Budapest University of Technology and Economics
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Configurable Stochastic Analysis Framework for Asynchronous Systems

Scientific Students' Associations Report

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Contents

Összefoglaló   A technológia fejlődésével a számítógépes rendszerek alkalmazási köre ma már olyan biztonságkritikus rendszerekre is kiterjed, amelyek helyes működésétől sokszor teljes vállalatok sorsa, vagy akár emberéletek is függhetnek. Az ilyen rendszerek méretének és bonyolultságának növekedtével szükségessé vált a megbízható automatikus módszerek kifejlesztése a rendszer (biztonság szempontjából) kritikus tulajdonságai teljesülésének ellenőrzésére, illetve a megbízhatósági és teljesítményjellemzők kiszámítására. Míg a formális módszerekkel matematikai igényességgel igazolható a tervezési folyamat helyessége, a sztochastikus technikák lehetővé teszik a modell kibővítését dinamikus, kvantiatív jellemzőkkel.

A modellek métetének és bonyolultságának növekedése nagyságrendekkel emelheti meg a modell lehetséges futásidejű konfigurációinak számát. Ez a jelenség az *állapottér-robbanás*, mely mind a modellelenőrzés alapú formális verifikáció, mind a sztochasztikus analízis során nehézséget jelent. A modellellenőrzésnél, mely a formalizált követelmények verifikálásához felderíti a modell állapothalmazát, a *szaturációs* használó szimbolikus technikák segítik a rendkívül nagy méretű állapotterek hatékony kezelését. A sztochastikus analízis erre a célra általában lineáris algebrai mátrix-dekompozíciókat alkalmaz.

Abstract   Ensuring correctness of critical systems, like safety-critical, distributed and cloud applications necessitates the analysis of functional and also qualitative aspects. Typical qualitative questions regarding dependability and performability are answered by stochastic analysis. However, the complexity of such systems raised the need for rigorous analysis methods. Stochastic analysis methods are highly sensitive to the number of possible behaviours of the systems. Recent critical systems are used to be distributed/asynchronous leading to the well-known phenomenon of state space explosion often preventing the success of stochastic analysis. In addition, temporal characteristics of the components can easily lead to huge computational overhead.

Calculation of dependability and performability measures can be reduced to steady-state and transient solutions of Markovian models. Various approaches are known in the literature for these problems differing in the representation of the stochastic behaviour of the model and also differing in the applied numerical algorithms. Various characteristics of the models influence the overheads associated with these approaches, therefore no single best approach is known.

The prerequisite of Markovian analysis is the exploration of the set of reachable states i.e. the behaviours of the system. Symbolic approaches provide an efficient state space exploration and storage technique, however their application to support the vector operations and index manipulations extensively used by stochastic algorithms is cumbersome. The goal of our work is to introduce a framework that facilitates the analysis of complex, stochastic systems.

We propose a fully symbolic method to describe the stochastic behavior of the system. A new algorithm is introduced to transform the symbolic state space representation into a decomposed linear algebraic representation. This approach allows leveraging existing symbolic techniques for complex systems, such as the specification of properties with *Computational Tree Logic* (CTL) expressions.

The decomposition method is implemented for steady-state reward and sensitivity analysis, transient reward analysis and mean-time-to-first-failure analysis of stochastic models in the *stochastic Petri net* (SPN) Markov reward model formalism. The analysis tool is integrated into the Petridotnet 1.3[1] modeling and analysis applications along with a flexible linear algebra optimized for decompositions and calculations in used stochastic analysis which allows the user to fine-tune the analysis for the model under study.

The performance of analysis algorithms and decompositions is studied for multiple benchmark models and case studies and compared to other available analysis tools.

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[1]`https://inf.mit.bme.hu/research/tools/petridotnet`

Chapter 1

Introduction

Árvíztűrő tükörfúrógép

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Chapter 2

Background

2.1 Petri nets

2.2 Continous-time Markov chains

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Dayar (2012)

2.3 Stochastic Petri nets

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. $\sin^2(\alpha) + \cos^2(\beta) = 1$. If you read this text, you will get no information $E = mc^2$. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text *like this* gives you information about the selected font, how the letters are written and an impression of the look. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$. This text should contain *all letters of the alphabet* and it should be written in of the original language. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$. There is no need for special content, but the length of words should match the language. $a\sqrt[n]{b} = \sqrt[n]{a^n b}$.

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Theorem 2.1. *Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.* $\sin^2(\alpha) + \cos^2(\beta) = 1$. *If you read this text, you will get no information* $E = mc^2$. *Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text* like this *gives you information about the selected font, how the letters are written and an impression of the look.* $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$. *This text should contain* all letters of the alphabet *and it should be written in of the original language.* $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$. *There is no need for special content, but the length of words should match the language.* $a\sqrt[n]{b} = \sqrt[n]{a^n b}$.

2.4 Example section

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2.4.1 Example subsection

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2.4.2 Example subsection

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- First item in a list

- Second item in a list

- Third item in a list

- Fourth item in a list

- Fifth item in a list

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Chapter 3

Stochastic analysis

3.1 Steady-state analysis

3.2 Transient analysis

3.2.1 Transient probability calculation

3.2.2 Accumulated probability calculation

3.3 Rewards and sensitivity

3.3.1 Stochastic reward nets

3.3.2 Sensitivity of rewards

Chapter 4

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4.1.1 Explicit state-space exploration

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Performability of clouds

7.2 Baselines

7.2.1 PRISM

7.2.2 SMART

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Chapter 8

Conclusion

8.1 Future work

References

Dayar, Tugrul (2012). *Analyzing Markov chains using Kronecker products: theory and applications*. Springer.