

CP322 Assignment 2

Part 1

Part 1

a. Given dataset

x	t
-1	1
1	0
3	1

why this isn't linearly separable



0(.)
1(.)

b. Apply the feature map

$$\psi(x) = \begin{pmatrix} \psi(x) \\ \psi(x) \end{pmatrix} = \begin{pmatrix} x \\ x^2 \end{pmatrix}$$

x	t	transformed
-1	1	$(-1, 1)$
1	0	$(1, 1)$
3	1	$(3, 9)$

Constraints on w_1 and w_2

The linear classifier in the transformed space:

$$w_1 x + w_2 x^2 = 0$$

The classification rule:

if $w_1 x + w_2 x^2 \geq 0$, classify as 1

if $w_1 x + w_2 x^2 < 0$, classify as 0

Point $(-1, 1)$, class 1.

$$\begin{aligned} w_1(-1) + w_2(1) &\geq 0 \\ -w_1 + w_2 &\geq 0 \end{aligned}$$

Point $(1, 1)$, class 0

$$\begin{aligned} w_1(1) + w_2(1) &< 0 \\ w_1 + w_2 &< 0 \end{aligned}$$

Point $(3, 9)$, class 1.

$$\begin{aligned} w_1(3) + w_2(9) &\geq 0 \\ 3w_1 + 9w_2 &\geq 0 \\ w_1 + 3w_2 &\geq 0 \end{aligned}$$

We can now use system of equations to find the values of w_1 and w_2 where they meet the constraints.

$$\textcircled{1} \quad w_2 \geq w_1$$

$$\textcircled{2} \quad w_1 + w_2 < 0$$

$$\textcircled{3} \quad w_1 + 3w_2 \geq 0$$

$$w_2 \geq w_1$$

$$w_1 + w_2 < 0$$

Substitute $w_2 = w_1$ into $w_1 + w_2 < 0$

$$w_1 + w_1 < 0$$

$$2w_1 < 0$$

$$w_1 < 0 \quad (w_1 \text{ must be a negative number})$$

We can also say $w_2 < -w_1$ using $\textcircled{2}$

$$\text{So} \quad w_1 \leq w_2 < -w_1$$

$$w_1 + 3w_2 \geq 0 \quad \textcircled{3}$$

$$\text{Since } w_1 = w_2$$

$$w_1 + 3w_1 \geq 0$$

$$4w_1 \geq 0$$

Since we know from above w_1 must be negative we can't say $w_1 = w_2$ since we will get a sum < 0 .

$$w_1 + 3w_2 \geq 0$$

$$w_1 \geq -3w_2$$

We need to find some w_1 and w_2 that meet these conditions

$$w_1 < 0$$

$$w_1 \geq -3w_2$$

$$w_1 \leq w_2 < -w_1$$

$$\text{let } w_1 = -1$$

$$\frac{-1}{-3} \geq \frac{-3w_2}{-3}$$

$$\frac{1}{3} \geq w_2$$

$$\frac{1}{3} = w_2$$

$$w_1 \leq w_2 < -w_1$$

$$-1 \leq \frac{1}{3} < 1$$

\therefore all 3 conditions are satisfied.

So a possible pair of (w_1, w_2)

$$\text{is } \underline{\underline{(-1, 1/3)}}$$

Part 2

i, Weights, z-value, y-value, loss-value

0.

Weights: [-0.60630875 1.99369125 1.19760444 -2.81125415]

Z-value: [-0.6 0.6 1.4 -0.2],

Y-value[0.35434369 0.64565631 0.80218389 0.450166],

Loss_value:[0.43748795 0.43748795 0.22041741 0.59813887]

1.

Weights: [-0.61231903 1.98759507 1.19542105 -2.82234598]

Z-value: [-0.60630875 0.59129569 1.38738251 -0.2262672],

Y-value[0.35290168 0.64366238 0.80017405 0.44367331],

Loss_value:[0.43525704 0.44058094 0.22292602 0.58639958]

2.

Weights: [-0.61804304 1.98170385 1.19344109 -2.83328143]

Z-value: [-0.61231903 0.58310202 1.37527604 -0.2516489],

Y-value[0.35153038 0.64178087 0.79823124 0.43741769],

Loss_value:[0.43314012 0.44350836 0.22535695 0.57521783]

3.

Weights: [-0.62349252 1.97601023 1.19165612 -2.84406619]

Z-value: [-0.61804304 0.57539805 1.36366081 -0.27617953],

Y-value[0.35022666 0.64000781 0.79635403 0.43139066],

Loss_value:[0.43113169 0.4462749 0.22771143 0.56456166]

4.

Weights: [-0.62867881 1.97050711 1.19005805 -2.85470579]

Z-value: [-0.62349252 0.56816361 1.35251771 -0.29989236],

Y-value[0.34898755 0.63833933 0.79454094 0.4255838],

Loss_value:[0.42922651 0.44888527 0.22999077 0.55440105]

5.

Weights: [-0.63361283 1.96518763 1.18863904 -2.86520551]

Z-value: [-0.62867881 0.56137924 1.3418283 -0.32281944],

Y-value[0.34781017 0.63677161 0.79279044 0.41998878],

Loss_value:[0.42741962 0.45134423 0.23219635 0.54470784]

6.

Weights: [-0.63830511 1.96004517 1.18739158 -2.87557044]

Z-value: [-0.63361283 0.5550262 1.3315748 -0.34499167],

Y-value[0.34669179 0.63530092 0.79110101 0.41459745],

Loss_value:[0.42570627 0.4536565 0.23432963 0.53545555]

7.

Weights: [-0.64276577 1.95507335 1.18630844 -2.88580549]
Z-value: [-0.63830511 0.54908647 1.32174006 -0.36643881],
Y-value [0.34562977 0.63392362 0.78947106 0.40940181],
Loss_value: [0.42408199 0.45582681 0.2363921 0.52661937]

8.

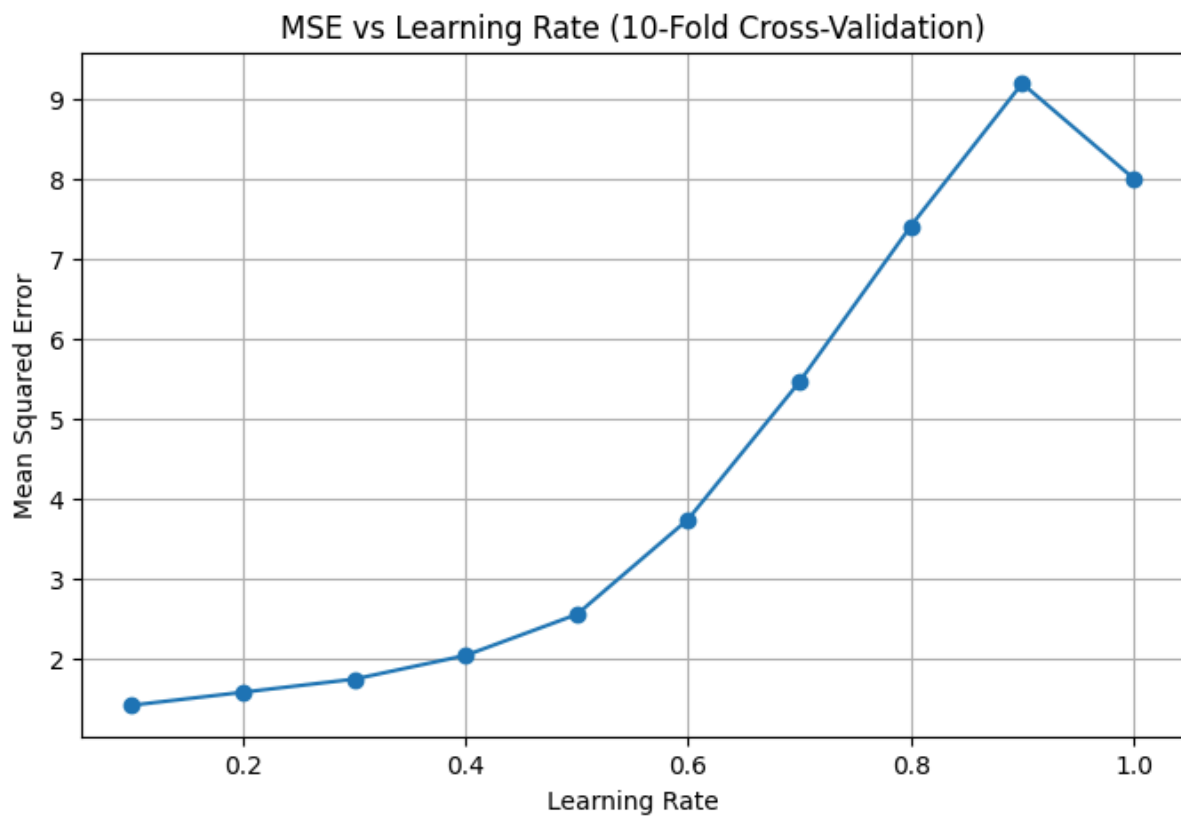
Weights: [-0.64700454 1.95026602 1.18538269 -2.89591534]
Z-value: [-0.64276577 0.54354267 1.31230758 -0.38718947],
Y-value [0.3446216 0.63263615 0.78789904 0.40439406],
Loss_value: [0.4225425 0.45785983 0.23838532 0.51817601]

9.

Weights: [-0.65103079 1.94561727 1.18460764 -2.9059045]
Z-value: [-0.64700454 0.53837815 1.30326148 -0.40727117],
Y-value [0.34366487 0.63143505 0.78638337 0.39956662],
Loss_value: [0.42108376 0.45976019 0.24031085 0.51010359]

Bolded is the answer after 10 iterations.

Part 3



Best Learning Rate: 0.1

Cross-Validation MSE: 1.413572106992406

Test MSE: 1.5318262703219885