TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division
2067 Ashadh

I-62	BE	Full Marks	80
Programme	All (Except B.Arch.)	Pass Marks	32
Year / Part	I / I	Time	3 hrs.

Subject: - Engineering Mathematics I

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt All questions.*
- ✓ *The figures in the margin indicate Full Marks.*
- ✓ *Assume suitable data if necessary.*

1. If $y = e^{a\tan^{-1}x}$, prove that $(1+x^2)y_{n+2} + (2nx + 2x - a)y_{n+1} + n(n+1)y_n = 0$.
2. State and prove Lagrange's mean value theorem.
3. Evaluate $\lim_{x \to 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{x}}$
4. Find the asymptotes of the curve $(x+y)^2(x+2y+z) = x + 9y - 2$.
5. Find the radius of curvature of the curve $r = a(1 - \cos\theta)$.
6. Apply the method of differentiation under integral sign to evaluate $\int_0^\infty \frac{\tan^{-1}(ax)}{x(1+x^2)}dx$.
7. Prove that $\int_0^{\pi/2} \frac{\sin^2 x dx}{\sin x + \cos x} = \frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$.
8. Use Gamma function to prove $\int_0^{\pi/6} \cos^4 3\theta . \sin^2 6\theta = \frac{5\pi}{192}$.
9. Find, by method of integration, the area of the loop of the curve $ay^2 = x^2(a - x)$.
10. Solve the differential equation $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$.
11. Solve $y = yp^2 + 2px$, where $p = dy/dx$
12. Solve $(D^2 - 3D + 2)y = x^2 + x$
13. Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference of temperature between the object and its surroundings. Supposing water at 100°C cools to 80°C in 10 minutes, in a room temperature of 30°C, find when the temperature of water will become 40°C?

OR

Solve the differential equation $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$.

14. Find the condition that the line $lx + my + n = 0$ may be the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
15. Derive the equation of a hyperbola in standard form.
16. Find the centre, length of axes and eccentricity of the conic $2x^2 + 3y^2 - 4x - 12y + 13 = 0$.

OR

Identify and sketch the conic $r = \frac{10}{3 + 2\cos\theta}$.

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