MŰEGYETEM 1782

Budapest University of Technology and Economics
Faculty of Electrical Engineering and Informatics
Department of Measurement and Information Systems

Configurable Stochastic Analysis Framework for Asynchronous Systems

Scientific Students' Associations Report

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Contents

Összefoglaló A kritikus rendszerek – biztonságkritikus, elosztott és felhőalkalmazások – helyességének biztosításához szükséges a funkcionális és nemfunkcionális követelmények matematikai igényességű ellenőrzése. Számos, szolgáltatásbiztonsággal és teljesítményvizsgálattal kapcsolatos tipikus kérdés általában sztochasztikus analízis segítségével válaszolható meg.

A kritikus rendszerek elosztott és aszinkron tulajdonságai az *állapottér robbanás* jelenségéhez vezetnek. Emiatt méretük és komplexitásuk gyakran megakadályozza a sikeres sztochasztikus analízist, melynek számításigénye nagyban függ a lehetséges viselkedések számától. A modellek komponenseinek jellegzetes időbeli viselkedése a számításigény további jelentős növekedését okozhatja.

A szolgáltatásbiztonsági és teljesítményjellemzők kiszámítása markovi modellek állandósult állapotbeli és tranziens megoldását igényli. Számos eljárás ismert ezen problémák kezelésére, melyek eltérő reprezentációkat és numerikus algoritmusokat alkalmaznak; ám a modellek változatos tulajdonságai miatt nem választható ki olyan eljárás, mely minden esetben hatékony lenne.

A markovi analízishez szükséges a modell lehetséges viselkedéseinek, azaz állapotterének felderítése, illetve tárolása, mely szimbolikus módszerekkel hatékonyan végezhető el. Ezzen szemben a sztochasztikus algoritmusokban használt vektor- és indexműveletek szimbolikus megvalósítása nehézkes. Munkánk célja egy olyan, integrált keretrendszer fejlesztése, mely lehetővé teszi a komplex sztochasztikus rendszerek kezelését a szimbolikus módszerek, hatékony mátrix reprezentációk és numerikus algoritmusok előnyeinek ötvözésével.

Egy teljesen szimbolikus algoritmust javasolunk a sztochasztikus viselkedéseket leíró mátrix-dekompozíciók előállítására a szimbolikus formában adott állapottérből kiindulva. Ez az eljárás lehetővé teszi a temporális logikai kifejezéseken alapuló szimbolikus technikák használatát.

A keretrendszerben megvalósítottuk a konfigurálható sztochasztikus analízist: megközelítésünk lehetővé teszi a különböző mátrix reprezentációk és numerikus algoritmusok kombinált használatát. Az implementált algoritmusokkal állandósult állapotbeli költség- és érzékenység analízis, tranziens költséganalízis és első hiba várható bekövetkezési idő analízis végezhető el sztochasztikus Petri-háló (SPN) markovi költségmodelleken. Az elkészített eszközt integráltuk a PetriDotNet modellező szoftverrel. Módszerünk gyakorlati alkalmazhatóságát szintetikus és ipari modelleken végzett mérésekkel igazoljuk.

Abstract Ensuring the correctness of critical systems—such as safety-critical, distributed and cloud applications—requires the rigorous analysis of the functional and extra-functional properties of the system. A large class of typical quantitative questions regarding dependability and performability are usually addressed by stochastic analysis.

Recent critical systems are often distributed/asynchronous, leading to the well-known phenomenon of *state space explosion*. The size and complexity of such systems often prevents the success of the analysis due to the high sensitivity to the number of possible behaviors. In addition, temporal characteristics of the components can easily lead to huge computational overhead.

Calculation of dependability and performability measures can be reduced to steady-state and transient solutions of Markovian models. Various approaches are known in the literature for these problems differing in the representation of the stochastic behavior of the models or in the applied numerical algorithms. The efficiency of these approaches are influenced by various characteristics of the models, therefore no single best approach is known.

The prerequisite of Markovian analysis is the exploration of the state space, i.e. the possible behaviors of the system. Symbolic approaches provide an efficient state space exploration and storage technique, however their application to support the vector operations and index manipulations extensively used by stochastic algorithms is cumbersome. The goal of our work is to introduce a framework that facilitates the analysis of complex, stochastic systems by combining together the advantages of symbolic algorithms, compact matrix representations and various numerical algorithms.

We propose a fully symbolic method to explore and describe the stochastic behaviors. A new algorithm is introduced to transform the symbolic state space representation into a decomposed linear algebraic representation. This approach allows leveraging existing symbolic techniques, such as the specification of properties with *Computational Tree Logic* (CTL) expressions.

The framework provides configurable stochastic analysis: an approach is introduced to combine the different matrix representations withq numerical solution algorithms. Various algorithms are implemented for steady-state reward and sensitivity analysis, transient reward analysis and mean-time-to-first-failure analysis of stochastic models in the *Stochastic Petri Net* (SPN) Markov reward model formalism. The analysis tool is integrated into the PetriDotNet modeling application. Benchmarks and industrial case studies are used to evaluate the applicability of our approach.

Chapter 1

Introduction

Árvíztűrő tükörfúrógép

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Chapter 2

Background

2.1 Petri nets

2.2 Continous-time Markov chains

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Dayar (2012)

2.3 Stochastic Petri nets

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Theorem 2.1. *Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.* $\sin^2(\alpha) + \cos^2(\beta) = 1$. *If you read this text, you will get no information* $E = mc^2$. *Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text* like this *gives you information about the selected font, how the letters are written and an impression of the look.* $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$. *This text should contain* all letters of the alphabet *and it should be written in of the original language.* $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$. *There is no need for special content, but the length of words should match the language.* $a\sqrt[n]{b} = \sqrt[n]{a^n b}$.

2.4 Example section

And after the second paragraph follows the third paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. $\sin^2(\alpha) + \cos^2(\beta) = 1$. If you read this text, you will get no information $E = mc^2$. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text *like this* gives you information about the selected font, how the letters are written and an impression of the look. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$. This text should contain *all letters of the alphabet* and it should be written in of the original language. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$. There is no need for special content, but the length of words should match the language. $a\sqrt[n]{b} = \sqrt[n]{a^n b}$.

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2.4.1 Example subsection

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2.4.2 Example subsection

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- First item in a list
- Second item in a list
- Third item in a list
- Fourth item in a list
- Fifth item in a list

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Chapter 3

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References

Dayar, Tugrul (2012). *Analyzing Markov chains using Kronecker products: theory and applications*. Springer.