

Exam.	Back		
Level	BE	Full Marks	80
Programme	ALL (Except B. Arch)	Pass Marks	32
Year / Part	I / I	Time	3 hrs.

Subject: - Engineering Mathematics I (SH401)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

- State Leibnitz's theorem. If $y^{\frac{1}{m}} + y^{\frac{-1}{m}} = 2x$, show that $(x^2 - 1)y_2 + xy_1 - m^2y = 0$ and hence prove that $(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$. [2+3]
- State Roller's theorem. Does the theorem hold when the function is not continuous at the end points? Justify your answer. Verify the theorem for $f(x) = x^2 - 4x + 3$ on $[1, 3]$. [5]
- State L-Hospital's theorem and evaluate $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x^2}}$ [5]
- Find the asymptotes of curve $4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 - 1 = 0$ [5]
- Find the pedal equation of the curve $y^2 = 4c(x + c)$ [5]
- Show that $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx = \frac{1}{\sqrt{2}} \log(\sqrt{2} + 1)$ [5]
- Evaluate, by using differentiation under the sign of integration $\int_0^{\infty} \frac{\log(1 + a^2x^2)}{1 + b^2x^2} dx$ [5]
- Define Beta-Gamma function and use it to evaluate $\int_0^{\frac{\pi}{6}} \cos^4 3\theta \cdot \sin^2 6\theta \cdot d\theta$ [5]
- Find the surface area of the solid generated by the revolution of the cardioids $r = a(1 + \cos\theta)$ about the initial line. [5]
- Transform the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + 2 = 0$ by translating the axes into an equation with linear term missing. [5]
- Derive the standard equation of hyperbola. [5]
- Find the centre, Length of axes and eccentricity of the conic $9x^2 + 4xy + 6y^2 - 22x - 16y + 9 = 0$ [5]

OR

Describe and sketch the graph of the equation $r = \frac{12 \sec \theta}{2 + 3 \sec \theta}$

- Solve $\frac{dy}{dx} + \frac{\sin 2y}{x} = x^3 \cos^2 y$ [5]
- Solve the differential equation of $xp^2 - 2yp + ax = 0$ [5]
- Solve $(D^2 - 1)y = \sinh(x)$ [5]
- $(x^2 D^2 + xD + 1)y = \sin(\log x^2)$ [5]