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Budapest University of Technology and Economics
Faculty of Electrical Engineering and Informatics
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Configurable Stochastic Analysis Framework for Asynchronous Systems

Scientific Students' Associations Report

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Összefoglaló A technológia fejlődésével a számítógépes rendszerek alkalmazási köre ma már olyan biztonságkritikus rendszerekre is kiterjed, amelyek helyes működésétől sokszor teljes vállalatok sorsa, vagy akár emberéletek is függhetnek. Az ilyen rendszerek méretének és bonyolultságának növekedtével szükségessé vált a megbízható automatikus módszerek kifejlesztése a rendszer (biztonság szempontjából) kritikus tulajdonságai teljesülésének ellenőrzésére, illetve a megbízhatósági és teljesítményjellemzők kiszámítására. Míg a formális módszerekkel matematikai igényességgel igazolható a tervezési folyamat helyessége, a sztochastikus technikák lehetővé teszik a modell kibővítését dinamikus, kvantiatív jellemzőkkel.

A modellek métetének és bonyolultságának növekedése nagyságrendekkel emelheti meg a modell lehetséges futásidejű konfigurációinak számát. Ez a jelenség az *állapottér-robbanás*, mely mind a modellelenőrzés alapú formális verifikáció, mind a sztochasztikus analízis során nehézséget jelent. A modellellenőrzésnél, mely a formalizált követelmények verifikálásához felderíti a modell állapothalmazát, a *szaturációs* használó szimbolikus technikák segítik a rendkívül nagy méretű állapotterek hatékony kezelését. A sztochastikus analízis erre a célra általában lineáris algebrai mátrix-dekompozíciókat alkalmaz.

Abstract Ensuring the correctness of critical systems—such as safety-critical, distributed and cloud applications—requires the rigorous analysis of the functional and extra-functional properties of the system. A large class of typical quantitative questions regarding dependability and performability are usually addressed by stochastic analysis.

However, the size and complexity of such systems often prevents the success of the analysis as they are highly sensitive to the number of possible behaviors. Recent critical systems are used to be distributed/asynchronous leading to the well-known phenomenon of state space explosion. In addition, temporal characteristics of the components can easily lead to huge computational overhead.

Calculation of dependability and performability measures can be reduced to steady-state and transient solutions of Markovian models. Various approaches are known in the literature for these problems differing in the representation of the stochastic behavior of the models and also differing in the applied numerical algorithms. Various characteristics of the models influence overheads associated with these approaches, therefore no single best approach is known.

The prerequisite of Markovian analysis is the exploration of the set of reachable states, i.e. the behaviors of the system. Symbolic approaches provide an efficient state space exploration and storage technique, however their application to support the vector operations and index manipulations extensively used by stochastic algorithms is cumbersome. The goal of our work is to introduce a framework that facilitates the analysis of complex, stochastic systems by combining together the advantages of symbolic algorithms, compact matrix representations and various numerical algorithms.

We propose a fully symbolic method to describe the stochastic behaviors. A new algorithm is introduced to transform the symbolic state space representation into a decomposed linear algebraic representation. This approach allows leveraging existing symbolic techniques, such as the specification of properties with *Computational Tree Logic* (CTL) expressions.

The framework provides configurable stochastic analysis: an approach is introduced to combine the various matrix representations with the numerical solution algorithms. Various algorithms are implemented for steady-state reward and sensitivity analysis, transient reward analysis and mean-time-to-first-failure analysis of stochastic models in the *Stochastic Petri Net* (SPN) Markov reward model formalism. The analysis tool is integrated into the PetriDotNet modeling application. Benchmarks and industrial case studies are used to evaluate the applicability of our approach.

Chapter 1

Introduction

Árvíztűrő tükörfúrógép

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Chapter 2

Background

2.1 Petri nets

2.2 Continous-time Markov chains

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Dayar (2012)

2.3 Stochastic Petri nets

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. $\sin^2(\alpha) + \cos^2(\beta) = 1$. If you read this text, you will get no information $E = mc^2$. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text *like this* gives you information about the selected font, how the letters are written and an impression of the look. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$. This text should contain *all letters of the alphabet* and it should be written in of the original language. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$. There is no need for special content, but the length of words should match the language. $a\sqrt[n]{b} = \sqrt[n]{a^n b}$.

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Theorem 2.1. *Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.* $\sin^2(\alpha) + \cos^2(\beta) = 1$. *If you read this text, you will get no information* $E = mc^2$. *Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text* like this *gives you information about the selected font, how the letters are written and an impression of the look.* $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$. *This text should contain* all letters of the alphabet *and it should be written in of the original language.* $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$. *There is no need for special content, but the length of words should match the language.* $a\sqrt[n]{b} = \sqrt[n]{a^n b}$.

2.4 Example section

And after the second paragraph follows the third paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. $\sin^2(\alpha) + \cos^2(\beta) = 1$. If you read this text, you will get no information $E = mc^2$. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text *like this* gives you information about the selected font, how the letters are written and an impression of the look. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$. This text should contain *all letters of the alphabet* and it should be written in of the original language. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$. There is no need for special content, but the length of words should match the language. $a\sqrt[n]{b} = \sqrt[n]{a^n b}$.

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2.4.1 Example subsection

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2.4.2 Example subsection

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- First item in a list
- Second item in a list
- Third item in a list
- Fourth item in a list
- Fifth item in a list

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Chapter 3

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3.1 Steady-state analysis

3.2 Transient analysis

3.2.1 Transient probability calculation

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4.1.1 Explicit state-space exploration

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8.1 Future work

References

Dayar, Tugrul (2012). *Analyzing Markov chains using Kronecker products: theory and applications*. Springer.