



Budapest University of Technology and Economics
Faculty of Electrical Engineering and Informatics
Department of Measurement and Information Systems

Steady-state and Transient Analysis of Stochastic Petri Nets

Scientific Students' Associations Report

Authors:

Attila Klenik
Kristóf Marussy

Supervisors:

András Vörös
Vince Molnár
Miklós Telek

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Kivonat

Jelen dokumentum egy diplomaterv sablon, amely formai keretet ad a BME Villamosmérnöki és Informatikai Karán végző hallgatók által elkészítendő szakdolgozatnak és diplomatervnek. A sablon használata opcionális. Ez a sablon \LaTeX alapú, a *TeXLive* \TeX -implementációval és a PDF- \TeX fordítóval működőképes.

Abstract

This document is a \LaTeX -based skeleton for BSc/MSc theses of students at the Electrical Engineering and Informatics Faculty, Budapest University of Technology and Economics. The usage of this skeleton is optional. It has been tested with the *TeXLive* \TeX implementation, and it requires the PDF- \LaTeX compiler.

Chapter 1

Introduction

Árvíztűrő tükörfúrógép

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Chapter 2

Background

2.1 Petri nets

2.2 Continous-time Markov chains

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Dayar (2012)

2.3 Stochastic Petri nets

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. $\sin^2(\alpha) + \cos^2(\beta) = 1$. If you read this text, you will get no information $E = mc^2$. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text *like this* gives you information about the selected font, how the letters are written and an impression of the look. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$. This text should contain *all letters of the alphabet* and it should be written in of the original language. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$. There is no need for special content, but the length of words should match the language. $a \sqrt[n]{b} = \sqrt[n]{a^n b}$.

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Theorem 2.1. *Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. $\sin^2(\alpha) + \cos^2(\beta) = 1$. If you read this text, you will get no information $E = mc^2$. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$. This text should contain all letters of the alphabet and it should be written in of the original language. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$. There is no need for special content, but the length of words should match the language. $a \sqrt[n]{b} = \sqrt[n]{a^n b}$.*

2.4 Example section

And after the second paragraph follows the third paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. $\sin^2(\alpha) + \cos^2(\beta) = 1$. If you read this text, you will get no information $E = mc^2$. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text *like this* gives you information about the selected font, how the letters are written and an impression of the look. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$. This text should contain *all letters of the alphabet* and it should be written in of the original language. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$. There is no need for special content, but the length of words should match the language. $a \sqrt[n]{b} = \sqrt[n]{a^n b}$.

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2.4.1 Example subsection

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2.4.2 Example subsection

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- First item in a list
- Second item in a list
- Third item in a list
- Fourth item in a list
- Fifth item in a list

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Chapter 3

Stochastic analysis

3.1 Steady-state analysis

3.2 Transient analysis

3.2.1 Transient probability calculation

3.2.2 Accumulated probability calculation

3.3 Rewards and sensitivity

3.3.1 Stochastic reward nets

3.3.2 Sensitivity of rewards

Chapter 4

Efficient generation and storage of continuous-time Markov chains

4.1 State-space exploration

4.1.1 Explicit state-space exploration

4.1.2 Symbolic methods

Multivalued decision diagrams

Edge-labeled decision diagrams

4.2 Storage of generator matrices

4.2.1 Explicit matrix storage

Dense matrices

Sparse matrices

Column major versus row major storage

4.2.2 Kronecker decomposition

4.2.3 Block Kronecker decomposition

4.3 Matrix composition

4.3.1 Generating sparse matrices from symbolic state spaces

4.3.2 Explicit block Kronecker decomposition

4.3.3 Symbolic block Kronecker decomposition

Chapter 5

Algorithms for stochastic analysis

5.1 Steady-state analysis

5.1.1 Explicit solution by LU decomposition

5.1.2 Stationary iterative methods

Power iteration

Jacobi iteration and Jacobi over-relaxation

Gauss-Seidel iteration and successive over-relaxation

5.1.3 Krylov subspace methods

Biconjugate gradient stabilized (BiCGSTAB)

5.2 Transient analysis

5.2.1 Uniformization

Calculation of uniformization weights

- Weights for transient probability with *trimming*
- Weights for accumulated probability

Steady-state detection

5.3 Processing results

5.3.1 Calculation of rewards

Symbolic storage of reward functions

5.3.2 Calculation of sensitivity

Sensitivity of state probabilities

Sensitivity of rewards

Chapter 6

Configurable stochastic analysis

6.1 Matrix storage and algorithm selection in practice

6.2 Implementation of configurable workflows

Chapter 7

Evaluation

7.1 Benchmark models

7.1.1 Synthetic models

Resource sharing

Kanban

Dining philosophers

7.1.2 Case studies

Performability of clouds

7.2 Baselines

7.2.1 PRISM

7.2.2 SMART

7.3 Results

Chapter 8

Conclusion

8.1 Future work

References

Dayar, Tugrul (2012). *Analyzing Markov chains using Kronecker products: theory and applications*. Springer.