Types of Matrix ;

Zero matrix = $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times 3}$

Identity matrix = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{\substack{3\times 3 \\ r\times c}}$

Symmetric matrix = $\begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 6 \\ -1 & 6 & 5 \end{bmatrix}$

Diagonal matrix = $\begin{bmatrix} 6 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

upper Triangular matrix = $\begin{bmatrix} 6 & -1 & 5 \\ 0 & 4 & 2 \\ 0 & 0 & 2 \end{bmatrix}$

Lower Triangular matrix = $\begin{bmatrix} 2 & 0 & 0 \\ 6 & 4 & 0 \\ -1 & 5 & 2 \end{bmatrix}$

Inverse of Matrix = $AA^{-1} = A^{-1}A = I$ (Identity matrix)

Note: Determinant of square matrix should not be zero in inverse or else inverse doesn't exist.

Inverse is always for square matrix.

Transpose of matrix = for matrix A, transpose is "A^T"

$$A \neq A^T$$

$$A = \begin{bmatrix} 6 & 2 & 1 \\ 3 & 5 & 9 \\ 7 & 4 & 8 \end{bmatrix} ; \quad A^T = \begin{bmatrix} 6 & 3 & 7 \\ 2 & 5 & 4 \\ 1 & 9 & 8 \end{bmatrix}$$

If matrix is Symmetric & square then

"$A = A^T$"

Determinants

$$A = \begin{bmatrix} m & n & o & p \\ q & r & s & t \\ u & v & w & x \\ y & z & a & b \end{bmatrix}_{4\times4} \Rightarrow \text{Matrix}$$

$$|A| = \begin{vmatrix} m & n & o & p \\ q & r & s & T \\ u & v & w & x \\ y & z & a & b \end{vmatrix}_{4\times4} \Rightarrow \text{Determinant}$$

$$\cancel{= m \begin{vmatrix} r & s & T \\ u & v & w \end{vmatrix}}$$

$\boxed{\cancel{\text{First method}}}$

$$= m \begin{vmatrix} r & s & t \\ v & w & x \\ z & a & b \end{vmatrix} - n \begin{vmatrix} q & s & T \\ v & w & x \\ y & a & b \end{vmatrix} + o \begin{vmatrix} q & r & t \\ u & v & x \\ y & z & b \end{vmatrix} - p \begin{vmatrix} q & r & s \\ u & v & w \\ y & z & a \end{vmatrix} = |A|$$

~~Second method~~

Finding determinant of matrix;

$$\left.\begin{aligned} A &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2} \\ |A| &= \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \\ |A| &= a_{11}\cdot a_{22} - a_{21}\cdot a_{12} \end{aligned}\right\} \text{for } 2\times 2 \text{ matrix}$$

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}_{3\times 3}$$

$$|A| = a_{11}\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12}\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13}\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

Find Inverse of matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}$$

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

method 1

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or

$$A^{-1} = \frac{\text{adj}(A)}{\det(A)} = \cancel{\text{adj}}\ \cancel{\text{adj}} \begin{bmatrix} -\frac{33}{36} & \frac{12}{36} & \frac{3}{36} \\ -6/36 & 12/36 & -6/36 \\ 27/36 & -12/36 & 3/36 \end{bmatrix}$$

→ Transpose of minors matrix

$$\text{adj}(A) = \det\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 2 & 9 \end{vmatrix} = -36$$

$\therefore M_{1,1} = 33$ $\quad M_{3,1} = -3$

$\therefore M_{1,2} = -6$ $\quad M_{3,2} = -6$

$\therefore M_{1,3} = -27$ $\quad M_{3,3} = -3$

$\therefore M_{2,1} = 12$

$M_{2,2} = -12$

$M_{2,3} = -12$

$$\therefore \begin{bmatrix} 33 & -6 & -27 \\ 12 & -12 & -12 \\ -3 & -6 & -3 \end{bmatrix}$$

Transpose this and result is adj(A)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Inverse is $\Rightarrow$

$$A^{-1} = \frac{1}{|A|} \begin{vmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{12} \\ a_{33} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ \begin{vmatrix} a_{23} & a_{21} \\ a_{33} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{11} \\ a_{23} & a_{21} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{11} \\ a_{32} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{vmatrix}$$

or

$A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$, Let $A = IA$

$$\therefore \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$\therefore R_1 \to (1/2) R_1$,

$$\begin{bmatrix} 1 & 1/2 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} A$$

$R_2 \to R_2 - 7R_1;$

$$\begin{bmatrix} 1 & 1/2 \\ 0 & 1/2 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ -7/2 & 1 \end{bmatrix} A$$

$R_2 \to 2R_2;$

$$\begin{bmatrix} 1 & 1/2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ -7 & 2 \end{bmatrix} A$$

$R_1 \to R_1 - (1/2)R_2;$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} A$$

$\therefore \; I = A^{-1}A$

$\therefore \; A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$

Cofactor $= C_{ij} = (-1)^{i+j} M_{ij}$

M_{ij} = Minor, Transpose means rows converted to Columns.

Properties:

$\lambda(A+B) = \lambda A + \lambda B$

$\lambda A = A \lambda$

$AB \neq BA$

$(AB)C = A(BC)$

$AI_n = A = I_n A$

for Every A ($|A| \neq 0$), there exists a B

where $AB = I_n = BA$, i.e. $B = A^{-1}$ or $A = B^{-1}$.

$A'' = A = A^{TT}$

$(AB)' = A'B'$

if $A' = A$, it is Symmetric

if $A' = -A$, it is skew Symmetric

$|A| = |A'|$

if two rows or columns are equal $|A| = 0$