This is the softmax workbook for ECE C147/C247 Assignment #2

Please follow the notebook linearly to implement a softmax classifier.

Please print out the workbook entirely when completed.

The goal of this workbook is to give you experience with training a softmax classifier.

```python
import random
import numpy as np
from utils.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

%matplotlib inline
%load_ext autoreload
%autoreload 2
```

```
The autoreload extension is already loaded. To reload it, use:
  %reload_ext autoreload
```

```python
#Downloading the CIFAR-10 data
import shutil
import urllib.request
urllib.request.urlretrieve(
    "http://www.cs.toronto.edu/~kriz/cifar-10-python.tar.gz",
"utils/datasets/cifar-10-python.tar.gz")

#Unzipping the downloaded file
shutil.unpack_archive(
    "utils/datasets/cifar-10-python.tar.gz", "utils/datasets/")

def get_CIFAR10_data(num_training=49000, num_validation=1000,
num_test=1000, num_dev=500):
    """
    Load the CIFAR-10 dataset from disk and perform preprocessing to
prepare
    it for the linear classifier. These are the same steps as we used
for the
    SVM, but condensed to a single function.
    """
    # Load the raw CIFAR-10 data
    cifar10_dir = 'utils/datasets/cifar-10-batches-py'  # You need to
update this line
    X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

    # subsample the data
```

```
    mask = list(range(num_training, num_training + num_validation))
    X_val = X_train[mask]
    y_val = y_train[mask]
    mask = list(range(num_training))
    X_train = X_train[mask]
    y_train = y_train[mask]
    mask = list(range(num_test))
    X_test = X_test[mask]
    y_test = y_test[mask]
    mask = np.random.choice(num_training, num_dev, replace=False)
    X_dev = X_train[mask]
    y_dev = y_train[mask]

    # Preprocessing: reshape the image data into rows
    X_train = np.reshape(X_train, (X_train.shape[0], -1))
    X_val = np.reshape(X_val, (X_val.shape[0], -1))
    X_test = np.reshape(X_test, (X_test.shape[0], -1))
    X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

    # Normalize the data: subtract the mean image
    mean_image = np.mean(X_train, axis = 0)
    X_train -= mean_image
    X_val -= mean_image
    X_test -= mean_image
    X_dev -= mean_image

    # add bias dimension and transform into columns
    X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
    X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
    X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
    X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])

    return X_train, y_train, X_val, y_val, X_test, y_test, X_dev,
y_dev


# Invoke the above function to get our data.
X_train, y_train, X_val, y_val, X_test, y_test, X_dev, y_dev =
get_CIFAR10_data()
print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
print('dev data shape: ', X_dev.shape)
print('dev labels shape: ', y_dev.shape)
```

```
Train data shape:  (49000, 3073)
Train labels shape:  (49000,)
```

```
Validation data shape:  (1000, 3073)
Validation labels shape:  (1000,)
Test data shape:  (1000, 3073)
Test labels shape:  (1000,)
dev data shape:  (500, 3073)
dev labels shape:  (500,)
```

Training a softmax classifier.

The following cells will take you through building a softmax classifier. You will implement its loss function, then subsequently train it with gradient descent. Finally, you will choose the learning rate of gradient descent to optimize its classification performance.

```
from nndl import Softmax

import subprocess

# List of pip commands

# Declare an instance of the Softmax class.
# Weights are initialized to a random value.
# Note, to keep people's first solutions consistent, we are going to
use a random seed.

np.random.seed(1)

num_classes = len(np.unique(y_train))
num_features = X_train.shape[1]

softmax = Softmax(dims=[num_classes, num_features])
```

Softmax loss

```
## Implement the loss function of the softmax using a for loop over
#  the number of examples

loss = softmax.loss(X_train, y_train)

print(loss)

2.3277607028048966
```

Question:

You'll notice the loss returned by the softmax is about 2.3 (if implemented correctly). Why does this make sense?

Answer:

Intially, no training is done on the Softmax. Thus, as the weights are initalized to zero, the value of the loss according to the softmax is: -1/m *log((1/10)m) = log(10), which is close to 2.3

Softmax gradient

```
## Calculate the gradient of the softmax loss in the Softmax class.
# For convenience, we'll write one function that computes the loss
#   and gradient together, softmax.loss_and_grad(X, y)
# You may copy and paste your loss code from softmax.loss() here, and then
#   use the appropriate intermediate values to calculate the gradient.

loss, grad = softmax.loss_and_grad(X_dev,y_dev)

# Compare your gradient to a gradient check we wrote.
# You should see relative gradient errors on the order of 1e-07 or less if you implemented the gradient correctly.
softmax.grad_check_sparse(X_dev, y_dev, grad)
```

```
numerical: -0.659919 analytic: -0.659919, relative error: 4.811308e-08
numerical: -0.887144 analytic: -0.887144, relative error: 1.872054e-08
numerical: 0.593711 analytic: 0.593711, relative error: 1.212518e-08
numerical: 2.179519 analytic: 2.179519, relative error: 3.471660e-10
numerical: -0.557324 analytic: -0.557324, relative error: 1.079517e-07
numerical: 1.998294 analytic: 1.998293, relative error: 1.228015e-08
numerical: -0.522391 analytic: -0.522392, relative error: 1.038401e-07
numerical: -1.286610 analytic: -1.286610, relative error: 2.583998e-09
numerical: 0.518052 analytic: 0.518052, relative error: 9.485669e-09
numerical: -4.808219 analytic: -4.808220, relative error: 1.449431e-08
```

A vectorized version of Softmax

To speed things up, we will vectorize the loss and gradient calculations. This will be helpful for stochastic gradient descent.

```
import time

## Implement softmax.fast_loss_and_grad which calculates the loss and gradient
#    WITHOUT using any for loops.

# Standard loss and gradient
tic = time.time()
loss, grad = softmax.loss_and_grad(X_dev, y_dev)
toc = time.time()
print('Normal loss / grad_norm: {} / {} computed in {}s'.format(loss, np.linalg.norm(grad, 'fro'), toc - tic))
```

```
tic = time.time()
loss_vectorized, grad_vectorized = softmax.fast_loss_and_grad(X_dev,
y_dev)
toc = time.time()
print('Vectorized loss / grad: {} / {} computed in
{}s'.format(loss_vectorized, np.linalg.norm(grad_vectorized, 'fro'),
toc - tic))

# The losses should match but your vectorized implementation should be
much faster.
print('difference in loss / grad: {} /{} '.format(loss -
loss_vectorized, np.linalg.norm(grad - grad_vectorized)))

# You should notice a speedup with the same output.
```

```
Normal loss / grad_norm: 2.329781476362285 / 400.2127530552219
computed in 0.07850503921508789s
Vectorized loss / grad: 2.329781476362286 / 400.212753055222 computed
in 0.024672985076904297s
difference in loss / grad: -1.3322676295501878e-15
/3.705124159971763e-13
```

Stochastic gradient descent

We now implement stochastic gradient descent. This uses the same principles of gradient descent we discussed in class, however, it calculates the gradient by only using examples from a subset of the training set (so each gradient calculation is faster).

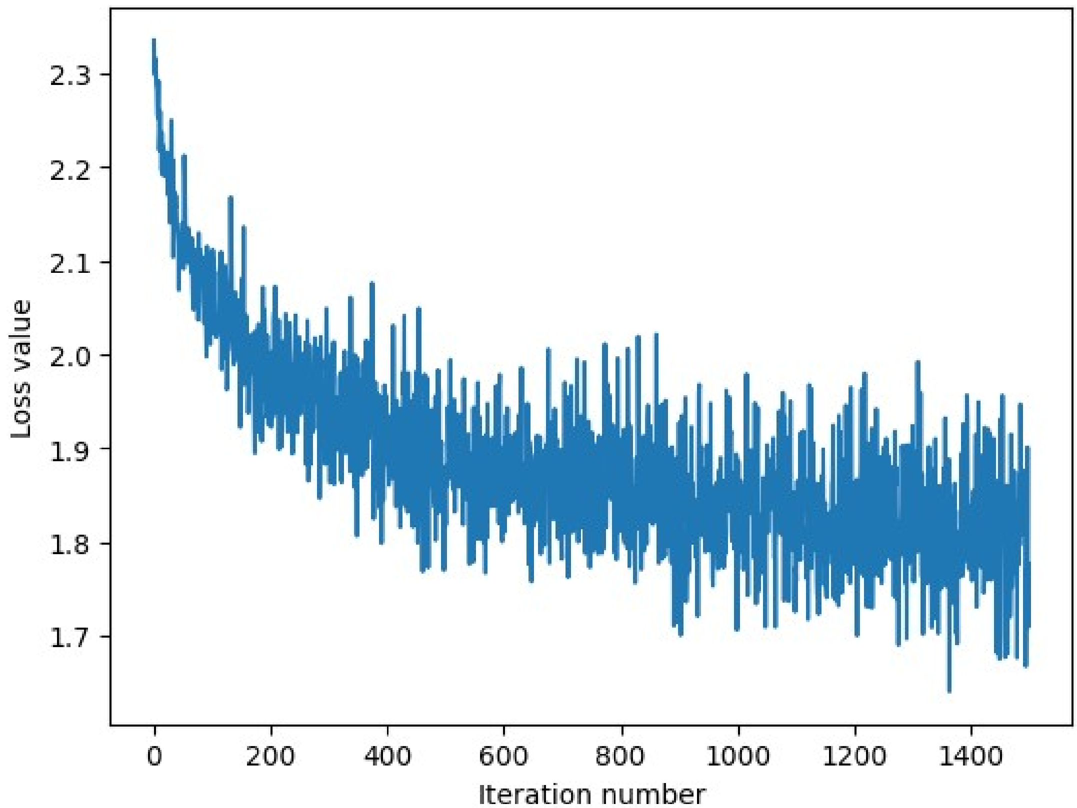
```
# Implement softmax.train() by filling in the code to extract a batch
of data
# and perform the gradient step.
import time


tic = time.time()
loss_hist = softmax.train(X_train, y_train, learning_rate=1e-7,
                      num_iters=1500, verbose=True)
toc = time.time()
print('That took {}s'.format(toc - tic))

plt.plot(loss_hist)
plt.xlabel('Iteration number')
plt.ylabel('Loss value')
plt.show()
```

```
iteration 0 / 1500: loss 2.3353835450891545
iteration 100 / 1500: loss 2.0225093946317187
iteration 200 / 1500: loss 1.982172871654982
iteration 300 / 1500: loss 1.9356442081331486
```

```
iteration 400 / 1500: loss 1.882893396815689
iteration 500 / 1500: loss 1.81818696973945
iteration 600 / 1500: loss 1.874513153185746
iteration 700 / 1500: loss 1.8361832500173585
iteration 800 / 1500: loss 1.8584086819212184
iteration 900 / 1500: loss 1.9275087067564147
iteration 1000 / 1500: loss 1.824667969507725
iteration 1100 / 1500: loss 1.7731817984393607
iteration 1200 / 1500: loss 1.8636308568113118
iteration 1300 / 1500: loss 1.924074621260815
iteration 1400 / 1500: loss 1.7846918635831293
That took 9.59458303451538s
```



Evaluate the performance of the trained softmax classifier on the validation data.

```
## Implement softmax.predict() and use it to compute the training and
testing error.

y_train_pred = softmax.predict(X_train)
print('training accuracy:
{}'.format(np.mean(np.equal(y_train,y_train_pred), )))
y_val_pred = softmax.predict(X_val)
```

```
print('validation accuracy: {}'.format(np.mean(np.equal(y_val,
y_val_pred)), ))
```

```
training accuracy: 0.37881632653061226
validation accuracy: 0.39
```

Optimize the softmax classifier

```
np.finfo(float).eps
```

```
2.220446049250313e-16
```

```
# ================================================================ #
# YOUR CODE HERE:
#   Train the Softmax classifier with different learning rates and
#     evaluate on the validation data.
#   Report:
#     - The best learning rate of the ones you tested.
#     - The best validation accuracy corresponding to the best
validation error.
#
#   Select the SVM that achieved the best validation error and report
#     its error rate on the test set.
# ================================================================ #

learning_rates = np.linspace(2, -8, 12)
learning_rates = 10**learning_rates
print(learning_rates)

num_iters = 1500
num_rates = len(learning_rates)
loss_histories = np.zeros((num_rates, num_iters), dtype=float)
validation_accuracies = np.zeros(num_rates, dtype=float)
for i in range(num_rates):
    loss_histories[i] = softmax.train(X_train, y_train,
learning_rate=learning_rates[i],
                                      num_iters=num_iters,
verbose=True)
    y_pred = softmax.predict(X_val)
    accuracy = np.mean(y_pred == y_val)
    validation_accuracies[i] = accuracy

best_lr = learning_rates[np.argmax(validation_accuracies)]
best_accuracy =
validation_accuracies[np.argmax(validation_accuracies)]

print("Best learning rate:", best_lr)
print("Best validation accuracy:", best_accuracy)
# ================================================================ #
```

```
# END YOUR CODE HERE
# ================================================================ #
```

```
[1.00000000e+02 1.23284674e+01 1.51991108e+00 1.87381742e-01
 2.31012970e-02 2.84803587e-03 3.51119173e-04 4.32876128e-05
 5.33669923e-06 6.57933225e-07 8.11130831e-08 1.00000000e-08]
iteration 0 / 1500: loss 2.37685010419801l
iteration 100 / 1500: loss 21930450.818899762
iteration 200 / 1500: loss 18002661.772628788
iteration 300 / 1500: loss 30139112.424423013
iteration 400 / 1500: loss 35643494.85798577
iteration 500 / 1500: loss 22668817.58273412
iteration 600 / 1500: loss 41879530.25533271
iteration 700 / 1500: loss 23980976.744694404
iteration 800 / 1500: loss 28425963.735318568
iteration 900 / 1500: loss 41653045.50052511
iteration 1000 / 1500: loss 42389915.23338503
iteration 1100 / 1500: loss 20859953.49689545
iteration 1200 / 1500: loss 22777862.354245503
iteration 1300 / 1500: loss 21338356.136914648
iteration 1400 / 1500: loss 38965792.72549838
iteration 0 / 1500: loss 2.431368148995201
iteration 100 / 1500: loss 3911024.0104619255
iteration 200 / 1500: loss 3483064.3656539526
iteration 300 / 1500: loss 3829290.333834709
iteration 400 / 1500: loss 4001500.0194998938
iteration 500 / 1500: loss 2473336.2546379687
iteration 600 / 1500: loss 3306166.6987504843
iteration 700 / 1500: loss 3536105.661674255
iteration 800 / 1500: loss 2981740.0236274083
iteration 900 / 1500: loss 3172550.679629564
iteration 1000 / 1500: loss 4485552.570294583
iteration 1100 / 1500: loss 2403146.447038425
iteration 1200 / 1500: loss 2428078.177848376
iteration 1300 / 1500: loss 3443958.519265981
iteration 1400 / 1500: loss 3076566.152687497
iteration 0 / 1500: loss 2.363535760198207
iteration 100 / 1500: loss 372500.94385254284
iteration 200 / 1500: loss 367745.3050547632
iteration 300 / 1500: loss 266145.98215907655
iteration 400 / 1500: loss 386647.6504217968
iteration 500 / 1500: loss 389080.4494084273
iteration 600 / 1500: loss 314276.8420210867
iteration 700 / 1500: loss 266822.1915628476
iteration 800 / 1500: loss 424908.9585048904
iteration 900 / 1500: loss 347603.45680869895
iteration 1000 / 1500: loss 415194.3586194626
iteration 1100 / 1500: loss 284708.4698438062
iteration 1200 / 1500: loss 299317.73308567784
iteration 1300 / 1500: loss 336885.17984189413
```

```
iteration 1400 / 1500: loss 550591.4480033843
iteration 0 / 1500: loss 2.3577965172090742
iteration 100 / 1500: loss 49415.40619365274
iteration 200 / 1500: loss 48094.422779138775
iteration 300 / 1500: loss 62282.17075980758
iteration 400 / 1500: loss 55219.77597413651
iteration 500 / 1500: loss 79037.59562536365
iteration 600 / 1500: loss 46623.959253629255
iteration 700 / 1500: loss 55592.77071718095
iteration 800 / 1500: loss 50293.75325395955
iteration 900 / 1500: loss 45938.17900252937
iteration 1000 / 1500: loss 60126.71120511817
iteration 1100 / 1500: loss 52526.98692114738
iteration 1200 / 1500: loss 71199.01830421321
iteration 1300 / 1500: loss 47138.68953778818
iteration 1400 / 1500: loss 83020.0362768881
iteration 0 / 1500: loss 2.3623163529832842
iteration 100 / 1500: loss 7386.785167363726
iteration 200 / 1500: loss 6561.724714169041
iteration 300 / 1500: loss 7762.457975035249
iteration 400 / 1500: loss 8185.593338401697
iteration 500 / 1500: loss 9241.982105076282
iteration 600 / 1500: loss 6448.578867873589
iteration 700 / 1500: loss 5490.309019938016
iteration 800 / 1500: loss 5302.935330988444
iteration 900 / 1500: loss 6600.164089494958
iteration 1000 / 1500: loss 4467.617866491232
iteration 1100 / 1500: loss 7304.733435829171
iteration 1200 / 1500: loss 9017.892633791746
iteration 1300 / 1500: loss 6068.499578394603
iteration 1400 / 1500: loss 5582.4341722882955
iteration 0 / 1500: loss 2.3303211245338518
iteration 100 / 1500: loss 702.8104745719538
iteration 200 / 1500: loss 1003.0492399680344
iteration 300 / 1500: loss 700.1187822421518
iteration 400 / 1500: loss 835.1644800620383
iteration 500 / 1500: loss 446.30354572286905
iteration 600 / 1500: loss 982.7065184736753
iteration 700 / 1500: loss 879.9026852273258
iteration 800 / 1500: loss 822.1161610487652
iteration 900 / 1500: loss 579.046723437298
iteration 1000 / 1500: loss 536.6872540150157
iteration 1100 / 1500: loss 703.7149140578282
iteration 1200 / 1500: loss 756.385763502355
iteration 1300 / 1500: loss 648.743959299979
iteration 1400 / 1500: loss 568.723610575774
iteration 0 / 1500: loss 2.3819157186171527
iteration 100 / 1500: loss 101.90366538590519
iteration 200 / 1500: loss 118.06337378241717
```

```
iteration 300 / 1500: loss 77.94405186044338
iteration 400 / 1500: loss 103.57086415363067
iteration 500 / 1500: loss 88.07708962144505
iteration 600 / 1500: loss 87.00524664953944
iteration 700 / 1500: loss 99.1464793458803
iteration 800 / 1500: loss 80.50596718681074
iteration 900 / 1500: loss 54.873063256993994
iteration 1000 / 1500: loss 138.13711973359645
iteration 1100 / 1500: loss 120.9919651195003
iteration 1200 / 1500: loss 92.57520058567
iteration 1300 / 1500: loss 133.3349194177065
iteration 1400 / 1500: loss 81.83726018936659
iteration 0 / 1500: loss 2.3484318532800312
iteration 100 / 1500: loss 12.182852376071834
iteration 200 / 1500: loss 12.087503451124547
iteration 300 / 1500: loss 10.615130409607861
iteration 400 / 1500: loss 13.84461942104537
iteration 500 / 1500: loss 12.571806312922758
iteration 600 / 1500: loss 6.984199121128224
iteration 700 / 1500: loss 8.64487613420592
iteration 800 / 1500: loss 14.502744343807787
iteration 900 / 1500: loss 10.463870556146347
iteration 1000 / 1500: loss 13.839418943398323
iteration 1100 / 1500: loss 8.439131732079757
iteration 1200 / 1500: loss 25.471756400015657
iteration 1300 / 1500: loss 11.112198046857
iteration 1400 / 1500: loss 12.513600477833668
iteration 0 / 1500: loss 2.3884800630134353
iteration 100 / 1500: loss 1.9244324754685642
iteration 200 / 1500: loss 1.8307907909717744
iteration 300 / 1500: loss 1.9401241351781335
iteration 400 / 1500: loss 1.9390054291197396
iteration 500 / 1500: loss 1.7642694060698612
iteration 600 / 1500: loss 1.914377770811749
iteration 700 / 1500: loss 2.140990947900194
iteration 800 / 1500: loss 1.823280960376048
iteration 900 / 1500: loss 1.7823081429383212
iteration 1000 / 1500: loss 1.8361834433051791
iteration 1100 / 1500: loss 1.7918254575079688
iteration 1200 / 1500: loss 1.6720707159338417
iteration 1300 / 1500: loss 1.67405276230121
iteration 1400 / 1500: loss 1.8196460558278313
iteration 0 / 1500: loss 2.3738551189467474
iteration 100 / 1500: loss 1.8800757751651815
iteration 200 / 1500: loss 1.9032012169003765
iteration 300 / 1500: loss 1.7552064783210315
iteration 400 / 1500: loss 1.8057427708923495
iteration 500 / 1500: loss 1.703297149344325
iteration 600 / 1500: loss 1.8106431097810396
```

```
iteration 700 / 1500: loss 1.781909808759197
iteration 800 / 1500: loss 1.8347189009757465
iteration 900 / 1500: loss 1.772411828877675
iteration 1000 / 1500: loss 1.7426963229988734
iteration 1100 / 1500: loss 1.7939978026703687
iteration 1200 / 1500: loss 1.7390890428035715
iteration 1300 / 1500: loss 1.6237130713164825
iteration 1400 / 1500: loss 1.6319798366216436
iteration 0 / 1500: loss 2.381247622289375
iteration 100 / 1500: loss 2.0597882886583454
iteration 200 / 1500: loss 2.0907885620978175
iteration 300 / 1500: loss 1.908021469908436
iteration 400 / 1500: loss 1.891647808828347
iteration 500 / 1500: loss 1.8553358364243135
iteration 600 / 1500: loss 2.018174773065773
iteration 700 / 1500: loss 1.9029340789229914
iteration 800 / 1500: loss 1.8551084631237507
iteration 900 / 1500: loss 1.9528490745867333
iteration 1000 / 1500: loss 1.8609197826141204
iteration 1100 / 1500: loss 1.8487548977230108
iteration 1200 / 1500: loss 1.8788884653758466
iteration 1300 / 1500: loss 1.876206304543427
iteration 1400 / 1500: loss 1.8559469827187676
iteration 0 / 1500: loss 2.472523652438493
iteration 100 / 1500: loss 2.310952064158016
iteration 200 / 1500: loss 2.261801262801085
iteration 300 / 1500: loss 2.172240024995832
iteration 400 / 1500: loss 2.2023515733714847
iteration 500 / 1500: loss 2.170122715607117
iteration 600 / 1500: loss 2.147718560680545
iteration 700 / 1500: loss 2.145091568186487
iteration 800 / 1500: loss 2.1314891893644052
iteration 900 / 1500: loss 2.130507158028264
iteration 1000 / 1500: loss 2.0524036308637954
iteration 1100 / 1500: loss 2.054309380022291
iteration 1200 / 1500: loss 2.1269059271129307
iteration 1300 / 1500: loss 2.0956204583447713
iteration 1400 / 1500: loss 2.0905403988265276
Best learning rate: 6.579332246575682e-07
Best validation accuracy: 0.395
```

According to the output above, the best validation accuracy is when the learning rate is around 6.579332246575682e-07 out of 12 different learning rates. The best validation accuracy that was achieved for this learning rate = 0.395