Assignment-3

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Breaker Code: P1 P2 P3 P4
Colour codes: R – red, B – blue, G – green, Y – yellow, O – orange, P – purple, W – white, K – black
Color and Position encoding in variable (1-8 for P1, 9-16 for P2, 17-24 for P3 and 25-32 for P4)

P1R	P1B	P1G	P1Y	P1O	P1P	P1W	P1K
P2R	P2B	P2G	P2Y	P2O	P2P	P2W	P2K
P3R	P3B	P3G	P3Y	P3O	P3P	P3W	P3K
P4R	P4B	P4G	P4Y	P4O	P4P	P4W	P4K

Interpretation of above encoding:
e.g., P1R- Color R is at position P1

Types of Clauses added for Task-1

1) At least one color should be selected for each position/index.

(P1R ∨ P1B ∨ P1G ∨ P1Y ∨ P1O ∨ P1P ∨ P1W ∨ P1K)
∧ (P2R ∨ P2B ∨ P2G ∨ P2Y ∨ P2O ∨ P2P ∨ P2W ∨ P2K)
∧ (P3R ∨ P3B ∨ P3G ∨ P3Y ∨ P3O ∨ P3P ∨ P3W ∨ P3K)
∧ (P4R ∨ P4B ∨ P4G ∨ P4Y ∨ P4O ∨ P4P ∨ P4W ∨ P4K)

2)No Duplicate colors are allowed:

P1R -> (¬P2R) ∧ (¬P3R) ∧ (¬P4R)
P2R -> (¬P1R) ∧ (¬P3R) ∧ (¬P4R)
P3R -> (¬P1R) ∧ (¬P2R) ∧ (¬P4R)
P4R -> (¬P1R) ∧ (¬P2R) ∧ (¬P3R). (Similarly for other colors)

Tseytin Transformation of [P1R -> (¬P2R) ∧ (¬P3R) ∧ (¬P4R)]

(¬P1R ∨ ¬P2R) ∧ (¬P1R ∨ ¬P3R) ∧ (¬P1R ∨ ¬P4R)

3)No two colors should be selected for the same index:

e.g., P1R -> (¬P1B) ∧ (¬P1G) ∧ (¬P1Y) ∧ (¬P1O) ∧ (¬P1P) ∧ (¬P1W) ∧ (¬P1K)

4)Color at index i is responsible for Black feedback (Defined 4 variable B1…B4):

e.g., [R B G Y]

B1 <-> (P1R)
B2 <-> (P2B)
B3 <-> (P3G)
B4 <-> (P4Y)

Exactly one Black feedback:

(B1 ∧ ¬B2 ∧ ¬B3 ∧ ¬B4) ∨ (¬B1 ∧ B2 ∧ ¬B3 ∧ ¬B4) ∨ (¬B1 ∧ ¬B2 ∧ B3 ∧¬ B4)
∨ (¬B1 ∧ ¬B2 ∧ ¬B3 ∧ B4)

Exactly two Black feedback:

(B1 ∧ B2 ∧ ¬B3 ∧ ¬B4) ∨ (B1 ∧ ¬B2 ∧ B3 ∧ ¬B4) ∨ (B1 ∧ ¬B2 ∧ ¬B3 ∧ B4)
∨ (¬B1 ∧ B2 ∧ B3 ∧ ¬B4) ∨ (¬B1 ∧ B2 ∧ ¬B3 ∧ B4) ∨ (¬B1 ∧ ¬B2 ∧ B3 ∧ B4)

Exactly three Black feedback:

(B1 ∧ B2 ∧ B3 ∧ ¬B4) ∨ (B1 ∧ B2 ∧ ¬B3 ∧ B4) ∨ (B1 ∧ ¬B2 ∧ B3 ∧ B4) ∨ (¬B1 ∧ B2 ∧ B3 ∧ B4)

Exactly four Black feedback:

(B1 ∧ B2 ∧ B3 ∧ B4)

5)Color at index i is responsible for White feedback (Defined 4 variable W1…W4):

e.g., [R B G Y]

W1 <-> (P2R ∨ P3R ∨ P4R)
W2 <-> (P1B ∨ P3B ∨ P4B)
W3 <-> (P1G ∨ P2G ∨ P4G)
W4 <-> (P1Y ∨ P2Y ∨ P3Y)

Exactly one White feedback:

(W1 ∧ ¬W2 ∧ ¬W3 ∧ ¬W4) ∨ (¬W1 ∧ W2 ∧ ¬W3 ∧ ¬W4) ∨ (¬W1 ∧ ¬W2 ∧ W3 ∧¬ W4)
∨ (¬W1 ∧ ¬W2 ∧ ¬W3 ∧ W4)

Exactly two White feedback:

(W1 ∧ W2 ∧ ¬W3 ∧ ¬W4) ∨ (W1 ∧ ¬W2 ∧ W3 ∧ ¬W4) ∨ (W1 ∧ ¬W2 ∧ ¬W3 ∧ W4)
∨ (¬W1 ∧ W2 ∧ W3 ∧ ¬W4) ∨ (¬W1 ∧ W2 ∧ ¬W3 ∧ W4) ∨ (¬W1 ∧ ¬W2 ∧ W3 ∧ W4)

Exactly three White feedback:

(W1 ∧ W2 ∧ W3 ∧ ¬W4) ∨ (W1 ∧ W2 ∧ ¬W3 ∧ W4) ∨ (W1 ∧ ¬W2 ∧ W3 ∧ W4)
∨ (¬W1 ∧ W2 ∧ W3 ∧ W4)

Exactly four White feedback:

(W1 ∧ W2 ∧ W3 ∧ W4)

6)Color at index i is not present in hidden code. (C=4-(B+W)) feedback (Defined 4 variable C1...C4):

e.g., [R B G Y]

C1 <-> (¬P1R ∧ ¬P2R ∧ ¬P3R ∧ ¬P4R)
C2 <-> (¬P1B ∧ ¬P2B ∧ ¬P3B ∧ ¬P4B)
C3 <-> (¬P1G ∧ ¬P2G ∧ ¬P3G ∧ ¬P4G)
C4 <-> (¬P1Y ∧ ¬P2Y ∧ ¬P3Y ∧ ¬P4Y)

Exactly one color-change feedback:

(C1 ∧ ¬C2 ∧ ¬C3 ∧ ¬C4) ∨ (¬C1 ∧ C2 ∧ ¬C3 ∧ ¬C4) ∨ (¬C1 ∧ ¬C2 ∧ C3 ∧¬ C4)
∨ (¬C1 ∧ ¬C2 ∧ ¬C3 ∧ C4)

Exactly two color-change feedbacks:

(C1 ∧ C2 ∧ ¬C3 ∧ ¬C4) ∨ (C1 ∧ ¬C2 ∧ C3 ∧ ¬C4) ∨ (C1 ∧ ¬C2 ∧ ¬C3 ∧ C4)
∨ (¬C1 ∧ C2 ∧ C3 ∧ ¬C4) ∨ (¬C1 ∧ C2 ∧ ¬C3 ∧ C4) ∨ (¬C1 ∧ ¬C2 ∧ C3 ∧ C4)

Exactly three color-change feedbacks:

(C1 ∧ C2 ∧ C3 ∧ ¬C4) ∨ (C1 ∧ C2 ∧ ¬C3 ∧ C4) ∨ (C1 ∧ ¬C2 ∧ C3 ∧ C4) ∨ (¬C1 ∧ C2 ∧ C3 ∧ C4)

Exactly four color-change feedbacks:

(C1 ∧ C2 ∧ C3 ∧ C4)

Types of Clauses added for Task-2

1) At least one color should be selected for each position/index.

(P1R ∨ P1B ∨ P1G ∨ P1Y ∨ P1O ∨ P1P ∨ P1W ∨ P1K)
∧ (P2R ∨ P2B ∨ P2G ∨ P2Y ∨ P2O ∨ P2P ∨ P2W ∨ P2K)
∧ (P3R ∨ P3B ∨ P3G ∨ P3Y ∨ P3O ∨ P3P ∨ P3W ∨ P3K)
∧ (P4R ∨ P4B ∨ P4G ∨ P4Y ∨ P4O ∨ P4P ∨ P4W ∨ P4K)

2)No two colors should be selected for the same index:

P1R -> (¬P1B) ∧ (¬P1G) ∧ (¬P1Y) ∧ (¬P1O) ∧ (¬P1P) ∧ (¬P1W) ∧ (¬P1K)

3)Color at index i is responsible for Black feedback (Defined 4 variable B1…B4):

e.g., [R B G Y]

B1 <-> (P1R)
B2 <-> (P2B)
B3 <-> (P3G)
B4 <-> (P4Y)

Exactly one Black feedback:

(B1 ∧ ¬B2 ∧ ¬B3 ∧ ¬B4) ∨ (¬B1 ∧ B2 ∧ ¬B3 ∧ ¬B4) ∨ (¬B1 ∧ ¬B2 ∧ B3 ∧¬ B4)
∨ (¬B1 ∧ ¬B2 ∧ ¬B3 ∧ B4)

Exactly two Black feedback:

(B1 ∧ B2 ∧ ¬B3 ∧ ¬B4) ∨ (B1 ∧ ¬B2 ∧ B3 ∧ ¬B4) ∨ (B1 ∧ ¬B2 ∧ ¬B3 ∧ B4)
∨ (¬B1 ∧ B2 ∧ B3 ∧ ¬B4) ∨ (¬B1 ∧ B2 ∧ ¬B3 ∧ B4) ∨ (¬B1 ∧ ¬B2 ∧ B3 ∧ B4)

Exactly three Black feedback:

(B1 ∧ B2 ∧ B3 ∧ ¬B4) ∨ (B1 ∧ B2 ∧ ¬B3 ∧ B4) ∨ (B1 ∧ ¬B2 ∧ B3 ∧ B4) ∨ (¬B1 ∧ B2 ∧ B3 ∧ B4)

Exactly four Black feedback:

(B1 ∧ B2 ∧ B3 ∧ B4)

4)Color at index i is responsible for White feedback (Defined 4 variable W1…W4):

e.g., [R B G Y]

W1 <-> (P2R ∨ P3R ∨ P4R)
W2 <-> (P1B ∨ P3B ∨ P4B)
W3 <-> (P1G ∨ P2G ∨ P4G)
W4 <-> (P1Y ∨ P2Y ∨ P3Y)

Exactly one White feedback:

(W1 ∧ ¬W2 ∧ ¬W3 ∧ ¬W4) ∨ (¬W1 ∧ W2 ∧ ¬W3 ∧ ¬W4) ∨ (¬W1 ∧ ¬W2 ∧ W3 ∧¬ W4)
∨ (¬W1 ∧ ¬W2 ∧ ¬W3 ∧ W4)

Exactly two White feedback:

(W1 ∧ W2 ∧ ¬W3 ∧ ¬W4) ∨ (W1 ∧ ¬W2 ∧ W3 ∧ ¬W4) ∨ (W1 ∧ ¬W2 ∧ ¬W3 ∧ W4)
∨ (¬W1 ∧ W2 ∧ W3 ∧ ¬W4) ∨ (¬W1 ∧ W2 ∧ ¬W3 ∧ W4) ∨ (¬W1 ∧ ¬W2 ∧ W3 ∧ W4)

Exactly three White feedback:

(W1 ∧ W2 ∧ W3 ∧ ¬W4) ∨ (W1 ∧ W2 ∧ ¬W3 ∧ W4) ∨ (W1 ∧ ¬W2 ∧ W3 ∧ W4)
∨ (¬W1 ∧ W2 ∧ W3 ∧ W4)

Exactly four White feedback:

(W1 ∧ W2 ∧ W3 ∧ W4)

5) No color from the previous guess can be repeated at index i. (Defined 4 variable D1...D4)

e.g., [R B G Y]

D1 <-> (¬P1R ∧ ¬P1B ∧ ¬P1G ∧ ¬P1Y)

D2 <-> (¬P2R ∧ ¬P2B ∧ ¬P2G ∧ ¬P2Y)

D3 <-> (¬P3R ∧ ¬P3B ∧ ¬P3G ∧ ¬P3Y)

D4 <-> (¬P4R ∧ ¬P4B ∧ ¬P4G ∧ ¬P4Y)

Exactly one color-change feedback:

(D1 ∧ ¬D2 ∧ ¬D3 ∧ ¬D4) ∨ (¬D1 ∧ D2 ∧ ¬D3 ∧ ¬D4) ∨ (¬D1 ∧ ¬D2 ∧ D3 ∧¬ D4)
∨ (¬D1 ∧ ¬D2 ∧ ¬D3 ∧ D4)

Exactly two color-change feedbacks:

(D1 ∧ D2 ∧ ¬D3 ∧ ¬D4) ∨ (D1 ∧ ¬D2 ∧ D3 ∧ ¬D4) ∨ (D1 ∧ ¬D2 ∧ ¬D3 ∧ D4)
∨ (¬D1 ∧ D2 ∧ D3 ∧ ¬D4) ∨ (¬D1 ∧ D2 ∧ ¬D3 ∧ D4) ∨ (¬D1 ∧ ¬D2 ∧ D3 ∧ D4)

Exactly three color-change feedbacks:

(D1 ∧ D2 ∧ D3 ∧ ¬D4) ∨ (D1 ∧ D2 ∧ ¬D3 ∧ D4) ∨ (D1 ∧ ¬D2 ∧ D3 ∧ D4) ∨
(¬D1 ∧ D2 ∧ D3 ∧ D4)

Exactly four color-change feedbacks:

(D1 ∧ D2 ∧ D3 ∧ D4)

Note: These are the higher-level clauses which will be included in the SAT solver. However, in actual implementation, it is required to use Tseytin Transformation to convert the clauses into CNF form.