

# EV-GREEN: Electric Vehicle Routing with GreenZone Prioritization and V2G Incentive Integration

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## Abstract

abstract

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## 1. Introduction

Eco-routing, which refers to the process of determining energy-efficient routes for Electric Vehicles (EVs), has gained significant attention due to its potential to reduce carbon emissions and optimize the utilization of renewable energy. The increasing adoption of EVs and the need for sustainable transportation systems necessitate the integration of energy-saving routing techniques that minimize the environmental impact of road traffic. Eco-routing aims to find paths that not only reduce fuel consumption and emissions but also consider the energy efficiency of the vehicle, the availability of charging infrastructure, and the integration of renewable energy sources. Furthermore, as the global push for decarbonization intensifies, eco-routing becomes a key solution to support sustainable urban mobility and smart city initiatives.

In addition, the advent of Vehicle-to-Grid (V2G) technologies provides an opportunity to enhance the environmental benefits of EVs. V2G enables EVs to discharge energy back to the grid, offering financial incentives to owners and contributing to grid stability. Thus, eco-routing can be enhanced by considering V2G incentives, where EVs are directed through charging stations that maximize these incentives, all while adhering to energy consumption

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constraints and ensuring minimal environmental impact. The combination of eco-routing with V2G incentives is crucial for fostering a more sustainable, integrated transportation and energy system.

While eco-routing algorithms have been extensively researched, there are several challenges when attempting to develop models that simultaneously consider energy efficiency, V2G incentives, and GreenZone compliance.

- **Balancing Energy Efficiency and V2G Incentives:** The main challenge lies in balancing the trade-off between minimizing energy consumption during travel and optimizing V2G incentives. On one hand, the goal is to minimize the total energy consumption and reduce greenhouse gas emissions. On the other hand, EVs can contribute to the grid by discharging energy, but this process can deplete their battery charge, affecting travel efficiency and potentially causing range anxiety for drivers. Developing an eco-routing model that effectively balances these competing objectives is a complex task.
- **GreenZone Compliance:** GreenZones represent urban areas where driving is encouraged to be more environmentally friendly, often through low-emission zones or areas with abundant renewable energy sources. However, these zones are dynamic and may change depending on factors such as the availability of renewable energy or traffic conditions. Thus, an eco-routing model must adapt to these changing GreenZone boundaries, ensuring that routes through these areas are prioritized while still maintaining energy efficiency and minimizing penalties for non-compliance.
- **Practical Constraints:** In real-world applications, practical constraints such as battery state-of-charge (SOC) limits, the availability of charging stations, and the energy consumption profiles of EVs further complicate the optimization process. These constraints must be carefully modeled and incorporated into the eco-routing algorithms to ensure that the routes selected are feasible and do not lead to excessive energy consumption or depletion of battery reserves.

Despite the growing body of research on eco-routing and energy optimization for EVs, there is a noticeable gap in literature regarding the integration of V2G incentives and GreenZone prioritization into eco-routing models. While previous studies have focused on energy-efficient routing, the

potential benefits of incorporating V2G technologies, where EVs contribute energy back to the grid in exchange for incentives, remain underexplored in the context of eco-routing. Additionally, GreenZone prioritization, which encourages routing through environmentally friendly areas with renewable energy availability, is rarely considered alongside V2G incentives in existing models. The lack of a unified framework that incorporates these elements presents a significant gap in the research that needs to be addressed.

Furthermore, existing approaches often struggle with scalability issues, especially when considering large-scale networks or real-time applications. Although Integer Linear Programming (ILP) formulations have been successfully applied to small-scale eco-routing problems, their application to large-scale problems involving V2G incentives and dynamic GreenZone updates remains limited. The computational complexity of ILP models becomes a significant challenge as the size of the problem increases, making heuristic solutions a promising alternative for real-time optimization.

**Contributions:** In this paper, we propose a novel solution that addresses the aforementioned challenges by combining an ILP formulation with a heuristic algorithm to handle large-scale, real-time eco-routing problems. Our main contributions are as follows:

- **A Novel ILP Formulation for GreenZone Prioritization with V2G Incentives:** We introduce a new ILP-based eco-routing model that optimizes the selection of routes based on energy efficiency, V2G incentives, and GreenZone compliance. This formulation accounts for dynamic updates to GreenZones and incorporates real-time V2G rewards, while ensuring that the energy consumption of the EVs is minimized, and the battery state-of-charge remains within feasible limits.
- **Development of a Heuristic Algorithm for Large-Scale, Real-Time Applications:** We design a hybrid heuristic algorithm that combines Dijkstra’s shortest-path algorithm with Ant Colony Optimization (ACO) to address the scalability issues associated with exact ILP solutions. The proposed heuristic solution is capable of finding near-optimal solutions in large-scale, real-time eco-routing scenarios, making it suitable for dynamic and complex transportation networks.

Through these contributions, this work not only fills a critical gap in

eco-routing research but also offers practical solutions for integrating V2G incentives and GreenZone prioritization in future transportation systems.

## 2. Problem Description and System Model

This research addresses the problem of Electric Vehicle (EV) eco-routing, incorporating the following features:

- **GreenZones:** Regions with higher availability of renewable energy resources and lower emissions are designated as GreenZones. Vehicles are incentivized to route through these zones to maximize environmental benefits.
- **Vehicle-to-Grid (V2G) Operations:** EVs can discharge surplus energy to the grid at designated V2G-enabled charging stations. These interactions help stabilize the grid, particularly during peak demand hours, and are rewarded with monetary incentives proportional to the energy contributed.
- **Grid Interaction:** The system dynamically interacts with grid operators to receive real-time data on renewable energy availability, energy prices, and V2G demand, influencing the routing decisions.

The goal is to develop a routing mechanism that minimizes the energy consumption of EVs while maximizing rewards from V2G participation and ensuring compliance with GreenZone prioritization policies.

### 2.1. Assumptions

The proposed system operates under the following assumptions:

- **Dynamic GreenZones:** GreenZones are pre-identified based on regions with abundant renewable energy generation and favorable environmental conditions. These zones are dynamically updated based on real-time data.
- **Proportional Incentives:** The monetary incentives for V2G operations are directly proportional to the amount of energy discharged by the EVs to the grid.

- **Charging Station Constraints:** Not all charging stations support V2G operations. Only designated V2G-enabled stations can facilitate energy discharge to the grid.
- **Vehicle Constraints:** The energy discharged by the EV must not deplete its state of charge (SOC) below a predefined threshold required for safe operation.

### 3. Integer Linear Programming (ILP) Formulation

#### 3.1. Decision Variables

- Binary variable  $x_{ij}$  indicates whether the route segment between node  $i$  and node  $j$  is included in the chosen path:

$$x_{ij} = \begin{cases} 1, & \text{if segment } (i, j) \text{ is selected,} \\ 0, & \text{otherwise.} \end{cases}$$

- Binary variable  $y_k$  indicates whether the vehicle stops at the V2G-enabled charging station  $k$ :

$$y_k = \begin{cases} 1, & \text{if the vehicle stops at station } k, \\ 0, & \text{otherwise.} \end{cases}$$

- Continuous variable  $E_{\text{out}}^k$  represents the amount of energy discharged to the grid at station  $k$  (in kWh).
- Continuous variable  $E_{\text{in}}^k$  represents the amount of energy recharged from the grid at station  $k$  (in kWh).

#### 3.2. Objective Function

The objective is to maximize the overall benefit, comprising three components:

1. **Energy Efficiency:** Minimize the total energy consumption  $E_{ij}$  over the selected route segments.
2. **V2G Rewards:** Maximize the monetary rewards earned from discharging energy to the grid.

3. **GreenZone Compliance:** Maximize the benefits associated with routing through GreenZones.

The objective function is formulated as:

$$\text{Maximize: } B = \alpha \sum_k r_k \cdot E_{\text{out}}^k - \beta \sum_{i,j} x_{ij} \cdot E_{ij} + \gamma \cdot \text{GreenZone Benefit},$$

where  $\alpha, \beta, \gamma$  are weights balancing the objectives.

### 3.3. Constraints

1. **Path Continuity:** Ensure the selected segments form a valid path from source  $s$  to destination  $d$ :

$$\sum_j x_{sj} = 1, \quad \sum_i x_{id} = 1, \quad \sum_j x_{ij} - \sum_j x_{ji} = 0 \quad \forall i \notin \{s, d\}.$$

2. **Energy Balance:** Maintain energy balance at each charging station  $k$ :

$$E_{\text{SOC}}^k + E_{\text{out}}^k - E_{\text{in}}^k = \text{SOC}_k.$$

3. **Battery Constraints:** Ensure the SOC remains within allowable limits:

$$E_{\min} \leq \text{SOC}_k \leq E_{\max}.$$

4. **V2G Station Constraints:** Limit energy discharge to stations that support V2G:

$$E_{\text{out}}^k \leq y_k \cdot C_k.$$

5. **GreenZone Penalty:** Impose penalties for non-compliance with GreenZone routing:

$$\sum_{i,j} x_{ij} \cdot p_{ij} \leq \Theta.$$

### 3.4. Model Complexity

The ILP formulation involves both binary and continuous decision variables, making it a mixed-integer linear programming (MILP) problem. The complexity grows with the size of the network, as the number of route segments and charging stations increases. Solving this model for large-scale networks requires significant computational resources due to:

- The exponential growth in the number of feasible routes.
- The integration of non-linear energy and reward terms, which are linearized for tractability.
- Real-time updates to GreenZones and V2G incentives, necessitating frequent re-optimization.

While the ILP provides an exact solution, heuristic methods are proposed to address scalability challenges for real-time applications.

#### 4. Heuristic Solution Approach

Scalability issues arise when using exact Integer Linear Programming (ILP) solutions for large-scale networks. The computational complexity of solving the ILP increases exponentially with the size of the problem, particularly when the number of nodes and edges in the network is large. This makes ILP-based solutions impractical for real-time routing and V2G decision-making in dynamic and large transportation networks. Therefore, a heuristic approach is proposed to provide approximate solutions with lower computational cost while maintaining solution quality.

The proposed heuristic solution is a hybrid approach that combines multiple smaller algorithms, each contributing to solving a specific aspect of the problem. The algorithms work together iteratively to improve the solution. The main algorithms involved are:

- **Shortest-Path Algorithm (Dijkstra’s Algorithm):** Computes an initial feasible path considering GreenZone and energy constraints.
- **Ant Colony Optimization (ACO):** Refines the initial path by exploring multiple path alternatives to optimize V2G rewards, energy consumption, and GreenZone compliance.
- **Penalty Minimization Algorithm:** Minimizes penalties for bypassing GreenZones by adjusting the path.
- **V2G Reward Maximization Algorithm:** Optimizes the energy discharged to V2G-enabled stations to maximize rewards while maintaining a safe state of charge.

Symbol	Definition
$x_{ij}$	Binary decision variable indicating whether the route segment between node $i$ and node $j$ is selected.
$y_k$	Binary decision variable indicating whether the vehicle stops at the V2G-enabled charging station $k$ .
$E_{\text{out}}^k$	Continuous variable representing the amount of energy discharged to the grid at station $k$ (in kWh).
$E_{\text{in}}^k$	Continuous variable representing the amount of energy recharged from the grid at station $k$ (in kWh).
$E_{ij}$	Energy consumption on route segment $(i, j)$ (in kWh).
$\text{SOC}_k$	State of charge of the vehicle's battery after leaving station $k$ (in kWh).
$E_{\text{SOC}}^k$	State of charge of the vehicle's battery upon arrival at station $k$ (in kWh).
$E_{\text{min}}$	Minimum allowable state of charge of the vehicle's battery (in kWh).
$E_{\text{max}}$	Maximum capacity of the vehicle's battery (in kWh).
$C_k$	Maximum energy discharge capacity of the V2G-enabled charging station $k$ (in kWh).
$r_k$	Monetary incentive for discharging energy to the grid at station $k$ (per kWh).
$p_{ij}$	Penalty incurred for not traversing through a GreenZone segment $(i, j)$ .
$\Theta$	Maximum allowable penalty for non-compliance with GreenZone prioritization.
$\alpha$	Weight for the V2G reward component in the objective function.
$\beta$	Weight for the energy consumption component in the objective function.
$\gamma$	Weight for the GreenZone compliance benefit in the objective function.
$s$	Source node in the routing network.
$d$	Destination node in the routing network.

Table 1: Symbol Table for the ILP Formulation



#### 4.1. Shortest-Path Algorithm (Dijkstra's Algorithm)

The first step in the heuristic approach is to compute an initial feasible path from the source node  $s$  to the destination node  $d$ . Dijkstra's algorithm is employed for this task, modified to consider the GreenZone regions and energy consumption constraints. The algorithm iteratively explores the graph by evaluating the shortest path while taking into account the energy consumption along each edge and the GreenZone compliance. This results in an initial path that satisfies basic routing requirements, ensuring the EV moves through GreenZones wherever possible and consumes energy efficiently. The key role of this algorithm is to provide a feasible starting solution for further optimization.

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#### **Algorithm 1:** Shortest-Path Algorithm (Dijkstra's Algorithm)

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**Input:** Source node  $s$ , Destination node  $d$ , Graph  $G(V, E)$ , Energy consumption  $E_{ij}$ , GreenZones  $G$   
**Output:** Initial path  $P_0$

- 1 Initialize distances from  $s$  to all nodes as infinity;
- 2 Set distance from  $s$  to  $s$  as 0;
- 3 Add source node  $s$  to the priority queue;
- 4 **while** *priority queue is not empty* **do**
- 5     Pop the node  $u$  with the smallest distance;
- 6     **for** *each neighbor  $v$  of  $u$*  **do**
- 7         Calculate energy consumption  $E_{uv}$  for edge  $(u, v)$ ;
- 8         If the node  $v$  is not in the GreenZone and does not meet the energy constraints, skip it;
- 9         Update the distance for  $v$  if a shorter path is found;
- 10        Add node  $v$  to the priority queue;
- 11 Return the path  $P_0$ ;

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#### 4.2. Ant Colony Optimization (ACO)

Once an initial path is computed using Dijkstra's algorithm, the next step is to refine the path through the Ant Colony Optimization (ACO) algorithm. ACO is a probabilistic optimization technique inspired by the foraging behavior of ants. It simulates a colony of ants searching for the optimal route by exploring multiple alternatives. Each ant starts from the source and explores the network, depositing pheromones on the edges that lead to better paths. Over multiple iterations, paths with higher rewards (including V2G

incentives) and better energy efficiency are reinforced. The goal is to find an optimal path that balances energy consumption, V2G rewards, and compliance with GreenZones. ACO explores the solution space by adjusting the pheromone levels, thus leading to a better solution over time.

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**Algorithm 2:** Ant Colony Optimization (ACO) Refinement

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**Input:** Initial path  $P_0$ , Graph  $G(V, E)$ , Number of ants  $k$ ,  
Pheromone levels  $\tau$ , V2G reward function  $r_k$

**Output:** Refined path  $P_r$

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1 for each ant do
2   Set current node to source node  $s$ ;
3   while ant has not reached destination  $d$  do
4     Evaluate all possible paths and select the next node based on
       pheromone levels and energy consumption;
5     Deposit pheromone on the selected path segment;
6   Update pheromone levels based on path quality and V2G reward
       maximization;
7 Return the best path  $P_r$ ;

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#### 4.3. Penalty Minimization Algorithm

The Penalty Minimization Algorithm is applied after ACO refinement to reduce penalties associated with bypassing GreenZones. In this phase, the path is analyzed for any GreenZone non-compliance, and penalties for such deviations are computed. The algorithm iterates through the path and checks each route segment for potential penalties. If a segment bypasses a GreenZone, the penalty is computed based on the severity of the non-compliance. The path is adjusted by replacing non-compliant segments with alternatives that reduce penalties while maintaining energy efficiency and maximizing V2G rewards. This step ensures that the final path complies as much as possible with GreenZone regulations.

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**Algorithm 3:** Penalty Minimization

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**Input:** Refined path  $P_r$ , GreenZone regions  $G$ , Penalty function  $p_{ij}$

**Output:** Path with minimized penalties

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1 for each path segment  $(i, j)$  in  $P_r$  do
2   Calculate penalty  $p_{ij}$  for bypassing a GreenZone;
3   If  $p_{ij} > 0$  then find alternative path segments that minimize  $p_{ij}$ ;
4 Return the adjusted path with minimized penalties;

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#### 4.4. V2G Reward Maximization Algorithm

The final algorithm in the heuristic is the V2G Reward Maximization Algorithm. After determining the optimal path that minimizes penalties and energy consumption, this algorithm focuses on maximizing the rewards from Vehicle-to-Grid (V2G) interactions. At each V2G-enabled charging station along the path, the amount of energy to be discharged to the grid is calculated based on the V2G reward function  $r_k$ . The algorithm ensures that the EV's battery state of charge (SOC) remains within acceptable limits while maximizing the discharge to the grid. This phase enhances the overall rewards of the solution by optimizing the V2G discharge decisions.

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##### Algorithm 4: V2G Reward Maximization

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**Input:** Path  $P_r$ , V2G-enabled stations  $C_k$ , Reward function  $r_k$ ,  
Energy discharge limits  $E_{\min}, E_{\max}$

**Output:** Energy discharged to grid  $E_{\text{out}}^k$

- 1 **for** each V2G-enabled station  $k$  along the path **do**
  - 2     Calculate the maximum amount of energy  $E_{\text{out}}^k$  to be discharged  
      to maximize  $r_k$ ;
  - 3     Ensure that SOC remains within the bounds  
       $E_{\min} \leq \text{SOC}_k \leq E_{\max}$ ;
  - 4 **Return** the total energy discharged to the grid  $E_{\text{out}}^k$ ;
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#### 4.5. Complexity Analysis

The computational complexity of the heuristic approach depends on the individual algorithms involved:

- **Shortest-Path (Dijkstra):** The time complexity of Dijkstra's algorithm is  $O(|E| + |V| \log |V|)$ , where  $|E|$  is the number of edges and  $|V|$  is the number of vertices in the network. This provides an initial feasible solution.
- **Ant Colony Optimization (ACO):** The ACO algorithm involves multiple iterations over a search space and updates pheromone trails. Its time complexity is  $O(k \cdot m \cdot t)$ , where  $k$  is the number of ants,  $m$  is the number of paths to evaluate, and  $t$  is the number of iterations. This phase refines the path by exploring alternatives.

- **Penalty Minimization and V2G Reward Maximization:** These phases require checking each route segment and each V2G-enabled station, leading to a time complexity of  $O(|V|)$  and  $O(|C_k|)$ , where  $|V|$  is the number of nodes and  $|C_k|$  is the number of V2G stations along the path.

Thus, the overall complexity is dominated by the ACO phase, with an approximate time complexity of  $O(k \cdot m \cdot t)$ .

## 5. Analysis of the proposed solution

This section presents the key theorems and corollaries relevant to the problem of eco-routing with GreenZones and V2G incentives. These theorems and corollaries form the mathematical foundation for the optimization and heuristic approaches discussed earlier.

### 5.1. Theorem 1: Optimal Path Selection for Energy Minimization and GreenZone Compliance

**Statement:** Given a network of roads with EV fleet routing from source  $s$  to destination  $d$ , where GreenZone regions are dynamic, there exists an optimal path  $P^*$  that minimizes energy consumption while maintaining GreenZone compliance, subject to battery state-of-charge (SOC) constraints and GreenZone region penalties.

**Proof:**

1. **Problem Formulation:** Let the set of paths from  $s$  to  $d$  be  $P(s, d)$ , and each path  $P \in P(s, d)$  has associated energy consumption  $C_{\text{energy}}(P)$ , GreenZone compliance  $C_{\text{GreenZone}}(P)$ , and penalties  $P_{\text{penalty}}(P)$ .

2. **Cost Function:** The total cost  $C(P)$  of each path is:

$$C(P) = w_1 \cdot C_{\text{energy}}(P) + w_2 \cdot C_{\text{GreenZone}}(P) + w_3 \cdot P_{\text{penalty}}(P)$$

where  $w_1, w_2, w_3$  are the weights assigned to energy consumption, GreenZone compliance, and penalties, respectively.

3. **GreenZone Compliance Cost and Penalties:** The GreenZone compliance cost  $C_{\text{GreenZone}}(P)$  is a function of the path's traversal through GreenZone regions. If path  $P$  crosses  $m$  GreenZones, then:

$$C_{\text{GreenZone}}(P) = \sum_{i=1}^m C_{\text{GreenZone}}(i)$$

where  $C_{\text{GreenZone}}(i)$  is the compliance cost associated with crossing the  $i$ -th GreenZone, and  $C_{\text{GreenZone}}(i) = \alpha_i \cdot d_i$  where  $d_i$  is the distance through the  $i$ -th GreenZone, and  $\alpha_i$  is a constant that penalizes the deviation from ideal GreenZone compliance.

4. **Minimization of Energy Consumption:** Let  $C_{\text{energy}}(P)$  be the total energy consumption of path  $P$ , which depends on the path length, the road type, and the vehicle's energy efficiency. We define:

$$C_{\text{energy}}(P) = \sum_{e \in P} (\text{energy consumption of edge } e)$$

The energy consumption is a continuous function of the path, and can be expressed as:

$$C_{\text{energy}}(P) = \sum_{e \in P} \gamma_e \cdot L_e$$

where  $\gamma_e$  is the energy consumption per unit distance of edge  $e$ , and  $L_e$  is the length of edge  $e$ .

5. **SOC Constraints:** Let the battery state-of-charge (SOC) at any point along path  $P$  be  $\text{SOC}(P)$ . For each path  $P$ , the SOC must remain within limits:

$$E_{\min} \leq \text{SOC}(P) \leq E_{\max}$$

where  $E_{\min}$  and  $E_{\max}$  are the minimum and maximum SOC limits, respectively.

6. **Convexity and Existence of Optimal Path:** The objective function  $C(P)$  is convex in nature since both  $C_{\text{energy}}(P)$  and  $C_{\text{GreenZone}}(P)$  are convex, and penalties increase with the distance or deviation from GreenZone compliance. The SOC constraints are linear, which preserves the convexity of the problem. By the Weierstrass Extreme Value Theorem, since the objective function is convex and continuous, and the feasible set is non-empty and closed, there exists an optimal path  $P^*$  that minimizes energy consumption and maximizes GreenZone compliance.

Thus, the optimal path  $P^*$  exists and satisfies the energy and GreenZone compliance constraints.  $\square$

### 5.2. Corollary 1: Effect of Dynamic GreenZone Updates on Path Feasibility

**Statement:** If GreenZones are dynamically updated based on renewable energy availability, the feasibility of the optimal path  $P^*$  is preserved. However, the optimal path may change as GreenZone regions expand or contract.

**Proof:**

1. **Dynamic Update of GreenZones:** Let the GreenZone regions at time  $t$  be denoted by  $G_t$ , and let these regions evolve over time based on renewable energy availability. The GreenZone boundaries and compliance costs may change with each update.
2. **Feasibility of New Path After Update:** For each update, we must check the feasibility of the previously optimal path  $P^*$ . The feasible set for the optimization problem changes with the updated GreenZone regions, and the cost of each path  $C(P)$  is recalculated based on the new GreenZone compliance costs.
3. **Preservation of Feasibility:** Since the path selection problem is convex and the feasibility region remains non-empty after the update, the optimal path  $P^*$  may change but always exists. The updated GreenZone regions do not invalidate the feasibility of the problem.

Thus, after each update, the problem remains feasible, and a new optimal path can be computed.  $\square$

### 5.3. Theorem 2: V2G Reward Maximization Given Discharge and SOC Constraints

**Statement:** For an EV fleet discharging energy at V2G stations, the optimal amount of energy  $E_{\text{out}}^k$  to be discharged at each V2G station  $k$  can be found by solving a constrained optimization problem that maximizes the total reward, subject to SOC limits and discharge capacity limits at each V2G station.

**Proof:**

1. **Reward Function and Objective:** Let  $r_k(E_{\text{out}}^k)$  represent the reward function for discharging energy  $E_{\text{out}}^k$  at V2G station  $k$ . The total reward  $R_{\text{total}}$  is the sum of the individual rewards:

$$R_{\text{total}} = \sum_{k \in C} r_k(E_{\text{out}}^k)$$

where  $C$  is the set of all V2G-enabled stations along the path.

2. **Constraints on Energy Discharge:** The discharge of energy at each station  $k$  is bounded by:

$$0 \leq E_{\text{out}}^k \leq E_{\text{max}}^k$$

where  $E_{\text{max}}^k$  is the maximum allowable discharge at station  $k$ .

3. **SOC Constraints:** Let the state-of-charge (SOC) at any point along the path be  $\text{SOC}_k$ , where:

$$E_{\text{min}} \leq \text{SOC}_k \leq E_{\text{max}}$$

The discharge must ensure that the SOC remains within these bounds. The battery's SOC decreases as energy is discharged, and the discharge at each station must account for the remaining SOC.

4. **Convexity of the Reward Function:** Assume the reward function  $r_k(E_{\text{out}}^k)$  is concave, which is typically the case for V2G rewards. The total reward  $R_{\text{total}}$  is the sum of concave functions and is therefore concave.
5. **Solution Using KKT Conditions:** The optimization problem is convex, and the constraints are linear. By the Karush-Kuhn-Tucker (KKT) conditions, we can find the optimal discharge amount  $E_{\text{out}}^k$  at each V2G station that maximizes the total reward while respecting the energy discharge and SOC constraints.

Thus, the optimal discharge at each V2G station exists and can be obtained by solving this optimization problem.  $\square$

## 6. Experimental Setup

The aim of this experiment is to compare the performance of our proposed eco-routing solution, which incorporates Vehicle-to-Grid (V2G) incentives and GreenZone prioritization, with two state-of-the-art (SOTA) eco-routing strategies:

- The model-based eco-routing strategy developed by De Nunzio et al. [1].
- The combined eco-routing and power-train control strategy by Houshmand et al. [2].

In this section, we describe the datasets, parameters, evaluation metrics, and the implementation details of the experiment.

### *6.1. Simulated Data Generation*

Given that real-world data is often hard to obtain for large-scale simulations, we generated simulated data for our experiments. The data was designed to represent realistic urban environments and EV usage patterns.

#### *6.1.1. Road Network and Traffic Data*

We created a synthetic road network based on a simplified urban grid with the following parameters:

- Number of nodes: 50 (representing intersections or road junctions).
- Number of edges: 100 (representing roads between intersections).
- Road length: Randomly assigned between 0.5 km and 2.5 km.
- Traffic density: Simulated using a normal distribution, with a mean of 0.6 vehicles per kilometer (v/km) and a standard deviation of 0.1 v/km.
- Travel speed: Varies based on road type, with an average speed between 30 km/h and 50 km/h.

#### *6.1.2. Electric Vehicle Data*

We simulated a fleet of 100 EVs with the following characteristics:

- Average battery capacity: 40 kWh.
- Energy consumption: Randomly assigned between 0.15 kWh/km and 0.25 kWh/km, based on vehicle type and road conditions.
- Initial battery state-of-charge (SoC): Randomly set between 20% and 80%.
- Charging stations: Simulated by assigning 10 charging stations to the network, with varying capacities between 5 and 15 vehicles.



### 6.1.3. Renewable Energy and Grid Data

For the purpose of this experiment, renewable energy availability (solar and wind) was modeled using a sinusoidal pattern to simulate daily fluctuations:

- Solar energy availability: Varies between 0 kW and 5 kW, peaking at noon.
- Wind energy availability: Varies between 0 kW and 3 kW, peaking in the evening.
- Energy price for V2G discharge: \$0.10 per kWh when the grid requires energy.

The grid model for V2G incentives is based on real-time demand, with prices varying based on the grid's load (higher prices during peak demand).

### 6.2. Evaluation Metrics

We compare the following key performance indicators (KPIs) for all three methods:

- **Energy Consumption (EC)**: Total energy consumed by the EV during the trip (in kWh).
- **V2G Incentives (V2G)**: The monetary reward received for discharging energy back into the grid (in USD).
- **Travel Time (TT)**: Total time taken to complete the trip (in minutes).
- **GreenZone Compliance (GZC)**: The percentage of the route that passes through GreenZones (areas with high renewable energy).
- **Emissions Reduction (ER)**: The reduction in carbon emissions based on the route and energy consumption model (in kg CO<sub>2</sub>).
- **Route Cost (RC)**: The total cost (in USD) of the trip, factoring in energy consumption, V2G rewards, and GreenZone penalties.

### 6.3. Implementation Details

The implementation was carried out using Python with the following libraries:

- **networkx**: For graph-based route computations and network simulation.
- **numpy, pandas**: For data handling and numerical computations.
- **matplotlib, seaborn**: For plotting and visualizing results.
- **scipy**: For optimization and solving ILP formulations.
- **Google Maps API**: For real-time traffic data (optional for real-world deployment).

We used the Dijkstra algorithm for the shortest path computation in the SOTA methods and a hybrid approach combining Dijkstra with Ant Colony Optimization (ACO) for the proposed solution. For V2G incentives and GreenZone prioritization, we utilized a dynamic reward calculation system based on renewable energy availability and charging station usage.

### 6.4. Comparison Methods

Three methods were compared in the experiment:

1. **Proposed Method (PM)**: Our method incorporates GreenZone prioritization, V2G incentives, and dynamic energy consumption models.
2. **State-of-the-Art 1 (SOTA1)**: The model by De Nunzio et al. [1], which minimizes energy consumption in urban networks without considering V2G incentives or GreenZone prioritization.
3. **State-of-the-Art 2 (SOTA2)**: The combined eco-routing and power-train control strategy by Houshmand et al. [2], which incorporates power-train control but does not consider V2G incentives or GreenZone prioritization.

All methods were tested on the same road network with 100 simulated EVs. The routes were calculated for each EV in the fleet, and the total energy consumption, travel time, and V2G incentives were recorded.

### 6.5. Results and Discussion

The results of the experiment are shown in Table 2. The comparison highlights the advantages of our proposed method in terms of energy consumption, V2G incentives, GreenZone compliance, and emissions reduction.

Metric	Proposed Method	SOTA 1	SOTA 2	Improvement (PM vs. SOTA1)	Improvement (PM vs. SOTA2)
Energy Consumption (kWh)	4.8	5.5	5.2	12.7	7.7
V2G Incentives (\$)	2.1	0.0	0.0	100	100
Travel Time (minutes)	27.3	30.5	28.7	10.5	4.9
GreenZone Compliance (%)	90	0	0	100	100
Emissions Reduction (kg CO2)	15.8	10.4	12.1	51.9	30.6
Route Cost (USD)	5.4	5.7	5.6	5.3	3.6

Table 2: Comparison of the Proposed and SOTA Methods

From the results, we observe the following:

- **Energy Consumption:** The proposed method achieves a 12.7% reduction in energy consumption compared to the SOTA1 method, and a 7.7% reduction compared to SOTA2.
- **V2G Incentives:** The proposed method fully integrates V2G incentives, leading to a 100% improvement in V2G rewards compared to both SOTA methods.
- **Travel Time:** The proposed method reduces travel time by 10.5% compared to SOTA1 and by 4.9% compared to SOTA2.
- **GreenZone Compliance:** The proposed method adheres to GreenZone priorities 90% of the time, whereas both SOTA methods fail to prioritize these zones.
- **Emissions Reduction:** The proposed method achieves a 51.9% reduction in emissions compared to SOTA1 and 30.6% compared to SOTA2.
- **Route Cost:** The cost of the trip is reduced by 5.3% in the proposed method compared to SOTA1 and 3.6% compared to SOTA2, despite the V2G incentives and GreenZone penalties.

#### Credit author statement

Yanshul Sharma: Conceptualization, Methodology, Validation, Investigation,

Writing – original draft, Writing – review & editing, Supervision and Visualization.

Sanjay Moulik: Conceptualization, Methodology, Validation, Investigation and Visualization.

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