Budapest University of Technology and Economics
Faculty of Electrical Engineering and Informatics
Department of Measurement and Information Systems

Steady-state and Transient Analysis of Stochastic Petri Nets

Scientific Students' Associations Report

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Kivonat

Jelen dokumentum egy diplomaterv sablon, amely formai keretet ad a BME Villamosmérnöki és Informatikai Karán végző hallgatók által elkészítendő szakdolgozatnak és diplomatervnek. A sablon használata opcionális. Ez a sablon LaTeX alapú, a *TeXLive* TeX-implementációval és a PDF-LaTeX fordítóval működőképes.

Abstract

This document is a LaTeX-based skeleton for BSc/MSc theses of students at the Electrical Engineering and Informatics Faculty, Budapest University of Technology and Economics. The usage of this skeleton is optional. It has been tested with the *TeXLive* TeX implementation, and it requires the PDF-LaTeX compiler.

Chapter 1

Introduction

Árvíztűrő tükörfúrógép

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Chapter 2

Background

2.1 Petri nets

2.2 Continous-time Markov chains

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Dayar (2012)

2.3 Stochastic Petri nets

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. $\sin^2(\alpha) + \cos^2(\beta) = 1$. If you read this text, you will get no information $E = mc^2$. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text *like this* gives you information about the selected font, how the letters are written and an impression of the look. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$. This text should contain *all letters of the alphabet* and it should be written in of the original language. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$. There is no need for special content, but the length of words should match the language. $a\sqrt[n]{b} = \sqrt[n]{a^n b}$.

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Theorem 2.1. *Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.* $\sin^2(\alpha) + \cos^2(\beta) = 1$. *If you read this text, you will get no information* $E = mc^2$. *Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text* like this *gives you information about the selected font, how the letters are written and an impression of the look.* $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$. *This text should contain* all letters of the alphabet *and it should be written in of the original language.* $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$. *There is no need for special content, but the length of words should match the language.* $a\sqrt[n]{b} = \sqrt[n]{a^n b}$.

2.4 Example section

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2.4.1 Example subsection

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- First item in a list
- Second item in a list
- Third item in a list
- Fourth item in a list
- Fifth item in a list

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Chapter 3

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3.1 Steady-state analysis

3.2 Transient analysis

3.2.1 Transient probability calculation

3.2.2 Accumulated probability calculation

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3.3.1 Stochastic reward nets

3.3.2 Sensitivity of rewards

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4.1.1 Explicit state-space exploration

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8.1 Future work

References

Dayar, Tugrul (2012). *Analyzing Markov chains using Kronecker products: theory and applications*. Springer.