Math 61

UCLA Department of Mathematics

Fall 2024

Instructor: Oleg Gleizer

## Homework 2

- Circle or box your answers.
- Show work! Answers unsupported by work yield zero credit.
- No partial credit is given for the extra credit problem. It's all or nothing.

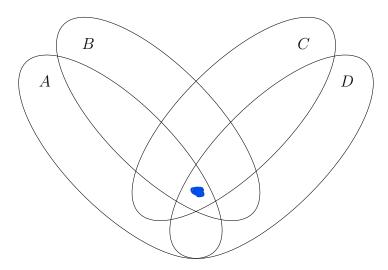
Problem 1 10 pts

Prove PIE for three sets, A, B, and C.

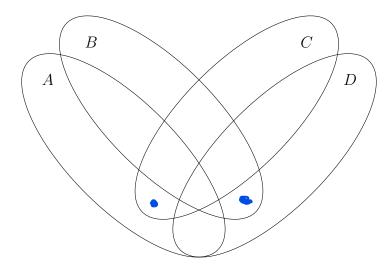
Problem 2 10 pts

The first sub-problem is worth 1 pt, the second 2 pts, the third 3 pts, the fourth 4 pts. The Venn diagrams below show intersections of four sets, A, B, C, and D.

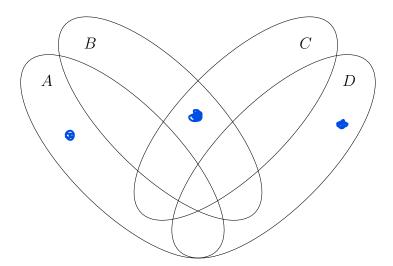
• Place one point on the diagram so that each of the four sets contains one point.



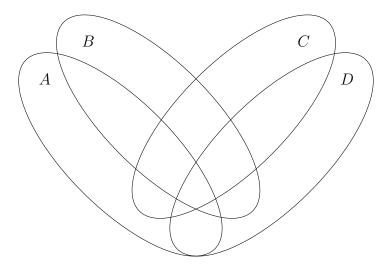
• Place two points on the diagram so that each of the four sets contains one point.



• Place three points on the diagram so that each of the four sets contains one point.



• Place four point on the diagram so that each of the four sets contains one point.



Problem 3 10 pts

Consider a chess board with two opposite squares, say the black ones, sawed off. Is it possible to cover all the remaining squares with dominoes equal in size to precisely two chess board squares so that the dominoes neither overlap nor stick out of the board? Why or why not?

Problem 4 10 pts

Show that  $\aleph_0 + \aleph_0 = \aleph_0$ .

Problem 5

10 pts

R is the set of all the sets that do not have themselves as elements:

$$R = \{S : S \not\in S\}$$

Is R an element of R? Why or why not?

Problem 6

!!!extra credit!!!

5 pts

Let  $\aleph_1 = 2^{\aleph_0}$ . Show that  $|\mathbb{R}| = \aleph_1$ .