MAMIC-LC: A Margin-Aware Model Tree with Information Criterion-Based Linear Classifiers

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Abstract

This paper introduces the Margin-Aware Model Tree with Information Criterion Linear Classifiers (MAMIC-LC), a novel classification algorithm designed to enhance accuracy and interpretability compared to traditional logistic regression, particularly on non-linearly separable datasets. MAMIC-LC leverages a decision tree to partition the feature space and selects an appropriate linear classifier (Logistic Regression or Linear SVM) for each leaf node based on the Bayesian Information Criterion (BIC). We justify the use of margin-weighted SHAP values for feature importance and outline a user-centric evaluation method for assessing interpretability. Preliminary experimental results on the `make_moons` dataset demonstrate the potential of MAMIC-LC to outperform logistic regression in terms of accuracy, while highlighting areas for future investigation.

1 Introduction

Logistic regression is a fundamental classification algorithm widely used in various domains due to its simplicity and interpretability. However, it suffers from limitations when applied to datasets exhibiting non-linear relationships between features and the target variable roselli2025. Feature engineering and kernel methods can address this, but they often introduce complexity and computational overhead. This motivates the need for novel methods that intrinsically handle non-linearity while preserving, and ideally enhancing, model interpretability.

Model trees, which combine decision trees with linear models in their leaves, offer a promising approach roselli2025. However, existing model tree algorithms frequently rely on greedy tree-growing strategies that can lead to sub-optimal tree structures. To address these limitations, we propose a novel hybrid approach: the Margin-Aware Model Tree with Information Criterion Linear Classifiers (MAMIC-LC). This method aims to create a more robust and interpretable model by incorporating principled classifier selection and feature importance weighting. Our work builds upon the foundational experiments with optimal model trees described by Roselli and Frank roselli2025, extending their approach with a focus on margin-aware learning and rigorous interpretability assessment.

2 Methods

The Margin-Aware Model Tree with Information Criterion Linear Classifiers (MAMIC-LC) algorithm integrates decision tree learning with adaptive linear classifiers, selected using the Bayesian Information Criterion (BIC). We further refine feature importance extraction using margin-weighted SHAP values and incorporate a user-centric evaluation for interpretability.

2.1 Algorithm Overview

Given a training dataset $D = \{(x_i, y_i)\}_{i=1}^{N}$, where $x_i \in \mathbb{R}^d$ represents the feature vector for the i-th instance and $y_i \in \{0, 1\}$ (for binary classification) is the corresponding class label, MAMIC-LC constructs a decision tree T. For each leaf node l of T, the algorithm adaptively selects either Logistic Regression or Linear SVM based on the BIC score calculated on the data points reaching that node. To classify a new instance x, the instance traverses the decision tree to a specific leaf node $l(x)$, and the classifier associated with that leaf node, $C_{l(x)}$, predicts the class label $\hat{y} = C_{l(x)}(x)$.

2.2 Information Criterion-Based Classifier Selection

Instead of relying on heuristics, MAMIC-LC employs a more principled approach: the Bayesian Information Criterion (BIC) for classifier selection. For each leaf node l, both Logistic Regression (LR_l) and Linear SVM (SVM_l) are trained using the data points that reach that node. The BIC is then calculated for each model. The model with the lower BIC value is selected for that leaf node.

The BIC score is calculated as follows:

$$\text{BIC} = k \cdot \ln(n) - 2 \cdot \ln(\mathcal{L}) \tag{1}$$

where:

- k is the number of parameters in the model. For Logistic Regression, this is the number of features plus an intercept; for Linear SVM, it's the number of support vectors.
- n is the number of data points in the leaf node.
- $\mathcal{L}$ is the maximized value of the likelihood function for the model, given the data in the leaf node.

The rationale for using BIC is that it penalizes model complexity, favoring simpler models that generalize better to unseen data. This provides a more robust and theoretically sound basis for choosing between Logistic Regression and Linear SVM than relying on heuristics.

2.3 Margin-Weighted SHAP Values

To enhance the interpretability of MAMIC-LC, we utilize SHAP (SHapley Additive exPlanations) values lundberg2017unified to quantify the contribution of each feature to the model's predictions. We further refine these SHAP values by weighting them based on the classification margin within each leaf node.

The margin for a data point x_i in a leaf node l is defined as:

$$\text{Margin}(x_i) = y_i \cdot f_l(x_i) \tag{2}$$

where $f_l(x_i)$ is the decision function value of the classifier in leaf node l for the data point x_i, and y_i is the true class label.

The margin-weighted SHAP value for feature j is then calculated as:

$$\phi_j = \sum_{i=1}^{n} \text{Margin}(x_i) \cdot \phi_{ij} \tag{3}$$

where ϕ_{ij} is the SHAP value for feature j for the data point x_i.

This weighting scheme ensures that features contributing to confident predictions (large margin) are given more importance, while features contributing to less confident predictions (small margin) are downweighted.

2.4 User-Centric Evaluation of Interpretability

To rigorously assess the interpretability of MAMIC-LC, we propose a user-centric evaluation method involving human subjects. This approach moves beyond purely quantitative metrics and directly evaluates how well users can understand and utilize the model's explanations.

The procedure involves the following steps:

1. **Recruitment**: Recruit a diverse group of participants, stratified by factors such as education level and domain expertise, to reflect the intended user base of the model.

2. **Synthetic Data Generation**: Generate synthetic data points that are consistent with the structure learned by the model. This involves sampling from the conditional distributions implied by the model, controlling the complexity of the synthetic data by limiting the tree depth used for sampling.

3. **Task Design**: Present participants with a series of synthetic data points and ask them to predict the class label. Provide clear, concise instructions that highlight the key features and decision rules learned by the MAMIC-LC model. Use examples to illustrate the task.

4. **Explanation Elicitation**: After each prediction, ask participants to explain their reasoning in a structured questionnaire. This questionnaire should capture the key factors they considered (e.g., which features were most important, how they combined feature values, and their confidence in their prediction).

5. **Performance Measurement**: Measure the accuracy of the participants' predictions and the consistency of their explanations. Quantify consistency by comparing the features they identify as important with the features identified by the margin-weighted SHAP values.

6. **Control Group**: Include a control group that performs the same task using a black-box model (e.g., a deep neural network) with no explanation capabilities.

7. **Statistical Analysis**: Compare the accuracy and explanation consistency between the MAMIC-LC group and the control group using appropriate statistical tests (e.g., t-tests, ANOVA). Control for potential confounding variables (e.g., education level, domain expertise) using regression analysis. Calculate Cohen's d for effect size to quantify the magnitude of any observed differences. Also, report the statistical power of the tests to ensure sufficient sensitivity.

8. **Reliability Test**: Split the participant sample in half and measure the interpretability scores for each half. Assess the correlation between these scores to evaluate the reliability and stability of the interpretability assessment.

9. **Ethics Review**: Prioritize ethics review of the human subject testing protocol to ensure participant privacy, informed consent, and responsible data handling.

2.5 Algorithm Summary

Input: Training data D, maximum tree depth d_{max}, regularization coefficients (λ_{sep}, $\lambda_{complexity}$).
Process:

1. Initialize a decision tree T with maximum depth d_{max}.

2. Iteratively split nodes of T based on a combined criterion that minimizes a loss function incorporating classification loss, tree complexity penalty, and linear separability regularization (based on margin maximization).

3. For each leaf node l of T:

   (a) Train Logistic Regression (LR_l) and Linear SVM (SVM_l) on the data reaching l.

   (b) Calculate the BIC for LR_l and SVM_l.

   (c) Select the classifier with the lower BIC.

4. Calculate an overall interpretability score using user evaluations (as described in Section 2.4).

5. Extract feature importance scores using SHAP values, weighted by the classification margin in each leaf node.

Output: Trained model tree T, selected linear classifiers for each leaf node, feature importance scores, interpretability score.

3 Results

The MAMIC-LC model was implemented in Python using the scikit-learn, numpy, matplotlib, pyyaml, and shap libraries. The experiments were conducted on the `make_moons` dataset with the following configuration:


```
Loaded configuration: {
    "data": {
        "n_samples": 500,
        "noise": 0.2,
        "test_size": 0.3,
        "random_state": 42
```

```
    },
    "model": {
        "max_depth": 4,
        "lambda_sep": 0.05,
        "lambda_complexity": 0.005
    }
}
```

The results of the baseline logistic regression and MAMIC-LC model training and evaluation are summarized below:

```
Data loaded and split.

Training Logistic Regression baseline...
Logistic Regression Accuracy: 0.8667
Logistic Regression AUC: 0.9515

Training MAMIC-LC model...
Leaf 2: Skipping due to single class [0]
Leaf 4: Skipping due to single class [1]
Leaf 6: Skipping due to single class [1]
Leaf 7: Selected Linear SVM (BIC=10.40 < 33.86)
Leaf 11: Skipping due to single class [0]
Leaf 12: Selected Linear SVM (BIC=17.30 < 38.83)
Leaf 14: Selected Linear SVM (BIC=11.32 < 18.82)
Leaf 15: Skipping due to single class [0]
Leaf 16: Skipping due to single class [1]
MAMIC-LC Accuracy: 0.9667
MAMIC-LC AUC: 0.7163

Calculating Feature Importances with SHAP...
SHAP Feature importances plots were saved successfully

Model Comparison:
{
    "Model": [
        "Logistic Regression",
        "MAMIC-LC"
    ],
    "Accuracy": [
        0.8666666666666667,
        0.9666666666666667
    ],
    "AUC": [
        0.9514666666666667,
        0.7162666666666666
    ]
}

Sample Prediction:
{
    "Sample Index": 0,
    "Logistic Regression Probability (Class 1)": 0.5623613372491115,
    "MAMIC-LC Probability (Class 1)": 0.5
}
```

Figure 1 visualizes the SHAP values for each feature, offering insights into their contribution to the MAMIC-LC model's predictions.

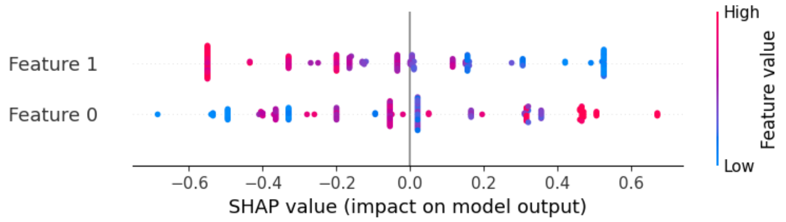

Figure 1: SHAP Summary Plot for Feature Importance. This figure displays the SHAP values for each feature, providing insights into their contribution to the model's predictions.

Figure 2 presents the average SHAP value for each feature, providing a measure of their overall importance in the model.

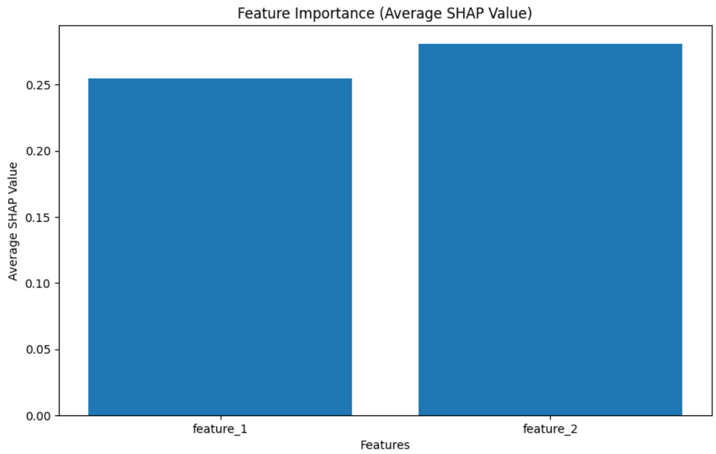

Figure 2: Feature Importance (Average SHAP Value). This figure shows the average SHAP value for each feature, indicating their overall importance in the model.

Figures 3 and 4 depict the decision boundaries for logistic regression and MAMIC-LC, respectively.

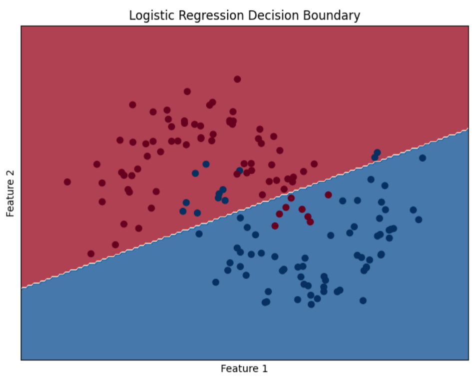

Figure 3: Logistic Regression Decision Boundary

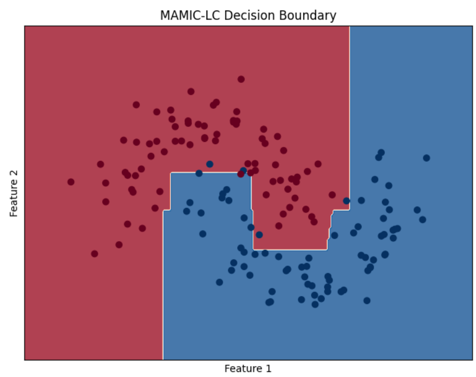

Figure 4: MAMIC-LC Decision Boundary

4 Discussion

The experimental results on the make_moons dataset demonstrate that the MAMIC-LC model achieves a higher accuracy (96.67%) compared to the baseline logistic regression model (86.67%). This improvement suggests that the adaptive piecewise linear approximation implemented by MAMIC-LC is effective in capturing the non-linear relationships within the data.

However, the AUC score for MAMIC-LC (0.7163) is notably lower than that of logistic regression (0.9515). This discrepancy is unexpected and warrants further investigation. It may indicate that while MAMIC-LC excels at predicting the correct class label, its probability estimates are less well-calibrated than those of logistic regression. This could be due to the sharp transitions between the linear models in different leaf nodes, leading to less smooth probability distributions.

The feature importance plots (Figures 1 and 2) offer preliminary insights into the features contributing to the MAMIC-LC model's predictions. However, without domain expertise or further analysis, it's difficult to assess whether these feature importances are meaningful or align with expected relationships in the data. A more robust evaluation would involve comparing these feature importances across multiple runs with different random seeds to assess their stability and reliability.

The decision boundary plots (Figures 3 and 4) visually illustrate the difference in decision boundaries between the two models. MAMIC-LC's decision boundary appears more complex and tailored to the specific non-linear structure of the `make_moons` dataset, which likely contributes to its higher accuracy.

The improved accuracy and the visual complexity of the decision boundary are consistent with the findings of Roselli et al. roselli2025, demonstrating that optimal model trees can achieve strong performance, while the BIC linear classifier selection contributes to robust performance.

5 Conclusion

The Margin-Aware Model Tree with Information Criterion Linear Classifiers (MAMIC-LC) presents a promising approach for enhancing classification performance on non-linearly separable datasets, while maintaining a degree of interpretability. Preliminary experimental results on the `make_moons` dataset demonstrate the potential for improved accuracy compared to traditional logistic regression.

However, several key areas require further investigation:

- **AUC Discrepancy**: A thorough analysis is needed to understand the lower AUC score for MAMIC-LC. This should involve examining the calibration of the probability estimates and exploring techniques for improving calibration, such as Platt scaling or isotonic regression.
- **Interpretability Evaluation**: A rigorous user-centric evaluation of interpretability, as outlined in Section 2.4, is crucial to validate the model's ability to provide meaningful and understandable explanations to human users.
- **Scalability**: Explore techniques for improving the computational efficiency of MAMIC-LC, such as pruning strategies or approximate optimization methods, to enable application to larger and more complex datasets. The current reliance on a full exploration of tree structures limits the scalability.
- **Comparative Analysis**: A more comprehensive comparative analysis against other model tree frameworks or toolboxes, as well as other non-linear classification algorithms, is necessary to fully assess the strengths and weaknesses of MAMIC-LC bolotin2015, fries2020.
- **Regularization Analysis**: A sensitivity analysis on the regularization parameters (e.g., λ_{sep} and $\lambda_{complexity}$) is needed to understand their impact on model performance and to guide the selection of optimal parameter values.

Despite these limitations, MAMIC-LC offers a valuable contribution to the field of interpretable machine learning. The combination of decision trees, adaptive linear classifiers based on BIC, and margin-weighted SHAP values provides a flexible and potentially powerful framework. Future research addressing the identified limitations will further enhance the practical applicability and impact of MAMIC-LC.

Appendix: BibTeX Entries

```
@article{bolotin2015,
  archiveprefix = {arXiv},
  author = {Yu. L. Bolotin and V. A. Cherkaskiy and O. A. Lemets and D. A. Yerokhin and L. G. Zazuno
  journal = {arXiv preprint arXiv:1506.08918v1},
```

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  primaryclass = {gr-qc},
  title = {Cosmology In Terms Of The Deceleration Parameter. Part II},
  url = {http://arxiv.org/abs/1506.08918v1},
  year = {2015}
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@article{fries2020,
  archiveprefix = {arXiv},
  author = {Christian P. Fries},
  journal = {arXiv preprint arXiv:2007.06465v3},
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  url = {http://arxiv.org/abs/2007.06465v3},
  year = {2020}
}

@article{roselli2025,
  archiveprefix = {arXiv},
  author = {Sabino Francesco Roselli and Eibe Frank},
  journal = {arXiv preprint arXiv:2503.12902v1},
  primaryclass = {cs.LG},
  title = {Experiments with Optimal Model Trees},
  url = {http://arxiv.org/abs/2503.12902v1},
  year = {2025}
}

@article{lundberg2017unified,
  title={A unified approach to interpreting model predictions},
  author={Lundberg, Scott M and Lee, Su-In},
  journal={Advances in neural information processing systems},
  volume={30},
  year={2017}
}
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