Electronics Lab Course
Experiment #1: Expansion of signals in conducters

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Contents

1 Theoretical background

1.1 Conducting properties

If the electrical properties of a double-cable are equal on the whole cable, it is called homogeneous. In this experiment, we work with such cables.
Capacitive and inductive properties of the cable are:

$$\begin{aligned} C &= \epsilon_r \epsilon_0 l \frac{2\pi}{\ln\left(\frac{r_a}{r_i}\right)} \\ L &= \mu_r \mu_0 \frac{\ln\left(\frac{r_a}{r_i}\right)}{2\pi} \end{aligned}$$

The four characteristics of a cable[1] grow proportional to it's length. A lossless cable can be approximated as a chain of many LC-links.

1.2 Expansion of waves in homogeneous cables

$$\begin{aligned} \frac{d^2}{dx^2}U - \gamma^2 U &= 0 \\ \gamma^2 &= z' \cdot y' \Rightarrow \text{damping} \\ \textit{solution}: \quad U(x,t) &= U_f(x,t) + U_b(x,t) \quad \text{f: forward, b: backwards} \\ I(x,t) &= I_f(x,t) + I_b(x,t) \end{aligned}$$

1.3 Phase velocity and wave resistance

The phase velocity is:

$$v_{Ph} = \frac{c_0}{\sqrt{\epsilon_r \mu_r}}$$

It is equal to the velocity of waves with equal wavelength in matter with equal ϵ_r and μ_r.
In lossless case the wave resistance is:

$$\begin{aligned} z &= \sqrt{\frac{L'}{C'}} \\ &= \sqrt{\frac{\mu_r \mu_0}{\epsilon_r \epsilon_0}} \cdot \frac{\ln\left(\frac{r_a}{r_i}\right)}{2\pi} \end{aligned}$$

[1] Resistance, inductance, capacity and loss

1.4 Cable termination and adjustment

Inside a cable there is not only the incoming, but also a reversal wave of voltage or current.
But depending on the termination it can be absorbed.

adjusted termination:	- terminal resistance = wave resistance
	- no reflexion
	- on the input like a cable of infinite length
open cable:	- infinite terminal resistance
	- incoming wave equal to reversal wave
	- factor of reflexion = +1
short circuit:	- terminal resistance = 0
	- reversal wave = −incoming wave
	- factor of reflexion = −1

2 Preperational exercises

2.1 1.A

To increase the delay, ϵ_r or μ_r must be increased. Those are proportional to C' and L' which are proportional to the delay. Increasing the length of the cable also increases the delay.

2.2 1.B

The impedance is proportional to μ_r and antiproportional to ϵ_r.

$$\Rightarrow Z \quad = \quad \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}}$$

2.3 1.C

Connected cable $\Rightarrow$ no reflexion.

$$\begin{aligned} \Rightarrow U_r &= I_r \\ &= 0 \\ \Rightarrow R_{in} &= \frac{U_f}{I_f} \\ &= \sqrt{\frac{R' + i\omega L'}{G' + i\omega C'}} \end{aligned}$$

Therefor R_{in} does not depend on the length of the cable.

2.4 1.D

$$Z \quad = \quad \frac{U_f}{I_f} \tag{1}$$

Without reversal wave: $Z_{in} = Z$
In this case:

$$Z_{in} \quad = \quad \frac{U_f(l) + U_r(l)}{I_f(l) + I_r(l)} \tag{2}$$

$$\begin{aligned} I_r(l) + I_f(l) &= 0 \\ U_r(l) - U_f(l) &= 0 \\ U_f(l) + U_r(l) &= (U_f(0) \exp -\gamma x + U_r(0) \exp \gamma x) \exp i\omega t \\ U_f(0) \exp -\gamma x &= U_r(0) \exp \gamma x \\ U_f(0) \exp -2\gamma x &= U_r(0) \end{aligned}$$

The current is analogue to this.
In combination with equation (2) and (1) we get:

$$
\begin{aligned}
Z_{in} &= \frac{U_f(l)}{I_f(l)}\,(1+\exp -2\gamma x) \\
Z_{in} &= Z\,(1+\exp -2\gamma x) \\
Y &= Y' = 0 \quad \text{(lossless cable)} \\
\Rightarrow \gamma &= 0 \\
\Rightarrow Z_{in} &= 2Z
\end{aligned}
$$

The impedance is independent of cable length, wavelength and frequency.

3 Experiment set-up

4 Procedure

4.1 1.5.1 Differentiator

The oscilloscope is triggered external and the rectangular signal is differentiated by a high-pass filter.
We observe, what happens if we use the RC-link with a build in Resistance of $2.2\,\mathrm{k\Omega}$.

4.2 1.5.2 Pulses in cables

4.3 1.5.3 Cable termination, delay

4.4 1.5.4 Clipcable, damping

4.5 1.5.5 $50\,\Omega$-Cable RG-58 C/U

5 Measurement

6 Evaluation

7 Conclusion