$$\mathfrak{H} = \mathrm{SL}_2(\mathbb{R})/\mathrm{SO}_2(\mathbb{R})$$

### Proposition

*Any* $\tau \in \mathfrak{H}$ *has a finite cyclic stabilizer subgroup.*

To prove it, we will use $\mathfrak{H} = \mathrm{SL}_2(\mathbb{R})/\mathrm{SO}_2(\mathbb{R})$.

## Proposition (Iwasawa decomposition)

*An element in* $\mathrm{SL}_2(\mathbb{R})$ *can be uniquely written as*

$$\begin{bmatrix} y & 0 \\ 0 & y^{-1} \end{bmatrix} \cdot \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

*with* $x \in \mathbb{R}$, $y \in \mathbb{R}_+$, $\theta \in S^1$.

In other words,

$$\mathrm{SL}_2(\mathbb{R})/\mathrm{SO}_2(\mathbb{R}) \simeq \mathbb{R} \times \mathbb{R}_+$$

or

$$\mathrm{SL}_2(\mathbb{R})/\mathrm{SO}_2(\mathbb{R}) \simeq \mathfrak{H}$$

with $\tau = x + yi$.

## Proposition

*Let* $\Gamma \subset \mathrm{SL}_2(\mathbb{R})$ *be a discrete subgroup. If* $\tau \in \mathfrak{H}$*, then*

$$\Gamma_\tau = \{\gamma \in \Gamma \colon \gamma\tau = \tau\}$$

*is finite cyclic.*

## Proof.

We have an injective homomorphism $\Gamma_\tau \to S^1$. A discrete subgroup of $S^1$ is finite and cyclic. $\square$