Subject: - Engineering Mathematics I (SH 401)

✓ Candidates are required to give their answers in their own words as far as practicable.
✓ Attempt **All** questions.
✓ **All** questions carry equal marks.
✓ Assume suitable data if necessary.
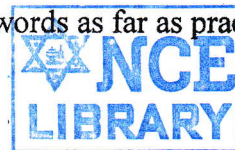
1. State Leibnitz's theorem. If $y = a \cos (\log x) + b \sin (\log x)$ then show that $x^2 y_{n+2} + (2n +1) xy_{n+1} x+ (n^2 +1) y_n = 0$.

2. Apply Maclaurin's series to find the expansion of $e^x \sec x$ as far as the term in x^3.

3. State L'Hopital's rule. Using it evaluate $\lim\limits_{x \to 0} \left(\dfrac{\sin x}{x} \right)^{\frac{1}{x^2}}$

4. Find the asymptotes of the curve
   $(x + y)^2 (x + 2y + 2) = x + 9y - 2$

5. Show that for the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, the radius of curvature at the extremity of the major axis is equal to half of the Latus rectum.

6. Integrate: $\displaystyle\int_0^{\frac{\pi}{2}} \dfrac{\cos x \, dx}{(1 + \sin x)(2 + \sin x)}$

7. Apply the rule of differentiation under integral sign to evaluate: $\displaystyle\int_0^\infty \dfrac{e^{-ax} \sin x}{x} dx$ and hence deduce that $\displaystyle\int_0^\infty \dfrac{\sin x}{x} dx = \dfrac{\pi}{2}$

8. Define Beta and Gamma functions. Evaluate: $\displaystyle\int_0^a x^4 \sqrt{a^2 - x^2} \, dx$

9. Show that the area of the astroid $x^{2/3} - y^{2/3} = a^{2/3}$ is $\dfrac{3\pi a^2}{8}$

   OR

   Find the volume of the solid of revolution of the cardoid $r = a(1+\cos\theta)$ about the initial line.

10. Solve: $x\dfrac{dy}{dx} + 2y = x^2 \log x$

11. Solve: $y = yp^2 + 2px$ where $p = \dfrac{dy}{dx}$

12. Solve: $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = e^{2x} \sin x$

13. Solve: $x\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 2\dfrac{y}{x} = \dfrac{1}{x^2}$

14. Derive the standard equation of an ellipse.

15. Through what angle should the axes be rotated to reduce the equation $3x^2 + 2xy + 3y^2 - \sqrt{2} \, x = 0$ in to one with the xy term missing?

16. Find the center, length of the axes and eccentricity of the conic
    $9x^2 + 4xy + 6y^2 - 22x - 16y + 9 = 0$

    OR

    Describe and sketch the graph of the equation $r = \dfrac{10}{3+2\cos\theta}$

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