

$$(2n-1)\frac{\pi}{2} < \beta l < n\pi$$

$$(2n-1)\frac{\lambda}{4} < l < n\frac{\lambda}{2} \leftarrow \text{for this length it will act like a capacitor.}$$

for open circuited line.

$$Z_{oc} = Z_0 \coth \gamma l.$$

$$Z_{oc} = jZ_0 \cot \beta l.$$

$$(2n-1)\frac{\pi}{2} < \beta l < n\pi$$

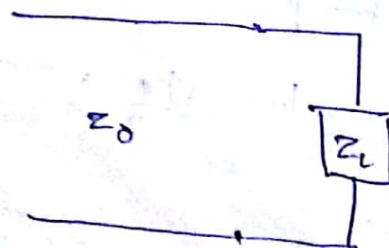
$$(2n-1)\frac{\lambda}{4} < l < n\frac{\lambda}{2} \leftarrow \text{Acts as inductor.}$$

also;

$$(n-1)\pi < \beta l < (2n-1)\frac{\pi}{2}$$

$$(n-1)\frac{\lambda}{2} < l < (2n-1)\frac{\lambda}{4} \leftarrow \text{Acts like capacitor.}$$

Q. The characteristic impedance of a transmission line working at 300 MHz is 50Ω and is connected to a load impedance of $(50 + j50)\Omega$. Calculate reflection coefficient and VSWR.



$$Z_0 = 50\Omega$$

$$Z_L = 50 + j50\Omega$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_L = \frac{50 + j50 - 50}{50 + j50 + 50}$$

$$\Gamma_L = \frac{j50}{100 + j50}$$

$$= 0.2 + j0.4$$

$$= 0.445 \angle 63^\circ$$

$$V_{SWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.445}{1 - 0.445}$$

$$= \frac{1.445}{0.555} = 2.64$$

$$Z_{in} = Z_0 \left\{ \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \right\}$$

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\gamma = \alpha + j\beta = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$v_p = \frac{\omega}{\beta}$$

$$\Gamma = \frac{V_{ref}}{V_{inc}} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$V_{SWR} = \frac{V_{max}}{V_{min}} = \frac{|V_1| + |V_2|}{|V_1| - |V_2|} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$\text{Return loss} \rightarrow R.L = -20 \log |\Gamma|$$

for loss less lines

$$(R=0 ; G=0)$$

$$Z_{in} = Z_0 \left\{ \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right\}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$\gamma = j\omega\sqrt{LC} \quad ; \quad \alpha = 0 ; \quad \beta = \omega\sqrt{LC}$$

$$v_p = \frac{1}{\sqrt{LC}}$$

$$Z_{sc} = jZ_0 \tan \beta l \quad ; \quad Z_{oc} = -jZ_0 \cot \beta l$$

Q. A 50Ω lossless transmission line has a phase constant of 4 rad/m , at a frequency of 10 MHz . Calculate the inductance & capacitance of the line.

$$\rightarrow \beta = 4 \text{ rad/m} \quad ; \quad Z_0 = 50 \Omega$$

$$f = 10 \text{ MHz} \quad \omega = 2\pi f = 20\pi \times 10^6 \text{ rad/s}$$

$$\begin{array}{l} \beta = \omega\sqrt{LC} \\ \Rightarrow 4 = 20\pi \times 10^6 \sqrt{LC} \\ \Rightarrow LC = \left(\frac{1}{5\pi \times 10^6} \right)^2 \\ \Rightarrow 2500 C^2 = \left(\frac{1}{5\pi \times 10^6} \right)^2 \\ \Rightarrow 50 C = \frac{1}{5\pi \times 10^6} \\ C = 1.27 \text{ nF} \end{array} \quad \left| \quad \begin{array}{l} Z_0 = \sqrt{\frac{L}{C}} \\ \Rightarrow 50 = \sqrt{\frac{L}{C}} \\ \Rightarrow 2500 = \frac{L}{C} \\ \Rightarrow L = 2500 C \\ L = 3.2 \mu\text{H} \end{array} \right.$$

Q. A lossless transmission line has a series inductance of 0.4 mH/km and capacitance of 0.2 nF/km . Calculate characteristic impedance & phase velocity.

$$\Rightarrow Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.4 \times 10^{-3}}{0.2 \times 10^{-9}}} = \sqrt{2 \times 10^6} = 1.414 \times 10^3 \Omega$$

$$\begin{aligned} V_p &= \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.4 \times 10^{-3} \times 0.2 \times 10^{-9}}} \\ &= \frac{1}{\sqrt{0.08 \times 10^{-12}}} \\ &= \frac{1}{\sqrt{8 \times 10^{-20}}} \\ &= \frac{10^{10}}{2.82} \\ &= 0.35 \times 10^{10} \text{ m/sec.} \end{aligned}$$

Q. A 30Ω transmission lossless T.L is connected to a load impedance of $(30 + j30) \Omega$. What is the input impedance of the transmission line at a point $\lambda/8$ away from the load.

$$\rightarrow Z_0 = 30 \Omega ; Z_L = 30 + j30 \Omega$$

$$Z_{in} = Z_0 \left\{ \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right\}$$

$$= Z_0 \left\{ \frac{Z_L + jZ_0 \tan \left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{8} \right)}{Z_0 + jZ_L \tan \left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{8} \right)} \right\}$$

$$= Z_0 \left(\frac{Z_L + jZ_0}{Z_0 + jZ_L} \right) = 30 \left\{ \frac{30 + j60}{30 + j30} \right\} = 160 - j30 \Omega$$

Q. The s.c & o.c impedances of T.L are $80 \angle -20^\circ$ and $20 \angle -30^\circ$ respectively. Determine the characteristic impedance of transmission.

$$Z_{sc} = 80 \angle -20^\circ ; Z_{oc} = 20 \angle -30^\circ$$

$$Z_{sc} \cdot Z_{oc} = Z_0^2$$

$$\Rightarrow 1600 \angle -50^\circ = Z_0^2$$

$$\Rightarrow Z_0 = 40 \angle -25^\circ \Omega$$

Q. A 40Ω T.L is terminated with a load impedance of $j25 \Omega$ and the line is operating at 150 MHz . Calculate the input impedance of the line at a distance of 50 cm away from the load. (lossless).

$$\rightarrow Z_0 = 40 \Omega ; Z_L = j25 \Omega$$

$$f = 150 \text{ MHz} \Rightarrow \omega = 300\pi \times 10^6 \text{ rad/sec.}$$

$$l = 50 \text{ cm} = 0.05 \text{ m.}$$

$$= \lambda/4$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{150 \times 10^6} = 2 \text{ m}$$

$$Z_{in} = Z_0 \left\{ \frac{Z_L + Z_0 \tan \beta l}{Z_0 + Z_L \tan \beta l} \right\}$$

$$= 40 \left\{ \frac{j25 + 40 \tan \left(\frac{2\pi}{2} \cdot \frac{1}{4} \right)}{40 + j25 \tan \left(\frac{2\pi}{2} \cdot \frac{1}{4} \right)} \right\}$$

$$= 40 \left\{ \frac{40^2}{j25} \right\} = -80$$

$$= 40 \times \frac{40}{j25}$$

$$= \frac{1600}{j25} = -64j \Omega$$

Q. A purely resistive load $Z_L = 60 \Omega$.

minimum & maximum voltages on T.L are $6 \mu V$ and $9 \mu V$ determine the load impedance

$$V_{\max} = 9 \mu V$$

$$\frac{9}{\max} V_{\min} = 6 \mu V$$

$$V_{\min}/V_{\max} = \frac{9}{6} = 1.5$$

$$V_{\min}/V_{\max} = \frac{1 + |r|}{1 - |r|}$$

$$\Rightarrow 1.5 (1 - |r|) = 1 + |r|$$

$$\Rightarrow 1.5 - 1.5|r| = 1 + |r|$$

$$\Rightarrow 0.5|r| = 0.5$$

$$\therefore |r| = 0.2$$

$$r = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Rightarrow 0.2 = \frac{Z_L - 60}{Z_L + 60}$$

$$\Rightarrow 0.2 Z_L + 12 = Z_L - 60$$

$$\Rightarrow 0.8 Z_L = 72$$

$$Z_L = 90 \Omega$$

Q. A distortionless line has $Z_0 = 70 \Omega$; $\alpha = 15 \text{ mNeper/m}$ and $v_p = 0.5 \times c$. Find the primary constants of transmission line.

$$\rightarrow Z_0 = 70 \Rightarrow \sqrt{\frac{L}{C}} = 70 \Rightarrow \frac{L}{C} = 4900$$

$$\alpha = 15 \text{ mNeper/m} \Rightarrow R \sqrt{\frac{C}{L}} = 15$$

$$\Rightarrow R^2 \cdot \frac{C}{L} = 15^2$$

$$\Rightarrow R^2 = 15 \times 4900$$

$$\Rightarrow R = \sqrt{15 \times 4900} = 105 \sqrt{15} \Omega = 105 \times 3.87 \Omega = 406.25 \Omega$$

$$= 406.25 \Omega$$

$$L = 4900 \times 950 \times 10^{-12} = 0.46 \mu\text{H/m}$$

$$\Rightarrow LC = 0.46 \times 10^{-16}$$

$$\Rightarrow 4900 C^2 = 0.46 \times 10^{-16}$$

$$C^2 = 9.07 \times 10^{-21} = 90.7 \times 10^{-22} \Rightarrow C = 9.52 \times 10^{-11} = 95.2 \text{ pF/m}$$

Q The attenuation of a 50Ω distortionless line is 0.01 dB/m . The line has a capacitance of 0.1 pF/m .
 i) Find primary constants.

ii) Find velocity of propagation

iii) Determine γ to which the amplitude the travelling wave decreases in 1 km & 5 km .

→
 i)

$$Z_0 = 50 \Omega \quad \alpha = 0.01 \text{ dB/m} \\ = 1.0152 \times 10^{-3} \text{ Np/m}$$

$$C = 0.1 \text{ pF/m}$$

$$\alpha = R \sqrt{\frac{C}{L}}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$\Rightarrow (1.0152 \times 10^{-3})^2 = R^2 \cdot \frac{0.1 \times 10^{-12}}{2.5 \times 10^{-10}} \Rightarrow (50)^2 = \frac{L}{0.1 \times 10^{-12}} \\ \Rightarrow L = 2500 \times 0.1 \times 10^{-12}$$

$$\Rightarrow 1.0327 \times 10^{-6} = R^2 \times \frac{0.1 \times 10^{-12}}{2.5} = 0.25 \text{ nH}$$

$$\Rightarrow R^2 = \sqrt{1.0327 \times 2.5 \times 10^{-3}}$$

$$= \sqrt{3.3175 \times 10^{-3}}$$

$$= 0.57 \times 10^{-2} \Omega$$

$$= 0.057 \Omega$$

$$v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25 \times 10^{-9} \times 0.1 \times 10^{-12}}}$$

$$= \frac{1}{\sqrt{25 \times 10^{-24}}}$$

$$= 0.2 \times 10^{11} \cdot 0.2 \times 10^{12}$$

$$= 2 \times 10^{11}$$

iii)

$$\frac{R}{L} = \frac{G}{C}$$

$$\Rightarrow \cancel{G} = \cancel{G}$$

We have,

for amplitude $\rightarrow V_1 e^{-\gamma l}$

where $\gamma = (\alpha + j\beta)$

amplitude

phase

$$\therefore V_1 e^{-\alpha l}$$

αl

for 1 km: $\alpha l = 1.152 \times 10^{-3} \times 1000$
 $= 1.152$

$$\therefore V_1 e^{-\alpha l} = 0.316 V_1$$

$$\begin{aligned} \% \text{ age decrease} &= \frac{V_1 - V_1 e^{-\alpha l}}{V_1} \times 100 \\ &= \frac{0.684 V_1}{V_1} \times 100 \\ &= 68.4\% \end{aligned}$$

for 5 km: $\alpha l = 5.76$

$$\therefore V_1 e^{-\alpha l} = 3.15 \times 10^{-3} V_1$$

$$\begin{aligned} \% \text{ age decrease} &= \frac{V_1 - V_1 e^{-\alpha l}}{V_1} \times 100 \\ &= \frac{99.68 V_1}{V_1} \times 100 \\ &= 99.68\% \end{aligned}$$

Q. Calculate the maximum and minimum impedances from a transmission line and their locations for $Z_L = 100$ and $Z_0 = 50$.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50}{150} = \frac{1}{3}$$

$$Z_{\max} = Z_0 (VSWR)$$

$$= Z_0 \cdot \left\{ \frac{1 + |\Gamma|}{1 - |\Gamma|} \right\}$$

$$= 50 \left\{ \frac{1 + \frac{1}{3}}{1 - \frac{1}{3}} \right\}$$

$$= 50 \cdot \frac{4}{2} = 100$$

$$Z_{\min} = \frac{Z_0}{VSWR} = \frac{50}{2} = 25$$