

AI5031: Machine learning, exercise sheet 2

1 1D derivatives

Compute the derivative $h'(x)$. Always give rule and decomposition!!

- a) $h(x) = \sin(\sin(x))$
- b) $h(x) = 2 \sin(x) + 5 \cos(x)$
- c) $h(x) = \alpha \sin(x) + \beta \cos(x)$
- d) $h(x) = \ln(\sin(\cos(5.2)))$
- e) $h(x) = \exp(x) \ln(x)$
- f) $h(x) = x \cos(x)$
- g) $h(x) = e^{\cos(x)}$

2 Preparations (on paper)

Compute the following derivatives!

- a) $\frac{\partial}{\partial x_3} \sum_{i=1}^5 x_i$
- b) $\frac{\partial}{\partial x_6} \sum_{i=1}^5 x_i$
- c) $\frac{\partial}{\partial x_3} \sum_{i=1}^5 (2x_i - 5)^2$
- d) $\frac{\partial}{\partial x_3} \sum_{i=1}^5 (x_i^2 - 5x_i)^2$
- e) $\frac{\partial}{\partial x_3} \cos(x_3)x_3$
- f) $\frac{\partial}{\partial x_3} \cos(x_3)x_4$
- g) $\frac{\partial}{\partial x_3} \cos(\exp(x_3))$

3 Vector-vector chain rule 1

Given the function $\vec{g}(\vec{x})$ with $g_i(\vec{x}) = x_i^2$, and $f(\vec{x}) = \sum_i \log x_i$: compute the partial derivative $\frac{\partial f(\vec{g}(\vec{x}))}{\partial x_1}$!

4 Vector-vector chain rule 2

We can chain the functions $\vec{y} = \vec{f}(\vec{w})$ with $f_i(\vec{w}) = w_i^3$, and $\tilde{\mathcal{L}}(\vec{y}) = \sum_i y_i$ as $\mathcal{L}(\vec{w}) \equiv \tilde{\mathcal{L}}(\vec{f}(\vec{w}))$. Compute the partial derivative $\frac{\partial \mathcal{L}}{\partial w_1}$! What does this remind you of?

5 Gradient descent on paper

Consider the function $f(\vec{x}) = x_1^2 + 2x_2^2$. Assuming a starting point of $\vec{x}(0) = [1, 3]$ and a step size of $\epsilon = 0.1$: perform 2 steps of gradient descent, that is, compute the values of $\vec{x}(1)$, $\vec{x}(2)$ by iteration. Verify that the value of f is decreasing!