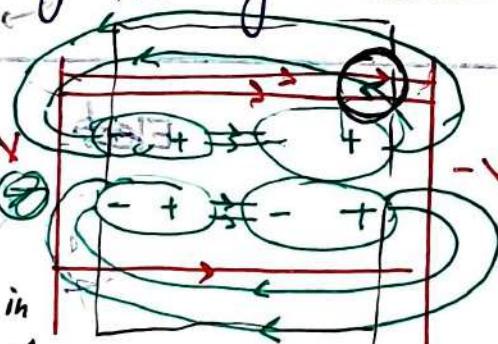
① Equipotential surface $\frac{dV}{dr} = 0$

not perpendicular at this point.

Free carriers \rightarrow steady state.

(Along Tangent voltage should not change).

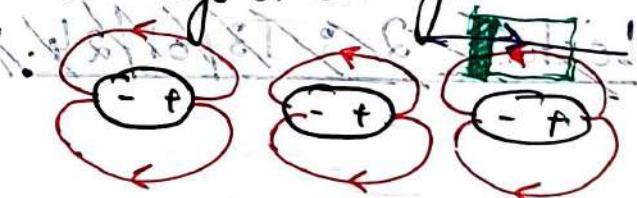
Can Dielectric constant be negative?



No field line exists in the heavily polarized atom. (No. of field lines reduction)

Less Accurate

Two ways of drawing this stuff.



More Accurate

Polarization is much less than Atomic Distance

Microscopic field is not uniform!!

Griffiths Electrodynamics :

$$\Rightarrow \mathbf{A} \times (\mathbf{B} + \mathbf{d}) = \mathbf{B} \cdot \mathbf{d} \mathbf{A} - \mathbf{C}(\mathbf{A} \cdot \mathbf{B}) \quad d\mathbf{l} = dx \hat{\mathbf{x}} + dy \hat{\mathbf{y}} + dz \hat{\mathbf{z}}$$

$$\vec{E} = \vec{r} - \vec{r}'$$

↓
field
Point.

source

Lecture - 3 : 15/01/24 :

Surprising : → Something as simple as polarization can cause doubts in mind.

→ Polarization extent is less / Pictures that seem counterintuitive.

Expected : → Twist in Dipole config. → Size of polarization matters.

→ Electric field will reduce - theory is Accurate.

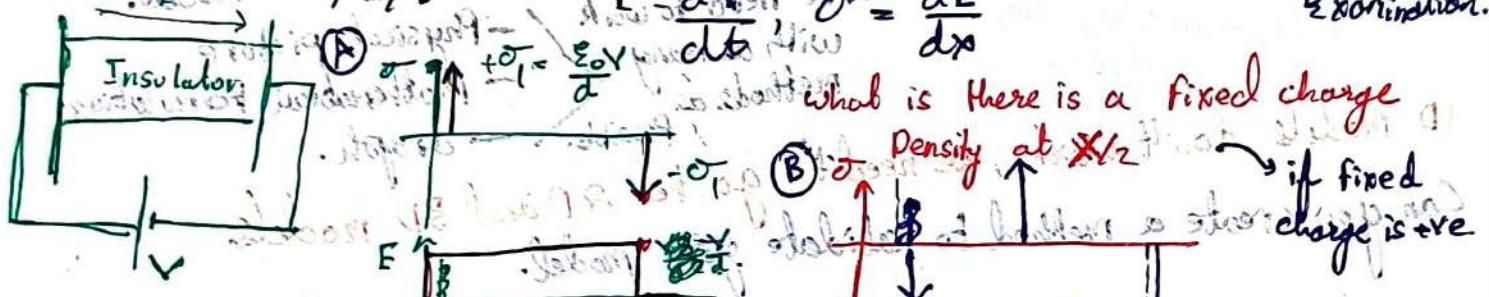
→ Avg field lines are straight.

Q) Do they matter?

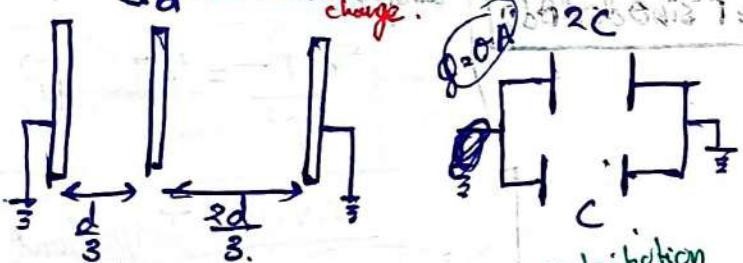
Now, 1D capacitor:

Superposition = Free charge response + Dielectric response.

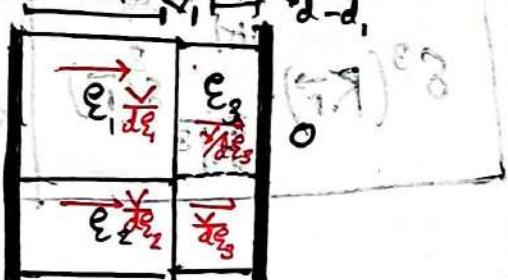
See σ , E , V . $\vec{E} = \frac{dV}{dx}$ or $\sigma = \frac{dE}{dx}$



not a : if it's an insulator with a fixed charge. Plate is a fixed charge. Does it reduce linearly? $\sigma(x)$ \rightarrow Fixed charge.

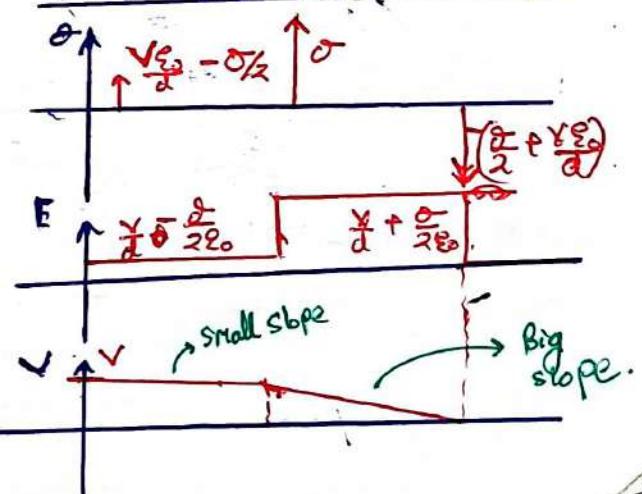


$$3A\frac{\sigma_0}{d} = 2C \text{ and } \frac{3AE_0}{d} = C$$

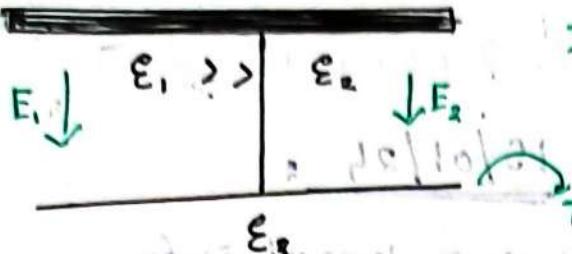


Distribution of charge density based on ratio.

MERGING THE TWO GRAPHS.



$$\rightarrow \frac{V - V_1}{\epsilon_1} + \frac{V_1 - 0}{\epsilon_{as}} = \frac{V - V_1}{\epsilon_2} + \frac{V_1}{\epsilon_3} \Rightarrow ? \rightarrow \text{no, not equipotential}$$

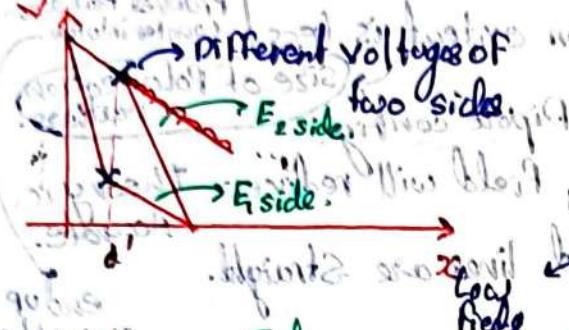


I know E_1 and E_2 far away, but they can't come to the interface with a discontinuity.
 $\rightarrow \epsilon - \text{discontinuity}$

This line is not equipotential.

* Capacitors where I can sneak in a metal plate.

* Can I sneak in two metal plates. no
 field is not perpendicular to the metal plates.



Why?

Circuit Model

detailed physics model.

Differential equation Solver.

We need to work with as many methods as possible.

- Physical picture
- Mathematical formulation
- graph.

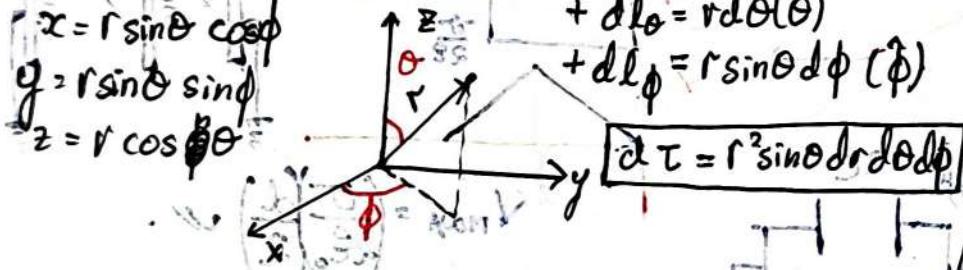
1D Models don't work, we need to go for 2D and 3D models

Spherical coordinates

$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$



Blows up at origin



$$\delta^3(\vec{r}) = \delta(x)\delta(y)\delta(z)$$

$$\int f(r) \delta^3(r=a) dT = f(a)$$

all space

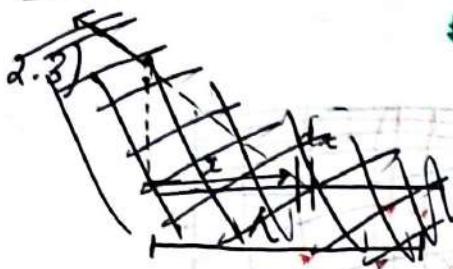
$$\nabla \cdot \frac{\hat{r}}{r^2} = 4\pi \delta^3(\vec{r})$$

Important

Blows up at origin

$$\delta(kr) = \frac{1}{|k|} \delta(r)$$

$$\delta^3(kr) = \frac{1}{|k|^3} \delta(r)$$



* $\frac{d\phi}{dx}$ is the factor by which ϕ varies with an infinitesimal change in x , i.e., dx .

$$\star \phi(x, y, z) \rightarrow \nabla \phi = \left(\frac{\partial \phi}{\partial x} \hat{i} + \frac{\partial \phi}{\partial y} \hat{j} + \frac{\partial \phi}{\partial z} \hat{k} \right)$$

$$\rightarrow dT = \frac{\partial T}{\partial x} (dx) + \frac{\partial T}{\partial y} (dy) + \frac{\partial T}{\partial z} (dz) \quad [dr = \left(\frac{dT}{dx} \right) dx]$$

$$= \left(\frac{\partial T}{\partial x} \hat{i} + \frac{\partial T}{\partial y} \hat{j} + \frac{\partial T}{\partial z} \hat{k} \right) \cdot (dx \hat{i} + dy \hat{j} + dz \hat{k}) = (\nabla T) \cdot (dl)$$

$$\cdot dT = \nabla T \cdot dl = |\nabla T| |dl| \cos \theta \rightarrow (\text{max. when } \theta=0), \text{ i.e., } dT \parallel \nabla T$$

* ∇T points in the direction of maximum increase of the function T . Moreover, the magnitude $|\nabla T|$ gives the slope (rate of increase) along this maximal direction.

$$\text{Stationary point} \leftarrow \nabla T = 0$$

* Del operator $\nabla = (F_1(x, y, z), F_2(x, y, z), F_3(x, y, z))$

∇T (gradient)

$\nabla \cdot \vec{v}$ (divergence)

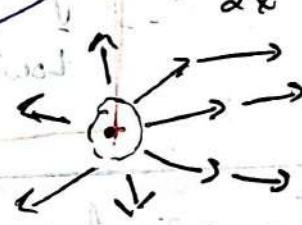
Divergence:

$$\nabla \times \vec{v} \quad (\text{curl}) \quad \nabla \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \quad (\text{Scalar}).$$

Vector Function

Vector Associated
with every point in
space.

It is the measure of the change
in value of the vector field in
every direction added.



It is the measure of how much the
vector \vec{v} spreads out (diverges) from
Point in question.

Positive divergence \Rightarrow Source.

Negative divergence \Rightarrow Sink.

$$\epsilon_R \div \frac{\vec{v}}{r^2} \Rightarrow \nabla \cdot \vec{v} = 0$$

except at origin where \vec{v} blows up as $r=0$

At that one point, divergence is infinite.

$$\text{Curl: } (\nabla \times \vec{v}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix}$$

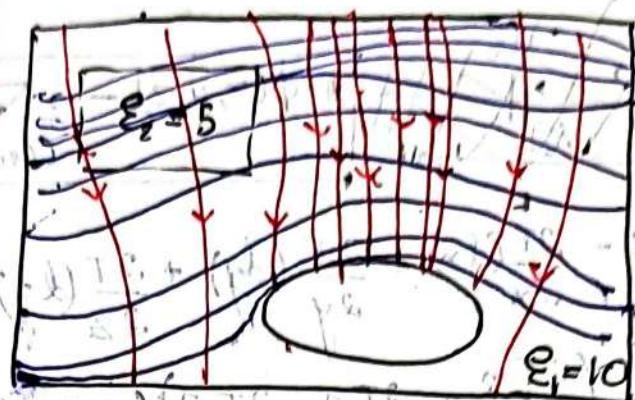
The measure of how much the vector \vec{v} swirls around a point in question.

* Intuition Doesn't work

When we guess — we need something to validate our guess
 Situation
 Then check why guess is wrong?

Why does it not make sense (to me).

- Why is there no electric field below the cavity.
- Why is the Potential at the boundary of the cavity zero?
- And not even below it? equi-potential.



Why it makes sense?

- Potential going from high to low, in a continuous gradient.
- Electric field intensity varies in Region of cavity vs zero. (Perpendicular field lines).
- Higher intensity of Electrical Field in Center $\Rightarrow (\partial V)_{\text{center}} > (\partial V)_{\text{cav}}$.

Higher Dielectric constant

↓
Lower Electric Field Intensity

Griffiths: Product Rules with vector operators

$$(i) \nabla(fg) = f \nabla g + g \nabla f$$

$$(ii) \nabla(A \cdot B) = A \times (\nabla \times B) + B \times (\nabla \times A) + (A \cdot \nabla)B + (B \cdot \nabla)A$$

$$(iii) \nabla \cdot (fA) = f(\nabla \cdot A) + A \cdot (\nabla f)$$

$$(iv) \nabla \cdot (A \times B) = B \cdot (\nabla \times A) - A \cdot (\nabla \times B)$$

$$(v) \nabla \times (fA) = f(\nabla \times A) - A \times (\nabla f)$$

$$(vi) \nabla \times (A \times B) = (B \cdot \nabla)A - (A \cdot \nabla)B + A(\nabla \cdot B) - B(\nabla \cdot A)$$

V.
Imp

Second Derivatives:

- (i) $\nabla \cdot (\nabla \phi) = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$ [Laplacian] Convenient extension occasionally.
- (ii) $\nabla \times (\nabla \phi) = 0$ [curl of Gradient is always zero] $\nabla^2 \vec{v} = \nabla^2 v_x \hat{i} + \nabla^2 v_y \hat{j} + \nabla^2 v_z \hat{k}$
- (iii) $\nabla (\nabla \cdot \vec{v}) \leftarrow$ Gradient of Divergence, $((v_x)_{xx} + (v_y)_{yy} + (v_z)_{zz})$
- (iv) $\nabla \cdot (\nabla \times \vec{v})$ [Divergence of curl is always zero] $((v_x)_{xy} + (v_y)_{yz} + (v_z)_{zx})$
- (v) $\nabla \times (\nabla \times \vec{v}) = \nabla(\nabla \cdot \vec{v}) - \nabla^2 \vec{v}$ (same as (i),(iii)) Identity.

Line Integrals: (Integral Calculus Req'd for Electrodynamics)

so, $\int_a^b \vec{v} \cdot d\vec{l}$ (for open paths) $\oint_L \vec{v} \cdot d\vec{l}$ (for closed paths)

To Perform this Integration, we have to ~~have to~~ Parameterize the path in terms of a single variable 't'. The differentiate this to get dl in terms of dt .

so, if path is $(x(t), y(t), z(t))$ then we get

$$*\int_a^{r(b)} \vec{v} \cdot d\vec{l} = \int_{r(a)}^{r(b)} \vec{v}(x(t), y(t), z(t)) \cdot (x'(t) dt, y'(t) dt, z'(t) dt)$$

Surface Integrals: $\int_S \vec{v} \cdot d\vec{a}$ or $\oint \vec{v} \cdot d\vec{a}$ [FLUX]

The sign of Surface integrals for open surfaces is intrinsically ambiguous. However for closed surfaces, we traditionally take outward positive. Again There is a special class of vectors whose surface integral doesn't depend on the particular surface chosen.

Parameterization of $S \equiv (x(u, v), y(u, v), z(u, v)) = \vec{r}(u, v)$

How to calculate $\int_Q \int \vec{F} \cdot (\vec{r}_u \times \vec{r}_v) du dv$ (As per MA 111)

Normally the line integral heavily depends on the prescribed Path P . But for a Special Class of vectors, the line integral only depends on the start and end points.

IF a vector can be written as a gradient of a scalar then we call it conservative.

i.e., if $\vec{v} = \nabla \phi$ then $\oint \vec{v} \cdot d\vec{l} = 0$

$$\text{And } \oint_a^b \vec{v} \cdot d\vec{l} = \phi(b) - \phi(a)$$

Why? Later! $\rightarrow d\phi$ Think!

$$d\vec{a} = \vec{r}_u \times \vec{r}_v$$

This is basically multiple Triple Integrals.

Volume Integrals: $\int_V \phi dV$ where $dV = dx dy dz$ // $\int_V \vec{v} dV = \int_V v_x dV \hat{i} + \int_V v_y dV \hat{j} + \int_V v_z dV \hat{k}$

* Fundamental Theorem of Calculus: $\int_a^b \left(\frac{dF}{dx} \right) dx = F(b) - F(a)$ OR $\int_a^b F(x) dx = F(b) - F(a)$, where $F(x) = \frac{dF}{dx}$

* Fundamental Theorem of Gradients: $d\phi = (\nabla \phi) \cdot (d\vec{l})$, $\int_a^b (\nabla \phi) \cdot d\vec{l} = \phi(b) - \phi(a)$

Thus $\int_a^b (\nabla \phi) \cdot d\vec{l} = \int_a^b d\phi = \phi(b) - \phi(a)$

Thus, Gradients have a special property that their line Integrals are path independent.

$$\int_V (\nabla \cdot \vec{v}) dV = \oint_S \vec{v} \cdot d\vec{a}$$

The integral of a derivative (Here, Divergence) over a Region (Here, Volume) is

The "boundary" of a line is just equal to the value of the two end points, but the boundary function at the boundary of a volume is a closed Surface (Here Surface S).

$\int f(\text{sources within the volume}) = \int (\text{flow out through the surface})$

\hookrightarrow (Total source/Drain)

* The Fundamental Theorem of Calculus

$$\int (\nabla \times \vec{V}) \cdot d\vec{a} = \oint \vec{V} \cdot d\vec{l}$$

total swirl in \vec{V} (sometimes called circulation).



* Integration by parts:

$$\int \nabla \phi \cdot d\vec{a} = \int \vec{v} \cdot d\vec{a} - \int \frac{du}{dx} \int v du dx$$

Again, here integral of a derivative (here, curl) over a region (here, a patch of surface, S) is equal to the value of the function at the boundary (here, the perimeter of the patch, P).

Corollary 1: $\int (\nabla \times \vec{V}) \cdot d\vec{a}$ depends only on the boundary line, not on the particular surface. (Boundary Value Problem)

(i) $\nabla \cdot (F \cdot A) = P(F \cdot A) + A \cdot (\nabla F)$

(ii) $\nabla \times (F \cdot A) = \vec{A} \times (\nabla F) + P(\nabla \times A)$

$$(i) \int \int F(\nabla \cdot A) d\tau = - \int A \cdot (\nabla F) d\tau + \int F \cdot A \cdot da$$

$$(ii) \int \int F(\nabla \times A) \cdot da = \int [A \times (\nabla F)] \cdot da + \int F \cdot A \cdot d\tau$$

\Rightarrow Divergence & Curl

Essentially what all Integration (of F) - (by parts) do.

The Theory of Vector Fields.

Well no, it is not a specified scalar or vector, in fact many such vector functions exist with both divergence and curl zero.

* To Solve a Differential Equation, you must also be supplied Appropriate Boundary conditions.

With this extra information Helmholtz Theorem guarantees that a field is uniquely determined by its divergence and curl!

* If the Divergence $D(r)$ and the curl $C(r)$ of a vector function $F(r)$ are specified, and if they both go to zero faster than $1/r^2$ as $r \rightarrow \infty$, and if $F(r)$ goes to zero as $r \rightarrow 0$ then F is uniquely given by,

$$F = \nabla U + \vec{v} \times \vec{w}$$

$$\left\{ U(r) = \frac{1}{4\pi} \int \frac{D(r')}{|r-r'|} d\tau' \right\}$$

Potentials \rightarrow Scalar Potential (V)

$\underline{\text{I.1}}$ Curl-less or irrotational fields

$$\vec{E}$$

- Scalar potential is not unique and depends on chosen reference.
- The above are spc cases.

$$\oint \vec{F} \cdot d\vec{l} = 0 \text{ for any closed loop.}$$

$$d) \vec{F} = -\nabla V \quad \text{where } V \text{ is some scalar function.}$$

$$c) \oint \vec{F} \cdot d\vec{a} = 0 \text{ for any closed surface}$$

The vector potential is not unique, the gradient of any scalar function μ $\vec{F} = \nabla \times \vec{A}$ where \vec{A} is some vector function independent of μ affecting the curl.

$$\vec{F} = -\nabla V + \nabla \times \vec{A}$$

(always)

Chapter 2.6 Electrostatics.

* We consider all source charges to be stationary.

$$\vec{F}_{\text{charge}} = \frac{kq}{r^2} \hat{r}$$

$$r = r - r' \quad \left(\begin{array}{l} \text{separation vector} \\ (\vec{r}) \end{array} \right) \quad E(r) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i^2} \hat{r}_i \quad (k = 1/4\pi\epsilon_0)$$

if $r \gg R$ then $\vec{E} \sim \vec{p}/r^2$

Plug in for various types of charges

$$E_{\perp} = \frac{k\lambda}{2} [\sin\beta + \sin\alpha]$$

$$E_{\parallel} = \frac{k\lambda}{2} [\cos\beta - \cos\alpha]$$

$$(\vec{E}_{\text{shell}})_{\text{in}} = 0 \quad (\vec{E}_{\text{shell}})_{\text{out}} = \frac{kq}{r^2}$$

Now, we know that $d\vec{a} = \vec{r} d\theta \hat{r} + r \sin\theta d\phi \hat{\phi}$.

$$S_o, \vec{E} = K \int \frac{q(r) d\tau}{r^2} \vec{r}$$

The Flux of an Electric field is the surface integral of the field at that surface.

$$(\vec{E}_{\text{in}})_{\text{sphere}} = \frac{kqr}{R^3} = \frac{p_r}{3\epsilon_0}$$

$$(\vec{E}_{\text{out}})_{\text{sphere}} = \frac{kq}{r^2} = \frac{\sqrt{p_r} R^3}{3\epsilon_0 r^2}$$

The Flux of an Electric field is the surface integral of the field at that surface.

$$\oint \vec{E} \cdot d\vec{a} = \int \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \left(r^2 \sin\theta d\theta d\phi \right) r^2 = \left(\frac{q}{\epsilon_0} \right) \star \star$$

* ~~The~~ The radius \rightarrow Field goes down by $\frac{1}{r^2}$ but surface area goes up by r^2 .

Vector Potential (A)

$\underline{\text{I.2}}$ Divergence-less or solenoidal fields

$$\vec{B}$$

a) $\nabla \cdot \vec{F} = 0$ everywhere

$$\oint \vec{F} \cdot d\vec{l} = 0$$

b) $\oint \vec{F} \cdot d\vec{a} = 0$ for any closed surface

$$\oint \vec{F} \cdot d\vec{a} = 0$$

The flux of any scalar function μ $\vec{F} = \nabla \times \vec{A}$ where \vec{A} is some vector function.

$$\vec{E} = \frac{\partial \vec{A}}{\partial r} = K \sum_i \frac{q_i}{r_i^2} \hat{r}_i$$

↓ Electric Field of "source charges"

$$E_{\text{ring}} = \frac{KQ}{(R^2 + z^2)^{3/2}}$$

$$E_{\text{disc}} = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{z}{\sqrt{R^2 + z^2}} \right]$$

Essence of Gauss's law:

The flux through any closed surface is representative of the charge inside. For

field lines must originate at a positive charge most either pass through the surface or terminate above the charge.

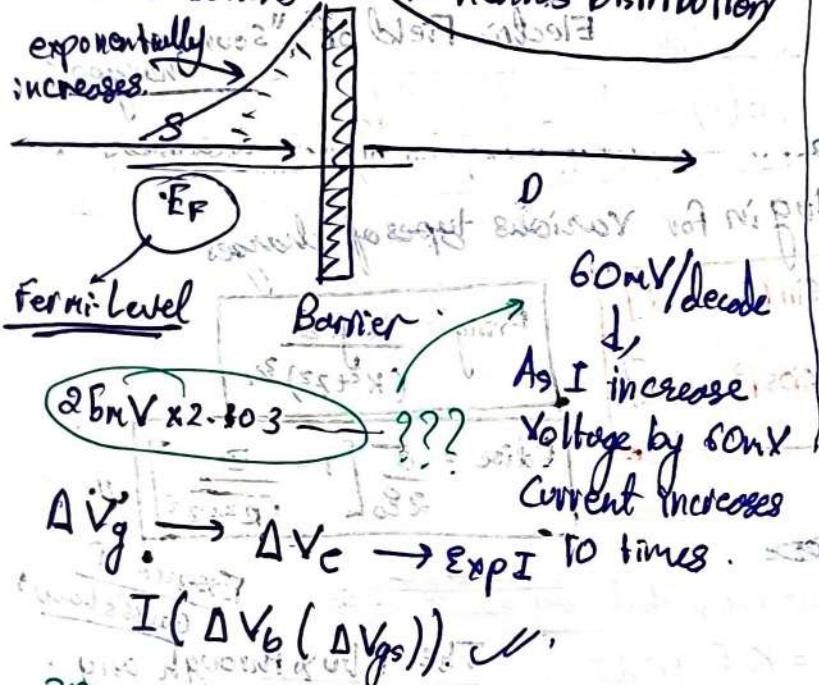
Lecture - 5 : 20/01/24



- only gate controlled non-linear resistor
- what is the efficiency of gate bias controlling the channel current — High
- what is the efficiency of drain bias controlling channel current — Low

Mother of All exponential Distributions exponentially increases.

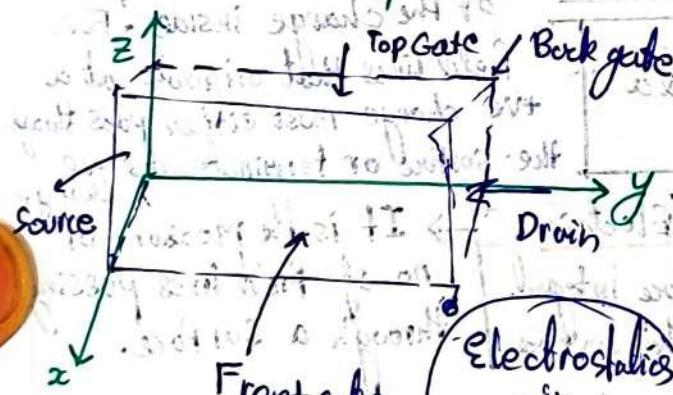
Maxwell-Boltzmann of Arrhenius Distribution



$$\Delta V_g \rightarrow \Delta V_c \rightarrow \text{exp} I \text{ to times.}$$

$$I(\Delta V_b (\Delta V_{gs}))$$

2D → fictitious problem that the fin is very tall



Sharp ON/OFF step function (controllable)

Good Switch

- No Resistance in ON state.
- Simple stops conduction completely in OFF state, i.e., no leakage current across it.
- can handle any amount of current.

* Ideally power needed to control the switch should be zero.

* High switching speed.

* Controlled by some external Voltage.

exponentials are closest to a very sharp take off

* Two states are like a Memory. UP or DOWN.

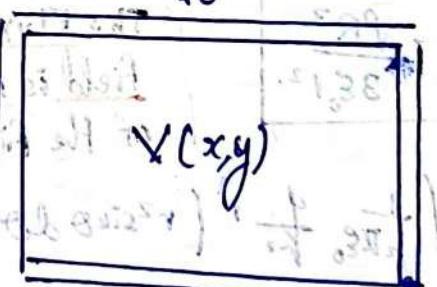
* If I move fwd then its response exponentially increases and when I move bwd exponentially decreases

I want such control

Ex: Diode I-V characteristics

Laplace Equation:

$$\nabla^2 V = 0$$



Free charge e^-

Bound charge & Doping

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0.$$

Learn how to solve.

$$Y \frac{d^2 X}{dx^2} + X \frac{d^2 Y}{dy^2} = 0$$

Divide by XY.

Separation of Variables

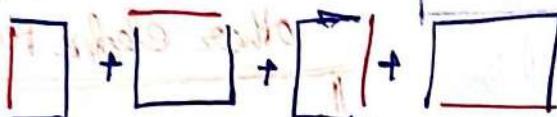
$$V(x, y) = X(x)Y(y)$$

$$\frac{1}{X} \frac{d^2 X}{dx^2} + \frac{1}{Y} \frac{d^2 Y}{dy^2} = 0$$

Equating each term to R² and -k²

$$\frac{d^2 X}{dx^2} = k^2 X$$

$$\frac{d^2 Y}{dy^2} = -k^2 Y$$

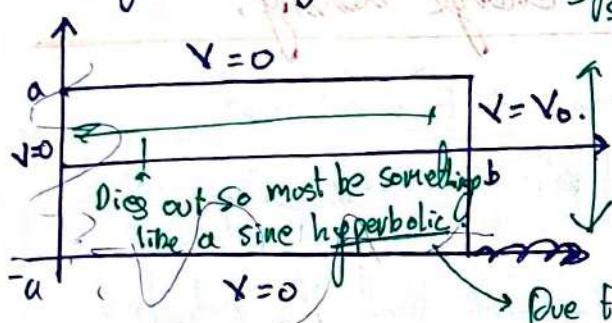


$\sin(k_x x)$
and
 $\cosh(k_x x)$

$\sin(k_y y)$
and
 $\cos(k_y y)$

Take one voltage at a time and solve.

Using Bounding Conditions:



Symmetric
Sine

$$V(x=0, y) = V(z, y = \pm a) = 0$$

Learn how to
solve these types
of PDEs.

$$V(x=b, y) = V_0.$$

Need to understand 2D PDE's
and learn the math and how it
is applied to Electrostatics.

Lecture 6 - 22/01/24

My solution after
calculation is
(To learn)

$$V(x, y) = \sum_{n=0}^{\infty} (-1)^n \frac{a^2 V_0}{a k_n} \cos(k_n y) \frac{\sinh(k_n x)}{\sinh(k_n b)}$$

Q.) Which is the most dominant term in terms of furthest effect from the "active electrode"?

$$k_n = \frac{\pi(2n+1)}{2a}$$

which term affects the voltage decay? } most representative term

$$n=0$$

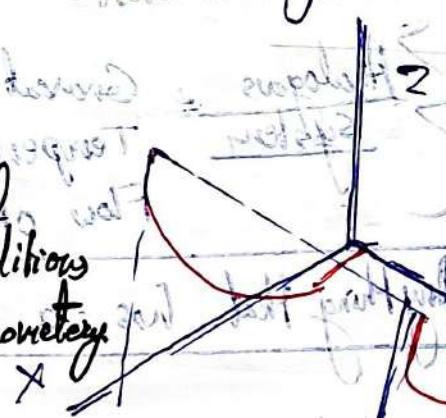
$k_n \rightarrow$ inverse wavelength \rightarrow corresponds to the slowest frequency

As $n(1)$, wavelength(l).

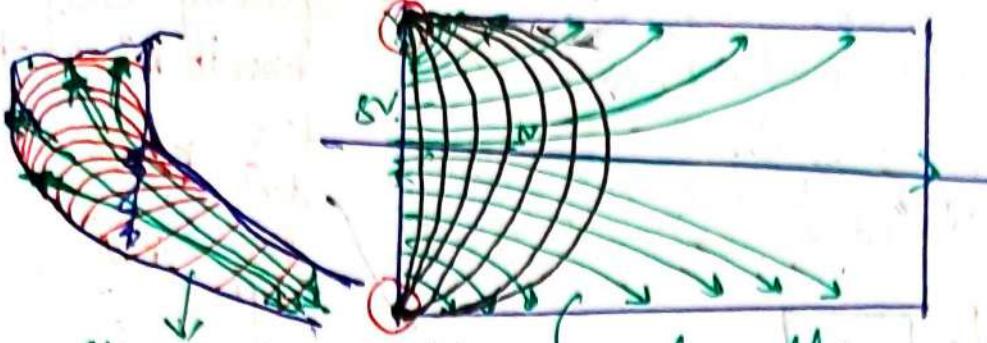
And thus the longest wavelength

These terms add up the infinite cos terms to give a Fourier representation of a square wave.

Fourier trick can be used to solve any boundary conditions for a regular rectangular Geometry.



Below a rectangle and a circle with a central node. A blue arrow points to the center of the rectangle. A red arrow points to the center of the circle. A green arrow points to the center of the rectangle. A blue arrow points to the center of the rectangle. A red arrow points to the center of the circle. A green arrow points to the center of the rectangle.



My guess was wrong.

Why needed to think more
(Rough/First intuition)

Again It's Actually completely wrong.

Should have drawn a 2D plane
and should have slowly mapped
out points of equal potential

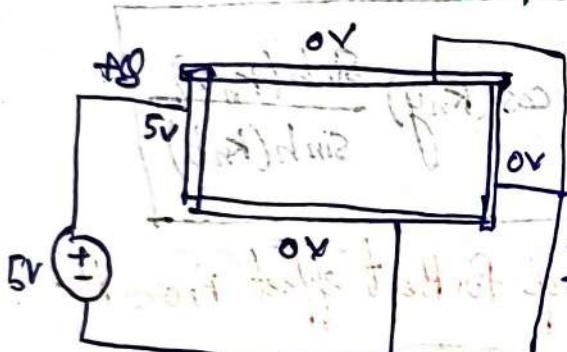
Very high electric field at the two corners such that the simulation overshadows all other electric field lines.

Shortest separation between two electrodes at the corners so highest charge density.

∇ is non-linear $\Rightarrow E$ is not const

but E is not constant despite of no charge.

* How is it possible that inspite of no charge in the capacitor that the potential is decreasing non linearly?



* How is it possible the new electric fields lines are being created w/o charge?

* In 1D laplace equation ensures constant E field when charge is zero.

* In 2D even w/o charge E field is position dependent, why?

Every point in the potential profile is a saddle point, i.e.

opposite curvature in x and y directions bcoz this is a charge free laplace system.

At that point the up curvature and down curvature must cancel out.

Laplace solution is valid when there is almost zero free charge.

$$\text{So, } \nabla^2 V = \nabla E = 0 \rightarrow \text{net divergence in } E \text{ field is zero.}$$

And, in 1D laplace equation electric is const.

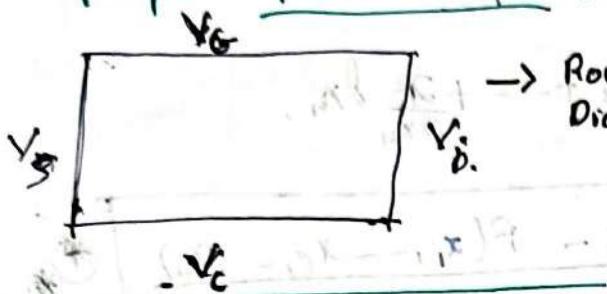
$\frac{\partial E}{\partial x} = 0$ implies that

Analogous to Gravitational (stress strain) System Temperature Flow of Water

Anything that has a scalar potential

To Show : How is Laplace Applied to FinFET Analysis?

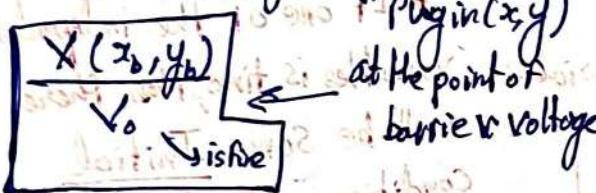
The Rectangular Geometry can be Applied to FinFET Analysis as we can Superpose & such solutions.



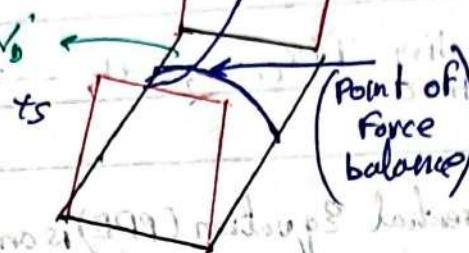
How will my barrier voltage be affected by variation in gate voltage at the boundary?

$$\frac{\Delta V_b}{\Delta V_G} = ?$$

We know $V(x, y)$ →



→ Rough Diagram



in our Analogy

"effective barrier" is the minimum barrier that an electron faces to cross from source to drain.

* The most Advanced Devices we can build (FinFET) in completely Analytically tractable.

→ Figure out this math and come up with a closed form Analytical Solution!

Q.) Can we make an effective FinFET model.

→ Using this Design a Good Transistor

Partial Differential Equations

* Some DE's have more than one independent variable. Unknowns are functions of more than one variable.

Ex) 1D heat equation: $u: [0, \infty) \times [0, L] \rightarrow \mathbb{R}$: $\left\{ \begin{array}{l} \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \text{ for } t > 0, x \in (0, L) \\ u(0, x) = u_0(x) \text{ for } x \in (0, L) \\ u(t, 0) = u(t, L) = 0 \text{ for } t > 0 \end{array} \right.$

Types of PDEs

linear
quasilinear
semilinear
non-linear

→ Existence and Uniqueness of solutions.

→ Solving PDEs analytically is generally based on finding a change of variable to transform the equations into something soluble or finding an integral part of the solution.

First order linear: $a \frac{\partial u}{\partial x} + b \frac{\partial u}{\partial y} = c$

Second order:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} + d \frac{\partial u}{\partial x} + e \frac{\partial u}{\partial y} + fu = g$$

P.T.O

Why Study PDE's \Rightarrow most mathematical physics is described by such equations.

↳ Typically a PDE will only be accessible to numerical solutions and analytical solutions in a practical or research are impossible.

Simply, $df = \sum_{i=1}^n \frac{\partial f}{\partial x_i} dx_i = \frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x_2} dx_2 + \dots + \frac{\partial f}{\partial x_n} dx_n$.

So, $\frac{\partial f}{\partial x_i} = \lim_{h \rightarrow 0} \frac{f(x_1, \dots, x_i + h, \dots, x_n) - f(x_1, \dots, x_i, \dots, x_n)}{h}$

- A Partial Differential Equation (PDE) is an equation for some quantity (u) which depends on independent variables $x_1, x_2, x_3, \dots, x_n$ (n ≥ 2) and involves derivatives $\frac{\partial u}{\partial x_i}$ of u w.r.t at least some of the independent variables.

Reigon of space Ω

x_1, x_2, \dots, x_n are space variables
If one of the independent variables is time, then there will be some Initial Conditions (IC's) to be satisfied.

Boundary $\rightarrow \partial \Omega \rightarrow$ Boundary conditions need to be satisfied.

BCs

* If there is more than one dependent variable \Rightarrow A system of PDE's arises.

Order $\hat{=} \text{order of the highest partial differential coefficient in the equation.}$

* Linear : Any boundary or Initial conditions and the equation do not include any product of dependent variables or their derivatives.

$$ax + \frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = 0 \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \phi(x, y)$$

(First Order Linear PDE) (Second Order Linear PDE)

* Non-Linear : a) Semilinear $\hat{=} (x+3) \frac{\partial u}{\partial x} + xy^2 \frac{\partial u}{\partial y} = u^3$

$$\frac{x \frac{\partial^2 u}{\partial x^2}}{\partial x^2} + (xy + y^2) \frac{\partial^2 u}{\partial y^2} + u \frac{\partial u}{\partial x} + u^2 \frac{\partial u}{\partial y} = 0$$

If coefficients of the Highest Derivative are functions of independent variables only.

b) quasilinear $\hat{=} \text{derivative of order } m \text{ have.}$

$$ax + \left[1 + \left(\frac{\partial u}{\partial y} \right)^2 \right] \frac{\partial^2 u}{\partial x^2} - \left[2 \left(\frac{\partial u}{\partial x} \right) \left(\frac{\partial u}{\partial y} \right) \right] \frac{\partial^2 u}{\partial x \partial y} + \left[1 + \left(\frac{\partial u}{\partial x} \right)^2 \right] \frac{\partial^2 u}{\partial y^2} = 0.$$

A PDE of order m if coefficients of derivatives of order m depend only on independent variables and derivatives of order $< m$.

Property of linear equation. if $v_1 \in \text{Solution set}(S)$
and $v_2 \in S$ then $\alpha v_1 + \beta v_2 \in S$. ($\alpha, \beta \in \mathbb{R}$)

Wave equation: $\frac{1}{c^2} \frac{\partial^2 v}{\partial t^2} = \nabla^2 u$

Heat Equation: $\frac{\partial u}{\partial t} = k \nabla^2 u = k \nabla \cdot (\nabla u)$

Schrodinger Equation: $\nabla^2 u + V u = i \frac{\partial u}{\partial t}$

Poisson and Laplace eqn: $\nabla^2 u = 0, \nabla^2 u = \phi$

Well posed \Rightarrow exists globally, uniquely and depends continuously on the assigned data.

Ex: Proving uniqueness in Poisson's equation: $\nabla^2 \Omega u = F$ in T where ∂T is the boundary of T .

$$\int_{\partial T} \omega \nabla \omega \cdot n \, ds = \int_T \nabla \cdot (\omega \nabla \omega) \, dV$$

0 since $\omega = 0$ at boundary. $= \int_T (\omega \nabla^2 \omega + (\nabla \omega)^2) \, dV$

$\rightarrow \int_{\partial T} \omega \frac{\partial \omega}{\partial n} \, ds = \int_T (\nabla \omega)^2 \, dV = 0$ but since $\omega = 0$ at ∂T , $\omega = \Omega_1 - \Omega_2; \nabla^2 \omega = 0$ in T

since $(\nabla \omega)^2 \geq 0 \rightarrow \nabla \omega = 0 \rightarrow \omega = \text{const.}$ And $\omega = 0$ on ∂T .

$\rightarrow \omega = 0 \Rightarrow \Omega_1 = \Omega_2$ (unique solution).

* Let Ω_1, Ω_2 be two solutions satisfying the equation. Then consider

$\nabla^2(\Omega_1 - \Omega_2) = F - F = 0$

Griffiths Continued:

Integral Form of Gauss Law:

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

Note: $\vec{E} = \sum E_i$

$$\Rightarrow \oint \vec{E} \cdot d\vec{a} = \oint \sum E_i \cdot d\vec{a} = \sum \oint E_i \cdot d\vec{a} = \sum q_i / \epsilon_0 = Q_{\text{enc}} / \epsilon_0$$

$\rightarrow \oint \vec{E} \cdot d\vec{a} = \int_T (\nabla \cdot \vec{E}) \, dT$

But, $\oint \vec{E} \cdot d\vec{a} = \frac{Q_{\text{enc}}}{\epsilon_0} = \int_V \rho \, dT$ $\Rightarrow \int_V (\nabla \cdot \vec{E}) \, dT = \int_V \left(\frac{\rho}{\epsilon_0} \right) \, dT \quad \forall \text{ Volume } V$

$$\Rightarrow \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

Differential form of Gauss's Law.

The Divergence of \vec{E} :

$$E(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{\mathbb{R}^3} \frac{\hat{r} \cdot \rho(r')}{r'^2} \, dT' \quad \nabla \cdot \vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{\mathbb{R}^3} \nabla \cdot \left(\frac{\hat{r}}{r'^2} \right) \rho(r') \, dT'$$

$$\Rightarrow \nabla \cdot \vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{\mathbb{R}^3} 4\pi r'^2 \delta(r) \rho(r') \, dT' = \frac{\rho(r)}{\epsilon_0}$$

OK!!

$$\int_V \nabla \cdot \vec{E} \, dT = \oint \vec{E} \cdot d\vec{a} = \int_V \frac{\rho(r)}{\epsilon_0} \, dT = \frac{Q_{\text{enc}}}{\epsilon_0} \quad [\text{Gauss's Law}]$$

$$\text{The Curl of } \vec{E} \Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \hat{r}$$

$$\text{So, } \int_a^b \vec{E} \cdot d\vec{l} \cdot dl = dr \hat{r} + r d\theta \hat{\theta} + r \sin\theta d\phi \hat{\phi}$$

$$\Rightarrow \int_a^b \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} dr = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r_a} - \frac{q}{r_b} \right) \Rightarrow$$

$$\oint_L \vec{E} \cdot d\vec{l} = 0$$

And by Stokes
Theorem

$$\nabla \times \vec{E} = 0$$

Electric Potential

\vec{E} : It is a very special vector field with a Zero curl. $\Rightarrow \vec{E} = -\nabla V$

$$V(r) = - \int_{\text{Ref}}^r \vec{E} \cdot dl$$

$$V(b) - V(a) = - \int_a^b \vec{E} \cdot dl + \int_a^b \vec{E} \cdot dl = - \int_a^b \vec{E} \cdot dl$$

for some scalar field V .

$$V(b) - V(a) = \int_a^b (\nabla V) \cdot dl$$

(Fundamental Theorem of Gradient)

* The three components of Electric Field are related to each other by curl formulation. Thus, the three of them can be represented by a single scalar field

$$\nabla \cdot \left(\frac{\partial E_x}{\partial y}, \frac{\partial E_y}{\partial x}, \frac{\partial E_y}{\partial z}, \frac{\partial E_z}{\partial y}, \frac{\partial E_z}{\partial x} \right) \rightarrow \text{This allows us to reduce a vector problem to a scalar one.}$$

Poisson's Equation and Laplace's Equation +

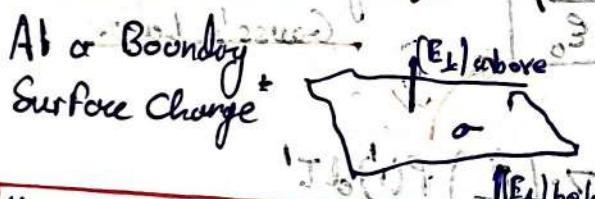
$$\vec{E} = -\nabla V, \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}, \nabla \times \vec{E} = 0 \Rightarrow \nabla^2 V = -\frac{\rho}{\epsilon_0} \quad (\text{Poisson's Eqn})$$

Thus we only need 1 eqn to define Potential whereas we need 2 eqns (div and curl) to define \vec{E}

For a Point Charge : $V(r) = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r} \Rightarrow \frac{1}{4\pi\epsilon_0} \int \frac{\rho(r')}{r'^2} dt' = V(r)$

$V(r) = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r'} \quad (\text{for a continuous charge distribution})$

$$\text{Ans} = \frac{\sigma}{2\pi} (\sqrt{R^2 + z^2} - R)$$



However, Potential is continuous across any boundary.

$$V_{\text{above}} - V_{\text{below}} = \int_a^b \vec{E} \cdot dl = 0$$

$$\Rightarrow V_{\text{above}} = V_{\text{below}}$$

$$(i) (E_{\perp})_{\text{above}} = (E_{\perp})_{\text{below}}, \quad (ii) (E_{\parallel})_{\text{above}} = (E_{\parallel})_{\text{below}}$$

so, $\nabla V_{\text{above}} - \nabla V_{\text{below}} = -\frac{\sigma}{\epsilon_0} \hat{n}$ in a direction

$$\text{And } \frac{\partial V_{\text{above}}}{\partial n} - \frac{\partial V_{\text{below}}}{\partial n} = -\frac{\sigma}{\epsilon_0} \hat{n}$$

where $\frac{\partial V}{\partial n} = \nabla V \cdot \hat{n}$ (normal derivative of V)

Work and Energy in Electrostatics :

$$\sqrt{b} - \sqrt{a} = \frac{W}{\phi}; \quad \frac{W}{\phi} = \sqrt{r} - \sqrt{\infty}$$

energy to assemble a system!

$W = \frac{1}{2} \sum_{i=1}^n q_i \sqrt{r_i}$ → Potential at point r_p due to all other charges. Discrepancy

The Energy of a Continuous Charge distribution :

$$W = \frac{1}{2} \int \rho V d\tau \xrightarrow{\rho = \epsilon_0 \nabla \cdot \vec{E}} W = \frac{\epsilon_0}{2} \int (\nabla \cdot \vec{E}) V d\tau$$

Since $\nabla \cdot \vec{E}$ is full Potential.
but $(\nabla V) = -\vec{E}$

Goes to zero over All space. ($V \rightarrow 0$)

$$= \frac{\epsilon_0}{2} \left[- \int \vec{E} \cdot (\nabla V) d\tau + \oint \vec{V} \cdot \vec{E} \cdot d\vec{a} \right]$$

$$\Rightarrow W = \frac{\epsilon_0}{2} \left[\int \vec{E}^2 d\tau + \oint \vec{V} \cdot \vec{E} d\vec{a} \right]$$

↓ increases ↓ decreases.

→ we can take V to trap all the charge but even if we increase the Area beyond the minimum required, the same Energy should be trapped in the Electric Field as $(P=0)$

$$W = \frac{\epsilon_0}{2} \int \vec{E}^2 d\tau = (\text{all space})$$

Energy is stored in the field with a density,

$$\frac{\epsilon_0}{2} \vec{E}^2 = \text{energy per unit volume.}$$

(Quadratic \Rightarrow Proportional terms.)

$$(V_{\text{solid sphere}}) = \frac{\rho}{2\epsilon_0} \left(R^2 - \frac{r^2}{3} \right) = \frac{Kq}{2R} \left(3 - \frac{r^2}{R^2} \right)$$

$$\rightarrow W_{\text{tot.}} = \frac{\epsilon_0}{2} \int \vec{E}^2 d\tau = \frac{\epsilon_0}{2} \int (\vec{E}_1 + \vec{E}_2) \cdot (\vec{E}_1 + \vec{E}_2) d\tau = \frac{\epsilon_0}{2} \int \vec{E}_1^2 + \vec{E}_2^2 + 2\vec{E}_1 \cdot \vec{E}_2 d\tau.$$

$$\rightarrow W_{\text{tot.}} = \epsilon W_1 + W_2 + \epsilon \int \vec{E}_1 \cdot \vec{E}_2 d\tau \rightarrow \text{Electrostatic Energy does not obey superposition principle.}$$

Conductors :

"A perfect conductor would ~~not~~ contain an unlimited supply of free charges." !!!

- (i) $E = 0$ inside a conductor: Field due to induced charges cancels out External Field.
- (ii) $\rho = 0$ inside a conductor: $\nabla \cdot E = \frac{\rho}{\epsilon_0} \Rightarrow$ If $E = 0 \rightarrow \boxed{\rho = 0}$.
- (iii) Any net charge resides on the surface.
- (iv) A conductor is on eqv potential. $\rightarrow \boxed{V(b) = V(a)}$ $\rightarrow E_{\text{tangential}} = 0$
- (v) E is perpendicular to the surface of the conductor, just outside the conductor.

! Free Charges cannot Flow in a conductor in Electrostatic conditions.

→ The charge on the conductor will seek the configuration that minimizes its PE.

④ The Electrostatic Energy of a Solid Object is minimum when the charge is spread over the surface.

Force on Conductor and capacitors

$$(E_{\text{outside}})_{\text{just}} = \frac{\sigma}{\epsilon_0} \hat{n} \quad \text{And} \quad \sigma = -\epsilon_0 \frac{\partial V}{\partial n}$$

These Equations allow us to calculate surface charge of the conductor provided we know E and V .

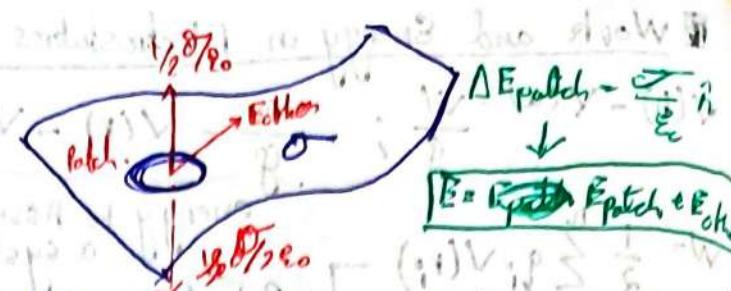
Capacitors.

$$\nabla \cdot \mathbf{V} = \nabla \cdot \mathbf{E} = - \int \mathbf{E} \cdot d\mathbf{l}$$

$$E \propto \rho \quad \text{and} \quad C = \frac{\rho}{\epsilon_0}$$

$$C_{\text{parallel}} = A \frac{\epsilon_0}{d} \quad C_{\text{sphere}} = \frac{4\pi \epsilon_0 a b}{b - a}$$

$$\text{Energy} = \frac{\rho^2}{2C} = \frac{1}{2} CV^2$$



$$(F)_{\text{unit area}} = \sigma E_{\text{average}} = \frac{\sigma}{2} (E_{\text{above}} + E_{\text{below}})$$

$$E_{\text{above}} = F_{\text{other}} + \frac{\sigma}{2\epsilon_0} \quad E_{\text{below}} = F_{\text{other}} - \frac{\sigma}{2\epsilon_0}$$

$$E_{\text{other}} = E_{\text{avg}} = \frac{E_{\text{above}} + E_{\text{below}}}{2}$$

$$So \quad F = \frac{1}{2\epsilon_0} \sigma^2$$

This amounts to an outward electrostatic pressure that tends to draw the conductor into the field. Given by, $P = \frac{\epsilon_0}{2} E^2$ just outside

Lecture 8 - 29/01/24

(The Boundary Defines Everything)

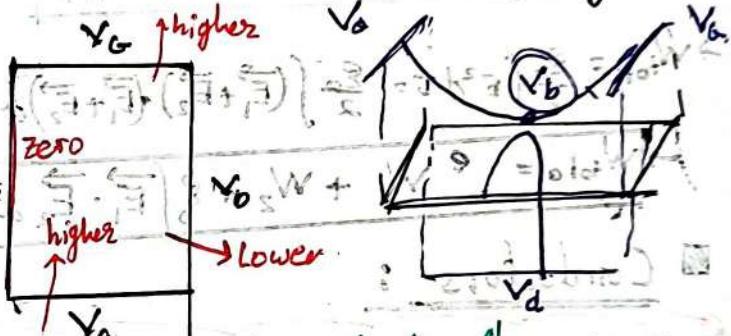
Electrostatics

Dielectric + Metal - Scale Free.

- 1D $G \parallel C_2 + C_3$
- 2D Laplace eqn
- 3D Finfets

* systems w/ length, scale factors

M + D + semiconductor aspects



Curvatures in orthogonal directions has to be equal and opposite

$$I_{S \rightarrow D} = I(V_B)$$

The current going from source to drain is controlled by the lowest barrier.

Does the e^- know the lowest barrier?

NOT
 $I = e^{-eV}/(eV + 1)$

If e^- is going from everywhere then why does it depend on minimum. Why not avg, max or any other function.

Transformed by an exponential
So V_b obs. affects the current function the most

* The other paths are there but not significant

$$SS = \frac{dV_b}{dV_G}$$

$$DIBL = \frac{dV_b}{dV_D}$$

$$i_{S \rightarrow D} = e^{-V/kT} \approx 25mV$$

→ Drain induced barrier lowering.

$$DIBL < 20mV/V/decade$$

The math goes back to exponential

* How can I flatten the exponential decay. → (Increase length L).

(From Square waves Resolved)
Using Fourier methods.

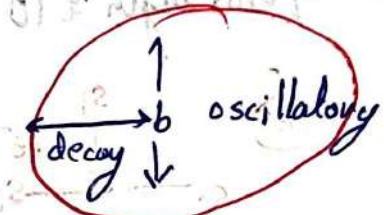
K_0 comes from width $\rightarrow K_0 = \frac{2\pi}{b} \rightarrow$ So increasing width will too.

If $b \downarrow$ → Length scale Reduced → Spatial frequency (\uparrow) → Decay length (\downarrow)

And if it decays faster
↓
Lower DIBL.

[Instead in the Decay orthogonal to $b \downarrow$ direction.]

$$V_0 e^{-x/b}$$



De

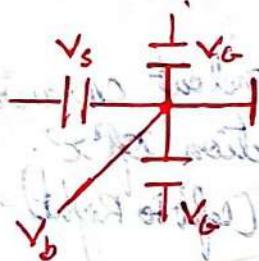
$$V(x, y) = \sum_{n=0}^{\infty} (-1)^n \frac{V_0}{ak_n} \cos(k_n y) \frac{\sin(k_n x)}{\sin(k_n b)}$$

N.B.

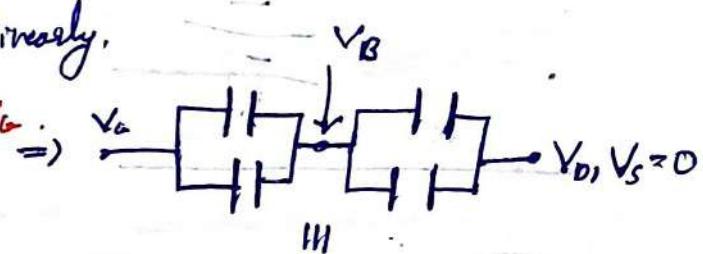
$$V(x, y)$$

→ If I increase V_a linearly how will V_b vary → linearly

However depends on y non-linearly.



What if $V_a = V_b$ and $V_D = V_s$.

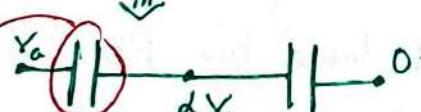
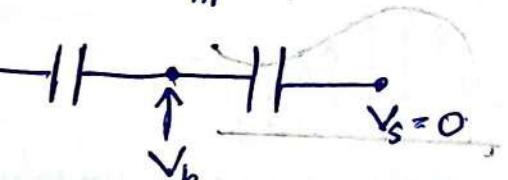


This is a Dumb down model of the potential profile of a FINFET.

Funky Capacitors

Is this own assumption or is it absolutely accurate mathematics

This is correct for the correct capacitances. The Potential in these capacitances is non-linear with position.



Can I formulate a completely linear model?

Finfets Fundamental length scale of scaling is geometric or structural.
 Scalable \rightarrow Material length scaling is absent. Finfets were created to the sole purpose to avoid geometric length scaling. (Shape is everything).

Screening in Semiconductors

(1)

Band Gap, Fermi level and Equilibrium

$$\text{Atoms/unit area} \approx 10^{22}/\text{cm}^2$$

$$\text{Atoms/unit length} \approx 10^7/\text{cm}$$

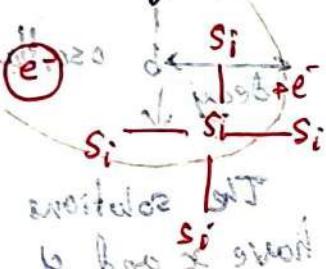
$$n \cdot p = n_e^2 \quad (\text{Intrinsic Semiconductor})$$

negative carriers
positive carriers
no. of bonds broken at a particular temperature

(↑) Increase $n \cdot p$ \leftarrow Limited Temperature

$$10^{22}/\text{cm}^3$$

$$= 1.8 \times 10^{14} \text{ cm}^{-3}$$



e^- plus a broken bond \downarrow
[Deionized water]

Any disturbance will finally

reach thermal equilibrium.

$$n = 10^{20}/\text{cm}^3$$

$$n = p = 10^{10}/\text{cm}^3$$

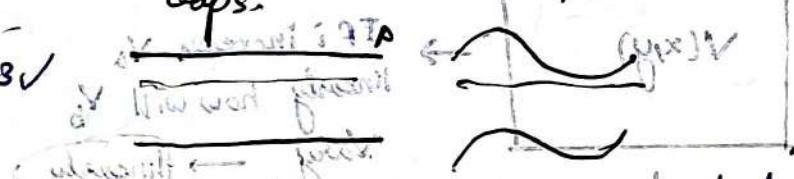
$$p = 10^{20}/\text{cm}^3$$

Resistivity of pure water at R.T.

[Deionized water]

Used to wash Silicon wafers.

(Ques.) Can we estimate n and p via band gaps.



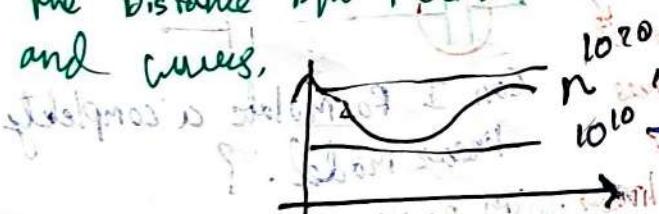
Boltzmann is linear in the exponential scale.

Can I quantitatively find out carrier concentration as a function of x ? (Left to Right)

Draw it EXACTLY AND QUANTITATIVELY



Iska log varies linearly with the distance b/w Fermi level and curves.



If a band bends, how many free carriers are generated.

Laplace — Scale free $\rightarrow \nabla^2 V = 0$ which is a simplification for only Metals and Insulators.

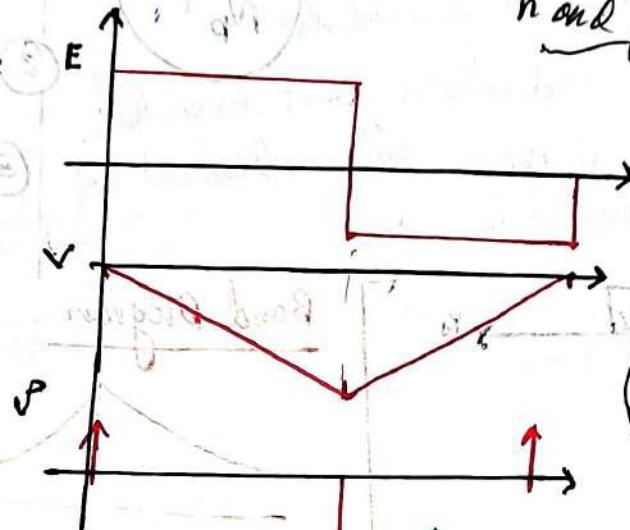
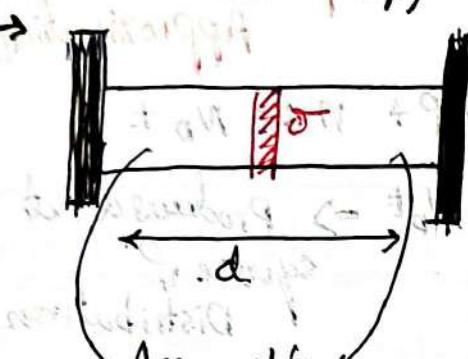
Scale Dependent $\rightarrow \nabla^2 V = -\frac{\rho}{\epsilon}$

\rightarrow metal + Insulator + Semiconductor.

\rightarrow Band Gap] some dependence of
 \rightarrow Fermi Level] n, p , some dependence of
 $n(E_F), p(E_F)$

if I know conduction band, valence band and fermi level \downarrow

I can calculate n and p in 1D.



As long as there is no current flow, it's equilibrium.

Assume it's a dielectric and study its Resistor.
 (But this is a semiconductor)

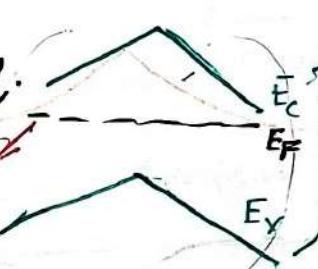
Band Diagram \downarrow

(It is about e^- energy)

Fermi level is flat in Equilibrium.

Electric field and potential change the Energy of the Vacuum level.

First calculate in log scale the compute in linear scale.



Are these consistent?

only if the semiconductor is not a semiconductor because now the conductor will have a response

Do you know how to calculate the carrier concentration.

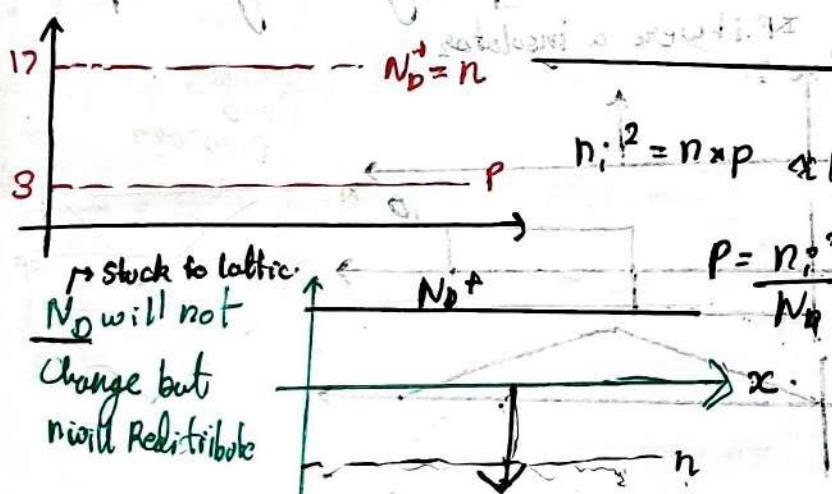
What is the correction in charge?

$$\left[-\frac{\rho}{\epsilon_0} (1 - p - n + N_D^+ - N_A^+) \right]$$

this must be added.

Semiconductor \downarrow

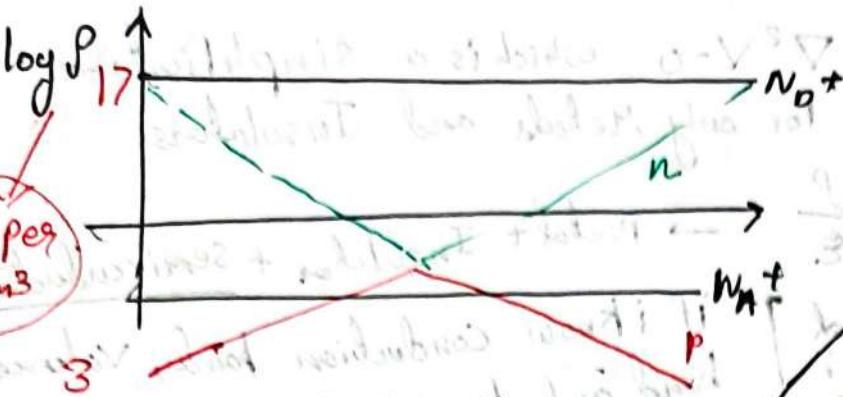
$$E_c \downarrow E_F \downarrow E_v$$



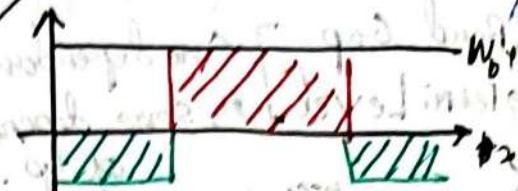
n-type semiconductor

Doping $N_D = 10^{17}/\text{cm}^3$.

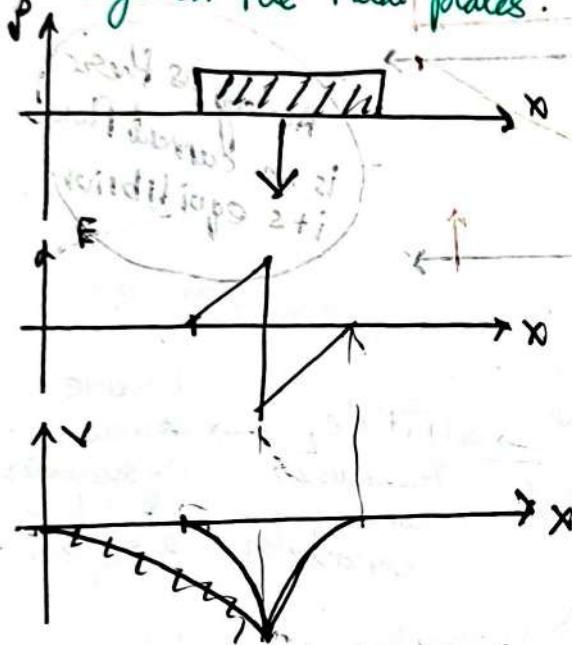
hand-pore-free e- and holes



① Electrons are pushed away from negative sheet charge. (-Q).

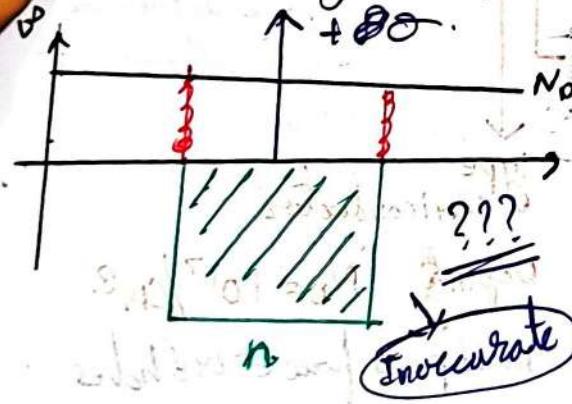


Now the +ve charge has been accounted for by the semiconductor. So, the metal conductors don't even feel the -ve charge in the middle and hence no charge on the metal plates. So,

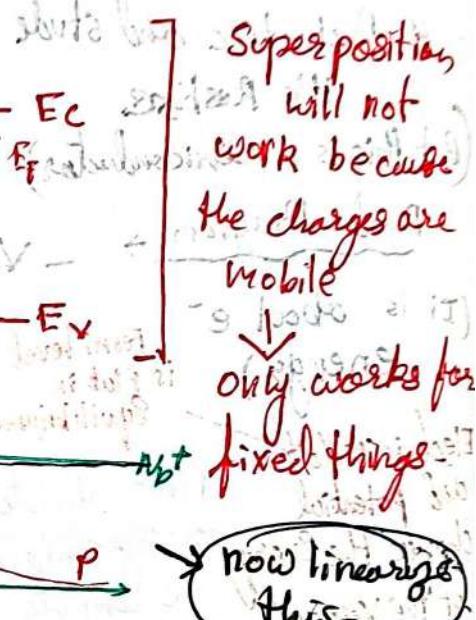


{ This is called Depletion Screening }

What if Q were a positive sheet of charge?

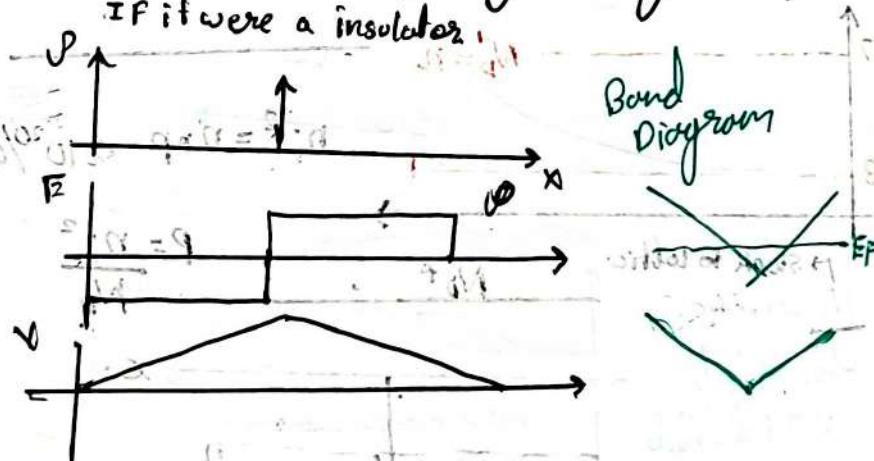


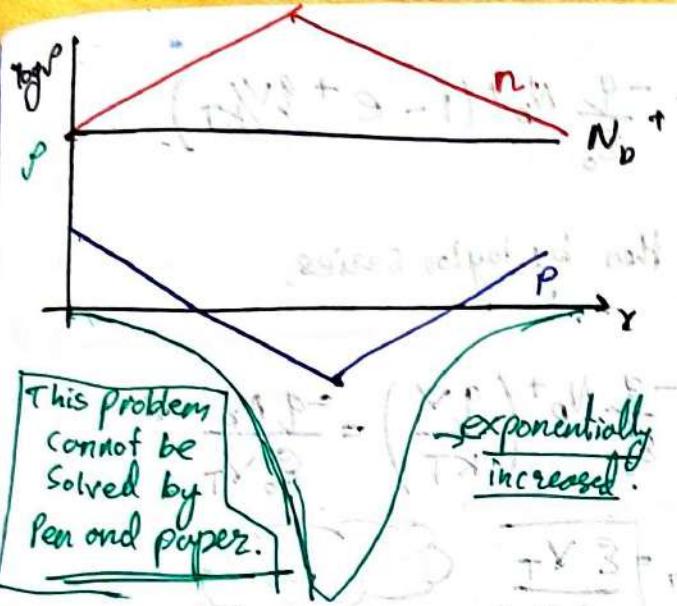
- ② $\exp(-Q/x) \ll N_D^+$.
- ③ $N_D^+ \rightarrow$ Produces a net square of Distribution.



This will give me a quadratic in the exponential scale which is a pretty strong drop.

IF it were an insulator.





- Not a nice square charge distribution
- ① what will an insulator do?
 - ② what will a semiconductor do?
 - ③ How to make the solutions consistent

Electrostatics + Devices

Gauss's Law

semiconductor carrier statistics.

Griffiths Chapter 3: Potentials

somewhat more tractable but $\rho(r)$ may vary.

How can I find electric field? $\rightarrow E(r) = \frac{q}{4\pi\epsilon_0} \int \frac{\rho(r')}{r'^2} dV' \rightarrow V(r) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(r')}{r'} dV'$

Too though to Integrate

"Electrostatics is the study of Laplace's Equation":

$$1D: \frac{d^2V}{dx^2} = 0 \rightarrow V(x) = mx + b \rightarrow \text{set boundary conditions.}$$

Notice: $V(x) = V(x+a) + V(x-a)$; Laplace's Equation is a kind of Averaging Instruction.

Poisson's

If there is no charge \downarrow
 $\nabla^2 V = 0$. Laplace's Equation.

Also, Laplace's Equation tolerates no local maxima/minima. All Extrema must be at the boundaries. Since If an extrema were to exist, it would violate the averaging condition.

Lecture 10 - 05/02/24 :-

- $E_c, E_V, E_V \leftrightarrow n, p$
- Doping charges, free carriers.

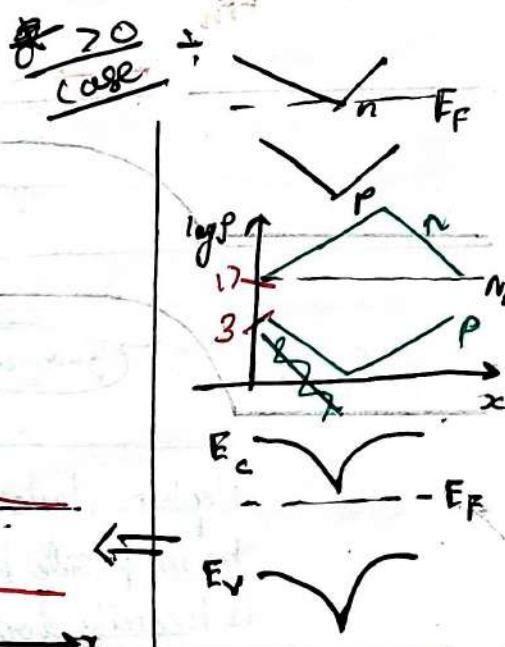
$$\frac{d^2V}{dx^2} = -\frac{\rho}{\epsilon_0} = -\frac{q}{\epsilon_0} (N_D^+ - N_A^+) + \rho(x) - n(x)$$

$$\frac{d^2V}{dx^2} = -\frac{q}{\epsilon_0} (N_D^+ - n(x))$$

$\log(\rho)$

17

3



$$\text{Now, } n = N_D + e^{-qV/kT} + qV/kT \rightarrow \frac{d^2V}{dx^2} = -\frac{q}{\epsilon_0} N_D (1 - e^{-qV/kT}).$$

2 cases

Free carrier
Dominates

Dopant
Dominates.

IF Dopant Dominates,

$$\frac{d^2V}{dx^2} = -\frac{qN_D}{\epsilon_0}$$

$$\rightarrow V(x) = \frac{qN_D}{\epsilon_0} \frac{x^2}{2} \Big|_0^L$$

$$X = \frac{qN_D L^2}{\epsilon_0 2} \rightarrow Z = \sqrt{\frac{2\epsilon_0}{qN_D}}$$

Depletion Length,

$$\Rightarrow \frac{d^2V}{dx^2} = -\frac{q}{\epsilon_0} N_D \left(\frac{qV}{kT} \right) = \frac{qN_D}{\epsilon_0 kT}$$

$$= \frac{qV}{L_n^2}$$

$$L_n = \sqrt{\frac{\epsilon_0 kT}{qN_D}}$$

Debye length

However my voltage can be very high,
we can't do math there

This is how much it takes to bend IV.

At OK, Debye length becomes 0 \rightarrow Perfect Screening.

Why does screening become worse with increase in Energy. You can squish the free carriers in one place due to their thermal energy.

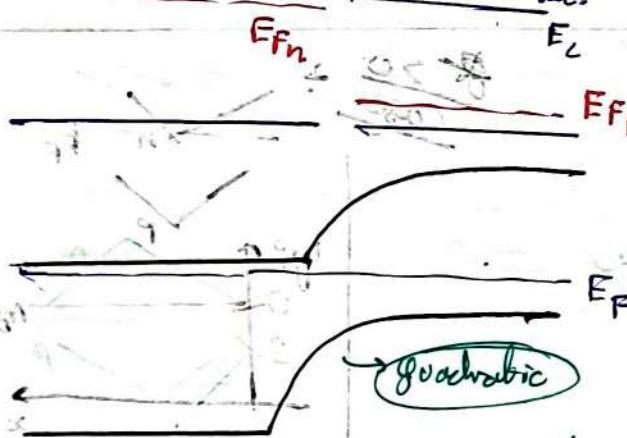


P-doping is $10^{13}/\text{cm}^3$

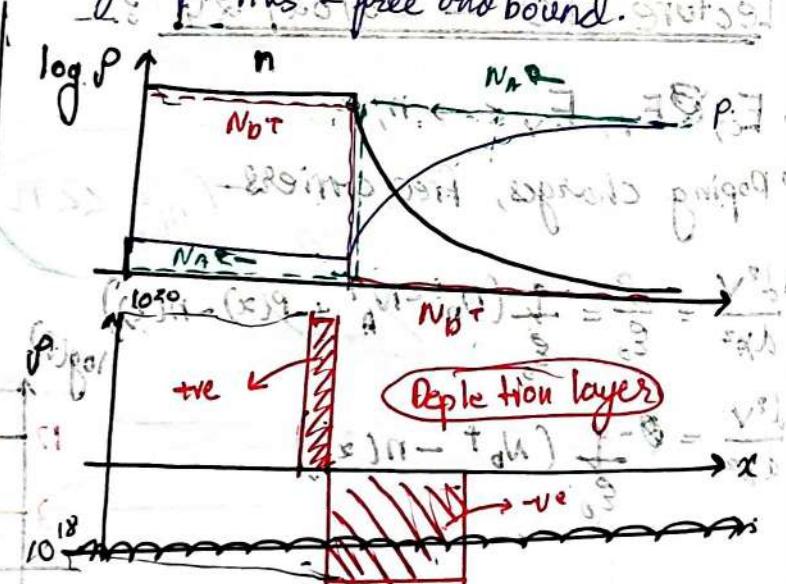
n+ doping is $10^{20}/\text{cm}^3$

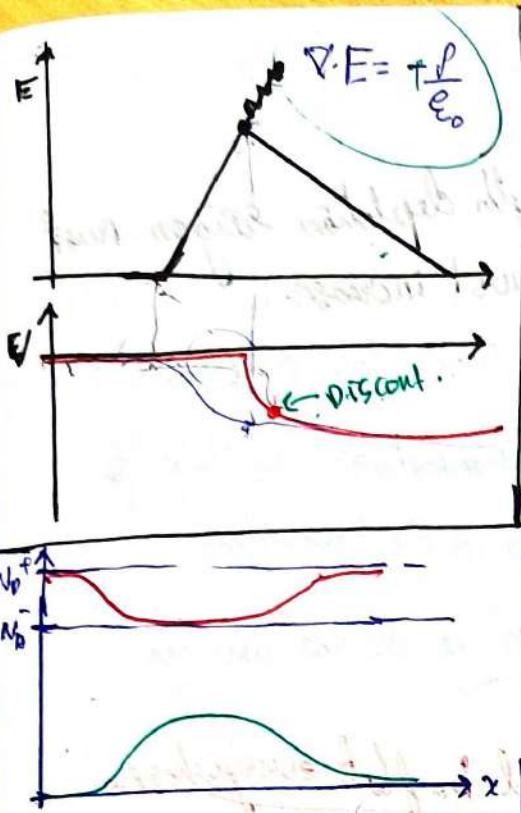
20 μm of n+ Write down the steps including Assumptions.

Evac. to draw the band diagram. Explicitly draw all charge profiles - free and bound.

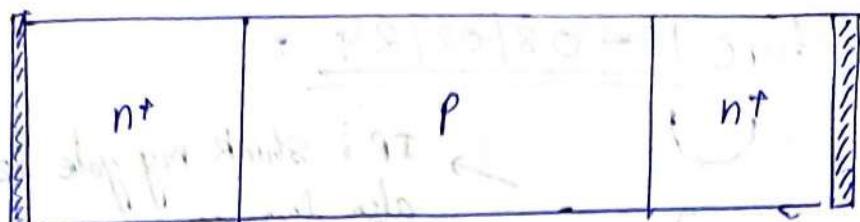


Depletion layer will be in p-side bcoz n is heavily doped





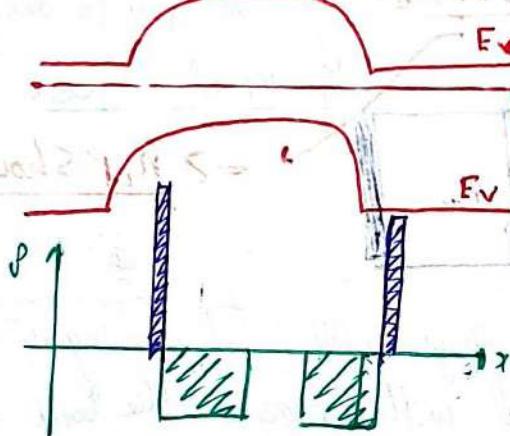
The n-type free carriers are greater than the p-type free carriers in the p-region.



20nm; notice the units.

Smaller than my depletion length?

What if my length was 20μm.



$$q = 1.6 \times 10^{-19} C$$

$$\epsilon_0 = 8.84 \times 10^{-12} F/cm$$

$$N_p = 10^{17}/cm^3$$

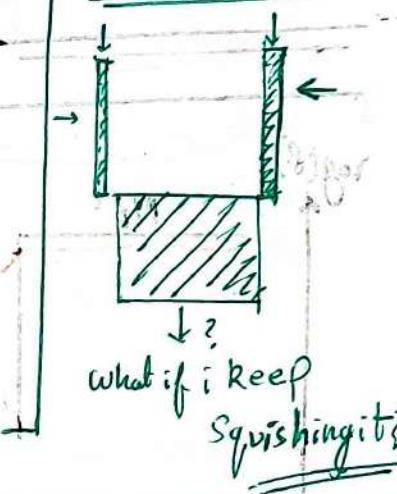
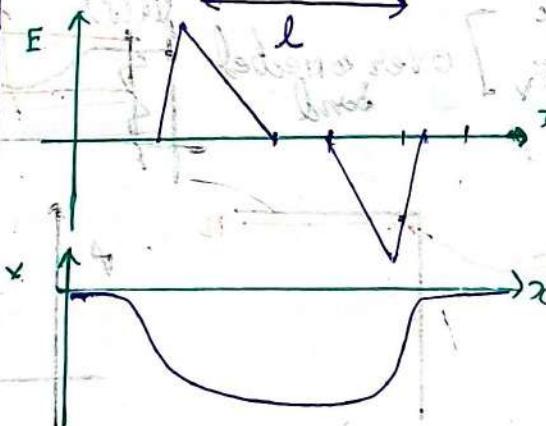
$$V = 1 V$$

$$(E_a)_{semi} = 11.9$$

$$L = 1.15 \times 10^5 cm$$

$$= 115 nm.$$

Now : take this and Reduce it to nanoscale.



what if i keep squishing it??

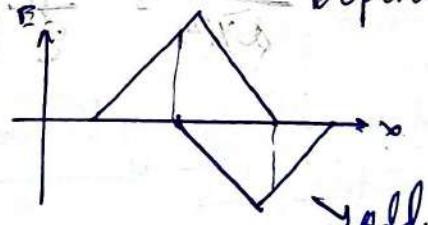
If we keep Reducing the length, the free carriers of n+ doping will over power the Dopant.

$$\frac{N_D}{l} \rightarrow \frac{n_D}{l}$$

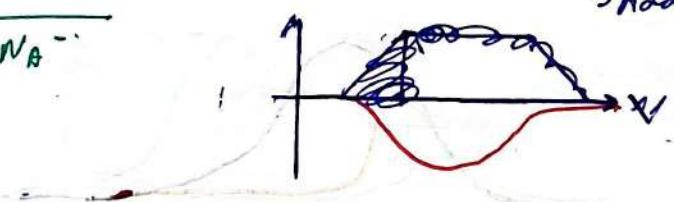
$\leftarrow (p) \rightarrow$

$$n^+ \rightarrow \leftarrow n^+$$

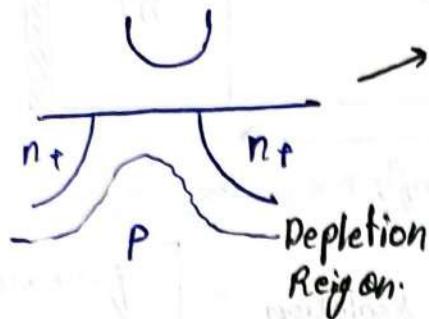
$$N_A^-$$



Add.



Lecture 11 - 08/02/24 :



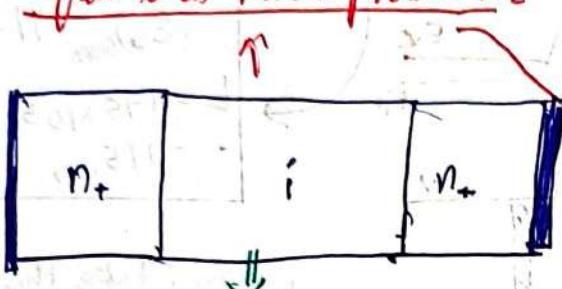
If the intrinsic Region has no charge and the bands are flat then this is problematic.

→ If I shrink my gate length depletion region must also decrease. So N_D must increase.

Doping largely varies Transistor electrostatics.

cause disparities in transistors as they lie in different places in the transistor.

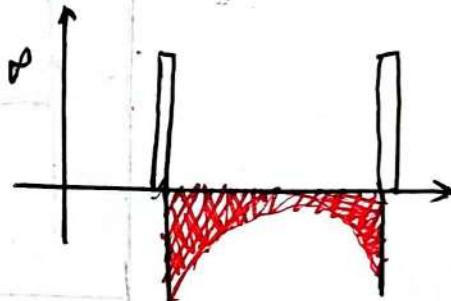
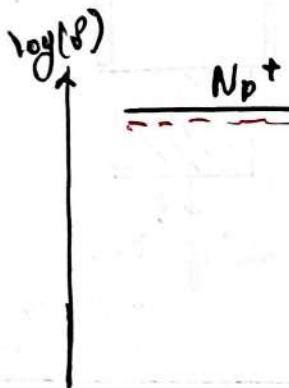
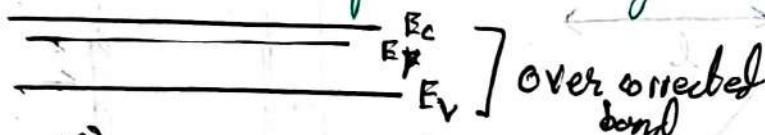
so we try to design a device which if Dopant Free:



→ n, p Should be flat everywhere.

The intrinsic Region will be filled with charge.

coming in e^- will feel ~~feel~~ more the band bend upwards.



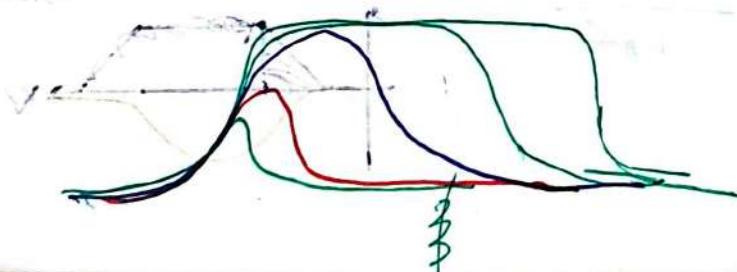
Can you solve Poisson's equation in the middle.

$$\frac{d^2V}{dx^2} = -\frac{q}{\epsilon} \left[-N_D^+ \exp\left(\frac{-qV(x)}{k_b T}\right) \right]$$

→ Numerical Solution

How will my barrier change with decrease in length in a npn transistor?

Two ways



Two types of monopoles carried by semiconductors. Dopeants and free charges.

Numerical Poisson Equation :

HOW DOES A SOLVER WORK!! —

A solver cannot solve big things.

1) Linearization

2) I will work in a scale free environment.

I don't have to worry about dimension in my equations.

$$\frac{d^2\phi}{dx^2} = -\frac{q\rho}{\epsilon} \text{ where } \rho = N_b + p(x) - N_A - n(x).$$

$$E_F = E_c + E_v$$

$$n = n_i \exp\left(\frac{E_F - E_F^0}{k_B T}\right) \Rightarrow \frac{n}{n_i} = \exp(\phi)$$

where $\phi = \frac{E_F - E_F^0}{k_B T}$

$$\text{Similarly } p = n_i \exp\left(\frac{E_i - E_F}{k_B T}\right) = n_i \exp(-\phi)$$

$\phi_{n \rightarrow i}$ → Iteration

PLs
Read Today's
Slides Very
Carefully.

Lecture 12 - 12/02/24 :

Numerical Solver → Depletion and Debye length

makes life simple (fairly square)

Doping → can integrate etc

For Debye length, $\rightarrow n, i, n \rightarrow$ while we could intuitively guess the profile, intuitively we couldn't provide and justification

Computational Nanoelectronics.

- D Vasileska, S.M Goodnick, G Klimeck
- CRC Press.
- Chapter 4.

\Rightarrow If I wanted to know the newer potential profile, I needed to know the older guess.

\Rightarrow linear equation

Can use Matrices.

A code to play with.

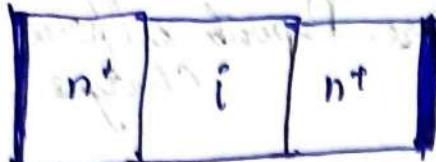
MATLAB code

LU Decomposition

What is the numerical value of the Depletion Region from graph ($\approx 12 \mu m$).

n-p-n → Depletion charge is higher than Free carrier charge.

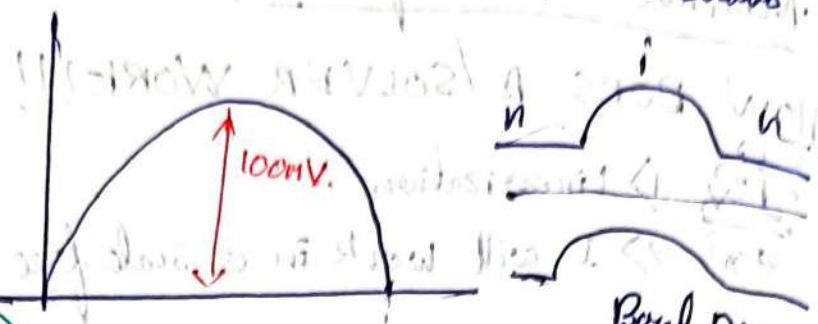
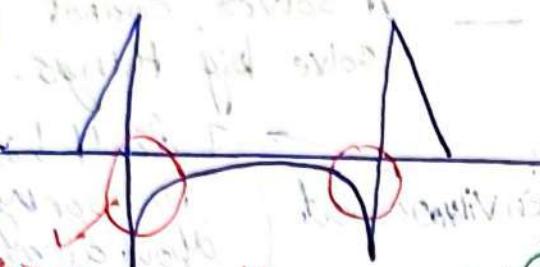
Depletion Approximation.



$V, \delta, \log(P), E.$

10 \rightarrow highly doped
20 \rightarrow pretty metallics.

P)



This shape is not known so we can't know the Electric field and Voltage. [Problem]



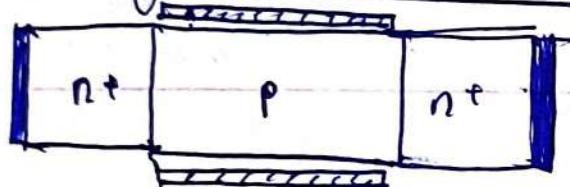
Free carriers.

Bond Diagram

* Electric field also has a weird shape which comes about the numerical solution.

* \rightarrow This was the reason we Required a Numerical Solution.

Let's say we have a FinFet.



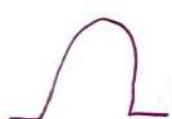
n+ doping is $10^{18}/\text{cm}^3$.

p vary from $[1, 16, 100] \times 10^{12}/\text{cm}^3$

Q.) Can we increase the barrier to kill the free carrier screening?

∇_T is a significant change in barrier potential.
(60mV).
 \Rightarrow 2mV change and I'm fine.

THE GATE (apply 0.4V) when off turns the FinFet off during which there are no carriers in the middle.



w/o Any Gate potential.



Currently

SS and DIBL is bad.

Can't even start computing in the current context.

IF I increase the barrier potential via the gate, I'll pull the carrier concentration down to negate this effect.

Make's Laplace Valid.

(If I increase the barrier potential by say, 300mV - 6 orders of ∇_T)
So my carrier concentration is pulled down and overpowered by the dopant.

OFF State's Where we can solve Laplace

(i) Then we can Approximate as Approximation Laplace since carriers are negligible.

(i) Laplace $V_b = 0$ @ $V_a = V_s = V_D = 0$.

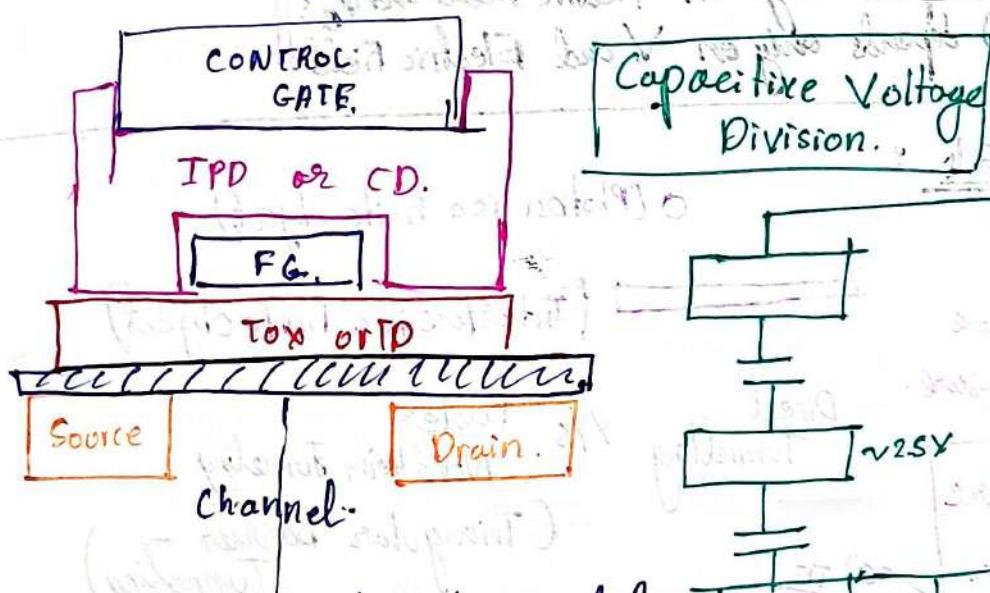
but Δ free $\approx 100mV$.

Nanocrystal based Flash Memory : An Application of Semiconductor Electrostatics;

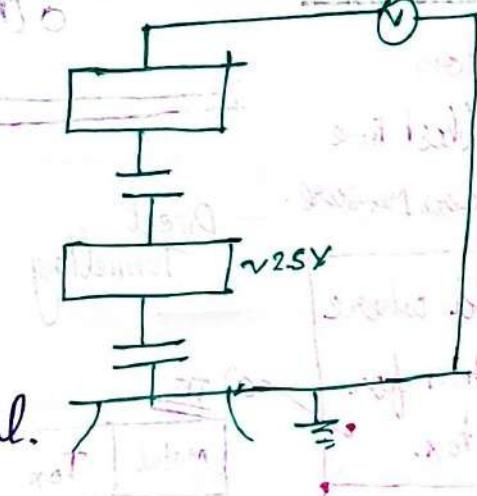
I need to push a voltage on the Floating gate

such that there is a high Electrical Field between the FG and the channel

no electrical leads

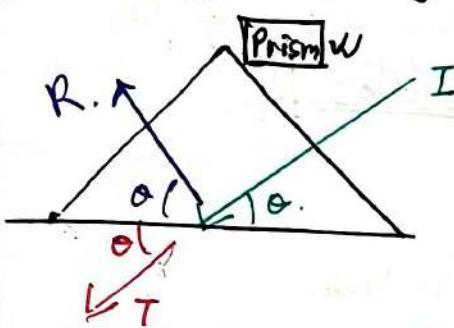


- IPD: Inter poly Dielectric.
- blw two polysilicons
- Manufacturing term.
- Insulator.



In ON state, channel & metal.

Tunneling (New physics)



The Light Analogy:

If we place a prism with light and go beyond a critical angle θ_c we get Total Internal Reflection.

Think: Where does the light "decide" to turn around

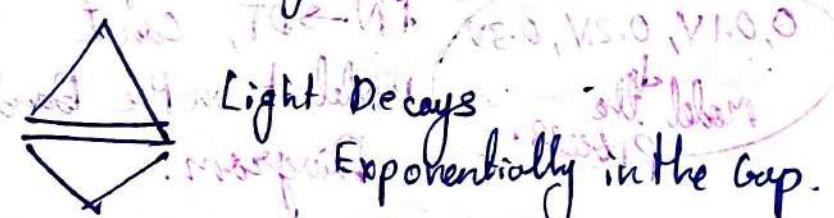
- At
 - before
 - After
 - OR irrelevant.
- The Surface.

What is an Interface?

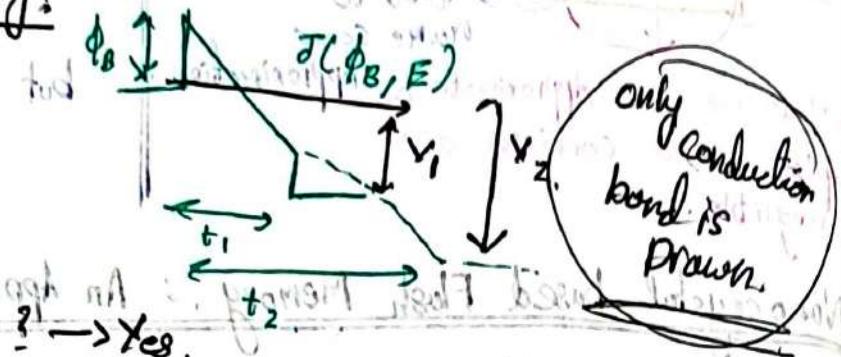
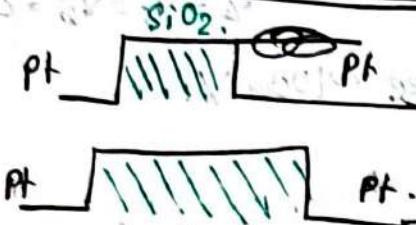
It is a fictitious plane that separates two materials.

Any wave can tunnel.

(Q.) What Experiment will I do to find out exactly where it is turning around?



A bit more on Tunneling?



Is Current \propto dependent? \rightarrow Yes.

Pair: If barrier is Triangular, can I claim:

(a) Current Density (J) depends only on V and ϕ_B ?

(b) Current Density (J) depends only on Electric field and ϕ_B ?

(c) Current Density (J) depends only on V and Electric Field?

Lecture 13 - 15/02/24.:

① Size of a photon

② what is the smallest time scale, the universe can measure.

Q.) what V and E field where $FN \rightarrow DT$ transition for different thickness t_{ox} .

③ Most of the current comes from the Fermi level electrons tunnelling through.

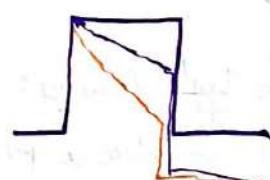
O (Photon is a finite object)

(Interference is a finite object)

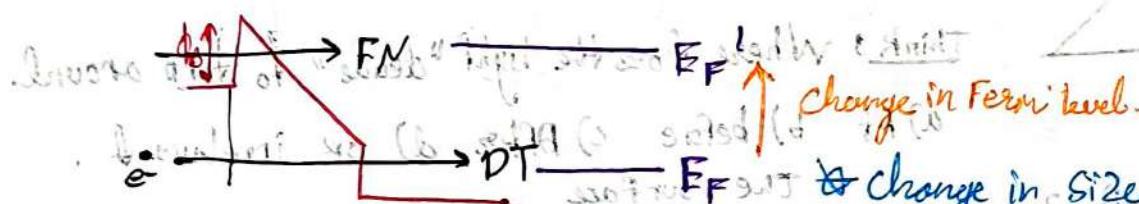
Direct Tunnelling

y/s Fowler Nordheim Tunnelling

(Triangular Barrier Tunnelling)



(BE) E field penetrates into the insulator!!!



change in size of the insulator will change the barrier bending and

At what point will I transition from $FN \rightarrow DT$, can I model it on the Band Diagram?

0, 0.1V, 0.2V, 0.3V

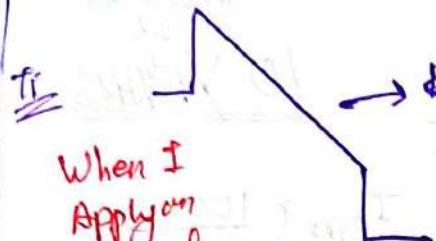
model the process.

Model it on the Band Diagram:

$$\phi_B = \frac{q}{8\pi\epsilon_0} \frac{V}{t_{ox}}$$

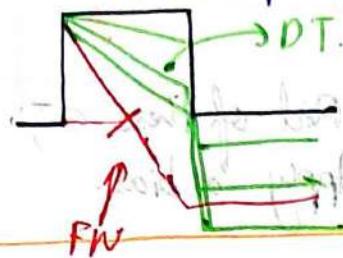
$$E = \frac{q}{8\pi\epsilon_0} \frac{V}{t_{ox}}$$

Now If I Apply a Voltage.



When I apply an external potential.

Linearly Increasing Potential profile.



If will get DT if I can see.

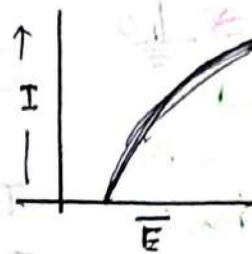
A Trapezoid within my barrier height

If I see a Triangle (tunneling Region).

within my ϕ_B Region then it will follow FN tunneling.

Lecture 14 -

19/08/2024



$$I = f(E)$$

$$\frac{dI}{dE} = \sigma E$$

I v/s σE

$$(1/R)$$

$$\frac{\partial I}{\partial E} \rightarrow \sigma A$$

$$A \frac{\partial E}{\partial I} \rightarrow P$$

$$\text{And } \left(\frac{\partial E}{\partial I} \right) \cdot \frac{1}{t_{ox}}$$

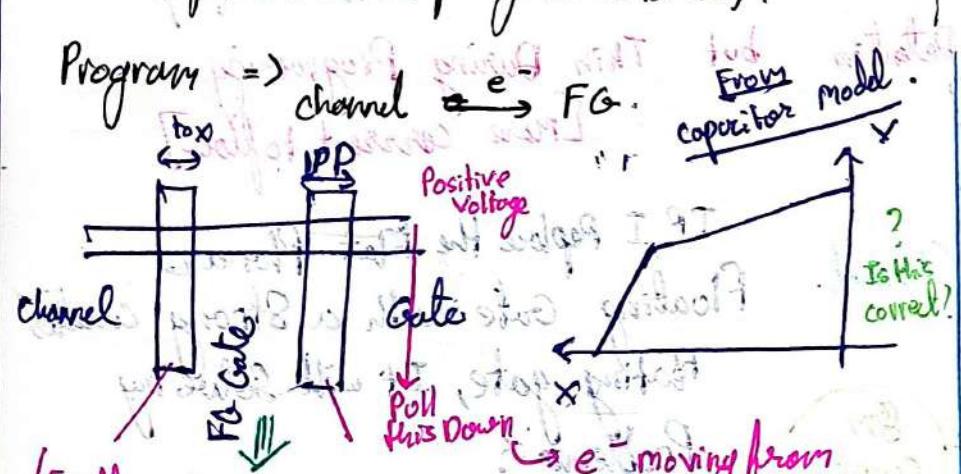


Flashing Memory Operations

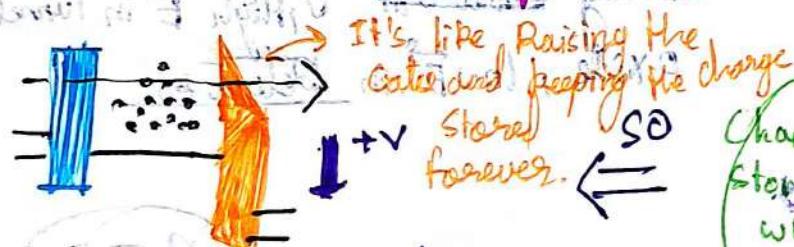
Channel, Floating gate and Real gate are all metallic.

(Assumption to Simplify calculation).

Program \Rightarrow



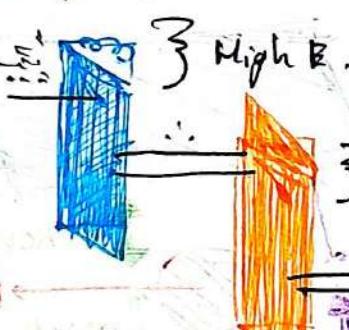
(Field in the two Insulators) e^- moving from Right to Left



I want to store this charge for a long time.

Program

Voltage (ϕ)



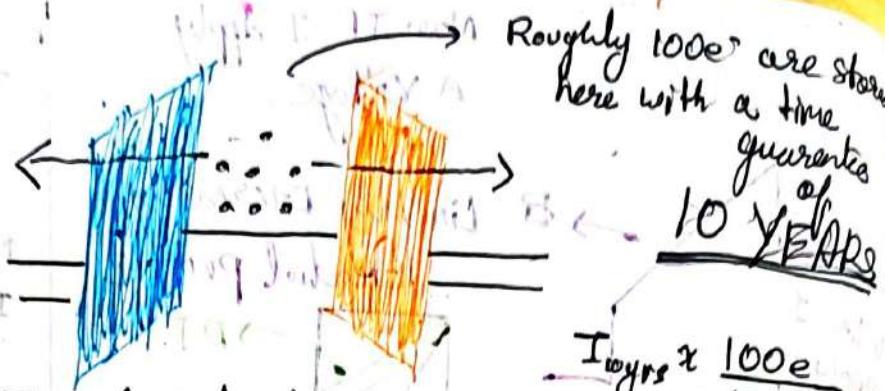
Now after e^- move, charge gets stored and the potential of the floating gate becomes more negative.

(Incoming current = outgoing current) \downarrow steady state.

RETENTION +



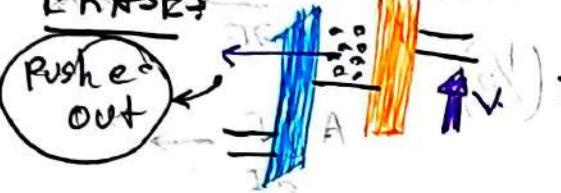
[A capacitive system with a pseudo voltage in between]



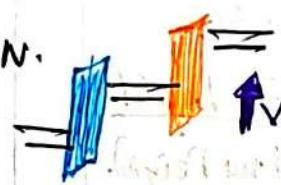
To Get Rid of these e^- , I must Apply a bias.

$$I_{10\text{ yrs}} \propto \frac{100e}{10\text{ yrs}} \times 10^{-24} \text{ A} \times 10^{-26} \text{ A}$$

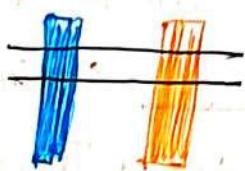
ERASE \rightarrow



THEN,



(Remove THEN Bias)



- Dielectrics have some length scale dependence for tunnelling. Decays exponentially with length.

Back to Stage 1

DESIGN PROBLEM

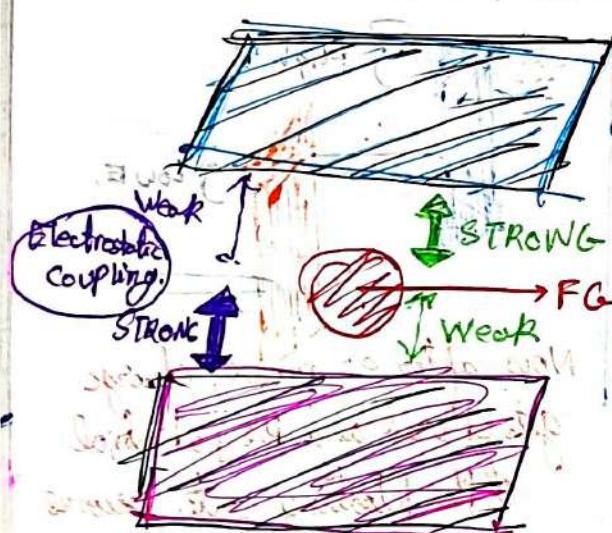
Write down the requirement of tunnel oxide v/s IPD. for Prog (18V) v/s Retention (10V) to set the thickness.

① IPD \downarrow

Is this a Design problem or a Dilemma?

② Tun. Oxide \downarrow Thick during Retention [Barrier] but Thin During Programming [more correct to place]

Thin During Programming [more correct to place]



Channel.

GM Coupling

If I Replace the Flat Planar Floating Gate with a Strong circuit Floating gate, It will Solve my Problem?

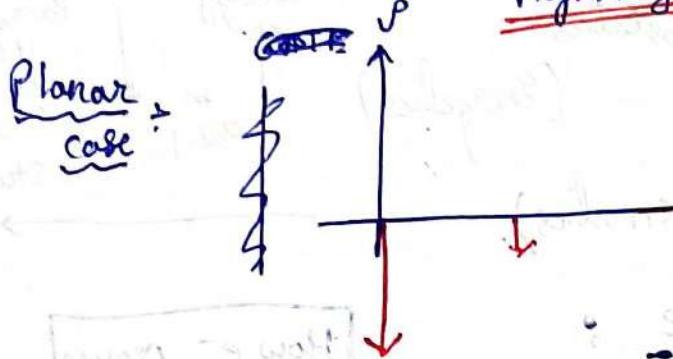
During Programming, High E in Tunnel Oxide, low E in Retention

Both Tunnelling and Electrostatics are a Function of Distance

spreads and grows over time
shift phs to not

Re-T.O.

NEXT TASK: Draw Electrostatics of a Nanocrystal Based Memory.



$$\nabla \vec{E} = \frac{\rho}{\epsilon_0}$$

Answers to what

E

(2 outputs) Help needles out ← Jeff, H

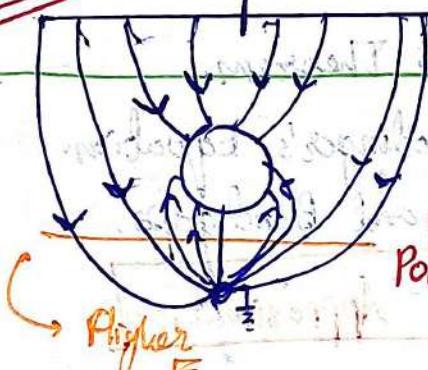
semiconductor island ← MGR H

Junction

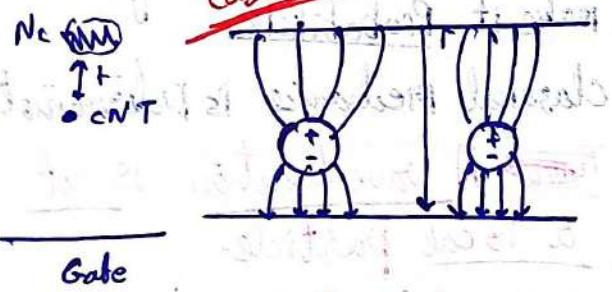
Floating gate
Voltage will be closer
to 18 V Itens
Since lower potential drop.

Smaller the T,
Higher the Electric field, more concentrated

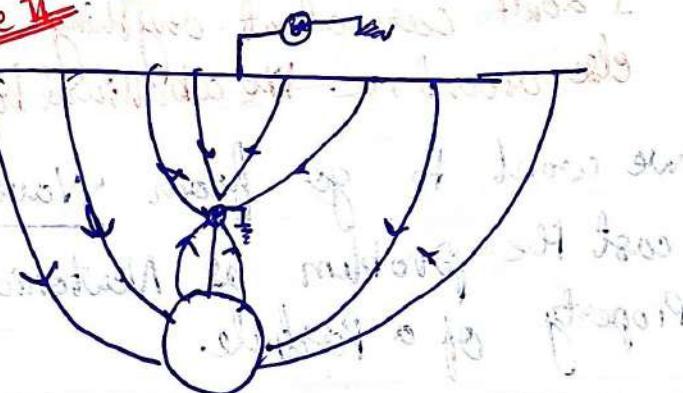
Case I



This is better
Since more
Electric field is
Pooled at the
blobs



Case II



- For intrinsic Semiconductor $\therefore E_F = \frac{E_c + E_v}{2} + \frac{3}{4} k_b T \ln \left(\frac{m_p}{m_n} \right)$
 - $n = n_i \exp \left(\frac{E_F - E_i^*}{k_b T} \right)$
 - $p = n_i \exp \left(\frac{E_p - E_F}{k_b T} \right)$
 - $E_F = E_i^* + \sinh^{-1} \left(\frac{N_D - N_A}{2n_i} \right)$
- we Apply charge neutrality

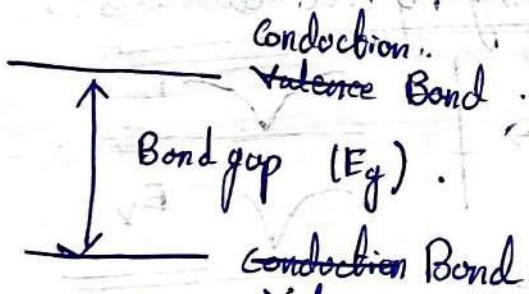
EE 724 Nano Electronics Quiz 2.

- In Metals $\therefore (G = \infty)$ - infinite conductivity
 - In Insulators $\therefore (G = 0)$ - zero conductivity
- No Band gap idea is needed.

→ For Semiconductors we need to acknowledge the presence of Bandgap.

- Free carriers (majority, minority and dopants)

- Fermi level:



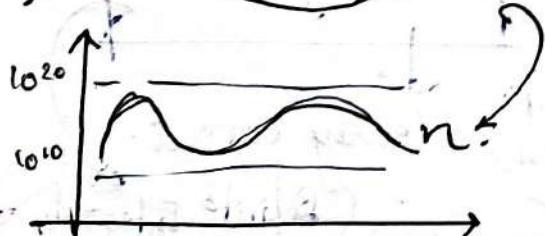
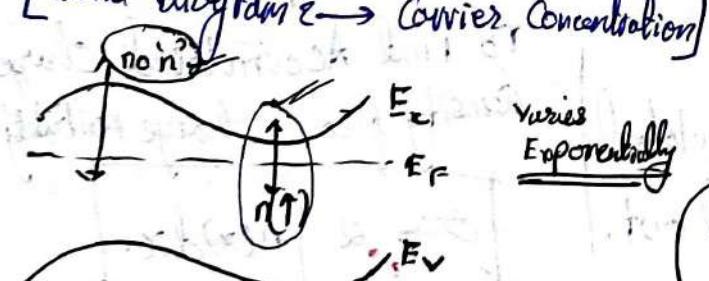
① Equilibrium $\therefore np = n_i^2$ OK!
(no excess carriers).

- Fermi level is common for n and p.
($\text{Flux}_{L \rightarrow R} = \text{Flux}_{R \rightarrow L}$).

② Conduction and Valence bands are shorted.

Basically means conduction and valence bands are at same potential if

can only be true for a semiconductor junction with no junctions. (semi)



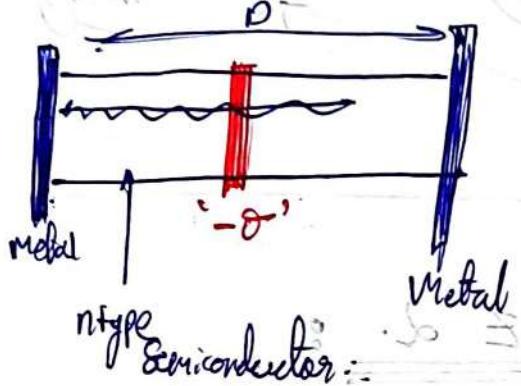
Possibly by

- fast Recombination/Generation
- Metal Semiconductor Generation
- Quickly Account for Excess
- Depleted Carriers

Screening by Depletion & Free Carriers.

Metals, Insulators - Scale free

Semiconductors - Scale Dependant



$$\delta = N_0 * L_{\text{Depletion}}$$

This is Screening by Depletion

The Electrode will participate in screening if $|L_{\text{Dep}}| \gg D$

Scale Dependancy

clearly if $D \gg |L_{\text{Dep}}|$

then electrode doesn't feel presence of sheet of charge.

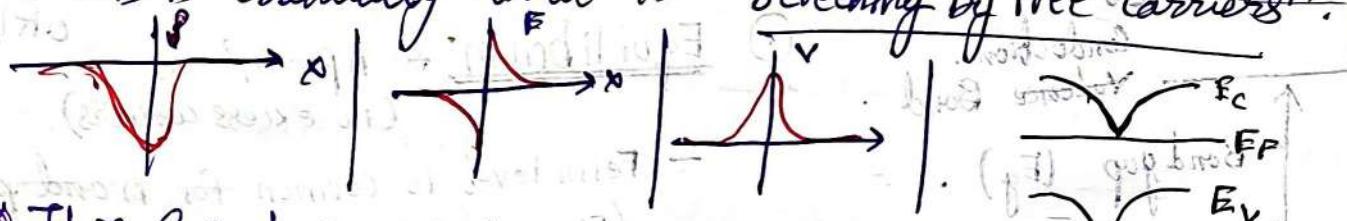
→ Cannot be solved by pen on paper.

Now what if $\delta(x=0) > 0$

We won't get a nice square distribution

as majority charge will increase exponentially at the center of the ~~center of the semiconductor~~.

• But this is essentially what is "Screening by Free Carriers".



* It is easy to see why the problem occurs.

All we know is the length scale (D) and δ .

From this we need to predict the redistribution of charges.

→ Clearly the model in this case is more complex and cannot be simply "guessed".

It is easy to see where the problem occurs.

• Near $x=0$ $\rho \approx n(x)$, → accumulated

To find accumulated charge density from charge neutrality

• Far from $x=0$ $\rho \approx 0$, → screened out.

$$\delta = 2 \int n(x) dx$$

→ The basic form of $n(x)$ and $V(x)$ is not known
→ we can't calculate

$n(x)$ must be known

Clearly correct

Definite Integral

However we can draw Bond Diagrams base on crude Approx of $n(x)$.

Debye Length v/s Depletion Length

$$\frac{d^2V}{dx^2} = -\frac{q\rho}{\epsilon_s} \text{ where } \rho = N_D + P(x) - n(x) + \sigma(x=0)$$

For Any case if
Appropriate
Representation

To get General soln we solve for the case when ($\sigma = 0$), i.e., at $x \neq 0$.
Then we can find particular solution. (initial solution by $\sigma = 0$?)

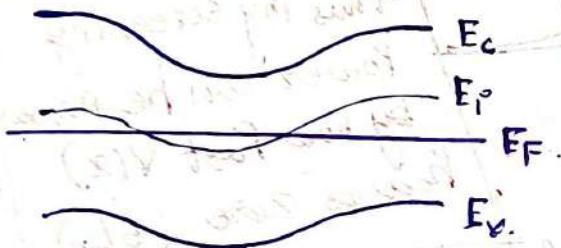
At most places $V = 0 \Rightarrow \rho$. So let's calculate ΔV instead which comes from the excess charge compared to net $\rho = 0$. ($\Delta \rho \rightarrow \Delta V$?)

$$\rho = N_D + \frac{n^2}{n(x)} - n(x) = N_D \left(1 - \exp\left(\frac{qV}{k_B T}\right) \right)$$

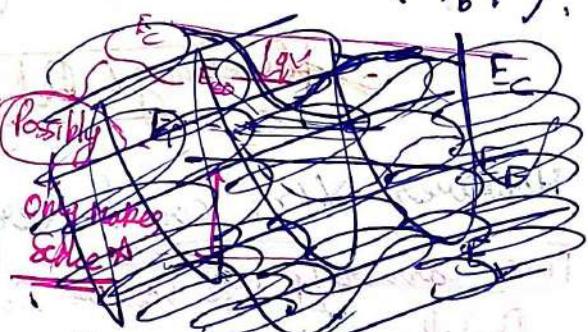
How does this even
make a difference?

$$\rightarrow qV = E_c - E_{co}$$

(i.e., the E_{co} bending)



$$n(x) = N_D \exp\left(\frac{E_F - E_i}{k_B T}\right)$$



Apply Taylor Approximation for small bending $V(x) \rightarrow 0$. $\rho = -N_D \left(\frac{qV}{kT} \right)$

$$\frac{d^2V}{dx^2} = \frac{q^2 N_D}{\epsilon_s k T} V \Rightarrow V(x) \sim \exp(x/L_D) \text{ for small charge perturbation}$$

$$L_D = \sqrt{\frac{\epsilon_s k T}{q^2 N_D}} = \sqrt{\frac{\epsilon_s V_T}{q N_D}}$$

→ Debye length.

{ Free Carrier Screening or Debye Length Screening is exponential with Screening length. This is very effective Screening cf. depletion. }

$$L_{dep} = \sqrt{\frac{2 \epsilon_s \phi_{bi}}{q N_D}}$$

$$\text{so } \frac{L_{dep}}{L_D} = \sqrt{\frac{2 \phi_{bi}}{V_T}}$$

at the same doping.

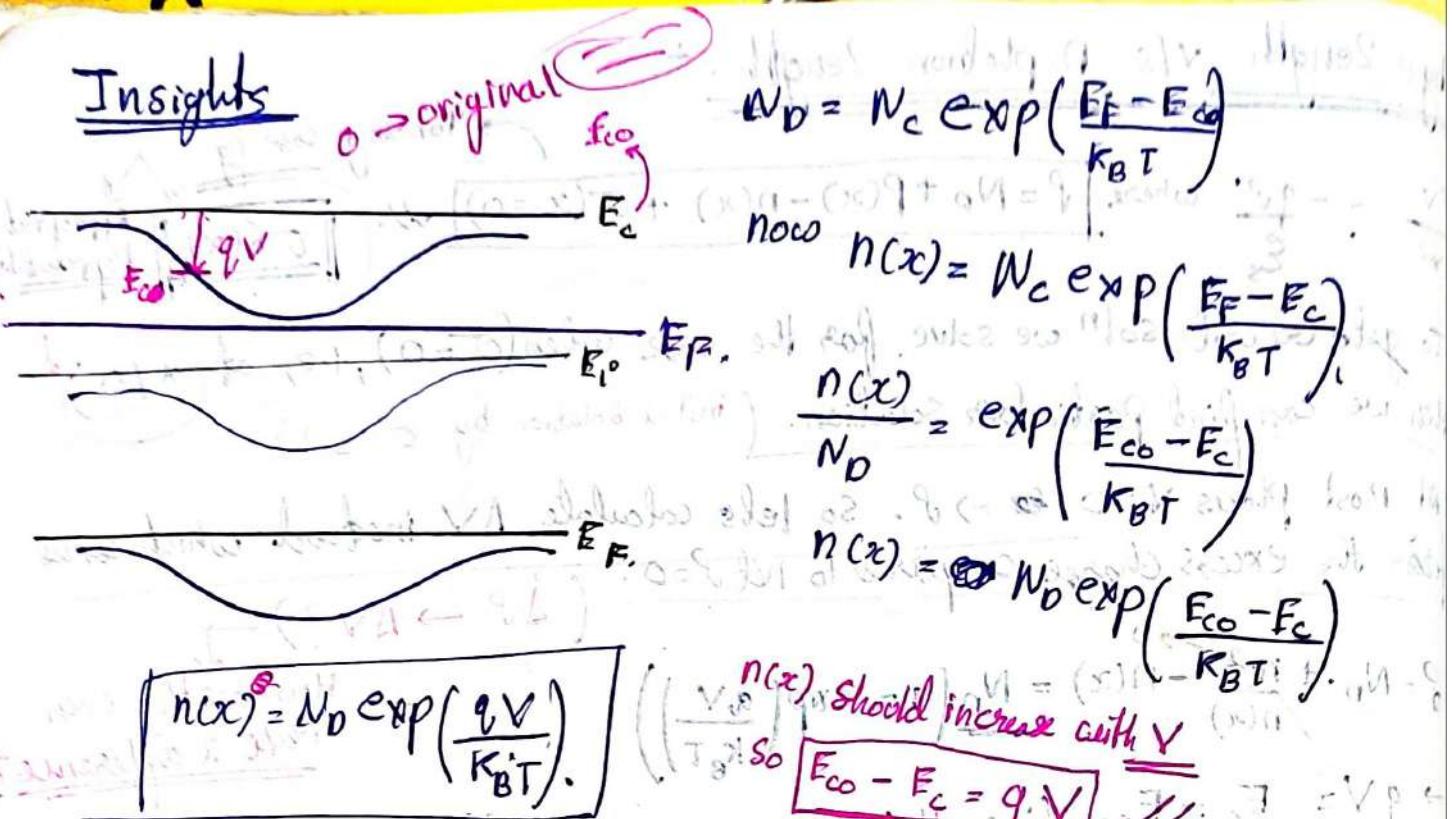
→ Debye length is smaller

→ Its Dependence is exponential

→ Temperature Dependent

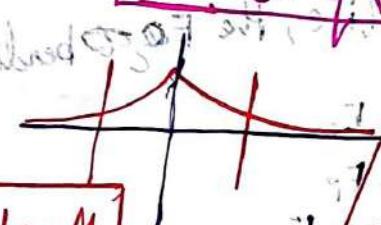
At $T=0$ $L_D \rightarrow 0$ but L_{dep} remains some. Physically why? Electrons are confined by well easily at $T=0$. No spread (diffusion) is needed.

Insights



Then for small $V(x)$ essentially

~~Screening & Debye Length~~



Thus my screening power can be seen by how fast $V(x)$ becomes zero. (SL_D)

However $V(x)$ can be very high. we cannot do much there.
Free carrier/Debye length screening \rightarrow exponential.
Depletion length screening \rightarrow quadratic.

Exact Solution is not easy. NUMERICAL SOLUTION OK!!

Temperature Dependence of Debye Length

$$L_D = \sqrt{\frac{eV_T}{qN_D}}$$

$$L_{dep} = \sqrt{\frac{qe\phi_b}{qN_D}}$$

$$L_D \propto \sqrt{T}$$

essentially the lower the temperature, the better the free carrier screening.

At lower Temperature Diffusion is tougher. Thus e- can more easily be localized at $x=0$.

(They don't have enough energy to overcome the barrier)

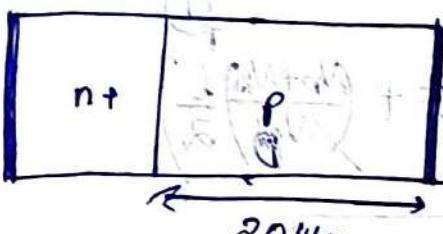
Thus at $T=0K$ we achieve PERFECT FREE CARRIER SCREENING.

Screening becomes worse with increase in Energy as you cannot squish the free carriers into one place due to their thermal energy.

$$P = E_D$$

A Few Interesting Examples.

A]



→ Depletion length, Band Diagrams, charge profiles.

Simple n+p junction

Inflection point

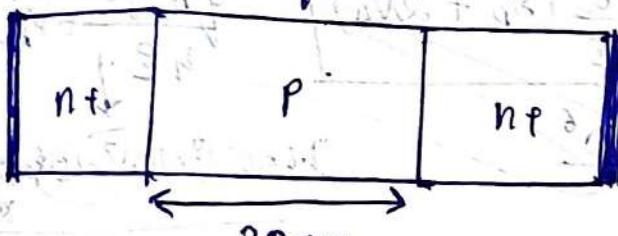
steep slope

E_c

E_F

E_v

B)



Notice the length

Depletion length = 115 nm

$$d \ll L_{\text{Dep.}} / \sqrt{V}$$

Consider it a Planar MOSFET without a gate electrode.

Given a n+/p/n+ device, there should be a barrier b/w S/D.

(What should be the doping for a 0.6 V barrier).

Q) The First q. is what is the current barrier height?

→ I will have to use the depletion approximation.

$$\rightarrow \text{so, } \delta = -qN_A$$

$$\rightarrow \nabla \cdot E = \frac{\delta}{\epsilon} = -\frac{qN_A}{\epsilon}$$

$$\frac{d^2V}{dx^2} = \frac{-\delta}{\epsilon} = \frac{+qN_A}{\epsilon}$$

$$N_D(2\frac{l}{\lambda}) = N_A(\frac{l}{\lambda})$$

$$x_n = N_A l$$

As the length scale decreases.

This is precisely the advantage I have here over case C

$$+\frac{qN_D}{\epsilon} \text{ with}$$

$$\frac{qN_D(x+x_{n_1})}{\epsilon}$$

$$-\frac{qN_A(x-\frac{l}{2})}{\epsilon}$$

$$\text{so, } -\frac{qN_A(-l)}{\epsilon} = \frac{qN_D(\frac{l}{2} + x_n)}{\epsilon}$$

$$N_D(x_n-l) = N_A l$$

$$(N_D + N_A)l = 2N_A N_D$$

$$\rightarrow x_n = \left(\frac{N_D + N_A}{2N_D} \right) l$$

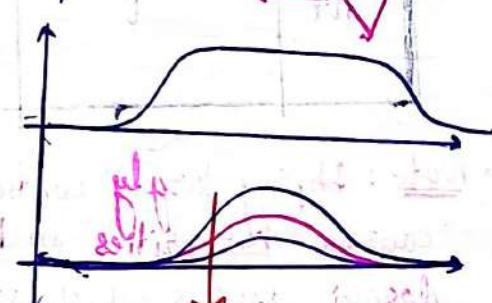
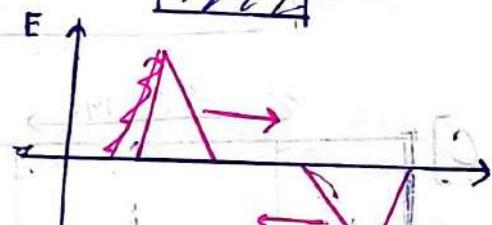
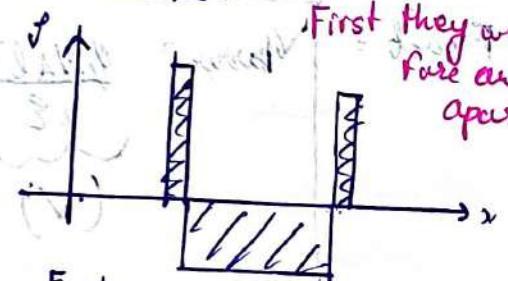
$$\frac{N_A l}{2} = N_D(x_n - \frac{l}{2})$$

Some eqn.

ϵ continuity

Charge conservation.

First they were far away apart.



As length scale decreases.

(So Barrier Decreases as the length scale Decreases)

Crazy part is that at one point n-carriers will dominate over p-carriers in the p-Doped Region. which is precisely the case here



$$\text{So far } V \rightarrow E_{max} = \frac{qN_A l}{2\epsilon}$$

$$\rightarrow V_{barrier} = \frac{1}{2} \left(x_n + \frac{l}{2} \right) E_{max} = \left(\frac{qN_A l}{2\epsilon} \right) \left(\frac{l}{2} + \frac{(N_D + N_A) l}{N_D} \frac{l}{2} \right) \quad (1)$$

$$\frac{qN_A l^2}{8\epsilon} \left(\frac{N_D + 2N_A}{N_D} \right) = \frac{q l^2 (N_D + 2N_A)}{8\epsilon} \quad \frac{qN_A l^2}{4\epsilon} = V_{barrier}$$

$$1.6 \times 10^{-19} \times (20)^2 \times 10^{-18} \times 10^{20} \times 10^6 = 1.6 \times 10^{-19} \times 10^{17} \times 10^6 \times 10^{-12}$$

$$4 \times 11.9 \times 8.85 \times 10^{-12}$$

$$V_{barrier} = 0.015 V$$

$E_{max} =$

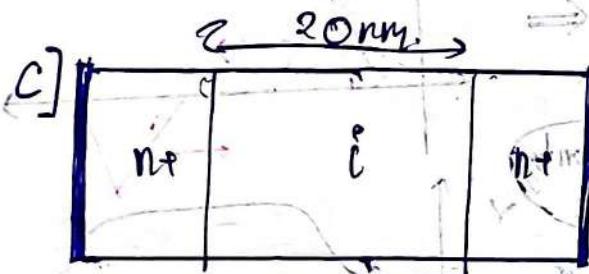
$$V_{barrier} = \frac{qN_A l^2}{4\epsilon} + \left(\frac{qN_A l^2}{8\epsilon} \right) \left(\frac{N_A}{N_D} \right) \quad (\checkmark) + \left(\frac{V}{2} \right) \left(\frac{N_A}{N_D} \right)$$

$$0.6 V \rightarrow N_A \approx 6 \times 10^{18}$$

$$4 \times 10^{18} = N_A$$

$$0.612 V$$

Approximately



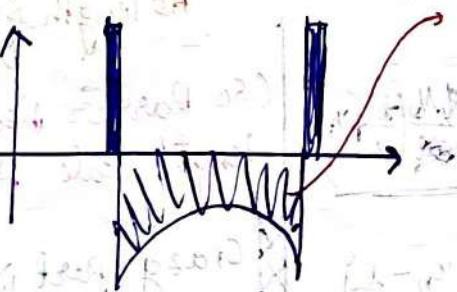
• Consider it a FinFET w/o a gate electrode.

• How much is the barrier b/w S/D.

• FOR SUCH A DEVICE WE CANNOT EMPLOY DEPLETION APPROXIMATION.

• Note: Doping largely controls Transistor electrostatics. It often ends up causing disparities in the transistor and thus we have always aimed to design devices which are dopant free.

Basically, if



This again cannot be modelled which means we will have to use a numerical solution.

$$\rightarrow \frac{d^2 V}{dx^2} = -\frac{q}{\epsilon} \left[N_D + \exp\left(\frac{-qV(x)}{kT}\right) \right]$$

P-I-O

Learnings obtained from this entire exercise.

1) Changing L_p in n-p-n structure $\Rightarrow V_{bi} \propto L_p$.

• only Doping Dependent built-in barrier or potential is the max potential possible.

• When $L_p \gg L_{dep}$: V_{bi} is L_p independent

but when $L_p \leq L_{dep}$, the V_{bi} reduces with L_p , i.e., L_p dependent. $\rightarrow V_{bi} = f(L_p)$

Q: Can I keep L_p const and reduce V_{bi} \rightarrow (by increasing L_{dep} which depends upon doping.)

2) 3 Types of Screening:

a) Screening by dielectric polarization. This happens due to the dielectric properties of the material. The Field lines will go all the way to the electrode.

b) Screening by Depletion. Free carriers Repelled because of local depletion. Ionic cores will screen out potential in Depletion Length, $L_{depletion}$ (material's length scale). $V(x)$ is quadratic.

c) Screening by Free carrier. Accumulation of majority carriers at the center. Free carrier screening will screen out electric field by Debye length (L_{Debye}). $V(x)$ is approx a exponential decay.

If all phenomenon are acting together, then effective screening is by the shortest length scale.

8) • $n(x) = N_c \times \exp\left(-\frac{(E_c(x) - E_F)}{kT}\right) \rightarrow$ Under Boltzmann Distribution and Eqn, $\log(n(x))$ has the same shape as E_c

• Types of screening

Address

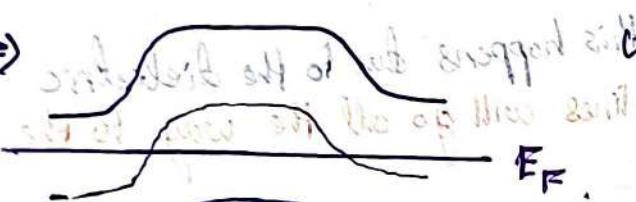
$$[3 + (2+1)^{2013} - (2+1)^{2012}] = \frac{2^{2013}}{2^{2012}} : 3 + 2^{2012} = \boxed{P.T.O.}$$

$$[3 + (2+1)^{2013} - (2+1)^{2012}] = \frac{2^{2013}}{2^{2012}}$$

$$(3 + 2^{2013})(2^{2012} - 1) = [3 + 2^{2013} - 2^{2012}] = \frac{2^{2013}}{2^{2012}}$$

Numerical Solution of Poisson Equation.

- 1) carrier concentrations in terms of potentials and apply charge neutrality to get exact form of Poisson's equation which is of the form,
$$\frac{d^2\phi}{dx^2} = f(\phi).$$
- 2) Perturb the poisson equation with small δ : $\phi_{\text{new}} = \phi_{\text{old}} + \delta$. (For this we need initial guess)
- 3) δ in terms of ϕ_{old} , ϕ_{new} . ϕ_{new} on one side of equality.
- 4) Normalize the variables.
- 5) Now difference discretize the potential equation in matrix form.

\Rightarrow  Use $\phi = \frac{E_F - E_i}{k_B T}$

$$\text{where } E_F - E_i = q(V_F + V)$$

so $\frac{n}{n_i} = e^{-\phi}$, $\frac{p}{n_i} = e^{-\phi}$ constant + variation in potential

$$\Rightarrow \left[\frac{d^2V}{dx^2} = \frac{1}{q} \left(-\frac{d^2E_i}{dx^2} \right) = \left(\frac{k_B T}{q} \right) \frac{d^2\phi}{dx^2} \right] = -\frac{\rho}{\epsilon} = (e^{-\phi} - e^{\phi} + C) \times \frac{q n_i}{\epsilon}$$

$$\Rightarrow \frac{d^2\phi}{dx^2} = -\frac{q^2 n_i}{K T \epsilon} (e^{-\phi} - e^{\phi} + C) \quad \text{where } C = \frac{N - N_A}{n_i}$$

Let $L_D^2 = \frac{K T \epsilon}{q^2 n_i}$, $\frac{V_I \epsilon}{q n_i} \Rightarrow \frac{d^2\phi}{d(x/L_D)^2} = e^{-\phi} - e^{\phi} + C$.

define $X = x/L_D \Rightarrow \frac{d^2\phi}{dX^2} = e^{-\phi} - e^{\phi} + C$.

$$\rightarrow \phi_{\text{new}} = \phi_{\text{old}} + \delta: \frac{d^2\phi_{\text{new}}}{dx^2} = [e^{-(\phi_{\text{old}} + \delta)} - e^{(\phi_{\text{old}} + \delta)} + C]$$

$$\frac{d^2\phi_{\text{new}}}{dx^2} = -[e^{-\phi_{\text{old}}(1-\delta)} - e^{\phi_{\text{old}}(1+\delta)} + C].$$

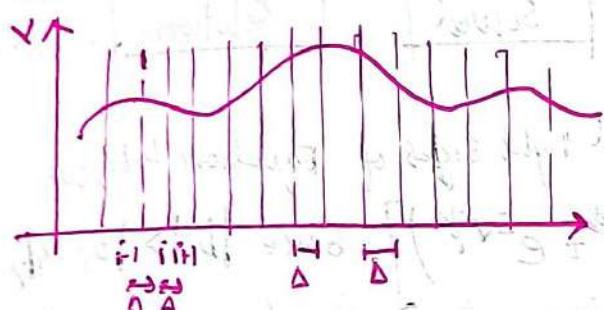
$$\frac{d^2\phi_{\text{new}}}{dx^2} = -\{[e^{-\phi_{\text{old}}} - e^{\phi_{\text{old}}} + C] - (\phi_{\text{new}} - \phi_{\text{old}})(e^{-\phi_{\text{old}}} + e^{\phi_{\text{old}}})\}$$

$$\begin{aligned} \frac{d^2 \phi_{\text{new}}}{dx^2} &= -\phi_{\text{new}} (e^{-\phi_{\text{old}}} + e^{\phi_{\text{old}}}) \\ &= -\{ (e^{-\phi_{\text{old}}} - e^{\phi_{\text{old}}} + c) + \phi_{\text{old}} (e^{-\phi_{\text{old}}} + e^{\phi_{\text{old}}}) \} \end{aligned}$$

So,

$$\frac{d^2 \phi_{\text{new}}}{dx^2} = -\phi_{\text{new}} (e^{-\phi_{\text{old}}} + e^{\phi_{\text{old}}}) = -(e^{-\phi_{\text{old}}} - e^{\phi_{\text{old}}} + c) \Rightarrow \phi_{\text{old}} (e^{-\phi_{\text{old}}} + e^{\phi_{\text{old}}})$$

Difference Discretization of Potential.



Basically Δ is mesh size. Potential is sampled at a distance of Δ from previous sample.

$i \rightarrow$ Label of the mesh element.

$$\frac{d^2 V}{dx^2} = \frac{(V_{i+1} - V_i)}{\Delta} - \frac{(V_i - V_{i-1})}{\Delta}$$

$$= \frac{V_{i+1} - 2V_i + V_{i-1}}{\Delta^2}$$

$n \rightarrow n^{\text{th}}$ potential guess. Found using the $n-1^{\text{th}}$ guess.

$$\text{So, } \frac{\phi_{i+1}^{n+1} - 2\phi_i^{n+1} + \phi_{i-1}^{n+1}}{\Delta^2} - \phi_i^{n+1} (p_i + n_i) = -[p_i - n_i + c_i] + \phi_i^n [p_i + n_i]$$

$$\Rightarrow \frac{1}{\Delta^2} \phi_{i+1}^{n+1} - \left(\frac{2}{\Delta^2} + p_i + n_i \right) \phi_i^{n+1} + \frac{1}{\Delta^2} \phi_{i-1}^{n+1} = -[p_i - n_i + c_i] + \phi_i^n [p_i + n_i]$$

p_i, n_i, c_i are the charge concentrations of the carriers at the i^{th} mesh element. According to the previous (i^{th} in this case, n^{th}) guess. Thus they are terms which depend on our previously evaluated potential.

$$\Rightarrow \frac{a^n}{\Delta^2} \phi_{i+1}^{n+1} - b^n \phi_i^{n+1} + c^n \phi_{i-1}^{n+1} = p^n$$

new potential, $n+1^{\text{th}}$ iteration.

old Potential.

Choose guess
Potential $V_{\text{old}}(x)$

Find $n(x), p(x)$ from
 V_{old} using carrier statistics

use V_{new} as
the next V_{old}

IF
Error > min

Use Poisson's Eqn
to calc. $V_{\text{new}}(x)$

$$E(x) = V_{\text{new}} - V_{\text{old}}$$

IF Error < min
Stop

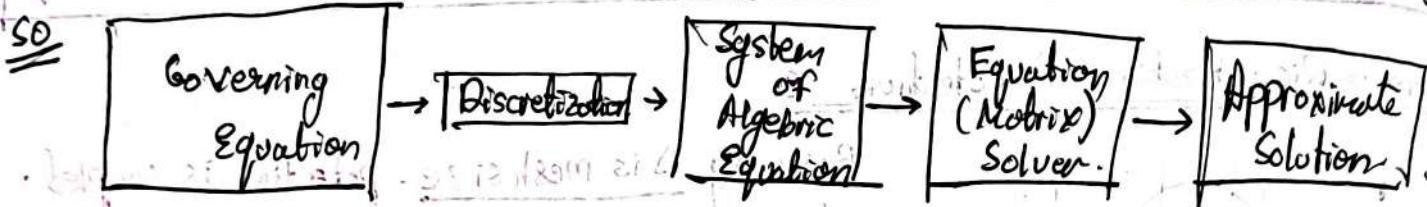
What is Finite difference discretization?

- Potential varies linearly b/w mesh points
- Electric field is constant between mesh points.

Linearization \rightarrow Diagonally dominant coefficient matrix A is obtained.

$$\frac{d^2 V^{\text{new}}}{dx^2} = \frac{q n_i}{\epsilon V_T} (e^{-V^{\text{old}}/V_T} + e^{V^{\text{old}}/V_T}) V^{\text{new}} \quad \text{and} \quad \frac{d^2 V^{\text{old}}}{dx^2} = -\frac{q n_i}{\epsilon V_T} (e^{-V^{\text{old}}/V_T} - e^{V^{\text{old}}/V_T} + c/n_i) - \frac{q n_i}{\epsilon V_T} (e^{-V^{\text{old}}/V_T} + e^{V^{\text{old}}/V_T})$$

Be unitless, use fundamental scale, $(V \rightarrow V_T), (x \rightarrow L), (n, p \rightarrow n_p)$ \rightarrow normalizing



\rightarrow By comparison with the terms of the left and right sides of Equation 4.12, we see that $a_i = c_i = \frac{1}{\Delta^2}$ and $b_i = -\left[\frac{2}{\Delta^2} + (e^{V_i^{\text{old}}} + e^{-V_i^{\text{old}}})\right]$. Since $|b_i| > |a_i|$, the matrix A (after linearization procedure) becomes diagonally dominant, which in turn leads to stable convergence. The residual of Equation 4.12 is calculated and convergence is achieved if the norm of the residual is smaller than a preset tolerance. In practice, one might simply check that the maximum absolute potential anywhere on the mesh is smaller than some preset tolerance.

Getting the Numerical Solution Using ~~P~~ LU Decomposition:

$$[P] * [V^{\text{new}}] = F[V^{\text{old}}]$$

↓ Tridiagonal matrix (calculated from prev. potential guess.) ↓ Unknown vector ↓ Known

$$[P]V = F \rightarrow \text{use LU Decomposition}$$

$$\Rightarrow LU V = F$$

first, $LA = F$ where $A = UV$
 isolve for V solve for U

Basic Equations of a Representative LU Decomposition:

$$\begin{bmatrix}
 a_1 & c_1 \\
 b_2 & a_2 & c_2 \\
 & \ddots & \ddots & \ddots
 \end{bmatrix} \begin{bmatrix}
 v_{1,1} \\
 v_{2,1} \\
 \vdots \\
 v_{n,1}
 \end{bmatrix} = \begin{bmatrix}
 f_{1,1} \\
 f_{2,1} \\
 \vdots \\
 f_{n,1}
 \end{bmatrix}$$

How will I model the tridiagonal matrix for linearized Poisson's equation?

will it be the same or will coefficients vary?

P-T.O.

$$\left(\begin{array}{cc|c} a_1 & c_1 & \\ b_2 & a_2 & c_2 \\ \hline b_{n-1} & a_{n-1} & c_{n-1} \\ b_n & a_n & \end{array} \right) = \left(\begin{array}{cc|c} 1 & & \\ & 1 & \\ & & 1 \\ \hline & & \\ & & 1 \\ & & \end{array} \right) \left(\begin{array}{cc|c} d_1 & c_1 & \\ d_2 & c_2 & \\ \hline d_{n-1} & c_{n-1} & \\ d_n & c_n & \end{array} \right)$$

② We can derive L, β by simple matrix multiplication.

$$\beta_{k+1} d_k = b_{k+1} \quad \text{where } 1 \leq k \leq n$$

$$\beta_{k+1} c_k + \alpha_{k+1} = a_{k+1}$$

The values of α and β are found by fwd and back substitution.



$$\begin{aligned} ① \alpha_1 &= a_1 \\ ② \beta_k &= \frac{b_k}{d_{k-1}} \\ ③ d_k &= a_k - \beta_k c_k \end{aligned}$$

Recursive Relation to solve for V in matrix form,

$$\left(\begin{array}{cc|c} \beta_1 & 1 & \\ \beta_2 & 1 & \\ \hline \beta_{n-1} & 1 & \\ \beta_n & 1 & \end{array} \right) \left(\begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \right) = \left(\begin{array}{c} f_1 \\ f_2 \\ \vdots \\ f_n \end{array} \right) \rightarrow A_1 = f_1$$

$$\rightarrow A_1 \beta_2 + A_2 = f_2$$

$$\rightarrow A_2 \times \beta_3 + A_3 = f_3$$

Solve Downwards

$$\left(\begin{array}{cc|c} d_1 & c_1 & \\ d_2 & c_2 & \\ \hline \vdots & \vdots & \\ d_{n-1} & c_{n-1} & \\ d_n & c_n & \end{array} \right) \left(\begin{array}{c} V_1 \\ V_2 \\ \vdots \\ V_{n-1} \\ V_n \end{array} \right) = \left(\begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \right) \rightarrow d_1 V_1 + c_1 V_2 = A_2$$

$$d_2 V_2 + c_2 V_3 = A_3$$

Solve upwards

$$d_n V_n = A_n$$

Essentially for $i=2$ to $i=n$

$$\rightarrow A_i = f_i$$

$$\rightarrow A_i = f_i - A_{i-1} \beta_{i-1}$$

And for $i=1$ to $i=n-1$

$$V_n = A_n / d_n$$

$$V_{n-i} = \frac{A_{n-i} - (n-i) V_{n-(i+1)}}{d_{n-i}}$$

In $n_p / p / n_t$ as $p(t)$, the charge density moves away from free carrier screening towards screening by Depletion.

Another way to push out the free carrier screening is to apply a gate voltage bias to increase the barrier to kill the free carriers. Screening. If I increase the barrier potential via the gate, it'll pull the carrier concentration down to negative thier effect. Makes Laplace Valid. (OFF state is where we can solve Laplace.)

(This is why first guess is Laplace soln)

LECTURE 15 - 09/03/24

Why e^- moves? \rightarrow Force / Electrostatics
 Energy. (Energetics)

How fast an e^- moves \rightarrow (Kinetics).

QM to Semi Classical Picture :

Kittel \rightarrow Free electron gas!! (chapter 6)

Kronning Penny Model

- It is the Kronning Penny Approximation of Periodic potential.
- Apply Bloch's Theorem.
- Solve Schrodinger's Equation.
- Get E vs k and Band gaps.

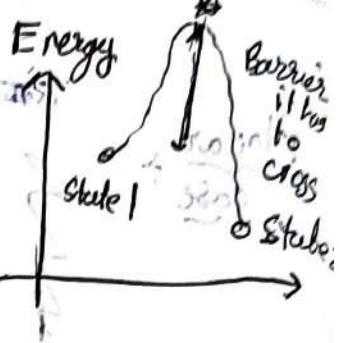
Independent e^- Approximation.

I don't care about anything else around me. like a billiard ball.

We want to go from Wave to particle. Then we can cast the problem as Newtonian mechanics. Mass is the fundamental property of a particle.

Game Plan :

- Electron in solid: Simplifying the QM Problem
- Perfect Crystal
 - Wave Packet is a particle
 - Particle Motion
 - Effective Mass
 - Electronic Structure
- Scattering
- Electron Transport in E Field



How e^- moves

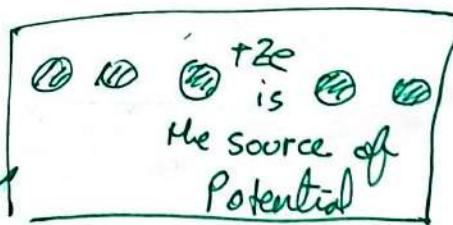
QM \rightarrow Bohr Schrodinger
 Classical \rightarrow Newton.

tunneling

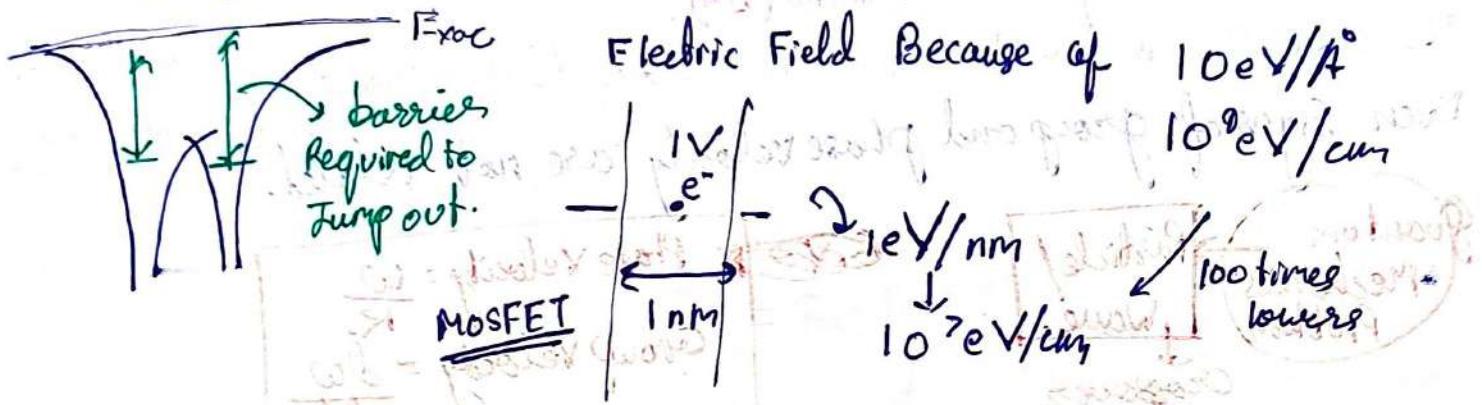
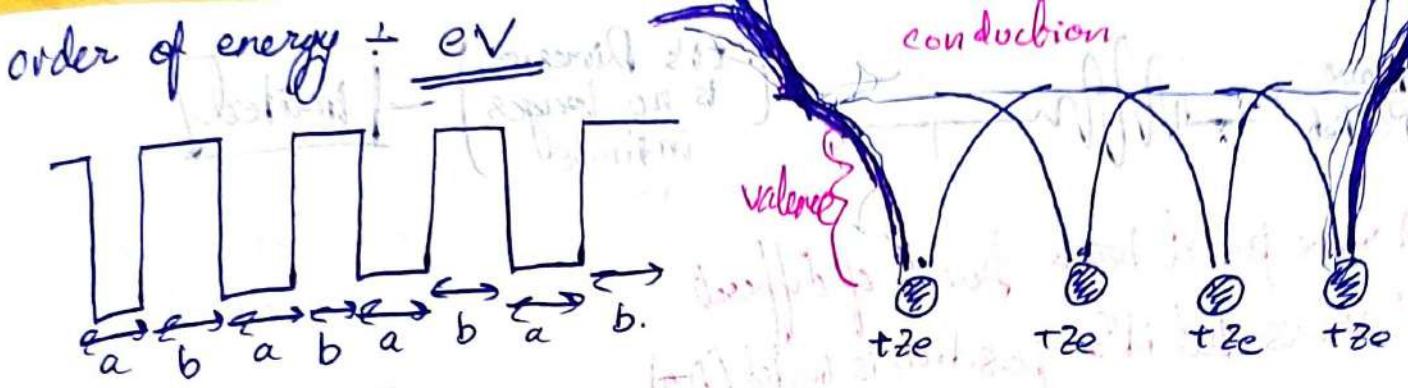
Because we deal with probabilities and uncertainties in the dynamics make it Probabilistic.
 Classical Mechanics is Deterministic.

~~Fundamental~~ A wave function is not a local particle.

(spread around and talk to everybody.)



The movement of the other nuclei is negligible.



!!! Any field we apply can be treated as a perturbation since they don't change the Electric field of crystal oscillations at all.

Nothing is majorly affecting the atomic potential. Everything is to 1% correction - External Electric field is pretty weak.

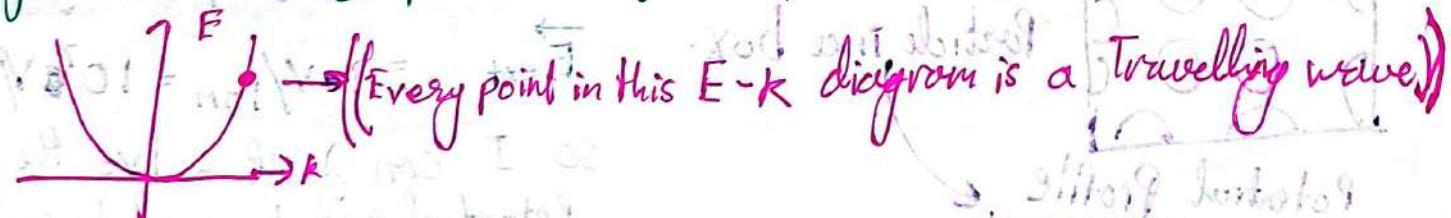
$$\psi_{\text{total}}(x,t) = \psi_{\text{lattice}} + \psi_{\text{vibration}} + \psi_{\text{external}}$$



$$E(k) = \left(\frac{\hbar^2}{2m}\right) k^2 \rightarrow \text{Energy v/s Momentum plot.}$$

\rightarrow mass is defined by the curvature

For a certain kind of e^- , the E field doesn't appear to be oscillatory. There is a mass, charge and they feel that the potential is just a ~~flat line~~ flat line.



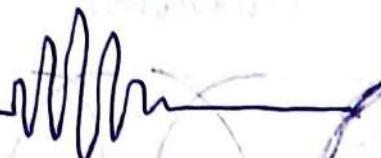
e^- gas in vacuum ??

wave \rightarrow particle
free e^- gas

e^- is free, nothing is confining it ~~now~~ where

Effect of Periodic Lattice

- Nearly free e^- gas
- Tight Binding.

Wave Packet  (It's dimension is no longer infinite) — limited.

A wave packet has a bunch of different k values but its position is limited (Δx).

$gm \rightarrow$ semi-classical
 $\psi \rightarrow$ wave packet

The crest relative to the center is moving to the right (Phase velocity + Group velocity). Even signs of group and phase velocity are not related.

Quantum Mechanics Problem → Particle/Wave.
Crossover

Phase Velocity = $\frac{\omega}{k}$
 Group Velocity = $\frac{dk\omega}{dt}$

• A parabola has a single mass. A Random Fk diagram has a different curvature at every point and hence a different mass everywhere.

Lecture 16 - 11/03/24

kittel chp 6 - Free e-gas

(visualize how an e- moves under a force)!!

+ (A software to play with) [1110](#)

Harmonic Disturbance of an Atom

(PPM) of an A

$(1 + d\Delta)$ (interatomic distance)

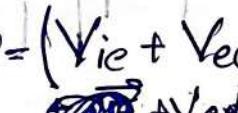
Thermal expansion.

$$H_{atom} = -\frac{(kZe^2)}{r}$$

Complex

$$\downarrow \text{He Atom.} \quad V = (V_{ie} + V_{ee})$$

(need to simplify to 1st order terms)

 + V_{ext}
 (external electric field)

$V_{ie} \sim$ Energy is 10eV Energy

1\AA is distance.

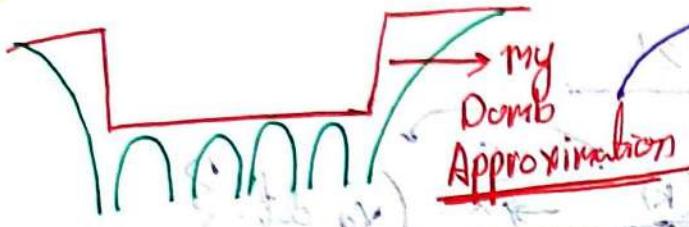
$$+ 10^9 \text{ eV/cm}$$

$$E_{ext} = 1 \text{ eV/nm} = 10^7 \text{ eV/cm}$$

so I can just solve the atomic potential and be good to go.



Potential Profile



My Dumb Approximation

Lots of knowledge to fall back on.

→ Gives us nice e^- independent states.

$$\nabla_g = h^{-1} \frac{d\epsilon}{dk}, \quad \boxed{\epsilon = -eE_g \delta t}$$

$$\delta\epsilon = \left(\frac{d\epsilon}{dk}\right) \delta k = \nabla_g \delta k$$

??

$$\frac{dk}{dt} = \frac{F}{m} = 0$$

$$\delta k = -\left(\frac{eE}{h}\right) \delta t$$

$$h \frac{dk}{dt} = F$$

Particle moves with const. velocity. $\rightarrow v_g(k_0)$

is v_g changing uniformly with time.

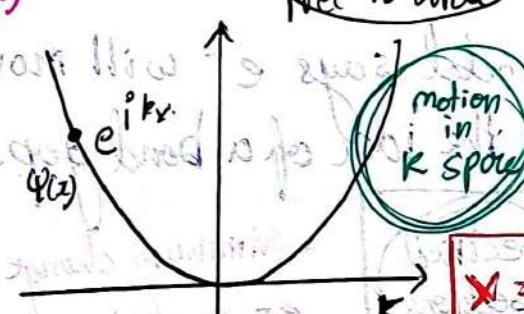
external Force

$\frac{dk}{dt} = \frac{F}{m} \rightarrow k$ varies uniformly with time.

v_g = varies uniformly with k .

\Rightarrow so v_g varies uniformly with time.

Red Dot → center of the box
Green Dot → centroid of e^-

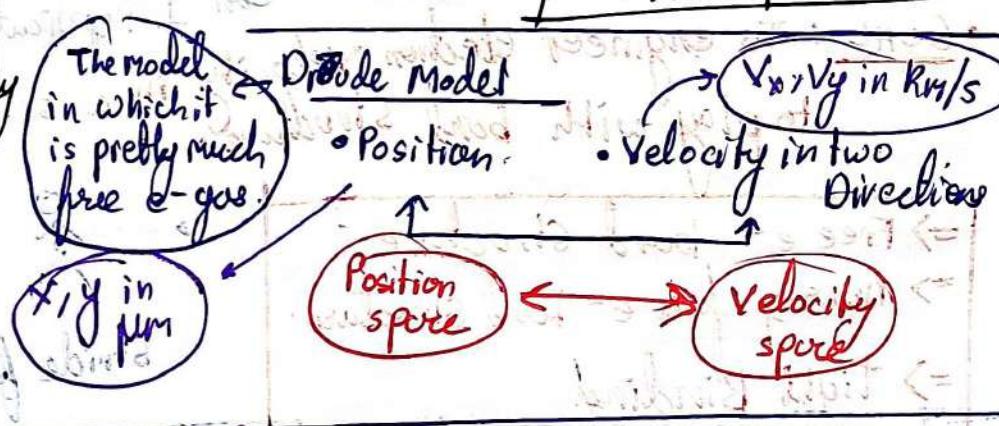


Two states don't talk to each other via electrostatics.
Vee → neglect

The other problem is e^- are waves here (not particles). How to make them particles? → Wave packets

$$X = \int v_g(k) dk + x_0$$

Parabolic \Rightarrow we can treat it as a newtonian particle.



Left Side

- Green dot is moving slowly.
- Velocity is moving particles

Thermal motion

Instantaneous change in Velocity

- Diffusion when no force.
(Disappear - Reappear)

Right Side

- There is a lot of flickering. For each particle \rightarrow when one value of k , it will stick at that point. \rightarrow when constant velocity (new value of k) \leftarrow Instantaneous change (Impulse)

$$T(\downarrow) \rightarrow \text{Diffusion}(\downarrow) \rightarrow \text{less flickering}(\downarrow)$$

LECTURE 17 — 14/08/24

As \mathbf{k} changes, $v_g(\mathbf{k}) = \frac{d\omega_f(\mathbf{k})}{d\mathbf{k}}$ $\rightarrow \mathbf{x} = \int v_g dt.$ }

$$F = \frac{d\mathbf{k}}{dt} = \hbar \frac{dk}{dt}$$

Thus we know position of a particle at any point of time.

- ① Real potential \rightarrow square well (PIB) $\rightarrow E(\mathbf{k})$ Diagram
- ② Wave \rightarrow Wave packets $\rightarrow v_g, m^*$.

\rightarrow Free electron model says e^- will move. But it does not use and promote the idea of a band gap.

Ch - 7 ÷ Kittel — Specified Sections
Ch - 8 ÷ Kittel —

• Minimum change to accommodate nearly free e^- model

Bond Structure.

I don't want to create new maths, can I replicate what I already have?

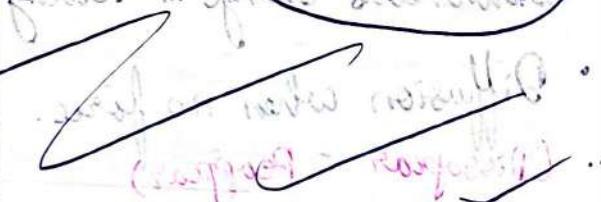
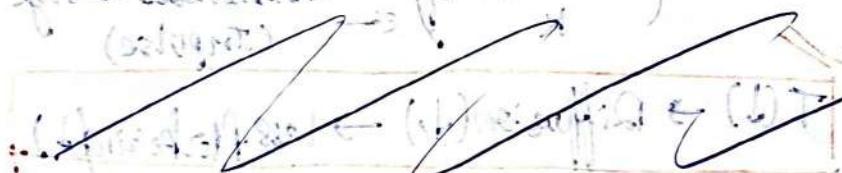
• GOAL : To engineer electron velocity in a solid, we must be able to play with bond structure.

\Rightarrow Free e^- band structure
 \Rightarrow nearly free e^- band structure.
 \Rightarrow Tight Binding!



so the wavelength of the potential and the wavelength of the e^- must match with each other (or integral multiple)

Real (x) v/s Reciprocal (\mathbf{k}) space: Block 3 — Intuition for Block's Theorem



Generating the central & quadrupole

KE:

$$U(x) = \sum_{G>0} U_G (e^{iGx} + e^{-iGx}) = 2 \sum_{G>0} U_G$$

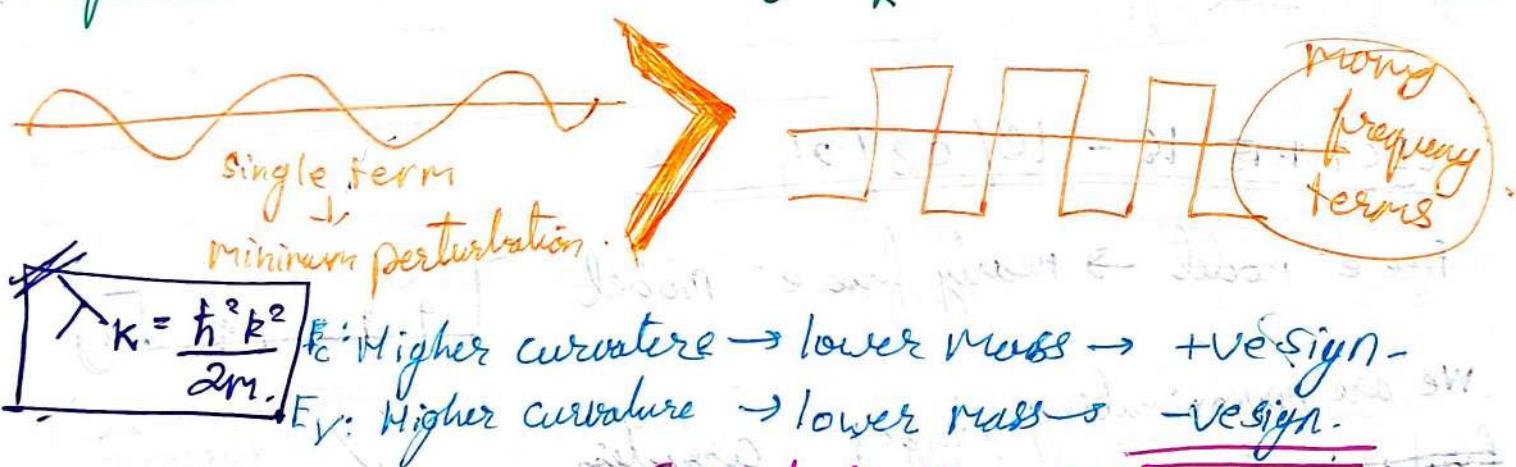
$$\Psi(x) = \sum_k C(k) e^{ikx}$$

$$KE: \frac{1}{2m} P^2 \Psi(x) = \frac{1}{2m} \left(-\hbar \frac{d}{dx} \right)^2 \Psi(x) = \frac{\hbar^2}{2m} \sum_k k^2 C(k) e^{ikx}$$

$$PE: \left(\sum_G U_G e^{iGx} \right) \Psi(x) = \sum_G \sum_k U_G e^{iGx} C(k) e^{ikx}$$

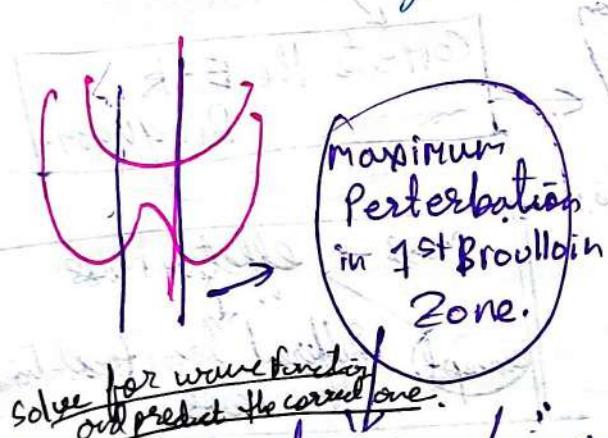
I want to collect all terms for a single oscillation k' and make them equal individually

- Schrodinger's Equation: $\sum_k \frac{\hbar^2 k^2}{2m} C(k) e^{ikx} + \sum_G \sum_k U_G C(k) e^{ikx}$



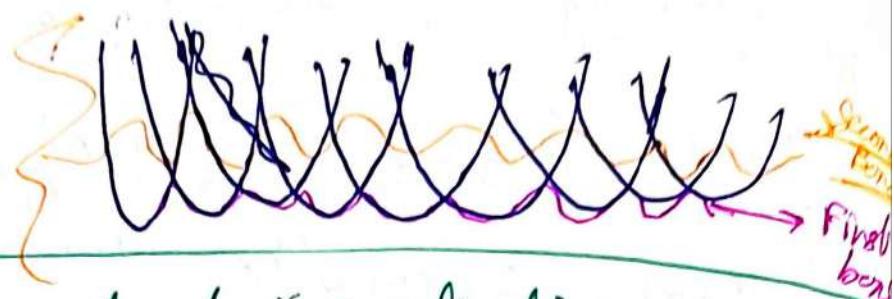
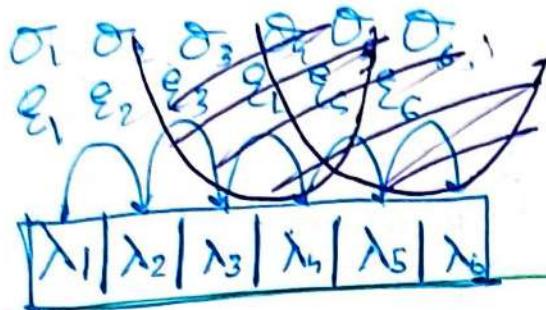
[Effectively the particle accelerates in the opposite direction of force.]

[Also effect on ~~together~~ momentum velocity relation.]



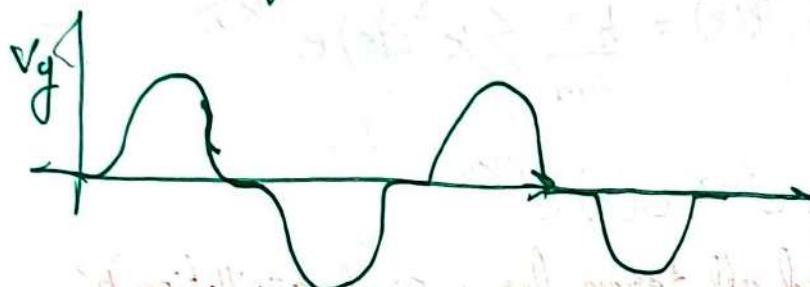
U_G at minima/maxima = 0 \rightarrow Standing wave at those points \rightarrow Particle does not move.

The E-K diagram only changes at and around the First Broullion Zone \rightarrow Split into two bands, masses change, E_C, E_V type of situation. $\boxed{E_g \propto 2U}$.



Force \rightarrow \mathbf{R} will move uniformly in one direction.

How to find out what will happen in real space \rightarrow plot group Velocity



For the above given band Diagram

A DC Electric field will produce an AC Voltage as on ~~an~~ applied electric field will produce an oscillation and not a steady current
 \Rightarrow [FREAKY!]

LECTURE 18 - 18/08/24

Free e^- model \rightarrow nearly free e^- model

We are approximating strong ~~functions~~ functions as parabolas locally.

Broullion Zone \rightarrow intersection of Parabolae

Deformation

Correction is only at/near zone boundaries.

Perturbation (sineoidal)

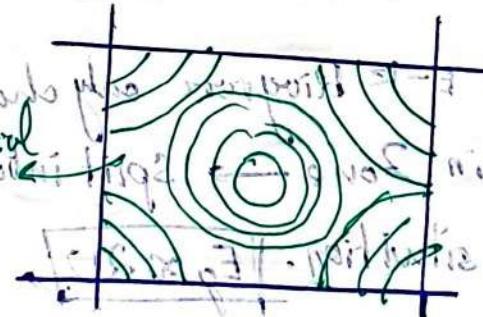
Corrects the E-k Diagram

Mass \rightarrow effective mass
 concept Different from traditional mass.

Free e^- v/s nearly free e^-

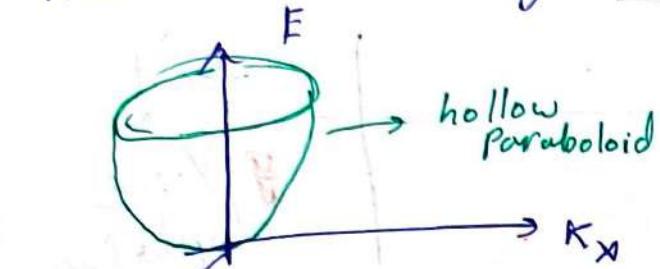
TRAJECTORY

\rightarrow no potential in case of free e^- .
 do around the equipotential lines.



2D-Broullion Zone of nearly free e^- model

Free $e^- \rightarrow$ E-K Diagram would be a hollow paraboloid



→ Equi potential lines would be nice concentric circles.

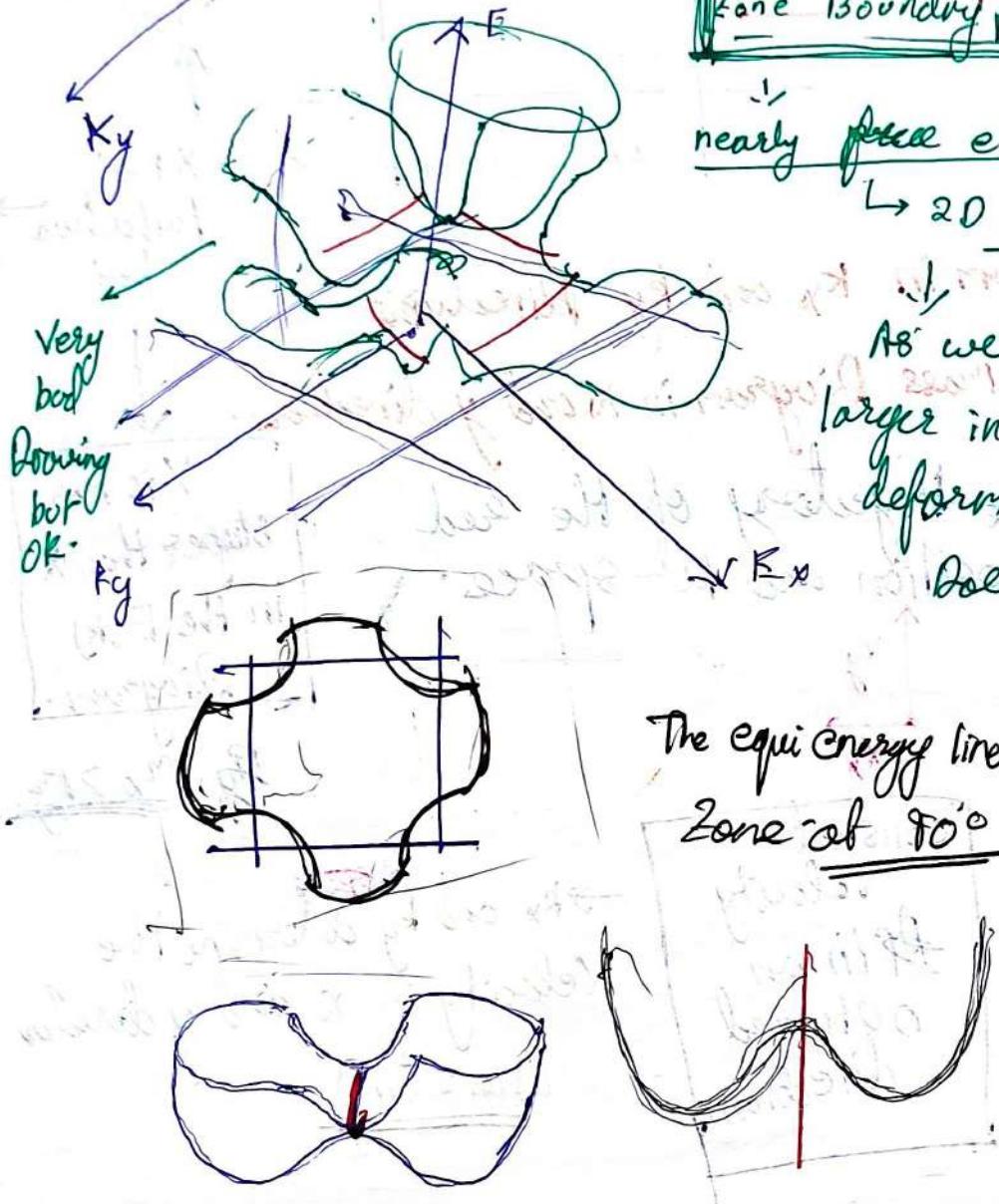
Zone Boundary \rightarrow sort of split up

nearly free e^- model

→ 2D nearly free e^- gas

As we go larger and larger in Energy the circles deform.

Does the Deformation make sense.



What Does the Deformation in 2D mean?

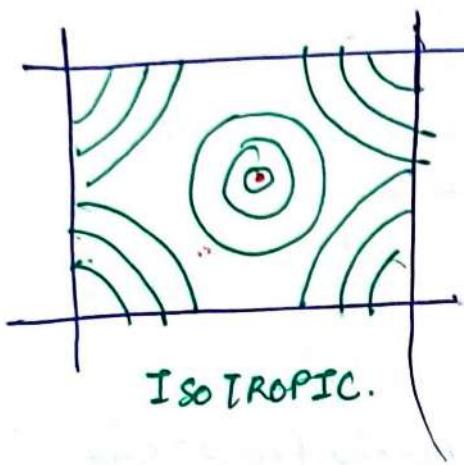
Gradient at wall of Broullion Zone is zero.

Group velocity in that direction is zero

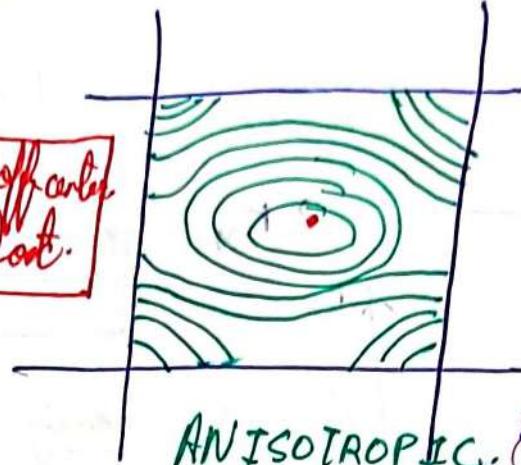
\Rightarrow Perpendicular to walls

Far away from broullion zone
very parabolic like
Close to broullion zone
distorted

Anisotropy + Effective mass



Slightly off-center
Red dot.

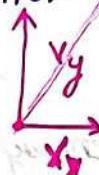


- Draw $E(k)$ Diagram in k_x and k_y Directions

- construct of effective mass Diagram in x and y directions.

Q.) what will be the trajectory of the red dot in both position and k -space.

$$N_g = \frac{dE}{dk}$$



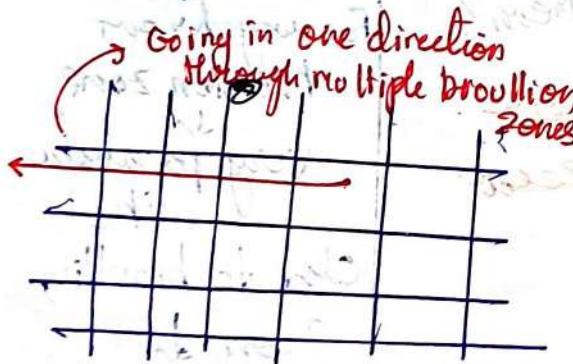
~~K space~~
nothing changes because there is no force.

Constant Velocity in an outward direction.

$\rightarrow k_x$ and k_y determine the velocity in x and y direction (momentum).

y is more steeper than x in the $E(k)$ Diagram.
 $Dg. m_x > m_y$

What if I perturb with a small ~~force~~ in the x -direction?



Essentially Periodic

$\frac{dx}{dt} \rightarrow$ Electric Field in x direction

$\frac{dk_x}{dt}$ is non zero and finite so stuff in the E_R diagram changes.

This is the motion of the particle

Force in a perfect crystal does not produce current. It's all ~~not~~ oscillating in the same place. We need scattering.

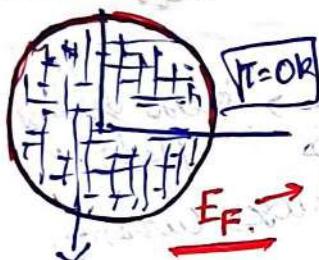
STRAIN
ENGINEERING

→ change the periodicity of a lattice

strain by introducing different atoms

There needs to be $e^- - e^-$ bumping around. If there is no scattering, there is no current. I get a velocity!! But everytime it reaches $F \rightarrow$ current \rightarrow I get velocity not acceleration. A particular value it gets rest.

Perfect Metal \Rightarrow Filled up to Fermi level. Perfectly Parabolic and.



no Brillouin zones.



k-space, these are k states (Reciprocal Lattice)

$T_e \rightarrow$ electron-electron collision

$T_i \rightarrow$ ion-electron collision



On application of E , charges near fermi level. When I increase Temp.

something about avg. Energy

If I increase scattering time

a lot, at some temperature it will still behave like perfect lattice

E : net movement of E-k diagram produced

→ Entire distribution

has moved and that will give you some current

LECTURE 19 - 21/03/24

single e^- motion \rightarrow complete today \rightarrow get a current. (move on)

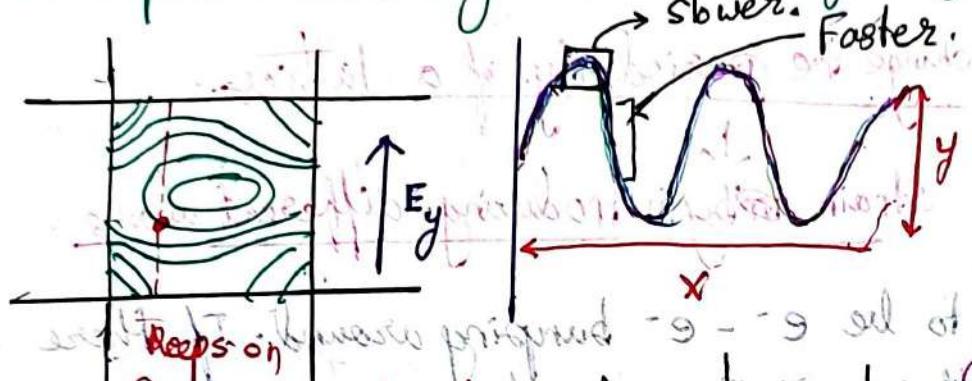
$$\frac{d\mathbf{v}_p}{dt} = \frac{e\mathbf{E}}{m^*}$$

\downarrow \mathbf{F}_y ($\mathbf{F}_y \propto dE/dk$)

$\rightarrow \mathbf{F}_y$ (what if applied field in y direction?)

{ Start looking of FULL Devices. 3

In a space which way will the trajectory go and in real space which way will the trajectory go?



Due to Frictional Processes when Electric field is applied

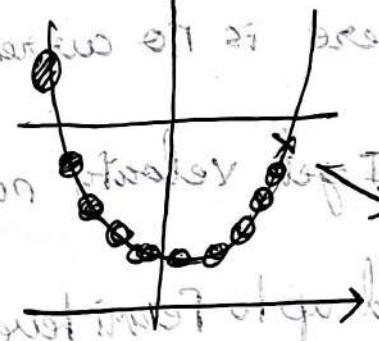
The current will be generated in direction opposite to Applied Electric Field.

Oscillatory motion

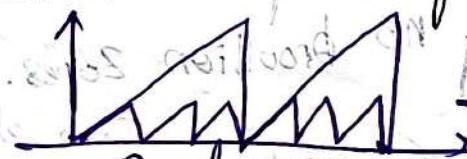
Repeating so k values in y repeat

$$\frac{dy}{dk} (\uparrow) (\downarrow)$$

Changes sign at a "center placed" 0



Distortions happen near the Fermi level.



Keeps getting Reset.

By Scattering "Friction" $\rightarrow e^- - e^-$ Repulsion \rightarrow causes current
Phonon \rightarrow DC scattering $e^- - \text{ion}$ Repulsion \rightarrow causes current.

\rightarrow called Coulomb

Interference. Roughness.

Scattering

Elastic & \rightarrow Does not accept Energy

Crystal

Absorb/Emit Energy.

$e^- \rightarrow$ elastic.

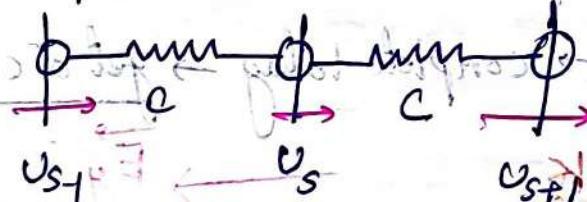
Launches \rightarrow Must Accept a Phonon.

Accept Friction

M: Phonons \rightarrow Phonons are collective atomic vibrations.

Acoustic phonons \rightarrow pressure waves \rightarrow same as sound.

What is the Energy E-K diagram for this guy?



They will have a displacement based on the positions.

Spring \rightarrow no longer at relaxation length.

C: Spring constant:

If c calculate the Force acting on each particle.

$$F = C(v_{s+1} - v_s) + C(v_{s-1} - v_s)$$

$$\rightarrow M \frac{d^2 v_s}{dt^2} = C(v_{s+1} + v_{s-1} - 2v_s) \quad \left. \begin{array}{l} \text{Force} \\ \text{Expression} \end{array} \right\}$$

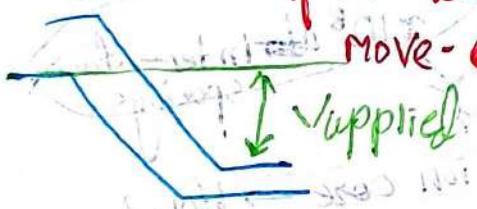
What happens to the vibrations if these are ions (+, -, +) and e⁻ passes through.

$v_s \rightarrow$ Sinusoid or Standing wave
as that point is not travelling anywhere
Fixed in space.

When an e⁻ comes close, + will attract and - will repel it.

Every atom will be completely out of phase all the time and this thus is not a pressure wave.

e⁻ if i launch an e⁻ from here, how will it move down the electric field



Si-Band Diagram

Highly reverse biased.

→ root

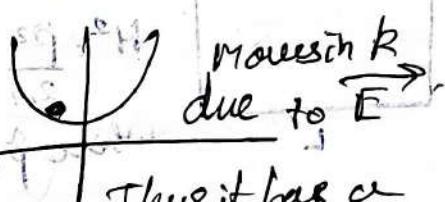
→ bounce

→ shoot and relax

How will it move down?

If probably shoots and relaxes but what is happening here and what happens in the E-k diagram.

I can compute trajectory in E-k diagram but not in E-x (band diagram).



Thus it has a kg in direction opposite to E

e⁻ hit the bond → At high Energy

Release of Phonons

Release on e⁻-hole pair.

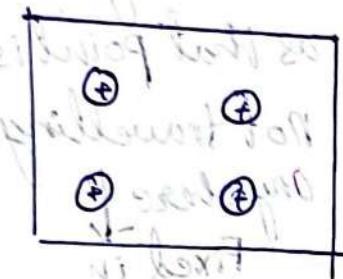
Release could emit a phonon

Probabilistic Case

E Phonons → Scattering to the lattice

If an e⁻ moves in a band diagram there is an E-R diagram in which the e⁻ is actually moving.

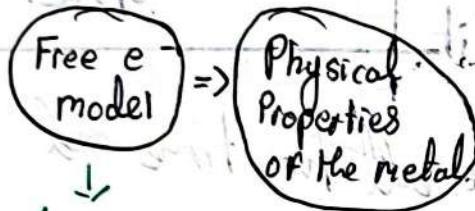
Kittel Chapter 6: Free e⁻ Fermi Gas.



→ Positive ions in a sea of conduction e⁻.

→ in Na: (+ve ions) ~ (15% of crystal) [BCC]

→ Noble Metals: +ve ions ions occupy larger % of crystal [FCC]



Valence e⁻ of the constituent atoms become free e⁻ and move freely about the volume of the metal.

charge distribution of Valence e⁻ reflects the strong electrostatic potential of the ion cores.

For a free e⁻ gas we don't consider the interaction of the e⁻ with ions in the lattice.

Conduction e⁻ can freely move very large distances in a straight path.

Moving undeflected by other e⁻ and ion core → How?

Why is condensed matter so transparent for conduction e⁻?

1D Energy Levels.

$$\frac{H\psi}{L} = \frac{\epsilon\psi}{L}$$

$$H\psi = \frac{p^2}{2m}$$

where $\vec{p} = -i\hbar\frac{d}{dx}$

$$H\psi_n = -\frac{\hbar^2}{2m} \frac{d^2\psi_n}{dx^2} = \epsilon_n \psi_n$$

(A) A conduction e⁻ is not deflected by ion cores arranged on a periodic lattice bcoz: matter waves can propagate freely in a periodic structure, as a consequence of mathematics.

(B) A conduction e⁻ is only scattered infrequently by other conduction e⁻. This property is a consequence of the Pauli Exclusion Principle.

- Each solution → orbitals
- N e⁻ → approx → Assign n e⁻ to n different e⁻
- The orbital model is exact only if there are no interactions b/w electrons.

$$\psi_n(0) = 0 \quad | \quad \psi_n = A \sin\left(\frac{2\pi x}{\lambda_n}\right)$$

$$\psi_n(L) = 0 \quad | \quad \rightarrow$$

$$\frac{1}{2}n\lambda_n = L$$

$$\epsilon_n = \frac{\hbar^2}{2m} \left(\frac{n\pi}{L} \right)^2$$

Pauli Exclusion Principle: no two electrons can have the same quantum numbers.

All of them obv.

The Fermi Energy ϵ_F is defined as the energy of the topmost filled level in the ground state of the $N e^-$ system.

\Rightarrow Each orbital can be occupied by at most 1 e^- . Spin? \div "A pair of orbitals labelled by quantum no. 'n' can accommodate 2 e^- , one with spin up and one with spin down."

$$SOL \div \epsilon_F = \frac{\hbar^2}{2m} \left(\frac{n_F \pi}{L} \right)^2 = \frac{\hbar^2}{2m} \left(\frac{N \pi}{2L} \right)^2$$

Degeneracy \rightarrow these pairs of orbitals have the same energy.
"no. of orbitals with the same energy"

Temperature Dependence \div The Ground State of the $N e^-$ system is absolute zero.

$$f(\epsilon) = \frac{1}{\exp[(\epsilon - \mu)/k_b T] + 1}$$

Probability that an orbital of energy ϵ will be occupied in an ideal e^- gas in thermal equilibrium.

$\mu \rightarrow$ function of Temperature

At absolute zero, $\mu = \epsilon_F$ OK!!

μ is chemical potential. ???

something new!!

Free e^- gas in 3 dimensions \div

$$-\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Psi_K(\vec{r}) = \epsilon_K \Psi_K(\vec{r})$$

e^- confined to a cube of length L .

$$\rightarrow \Psi_n(r) = A \sin(\pi n_x x/L)$$

$$\sin(\pi n_y y/L) \sin(\pi n_z z/L)$$

so for x, y, z $\Psi_n(x+l, y, z) = \Psi_n(x, y, z)$ and so on for y, z .

$$\Rightarrow \Psi_K(r) = \exp(i \vec{K} \cdot \vec{r})$$

$$K_x = 0, \pm \frac{2\pi}{L}, \pm \frac{4\pi}{L} \text{ and so on.}$$

(Similarly for K_y, K_z)

$$\epsilon_K = \frac{\hbar^2}{2m} (K_x^2 + K_y^2 + K_z^2)$$

Wave vector

$$\rightarrow \vec{V} = \frac{i\hbar \vec{K}}{m}$$

$$\hat{P} \Psi_K(\vec{r}) = -i\hbar \nabla \cdot \Psi_K(\vec{r})$$

* The Occupied orbitals may be represented by points inside a sphere.

The Energy at the surface of the sphere is the Fermi Energy. (At the surface)

K_F :

$$\epsilon_F = \frac{\hbar^2 K_F^2}{2m}$$

1 orbital = 1 allowed value of k_x, k_y and k_z .

Thus,

$$2 \cdot \left(\frac{4\pi k_F^3}{3} \right)$$

$$\frac{L^3 k_F^3}{3\pi^2} = \frac{V k_F^3}{3\pi^2} \Theta = N \quad (\text{no. of occupied states in that volume})$$

$$\Rightarrow R_p = \left(\frac{3\pi^2 N}{V} \right)^{1/3}$$

which only depends on the particle concentration.

Volume should be N for a $N e^-$ system.

Summary:

The E-K diagram of a free e^- gas is parabolic, and the \vec{k} is the momentum of the single e^- in that orbital.

Kittel Chapter 7: Energy bands.

* Nearly Free e^- model:

"Any simple attempt to improve upon the free e^- model is enormously profitless."

[Electrical conductivity]:

How do e^- 's respond to applied electric field.

In free e^- model, Energy values are continuously distributed essentially continuously from $0 \rightarrow \infty$.

$$E_K = \frac{\hbar^2}{2m} (k_x^2 + k_y^2 + k_z^2)$$

→ bond e^- are weakly perturbed by the periodic potential of the ion cores.

we shall see that electrons in crystals are arranged in energy bands separated by regions in energy for which no wavelike electron orbital exists.

(Energy gaps/band gaps) → result from the interaction of the conduction

Periodic lattice of the solid

electron waves with ion cores of the crystal.

If Energy Bands → completely filled or completely empty → insulator.

10-1 - 90-1
filled

↓
conductor

Slightly filled +
Slightly empty
(semiconductor)

Anything is possible.

Such E-K diagrams

→ effective mass

$$M_e < M_0$$

$$M_e > M_0$$

~~origin of
Energy gaps~~

LECTURE 20 - 28/03/24

Phonons	Electrons
Integer spin (Bosons)	Fermions → spin no. is conserved (cannot be destroyed/created)
Phonons can be created or destroyed	e ⁻ cannot be created or destroyed.

In similarities, both have effective mass, E-k Diagram, & momentum ($\hbar - k$).

At OK, the no. of states occupied by electrons define the fermi level.

* boson energy level is 0 because every state can have infinite particles

THEY DON'T HAVE A STACKING PROBLEM!!!

They don't need to gain Energy, they can occupy the same state and there's nothing stopping them from occupying the same state.

$$I_{\text{Linear}} = \mu_n C_{\text{ox}} \left(\frac{W}{L} \right) \left[(V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

Saturation

$$V_{Dsat} = V_G - V_T$$

$$I_{\text{sat}} = \frac{1}{2} \mu_n C_{\text{ox}} \left(\frac{W}{L} \right) (V_{GS} - V_{Tsat})^2$$

Pinchoff.

Note:

$$I_D = n q \sqrt{V_{DS}} = \frac{\mu}{L} V_{DS}$$

$V \approx V_{sat}$ ⇒ then what will happen to the

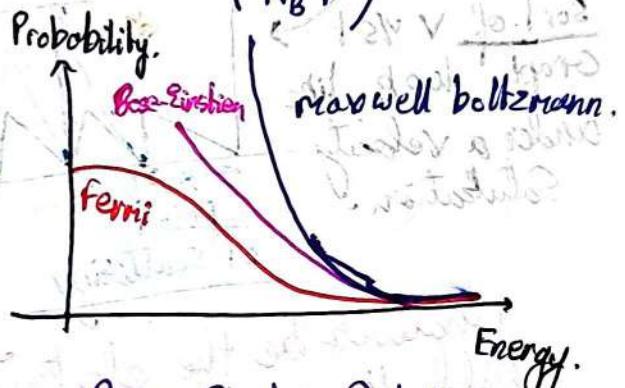
$$I_D - V_D$$

This is only for the case when V_D becomes equal to V_{sat} before $I_D = V_G - V_{Dsat}$

Phonons have a fermi level, but electrons bosons don't have a base level why is that?

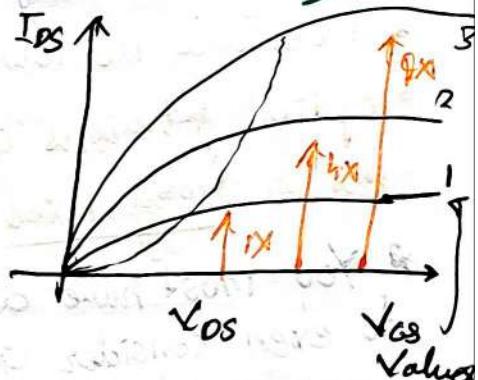
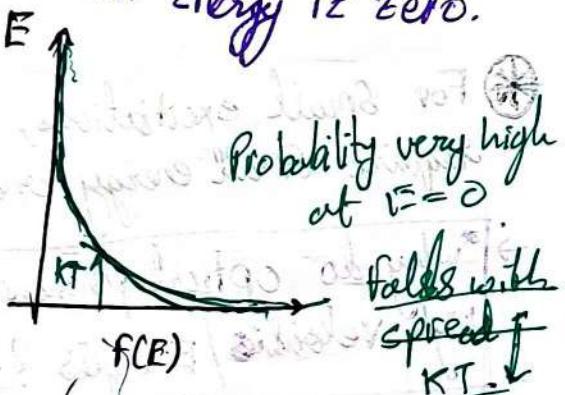
$$E_{\text{bose}} \rightarrow 0 \quad F_{\text{Fermi}} \gg 0$$

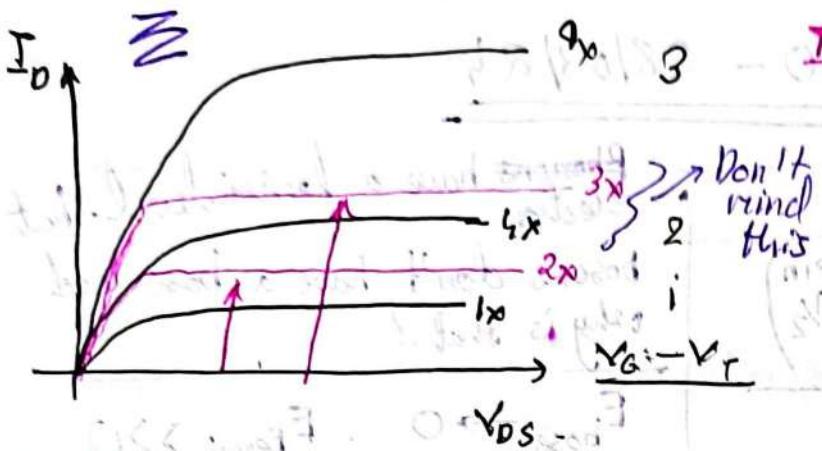
$$f(E) = \frac{1}{\exp \left(\frac{E - E_F}{k_B T} \right) + 1}$$



Bose-Einstein Distribution

mean Energy is zero.





It will now just increase linearly with $V_D - V_T$ \square

"why scattering time?
not length?"

Scattering \Rightarrow Idea of Particles
(Particle collisions)

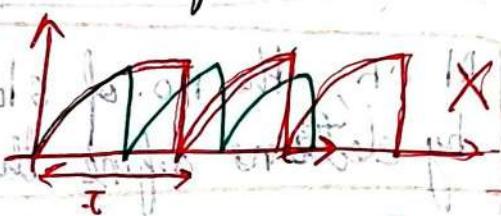
- What must this sort of V vs I graph look like under a velocity saturation?



whatever be the electric field, the scattering time must be constant

Energy balance: $\Delta E_e = E_i - E_f = E_{ph}$
Momentum balance: $\Delta p_e = p_i - p_f = p_{ph}$

$\textcircled{T} \rightarrow$ Scattering time



scattering time must reduce,
Because at V_{sat} it must scatter no matter what happens

Why??

Violated

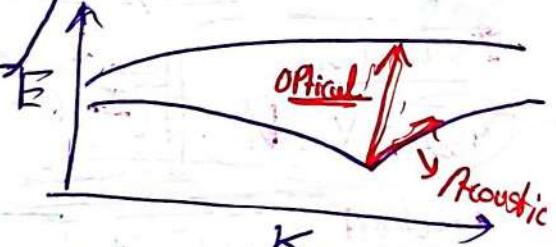
\rightarrow "selective Relaxations"

* For small excitations, only Acoustic phonons work because they require small energy and momentum.

* why do optical phonons only get activated at high velocities/Energies ??

"optical phonons v/s Acoustic phonons"

- There are no low energy optical phonons.
- optical phonons can have many momentum but a very fixed Energy



(E-k Diagram of Phonon??)

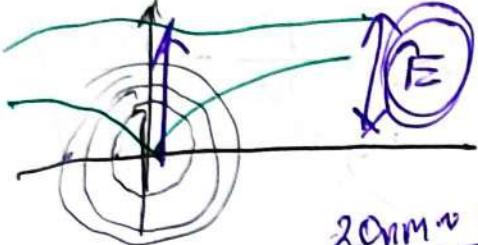
* You must have a certain amount of Energy to even consider emitting an optical phonon.

It hurts the feelings of poor Acoustic Phonons [Legit Poor in terms of Energy]

Gys!!!
Please Don't be
on Energy Elitist.

You Energy Peasant! \square

Stay in your trash Acoustic Village if you don't even have ~~100~~ this small Energy.



$$\frac{\text{Velocity overshoot}}{\text{Vovershoot}} \div \text{Vovershoot} > \text{Vsat}$$

* If your Device is very small then e^- will never scatter in this device as scattering length is comparable to length of Device.

Ballistic transport

never scatters

* Since there is only one scattering event, we will get twice the velocity we expect. This causes velocity overshoot.

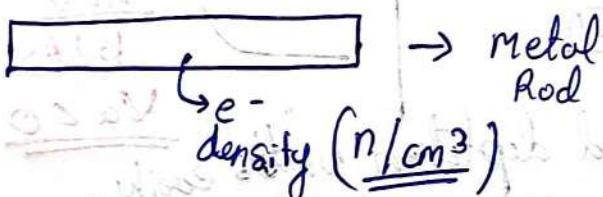
(D) crystals 2 places to scatter

"Learn how to use E-R Diagrams to do Energy and Momentum conservation".

LECTURE 21 - 01/04/24

→ PN Junctions 2-1

"Current Transport"



→ metal

Rod (length L)

→ voltage 'V' applied across it.

• If I have any device how am I gonna know where the current is going to be blocked?

Current through the device? → Yes // Acts like a

Electric field inside metal? → Resistor $\rightarrow J = n e V q$

It is the only source of blockage!

→ Yes → For Drift current.

↙ Potential Gradient inside metal.

[But typically we don't have an Electric field inside a metal].

→ Is the carrier concentration uniform across the metal?

E_{ext} is const. // charge cannot accumulate at a point if there is no uniform electric field. Thus charge concentration is uniform across the metal. No diffusion then? since no conc.

• Point: - Estimate the drift and diffusion current magnitudes, which will drift or diffusion dominate?

$$I_{\text{drift}} = q n V = q n \frac{N V}{L}$$

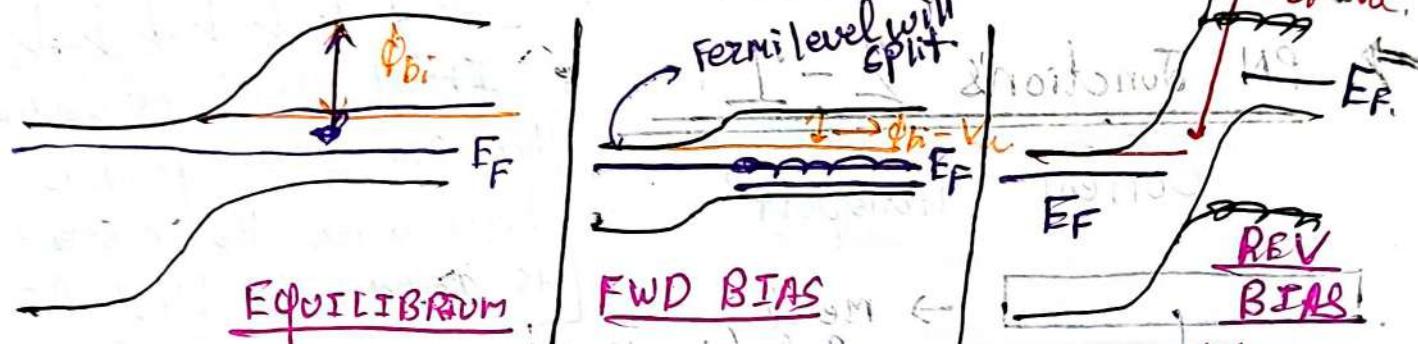
Einstein Relationship: $\frac{D}{\mu} = \frac{kT}{q}$

$$I_{\text{diffusion}} = -q \frac{D n}{dx} = \frac{q D n}{L} = \frac{q N V r n}{L}$$

So $\frac{I_{\text{drift}}}{I_{\text{diffusion}}} = \frac{V}{V_T}$ → As long as my $V > V_T$, my drift will dominate the diffusion current.

* Thus far every device, i need to calculate whether drift or diffusion will dominate and this is very painful.

PN-JUNCTION $\doteq N_A = N_D = 10^{15} / \text{cm}^3$



* can calculate built-in potentials and depletion widths easily.

* In Equilibrium, show that if $I = 0$ then E_F is flat.

$$\vec{E} = -\frac{dV}{dx} \quad J = n q \mu_n \vec{E} + q D_n \frac{dn}{dx} \rightarrow \vec{E} = -\frac{D_n}{\mu_n \sigma n} \frac{dn}{dx} = -\frac{kT}{q} \frac{d \ln(n)}{dx}$$

$$n = n_i e^{\frac{E_F - E_i}{kT}} \rightarrow \ln n = \ln n_i + \frac{E_F - E_i}{kT}$$

$$\text{But } \frac{dE_i}{dx} = -q \frac{dV}{dx} = q \vec{E} = -kT \frac{d \ln(n)}{dx} \rightarrow \frac{d \ln(n)}{dx} = \frac{1}{kT} \left(\frac{dE_F}{dx} - \frac{dE_i}{dx} \right)$$

$$\rightarrow \text{Putting this in (Red) eq } \frac{d \ln(n)}{dx} = \frac{dE_F}{dx} = 0$$

* This comes from the fact that Boltzmann statistics is roughly correct.

Steps to Calculate Current in FWD Bias:

(1) Continuity Equation for minority carriers with low level injection
 (i) no Electric field in bulk. (ii) Eq^m concentration doesn't change with x (iii) low level injection

(2) Quasi Fermi levels are flat across the depletion region e^- Fermi h hole Fermi

③ no generation in depletion Region

$$\text{so, } \frac{d^2 (\delta P_n(x'))}{dx'^2} = \frac{\delta S P_n(x')}{D_p T_p}$$

Apply boundary conditions. And Identify minority carrier concentration gradient.

④ calculate Diffusion/Drift current components at the two interfaces and add them up. $J = J_p + J_n$.

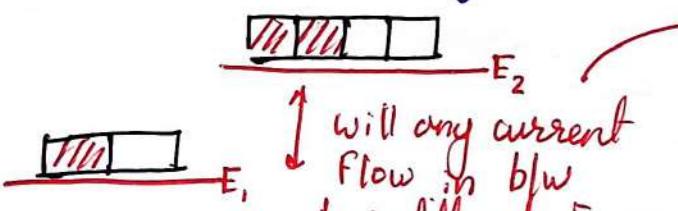
Flat Fermi Levels (Mott's Proof)

$$J_{L-R} = k n_L f_i \cdot n_R (1 - \frac{f_i}{2}) \rightarrow f_i = f_2$$

$$J_{R-L} = k n_R (f_i) \cdot n_L (1 - f_i)$$

→ This is the math b/w source states and sink states.

Fermi level is the supplier of density to a state. (At a particular Energy level)
If the Fermi level is flat, every point is at a given Energy level. will not exchange current.

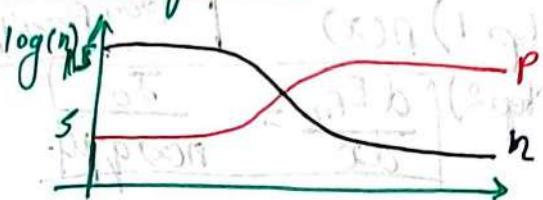


Drift/Diffusion → Force based current flow.

I'm trying to argue an Energy based current flow, i.e., if the Energy is same at both sides, no current can flow. [ENERGY BALANCE]

current is only zero when the field is countered by an exponentially changing carrier concentration.

For Eq vi:



- An Exponential carrier concentration opposes a linear voltage change.

- It is not trivial to push back against drift.

is there any reason to relax?



- Also, Fermi function must be maintained so a Relaxation will automatically be compensated by an excitation.

Different Voltages at two ends of the device cause the formation of Quasi Fermi levels

$E_{F_n} \rightarrow e^-$ in conduction ($n(E_{F_n})$)

$E_{F_p} \rightarrow$ holes in valence ($p(E_{F_p})$).

$n \cdot p \neq n_i^2 \rightarrow$ Thus we need two Fermi levels

$$J_p = nq\mu \frac{dE_{Fn}}{dx}$$

Similarly J_n .

I can get any variation of

Step 1) $n(x)$ ↑ For a Fermi level

Step 2) $\frac{dE_{Fn}}{dx} = \frac{J_0}{n(x)q\mu}$

Thus need to find the most Resistive part. (minority carriers) ↴ charge concentration loss.

most of the potential is dropped in the Resistive part.

- Thus, we have calculated current via Energies instead of Force Balance

Symbol of resistivity scattered



and term without resistivity
resistivity is due to scattering
but scattering at the interface the
conductors are

all to this out to especially because
will be scattered in same effect
about effect

(, 17) resistance in \rightarrow

((, 17) conductance \rightarrow)

out conductance \rightarrow

\rightarrow I thus have formulated a purely drift based definition of current.

I can calculate the current via drift based the gradient of the quasi Fermi level and Vice Versa.

carrier concentration high \rightarrow intrinsic
carrier concentration low \rightarrow Resistive

EE724: QM to Semi-classical Picture

Outline: e^- in solid - Simplifying the QM Problem.

- Perfect Crystal:
 — wave packet is particle — Particle motion
 — Effective mass — Electronic Structure.
- Scattering:
- e^- transport by \vec{E} -Field.

GOAL → Set up mechanics of e^-
 → Now, does an e^- move under force. (need speed) ✓
 (for a good transistor).

motion
 of e^- ? → Eigen Energies (Allowed values of Energies that the e^- can have) → Solution of Energies of the e^- (in the solid) by solving Hamiltonian.

Many e^- → It's too complicated to solve for more than 1 e^-

INDEPENDENT
 e^-
 APPROXIMATION.

⇒ So essentially we are going to find all the states in the solid disregarding e^-e^- interaction.

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{2x^2} + V\Psi.$$

Schrödinger's Equation

e^- (I have an e^- in a solid)

What are the influences on the independent e^- that need to be considered?

Relevant interactions in a solid which an e^- sees?

Periodic Potential by lattice.

$$U_{\text{total}}(x, t) = U_{\text{lattice}} + U_{\text{vibrations}} + U_{\text{External}}$$

Extent of interaction?
 An External Electric field is just a small perturbation over the pre-existing electric field due to the ions of the lattice.

* $U_{\text{lattice}} \gg U_{\text{vibrations}}, U_{\text{External}}$
 (Perturbation Approach).

- $E_{\text{ext}} \approx 10^5 \text{ V/cm}$
- interatomic spacing $\approx 10^{-8} \text{ cm}$.
- Max Voltage Drop $\approx 0.1 \text{ V per atom}$.

"Particle like Interactions"

Free e^- gas (metals) \rightarrow The nuclear charge is uniform (not periodic)

$U \rightarrow 0$ everywhere and $V \rightarrow \infty$ at boundary \rightarrow continuous spectrum of Energy levels.

$$\psi(x) = e^{ikx}$$

where

$$E = \frac{\hbar^2 k^2}{2m}$$

wavefunction at a particular Energy is traveling wave spread all over solid.
(not very particle like).

$$\psi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x, t) e^{-ikx} dx$$

Can be derived from
general Equation
of a wave.

$$\text{Travelling wave} = \psi(x, t) = e^{i(kx - \omega t)}$$

by eqn of
travelling wave • {Phase velocity $\frac{V_p}{\lambda} = \frac{\omega}{k}$ }

Energy of an e^- can mainly be attributed to its Kinetic energy $[U=0]$

$$\frac{1}{2} m V_p^2 = \frac{\hbar^2 k^2}{2m}$$

$$m \Rightarrow V_p = \frac{\hbar k}{m} = \frac{\omega}{k} \Rightarrow \omega = \frac{\hbar \cdot k^2}{m}$$

Group Velocity:

$\phi(k)$ is a narrow distribution of k_0 .

$$\psi(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k) e^{i(kx - \omega t)} dk.$$

$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k) e^{ikx} dk$$

$$F(t) = e^{-i\omega t}$$

Fourier Transform (from Schrodinger's equation)

$\omega(k) \rightarrow \text{Dispersion Relation}$

• Applying Taylor Expansion, $\omega(k) \approx \omega_0 + \frac{d\omega}{dk} (k - k_0)$

At $\omega = \omega_0$ [Essentially around $k = k_0$]

$(\omega_0) \leftrightarrow (k_0) \rightarrow$ Corresponding Values

• Change Variables from k to s where $s = k - k_0 \Rightarrow k = s + k_0$

$$\psi(x, t) \cong \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k_0 + s) e^{i[(k_0 + s)x - (\omega_0 + (\omega')_0 s)t]} ds$$

$$\text{At } t=0: \psi(x,0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k_0+s) e^{i(k_0+s)x} ds.$$

$$\text{At } t=t: \psi(x,t) = \frac{1}{\sqrt{2\pi}} e^{i(-\omega t + k_0(\omega')_0 t)} \int_{-\infty}^{\infty} \phi(k_0+s) e^{i(k_0+s)(x-(\omega')_0 t)} ds.$$

Time dependent phase shift
does not affect $|\psi|^2$

Integral where x shifted to $x - (\omega')_0 t$ after time t .

Based on group velocity, the particle velocity and momentum can be determined.

→ Wave packet is travelling at Group Velocity $\frac{d\omega}{dR}|_{\omega=\omega_0}$

- The s -dependent part shows the time dependent shift in the $\phi(k_0+s)$ diagram.
- Thus the ~~phase~~ group velocity at point is given by the slope of the E-k diagram.
- Phase velocity and group velocity are not correlated as such. They can have both different magnitudes as well as signs.
- "Group Velocity is the critical parameter determining particle velocity."

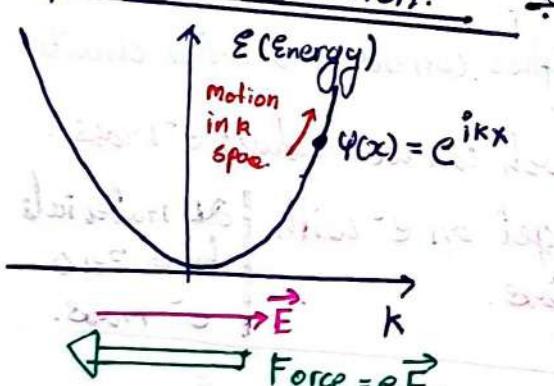
For E-k diagram,
(Parabolic)

$$\frac{\omega}{k} = \frac{\hbar k}{m}$$

$$\text{and } \frac{d\omega}{dR} = \frac{2\hbar k}{m}$$

$$\text{thus } \frac{v_g}{v_p} = 2.$$

Equation of Motion.



1. Force $e\vec{E}$

2. Particle Velocity $\vec{v} = \frac{1}{2} m \omega^2 \vec{k} \rightarrow \frac{dE}{dk} = \frac{m\omega}{k^2} \frac{d\omega}{dk}$

$$\rightarrow v_g = \frac{k^2}{m\omega} \frac{dE}{dk} = \frac{1}{\hbar} \frac{dE}{dk} \quad \boxed{v_g = \frac{1}{\hbar} \frac{dE}{dk}}$$

3. Work done $\delta E = -e\vec{E} \cdot \vec{v}_g \delta t$

4. Force changes k

$$\vec{F} = \hbar \frac{d\vec{k}}{dt}$$

$\rightarrow k$ vector changes in time with the application of Force $e\vec{E}$. $= \left(\frac{dE}{dk} \right) \delta k = \hbar v_g \delta k$

Motion in Absence of Force

$F=0 \Rightarrow \frac{d\mathbf{k}}{dt} = 0 \Rightarrow \mathbf{k} = \mathbf{k}_0 \rightarrow$ no motion in \mathbf{k} -space.

Thus $\mathbf{v}_g(\mathbf{k}) = \frac{d\omega}{dk} \Big|_{\mathbf{k}=\mathbf{k}_0}$ or $\mathbf{v}_g(\mathbf{k}_0) \Rightarrow$ in real space const. velocity is observed.

Motion in Presence of Force

$$\mathbf{k}(t) = \mathbf{k}_0 + \frac{\mathbf{F}}{\hbar} t$$

Then

$$x = x_0 + \int \mathbf{v}_g(\mathbf{k}) dt$$

Bloch model!!!

$$\mathbf{v}_g(\mathbf{k}) = \frac{d\omega}{dk} \Big|_{\mathbf{k}=\mathbf{k}_0 + \frac{\mathbf{F}}{\hbar} t}$$

→ SSS software

$$\text{Effective mass} \div \mathbf{v}_g = \frac{1}{\hbar} \frac{d\epsilon}{dk} \rightarrow \frac{d\mathbf{v}_g}{dt} = \frac{1}{\hbar} \frac{d^2\epsilon}{dk^2} = \frac{1}{\hbar} \left(\frac{\partial^2 \epsilon}{\partial k^2} \right) \left(\frac{\partial k}{\partial t} \right)$$

$$\rightarrow \frac{d\mathbf{v}_g}{dt} = \left(\frac{1}{\hbar^2} \frac{\partial^2 \epsilon}{\partial k^2} \right) \frac{1}{m_{\text{eff}}} \rightarrow m^* = \left(\frac{1}{\hbar^2} \frac{d^2 E}{dk^2} \right)^{-1}$$

$$\rightarrow m^* = \frac{\hbar^2}{(d^2 E / dk^2)}$$

$$m^* \propto \frac{1}{\text{Curvature of E-k Diagram}}$$

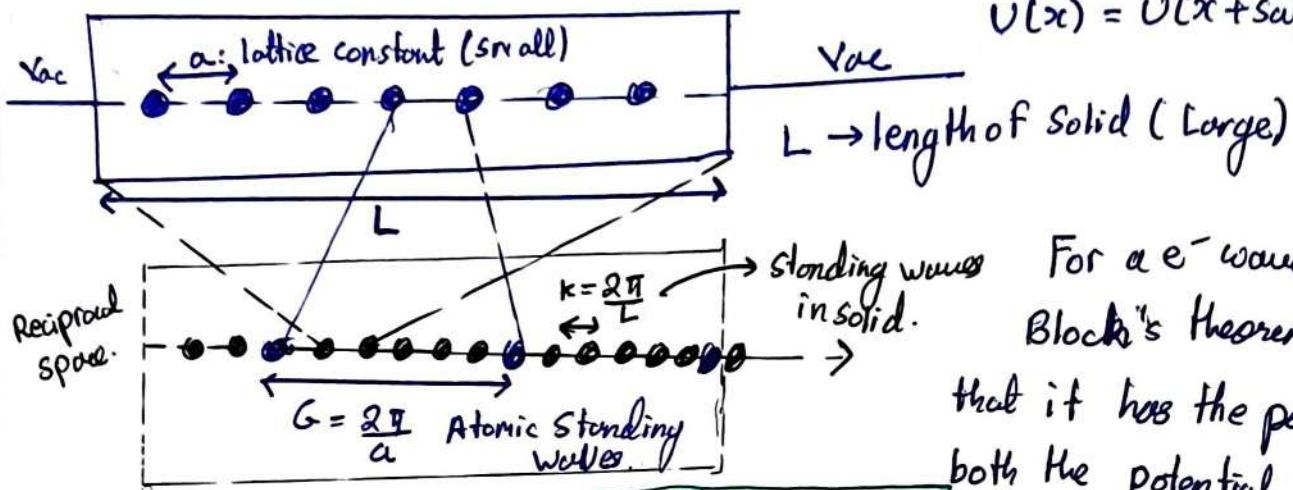
Thus, the mass depends on the local "shape" of the E-k diagram.
(not the actual e-mass).

1. Mass is the fundamental property which relates acceleration caused by a Force or Velocity due to Energy Imported.
2. Higher velocity (For some Force) may imply higher current and faster circuits.
3. How to increase e- velocity? → By how much can we reduce e-mass.
a) Can we get an e- with zero mass.
b) 2D materials have zero e-mass.

Bond Structure → to engineer velocity in a solid, we must be able to play with bond structure

- ↳ a) Free e- band structure.
- b) Nearly Free e- band structure
- c) Tight Binding.

Real (x) v/s Reciprocal (R) Space: Bloch.



$$\Psi_R(\vec{r}) = V_k(\vec{r}) e^{i\vec{k} \cdot \vec{r}}$$

$$\text{In 1D} \therefore \Psi_R(x) = V_k(x) e^{ikx}$$

As far as I know the value of k comes from equating the wave functions at the two ends of the lattice.

For an e^- wave to satisfy Bloch's theorem, we ensure that it has the periodicity of both the potential as well as the reciprocal space. And G describes $\frac{1}{2}$ Brillouin Zone