**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

*μ* = 45 minutes, *σ* = 8 minutes, x = 60-10 = 50 minutes

z = (50-45)/(8)

z = 5/8 = 0.625 , corresponding probability = 0.2324

Probability that the service manager cannnot meet his commitment = 1-(0.5+0.2324) =0.2676

Answer - Option B

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44. - False

44 is 1 *σ* away from the mean 38. In normal distribution, 68.26% of total population resides between 1 *σ* above and below mean. So % of employees between 38 and 44 is equal to 34.13% of total population. i.e = 137

% employees older than 44 will be (50-34.15)% of total population = 64. So the statement is false.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees. - True

X = 30, z = (30-38/6) = -1.33

Area under the curve for z = -1.33 = 0.4082, so probabiltiy of finding employees under age 30 will be = 0.5 - 0.4082 = 0.9

Hence expected number of employess = 0.9\*400 = 36.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then –

2X1 ~ *N*( 2μ, 22σ2) and

*X*1 + *X*2 ~ *N*( µ1 + µ2, σ12 + σ 2 2) , however for the given question means and SDs are equal.

Hence, *X*1 + *X*2 ~ *N*( 2µ, 2σ2)

i.e. after the linear transformation, the means will be same but the SDs will be different.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

μ = 100

σ2= 20, σ = 20

a/b = 100 ± σz (at 99% CI)

z = ± 2.57

a = 100 – 2.57 \* 20 = 48.6

b = 100 + 2.57 \* 20 = 151.5.

Answer - Option D

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Combined Profit = Profit 1+Profit 2 = Profit1 ~ N(5, 32)+ Profit2 ~ N(7, 42)

Combined Profit = Profit~ N (5+7 32+42) = Profit ~N(12,52)

μ = 12

σ = 5

Range = μ± 1.96 σ

Range = 12±1.96\*5 = Between 2.2 - 21.8 (in mn USD terms)

Range = 99 mn Rs – 981 mn Rs.

1. Specify the 5th percentile of profit (in Rupees) for the company

2.5 percentile of the profit is = 99mn. Hence 5th percentile of the profit = 99\*8 = 198 mn Rupees.

1. Which of the two divisions has a larger probability of making a loss in a given year?

P(x<0)

Z1 = (0 -5)/3 = -1.66, hence P1(loss) = 0.45

Z2 = (0 -7)/4 = - 1.75, hence P2(loss) = 0.46

Probability of making a loss is higher for the second division.