MŰEGYETEM 1782

Budapest University of Technology and Economics
Faculty of Electrical Engineering and Informatics
Department of Measurement and Information Systems

Stochastic Analysis of Complex Systems

Scientific Students' Associations Report

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Összefoglaló

A technológia fejlődésével a számítógépes rendszerek alkalmazási köre ma már olyan biztonságkritikus rendszerekre is kiterjed, amelyek helyes működésétől sokszor teljes vállalatok sorsa, vagy akár emberéletek is függhetnek. Az ilyen rendszerek méretének és bonyolultságának növekedtével szükségessé vált a megbízható automatikus módszerek kifejlesztése a rendszer (biztonság szempontjából) kritikus tulajdonságai teljesülésének ellenőrzésére, illetve a megbízhatósági és teljesítményjellemzők kiszámítására. Míg a formális módszerekkel matematikai igényességgel igazolható a tervezési folyamat helyessége, a sztochastikus technikák lehetővé teszik a modell kibővítését dinamikus, kvantiatív jellemzőkkel.

A modellek méteténék és bonyolultságának növekedése nagyságrendekkel emelheti meg a modell lehetséges futásidejű konfigurációinak számát. Ez a jelenség az *állapottér-robbanás*, mely mind a modellelenőrzés alapú formális verifikáció, mind a sztochasztikus analízis során nehézséget jelent. A modellellenőrzésnél, mely a formalizált követelmények verifikálásához felderíti a modell állapothalmazát, a *szaturációs* használó szimbolikus technikák segítik a rendkívül nagy méretű állapotterek hatékony kezelését. A sztochastikus analízis erre a célra általában lineáris algebrai mátrix-dekompozíciókat alkalmaz.

Abstract

The operation of entire corporations or even human life may depend on the correctness of safety-critical systems which became a prominent application area of computer systems due to the advancements in technology. The size and complexity of these systems is increasing, which creates a need for the development of trustable, automatic methods for verification of critical system properties and estimation of dependability and performability measures. Formal methods guarantee the correctness of the design process with mathematical rigor, while stochastic modeling allows extending models with quantitative properties.

The increase of model size and complexity can cause orders of magnitudes growth in the space of modeled possible runtime configurations. This phenomenon, called *state space explosion*, poses a difficulty for both formal verification based on model checking and for quantitative performance analysis. In model checking, which enumerates the possible states of the model to verify formalized system properties, symbolic approaches based on the *saturation* algorithm can overcome even exceptionally large state spaces with efficiency. In stochastic analysis decompositions based on linear algebra are used.

We propose a fully symbolic method of creating a description of the stochastic behavior of the system to be used in analysis calculations in decomposed form. This approach allows leveraging existing symbolic techniques of model checking for complex systems, such as the specification of the analyzed model properties with *continous-time temporal logic* (CTL) expressions.

The decomposition method is implemented for steady-state reward and sensitivity analysis, transient reward analysis and mean-time-to-first-failure analysis of stochastic models in the *stochastic Petri net* (SPN) Markov reward model formalism. The analysis tool is integrated into the Petridotnet 1.3[1] modeling and analysis applications along with a flexible linear algebra optimized for decompositions and calculations in used stochastic analysis which allows the user to fine-tune the analysis for the model under study.

The performance of analysis algorithms and decompositions is studied for multiple benchmark models and case studies and compared to other available analysis tools.

[1] `https://inf.mit.bme.hu/research/tools/petridotnet`

Chapter 1

Introduction

Árvíztűrő tükörfúrógép

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Chapter 2

Background

2.1 Petri nets

2.2 Continous-time Markov chains

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Dayar (2012)

2.3 Stochastic Petri nets

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Theorem 2.1. *Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.* $\sin^2(\alpha) + \cos^2(\beta) = 1$. *If you read this text, you will get no information* $E = mc^2$. *Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text* like this *gives you information about the selected font, how the letters are written and an impression of the look.* $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$. *This text should contain* all letters of the alphabet *and it should be written in of the original language.* $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$. *There is no need for special content, but the length of words should match the language.* $a\sqrt[n]{b} = \sqrt[n]{a^n b}$.

2.4 Example section

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2.4.1 Example subsection

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2.4.2 Example subsection

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- First item in a list
- Second item in a list
- Third item in a list
- Fourth item in a list
- Fifth item in a list

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References

Dayar, Tugrul (2012). *Analyzing Markov chains using Kronecker products: theory and applications*. Springer.