INSTITUTE OF ENGINEERING
Examination Control Division
2069 Ashad

Level	BE	Full Marks	80
Programme	All(Except B. Arch.)	Pass Marks	32
Year / Part	I / I	Time	3 hrs.

Subject: - Engineering Mathematics *(SH 401)*

✓ Candidates are required to give their answers in their own words as far as practicable.
✓ Attempt **All** questions.
✓ **All** questions carry equal marks.
✓ Assume suitable data if necessary.

1. If $y = (x^2-1)^n$, prove that $(x^2-1)y_{n+2}+2xy_{n+1}-n(n+1)y_n = 0$.

2. State and prove Lagrange's mean value theorem and verify $f(x) = \log x$, $x \in [i, e]$.

3. Evaluate $\displaystyle \lim_{x \to 0}\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$.

4. Find the asymptotes of the curve $x(x-y)^2 - 3(x^2-y^2) + 8y = 0$.

5. Find the tangent at (a, b) to the curve $\left(\dfrac{x}{a}\right)^3 + \left(\dfrac{y}{b}\right)^3 = 2$.

6. Evaluate $\displaystyle \int_\infty^\alpha \frac{dx}{x^3}$.

7. Use Gamma function to prove $\displaystyle \int_0^\pi \sin^6 \frac{x}{2} \cos^6 \frac{x}{2}\, dx = \frac{5\pi}{2^{11}}$.

8. Use method of differentiation under integral sign, evaluate $\displaystyle \int_0^\alpha \frac{\tan^{-1}(ax)}{x(1+x^2)}\, dx$.

9. Find the area between the curve and its asymptotes $y^2(a-x) = x^3$.

OR

Find the volume of the ellipscoid formed by the revolution of an ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

10. Transform the equation $3x^2 - 2xy + 4y^2 + 8x - 10y + 8 = 0$ by translating the axis into an equation with linear term missing.

11. Find the equation of ellipse whose centre is origin and whose axis are the axis of coordinates and passes through the pair of curves $(1, 6)$ and $(2, 3)$.

12. Prove that the product of the semi axis of conic $5x^2 + 6xy + 5y^2 + 12x + 4y - 4 = 0$ is 3.

13. Solve the differential equation $x\, dy - y\, dx = \sqrt{x^2 + y^2}\, dx$.

14. Find the general solution of the differential equation $xy^2(p^2+2) = 2py^3 + x^3$

15. Find the general solution of the differential equation $(x^2 D^2 + 4xD + 2)y = e^x$.

16. A tank contains 1000 liters of fresh water. Salt water which contains 150gms of salt per liter, runs into it at the rate of 5 liter per minute and well-stirred mixture runs out of it at the same rate. When will the tank contain 5000gms of salt?

OR

Solve $\dfrac{d^2 y}{dx^2} - y = x^2 \cos x$.

***