Latin and Greco-Latin Square Designs

Example in Education Using a Latin Square

Assume you are interested in the achievement of students under three methods of instruction for a required course in biology (self, computer, and classroom), interested in differences of these instruction modes for three colleges within a university (agriculture, education, engineering) and three types of students (in-state, out-of-state, out-of-country). We could use a completely balanced 3-way analysis of variance design with Factor A = instructional mode, Factor B = College and Factor C = type of student. There would be 27 experimental units (samples of subjects) in this design. On the other hand we might employ the following design:

	FACTOR A (Instruction)				
	Self	Computer	Classroom		
FACTOR B		•			
(College)					
Agriculture	C2	C1	C3		
Education	C1	C3	C2		
Engineering	C3	C2	C1		

In this design C1 is the in-state student unit, C2 is the out-of-state student unit and C3 is the out-of-country student unit. There are only 9 units in this design as contrasted with 27 units in the completely balanced design. Note that each type of student receives each type of instruction. Also note however that, wthin a college, students of each type do NOT receive each type of instruction. We will have to assume that the interaction of college and type of instruction, the interaction of college and type of student, the interaction of type of instruction and type of student and the triple interaction of College, instruction and student are small or do not exist. We are primarily interested in the main effects, that is differences among student types, types of instruction and colleges on the achievement scores obtained in the biology course. We might use Plan 1 described below.

Plan 1 by B.J. Winer

We have prepared an example file for you to analyze with OpenStat. Open the file labeled LatinSqr.tex in your set of sample data files. We have entered four cases for each unit in our design for instructional mode, college and home residence. Once you have loaded the file, select the Latin squares designs option under the sub-menu for comparisons under the Analyses menu. You should see the form below for selecting the Plan 1 analysis.

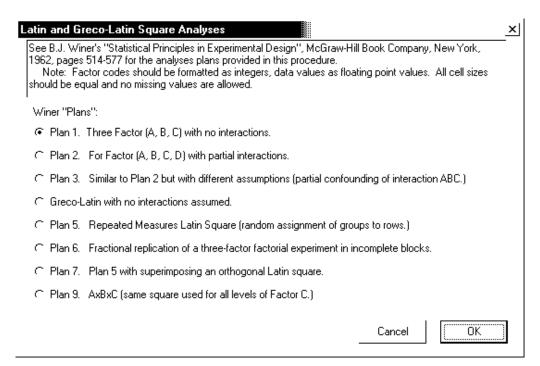


Figure 1. Latin and Greaco-Latin Squares Form

When you have selected Plan 1 for the analysis, click the OK button to continue. You will then see the form below for entering the specifications for your analysis. We have entered the variables for factors A, B and C and entered the number of cases for each unit:

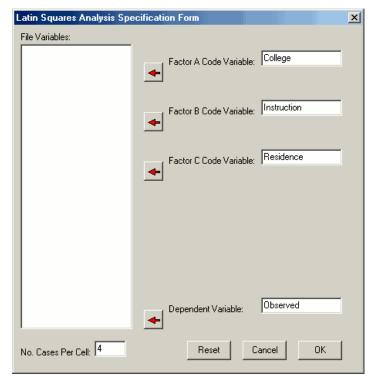


Figure 2. Latin Squares Analysis Dialog

We have completed the entry of our variables and the number of cases and are ready to continue. When you press the OK button, the following results are presented on the output page:

Latin Square Analysis Plan 1 Results

Source	SS	DF	MS	F	Prob.>F
Factor A Factor B Factor C Residual Within Total	92.389 40.222 198.722 33.389 99.500 464.222	2 2 2 2 2 27 35	46.194 20.111 99.361 16.694 3.685	12.535 5.457 26.962 4.530	0.000 0.010 0.000 0.020

Cell means a	nd totals			
Instruction	1	2	3	Total
College 1 2 3	2.750 8.250 1.500	10.750 2.250 1.500	3.500 1.250 2.250	5.667 3.917 1.750
Total	4.167	4.833 	2.333	3.778
Residence	1	2	3	Total
	2.417	1.833	7.083	3.778

A partial test of the interaction effects can be made by the ratio of the MS for residual to the MS within cells. In our example, it appears that our assumptions of no interaction effects may be in error. In this case, the main effects may be confounded by interactions among the factors. The results may never the less suggest differences do exist and we should complete another balanced experiment to determine the interaction effects.

Plan 2

We have included the file "LatinSqr2.tex" as an example for analysis. Load the file in the grid and select the Latin Square Analyses, Plan 2 design. The form below shows the entry of the variables and the sample size for the analysis:

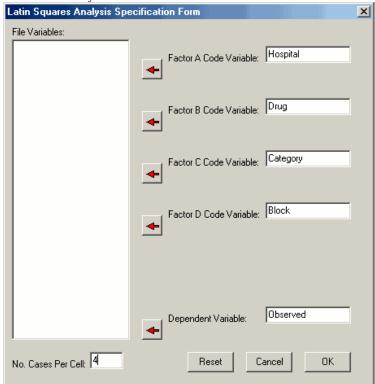


Figure 3. Four Factor Latin Square Design Form

When you click the OK button, you will see the following results:

Latin Square Analysis Plan 2 Results

Source	 	DF	MS	F	Prob.>F
Factor A Factor B Factor C Factor D A x D B x D C x D Residual Within Total	148.028 5.444 66.694 18.000 36.750 75.000 330.750 66.778 199.000 946.444	2 2 1 2 2 2 4 54 71	74.014 2.722 33.347 18.000 18.375 37.500 165.375 16.694 3.685	20.084 0.739 9.049 4.884 4.986 10.176 44.876 4.530	0.000 0.483 0.000 0.031 0.010 0.000 0.000 0.003

Experimental Design for block 1

Drug	1	2	3	
Hospital				
1	C2	С3	C1	
2	С3	C1	C2	
3	C1	C2	С3	

Experimental Design for block 2

D	rug	1	2	3
Hospi 1 2 3	tal	C2 C3 C1	C3 C1 C2	C1 C2 C3

BLOCK 1

Cell means and totals

Drug	1	2	3	Total
Hospital				
1	2.750	10.750	3.500	5.667
2	8.250	2.250	1.250	3.917
3	1.500	1.500	2.250	1.750
Total	4.167	4.833	2.333	4.278

BLOCK 2

Cell means and totals

Drug	1	2	3	Total
Hospital 1 2 3 Total	9.250 3.750 2.500 5.167	2.250 4.500 3.250 3.333	3.250 11.750 2.500 5.833	4.917 6.667 2.750 4.278
Category	1	2 2	3	Total
	2.917	4.958	4.958	4.278

Notice that the interactions with Factor D are obtained. The residual however indicates that some of the other interactions confounded with the main factors may be significant and, again, we do not know the portion of the differences among the main effects that are potentially due to interactions among A, B, and C.

Plan 3 Latin Squares Design

The file "LatinSqr3.tex" contains an example of data for the Plan 3 analysis. Following the previous plans, we show below the specifications for the analysis and results from analyzing this data:

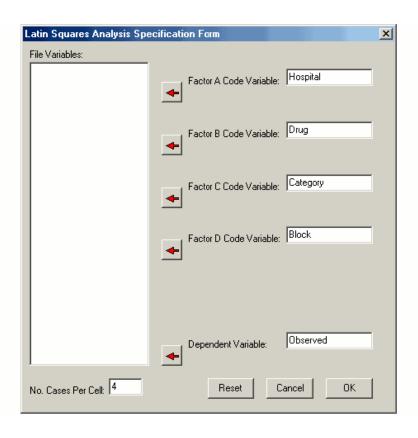


Figure 4. Another Latin Square (Plan 3) Dialog Form

Latin Square Analysis Plan 3 Results

Source	SS	DF	MS	F	Prob.>F
Factor A	26.963	2	13.481	3.785	0.027
Factor B	220.130	2	110.065	30.902	0.000
Factor C	213.574	2	106.787	29.982	0.000
Factor D	19.185	2	9.593	2.693	0.074
АхВ	49.148	4	12.287	3.450	0.012
A x C	375.037	4	93.759	26.324	0.000
вхС	78.370	4	19.593	5.501	0.001
AxBxC	118.500	6	19.750	5.545	0.000
Within	288.500	81	3.562		
Total	1389.407	107			

Experimental Design for block 1

	Drug	1	2	3
1	pital 1 2 3	C1 C2 C3	C2 C3 C1	C3 C1 C2

Experimental Design for block 2

Drug	1	2	3	
Hospital 1 2 3	C2 C3 C1	C3 C1 C2	C1 C2 C3	

Experimental Design for block 3

Drug	1 	2	3	
Hospital 1 2 3	C3 C1 C2	C1 C2 C3	C2 C3 C1	

BLOCK 1

Cell means and totals

Drug	1	2	3	Total
Hospital				
1	2.750	1.250	1.500	1.833
2	3.250	4.500	2.500	3.417
3	10.250	8.250	2.250	6.917
Total	5.417	4.667	2.083	4.074

BLOCK 2

Cell means and totals

Drug	1	2	3	Total
Hospital				
1	10.750	8.250	2.250	7.083
2	9.250	11.750	3.250	8.083
3	3.500	1.750	1.500	2.250
Total	7.833	7.250	2.333	4.074

BLOCK 3

Cell means and totals

Cell means a	nd totals			
Drug	1	2	3	Total
Hospital				
1	3.500	2.250	1.500	2.417
2	2.250	3.750	2.500	2.833
3	2.750	1.250	1.500	1.833
Total	2.833	2.417	1.833	4.074
Means for ea	ch variab	le		
Hospital	1	2	3	Total
	2 770	4 770	2 ((7	4 074
	3.778	4.778	3.667	4.074
Drug	1	2	3	Total
	5.361	4.778	2.083	4.074
Category	1	2	3	Total
	4.056	5.806	2.361	4.074
Block	1	2	3	Total
	4.500	4.222	3.500	4.074

Here, the main effect of factor D is partially confounded with the ABC interaction.

Analysis of Greco-Latin Squares

The file labeled "LatinGreco.tex" contains sample data for a Greco-Latin design analysis.

The specifications for the analysis are entered as:

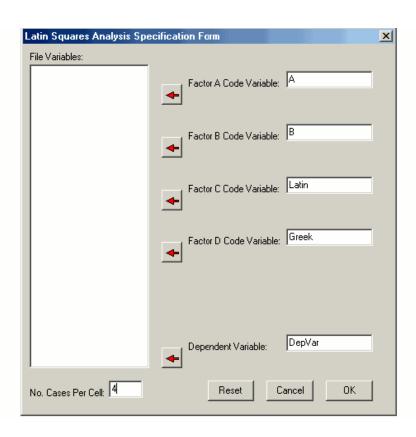


Figure 5. Latin Square Design Form

The results are obtained as:

Greco-Latin Square Analysis (No Interactions)

Source	SS	DF	MS	F	Prob.>F
Factor A	64.889	2	32.444	9.733	0.001
Factor B	64.889	2	32.444	9.733	0.001
Latin Sqr.	24.889	2	12.444	3.733	0.037
Greek Sqr.	22.222	2	11.111	3.333	0.051
Residual	_	_	-	_	_
Within	90.000	27	3.333		
Total	266.889	35			

Experimental	Design fo	or Latin	Square	
В	1 2	3	-	
A 1 2 3	C1 C2 C2 C3 C3 C1	C1	-	
Experimental	Design fo	or Greek	Square	
В	1 2	3		
A 1 2 3	C1 C2 C3 C1 C2 C3	C2	-	
Cell means a	nd totals			
В	1	2	3	Total
A 1 2 3 Total	6.000	12.000	7.000 8.000 10.000 8.333	
Means for ea	ch variab	le		
A	1	2	3	Total
	5.667	8.667 	8.333	7.556
В	1	2	3	Total
	5.667 	8.667	8.333	7.556
Latin	1 	2	3	Total
	6.667	7.333	8.667	7.556
 Greek		2	3	Total
	8.667	7.000	7.000	7.556

Notice that in the case of 3 levels that the residual degrees of freedom are 0 hence no term is shown for the residual in this example. For more than 3 levels the test of the residuals provides a partial check on the assumptions of negligible interactions. The residual is sometimes combined with the within cell variance to provide an over-all estimate of variation due to experimental error.

Plan 5 Latin Square Design

The specifications for the analysis of the sample file "LatinPlan5.tex" is shown below:

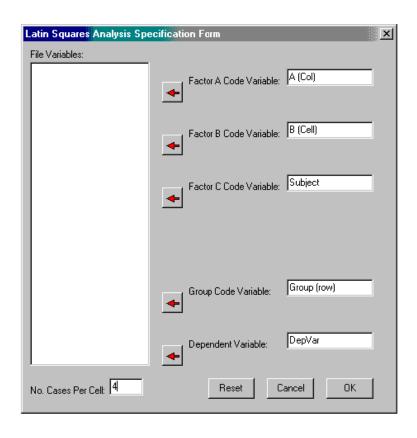


Figure 6. Latin Square Plan 5 Form

If you examine the sample file, you will notice that the subject Identification numbers (1,2,3,4) for the subjects in each group are the same even though the subjects in each group are different from group to group. The same ID is used in each group because they become "subscripts" for several arrays in the program. The results for our sample data are shown below:

Sums for ANOVA Analysis

Group (rows) times A Factor (columns) sums with 36 cases.

Vari	ables
vari	antes

	1	2	3	Total
1	14.000	19.000	18.000	51.000
2	15.000	18.000	16.000	49.000
3	14.000	21.000	18.000	53.000
Total	43.000	58.000	52.000	153.000

Group (rows) times B (cells Factor) sums with 36 cases.

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VA	r I	\sim	r) I	2.5

	1	2	3	Total
1	19.000	18.000	14.000	51.000
2	15.000	18.000	16.000	49.000
3	18.000	14.000	21.000	53.000
Total	52.000	50.000	51.000	153.000

Groups (rows) times Subjects (columns) matrix with 36 cases.

Variables	
variables	

		1	2	3	4
Total					
	1	13.000	11.000	13.000	14.000
51.000					
	2	10.000	14.000	10.000	15.000
49.000					
	3	13.000	9.000	17.000	14.000
53.000	_				
Tot	tal	36.000	34.000	40.000	43.000
153.000					

Latin Squares Repeated Analysis Plan 5 (Partial Interactions)

Source	SS	DF	MS	F	Prob.>F
Betw.Subj. Groups Subj.w.g.	20.083 0.667 19.417	11 2 9	0.333 2.157	0.155	0.859
Within Sub Factor A Factor B Factor AB	36.667 9.500 0.167 1.167	24 2 2	4.750 0.083 0.583	3.310 0.058 0.406	0.060 0.944 0.672
Error w.	25.833	18	1.435	0.400	0.072

Total 	56.7 		35 		
Experimental	Desi			Square	
A (Col)		2	3		
Group (row)					
	вЗ				
2					
3	В2 	В3 			
Cell means a	nd to	tals			
A (Col)			2	3	Total
Group (row)					
1	3.5	00	4.750	4.500	4.250
				4.000	
		\circ		4	1 117
				4.500	
				4.333	
Total Means for each	3.5 ch va 1	83 riabl 	4.833 Le	4.333	4.250 Total
Total Means for each	3.5 ch va 1 4.3	83 riabl 	4.833 Le	4.333	4.250 Total
Total Means for each	3.5 ch va 1 4.3	83 riabl 33 	4.833 	4.333	4.250 Total
Total Means for each	3.5 ch va 1 4.3 1	83 riabl 33 	4.833 Le 2 4.167	4.333 3 3 4.250	4.250 Total 4.250
Total Means for each A (Col) B (Cell) Group (row)	3.5 	83 riabl 33 50 	4.833 	4.333 3 3 3 4.250 3 4.417	4.250 Total 4.250 Total 4.250

Plan 6 Latin Squares Design

LatinPlan6.tex is the name of a sample file which you can analyze with the Plan 6 option of the Latin squares analysis procedure. Shown below is the specification form for the analysis of the data in that file:

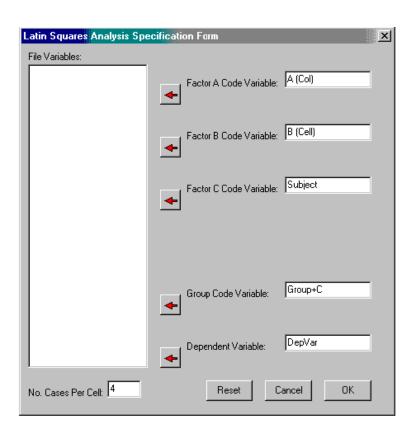


Figure 7. Latin Square Plan 6 Form

The results obtained when you click the OK button are shown below:

Latin Squares Repeated Analysis Plan 6

Sums for ANOVA Analysis

Group - C (rows) times A Factor (columns) sums with 36 cases.

Variables				
	1	2	3	Total
1	23.000	16.000	22.000	61.000
2	22.000	14.000	18.000	54.000
3	24.000	21.000	21.000	66.000
Total	69.000	51.000	61.000	181.000

Group - C (rows) times B (cells Factor) sums with 36 cases.

Variables				
	1	2	3	Total
1	16.000	22.000	23.000	61.000
2	22.000	14.000	18.000	54.000
3	21.000	24.000	21.000	66.000
Total	59.000	60.000	62.000	181.000

Group - C (rows) times Subjects (columns) matrix with 36 cases.

Variable	es				
		1	2	3	4
Total					
C1 000	1	16.000	14.000	13.000	18.000
61.000	2	12.000	13.000	14.000	15.000
54.000					
CC 000	3	18.000	19.000	11.000	18.000
66.000 Tot 181.000	al	46.000	46.000	38.000	51.000

Latin Squares Repeated Analysis Plan 6

Source	SS	DF	MS	F	Prob.>F
Betw.Subj. Factor C Subj.w.g.	26.306 6.056 20.250	11 2 9	3.028 2.250	1.346	0.308

Within Sub	70.667	24			
Factor A	13.556	2	6.778	2.259	0.133
Factor B	0.389	2	0.194	0.065	0.937
Residual	2.722	2	1.361	0.454	0.642
Error w.	54.000	18	3.000		
Total	96.972	35			

Experimental Design for Latin Square

A	(Col)	1	2	3	

С				
1	В3	В1	В2	
2	В1	В2	В3	
3	В2	В3	В1	
	1 2	1 B3 2 B1	1 B3 B1 2 B1 B2	1 B3 B1 B2 2 B1 B2 B3

Cell means and totals

A (Col)	1	2	3	Total
Group+C 1 2 3	5.750 5.500 6.000 5.750	4.000 3.500 5.250 4.250	5.500 4.500 5.250 5.083	5.083 4.500 5.500 5.028
Total	5.750	4.250	5.083	5.028

Means for each variable

A (Col)	1	2	3	Total
	4.917	5.000	5.167	5.028

B (Cell)	1	2	3	Total
	5.083	4.500	5.500	5.028

Group+C	1	2	3	Total

5.083 4.500 5.500 5.028

Plan 7 for Latin Squares

Shown below is the specification for analysis of the sample data file labeled LatinPlan7.tex and the results of the analysis:

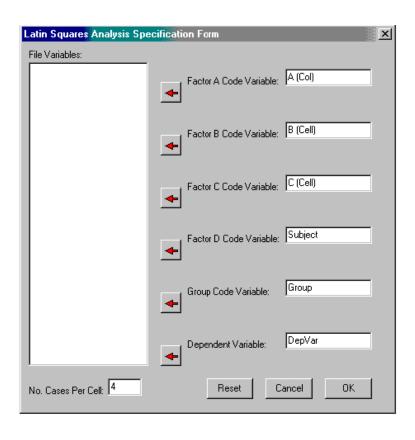


Figure 8. Latin Squares Repeated Analysis Plan 7 (Superimposed Squares)

Sums for ANOVA Analysis

Group (rows) times A Factor (columns) sums with 36 cases.

Variables				
	1	2	3	Total
1	23.000	16.000	22.000	61.000
2	22.000	14.000	18.000	54.000
3	24.000	21.000	21.000	66.000
Total	69.000	51.000	61.000	181.000

Group (rows) times B (cells Factor) sums with 36 cases.

Variables				
	1	2	3	Total
1	23.000	16.000	22.000	61.000
2	18.000	22.000	14.000	54.000
3	21.000	21.000	24.000	66.000
Total	62.000	59.000	60.000	181.000

Group (rows) times C (cells Factor) sums with 36 cases.

Variables				
	1	2	3	Total
1	23.000	22.000	16.000	61.000
2	14.000	22.000	18.000	54.000
3	21.000	21.000	24.000	66.000
Total	58.000	65.000	58.000	181.000

Group (rows) times Subjects (columns) sums with 36 cases.

Variables						
		1	2	3	4	
Total						
	1	16.000	14.000	13.000	18.000	
61.000						
	2	12.000	13.000	14.000	15.000	
54.000						
	3	18.000	19.000	11.000	18.000	
66.000						
Tot	tal	46.000	46.000	38.000	51.000	
181.000						

Latin Squares Repeated Analysis Plan 7 (superimposed squares)

Source	SS	DF	MS	F	Prob.>F
Betw.Subj. Groups Subj.w.g.	26.306 6.056 20.250	11 2 9	3.028 2.250	1.346	0.308
Within Sub Factor A Factor B Factor C residual Error w. Total	70.667 13.556 0.389 2.722 - 54.000 96.972	24 2 2 2 0 18 35	6.778 0.194 1.361 - 3.000	2.259 0.065 0.454	0.133 0.937 0.642

Experimental Design for Latin Square

 A	(Col)	1	2	3	
 	Group				
	5. \square	BC11	BC23	BC32	
	5. 🗆	BC22	BC31	BC13	

5.□ BC33 BC12 BC21

Cell means and totals

A (Col)	1	2	3	Total
Group	5.750	4.000	5.500	5.083
3 Total	5.500 6.000 5.750	3.500 5.250 4.250	4.500 5.250 5.083	4.500 5.500 5.028

Means for each variable

A (Col)	1	2	3	Total
	5.750	4.250	5.083	5.028
B (Cell)	1	2	3	Total
	5.167	4.917	5.000	5.028
C (Cell)	1	2	3	Total
	4.833	5.417	4.833	5.028

Group	1	2	3	Total
	5.083	4.500	5.500	5.028

Plan 9 Latin Squares

The sample data set labeled "LatinPlan9.tex" is used for the following analysis. The specification form shown below has the variables entered for the analysis. When you click the OK button, the results obtained are as shown following the form.

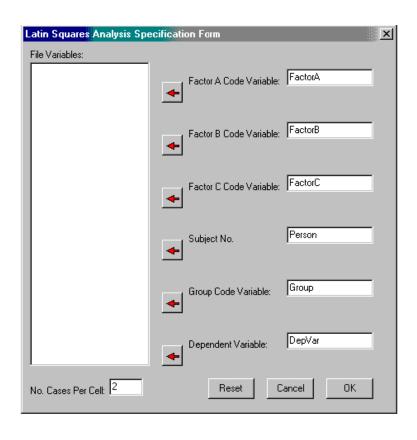


Figure 9. Latin Squares Repeated Analysis Plan 9

Sums for ANOVA Analysis

ABC matrix

\subset	1	P1791	- 1

	1	2	3
1	13.000	3.000	9.000
2	6.000	9.000	3.000
3	10.000	14.000	15.000

C level 2

	1	2	3
1	18.000	14.000	18.000
2	19.000	24.000	20.000
3	8 000	11 000	10 000

C level 3

	1	2	3
1	17.000	12.000	20.000
2	14.000	13.000	9.000
3	15.000	12.000	17.000

AB sums with 18 cases.

Variables

	1	2	3	B Total
1	48.000	29.000	47.000	124.000
2	39.000	46.000	32.000	117.000
3	33.000	37.000	42.000	112.000
Total	120.000	112.000	121.000	353.000

AC sums with 18 cases.

Variables

	1	2	3	Total
1	25.000	50.000	49.000	124.000
2	18.000	63.000	36.000	117.000
3	39.000	29.000	44.000	112.000
Total	82.000	142.000	129.000	353.000

BC sums with 18 cases.

Variables

1 2 3 Total

1 2 3 Total	29.000 26.000 27.000 82.000	45.000 49.000 48.000 142.000	46.000 37.000 46.000 129.000	120.000 112.000 121.000 353.000
RC sums with	18 cases.			
Variables 1 2 3 Total	1 16.000 37.000 29.000 82.000	2 42.000 52.000 48.000 142.000	3 36.000 47.000 46.000 129.000	Total 94.000 136.000 123.000 353.000
Group totals w	vith 18 va	lid cases.		
Variables 5	1	2	3	4
52.000	16.000	37.000	29.000	42.000
Variables Total	6	7	8	9
353.000	48.000	36.000	47.000	46.000
Subjects sums	with 18 v	alid cases.		
Variables 5	1	2	3	4
15.000	7.000	9.000	14.000	28.000
Variables 10	6	7	8	9
30.000	21.000	16.000	21.000	22.000
Variables	11	12	13	14
15 23.000	28.000	19.000	10.000	19.000
Variables	16 25.000	17 28.000	18 18.000	Total 0.000

Latin Squares Repeated Analysis Plan 9

Source	SS	DF	MS	F	Prob.>F
Betw.Subj. Factor C	267.426 110.704	17 2	55.352	5.058	0.034
Rows	51.370	2	25.685 1.713	2.347	0.151
C x row Subj.w.g.	98.500	9	10.944	0.137	0.933
Within Sub	236.000	36			
Factor A Factor B	4.037 2.704	2	2.019 1.352	0.626 0.420	0.546
Factor AC	146.519	4	36.630	11.368	0.000
Factor BC AB prime	8.519 7.148	4 2	2.130 3.574	0.661 1.109	0.627 0.351
ABC prime Error w.	9.074 58.000	4 18	2.269 3.222	0.704	0.599
Total	503.426	53			

Experimental Design for Latin Square

FactorA	1	2	3	
Group 1 2 3 4 5 6	B2 B1 B3 B2 B1 B3 B2	B3 B2 B1 B3 B2 B1 B3	B1 B3 B2 B1 B3 B2 B1	
8	В1	В2	В3	
9	В3	В1	В2	

Latin Squares Repeated Analysis Plan 9

Means for ANOVA Analysis

ABC matrix

C level 1 1 2 3	1	2	3
	6.500	1.500	4.500
	3.000	4.500	1.500
	5.000	7.000	7.500
C level 2	1	2	3
	9.000	7.000	9.000
	9.500	12.000	10.000

3	4.000	5.500 5.000		
C level 3 1 2 3	1 8.500 7.000 7.500	2 3 6.000 10.000 6.500 4.500 6.000 8.500		
AB Means with	. 54 case	s.		
Variables 1 2 3 Total	1 8.000 6.500 5.500 6.667	2 4.833 7.667 6.167 6.222	3 7.833 5.333 7.000 6.722	4 6.889 6.500 6.222 6.537
AC Means with	. 54 case	s.		
Variables 1 2 3 Total	1 4.167 3.000 6.500 4.556	2 8.333 10.500 4.833 7.889	3 8.167 6.000 7.333 7.167	4 6.889 6.500 6.222 6.537
BC Means with	. 54 case	s.		
Variables 1 2 3 Total	1 4.833 4.333 4.500 4.556	2 7.500 8.167 8.000 7.889	3 7.667 6.167 7.667 7.167	4 6.667 6.222 6.722 6.537
RC Means with	. 54 case	s.		
Variables 1 2 3	1 2.667 6.167 4.833	2 7.000 8.667 8.000	3 6.000 7.833 7.667	4 5.222 7.556 6.833

Total	4.556	7.889	7.167	6.537
Group Means w	ith 54 val	id cases.		
Variables 5	1	2	3	4
8.667	2.667	6.167	4.833	7.000
Variables Total	6	7	8	9
6.537	8.000	6.000	7.833	7.667
Subjects Mean	s with 54	valid cases.		
Variables 5	1	2	3	4
7.500	3.500	4.500	7.000	14.000
Variables 10	6	7	8	9
15.000	10.500	8.000	10.500	11.000
Variables	11	12	13	14
15 11.500	14.000	9.500	5.000	9.500
Variables	16 12.500	17 14.000	18 9.000	Total 6.537