Electronics Lab Course
Experiment #1: Expansion of signals in conducters

Christopher Jörn

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Contents

1 Theoretical background

1.1 Conducting properties

If the electrical properties of a double-cable are equal on the whole cable, it is called homogeneous. In this experiment, we work with such cables.
Capacitive and inductive properties of the cable are:

$$\begin{aligned} C &= \epsilon_r \epsilon_0 l \frac{2\pi}{\ln\left(\frac{r_a}{r_i}\right)} \\ L &= \mu_r \mu_0 \frac{\ln\left(\frac{r_a}{r_i}\right)}{2\pi} \end{aligned}$$

The four characteristics of a cable[1] grow proportional to it's length. A lossless cable can be approximated as a chain of many LC-links.

1.2 Expansion of waves in homogeneous cables

$$\begin{aligned} \frac{d^2}{dx^2}U - \gamma^2 U &= 0 \\ \gamma^2 &= z' \cdot y' \Rightarrow \text{damping} \\ solution: \quad U(x,t) &= U_f(x,t) + U_b(x,t) \quad \text{f: forward, b: backwards} \\ I(x,t) &= I_f(x,t) + I_b(x,t) \end{aligned}$$

1.3 Phase velocity and wave resistance

The phase velocity is:

$$v_{Ph} = \frac{c_0}{\sqrt{\epsilon_r \mu_r}}$$

It is equal to the velocity of waves with equal wavelength in matter with equal ϵ_r and μ_r.
In lossless case the wave resistance is:

$$\begin{aligned} z &= \sqrt{\frac{L'}{C'}} \\ &= \sqrt{\frac{\mu_r \mu_0}{\epsilon_r \epsilon_0}} \cdot \frac{\ln\left(\frac{r_a}{r_i}\right)}{2\pi} \end{aligned}$$

[1]Resistance, inductance, capacity and loss

1.4 Cable termination and adjustment

Inside a cable there is not only the incoming, but also a reversal wave of voltage or current.
But depending on the termination it can be absorbed.

adjusted termination:	- terminal resistance = wave resistance
	- no reflexion
	- on the input like a cable of infinite length
open cable:	- infinite terminal resistance
	- incoming wave equal to reversal wave
	- factor of reflexion = +1
short circuit:	- terminal resistance = 0
	- reversal wave = −incoming wave
	- factor of reflexion = −1

2 Preperational exercises

2.1 1.A

To increase the delay, ϵ_r or μ_r must be increased. Those are proportional to C' and L' which are proportional to the delay. Increasing the length of the cable also increases the delay.

2.2 1.B

The impedance is proportional to μ_r and antiproportional to ϵ_r.

$$\Rightarrow Z = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}}$$

2.3 1.C

Connected cable $\Rightarrow$ no reflexion.

$$\begin{aligned} \Rightarrow U_r &= I_r \\ &= 0 \\ \Rightarrow R_{in} &= \frac{U_f}{I_f} \\ &= \sqrt{\frac{R' + i\omega L'}{G' + i\omega C'}} \end{aligned}$$

Therefor R_{in} does not depend on the length of the cable.

2.4 1.D

$$Z = \frac{U_f}{I_f} \tag{1}$$

Without reversal wave: $Z_{in} = Z$
In this case:

$$Z_{in} = \frac{U_f(l) + U_r(l)}{I_f(l) + I_r(l)} \tag{2}$$

$$\begin{aligned} I_r(l) + I_f(l) &= 0 \\ U_r(l) - U_f(l) &= 0 \\ U_f(l) + U_r(l) &= (U_f(0)\exp -\gamma x + U_r(0)\exp \gamma x)\exp i\omega t \\ U_f(0)\exp -\gamma x &= U_r(0)\exp \gamma x \\ U_f(0)\exp -2\gamma x &= U_r(0) \end{aligned}$$

The current is analogue to this.
In combination with equation (2) and (1) we get:

$$\begin{aligned}
Z_{in} &= \frac{U_f(l)}{I_f(l)}\,(1+\exp -2\gamma x) \\
Z_{in} &= Z\,(1+\exp -2\gamma x) \\
Y &= Y' = 0 \quad \text{(lossless cable)} \\
\Rightarrow \gamma &= 0 \\
\Rightarrow Z_{in} &= 2Z
\end{aligned}$$

The impedance is independent of cable length, wavelength and frequency.

3 Procedure

3.1 1.5.1 Differentiator

The oscilloscope is triggered external and the rectangular signal is differentiated by a high-pass filter.
We observe, what happens if we use the RC-link with a build in Resistance of $2.2\,\mathrm{k\Omega}$.

3.2 1.5.2 Pulses in cables

To understand how pulses spread in cables with both ends open, we observe the voltage of pulses send through a $50\,\Omega$-cable at the beginning and the middle of the cable. The input resistance is very large compared to the $50\,\Omega$ which allows the pulses to get into the cable but also allows us to approximate the cable terminal open.

3.3 1.5.3 Cable termination, delay

3.3.1 a)/b)

The delay cable terminal is open

3.3.2 c)

The delay cable terminal is short-circuited.

3.3.3 d)

Varying frequencies.

3.4 1.5.4 Clipcable, damping

Often long pulses shall be shortened to a defined length. To do so we use clip cables, which use reflexion to shorten the pulses.

3.4.1 a)

Open terminal.

3.4.2 b)

Short-circuited clip cable.

3.4.3 c)/d)

We vary the frequencies and observe the distances between pulses. Then we use a $2\,\mathrm{m}$ clip cable and consider on what thepulse length depends.

3.4.4 e)

The specific damping of the HH 2500 shall be determined. To do so, we must measure the ratio of the top-stage to the stage one step below.

3.5 1.5.5 $50\,\Omega$-Cable RG-58 C/U

In most cases we want the cable to damp or distort incoming pulses as little as possible. In reality this is for different cables only possible for certain bandwidths.
Modern transmission cables are often used with a large bandwidth and have little wave resistance ($\approx 50\,\Omega$) and also little delays.

4 Measurement

4.1 1.5.1

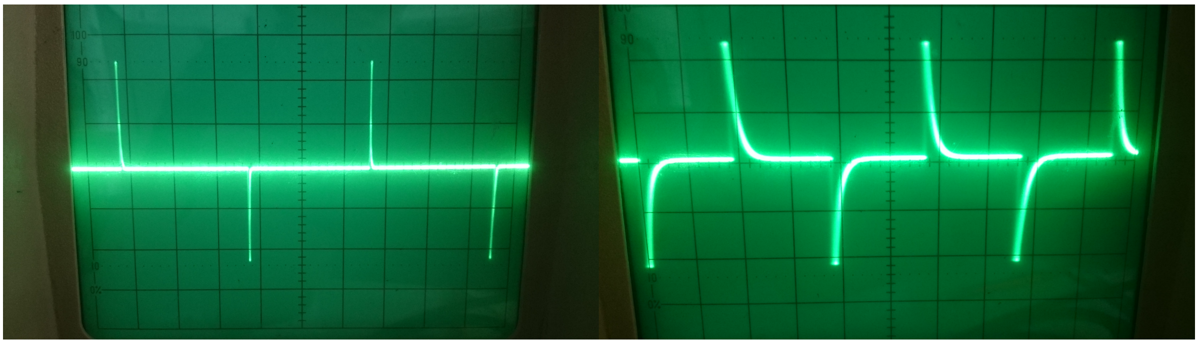

Figure 1: Rectangular signal (left) without RC-link $t/1\,\mu s$ and $U/0.2\,V$. With RC-link (right) $t/1\,\mu s$ and $U/2\,V$

4.2 1.5.2

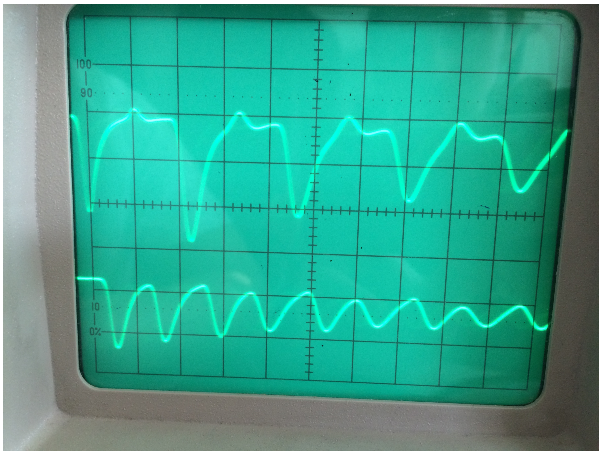

Figure 2: Signal: rectangular w. 100 kHz Scale: $U/5\,mV$ and $t/0.05\,\mu s$

4.3 1.5.3

4.3.1 a)/b)

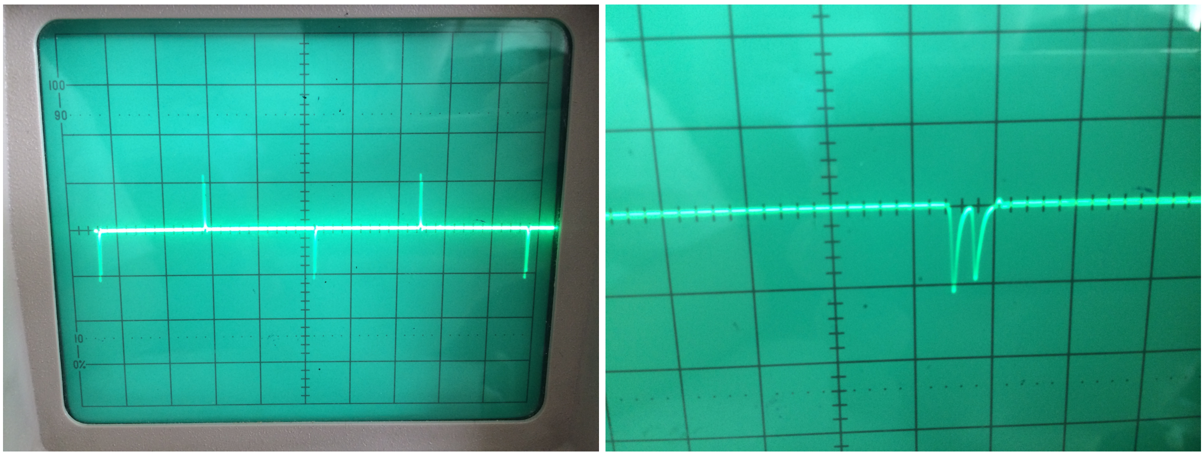

Figure 3: Signal: rectangular w. 100 kHz Scale: $U/5\,\mathrm{mV}$ and $t/2\,\mu\mathrm{s}$ for the left picture and Signal: rectangular w. 100 kHz Scale: $U/5\,\mathrm{mV}$ and $t/0.02\,\mu\mathrm{s}$ for the right picture

4.3.2 c)

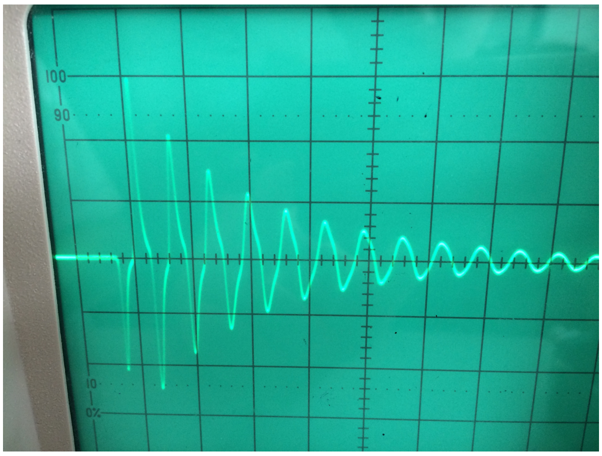

Figure 4: short circuited without Resistor. Scale: $U/5\,\mathrm{mV}$ and $t/2\,\mu\mathrm{s}$

5 Evaluation

5.1 1.5.1

It is well observable, that the peaks shrink when the $2.2\,\mathrm{k}\Omega$ RC-link is build in (enlarged resolution at the osci-picture). The resistor damps the signal. It is also well observable that the pulses decrease exponential. This damping comes through reflexion at both ends of the cable.