CONTENT

40+

APPLIED MECHANICS

Bachelor of Engineering

CHAPTER 1, 2, 3 Basic Concept of Statics, Static Equilibrium & Force Acting on Structures

DEFINITIONS

Force
Any action of a body on another which tends to change the state of rest or of motion of another body is called force.

System of force
When more than one force acts on a body at a particular instant they are said to constitute a system of forces.

Coplanar forces
If the forces of system of forces all lie on the same plane, they are said to be coplanar forces.

Non-coplanar or spatial forces
When all the forces lie on different planes is called non–coplanar or spatial forces.

Concurrent forces
When the lines of action of all the forces intersect at a point is called concurrent forces.

Non-concurrent forces
When the lines of action of all the forces do not intersect at a point is called non–concurrent forces.

Resultant forces
A single equivalent forces which produces the same effect on the body as that of all given forces is called resultant of force.

Rectangular component of a force
The rectangular components of a force are obtained by drawing lines parallel to the axes or by projecting the force onto x and y axes as shown in the figure.

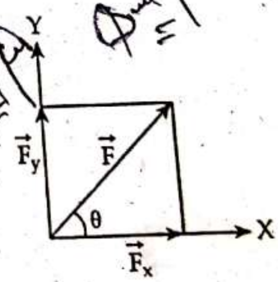
$$\vec{F} = \vec{F}_x + \vec{F}_y$$

$$\vec{F}_x = F\cos\theta$$

$$\vec{F}_y = F\sin\theta$$

$$|\vec{F}| = F = \sqrt{F_x^2 + F_y^2}$$

$$\theta = \tan^{-1}\left[\frac{F_y}{F_x}\right]$$



The principle of transmissibility
It states that the conditions of equilibrium or motion of a rigid body remains unchanged if a force acting at a given point of the rigid body is replaced by a force of the same magnitude and direction, but acting at a different point, provided that the two forces have the same line of action.

EXAM SOLUTION

1. What are the main characteristics of couple? [2061 Baishakh]

Solution:

The main characteristics of couple are;

i) Two forces must be involved in formulation of couple.
ii) Forces must be of same magnitude in couple.
iii) Forces must have parallel line of application and must be opposite to each other.
iv) Couple must rotate the body about the fixed axis either in clockwise or anticlockwise.
v) The algebraic sum of the moments of the forces, constructing the couple, about any point is the same, and equal to the moment of the couple itself.
vi) A couple is balanced by another couple but of opposite sense.
vii) Any number of coplanar couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.

2. Define moment of a force and couple. A p is directed from point A (2, 2, 5) m toward a point B (– 4, 5, – 2) m. If it causes a moment about z-axis, M_z = 2 kNm. Determine the moment of p about x-axis and y-axis. [2062 Poush]

Solution:

Refer definition part for moment of force and couple.

The position vector of given points A and B are;

$$\overrightarrow{OA} = 2\hat{i} + 2\hat{j} + 5\hat{k}$$

$$\overrightarrow{OB} = -4\hat{i} + 5\hat{j} - 2\hat{k}$$

Now,

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$= (-4\hat{i} + 5\hat{j} - 2\hat{k}) - (2\hat{i} + 2\hat{j} + 5\hat{k})$$

$$= -6\hat{i} + 3\hat{j} - 7\hat{k}$$

Now, unit vector of $\overrightarrow{AB}$;

$$\hat{n} = \frac{-6\hat{i} + 3\hat{j} - 7\hat{k}}{\sqrt{(-6)^2 + (3)^2 + (-7)^2}} = -0.619\,\hat{i} + 0.31\,\hat{j} - 0.72\,\hat{k}$$

... vector from as;

$$= (-1.44 - 1.55)p\,\hat{i} - (-1.44 + 3.095)p\,\hat{j} + (0.62 + 1.238)\,p\,\hat{k}$$
$$= -2.99p\,\hat{i} - 1.655\,p\,\hat{j} + 1.858\,p\hat{k}$$

Now, moment along z–axis is;

$$\vec{M_0}.\hat{k} = 1.858\text{ p} \qquad (1)$$

According to question; we have,

$$\vec{M_0}.\hat{k} = 2$$

so, equation (1) becomes;

$$1.858\text{ p} = 2\text{ kNm}$$

or, $\text{p} = 1.076\text{ kN}$

Moment about x and y axes are;

$$\vec{M_0}.\vec{i} = -2.99 \times 1.076 = -3.22\text{ kNm}$$
$$\vec{M_0}.\vec{j} = -1.655 \times 1.076 = -1.78\text{ kNm}$$

3. **What is a free–body diagram? Discuss about the principle of equilibrium of rigid bodies.** **[2064 Jestha, 2061 Baishakh]**

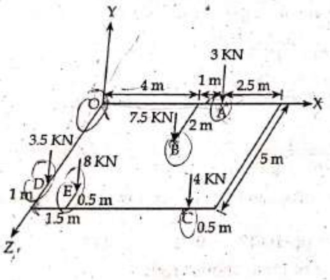
Solution:

A free- body diagram is a picture of physical situation to be analyzed which depicts all the relevant forces acting on the object of interest. A practical definition of the free–body diagram may be written in simple form as,

$$\boxed{\text{Free-body diagram}} = \boxed{\text{Space diagram}} - \boxed{\text{Supports and reactions}} + \boxed{\text{Reaction}}$$

The principle of equilibrium states that, a stationary body which is subjected to coplanar forces will be in equilibrium if the algebraic sum of all the external forces and moment are zero.

4. **A rectangular slab 5 m × 7.5 m sports five columns which exert force on the slab the forces as indicated in figure below. Determine the magnitude and point of application of the resultant of the forces.** **[2064 Jestha]**



Solution:

Co-ordinate axes are chosen as shown in the figure.

Co-ordinate of O = (0, 0, 0)
Co-ordinate of A = (5, 0, 0)
Co-ordinate of B = (4, 0, 2)
Co-ordinate of C = (7, 0, 5)

Co-ordinate of D = (1.5, 0, 4.5)
Co-ordinate of E = (0, 0, 4)

Resultant force, $\vec{F_R} = [-3\hat{j} - 7.5\,\hat{j} - 4\,\hat{j} - 8\,\hat{j} - 3.5\,\hat{j}] = -26\,\hat{j}$ kN

Let the resultant of force passes through point (x, y, z). Then,

Position vector, $\vec{r} = (x\,\vec{i} + y\,\hat{j} + z\,\hat{k})$ m

Taking moment about origin 'O'; we get,

Moment due to resultant force = sum of moment due to individual forces

i.e., $\vec{r} \times \vec{F} = \vec{r_1} \times \vec{F_1} + \vec{r_2} \times \vec{F_2} + \vec{r_3} \times \vec{F_3} + \vec{r_4} \times \vec{F_4} + \vec{r_5} \times \vec{F_5}$

or, $(x\,\hat{i} + y\,\hat{j} + z\,\hat{k}) \times (-26\hat{j}) = (5\hat{i} \times (-3\,\hat{j}) + (4\hat{i} + 2\,\hat{k}) \times (-7.5\hat{j})$
$\quad + (7\,\hat{i} + 5\hat{k}) \times (-4\hat{j}) + 4\,\hat{k} \times (-3.5\,\hat{j}) + (1.5\,\hat{i} + 4.5\hat{k}) \times (-8\hat{j})$

or, $-26\,x\hat{k} + 26z\hat{i} = -15\,\hat{k} - 30\hat{k} + 15\hat{i} + (-28\,\hat{k} + 20\hat{i}) + 14\,\hat{i} - 12\hat{k} + 36\hat{i}$

or, $-26\,x\hat{k} + 26z\hat{i} = 85\,\hat{i} + 85\hat{k}$

Hence equating coefficient of $\hat{i}$ and $\hat{k}$; we get,

$$26z = 85$$

$\therefore \quad z = 3.3$ m

Again,

$$-26x = -85$$

$\therefore \quad x = 3.3$ m

Hence, the resultant forces posses through the point (3.3, 0, 3.3) m.

5. **State the principle of transmissibility of forces and also discus equivalent forces with sketch.** **[2064 Jestha, 2061 Baishakh]**

Solution:

Refer definition for principle of transmissibility of forces.

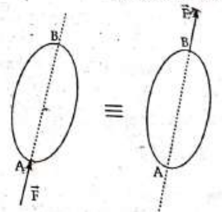
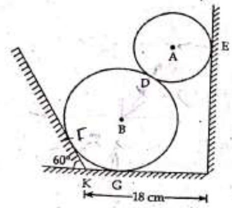


Figure: Equivalent forces

The effect of force $\vec{F}$ acting at 'A' will be same as the effect of $\vec{F}$ acting at 'B' on rigid body and are said to be equivalent force.

6. **Two cylinders 'A' and 'B' rest in a channel as shown in figure below 'A' has a diameter of 10 cm and weight 20 kg. 'B' has diameter 18 cm and weight 50 kg. The channel is 18 cm wide at the bottom with one side vertical and other side at 120° as shown. Determine the reactions at four contact points.** **[2064 Falgun]**



Solution:

Given that;

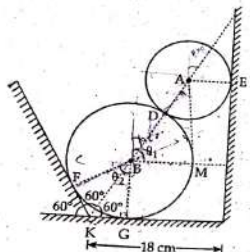


Figure: (a) Construction to be applied

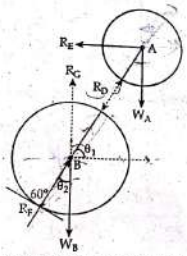


Figure: (b) Free body diagram

Diameter of A, $(D_A) = 10$ cm

so, Radius of A $(r_A) = 5$ cm

Diameter of B $(D_B) = 18$ cm

so, Radius of B $(r_B) = 9$ cm

Weight of A = 20 kg

Weight of B = 50 kg

From figure (a) in right angled ΔBKG; we have,

$\angle KBG = 30°$

Since, from ΔKBG; we have,

$$\tan 30° = \frac{KG}{BG} = \frac{KG}{9}$$

$\therefore$ KG = 5.2 cm

and, $BM = 18 - kG - r_A = 18 - 5.2 - 5 = 7.8$ c

Now, To calculate θ_1 and θ_2 in Δ BAM

$$\cos \theta_1 = \frac{BM}{AB} = \frac{7.8}{14}$$

$\therefore$ $\theta_1 = 56.14°$

and, $\theta_2 = 30° + 30° = 60°$

Now, from free-body diagram; we have,

Using the condition of equilibrium for sphere 'A' first of all;

$\angle BAM = 90° - 56.14° = 33.86°$

$(\overset{+}{\rightarrow}) \sum F_x = 0;$

or, $R_D \sin 33.86° - R_E = 0$ (1)

and, $(+\uparrow) \sum F_y = 0;$

or, $R_D \cos 33.86° - W_A = 0$

or, $R_D = \frac{W_A}{\cos 33.86°} = \frac{20}{0.83}$

$\therefore$ $R_D = 24.1$ kg = 236.42 N

From equation (1); we get,

$R_E = R_D \sin 33.86° = 24.1 \sin 33.86°$

$\therefore$ $R_E = 13.4$ KG = 131.45 N

Using the condition of equilibrium for sphere 'B'; we have,

$(\overset{+}{\rightarrow}) \sum F_x = 0;$

$R_f \sin 60° - R_D \cos 56.14° = 0$

or, $R_F = \frac{24.1 \cos 56.14°}{\sin 60°} = 15.5$ KG = 152.05 N

Again,

$(+\uparrow) \sum F_y = 0;$

or, $R_G - W_B - R_D \sin 56.14° + R_f \cos 60° = 0$

or, $R_G = 50 + 24.1 \sin 56.14° - 15.5 \cos 60°$

$\therefore$ $R_G = 62.3$ KG = 611.16 N

so, the reactions are;

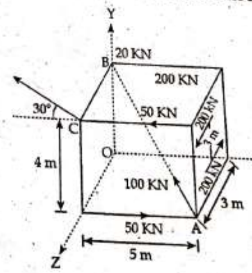
$R_D = 236.42$ N

$R_E = 131.45$ N

$R_F = 152.05$ N

and, $R_G = 611.16$ N

Find the magnitude and direction of the resultant forces and moment of the following system about the point 'O' as shown in the figure. [2065 Shrawan]



Solution:

Here,

Co-ordinates of A = (5, 0, 3)

Co-ordinates of B = (0, 4, 0)
Co-ordinates of C = (5, 4, 3)

Unit vector and force along AB and $\hat{F}_1$ and $\vec{F}_1$ respectively, then,

$$\vec{F}_1 = F_1\hat{F} = \frac{100(-5\hat{i} + 4\hat{j} - 3\vec{k})}{\sqrt{(-5)^2 + (4)^2 + (-3)^2}}$$

$$\vec{F}_1 = (-70.7\vec{i} + 56.56\vec{j} - 42.4\vec{k})\ \text{KN}$$

Again, unit vector and force along OC be $\vec{f}_2$ and $\vec{F}_2$ respectively then,

$$\vec{F}_2 = F_2\hat{f}_2 = \frac{30(4\hat{j} + 2\hat{k})}{\sqrt{(4)^2 + (2)^2}} = (26.83\vec{j} + 13.42\vec{k})\ \text{KN}$$

Resultant force about 0,

i.e., $\vec{F}_0 = \vec{F}_1 + \vec{F}_2$

$$= -70.7\vec{i} + 56.56\vec{j} - 42.4\vec{k} + 26.83\vec{j} + 13.42\vec{k}$$

$$= -70.7\vec{i} + 83.4\vec{j} - 29\vec{k}$$

Moment about 0;

i.e., $\vec{M}_0 = \vec{r}_1 \times \vec{F}_1 + \vec{c}_1 + \vec{c}_2$

$$= (5\hat{i} + 3\hat{k}) \times (-70.7\hat{i} + 83.4\hat{j} - 29\hat{k}) + 600\hat{i} + 200\hat{k}$$

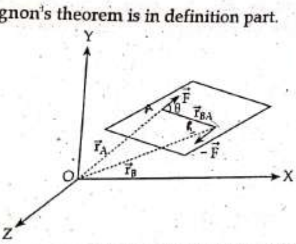
$$= 417\hat{k} + 145\hat{j} - 212.1\hat{j} - 250.2\hat{i} + 600\hat{i} + 200\hat{k}$$

$$= (349.8\hat{i} - 67.1\hat{j} + 1217\hat{k})\ \text{KNm}$$

8. State the Varignon's theorem and also prove that a couple is a free vector. [2065 Shrawan]

Solution:

Statement of Varignon's theorem is in definition part.



Consider two equal and opposite forces acting at points 'A' and 'B'; whose respective position vectors are $\vec{r}_A$ and $\vec{r}_B$ as shown in figure then, the sum of the moment of these two forces about the origin 'O' is given as;

$$\vec{M}_0 = [\vec{r}_A \times \vec{F}] + [\vec{r}_B \times (-\vec{F})]$$

$$= [\vec{r}_A - \vec{r}_B] \times \vec{F} = \vec{r}_{BA} \times \vec{F}$$

The moment vector thus obtained moment of a couple. Its magnitude is;

$$|\vec{M}_0| = F\, r_{BA} \sin(180° - \theta)$$

Note

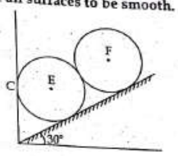
While taking the cross product of two vectors, the angle between their positive senses needs to be considered. Thus, the angle between $\vec{r}_{BA}$ and $\vec{F}$ is $(180° - \theta)$.

$$|\vec{M}_0| = Fr_{BA}\sin\theta = Fd$$

$[\because r_{AB}\sin\theta = d]$

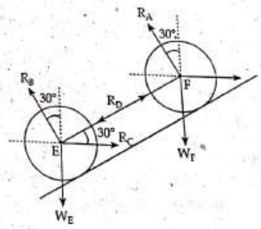
Since, $\vec{r}_{BA}$ the difference between two position vectors is independent of the choice of the origin 'O' of co-ordinate axes, the moment of couple remains the same irrespective of the reference point chosen. Hence, $\vec{M}$ is free vector.

9. Two identical rollers, each of weight (w) = 100N are supported by an inclined plane and vertical wall as shown in figure. Find the reaction at the contact points. Assume all surfaces to be smooth. [2066 Shrawan]



Solution:

Free-body diagram of figure is given as;
Given rollers are identical;



Weights of rollers (W) = 100 N

Since the rollers are identical diameter of both the rollers are same.

i.e., $R_{EF} = R_{FE} = R_D$

$W_E = W_F = 100\ \text{N}$

Now, from above free-body diagram using condition of equilibrium for Roller 'F'

$$(\overset{+}{\rightarrow})\sum F_x = 0;$$

or, $-R_A\sin 30° + R_D\cos 30° = 0$

or, $R_D = R_A \tan 30°$ (1)

$(+\uparrow)\sum F_y = 0$

$R_A \cos 30° - W_F + R_D \sin 30° = 0$

From equation (1) above expression can be written as;

$R_A \cos 30° - W_F + R_A \tan 30° \sin 30° = 0$

or, $R_A(\cos 30° + \tan 30° \sin 30°) = W_F$

$\therefore \quad R_A = \dfrac{100}{1.15} = 86.6\text{ N}$

Now,

$R_D = R_A \tan 30° = 86.6 \tan 30° = 50\text{ N}$

Now, using condition of equilibrium for Roller 'E'; we have,

$(\overset{+}{\rightarrow})\sum F_x = 0;$

or, $-R_D \cos 30° - R_B \sin 30° + R_C = 0$

or, $R_C = 50 \cos 30° + R_B \sin 30°$ (2)

$(+\uparrow)\sum F_y = 0;$

$R_B \cos 30° - R_D \sin - W_E = 0$

or, $R_B = \dfrac{R_D \sin 30° + W_E}{\cos 30°} = \dfrac{50 \sin 30° + 100}{\cos 30°} = 144.33\text{ N}$

From equation (2); we get,

$R_C = 50 \cos 30° + 144.34 \sin 30°$

$R_C = 115.5\text{ N}$

So, the reaction are as follows;

$R_A = 86.6\text{ N}$

$R_B = 144.3\text{ N}$

$R_C = 115.5\text{ N}$

and, $R_D = 50\text{ N}$

10. Define free-body diagram with a suitable example. Also mention the points to be considered while drawing free body diagram. [2066 Shrawan]

Solution:

Free body diagram

A sketch of the isolated body showing all the forces acting on it by vectors is called a free body diagram. *For example;*

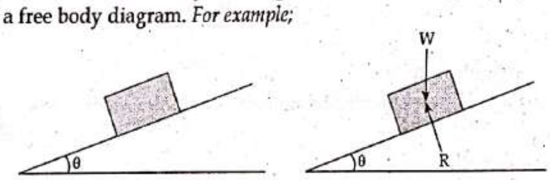


Figure: Block resting on a smooth inclined plane and its free body diagram

For the remaining part

See the solution of Q. no. 15 on page no. 13

11. Define moment of a force about a point and about an axis. What is the geometrical interpretation of scalar triple product? [2066 Chaitra]

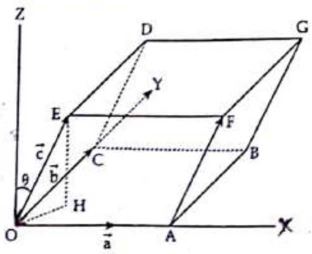
Solution:

Refer definition part for moment of a force about a point.

When the moment of force about a point is multiplied by direction of axis then it is called moment of force about an axis.

Geometrical interpretation of scalar triple product

If we assume $\vec{a}$ and $\vec{b}$ to lie on the x – y plane, then $\vec{a} \times \vec{b}$ is a vector whose magnitude, *i.e.,* $|\vec{a} \times \vec{b}|$ is the areas of the parallelepiped OABC and its direction is perpendicular to x – y plane. Then,



$(\vec{a} \times \vec{b}) \times \vec{c} = |\vec{a} \times \vec{b}|\ |\vec{c}| \cos \theta$

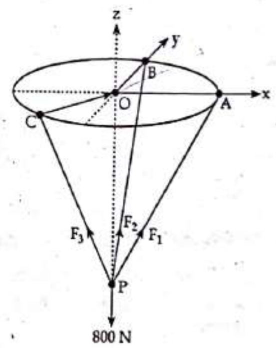
$= [\text{Magnitude of } \vec{a} \times \vec{b}]\ [\text{Magnitude of } \vec{c}]\ [\cos \theta]$

$= [\text{area of parallelogram OABC}]\ [\text{height EH}]$

$= \text{volume of parallelepiped with side } \vec{a}, \vec{b} \text{ and } \vec{c}$

12. A body weighing 800 N is hung from a horizontal ring 6 m in diameter by means of three chords each 5m long on the ring, two of the chords are placed 90° apart and the point of attachment of the third chord bisects the remaining arc of the ring. Find the tension in each chord. [2067 Ashadh]

Solution:



With reference to the figure where the body is represented by a particle P;

$\overrightarrow{OA} = 3\vec{i}$

$\overrightarrow{OB} = 3\vec{j}$

$\overrightarrow{OC} = (-3\cos 45°)\vec{i} - (3\sin 45°)\vec{j} = -\dfrac{3}{\sqrt{2}}\vec{i} - \dfrac{3}{\sqrt{2}}\vec{j}$

and, $\overrightarrow{OP} = -5\vec{k}$

From these above vectors; we have,

$$\overrightarrow{PA} = 3\vec{i} + 5\vec{k}$$
$$\overrightarrow{PB} = 3\vec{j} + 5\vec{k}$$

and, $\overrightarrow{PC} = -2.12\vec{i} - 2.12\vec{j} + 5\vec{k}$

and, the unit vectors along these chords are;

$$\hat{f}_1 = \frac{3\vec{i} + 5\vec{k}}{\sqrt{(3)^2 + (5)^2}} = 0.51\vec{i} + 0.86\vec{k}$$

$$\hat{f}_2 = \frac{3\vec{j} + 5\vec{k}}{\sqrt{(3)^2 + (5)^2}} = 0.51\vec{j} + 0.86\vec{k}$$

and, $\hat{f}_3 = \dfrac{-2.12\vec{i} - 2.12\vec{j} + 5\vec{k}}{\sqrt{(-2.12)^2 + (-2.12)^2 + (5)^2}} = -0.36\vec{i} - 0.36\vec{j} + 0.86\vec{k}$

Let, the tension in these cords be F_1, F_2 and F_3 respectively in the magnitude. Vectorially;

$$\vec{F}_1 = F_1\left(0.51\vec{i} + 0.86\vec{k}\right)$$
$$\vec{F}_2 = F_2\left(0.51\vec{j} + 0.86\vec{k}\right)$$

and, $\vec{F}_3 = F_3\left(-0.36\vec{i} - 0.36\vec{j} + 0.86\vec{k}\right)$

For equilibrium of the particle;

$$\Sigma F = 0$$

or, $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 - 800k = 0$

or, $(0.51F_1 - 0.36F_3)\vec{i} + (0.51F_2 - 0.36F_3)\vec{j} + (0.86F_1 + 0.86F_2 + 0.86F_3 - 800)\vec{k} = 0$

Equating co-efficient of $\vec{i}$, $\vec{j}$ and $\vec{k}$; we get,

$$0.51F_1 - 0.36F_3 = 0 \quad (1)$$
$$0.51F_2 - 0.36F_3 = 0 \quad (2)$$
$$0.86F_1 + 0.86F_2 + 0.86F_3 = 800 \quad (3)$$

Solving equations (1), (2) and (3); we get,

$$F_1 = 272\text{ N}$$
$$F_2 = 272\text{ N}$$

and, $F_3 = 386\text{ N}$

13. Define rigid and deformable body. Explain principles of free body diagram and static equilibrium while solving problem in statics? Support your answer with example. **[2067 Ashadh]**

Solution:

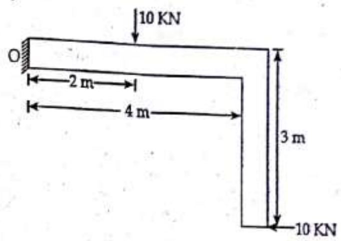
Deformable body is the combination of two or more particles which are so interconnected that they change their relative position when force is applied on it. Otherwise it is called rigid body. Principles of free-body diagram are as follows.

i) A body to be isolated for consideration may be any portion of system.
ii) The free body drawn should have no external connections.
iii) Any adopted co-ordinate system whose axes are not in horizontal and vertical direction should be shown.

iv) Each applied load should be indicated including weight of isolated body.
v) All the dimensions should be indicated including the slope.
vi) Action and reaction with an arrow head along with labeling should be used.

The principle of static equilibrium states that, a stationary body which is subjected to coplanar forces will be in equilibrium if the algebraic sum of all the external forces and moments are zero.

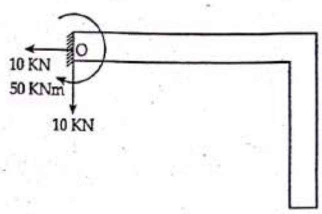
14. Resolve the force system as shown in the figure below into an equivalent force couple system about O. **[2067 Mangshir]**



Solution:

Both the forces (i) and (ii) as shown in figure below gives forces and couples on point 'O'.

Force (i) gives one vertical down ward force of 10 kN and a clockwise couple of 20 kNm. Similarly; force (ii) gives axial force of 10 KN and a couple of 30 kNm at point 'O'. So, the net couple on point 'O' is 50 kNm (clockwise) and one axial force of 10 kN and one vertical force of 10 kN downward as shown in the figure.



Force (i) gives;

$$\text{Couple} = 10 \times 2 = 20\text{ kNm}$$

Force (ii) gives;

$$\text{Couple} = 10 \times 3 = 30\text{ kNm}$$

Since, both the couple are in same direction so we can add it;

$$20 + 30 = 50\text{ kNm}$$

15. Explain free body diagram with suitable examples. **[2066 Shrawan, 2067 Mangshir]**

Solution:

A sketch of the isolated body showing all the forces acting on it by vectors is called a free body diagram. The following steps are important for construction of free-body diagram.

i) A body to be isolated for consideration may be any portion of system.
ii) The free-body drawn should have no external supports or connection
iii) Any adopted co-ordinate system whose axes are not in horizontal and vertical direction should be shown.
iv) Each applied load should be indicated including weight of isolated body.
v) All the dimensions should be indicated including the slope.
vi) Action and reaction with an arrow head along with labeling should be used.

For example;

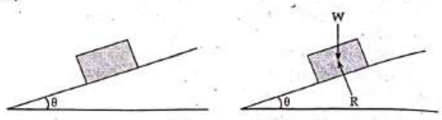
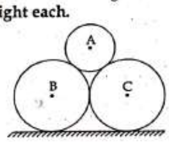


Figure: Block resting on a smooth inclined plane and its free body diagram

16. Find the contact forces of the three bodies as shown in figure below. Body A has 20 cm diameter and 60 N weight and bodies B and C have 30 cm diameter and 100N weight each. [2067 Mangshir]



Solution:

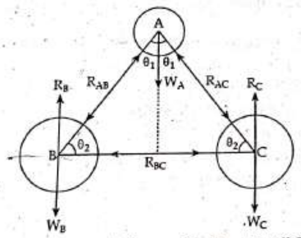


Figure: Free body diagram of bodies A, B and C

For calculation of θ_1 and θ_2;

Given that;

$MB = 15$ cm

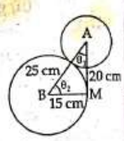
$AB = 10\text{ cm} + 15\text{ cm} = 25$ cm

$AM = \sqrt{AB^2 - BM^2} = \sqrt{25^2 - 15^2} = 20$ cm

$\tan\theta_1 = \dfrac{15}{20}$

$\therefore \quad \theta_1 = 36.87°$

$\tan\theta_2 = 90° - 36.87° = 53.13°$



Now, for body A;

$(^+\!\!\rightarrow) \sum F_x = 0;$

$|\vec{R}_{AB}| \sin 36.87° - |\vec{R}_{AC}| \sin 36.87° = 0$ (1)

$(^+\!\uparrow) \sum F_y = 0;$

$R_{AB} \cos 36.87° + R_{AC} \cos 36.87° = 60$ N (2)

Solving equation (1) and (2); we get,

$R_{AB} = 37.5$N, $R_{AC} = 37.5$ N

For body 'B';

$R_B = W_B + R_{AB} \sin 53.13° = 100 + 37.5 \sin 53.13° = 130$ N

$R_{BC} = -R_{AB} \cos 53.13° = -37.5 \cos 53.13° = -22.5$ N

For Body 'C'

$R_C = R_{AC} \sin 53.13° + W_C = 37.5 \sin 53.13° + 100 = 130$ N

So, contact forces are as follows;

$R_{AB} = 37.5$ N

$R_{AC} = 37.5$ N

$R_{BC} = -22.5$ N

$R_B = 130$ N

and, $R_C = 130$ N

17. Write the principles of transmissibility and define couples with suitable example. [2067 Mangshir]

Solution:

Refer definition part for principles of transmissibility and definition of couples. The examples of principle of transmissibility and couple are shown in the figure.

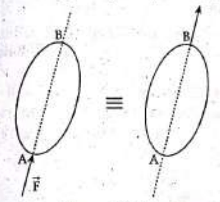


Figure: Transmissibility of forces

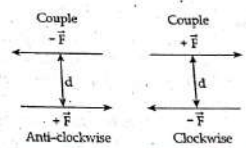
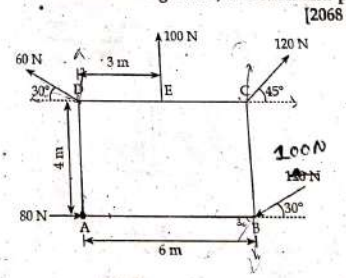


Figure: Clockwise and anti-clockwise couple

18. A plate of size 6 m × 4 m is acted upon by a set of forces in its plane as shown in figure below. Determine magnitude, direction and position of resultant force. [2068 Baishakh]



Solution:

Let us consider 'A' as origin. Then, the plate rest at xy plane such that AB as x-axis and AD as y-axis.

Now, resultant of force system is given by;

$$\vec{R} = 80\vec{i} - 100\cos 30°\vec{i} - 100\sin 30°\vec{j} + 120\cos 45°\vec{i} + 120\sin 45°\vec{j} + 100\hat{j} - 60\cos 30°\vec{i} + 60\sin 30°\vec{j}$$

$\therefore \quad R = (26.3\vec{i} + 164.85\vec{j})$ N

Now, let us consider the resultants acts at the point (x, y). Taking the moment about A (0, 0).

$$(x\vec{i} + y\vec{j}) \times (26.3\vec{i} + 164.85\vec{j}) = 6\vec{i} \times (-100\sin 30°\vec{j}) + (4\vec{j} + 6\vec{i}) \times 120\sin 45°\vec{j} + (6\vec{i} + 4\vec{j}) \times 120\cos 45\vec{i} + (3\vec{i} + 4\vec{j}) \times 100\vec{j} + (-4\vec{j}) \times (60\cos 30°\vec{i})$$

or, $164.85x\vec{k} - 26.3y\vec{k} = -300\vec{k} + 509.12\vec{k} - 339.4\vec{k} + 300\vec{k} + (207.85\vec{k})$

or, $164.85x - 26.3y = 377.57$

or, $\frac{x}{2.3} + \frac{y}{-14.36} = 1$

Hence, line of action cuts the edge AB at the distance 2.3 on in '+' ve x-axis from point 'A' and cut the side AB at the distance 14.36 m below from point 'A'.

19. **What is the equilibrium of a body? Write the condition of equilibrium of a particle.** **[2068 Baishakh]**

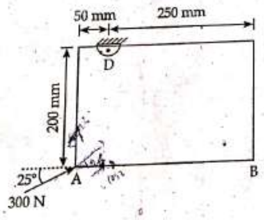
Solution:

A stationary body which is subjected to coplanar forces will be in equilibrium if the algebraic sum of all the external forces is zero and also algebraic sum of moments of all the external forces about any point in the plane is zero.

The conditions of equilibrium are;

$$\sum F_x = 0, \sum F_y = 0 \text{ and } \sum M_z = 0$$

20. **A 300 N force is applied at 'A' as shown. Determine (i) moment of 300 N force about 'D' (ii) smallest force applied at 'B' that creates same moment about 'D'.** **[2068 Magh]**



Solution:

i) Moment at D due to 300N force is;

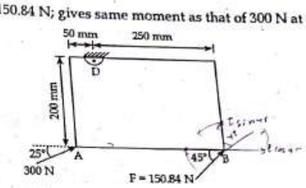
$M_D = -300\cos 25° \times 0.2 + 300\sin 25° \times 0.05 = -48$ Nm

ii) Smallest force applied at 'B' that creates same moment about 'D' must be at 45° as shown in the figure.

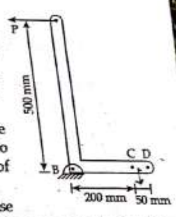
and, Magnitude of force $= -F\cos 45 \times 0.2 - F\sin 45° \times 0.25$
$= -48$

$F = \frac{48}{0.3182} = 150.84$ N

Force (F) = 150.84 N; gives same moment as that of 300 N at point 'D'.



21. **The 80N horizontal force 'P' acts on a bell crank as shown; (i) Replace p with an equivalent force couple system at B, (ii) Find two vertical forces at C and D that are equivalent to the couple found in part (i).** **[2068 Magh]**



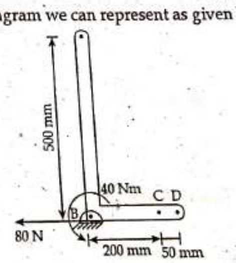
Solution:

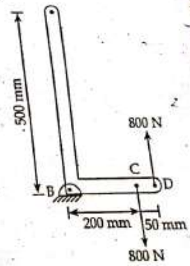
i) As, we know force can be converted into force and couple. Here, force 'p' can be transferred to point 'B' but there must be a couple of magnitude.
$P \times 0.5m = 80 \times 0.5 = 40$ Nm in anticlockwise direction (anticlockwise direction is because 'p' gives moment at point 'B' in anticlockwise direction.)

ii) For the vertical forces at point c and which forms couple in clockwise direction and having magnitude 40 Nm can be calculated by dividing this couple with distance between 'C' and 'D'. Mathematically;

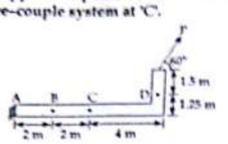
$$\text{Required vertical force at 'C' and 'D'} = \frac{40}{\text{distance betweeen points C and D}} = \frac{40}{0.05} = 800 \text{ N}$$

So, in diagram we can represent as given below.





22. **A 160 N force 'P' is applied at point 'A' of a structure member replace 'P' with equivalent force-couple system at 'C'.** **[2069 Bhadra]**



Solution:

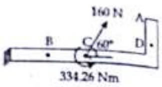
P force can be replaced at point 'c' as follows.

160 N load inclined at 60° is applied at 'c' and the couple of magnitude is:

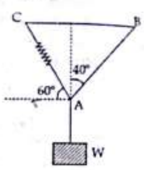
$P \cos 60° \times 2.75 - (P \sin 60° \times 4)$

$= 160 \cos 60° \times 2.75 - 160 \sin 60° \times 4$

$= -334.26$ Nm is applied at point 'C' as shown in the figure.



23. **If W is the weight of body applied as shown in the figure. Find the tension in the cable AB. Spring constant K = 100 N/m underformed length of spring is equal to 3m. When (a) W = 120 N (b) W = 160 N** **[2069 Bhadra]**



Solution:

When W = 120 N then,

Let T_1 be tension in AC and T_2 be tension in cable AB.

$(^+\rightarrow) \sum F_x = 0;$

or, $-T_1 \cos 60° + T_2 \sin 40° = 0$

or, $-0.5T_1 + 0.64T_2 = 0$ (1)

and, $(^+\uparrow) \sum F_y = 0;$

or, $T_1 \sin 60° + T_2 \cos 40 = 120$

or, $0.87 T_1 + 0.766 T_2 = 120$ (2)

Solving equation (1) and (2); we get,

$T_1 = 81.72$ N

and, $T_2 = 63.84$ N

so, Tension in cable AB = 63.84 N

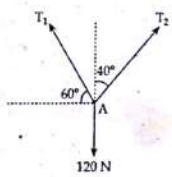


Figure: Free body diagram

When load is 160 N then, let tension in cable AC = T_1 and AB = T_2 then,

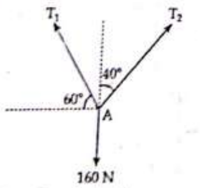


Figure: Free body diagram

$(^+\rightarrow) \sum F_x = 0;$

or, $-T_1 \cos 60° + T_2 \sin 40° = 0$

or, $-0.5T_1 + 0.64T_2 = 0$ (1)

and, $(^+\uparrow) \sum F_y = 0;$

or, $T_1 \sin 60° + T_2 \cos 40° = 160$ N

or, $0.87T_1 + 0.766T_2 = 160$ N (2)

Solving equation (1) and (2); we get,

$T_1 = 108.85$ N

and, $T_2 = 85.12$ N

24. **Describe briefly; the concept of particle, rigid body and deformable body** **[2069 Chaitra]**

Solution:

Particle means a very small amount of matter which may be assumed to occupy a single point in space.

For the remaining portion

See the solution of Q. no. 13 on page no. 12

25. **Describe free body diagram and physical meaning of equilibrium. Also describe the importance of free-body diagram and equilibrium in structural analysis.** **[2069 Chaitra]**

Solution:

Free body diagram

See the solution of Q. no. 15 on page no. 13

Physical meaning of equilibrium is that the system of the external forces will import no translational or rotational motion to the body considered.

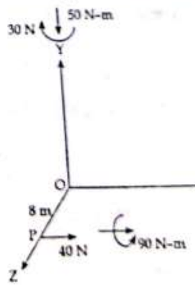
Free body diagram is important because of following causes:

i) We can analyze the body by isolating it from the system of bodies.
ii) We can easily apply equilibrium equation on free body diagram.

Importance of equilibrium in structure analysis is as given below:

i) We can analyze the structure by isolating some part of it and applying equilibrium equation.
ii) It becomes easy to calculate forces in structural member and reactions at support by applying equilibrium equation.

26. **Replace the two wrenches as shown in figure by a single equivalent wrench and determine (a) resultant force, (b) indicate its line of action.** **[2069 Chaitra]**



Solution:

a) First of all reducing the given system of wrench into force-couple system at 'O'.

This force-couple system consists of a force $\vec{R}$ and couple vector $\vec{M}_O^R$ defined as follows;

$$\vec{R} = \Sigma \vec{F}$$

and, $\vec{M}_O^R = \Sigma(\vec{r} \times \vec{F})$

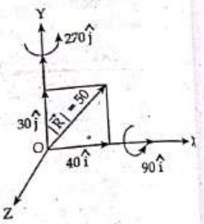
Computations are arranged in tabular form

$\vec{r}$, m	$\vec{F}$, N	$\vec{r} \times \vec{F}$, N-m
0	$30\hat{j}$	0
$8\hat{k}$	$40\hat{i}$	$320\hat{j}$
		$90\hat{i}$
		$-50\hat{j}$

$$\vec{R} = 40\hat{i} + 30\hat{j} \qquad \vec{M}_O^R = 90\hat{i} + 270\hat{j}$$

Magnitude of $\vec{R}$

$$|\vec{R}| = R = \sqrt{(40)^2 + (30)^2} = 50\text{ N}$$



b) Resultant force $\vec{R} = 40\hat{i} + 30\hat{j}$ and the adjoining sketch that the resultant force $\vec{R}$ has the magnitude R = 50 N lies in the xy plane and forms angles of 45° with x and y axes. Thus,

$$R = 50\text{ N}$$

$$\theta_x = \theta_y = 45^\circ$$

and, $\theta_z = 90^\circ$

For calculation of wrench;

$$\text{Pitch of wrench (P)} = \frac{\vec{R} \cdot \vec{M}_O^R}{R^2}$$

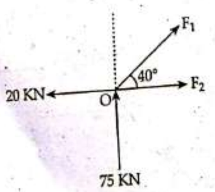
$$= \frac{(40\hat{i} + 30\hat{j}) \cdot (90\hat{i} + 270\hat{j})}{(50)^2} = \frac{3600 + 8100}{2500} = 4.68$$

Resolving $\vec{M}_O^R$ in direction of resultant force $\vec{R}$

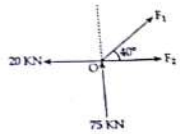
$$\vec{M}_1 = 4.68 \times (40\hat{i} + 30\hat{j}) = 187.2\hat{i} + 140.4\hat{j}$$

and, $\vec{R} = 40\hat{i} + 30\hat{j}$ is new wrench.

27. Determine the value of F_1 and F_2 if the forces shown in figure below ar[e] in equilibrium. [2069 Chaitr[a]]



Solution:



Resolving forces along x-direction *i.e.*,

$(^+\rightarrow) \Sigma F_x = 0$

or, $F_2 - 20 + F_1 \cos 40^\circ = 0$ (1)

Resolving forces along y-direction; we have,

$(^+\uparrow) \sum F_y = 0$

or, $F_1 \cos 60^\circ + 75 = 0$

or, $F_1 = \frac{-75}{\cos 60^\circ}$

$\therefore$ $F_1 = -150$ KN (2)

Putting value of F_1 from equation (2) to equation (1); we get,

$F_2 - 20 + (-150)\cos 40^\circ = 0$

or, $F_2 = 20 + 150 \cos 40^\circ$

$\therefore$ $F_2 = 134.91$ KN

28. Describe the scope of applied mechanics in engineering. **[2070 Ashadh]**

Solution:

The scope of applied mechanics in engineering can be described by following point.

i) It has scope to describe and predicts the condition of rest or motion of bodies under the action of forces.
ii) It has scope for application of equation of equilibrium to rigid bodies.
iii) It has scope to know the Mechanics of fluids.
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29. What is the physical meaning of equilibrium and why it is important in structure? How can we draw good free Body Diagram? Explain with suitable examples. **[2070 Ashadh]**

Solution:

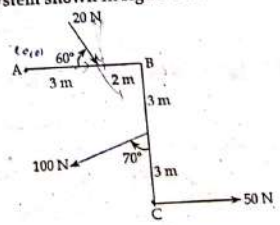
Physical meaning and importance of equilibrium

See the solution of Q. no. 22 on page no. 17

Process to draw well free body diagram

See the solution of Q. no. 15 on page no. 13

30. Determine magnitude, direction and line of action of the resultant of forces acting in the system shown in figure below. **[2070 Ashadh]**



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$\vec{R} = 40\hat{i} + 30\hat{j}$ $\quad \vec{M}_O^R = 90\hat{i} + 270\hat{j}$

Magnitude of $\vec{R}$

$|\vec{R}| = R = \sqrt{(40)^2 + (30)^2} = 50\text{ N}$

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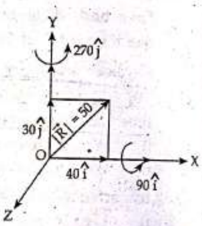

$R = 50\text{ N}$

$\theta_x = \theta_y = 45°$

and, $\theta_z = 90°$

For calculation of wrench;

$$\text{Pitch of wrench (P)} = \frac{\vec{R} \cdot \vec{M}_O^R}{R^2}$$

$$= \frac{(40\hat{i} + 30\hat{j}) \cdot (90\hat{i} + 270\hat{j})}{(50)^2} = \frac{3600 + 8100}{2500} = 4.68$$

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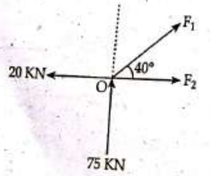
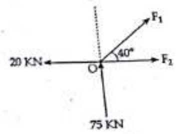


Solution:



Resolving forces along x-direction *i.e.,*

$(\overset{+}{\rightarrow}) \Sigma F_x = 0$

or, $F_2 - 20 + F_1 \cos 40° = 0$ (1)

Resolving forces along y-direction; we have,

$(+\uparrow) \sum F_y = 0$

or, $F_1 \cos 60° + 75 = 0$

or, $F_1 = \frac{-75}{\cos 60°}$

$\therefore$ $F_1 = -150\text{ KN}$ (2)

Putting value of F_1 from equation (2) to equation (1); we get,

$F_2 - 20 + (-150) \cos 40° = 0$

or, $F_2 = 20 + 150 \cos 40°$

$\therefore$ $F_2 = 134.91\text{ KN}$

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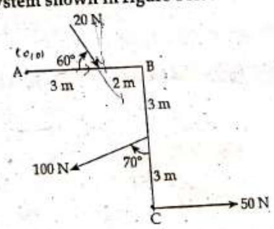


Solution:

Let us consider 'A' as origin.

Resolving all the forces in x and y direction respectively; we get,

$$(^+\rightarrow)\ \vec{F}_x = 20\cos 20°\,\hat{i} - 100\cos 20°\,\hat{i} + 50\,\hat{i} = -33.9\hat{i}$$

$$(^+\uparrow)\ \vec{F}_y = -20\sin 60°\hat{j} - 100\cos 70°\,\hat{j} = -51.52\hat{j}$$

Resultant of force system is given by;

$$\vec{R} = \vec{F}_x + \vec{F}_y = -33.97\,\hat{i} - 51.52\hat{j}$$

Now, magnitude of resultant cab be obtained by;

$$\vec{R} = |\vec{R}| = \sqrt{(-33.97)^2 + (-51.52)^2} = 61.61 \text{ N}$$

Now, direction of resultant force;

$$\tan\theta = \frac{F_y}{F_x} = \left(\frac{-51.52}{-33.97}\right)$$

or, $\theta = \tan^{-1}\left(\frac{51.52}{33.97}\right) = 56.6°$

Let us consider the resultant cuts at the point (x, y). Taking the moment about A (0, 0); we get,

$$(x\hat{i} + y\hat{j})(-33.97\hat{i} - 51.52\hat{j}) = -3\hat{i} \times 20\sin 60°\hat{j} + (5\hat{i} - 3\hat{j}) \times -100\cos 20°\,\hat{i} + (5\hat{i} - 3\hat{j}) \times -100\cos 70°\,\hat{j} + (5\hat{i} - 6\hat{j}) \times 50\hat{i}$$

or, $-51.52x\hat{k} + 33.97y\hat{k} = -60\sin 60°\,\hat{k} + (-300\cos 20°\,\hat{k}) + (-500\cos 70°\,\hat{k}) + 300\hat{k}$

or, $(-51.52x + 33.97y)\hat{k} = -100.95\hat{k}$

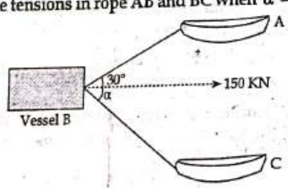
Equating the coefficient of $\hat{k}$; we get,

$$-51.52x + 33.97y = -100.95$$

or, $\frac{X}{1.96} + \frac{y}{-2.97} = 1$

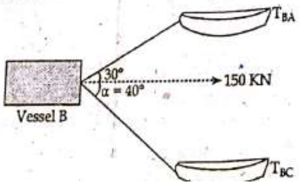
Hence line of action cuts the edge AB at the distance 2.3 on '+'ve x-axis from point A and passes through the point (0, -2.97).

31. **A commercial vessel is being pulled into labour for unloading by two tugboats as shown in figure knowing the vessel requires 150 KN along its axis to move it steadily. Compute the tensions in rope AB and BC when $\alpha = 40°$. [2070 Ashadh]**



Solution:

Consider T_{AB} and T_{BC} be the tensions in the ropes as shown in the figure.



According to question;

$$T_{AB}\cos 30° + T_{BC}\cos 40° = 150 \quad (1)$$

Again, Resolving T_{AB} and T_{BC} in r-direction

$$T_{AB}\sin 30° = T_{BC}\sin 40° \quad (2)$$

or, $T_{AB} = 2T_{BC}\sin 40°$ (3)

Putting the value of T_{AB} in equation (1); we get,

$$2T_{BC}\sin 40°\cos 30° + T_{BC}\cos 40° = 150$$

or, $1.88\,T_{BC} = 150$

or, $T_{BC} = 79.81$ kN

Again, putting the value of T_{AB} in equation (3); we get,

$$T_{AB} = 2 \times 79.81\sin 40° = 102.61 \text{ kN}$$

So, the tensions in rope AB, $T_{AB} = 102.6$ kN

The tensions in rope BC, $T_{BC} = 79.81$ kN

32. **What do you mean by rigid body? Why it is necessary to assume a body as "perfectly rigid" for the study of statics. [2070 Bhadra]**

Solution:

Rigid body

See the solution of Q. no. 13 on page no. 12

It is necessary to assume a body as "perfectly rigid" because of following causes;

i) Perfect body is combination of large number of particles occupying fixed position with respect to each other.
ii) Result obtained for a particle can be used directly in large number of problems dealing with the condition of rest or motion of actual bodies.
iii) Different laws can be applied on rigid body for example the principle of transmissibility.

33. **What is free body diagram? Why it is necessary to draw free body diagram in solving any structural problem? Also describe equation of equilibrium in the dimension. [2070 Bhadra]**

Solution:

Free body diagram

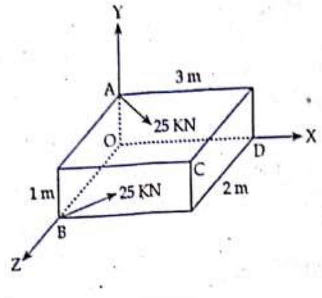
See the solution of Q. no. 15 on page no. 13

It is necessary to draw free body diagram in solving any structural problem because of following causes;

i) Free body diagram is sketch of the isolated body showing all the forces acting on it by vector.
ii) We can adopt any co-ordinate system whose axes are not in horizontal and vertical direction.
iii) Free body diagram indicates each applied load including the weight of isolated body.
iv) Free body diagram indicates all the dimensions including slope.

Refer definition part for equation of equilibrium in two dimensions.

34. **If two forces of same magnitude 25 kN act at points A and B as shown in figure and force of A passes through C and force at B passes through D. (a) Find equivalent force couple system at 'O'. (b) Find equivalent wrench and give pitch and axis of wrench. [2070 Bhadra]**

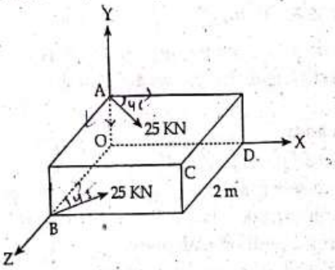


Solution:

a) Determining the force couple system at 'O'

Considering $\vec{F}_1$ and $\vec{F}_2$ as forces acting at points 'A' and 'B' respectively.

Position vectors of A and B are;



$\overrightarrow{OA} = \hat{j}$

$\overrightarrow{OB} = 2\hat{k}$

so, $\vec{F}_1 = 25\cos 45°\,\hat{i} + 25\cos 45°\,\hat{k}$

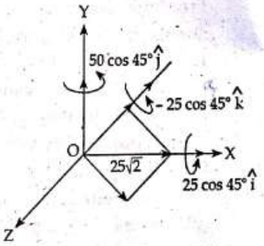
and, $\vec{F}_2 = 25\cos 45°\,\hat{i} - 25\cos 45°\,\hat{k}$

The resultant force $\vec{R}$ of the two forces $\vec{F}_1$ and $\vec{F}_2$ and their moment resultant $\vec{M}_O^R$ about 'O' are;

$\vec{R} = \vec{F}_1 + \vec{F}_2$

$= 25\cos 45°\,\hat{i} + 25\cos 45°\,\hat{k} + 25\cos 45°\,\hat{i} - 25\cos 45°\,\hat{k}$

or, $\vec{R} = 50\cos 45°\hat{i}$ (1)



and, Magnitude of $\vec{R}$ is;

$|\vec{R}| = R = \sqrt{(50\cos 45°)^2}$

$= 50\cos 45° = 25\sqrt{2}$

$\vec{M}_O^R = \overrightarrow{OA} \times \vec{F}_1 + \overrightarrow{OB} \times \vec{F}_2$

$= \hat{j}(25\cos 45°\,\hat{i} + 25\cos 45°\,\hat{k}) + 2\hat{k}(25\cos 45°\,\hat{i} - 25\cos 45°\,\hat{k})$

$= (-25\cos 45°\hat{k} + 25\cos 45°\hat{i} + 50\cos 45°\hat{j} - 0)$

$= 25\cos 45°\,\hat{i} + 50\cos 45°\,\hat{j} - 25\cos 45°\,\hat{k}$ (2)

b) Resultant force equation (1) and the adjoining sketch that the resultant $\vec{R}$ has the magnitude $R = 25\sqrt{2}$ directed toward positive x-axis and forms angle 0° with x-axis. Thus,

$R = 25\sqrt{2}$

$\theta_x = 0$

$\theta_y = 90°$

and, $\theta_z = 90°$

$$\text{Pitch of wrench (P)} = \frac{\vec{R}\cdot\vec{M}_O^R}{R^2} = \frac{(50\cos 45°\,\hat{i}\cdot(25\cos 45°\,\hat{i} + 50\cos 45°\,\hat{j} - 25\cos 45°\,\hat{k})}{(25\sqrt{2})^2}$$

or, $$P = \frac{(50\cos 45° \times 25\cos 45° - 0 - 0)}{(25)^2 \times 2} = \frac{625}{625 \times 2} = \frac{1}{2}$$

Axis of wrench

Resolving moment in direct of force

$\vec{M}_1 = P\vec{R} = \frac{1}{2}50\cos 45°\,\hat{i} = 25\cos 45°\,\hat{i}$ (3)

To find the point where the axis of the wrench intersect the yz plane, we express that the moment of the wrench about 'O' is equal to the moment resultant $\vec{M}_O^R$ of the original system;

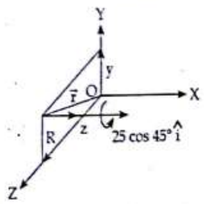
$\vec{M}_1 = \vec{r} \times \vec{R} = \vec{M}_O^R$

or, noting that $\vec{r} = y\hat{j} + z\hat{k}$ and substituting the values of $\vec{R}$, $\vec{M}_O^R$ and $\vec{M}_1$ from equation (1), (2) and (3); we get,

or, $25\cos 45°\,\hat{i} + [y\hat{j} + z\hat{k}][50\cos 45°\,\hat{i}] = 25\cos 45°\,\hat{i} + 50\cos 45°\,\hat{j} - 25\cos 45°\,\hat{k}$

or, $25\cos 45°\,\hat{i} - 50y\cos 45°\,\hat{k} + 50z\cos 45°\,\hat{j} = 25\cos 45°\,\hat{i} + 50\cos 45°\,\hat{j} - 25\cos 45°\,\hat{k}$

Equating the coefficients of $\hat{k}$ and then coefficient of $\hat{j}$; we find,



$-50y\cos 45° = -25\cos 45°$

or, $y = \frac{1}{2}$

and, $50z\cos 45° = 50\cos 45°$

i.e., $z = 1$

35. Differentiate between rigid body and deformable body. **[2070 Magh]**

Solution:

A rigid body may be defined as a body which does not deform or the distance between any two points of the body does not change under the action of an applied force. A deformable body may be defined as a body which deform or the relative positions of any two particles change under the action of the forces.

36. Explain about the physical meaning of equilibrium. Define force body diagram and concept of particle. **[2070 Magh]**

Solution:

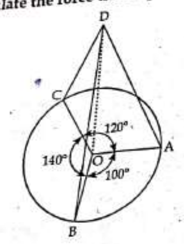
For the first part

See the solution of Q. no. 25 on page no. 19

For the second part

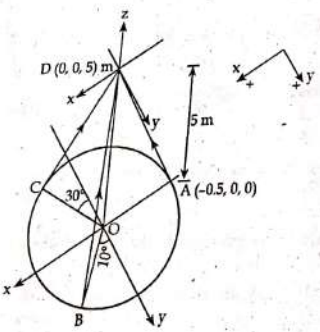
The particle is defined as an object whose mass is concentrated at a point. This assumption is made when the size of a body is negligible and is irrelevant to the description of the motion of the body.

37. **A homogeneous circular plate of mass 50 kg is supported by three wires. The angular distance between the points of attachment on the circumference of the plate with respect to centre of the plate makes an angle of 100° while other two angular distances are 120° and 140° as shown in the figure. The three wires are attached to a single point on the ceiling which is 5 m vertically above the centroid of the plate. The plate has diameter of 1 m. Calculate the force developed in each wire.** [2070 Magh]



Solution:

The free body diagram of circular plate is shown below. Let us the axes be x and y such that the plate rests on x-y plane. The weight of the plate acts on z-direction.



Here, given data are:

Diameter of the plate = 1 m

∴ Radius of the plate = 0.5 m

and, Mass of the plate = 50 kg

The co-ordinate of B and C are calculated as;

$$\text{Co-ordinate of C} = (0.5\sin 30°,\ -0.5\cos 30°,\ 0) = (0.25,\ -0.433,\ 0)\text{ m}$$

$$\text{Co-ordinate of B} = (0.5\sin 10°,\ 0.5\cos 10°,\ 0) = (0.0868,\ 0.492,\ 0)\text{ m}$$

The forces developed in the wires are $\vec{F}_{AD}$, $\vec{F}_{CD}$ and $\vec{F}_{BD}$.

Now,

$$\text{Unit vector along force } F_{AD} = \frac{(0+0.5)\vec{i} + (0-0)\vec{j} + (5-0)\vec{k}}{\sqrt{(0.5)^2 + (5)^2}} = 0.0995\vec{i} + 0.995\vec{k}$$

$$\therefore \quad \vec{F}_{AD} = F_{AB}\hat{n} = F_{AD}\left(0.0995\vec{i} + 0.995\vec{k}\right)$$

$$\text{Unit vector along force } F_{CD} = \hat{n} = \frac{(0-0.25)\vec{i} + (0+0.433)\vec{j} + (5-0)\vec{k}}{\sqrt{(-0.25)^2 + (0.433)^2 + (5)^2}} = -0.0497\vec{i} + 0.0861\vec{j} + 0.995\vec{k}$$

$$\therefore \quad \vec{F}_{CD} = F_{CD}\hat{n} = F_{CD}\left(-0.0497\vec{i} + 0.0861\vec{j} + 0.995\vec{k}\right)$$

$$\text{and, Unit vector along force } F_{BD} = \hat{n} = \frac{(0-0.0868)\vec{i} + (0-0.492)\vec{j} + (5-0)\vec{k}}{\sqrt{(-0.0868)^2 + (-0.492)^2 + (5)^2}} = -0.0173\vec{i} - 0.0979\vec{j} + 0.995\vec{k}$$

$$\therefore \quad \vec{F}_{BD} = F_{BD}\hat{n} = F_{BD}\left(-0.0173\vec{i} - 0.0979\vec{j} + 0.995\vec{k}\right)$$

and, Weight $(W) = 50 \times 9.81(-k) = -490.5\vec{k}$

Equation of equilibrium gives;

$$\sum F = 0$$

$$\text{or,} \quad \vec{F}_{AD} + \vec{F}_{CD} + \vec{F}_{BD} + W = 0$$

$$\text{or,} \quad F_{AD}\left(0.0995\vec{i} + 0.995\vec{k}\right) + F_{CD}\left(-0.0497\vec{i} + 0.0861\vec{j} + 0.995\vec{k}\right) F_{BD}\left(-0.0173\vec{i} - 0.0979\vec{j} + 0.995\vec{k}\right) - 490.5\vec{k} = 0$$

$$\text{or,} \quad (0.0995F_{AD} - 0.0497F_{CD} - 0.0173F_{BD})\vec{i} + (0.0861F_{CD} - 0.0979F_{BD})\vec{j} + (0.0995F_{AD} + 0.995F_{CD} - 0.995F_{BD} - 490.5)\vec{k} = 0$$

Equating co-efficient of $\vec{i}$, $\vec{j}$ and $\vec{k}$; we get,

$$0.0995F_{AD} - 0.0173F_{BD} - 0.0497F_{CD} = 0 \quad (1)$$

$$0 - 0.0979F_{BD} + 0.0861F_{CD} = 0 \quad (2)$$

$$0.995F_{AD} + 0.995F_{BD} + 0.995F_{CD} = 490.5 \quad (3)$$

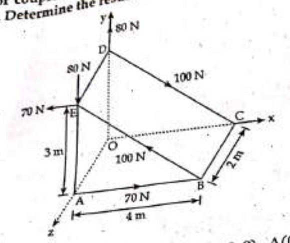
On solving; we get,

$$F_{AD} = 127.03\text{ N}$$

$$F_{BD} = 171.24\text{ N}$$

and, $F_{CD} = 194.70$ N

38. **Three pairs of couples are acted on the triangular block as shown in the figure below. Determine the resultant of them.** **[2070 Magh]**



Solution:

The co-ordinates of the various points are O(0, 0, 0), A(0, 0, 2), B(4, 0, 2), C(4, 0, 0), D(0, 3, 0) and E(0, 3, 2).

Let the given forces are represented by their directions are:

$$F_{OD} = 80\text{ N}$$
$$F_{DC} = 100\text{ N}$$
$$F_{BE} = 100\text{ N}$$
$$F_{EA} = 80\text{ N}$$
$$F_{AB} = 70\text{ N}$$

and, $F_E = 70\text{ N}$

Unit vector along the direction of the forces are determined as:

$$\hat{r}_{OD} = \frac{(0-0)\vec{i} + (3-0)\vec{j} + (0-0)\vec{k}}{\sqrt{(0-0)^2 + (3-0)^2 + (0-0)^2}} = \vec{j}$$

$$\therefore \quad \vec{F}_{OD} = \hat{r}_{OD}F_{OD} = 80\vec{j}$$

$$\hat{r}_{DC} = \frac{(4-0)\vec{i} + (0-3)\vec{j} + (0-0)\vec{k}}{\sqrt{(4-0)^2 + (0-3)^2 + (0-0)^2}} = 0.8\vec{i} - 0.6\vec{j}$$

$$\therefore \quad \vec{F}_{DC} = \hat{r}_{DC}F_{DC} = 100(0.8\vec{i} - 0.6\vec{j}) = 80\vec{i} - 60\vec{j}$$

$$\hat{r}_{BE} = \frac{(0-4)\vec{i} + (3-0)\vec{j} + (2-2)\vec{k}}{\sqrt{(0-4)^2 + (3-0)^2 + (2-2)^2}} = -0.8\vec{i} + 0.6\vec{j}$$

$$\therefore \quad \vec{F}_{BE} = \hat{r}_{BE}F_{BE} = 100(-0.8\vec{i} + 0.6\vec{j}) = -80\vec{i} + 60\vec{j}$$

$$\hat{r}_{EA} = \frac{(0-0)\vec{i} + (0-3)\vec{j} + (2-2)\vec{k}}{\sqrt{(0-0)^2 + (0-3)^2 + (2-2)^2}} = -\vec{j}$$

$$\therefore \quad \vec{F}_{EA} = \hat{r}_{EA}F_{EA} = -80\vec{j}$$

Similarly;

$$\vec{F}_{AB} = 70\vec{i}$$

and, $\vec{F}_E = -70\vec{i}$

Let $\vec{r}$ be the position vector of the points with respect to O.

$$\vec{r}_{OD} = 3\vec{j}$$
$$\vec{r}_{OB} = 4\vec{i} + 2\vec{k}$$
$$\vec{r}_{OE} = 3\vec{j} + 2\vec{k}$$

and, $\vec{r}_{OA} = 2\vec{k}$

Taking the moments about point O; we have,

$$\begin{aligned}\overrightarrow{M} &= \vec{r}_{OD} \times \vec{F}_{OD} + \vec{r}_{OD} \times \vec{F}_{DC} + \vec{r}_{OB} \times \vec{F}_{BE} + \vec{r}_{OE} \times \vec{F}_{EA} + \vec{r}_{OE} \times \vec{F}_{E} \\ &\qquad + \vec{r}_{OA} \times \vec{F}_{AB} \\ &= 3\vec{j} \times 80\vec{j} + 3\vec{j} \times (80\vec{i} - 60\vec{j}) + (4\vec{i} + 2\vec{k}) \times (-80\vec{i} + 60\vec{j}) + (3\vec{j} + 2\vec{k}) \times \\ &\qquad (-80\vec{j}) + (3\vec{j} + 2\vec{k}) \times (-70\vec{j}) + 2\vec{k} \times 70\vec{i} \\ &= 0 - 240\vec{k} + 0 + 0 + 240\vec{k} - 160\vec{j} - 120\vec{i} + 0 + 160\vec{i} + 210\vec{k} - 140\vec{j} + 140\vec{j}\end{aligned}$$

$$\therefore \quad \overrightarrow{M}_0 = (40\vec{i} - 160\vec{j} + 210\vec{k})\text{ Nm}$$

Alternatively;

Resultant of couples can be determined by taking moments about E also.

Taking moments of E; we get,

$$\begin{aligned}\overrightarrow{M} &= \vec{r}_{EA} \times \vec{F}_{AB} + \vec{r}_{EO} \times \vec{F}_{OD} + \vec{r}_{ED} \times \vec{F}_{DC} + \vec{r}_{EB} \times \vec{F}_{BE} + \vec{r}_{E} \times \vec{F}_{E} \\ &= -3\vec{j} \times 70\vec{i} + (-3\vec{j} - 2\vec{k}) \times 80\vec{j} + (-2\vec{k}) \times (80\vec{i} - 60\vec{j}) \\ &= 210\vec{k} + 160\vec{i} - 160\vec{j} - 120\vec{i}\end{aligned}$$

$$\therefore \quad \overrightarrow{M} = (40\vec{i} - 160\vec{j} + 210\vec{k})\text{ Nm}$$

39. **Describe the scope and importance of applied mechanics in engineering study. Define free body diagram with examples.** **[2070 Chaitra]**

Solution:

The scopes of the applied mechanics are as follows:

i) To describe and predicts the conditions of rest or motion of bodies under the action of forces.
ii) For the application of equation of equilibrium to the rigid bodies.
iii) To know about the mechanics of the fluids.
iv) For the study of physical phenomena like compressibility.

The applied mechanics is the very important subject for all the engineers. The engineers taking the help of the applied mechanics are able to design of different structures such as towers, bridges, chimneys, metro designing and different civil engineering works.

Similarly, the engineer can also design different machines and foundation of machines.

Such a diagram of the body in which the body under consideration is freed from all contact surfaces and all the forces acting on it (including reaction at contact surfaces) are drawn, is called **free body diagram**.

In plotting free body diagram care is given about the contact points. The contact points when the body is isolated will have the reaction forces. If we remove the support and replace; then, by reaction which they exert on the body.

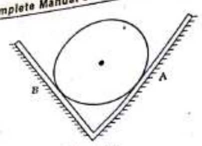


Figure: (a)

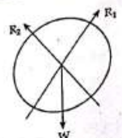
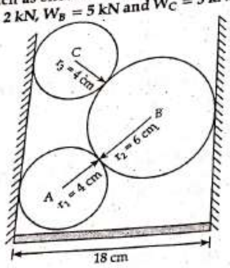


Figure: (b) Free body diagram

The contact points are A and B. When the body is isolated the reaction R_1 from B and R_2 from A will act towards centre and also weight of the body towards centre will act.

40. Determine the reactions at the contact points, if three cylinders are piled in a rectangular ditch as shown in the figure. Given that the weight of the cylinders are $W_A = 2$ kN, $W_B = 5$ kN and $W_C = 3$ kN. [2070 Chaitra]



Solution:

The free body diagram of the given figure is drawn below. The forces involved are the weights W_A, W_B and W_C of the three cylinders and reactions are R_1, R_2, R_3, R_4, R_5 and R_6 at various points of contact.

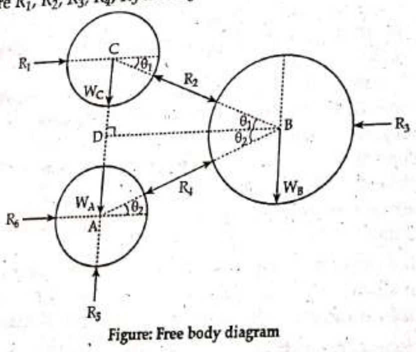


Figure: Free body diagram

From ΔCBD; we have,

$$\cos\theta_1 = \frac{BD}{BC} = \frac{18-6-4}{6+4} = 0.8$$

$\therefore \quad \theta_1 = \cos^{-1}(0.8) = 36.87°$

and, $\sin\theta_1 = \sin(36.87°) = 0.6$

Similarly, from ΔABD; we have,

$$\cos\theta_2 = \frac{BD}{AB} = \frac{18-6-4}{6+4} = 0.8$$

$\therefore \quad \theta_1 = \cos^{-1}(0.8) = 36.87°$

and, $\sin\theta_2 = \sin(36.87°) = 0.6$

Now, considering the equilibrium of cylinder C as it involves only two unknown forces;

$$\sum F_y = 0$$

or, $R_2 \sin\theta_1 - W_C = 0$

or, $R_2 \times 0.6 - 3 = 0$

$\therefore \quad R_2 = 5$ kN

Also,

$$\sum F_x = 0$$

or, $R_1 - R_2 \cos\theta_1 = 0$

or, $R_1 - 5 \times 0.8 = 0$

$\therefore \quad R_1 = 4$ kN

Considering free body diagram of cylinder B; we have,

$$\sum F_y = 0$$

or, $R_4 \sin\theta_2 - R_2 \sin\theta_1 - W_B = 0$

or, $R_4 \times 0.6 - 5 \times 0.6 - 5 = 0$

$\therefore \quad R_4 = 13.33$ kN

Also,

$$\sum F_x = 0$$

or, $R_2 \cos\theta_1 + R_4 \cos\theta_2 - R_3 = 0$

or, $5 \times 0.8 + 13.33 \times 0.8 - R_3 = 0$

$\therefore \quad R_3 = 14.67$ kN

Considering free body diagram of cylinder A; we have,

$$\sum F_y = 0$$

or, $R_5 - R_4 \sin\theta_2 - W_A = 0$

or, $R_5 - 13.33 \times 0.6 - 2 = 0$

$\therefore \quad R_5 = 10$ kN

Also,

$$\sum F_x = 0$$

or, $R_6 - R_4 \cos\theta_2 = 0$

or, $R_6 - 13.33 \times 0.8 = 0$

$\therefore \quad R_6 = 10.67$ kN

41. How can you reduce a force into a force and couple? Obtain the resultant of the two pairs of wrench shown in the figure. Indicate its line of action. [2070 Chaitra]

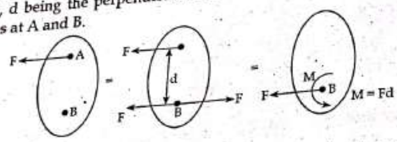
Solution:

Consider a force, F acting on a body at the point. This is to be replaced by force acting at some point B together with a couple as shown in the figure. Introduce two equal and opposite forces at B, each of magnitude F and acting parallel to the force at A.

From the principle of superposition, the second system of forces is equivalent to the single force acting at A of the three equal forces. Consider the two forces acting in opposite directions at points A and B. They form a couple of moment.

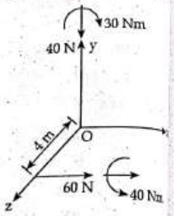
$$M = F \times d$$

Thus, the original force, F acting at the point can be replaced by a force, applied at another point B, together with a couple of magnitude $F \times d$; the distance, d being the perpendicular distance between the lines of action of the forces at A and B.



Second part

First we replace the two wrenches by an equivalent force couple system at O.



$$\vec{R} = 60\vec{i} + 40\vec{j}$$

$$(\vec{M}_0)_R = \overrightarrow{OA} \times \vec{F}_1 + C_1 + C_2$$

[Here, couple 30 Nm is negative since line of force 40 N and moment 30 N are in opposite direction]

or, $(\vec{M}_0)_R = 4\vec{k} \times 60\vec{i} + 40\vec{i} - 30\vec{j}$

$= 240\vec{j} + 40\vec{i} - 30\vec{j}$

$\therefore \quad (\vec{M}_0)_R = 40\vec{i} + 210\vec{j}$

and, $|\vec{R}| = R = \sqrt{(60)^2 + (40)^2} = 72.11$ N

To find the line of action

Magnitude of $(\vec{M}_0)_R$ along $\vec{R}$ *i.e.,* parallel to $\vec{R} = M_{11} = \vec{M} \cdot \vec{n}_R$

Here,

$$\vec{n}_R = \frac{\vec{R}}{R} = \frac{60\vec{i} - 40\vec{j}}{\sqrt{(60)^2 + (40)^2}} = 0.832\vec{i} + 0.555\vec{j}$$

$$\therefore \quad M_{11} = (\vec{M}_0)_R \times \vec{n}_R = (40\vec{i} + 210\vec{j}) \times (0.832\vec{i} + 0.555\vec{j})$$

$$= 40 \times 0.832 + 210 \times 0.555$$

$$= 149.83$$

$$\therefore \quad \vec{M}_{11} = M_{11} \times \vec{n}_R = 149.83 \times (0.832\vec{i} + 0.555\vec{j}) = 124.67\vec{i} + 83.16\vec{j}$$

Now, magnitude of $(\vec{M}_0)_R$ perpendicular to $\vec{R}$ is;

$$\vec{M}_1 = (\vec{M}_0)_R - \vec{M}_{11} = 40\vec{i} + 210\vec{j} - 124.67\vec{i} - 83.16\vec{j} = -84.67\vec{i} + 126.84\vec{j}$$

Intersection with xy plane

$$dx = -\frac{(M_1)_y}{R_z}$$

and, $dy = -\frac{M_1}{R_z}$

Intersection with xy plane is none because $R_z = 0$.

Intersection with yz plane

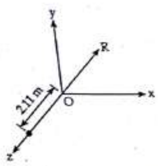
$$dy = -\frac{(M_1)_z}{R_x} = \frac{0}{60} = 0$$

$$dz = \frac{(M_1)_y}{R_z} = \frac{126.84}{60} = 2.11 \text{ m}$$

Intersection with xz plane

$$dx = \frac{(M_1)_z}{R_y} = \frac{0}{40} = 0$$

$$dz = -\frac{(M_1)_x}{R_y} = -\frac{84.67}{40} = 2.11 \text{ m}$$



Thus, line of action of resultants cuts yz plane at z = 2.11 m and xz plane is 2.11 m.

42. Why it is necessary to assume a solid body as a perfectly rigid in the engineering study. [2071 Shrawan]

Solution: See the solution of Q. no. 32 on page no. 23

43. What is free body diagram? The cylinder A and B rests in an inclined surface which makes an angle of 25° with horizontal as shown in the figure. Determine reaction at contact points. Take:
Weight of cylinder A (W_A) = 100 N
Weight of cylinder B (W_B) = 200 N
Radius of cylinder A (r_A) = 60 mm
Radius of cylinder B (r_B) = 90 mm [2071 Shrawan]

Solution:

Free body diagram

See the solution of Q. no. 39 on page no. 29

Second part

The given figure is;

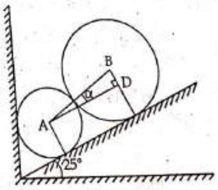


Figure: (a)

The free body diagram of the given figure is drawn.

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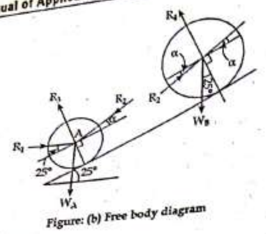


Figure: (b) Free body diagram

From figure (a); we have,

$$\sin\alpha = \frac{BD}{AB} = \frac{90-60}{90+60} = \frac{1}{5}$$

$\therefore \quad \alpha = 11.54°$

Now, referring to free body diagram of sphere B of figure (b); we have,

$$(^+\nwarrow)\sum F_y = 0$$

or, $R_4 + R_2 \sin\alpha - W_B \cos 25° = 0$

or, $R_4 + R_2 \times \frac{1}{5} - 200 \times 0.9063 = 0$

or, $R_4 + 0.2R_2 = 181.26$ (1)

$(^+\nearrow)\sum F_x = 0$

or, $R_2 + \cos\alpha - W_B \sin 25° = 0$

or, $R_2 \cos(11.54°) - 200 \times \sin 25° = 0$

or, $R_2 \times 0.9797 = 84.52$

$\therefore \quad R_2 = 86.27\ N$

Substituting the value of R_2 in the equation (1); we get,

$R_4 + 0.2 \times 86.27 = 181.26$

$\therefore \quad R_4 = 164.01\ N$

Similarly, considering the equilibrium of cylinder A; we have,

$(^+\nearrow)\sum F_x = 0$

or, $R_1 \cos 25° - R_2 \cos\alpha - W_A \sin 25° = 0$

or, $R_1 \times 0.9063 - 86.27\cos(11.54°) - 100\sin 25° = 0$

$\therefore \quad R_1 = 139.89\ N$

Also,

$$(^+\nwarrow)\sum F_y = 0$$

or, $R_3 - W_A \cos 25° - R_2 \sin\alpha - R_1 \sin 25° = 0$

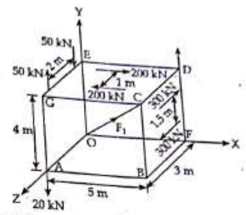
or, $R_3 - 100 \times \cos 25° - 86.27\sin(11.54°) - 139.89\sin 25° = 0$

$\therefore \quad R_3 = 167\ N$

44. Find the resultant of forces couple system at point A as shown in the figure. Take: $F_1 = 100$ kN and $F_2 = 300$ kN. Define a couple and show that couple is a free vector. **[2071 Shrawan]**

Solution:

The given figure is;



The co-ordinates of the corners of box are O(0, 0, 0), A(0, 0, 3), B(5, 0, 3), C(5, 4, 3), D(5, 4, 0), E(0, 4, 0), F(5, 0, 0) and G(0, 4, 3).

Different forces vectors are determined as;

$$\vec{F}_1 = F_1\hat{f} = F_1\frac{\overline{OC}}{OC} = 100\left(\frac{5\vec{i} + 4\vec{j} + 3\vec{k}}{\sqrt{(5)^2 + (4)^2 + (3)^2}}\right)$$

$$= 100\left(0.7071\vec{i} + 0.566\vec{j} + 0.424\vec{k}\right)$$

$$\therefore \quad \vec{F}_1 = 70.71\vec{i} + 56.6\vec{j} + 42.4\vec{k}$$

$$\vec{F}_2 = \vec{F}_{BE} = F_{BE}\hat{f} = F_{BE}\frac{\overline{BE}}{BE} = 300\left[\frac{(0-5)\vec{i} + (4-0)\vec{j} + (0-3)\vec{k}}{\sqrt{(0-5)^2 + (4-0)^2 + (0-3)^2}}\right]$$

$$= 300\left(-0.7071\vec{i} + 0.566\vec{j} - 0.424\vec{k}\right)$$

$$\therefore \quad \vec{F}_{BE} = -212.12\vec{i} + 169.8\vec{j} - 127.2\vec{k}$$

$$\vec{F}_{DF} = F_{DF}\hat{f} = F_{BE}\frac{\overline{DF}}{DF} = 20\left[\frac{(5-5)\vec{i} + (4-0)\vec{j} + (0-0)\vec{k}}{\sqrt{(5-5)^2 + (4-0)^2 + (0-0)^2}}\right]$$

$$\therefore \quad \vec{F}_{DF} = 20\vec{j}$$

$$\vec{F}_{GA} = F_{GA}\hat{f} = F_{GA}\frac{\overline{GA}}{GA} = 20\left[\frac{(0-0)\vec{i} + (0-4)\vec{j} + (3-3)\vec{k}}{\sqrt{(0-0)^2 + (0-4)^2 + (3-3)^2}}\right] = -20\vec{j}$$

and, couples are:

$C_1 = -300 \times 1.5 = -450\vec{i}$

$C_2 = -200 \times 1 = -200\vec{j}$

and, $C_3 = -50 \times 2 = -100\vec{i}$

Let $\vec{r}$ be the position vector of the points with respect to A; then,

$\vec{r}_{AB} = (5-0)\vec{i} + (0-0)\vec{j} + (3-3)\vec{k} = 5\vec{i}$

$\vec{r}_{AO} = (0-0)\vec{i} + (0-0)\vec{j} + (0-3)\vec{k} = -3\vec{k}$

$\vec{r}_{AG} = 4\vec{j}$

$\vec{r}_{AF} = 5\vec{i} - 3\vec{k}$

Now,

$$\text{Resultant of the force (R)} = \vec{F}_1 + \vec{F}_2 + \vec{F}_{DF} + \vec{F}_{GA}$$
$$= \left(70.71\vec{i} + 56.6\vec{j} + 42.4\vec{k}\right)$$
$$= \left(70.71\vec{i} + 56.6\vec{j} - 127.2\vec{k}\right) + 20\vec{j} + -20\vec{j}$$
$$+ \left(-212.12\vec{i} + 169.8\vec{j} - 127.2\vec{k}\right) \text{ kN}$$
$$= \left(-141.42\vec{i} + 226.4\vec{j} - 84.8\vec{k}\right) \text{ kN}$$

Now, taking moment about A; we get,

$$M_A = r_{GA} \times F_{GA} + r_{AB} \times F_{BE} + r_{AO} \times F_{OC} + r_{AF} \times F_{FD} + C_1 + C_2 + C_3$$
$$= 4\vec{j} \times (-20\vec{j}) + 5\vec{i} \times \left(-212.12\vec{i} + 169.8\vec{j} - 127.2\vec{k}\right) + \left(-3\vec{k}\right)$$
$$\times \left(70.71\vec{i} + 56.6\vec{j} + 42.4\vec{k}\right) + \left(5\vec{i} - 3\vec{k}\right) \times 20\vec{j} + (-450\vec{i}) + (-200\vec{j}) + (-100\vec{j})$$
$$= 0 + 0 + 849\vec{k} + 636\vec{j} - 212.13\vec{j} + 169.8\vec{i} + 0 + 100\vec{k} + 60\vec{i} - 450\vec{i} - 200\vec{j} - 100\vec{i}$$

$$\therefore \quad M_A = -320\vec{i} + 223.87\vec{j} + 949\vec{k}$$

Couple

See the solution of Q. no. 8 on page no. 8

45. What is mechanics? Mention scope of applied mechanics in engineering. [2071 Bhadra]

Solution:

The mechanics is the branch of the science which deals with the behavior of a body under the action of system of forces and also analyzes the motion of a physical system.

Scope of applied mechanics in engineering

See the solution of Q. no. 39 on page no. 29

46. Illustrate equilibrium condition of a rigid body and concept of free body diagram with suitable examples. [2071 Bhadra]

Solution:

Equilibrium is the condition in which the resultant of all forces and moments acting on a body is zero. In other way, a body is in equilibrium if all forces and moments applied to it are in balance. We have the equilibrium equation are:

$$R = \Sigma F = 0$$
$$\text{and, } M = \Sigma M = 0$$

These requirements are both necessary and sufficient conditions for equilibrium.

For 3-D

If the summation is taken about x, y and z axes then, equations of equilibrium are $\Sigma F_x = 0$, $\sum F_y = 0$, $\Sigma F_z = 0$, $\Sigma M_x = 0$, $\sum M_y = 0$ and $\Sigma M_z = 0$.

For 2-D (Planar structure)

For a planar structure in the xy plane there can be no forces acting in z-direction nor any moments about x and y-direction. Moments M_z then represents the moment about z-axis on any point in the plane. Thus, for a planar structure; we have only three equation of the equilibrium.

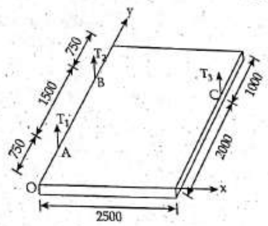
$$\Sigma F_x = 0$$
$$\sum F_y = 0$$
$$\text{and, } \Sigma F_z = 0$$

If all the forces acting on a two-dimensional structure are parallel, say parallel to the co-ordinate axis y; then term $\Sigma F_x = 0$ contains no term. Thus, there are only two effective equations of the equilibrium, *viz.*, $\sum F_y = 0$ and $\Sigma M_z = 0$ for these types of loading. Similarly, if all the forces located in a plane passes through a point, the summation of moments about this point would not contain any terms and only two equation of the equilibrium are available.

Second part

See the solution of Q. no. 39 on page no. 29

47. Three vertical wires are shown in the figure support a plate of 50 kg. Determine the tension in each wire. All dimensions are in mm. [2071 Bhadra]



Solution:

Let, O is the corner of the plate. Let the plate lies on the xy plane. Tension in the string in positive z-axis (upward) and weight of the plate in negative z-axis (downward).

At equilibrium conditions; we have,

$$\Sigma F = 0$$
$$\text{or, } T_1\vec{k} + T_2\vec{k} + T_3\vec{k} - W\vec{k} = 0$$
$$\text{or, } T_1 + T_2 + T_3 = W \qquad (1)$$

Also, taking moment about O; we have,

$$750\vec{j} \times T_1\vec{k} + 2250\vec{j} \times T_2\vec{k} + \left(2500\vec{i} + 2000\vec{j}\right) \times T_3\vec{k} + \left(1250\vec{i} + 1500\vec{j}\right) \times \left(-W\vec{k}\right) = 0$$
$$\text{or, } 750T_1\vec{i} + 2250T_2\vec{i} - 2500T_3\vec{j} + 2000T_3\vec{i} + 1250W\vec{j} - 1500W\vec{i} = 0$$

Equating coefficient of $\vec{i}$ and $\vec{j}$; we get,

$$750T_1 + 2250T_2 + 2000T_3 = 1500W \quad \text{...(2)}$$

and, $2500T_3 = 1250W$

or, $T_3 = \frac{W}{2} = \frac{50 \times 10}{2} = 250\ N$

Also, from equation (1); we get,

$$T_1 + T_2 + T_3 = W$$

or, $T_1 + T_2 + \frac{W}{2} = W \quad \text{...(4)}$

or, $T_1 = \frac{W}{2} - T_2$

Putting the value of T_1 in the equation (2); we get,

$$750\left(\frac{W}{2} - T_2\right) + 2250T_2 + 2000 \times \frac{W}{2} = 1500W$$

or, $375W - 750T_2 + 2250T_2 + 1000W = 1500W$

or, $1500T_2 = 125\ W$

∴ $T_2 = \frac{125 \times 50 \times 10}{1500}$

$= 41.67\ N$

and, $T_1 = \frac{50 \times 10}{2} - 41.67 = 208.33\ N$

48. Force $\vec{F}\left(3\vec{i} - 6\vec{j} + 4\vec{k}\right)$ N passes through point (6, 3, 2) m. Replace this force with an equivalent system, where the force $\vec{F}$ passes through point (2, 5, 10) m. [2071 Bhadra]

Solution:

Consider,

$\overrightarrow{OA} = 6\vec{i} + 3\vec{j} + 2\vec{k}$

$\overrightarrow{OB} = 2\vec{i} + 5\vec{j} + 10\vec{k}$

and, $\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$

$= 6\vec{i} + 3\vec{j} + 2\vec{k} - \left(2\vec{i} + 5\vec{j} + 10\vec{k}\right)$

$= 4\vec{i} - 2\vec{j} - 8\vec{k}$

The equivalent system consists of a force F acting at new point *i.e.*, (2, 5, 10) a couple $\vec{M}$. Thus,

$\vec{M} = \vec{r} \times \vec{F}$

$= \left(4\vec{i} - 2\vec{j} - 8\vec{k}\right) \times \left(3\vec{i} - 6\vec{j} + 4\vec{k}\right)$

∴ $\vec{M} = 40\vec{i} - 40\vec{j} - 18\vec{k}$

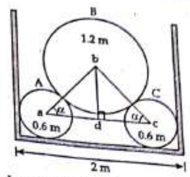
49. Describe the scope of applied mechanics. [2071 Bhadra]

Solution: See the solution of Q. no. 39 on page no. 29

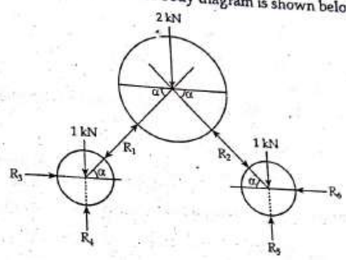
50. The cylinders A and C weight 1000 N each and the weight of cylinder B is 2000 N. Determine all forces exerted at the contact points. [2071 Magh]

Solution:

The given figure is;



Draw perpendicular bd to ac. The free body diagram is shown below.



Here,

$ac = 2 - 0.3 \times 2 = 1.4\ m$

Also, from Δabc; we have,

$$\cos\alpha = \frac{ad}{ab} = \frac{\frac{1.4}{2}}{(0.6 + 0.3)} = \frac{7}{9}$$

∴ $\alpha = 38.94°$

Considering free body diagram of cylinder B; we get,

$(^{+}\rightarrow) \sum H = 0$

or, $R_1 \cos\alpha - R_2 \cos\alpha = 0$

or, $R_1 = R_2$

$(^{+}\uparrow) \sum V = 0$

or, $R_1 \sin\alpha - R_2 \sin\alpha - W_B = 0$

or, $2R_1 \sin 38.94° = 2$

∴ $R_1 = 1.59\ kN$

∴ $R_1 = R_2 = 1.59\ kN$

Considering free body diagram of cylinder A; we get,

$(^{+}\rightarrow) \sum H = 0$

or, $R_3 - R_1 \cos\alpha = 0$

or, $R_3 - 1.59 \cos 38.94° = 0$

∴ $R_3 = 1.24\ kN$

$(^+\uparrow)\Sigma V = 0$

or, $R_4 - R_1 \sin\alpha - W_A = 0$

or, $R_4 - 1.59 \sin 38.94° - 1 = 0$

$\therefore$ $R_4 = 2\ kN$

Also, considering free body diagram of cylinder C; we get,

$(^+\rightarrow)\Sigma F_x = 0$

or, $R_2 \cos\alpha - R_6 = 0$

or, $1.59 \cos 38.94° - R_6 = 0$

$\therefore$ $R_6 = 1.24\ kN$

and, $(^+\uparrow)\sum F_y = 0$

or, $R_5 - R_2 \sin\alpha - 1 = 0$

or, $R_5 - 1.59 \sin 38.94° - 1 = 0$

$\therefore$ $R_5 = 2\ kN$

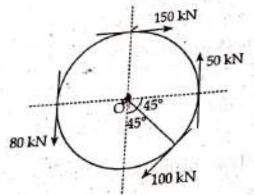
51. What is free body diagram and why it s used during analysis of structure? **[2071 Magh]**

Solution: See the solution of Q. no. 33 on page no. 23

52. Determine the resultant of the forces acting tangentially to a circle of radius 3 m as shown in the figure. What will be the location of the resultant with respect to centre of the circle? **[2071 Magh]**

Solution:

The given figure is;



Resolving the forces in x and y direction; we get,

$$(^+\rightarrow)\Sigma F_x = 150 - 100\cos 45° = 79.3\ kN(\rightarrow)$$

and, $$(^+\uparrow)\sum F_y = 50 - 80\sin 45° = -100.7\ kN = 100.7\ kN(\downarrow)$$

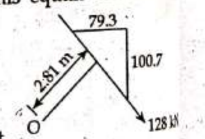
$\therefore$ Resultant force (R) $= \sqrt{(\Sigma F_x)^2 + \left(\sum F_y\right)^2} = \sqrt{(79.3)^2 + (-100.7)^2}$

$= 128\ kN$

The moment of R with respect to O is $R \times a$ and this equals the sum of moments of all the given forces about O. Hence,

$(^+\circlearrowleft) 128 \times a = 50 \times 3 + 80 \times 3 - 100 \times 3$

$\therefore$ $a = \frac{-360}{128} = 2.81\ m$



The resultant is shown in the figure, which is acting at a distance of 2.81 m from the centre, O of circle, causing negative moment.

53. Explain about the principles of applied mechanics. Why it is necessary to assume a solid body as 'perfectly rigid' for the study of statics? **[2071 Chaitra]**

Solution:

The study of applied mechanics depends upon few fundamental principles which are based on experimental observations. These are:

i) Newton's first law of motion

Everybody continues in a state of rest or uniform motion in a straight line unless an external unbalanced forces acts on it. Newton's first law contains the principle of equilibrium of forces.

ii) Newton's second law of motion

The rate of change of momentum of a body is directly proportional to the force acting on it and takes place in the direction of applied force. Thus,

$$F = \frac{mv - mu}{t} = \frac{m(v - u)}{t}$$

$\therefore$ $F = ma;$

where, F is the resultant force acting on a body of mass, m moving with acceleration, a.

This law forms the basis for most of the analysis in dynamics.

iii) Newton's third law of motion

The every action there is equal and opposite reaction which means that the forces of action and reaction between two bodies are equal in magnitude but opposite in direction.

iv) Newton's law of gravitation

Two particles are attracted towards each other along the line connecting them with a force whose magnitude is proportional to the product of their masses and inversely proportional to the square of the distance between them.

$$F = G\frac{m_1 m_2}{r^2}$$

The force of attraction exerted by the earth on a particle lying on its surface is governed by this law.

v) Principle of transmissibility

It states that, "the condition of equilibrium or of motion of rigid body will remain unchanged if the point of application of force acting on the rigid body is transmitted to act at any other points along its line of action".

vi) The parallelogram law

If two forces acting at a point are represented in magnitude and direction by the adjacent sides of parallelogram then the diagonal of the parallelogram passing through their point of intersection represent the resultant in both magnitude and direction.

54. Define free body diagram with example and explain about equation of static equilibrium for 2-D and 3-D analysis of a particle and a rigid body. **[2071 Chaitra]**

Solution:

Free body diagram with example

See the solution of Q. no. 5 on page no. 5

Second part

When a particle is at rest or moves with constant velocity, it is in equilibrium. This requires that all the forces acting on the particle from a zero resultant.

For 2-D

The two scalar equations of forces equilibrium can be applied with reference to an establish x, y co-ordinate system.

i.e., $\Sigma F_x = 0$

and, $\sum F_y = 0$

For 3-D

For the particle equilibrium equations are $\Sigma F = 0$, $\Sigma F_x = 0$, $\sum F_y = 0$ and $\Sigma F_z = 0$.

A rigid body is said to be in the equilibrium if it is at rest or can translate with constant velocity.

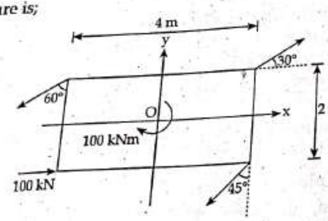
Rigid body equilibrium

See the solution of Q. no. 46 on page no. 36

55. **Describe the principle of transmissibility. Determine the magnitude, direction and position with respect to centre O of the resultant of the forces acting on the rectangular plate ABCD as shown in the figure below.** **[2071 Chaitra]**

Solution:

The given figure is;



Let us consider that x and y co-ordinate axes are chosen such that origin lies on point O. Thus, the plate ABCD lies in xy plane. Taking unit vectors $\vec{i}$ and $\vec{j}$ are in x-axis and y-axis respectively. Resolving all the forces on x-axis and y-axis; we get,

$$\Sigma F_x = (300\cos 30°)\vec{i} + 100\vec{i} - (200\sin 60°)\vec{i} - (150\sin 45°)\vec{i} = 80.54\vec{i}(\rightarrow)$$

and, $$\sum F_y = (300\sin 30°)\vec{j} - 200\cos 60°\vec{j} - (150\cos 45°)\vec{j} = -56.07\vec{j}$$

$\therefore$ $$R = \Sigma F_x + \sum F_y = 80.54\vec{i} - 56.07\vec{j}$$

$\therefore$ $$|R| = \sqrt{80.54 - 56.07} = 98.13 \text{ kN}$$

and, the direction is;

$$\tan\alpha = \frac{\sum F_y}{\sum F_x} = \frac{-56.07}{80.54} = -0.6962$$

$\therefore$ $\alpha = -34.84°$;

which lies in the fourth quadrant.

Taking moment about O; we get,

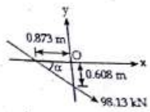
$$(x\vec{i} + y\vec{j}) \times (80.54\vec{i} - 56.07\vec{j}) = (2\vec{i} + 1.5\vec{j}) \times (80.54\vec{i} - 56.07\vec{j}) \times (300\cos 30°\vec{i} + 300\sin 30°\vec{j}) + (2\vec{i} - 1.5\vec{j}) \times (-150\cos 45°\vec{i} - 150\sin 45°\vec{j}) + (2\vec{i} + 1.5\vec{j}) \times (100\vec{i}) + (-2\vec{i} + 1.5\vec{j}) \times (-200\sin 60°\vec{i} - 200\cos 60°\vec{j}) - 100\vec{k}$$

or, $$-56.07x\vec{k} - 80.54y\vec{k} = 300\vec{k} - 389.71\vec{k} - 212.13\vec{k} - 159.01\vec{k} + 150\vec{k} + 200\vec{k} + 259.81\vec{k} - 100\vec{k}$$

or, $$-56.07x\vec{k} - 80.54y\vec{k} = 48.96\vec{k}$$

or, $$\frac{x}{-\frac{48.96}{56.07}} + \frac{y}{-\frac{48.96}{80.54}} = 1$$

or, $$\frac{x}{-0.873} + \frac{y}{-0.608} = 1$$



i.e., x-intercept = −0.873 m

and, y-intercept = −0.608 m

Alternatively (scalar method)

$$\Sigma F_x = 300\cos 30° + 100 - 200\sin 60° - 150\sin 45° = 80.54 \text{ kN}(\rightarrow)$$

$$\sum F_y = 300\sin 30° - 200\cos 60° - 150\cos 45° = -56.07 \text{ kN}$$

$\therefore$ $$R = \sqrt{(\Sigma F_x)^2 + \left(\sum F_y\right)^2} = 98.13 \text{ kN}$$

and, $$\tan\alpha = \frac{\sum F_y}{\sum F_x} = \frac{-56.07}{80.54} = -0.6962$$

$\therefore$ $\alpha = -34.84°$

Taking moment about O and taking anti-clockwise as positive;

$$\Sigma M_O = (200\sin 60°) \times 1.5 + (200\cos 60°) \times 2 + 100 \times 1.5 - (150\cos 45°) \times 2 - (150\sin 45°) \times 1.5 - (300\cos 30°) \times 1.5 + (300\sin 30°) \times 2 - 100$$

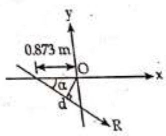
$\therefore$ $\Sigma M_O = 48.96$ kNm

Applying Varignon's theorem at O; we have,

$$R \times d = \Sigma M_O$$

or, $98.13d = 48.96$

$\therefore$ $d = 0.4989$ m



and, $$x = \frac{d}{\sin\alpha} = \frac{0.4989}{\sin(34.84°)} = 0.873 \text{ m}$$

and, $$y = \frac{d}{\cos\alpha} = \frac{0.4989}{\cos(34.84°)} = 0.608 \text{ m}$$

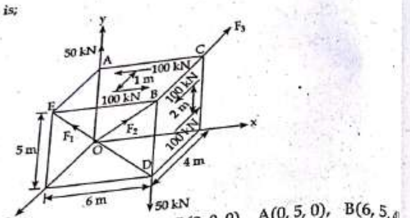
56. **What is mechanics? Mention scope of applied mechanics in engineering.** **[2072 Ashwin]**

Solution: See the solution of Q. no. 39 on page no. 29

57. **Define free body diagram with example. Explain about the principle of transmissibility. Determine the force couple system at origin of the given system. Take: $F_1 = 100$ kN, $F_2 = 300$ kN and $F_3 = 200$ kN** [2072 Ashwin]

Solution:

The given figure is;



The co-ordinates of corner boxes are O(0, 0, 0), A(0, 5, 0), B(6, 5, 4), C(6, 5, 0), D(6, 0, 4) and E(0, 5, 4).

Different forces vector are determined as;

$$\vec{F}_{OA} = \hat{r}F_{OA} = 50\vec{j}$$

$$\vec{F}_{OB} = \vec{F}_2 = \hat{r}F_{OB} = 300\left[\frac{(6-0)\vec{i} + (5-0)\vec{j} + (4-0)\vec{k}}{\sqrt{(6-5)^2 + (5-0)^2 + (4-0)^2}}\right]$$

$$= 300\left(0.684\vec{i} + 0.569\vec{j} + 0.456\vec{k}\right)$$

$$= 205.2\vec{i} + 170.7\vec{j} + 136.8\vec{k}$$

$$\vec{F}_{BC} = \vec{F}_3 = \hat{r}F_3 = 200\left[\frac{(6-6)\vec{i} + (5-5)\vec{j} + (0-4)\vec{k}}{\sqrt{(6-6)^2 + (5-5)^2 + (0-4)^2}}\right] = -200\vec{k}$$

$$\vec{F}_{BD} = \hat{r}F_{BD} = 50\left[\frac{(6-6)\vec{i} + (0-5)\vec{j} + (4-4)\vec{k}}{\sqrt{(6-6)^2 + (0-5)^2 + (4-4)^2}}\right] = -50\vec{j}$$

$$\vec{F}_{DE} = \vec{F}_1 = \hat{r}F_1 = 100\left[\frac{(0-6)\vec{i} + (5-0)\vec{j} + (4-4)\vec{k}}{\sqrt{(0-6)^2 + (5-0)^2 + (4-4)^2}}\right]$$

$$= 100(-0.768\vec{i} + 0.6401\vec{j}) = -76.8\vec{i} + 64.01\vec{j}$$

and, $C_1 = 100 \times 1 = 100\vec{j}$ kNm

$C_2 = 100 \times 2 = 200\vec{i}$ kNm

Let, $\vec{r}$ be the position vector with respect to O; then,

$$\vec{r}_{OA} = 5\vec{j}$$

$$\vec{r}_{OB} = 6\vec{i} + 5\vec{j} + 4\vec{k}$$

$$\vec{r}_{OD} = 6\vec{i} + 4\vec{k}$$

Taking moment about point O; we get,

$$\vec{M}_O = \vec{r}_{OA} \times \vec{F}_{OA} + \vec{r}_{OB} \times \vec{F}_{OB} + \vec{r}_{OB} \times \vec{F}_{BC} + \vec{r}_{OB} \times \vec{F}_{BD} + \vec{r}_{OD} \times \vec{F}_{DE} + C_1 + C_2$$

$$= (5\vec{j} \times 50\vec{j}) + \left(6\vec{i} + 5\vec{j} + 4\vec{k}\right) \times \left(205.2\vec{i} + 170.7\vec{j} + 136.8\vec{k}\right) + \left(6\vec{i} + 5\vec{j} + 4\vec{k}\right) \times \left(-200\vec{k}\right) + \left(6\vec{i} + 5\vec{j} + 4\vec{k}\right) \times (-50\vec{j}) + \left(6\vec{i} + 4\vec{k}\right) \times (-76.8\vec{i} + 64.01\vec{j}) + 100\vec{j} + 200\vec{i}$$

$$\therefore \quad \vec{M}_O = -854.84\vec{i} + 992.8\vec{j} - 600.54\vec{k} \text{ kNm}$$

58. **Determine reaction of all contact points. Assume all contact surface are smooth. Take weight of sphere A = 200 kN, weight of sphere B = 400 kN, radius of sphere A = 120 mm and radius of sphere B = 250 mm** [2072 Ashwin]

Solution:

The given figure is;

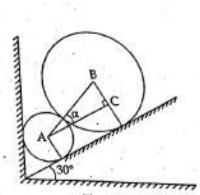


Figure: (a)

The free body diagram of given figure is drawn below.

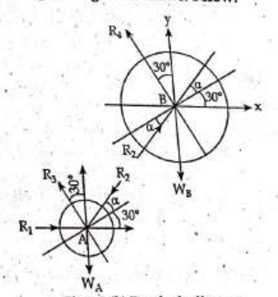


Figure: (b) Free body diagram

From ΔABC in the figure (a); we get,

$$\sin\alpha = \frac{250 - 120}{250 + 120} = 0.351$$

$\therefore \quad \alpha = 20.57°$

Now, considering equilibrium of block B; we get,

$(\overset{+}{\rightarrow}) \sum F_x = 0$

or, $R_2 \cos(30° + \alpha) - R_4 \sin 30° = 0$

or, $R_2 \cos(30° + 20.57°) - 0.5R_4 = 0$

or, $0.635R_2 - 0.5R_4 = 0$

$\therefore \quad R_4 = \frac{0.635R_2}{0.5}$

Also,

$(^+\uparrow)\sum F_y = 0$

or, $R_4 \cos 30° - W_B - R_2 \sin(30° + 20.57°) = 0$

or, $\left(\frac{0.635R_2}{0.5}\right)\cos 30° - 400 - 0.770R_2 = 0$

$\therefore \quad R_2 = 213.69\ kN$

From equation (1); we get,

$R_4 = \frac{0.635 \times 213.69}{0.5} = 217.38\ kN$

Again, considering equilibrium of block A; we get,

$(^+\uparrow)\sum F_y = 0$

or, $R_3 \cos 30° - R_2 \sin(30° + \alpha) - W_A = 0$

or, $R_3 \cos 30° - 213.69 \times 0.770 - 200 = 0$

$\therefore \quad R_3 = 421.43\ kN$

and, $(^+\rightarrow)\sum F_x = 0$

or, $R_1 - R_2 \cos(30° + \alpha) - R_3 \sin 30° = 0$

or, $R_1 - 213.69 \times 0.635 - 421.43 \times 0.5 = 0$

$\therefore \quad R_1 = 346.4\ kN$

59. Explain physical meaning of equilibrium and its application in structu[ral] engineering. [2072 Karti[k]]

Solution:

For the first part

See the solution of Q. no. 25 on page no. 19

For the second part

The primary use of equilibrium analysis is to evaluate the reaction a[nd] internal forces by forming a series of free body diagram. Also, the analy[sis] becomes simpler when the structure as well as load is co-planar a[nd] equation of equilibrium can be applied. Thus, using equation of equilibri[um] easily determined reaction of the structure.

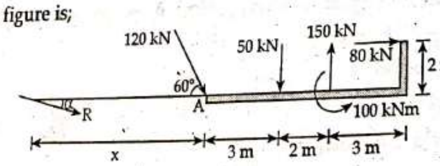
60. Differentiate between rigid body and deformable body. Also explain f[ree] body diagram. [2072 Karti[k]]

Solution: See the solution of Q. no. 13 on page no. 12

61. Determine the magnitude, direction and position of the resultant of t[he] system of forces with respect to point A as shown in the figure. [2072 Karti[k]]

Solution:

The given figure is;



Resolving the forces into x and y direction; we get,

$\sum F_x = 120 \cos 60° + 80 = 140\ kN(\rightarrow)$

and, $\sum F_y = -120 \cos 60° - 50 + 150 = -3.92\ kN = 3.92\ kN(\downarrow)$

$\therefore$ Resultant $(R) = \sqrt{(\sum F_x)^2 + \left(\sum F_y\right)^2} = \sqrt{(140)^2 + (3.92)^2} = 140.05\ kN$

and, $\tan \alpha = \frac{\sum F_y}{\sum F_x} = \frac{-3.92}{140} = -0.028$

$\therefore \quad \alpha = 1.60°$

Let the resultant lies x distance from A. Then, taking moment about A;

$\sum F_y \times x = -50 \times 3 + 150 \times 5 + 100 - 80 \times 2$

or, $R \sin \alpha \times x = 540$

or, $140.05 \sin(1.60°) \times x = 540$

$\therefore \quad x = 138.09\ m$

Thus, resultant R acting 138.09 m left to A.

62. How does a deformable body differ from rigid body? What were the assumptions made regarding rigid body for our present study? [2072 Magh]

Solution:

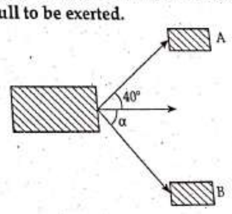
The following assumptions are made while considering the rigid body of the present study.

i) The body does not deform under the action of applied load.

ii) The distance between any two points of the body does not change under the action of applied load.

iii) We do not have to consider the type of material from which the body is made.

63. A vehicle needs 50 kN to be moved forward by two pullers A and B. Puller A is at 40° to the axis of movement. Compute the value of angle α for which puller B has to exert minimum force. Also compute the respective values of pull to be exerted. [2072 Magh]



Solution:

In order to exert a minimum force for puller B then F_B has to act perpendicular to puller A *i.e.,* perpendicular to F_A.

so, angle between F_A and $F_B = 90°$

Now,

$\alpha + 40° = 90°$

$\therefore \quad \alpha = 50°$

Resolving forces in x and y-direction; we get,

$$(^+\rightarrow) \sum F_x = 0 \quad (1)$$

or, $F_A \cos 40° + F_B \cos \alpha + 50 = 0$

or, $F_A \cos 40° + F_B \cos 50° = -50$

Also,

$$(^+\uparrow) \sum F_y = 0$$

or, $F_A \sin 40° - F_B \sin \alpha = 0$

or, $F_A \sin 40° - F_B \sin 50° = 0$

$\therefore \quad F_A = \dfrac{F_B \sin 50°}{\sin 40°} = 1.19 F_B$

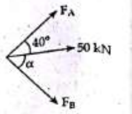


Figure: F.B.D. of pulley

Now,

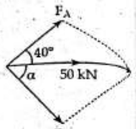
$1.19 F_B \cos 40° + F_B \cos 50° = -50$

$\therefore \quad F_B = -32.26 \text{ kN} = 32.26 \text{ kN} (\swarrow)$

and, $F_A = 1.19 \times (-32.26) = -38.39 \text{ kN} = 38.39 \text{ kN} (\nwarrow)$

Alternatively;

The parallelogram law of addition is shown in the figure.



Here,

$F_B = 50 \sin 40° = 32.12 \text{ kN}$

and, $F_A = 50 \cos 40° = 38.30 \text{ kN}$

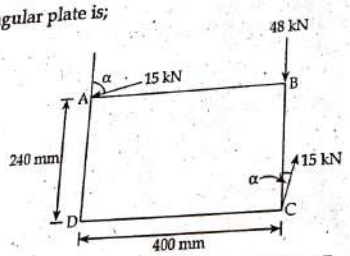
64. A rectangular plate is acted upon by the force and couple shown in the figure below. The system is to be replaced by a signal equivalent force.

i) For $\alpha = 40°$, specify the magnitude and the line of action of the equivalent force.

ii) Specify the value of α, if the line of action of the equivalent force is to intersect line CD 300 mm to the right of D. [2072 Magh]

Solution:

The given rectangular plate is;



i) Taking B as origin and summing forces vertically at B; then,

$$(^+\downarrow) \sum F_y = 48 + 15 \cos \alpha - 15 \cos \alpha = 48 \text{ kN}$$

and, taking moment about B; then,

$$\sum M_B = (0.4 \times 15 \cos 40°) + (0.24 \times 15 \sin 40°) = 6.91 \text{ kNm}$$

or, $\sum M_B = 6.91 \text{ kNm} (\circlearrowleft)$

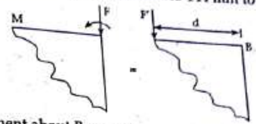
The single equivalent force F' is equal to F. Further for equivalence;

$\sum M_B = dF'$

or, $6.91 = d \times 48$

$\therefore \quad d = 0.144 \text{ m}$

and, line of action of F' intersects line AB 144 mm to right of A.



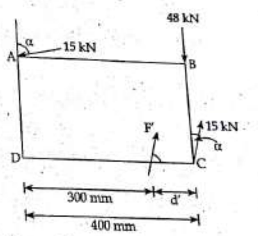
ii) Taking moment about B; we get,

$\sum M_B = F'd'$

or, $(0.4 \times 15 \cos \alpha) + (0.24 \times 15 \sin \alpha) = (0.4 - 0.3) \times 48$

or, $6 \cos \alpha + 3.6 \sin \alpha = 4.8$

or, $5 \cos \alpha + 3 \sin \alpha = 4$



Rearranging and squaring; we get,

$$25 \cos^2 \alpha = (4 - 3 \sin \alpha)^2$$

Using $\cos^2 \alpha = 1 - \sin^2 \alpha$ and expanding; we have,

$$25(1 - \sin^2 \alpha) = 16 - 24 \sin \alpha + 9 \sin^2 \alpha$$

or, $34 \sin^2 \alpha - 24 \sin \alpha - 9 = 0$

Then,

$$\sin \alpha = \frac{24 \pm \sqrt{(-24)^2 - [4 \times 34 \times (-9)]}}{2 \times 34}$$

Taking positive sign; we have,

or, $\sin \alpha = 0.97$

$\therefore \quad \alpha = \sin^{-1}(0.97) = 77.7°$

Taking negative sign; we have,

or, $\sin \alpha = -0.271$

$\therefore \quad \alpha = \sin^{-1}(-0.271) = -15.02°$

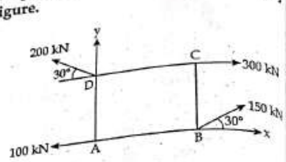
65. **Describe about fundamental principle of applied mechanics.** [2072 Chaitra]

Solution: See the solution of Q. no. 53 on page no. 41

66. **Write down the steps to be considered while drawing a free body diagram. Illustrate equilibrium condition of the particle and rigid body in two and three dimensional analysis.** [2072 Chaitra]

Solution: See the solution of Q. no. 13 and 54 on page no. 12 and 41 respectively

67. **Find the magnitude, direction and position of resultant forces of the following system as shown in the figure.** [2072 Chaitra]



Solution:

Let us consider that x and y co-ordinate axes are chosen such that origin lies on point A as shown in the figure. Thus, the plate ABCD lies on xy plane. Taking unit vectors $\vec{i}$ and $\vec{j}$ are along x-axis and y-axis respectively.

Resolving all forces along x and y-direction; we get,

$$(^{+}\rightarrow)\sum F_x = (150\cos 30°)\vec{i} + 300\vec{i} - (200\cos 30°)\vec{i} - 100\vec{i} = 156.7\vec{i}\text{ kN }(\rightarrow)$$

and, $$(^{+}\uparrow)\sum F_y = (150\sin 30°)\vec{j} + (200\sin 30°)\vec{j} = 175\vec{j}\text{ kN }(\uparrow)$$

Now,

$$\vec{R} = \sum F_x + \sum F_y = (156.7\vec{i} + 175\vec{j})\text{ kN}$$

$$\therefore \quad R = \sqrt{(\sum F_x)^2 + \left(\sum F_y\right)^2} = \sqrt{(156.7)^2 + (175)^2} = 234.90\text{ kN}$$

and, the direction is given by;

$$\tan\alpha = \frac{\sum F_y}{\sum F_x} = \frac{175}{156.7} = 1.12$$

$$\therefore \quad \alpha = \tan^{-1}(1.12) = 48.15°$$

Let's resultant lies $(x\vec{i} + y\vec{j})$ from A; then, using Varignon's theorem; we get,

$$(x\vec{i} + y\vec{j}) \times (156.7\vec{i} + 175\vec{j}) = 5\vec{i} \times (150\cos 30°\,\vec{i} + 150\sin 30°\,\vec{j})$$
$$+ (5\vec{i} + 3\vec{j}) \times 300\vec{i} + 3\vec{j} \times (-200\cos 30°\,\vec{i} + 200\sin 30°\,\vec{j})$$

or, $$175x\vec{k} - 156.7y\vec{k} = 375\vec{k} + 0 - 900\vec{k} + 519.62\vec{k}$$

or, $$175x\vec{k} - 156.7y\vec{k} = -5.38\vec{k}$$

or, $$\frac{x}{-\left(\frac{5.38}{175}\right)} + \frac{y}{\left(\frac{5.38}{156.7}\right)} = 1$$

or, $$\frac{x}{-0.031} + \frac{y}{0.034} = 1$$

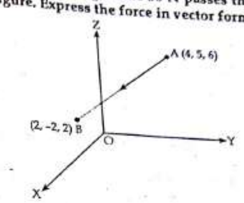
$\therefore$ x-intercept = −0.031

and, y-intercept = 0.034

Thus, resultant R cuts along x-axis at x = −0.031 m and along y-axis at y = 0.034 m.

ADDITIONAL PROBLEMS

1. **The line of action of a force of magnitude 50 N passes through points 'A' and 'B' as shown in figure. Express the force in vector form.**



Solution:

From the figure; we have,

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (2\vec{i} - 2\vec{j} + 2\vec{k}) - (4\vec{i} + 5\vec{j} + 6\vec{k}) = -2\vec{i} - 7\vec{j} - 4\vec{k}$$

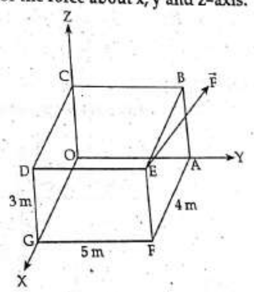
Therefore, unit vector along $\overrightarrow{AB}$ is;

$$\hat{n}_{AB} = \frac{-2\vec{i} - 7\vec{j} - 4\vec{k}}{\sqrt{(-2)^2 + (-7)^2 + (-4)^2}} = -0.241\vec{i} - 0.843\vec{j} - 0.482\vec{k}$$

Therefore the force can be expressed in vector form as;

$$\vec{F} = F\hat{n}_{AB}$$
$$= 50[-0.241\vec{i} - 0.843\vec{j} - 0.482\vec{k}]$$
$$= -12.05\text{ N}\vec{i} - 42.15\text{N}\vec{j} - 24.1\text{ N}\vec{k}$$

2. **A force $\vec{F} = (50\text{ N})\vec{i} + (75\text{ N})\vec{j} + (100\text{ N})\vec{k}$ acts as shown in figure below determine the moment of the force about x, y and z-axis.**



Solution:

The position vector of point of application of the force is;

$$\overrightarrow{OE} = 4\vec{i} + 5\vec{j} + 3\vec{k}$$

Therefore the moment of the force about the origin is given as;

$$\vec{M_0} = \vec{OE} \times \vec{F} = (4\vec{i} + 5\vec{j} + 3\vec{k}) \times (50\vec{i} + 75\vec{j} + 100\vec{k})$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 5 & 3 \\ 50 & 75 & 100 \end{vmatrix}$$

$$= [\vec{i}(500 - 225) - \vec{j}(400 - 150) + \vec{k}(300 - 250)]$$

$$= 275\vec{i} - 250\vec{j} + 50\vec{k}$$

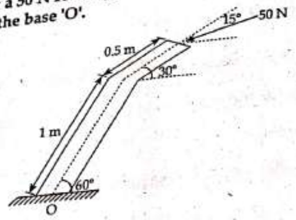
Hence, moment of force about x, y and z-axis are given by taking dot product of unit vectors along respective directions and moment vector,

i.e., $M_x = \vec{i} \cdot \vec{M_0} = 275$ Nm

$M_y = \vec{j} \cdot \vec{M_0} = -250$ Nm

$M_z = \vec{k} \cdot \vec{M_0} = 50$ Nm

3. **Find the moment of a 50 N force applied on a bent handle as shown in the figure below about the base 'O'.**



Solution:

From the figure, we can see that the inclination of the force with respect to the horizontal;

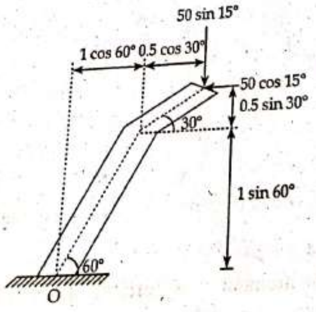
i.e., X-axis is 30° – 15° = 15°

Hence, resolving the force in to rectangular components along X and Y axes; we have,

$F_x = 50 \cos 15°$

and, $F_y = 50 \sin 15°$

The perpendicular distances of the components from the origin 'O' are in the figure below.

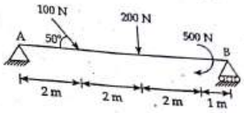


Taking the summation of the moments about 'O' considering the sign convention; we have,

$$M_0 = -50 \cos 15° [(0.5) \sin 30° + 1 \sin 60°] + 50 \sin 15° [0.5 \cos 30° + 1 \cos 60°]$$
$$= -41.83 \text{ Nm}$$

The negative sign indicates that the moment is in anti-clockwise direction.

4. **In figure shown, reduce the given system of forces acting on beam AB to (i) an equivalent force-couple system at 'A', and (ii) an equivalent force-couple system at 'B'.**



Solution:

Taking summation of all the forces along x-and y-directions,

$\Sigma F_x = 100 \cos 50° = 64.28$ N

$\Sigma F_y = -100 \sin 50° - 200 = -276$ N

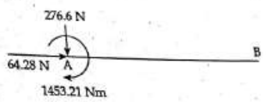
The negative sign indicates that the y-component of the resultant is directed along the negative y-direction.

i) Equivalent force couple system at A;

(Taking clockwise positive)

$\Sigma M_A = +100 \sin 50° \times 2 + 200 \times 4 + 500$

$= 1453.21$ Nm



The positive sign indicates clockwise moment.

Note

The couple 500 Nm is a free vector and that it can be placed anywhere on the beam. The equivalent force couple system is shown in the figure.

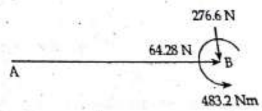
ii) Equivalent force-couple system at B;

Taking summation of the moments of all the forces about B

(Taking clockwise positive)

$\Sigma M_B = -100 \sin 50° \times 5 - 200 \times 3 + 500$

$= -483.02$ Nm



The negative sign indicates that it is in the anticlockwise direction.

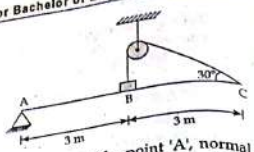
The equivalent force-couple system is shown in the figure.

5. **A beam AC hinged at 'A', is held in a horizontal position by a cable attached at end 'C' and passing over a smooth pulley as shown in figure. The free end of the cable is connected to a weight 2000 N that results on the beam. Determine the reaction at 'A' and tension in the cable. Neglect the weight of the beam.**

Solution:

To get a clear picture if we consider free-body diagrams for beam and weight separately as shown in the figure.

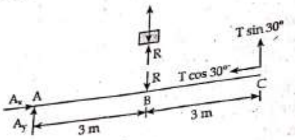
The forces acting on the weight resting on the beam are its own weight 'W'; tension 'T' due to its attachment with the cable and normal reaction 'R' exerted by the beam. As the cable passing over a smooth pulley, the tension at both ends will be equal to 'T'.



The forces acting on the beam are support reaction at the point 'A', normal reaction 'R' exerted by the weight on the beam and tension 'T' due to its attachment with the cable. 'T' has been resolved in to components along x and y directions.

Applying the equilibrium condition for the weight,

$(^+\uparrow)\ \Sigma F_y = 0;$



$T + R - 2000 = 0$ (1)

$\therefore\quad T + R = 2000\ N$

Similarly, applying the conditions of equilibrium for the beam,

$(^+\rightarrow)\ \Sigma F_x = 0;$

$A_x - T\cos 30^\circ = 0$ (2)

$\therefore\quad A_x = T\cos 30^\circ$

$(^+\uparrow)\ \Sigma F_y = 0$

$A_y + T\sin 30^\circ - R = 0$ (3)

$A_y = R - T\sin 30^\circ = R - 0.5\,T$

Taking moment about the hinge 'A' and equating it to zero,

$(^+\circlearrowleft)\ \Sigma M_A = 0;$

or, $T\sin 30^\circ \times 6 - R \times 3 = 0$ (4)

or, $T = R$

Substituting this in the equation (1); we get,

$T = R = 1000\ N$

From equation (2); we get

$A_x = T\cos 30^\circ$

$= (1000)\cos 30^\circ$

$= 866.03\ N$

From equation (3); we get,

$A_y = R - 0.5\,T$

$= 1000 - 0.5 \times 1000$

$= 500\ N$

6. Determine the required length of cord AC in figure below so that the 8 kg lamp is suspended in the position shown. The undeformed length of the spring AB is $l'_{AB} = 0.4$m, and the spring has a stiffness of $K_{AB} = 300$ N/m.

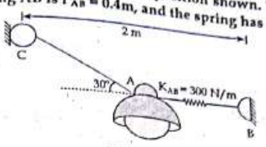


Figure: (a)

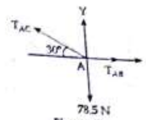


Figure: (b)

Solution:

If the force in spring AB is known, the stretch of the spring can be found (f = ks). Using the problem geometry it is then possible to calculate the required length of AC.

The lamp has a weight W = 8 (9.8) = 78.5 N. The free-body diagram of the ring at 'A' is shown in the figure above.

Equations of equilibrium

Using the x, y-axis; we have,

$(^+\rightarrow)\ \Sigma F_x = 0;$

or, $T_{AB} - T_{AC}\cos 30^\circ = 0$

$(^+\uparrow)\ \Sigma F_y = 0;$

or, $T_{AC}\sin 30^\circ - 78.5\ N = 0$

On solving; we obtain,

$T_{AC} = 157.0$ N and

and, $T_{AB} = 136.0$ N

The stretch of spring AB is;

$T_{AB} = K_{AB} S_{AB};$

or, $136 = 300\,(S_{AB})$

$\therefore\quad S_{AB} = 0.453$ m

so, Stretched length $(L_{AB}) = l'_{AB} + S_{AB}$

or, $L_{AB} = 0.4\ m + 0.453\ m = 0.853$ m

The horizontal distance from 'C' to 'B';

$2 = l_{AC}\cos 30^\circ + 0.853$

$\therefore\quad l_{AC} = 1.32$ m

7. Determine the magnitude of the resultant force and its direction measured counter-clockwise from positive x-axis.

Solution:

We have,

Forces along x-direction $(F_x) = F_1\sin 30^\circ + F_2\cos 45^\circ$

$= 250\sin 30^\circ + 375\cos 45^\circ = 390.16$ lb

Similarly,

Force along y-direction $(F_y) = F_1\cos 30^\circ - F_2\sin 45^\circ$

$= 250\cos 30^\circ - 375\sin 45^\circ$

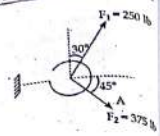
$= -48.6$ lb

Now,

Resultant force (R) $= \sqrt{F_x^2 + F_y^2}$

$= \sqrt{(390.16)^2 + (-48.6)^2}$

$= 393.17\ lb$



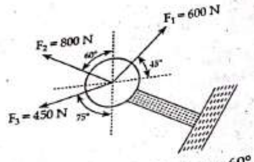
Now, we have direction,

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{-48.6}{390.16}\right) = 353°$

8. Determine the magnitude of the resultant force and its direction measured counter clockwise from the positive x-axis.

Solution:

We have,



Force along y-direction $(F_y) = F_1 \sin 45° + F_2 \cos 60° - F_3 \cos 75°$

$= 600 \sin 45° + 800 \cos 60° - 450 \cos 70°$

$= 707.8$ N

and, Force along X-direction $(F_x) = F_1 \cos 45° - F_2 \sin 60° - F_3 \sin 75°$

$= 600 \cos 45° - 800 \sin 60° - 450 \sin 75°$

$= -703.22$ N

Resultant force (R) $= \sqrt{(F_x)^2 + (F_y)^2} = \sqrt{(-703.22)^2 + (707.8)^2} = 997.7$

Now,

Direction of resultant force $(\theta) = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{707.8}{-703.22}\right) = 135$

9. Two forces are applied at the end of a screw eye in order to remove post determine the angle $\theta (0 \le \theta \le 90°)$ and the magnitude of the force that the resultant force acting on the post is directed vertically upward has a magnitude of 750 N.

Solution:

We have,

Force along x-direction $(F_x) = 0 = F \sin 30° - 500 \sin \theta$

or, $F \sin 30 = 500 \sin \theta$

Force along y-direction $(F_y) = 750 = 500 \cos \theta + F \cos 30°$

Now,

$F \cos 30° = 750 - 500 \cos\theta$

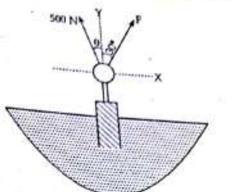
Squaring and adding equation (1) and (2); we get,

$500^2 \cos^2\theta + 500^2 \sin^2\theta = (F \cos 30° - 750)^2 + (F \sin 30°)^2$

or, $500^2 (\cos^2\theta + \sin^2\theta) = (F \cos 30°)^2 - 2 \times 750 \times F \cos 30° + (F \sin 30°)^2 + ($

or, $500^2 = F^2 - 1500\ F \cos 30° + (750)^2$

or, $F^2 - 1299.04\ F - 250000 + (750)^2 = 0;$



which is quadratic equation in 'F'; on solving; we get,

$F = 318.799 \approx 319$ N

Now, from equation (1); we get,

$500 \sin \theta = 319 \sin 30°$

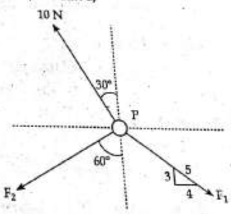
or, $\theta = \sin^{-1}\left(\frac{319 \times 0.5}{500}\right)$

$\therefore\ \theta = 18.6°$

10. Determine the magnitude of F_1 and F_2 so that the particle is in equilibrium.

Solution:

Using equilibrium equation; we have,



$(^+\rightarrow)\ \Sigma F_x = 0;$

$F_1 \cos 36.87 - F_2 \sin 60° - 400 \sin 30° = 0$

or, $0.8\ F_1 - 0.87\ F_2 - 200 = 0$ (1)

Again,

$\Sigma F_y = 0;$

$400 \cos 30° - F_2 \cos 60° - F_1 \sin 36.87° = 0$

or, $0.6F_1 + 0.5F_2 = -346.4 = 0$ (2)

On solving equation (1) and (2); we get,

$F_1 = 435.32$ lb

and, $F_2 = 170.41$ lb

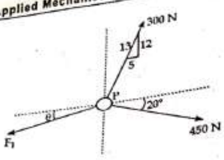
11. Determine magnitude and direction of F_1 so that particle 'p' is in equilibrium.

Solution:

Using equilibrium equation; we have,

$(^+\rightarrow)\ \Sigma F_x = 0;$

$300 \cos 67.38° + 450 \cos 20° - F_1 \cos \theta = 0$



(1)

$\therefore \quad F_1 \cos\theta = 538.247$

Again,

$\Sigma F_y = 0;$

$300 \sin 67.38° - 450 \sin 20° - F_1 \sin\theta = 0$ (2)

or, $F_1 \sin\theta = 123$

Squaring and adding equation (1) and (2); we have,

$F_1{}^2 = 123^2 + (538.247)^2$

$\therefore \quad F_1 = 552$ N

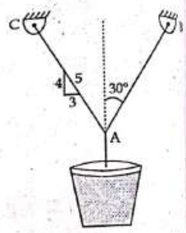
Now from equation (i); we get,

$552 \cos\theta = 538.247$

$\therefore \quad \theta = 12.9°$

12. **Determine the maximum weight of the flowerpot that can be supported without exceeding a cable tension of 50 lb in either cable AB or AC.**

Solution:



Let the tension in cable AB = 50 *lb*.

Now, tension in cable AC can be calculated as;

$T_{AB} \cos 30° + T_{AC} \sin (53.13°) - W = 0$ (1)

or, $50 \cos 30° + T_{AC} \sin (53.13°) - W = 0$

Now, force in x–direction;

$-T_{AC} \cos 53.13° + T_{AB} \sin 30° = 0$ (2)

$\therefore \quad T_{AC} = \dfrac{50 \sin 30°}{\cos 53.13°} = 41.67$

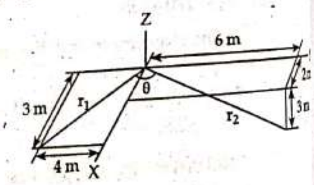
Now, from equation (1); we have,

$W = 50 \cos 30° + 41.67 \sin 53.13° = 76.6$ *lb*

13. **Determine the angle θ between the tails of the two vectors.**

Solution:

We have, position of $\vec{r_1}$ and $\vec{r_2}$ are $(3\vec{i} - 4\vec{j})$ and $(2\vec{i} + 6\vec{j} - 3\vec{k})$ respectively and magnitudes of $\vec{r_1}$ and $\vec{r_2}$ are;



$|\vec{r_1}| = \sqrt{3^2 + (-4)^2} = 5$

and, $|\vec{r_2}| = \sqrt{2^2 + 6^2 + (-3)^2} = 7$

Now, from definition of dot product;

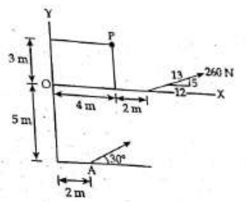
$\vec{r_1} . \vec{r_2} = |\vec{r_1}| \, |\vec{r_2}| \cos\theta$

or, $\cos\theta = \dfrac{\vec{r_1} . \vec{r_2}}{|\vec{r_1}| \, |\vec{r_2}|} = \dfrac{(3\vec{i} - 4\vec{j}).(2\vec{i} + 6\vec{j} - 3\vec{k})}{5 \times 7} = \dfrac{6 - 24}{35} = -\dfrac{18}{35}$

$\therefore \quad \theta = \cos^{-1}\left(-\dfrac{18}{35}\right) = 121°$

14. **Determine the magnitude and directional sense of the resultant moment of the forces about P.**

Solution:



Resolving 260 N and 400 N in horizontal and vertical component; we get,

$260 \cos 67.38°, 260 \sin 67.38°$ and $400 \cos 30°, 400 \sin 30°$ respectively

Now, moment at point 'P';

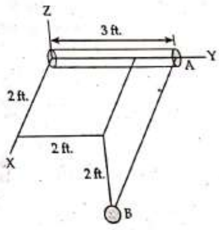
$M_p = -260 \cos 67.38° \times 3 - 260 \sin 67.38° \times 2 - 400 \cos 30° \times 8 + 400 \sin 30° \times 2$

$= -3151.28$ Nm $= 3.151$ kNm (anticlockwise)

15. **Determine the angle θ between the y–axis of the pole and the wire AB.**

Solution:

Position vector of A and B are $3\vec{j}$ and $(2\vec{i} + 2\vec{j} - 2\vec{k})$ respectively.



i.e., $\overrightarrow{OA} = 3\vec{j}$

or, $\overrightarrow{OB} = (2\vec{i} + 2\vec{j} - 2\vec{k})$

Now,

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= 2\vec{i} + 2\vec{j} - 2\vec{k} - 3\vec{j}$

$= 2\vec{i} - \vec{j} - 2\vec{k}$

Magnitudes of $\overrightarrow{OA}$ and $\overrightarrow{AB}$ are;

$|\overrightarrow{OA}| = \sqrt{3^2} = 3$

$|\overrightarrow{AB}| = \sqrt{2^2 + (-1)^2 + (-2)^2} = \sqrt{4 + 1 + 4} = 3$

Now, dot product between $\overrightarrow{OA}$ and $\overrightarrow{AB}$ is;

$\overrightarrow{OA} . \overrightarrow{AB} = |\overrightarrow{OA}| \, |\overrightarrow{AB}| \cos\theta$

or, $\cos\theta = \frac{(3\vec{j}) \cdot (2\vec{i} - \vec{j} - 2\vec{k})}{3 \times 3} = \frac{-3}{3 \times 3} = -\frac{1}{3}$

$\therefore \quad \theta = \cos^{-1}\left(-\frac{1}{3}\right) = 70.5°$

- **Note**
  $180° - 109.47° = 70.5°$

16. **Determine moment of the force at 'A' about point O. Express the result as a Cartesian vector.**

Solution:

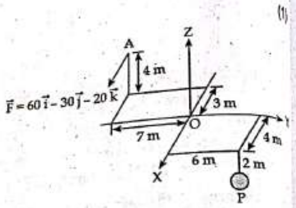
Position vector of A is;

$$\vec{OA} = -3\vec{i} - 7\vec{j} + 4\vec{k}$$

We have, moment about O;

$$\vec{M_o} = \vec{r} \times \vec{F} \quad (1)$$

i.e., $\vec{M_o} = \vec{OA} \times \vec{F}$



Given that;

$$\vec{F} = 60\vec{i} - 30\vec{j} - 20\vec{k}$$

From equation (1); we have,

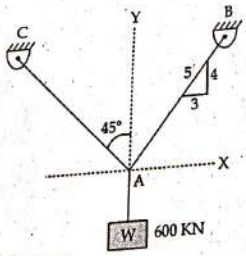
$$\vec{M_o} = \vec{OA} \times \vec{F}$$

or, $\vec{M_o} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & -7 & 4 \\ 60 & -30 & -20 \end{vmatrix}$

or, $\vec{M_o} = (140 + 120)\vec{i} - (60 - 240)\vec{j} + (90 + 420)\vec{k}$

$\therefore \quad \vec{M_o} = 260\vec{i} + 180\vec{j} + 510\vec{k}$

17. **Determine the tension in the cables AB and AC when the cables can support weight of 600 KN as shown in figure. Supposing the deflection in cables is negligible.**



Solution:

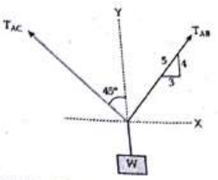
Free body diagram of given figure is given as;

Now, using condition of equilibrium; we have,

$(^+\rightarrow)\ \Sigma F_x = 0$

$T_{AB} \cos 53.13° - T_{AC} \sin 45° = 0$

or, $0.6\,T_{AB} - 0.707\,T_{AC} = 0 \quad (1)$



Again,

$\Sigma F_y = 0$

$T_{AC} \cos 45° + T_{AB} \sin 53.13° - W = 0$

or, $0.8\,T_{AB} + 0.707\,T_{AC} - 600 = 0 \quad (2)$

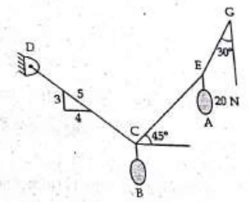
On solving equation (1) and (2); we get,

$T_{AB} = 428.57$ KN

and, $T_{AC} = 363.71$ KN

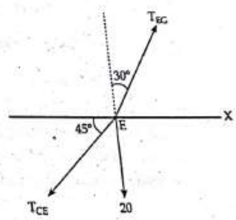
So, tension in cables AB and AC are 428.57 KN and 363.71 KN respectively.

18. **If the sack at 'A' in figure has a weight of 20N, determine the weight of the sack at 'B' and the force in each cord needed to hold the system in the equilibrium position shown.**



Solution:

There are three forces acting on E, as shown in figure. Establishing the x, y axes as shown in figure. Resolving each force into its x and y components using trigonometry; we have,



$(^+\rightarrow)\ \Sigma F_x = 0$

$T_{EG} \sin 30° - T_{EC} \cos 45° = 0 \quad (1)$

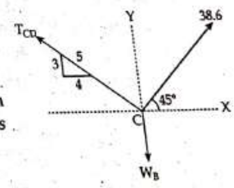
$(^+\uparrow)\ \Sigma F_y = 0$

$\cos 30° - T_{EC} \sin 45° - 20\text{ N} = 0 \quad (2)$

On solving equation (1) and (2); we get,

$T_{EC} = 38.6$ N

and, $T_{EG} = 54.6$ N



Free body diagram for point 'C' is as shown in figure. Establishing the x, y axes as shown in the figure.

$(^+\rightarrow)\ \Sigma F_x = 0$

$$38.6 \cos 45° - \frac{4}{5}\ T_{CD} \cos 36.87° = 0 \quad (3)$$

$(^+\uparrow)\ \Sigma F_y = 0$

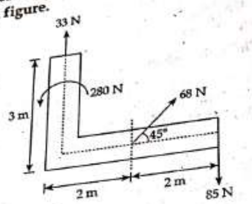
$$\frac{3}{5}\,T_{CD}\sin 36.87^\circ + 38.6\sin 45^\circ - W_B = 0 \qquad (4)$$

Solving equation (3) and substituting the result in to equation (4) gives;

$T_{CD} = 34.2$ N

and, $W_B = 47.8$N

19. Reduce the system of forces and couples to the simplest system using point 'A' as shown in figure.



Solution:

Transferring all forces to point A, the resultant force;

$$\vec{F_R} = 33\vec{i} - 85\vec{j} + 68\cos 45^\circ\ \vec{i} + 68\sin 45^\circ\ \vec{j}$$

$\therefore$ $\vec{F_R} = (48.08\ \vec{i} - 3.92\ \vec{j})$ N

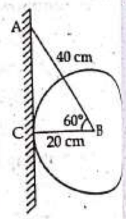
The resultant couple (moment about A) (Taking anticlockwise +ve) is;

$$\vec{C_R} = 4\vec{i}\times(-85\ \vec{j}) + 2\ \vec{i}\times(68\sin 45^\circ\ \vec{j}) + 280\ \vec{k}$$

$$= -340\vec{k} + 96.166\ \vec{k} + 280\ \vec{k}$$

$$= 36.17\ \vec{k}$$

20. A circular roller of weight 1000 N and radius 20 cm hangs by a tie rod AB = 40 cm and rests against a smooth vertical wall at C as shown in figure determine the tension in the tie rod and reaction at point C.



Solution:

Given that;

Radius of circular roller = 20 cm

Length of AB = 40 cm

$\therefore$ Length of AC = $\sqrt{40^2 - 20^2} = 20\sqrt{3}$

Now,

$$\tan\theta = \frac{p}{b} = \frac{20\sqrt{3}}{20} = \sqrt{3}$$

$\therefore$ $\theta = \tan^{-1}(\sqrt{3}) = 60^\circ$

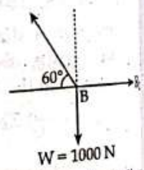


Figure: Free body diag[ram]

Free body diagram of the roller is given.

From free body diagram; we have,

$(^+\uparrow)\ \Sigma F_y = 0;\ T_{BA}\sin 60^\circ = 1000$

$\therefore$ $T_{BA} = 1154.7$ N

Again,

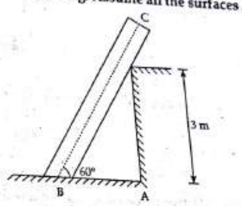
$(^+\rightarrow)\ \Sigma F_x = 0;$

$R_{CB} - T_{BA}\cos 60^\circ = 0$

or, $R_{CB} = 1154.7\cos 60^\circ$

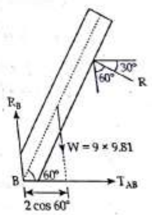
$\therefore$ $R_{CB} = 577.3$ N

21. Determine the tension in the cable AB which holds a post BC of 4 m length from sliding. The post has mass of 9 kg. Assume all the surfaces are smooth.



Solution:

Free body diagram of above figure;



$(^+\circlearrowleft)\ \Sigma M_B = 0;$

$$9\times 9.81\times 2\cos 60^\circ - R\times\frac{3}{\sin 60^\circ} = 0$$

$\therefore$ $R = 25.49$ N

$(^+\rightarrow)\ \Sigma F_x = 0;$

$T_{BA} - R\cos 30^\circ = 0$

$\therefore$ $T_{BA} = 22.07$ N

22. Compute a wrench out of a force $\vec{F} = 5\hat{i} + 7\hat{j} + 18\hat{k}$ acting at (4, 4, 5) and a couple $\vec{C} = 18\hat{i} + 7\hat{j} + 5\hat{k}$. Indicate its line of action.

Solution:

We have,

Force $\vec{F} = 5\hat{i} + 7\hat{j} + 18\hat{k}$ acting at (4, 4, 5)

and, couple $\vec{C} = 18\hat{i} + 7\hat{j} + 5\hat{k}$

Unit vector of $\vec{F}$ *i.e.,* $\hat{k} = \dfrac{5\hat{i}+7\hat{j}+18\hat{k}}{\sqrt{5^2+7^2+18^2}} = 0.25\hat{i} + 0.35\hat{j} + 0.9\hat{k}$

Magnitude of component of $\vec{C}$, along the direction of $\vec{F}$,

i.e., $C_f = \vec{C}\cdot\hat{i} = (18\hat{i} + 7\hat{j} + 5\hat{k})\cdot(0.25\hat{i} + 0.35\hat{j} + 0.9\hat{k})$

$= 4.5 + 2.45 + 4.5 = 11.45$

$\therefore$ Component of $\vec{C}$ along $\vec{F}$

i.e., $\vec{C_f} = 11.45(0.25\hat{i} + 0.35\hat{j} + 0.9\hat{k}) = 2.86\hat{i} + 4\hat{j} + 10.3\hat{k}$

Again, $\vec{C}_{nf}$ be the component of $\vec{C}$ normal to $\vec{F}$ then;

$$\vec{C}_{nf} = \vec{C} - \vec{C}_f = 18\hat{i} + 7\hat{j} + 5\hat{k} - 2.86\hat{i} - 4\hat{j} - 10.3\hat{k}$$
$$= (15.14\hat{i} + 3\hat{j} - 5.3\hat{k})$$

For nullifying $\vec{C}_{nf}$, let (x, y, z) be the new point, through which $\vec{F}$ must pass then,

$$\vec{r} \times \vec{F} = -\vec{C}_{nf}$$

or, $[(4-x)\hat{i} + (4-y)\hat{j} + (5-z)\hat{k}] \times (5\hat{i} + 7\hat{j} + 18\hat{k}) = -15.14\hat{i} - 3\hat{j} + 5.3\hat{k}$

or, $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ (4-x) & (4-y) & (5-z) \\ 5 & 7 & 18 \end{vmatrix} = -15.14\hat{i} - 3\hat{j} + 5.3\hat{k}$

or, $(72 - 18y - 35 + 7z)\hat{i} + (25 - 5z - 72 + 18x)\hat{j} + (28 - 7x - 20 + 5y)\hat{k} = -15.14\hat{i} - 3\hat{j} + 5.3\hat{k}$

Equating coefficient of like vector; we get,

$$31 - 18y + 7z = -15.14 \quad (1)$$
$$18x - 5z - 47 = -3 \quad (2)$$
$$8 - 7x + 5y = 5.3 \quad (3)$$

At xy plane z = 0, from equation (1), $y = \frac{1}{2} = 2.9$ and from equation (2);

$$x = \frac{44}{18} = 2.44$$

Hence the required wrench consists of force $\vec{F} = 5\hat{i} + 7\hat{j} + 18\hat{k}$ and a couple $\vec{C}_f = 2.86\hat{i} + 4\hat{j} + 10.3\hat{k}$ and the point on xy plane is;

$$x = 2.44$$

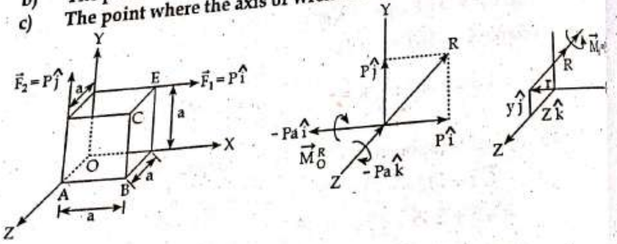
and, $y = 2.9$

Similarly;

At yz-plane; $x = 0, y = -0.54, z = -8.83$

and, At zx-plane; $y = 0, x = 0.385, z = -7.75$

22. **Two forces of the same magnitude 'P' act on a cube of side 'a' as shown. Replace the two forces by an equivalent wrench, and determine; Replace the two forces by an equivalent wrench, and determine the resultant force 'R'.**
    a) **The magnitude and direction of the resultant force 'R'.**
    b) **The pitch of the wrench.**
    c) **The point where the axis of wrench intersects the yz plane.**



Solution:

Equivalent force-couple system at 'O'

First, determining equivalent force couple system at origin 'O'. Consider $\vec{r}_E = a\hat{i} + a\hat{j}$ and $\vec{r}_D = a\hat{j} + a\hat{k}$ be position vectors of 'E' and 'D' respectively.

The resultant $\vec{R}$ of the two forces and their moment resultant $\vec{M}_O^R$ about 'O' are;

$$\vec{R} = \vec{f}_1 + \vec{f}_2 = P\hat{i} + P\hat{j} = P(\hat{i} + \hat{j}) \quad (1)$$
$$\vec{M}_O^R = \vec{r}_E \times \vec{f}_1 + \vec{r}_D \times \vec{f}_2 = (a\hat{i} + a\hat{j}) \times P\hat{i} + (a\hat{j} + a\hat{k}) \times P\hat{j}$$
$$= -Pa\hat{k} - Pa\hat{i} = -Pa(\hat{k} + \hat{i}) \quad (2)$$

a) Resultant force $\vec{R}$ It follows from equation (1) and the adjoining sketch that the resultant force $\vec{R}$ has the magnitude $R = P\sqrt{2}$, lies in the xy plane, and forms angle of 45° with 'x' and 'y' axes. Thus,

$$|\vec{R}| = R = P\sqrt{2}$$
$$\theta_x = \theta_y = 45°$$

and, $\theta_z = 90°$

b) Pitch of wrench

From formula; we have,

$$\text{Pitch (P)} = \frac{\vec{R} \cdot \vec{M}_O^R}{R^2}$$
$$= \frac{P(\hat{i} + \hat{j}) \cdot [-Pa(\hat{k} + \hat{i})]}{(P\sqrt{2})^2} = \frac{-P2a(1 + 0 + 0)}{2P^2}$$

$\therefore \quad P = \frac{-a}{2}$

c) Axis of wrench $(\vec{M}_1) = P\vec{R} = \frac{-a}{2}P(\hat{i} + \hat{j})$

$$= \frac{-Pa}{2}(\hat{i} + \hat{j}) \quad (3)$$

To find the point where the axis of the wrench intersects the yz plane, expressing the moment of the wrench about 'O' is equal to the moment resultant $\vec{M}_O^R$ of the original system.

$$\vec{M}_1 = \vec{r} \times \vec{R} = \vec{M}_O^R$$

where, $\vec{r} = y\hat{j} + z\hat{k}$.

Substituting for $\vec{R}$, $\vec{M}_O^R$ and $\vec{M}_1$ from equations (1), (2) and (3); we get,

$$\frac{-Pa}{2}(\hat{i} + \hat{j}) + (y\hat{j} + z\hat{k}) \times P(\hat{i} + \hat{j}) = -Pa(\hat{i} + \hat{k})$$

or, $\frac{-Pa}{2}\hat{i} - \frac{Pa}{2}\hat{j} - Py\hat{k} + Pz\hat{j} - Pz\hat{i} = -Pa\hat{k} - Pa\hat{i}$

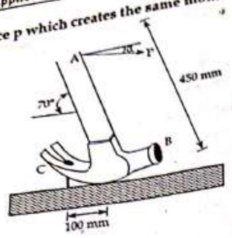
Equating the coefficients of 'K', and then the coefficients of 'j'; we find,

$$y = a$$

and, $z = \frac{a}{2}$

23. **It is known that a vertical force of 800 N is required to remove the nail at C from the board. As the nail first starts moving, determine;**
    a) **the moment about 'B' of the force exerted on the nail.**
    b) **the magnitude of the force 'p' which creates the same moment about 'B' if $\alpha = 10$.**

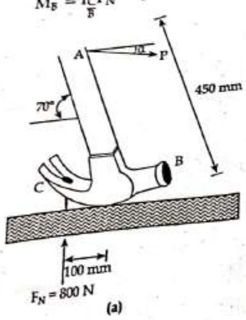
c) the smallest force p which creates the same moment about 'B'.



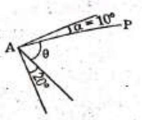
Solution:

a) We have,

$$M_B = r_{\frac{C}{B}} F_N = (0.1\ M)(800\ N) = 80\ NM$$



(a)



(b)

b) By Definition,

$$M_B = r_{\frac{A}{B}} P \sin\theta$$

$$\theta = 90° - (90° - 70°) - \alpha°$$

$$= 90° - 20° - 10°$$

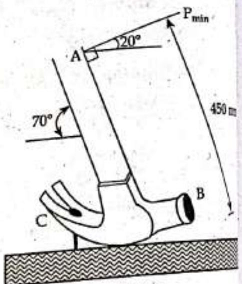
$$= 60°$$

$$\therefore\quad 80 = 0.45\,P \sin 60°$$

$$P = 205.28\ N$$

or, $P = 205\ N$

c) For P to be minimum; it must be perpendicular to the line joining points A and B. Thus, P must be directed as shown. Thus,



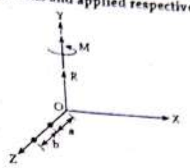
(c)

$$M_B = d\,P_{min} = r_{\frac{A}{B}} P_{Min}$$

$$80 = 0.45 P_{min}$$

$$\therefore\quad P_{Min} = 177.778\ N$$

24. Replace the wrench shown with an equivalent system consisting of two forces perpendicular to the z-axis and applied respectively at 'A' and 'B'.



Solution:

Consider

$$F_A = (F_A)_x\,\hat{i} + (F_A)_y\,\hat{j}$$

$$F_B = (F_B)_x\,\hat{i} + (F_B)_y\,\hat{j}$$

Then,

$\sum fx$;

$$(F_A)_x + (F_B)_x = 0$$

$$(F_A)_x = -(F_B)_x$$

$\sum fy$;

$$(F_A)_y + (F_B)_y = R$$

$$(F_A)_y = R - (F_B)_y$$

and, $\sum M_A$

$$b\,\hat{k} \times [(F_B)_x\,\hat{i} + (F_B)_y\,\hat{j}]$$

or, $a\,\hat{k} + R\,\hat{j} + M\,\hat{j}$

or, $b(F_B)_x\,\hat{j} + b(F_B)_y(-\hat{i}) = Ra\,\hat{i} + M\,\hat{j}$

Equating the coefficients of $\hat{i}$ and $\hat{j}$; we get,

$$-b(F_B)_y = Ra$$

or, $(F_B)_y = \frac{a}{b}R$

Then,

$$(F_A)_y = R - \left(-\frac{a}{b}R\right) = R\left(1 + \frac{a}{b}\right)$$

and, $b(F_B)_x = M$

$$(F_B)_x = \frac{M}{b}$$

Then,

$$(F_A)_x = \frac{-M}{b}$$

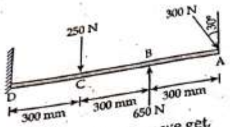
$$\therefore\quad \vec{F}_A = \frac{-M}{b}\hat{i} + R\left(1 + \frac{a}{b}\right)\hat{j}$$

$$\therefore\quad \vec{F}_B = \frac{M}{b}\hat{i} - \frac{a}{b}R\hat{j}$$

25. **In the figure (a) replace the three forces by an equivalent force and couple at D and (b) determine the distance x from D at which the resultant of the three forces acts.**

Solution:

The given figure is;



a) Resolving forces into x and y direction; we get,

$$\Sigma F_x = (300 \sin 30°)\vec{i} = 150\vec{i}$$

and, $\sum F_y = (650 - 250 - 300 \sin 30°)\vec{j} = 140.2\vec{j}$

$\therefore \quad R = \Sigma F_x + \sum F_y = 150\vec{i} + 140.2\vec{j}$

and, $|R| = \sqrt{(\Sigma F_x)^2 + \left(\sum F_y\right)^2} = \sqrt{(150)^2 + (140.2)^2} = 205.3$ N

and, Direction $\tan\theta = \dfrac{\sum F_y}{\sum F_x} = \dfrac{140.2}{150} = 0.935$

$\therefore \quad \theta = 43.06°$

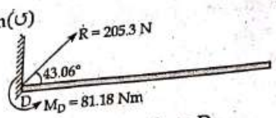
Taking moment about D; we get,

$$(^+\circlearrowleft) M_D = 250 \times \frac{300}{1000} - 650 \times \frac{600}{1000} + 300 \cos 30° \times \frac{900}{1000}$$

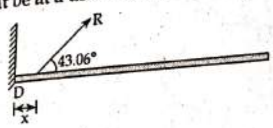
$\therefore \quad M_D = -81.18$ Nm

Thus,

$M_D = 81.18$ Nm($\circlearrowleft$)



b) Let the resultant be at a distance x from D.



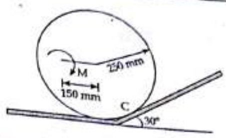
Taking moment about D; we get,

$$M_D = (R \sin 43.06°) \times x$$

or, $\quad 81.18 = (205.3 \times \sin 43.06°) \times x$

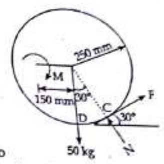
$\therefore \quad x = 0.573$ m

26. **Find the value of the couple M required to roll a 50 kg roller up the incline as shown in the figure. Also determine the contact force at C. The surface of the incline is sufficiently rough to prevent slipping.**



Solution:

The free body diagram of the given figure is drawn below when it is just about to roll over the floor.



Now, taking moment about C; we have,

$$\Sigma M_C = 0$$

or, $\quad M - (50 \times 9.81)CD = 0$

or, $\quad M = 50 \times 9.81 \times \left(\dfrac{250}{1000}\right) \times \sin 30° = 61.3$ Nm

For convenience, we choose the x-axis parallel to the incline plane and y-axis perpendicular to their incline plane.

Resolving the forces in y-direction; we get,

$$(^+\uparrow) \sum F_y = 0$$

or, $\quad N - 50 \times 9.81 \cos 30° = 0$

$\therefore \quad N = 424.7$ N

Resolving the forces in x-direction; we get,

$$(^+\nearrow) \Sigma F_x = 0$$

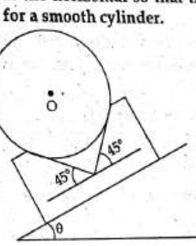
or, $\quad F - 50 \times 9.81 \sin 30° = 0$

$\therefore \quad F = 245.2$ N

Thus, reaction at C is;

$$R = \sqrt{N^2 + F^2} = \sqrt{(424.7)^2 + (245.2)^2} = 490.4 \text{ N}$$

27. **Find the angle of tilt θ with the horizontal so that the contact force at B will be one-third of that A for a smooth cylinder.**



Solution:

The free body diagram of the given figure is drawn below. Since, the cylinder is in equilibrium under the action of only three forces mg, R_A and R_B applying Lami's theorem; we get,

$$\frac{mg}{\sin 90°} = \frac{R_A}{\sin(90° + 45° - \theta)} = \frac{R_B}{\sin(90° + 45° + \theta)}$$

or, $\frac{R_A}{\sin(90° + 45° - \theta)} = \frac{R_B}{\sin(90° + 45° + \theta)}$

Substituting $R_B = \frac{R_A}{3}$; we get,

or, $\frac{R_A}{\sin(90° + 45° - \theta)} = \frac{R_A}{3\sin(90° + 45° + \theta)}$

or, $3\cos(45° + \theta) = \cos(45° - \theta)$

or, $3\cos 45° \cos\theta - 3\sin 45° \sin\theta = \cos 45° \cos\theta \sin 45° \sin\theta$

or, $2\cos 45° \cos\theta = 4\sin 45° \sin\theta$

or, $\tan\theta = \frac{1}{2}$

∴ $\theta = 26.56°$

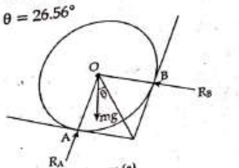


Figure: (a)

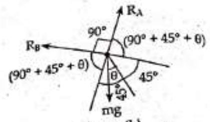
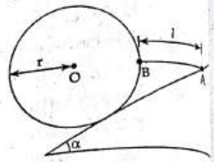


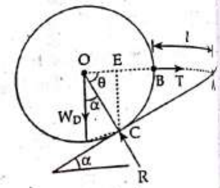
Figure: (b)

28. **A sphere of weight, W and radius, r is kept in equilibrium on a smooth inclined plane by means of a string of length, *l*. One end of the string is attached to the sphere and the other to the inclined plane as shown in the figure. Determine the tension of the string.**



Solution:

The figure shows the free body diagram of the sphere. The sphere is in equilibrium under the action of three forces: (a) its weight, W; (b) tension, T in the string and (c) reaction, R. Since, the lines of action of forces, R and W meet at O, so the action of the third force, T will pass through O.



Taking moment about C; we get,

$$\sum M_C = 0$$

or, $T \times CE - W \times CD = 0$

or, $T \times OC \times \sin\theta - W \times OC \times \sin\alpha = 0$

or, $T = W \times \frac{\sin\alpha}{\sin\theta}$

Also, in ΔAOC; we find that;

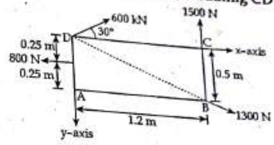
$$\cos\theta = \frac{OC}{OA} = \frac{r}{r + l}$$

or, $\sin\theta = \sqrt{1 - \cos^2\theta} = \frac{\sqrt{l^2 + 2lr}}{l + r}$

Substituting the value of cos θ in equation (1); we have,

∴ $T = \frac{W\sin\alpha(l + r)}{\left(\sqrt{l^2 + 2lr}\right)}$

29. **Four forces act on a rectangle in the same plane as shown. Find the magnitude and direction of resultant force. Also find intersection of the line of action of resultant with X and Y axes, assuming CD is origin.**



Solution:

Solving by scalar approach; we get,

Resolving forces into X and Y axes; we get,

$$\sum F_x = -800 + (1300)\cos\theta + 600\cos 30°$$

From geometry; we have,

$$\theta = \tan^{-1}\left(\frac{0.5}{1.2}\right) = 22.62°$$

∴ $\sum F_x = -800 + (1300)\cos(22.62°) + 600\cos 30° = 919.615$ N

and, $\sum F_y = -1300\sin(22.62°) + 600\sin 30° + 1500 = 1300$ N

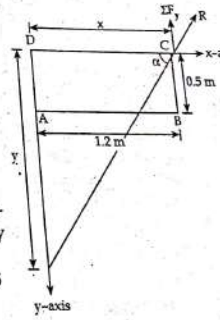
∴ $R = \sqrt{(\sum F_x)^2 + \left(\sum F_y\right)^2}$

$= \sqrt{(919.615)^2 + (1300)^2}$

$= 1592.386$ N

and, $\tan\alpha = \frac{\sum F_y}{\sum F_x} = \frac{1300}{919.615} = 1.414$

∴ $\alpha = 54.72°$



Assume that R intersect side DC (X-axis) at the distance x as shown. Apply principle of moment at D (⁺↺)

or, $1300 \times x = 1500 \times 1.2 - 800 \times 0.25$

∴ $x = 1.23$ m

i.e., R intersect just right of point C at a distance of 0.03 m from C.

Also, from the geometry; we have,

$$\tan\alpha = \frac{y}{x}$$

or, $\tan(54.72°) = \frac{y}{1.23}$

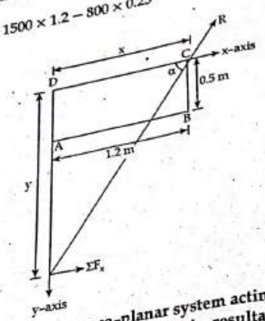
∴ $y = 1.74$ m

i.e., at 1.24 m below point A.

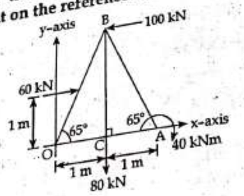
Alternatively, assuming R as shown in the figure and consider moment about D; we have,

$$919.615 \times y = 1500 \times 1.2 - 800 \times 0.25$$

$\therefore \quad y = 1.74$ m



30. Three forces and a couple from a co-planar system acting in XY plane on triangle as shown in the figure. Find their resultant. Also find the intersection of resultant on the reference axes.



Solution:

Resolving forces in X and Y direction using scalar approach; we get,

$$\sum F_x = 60 - 100 = -40 \text{ kN}$$

and, $\sum F_y = -80$ kN

$$\therefore \quad R = \sqrt{(\Sigma F_x)^2 + \left(\sum F_y\right)^2} = \sqrt{(-40)^2 + (-80)^2} = 89.443 \text{ kN}$$

and, $\tan\alpha = \dfrac{-80}{-40}$

$\therefore \quad \alpha = 63.43°$

Let R having Y intersect = x (as shown). Consider moment about (clockwise positive); we have,

$$40 \times y = 60 \times 1 - 100 \times BC + 40 + 80 \times 1$$

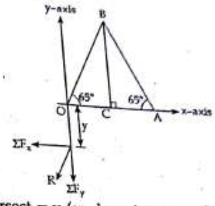
Here,

$$BC = OC \tan 65°$$

$\therefore \quad BC = 2.145$ m

$\therefore \quad y = \frac{1}{40}(60 - 100 \times 2.145 + 40 + 80) = -0.863$ m

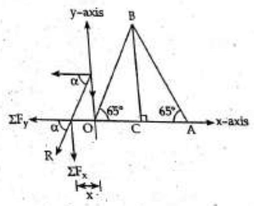
i.e., y = 0.863 m; above origin.



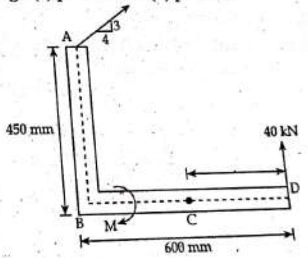
Let, R having X intersect = y (as shown). Consider moment about O (anti-clockwise positive); we have,

$$80 \times x = -80 \times 1 - 60 \times 1 + 100 \times BC - 40$$

$\therefore \quad x = 0.431$ m



31. A co-planar force system consists of two forces and a couple. Determine magnitude and sense (direction) of couple M so that the resultant of the system passes through (a) point D and (b) point B.



Solution:

a) Applying moment theorem about D; we get,

$(^{+}\circlearrowleft) 0 = M + 16 \times 0.45 + 12 \times 0.6$; resolving 20 kN force at A.

$\therefore \quad M = -14.4$

i.e., $M = 14.4$ kNm (↺)

b) Applying moment theorem about B (clockwise positive); we have,

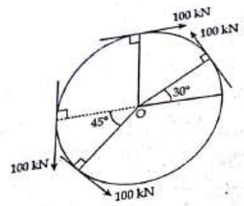
$0 = M - 40 \times 0.6 + 16 \times 0.45$

$\therefore$ $M = 16.8$ kNm (↻)

32. **Four forces acts tangentially to a 8 cm diameter circle as shown in the figure, what will be the magnitude and location of resultant with respect to O.**

Solution:

The given figure is;



Resolving force into x-direction; we get,

$(\overset{+}{\rightarrow}) \sum F_x = 100 + 100 \cos 45° = 120.71$ N

and, $\sum F_y = -100 - 100 \sin 45° + 100 \sin 60° = -84.108$ N

$\therefore$ $R = \sqrt{(\Sigma F_x)^2 + \left(\sum F_y\right)^2} = \sqrt{(120.71)^2 + (-84.108)^2} = 147.12$ kN

and, $\tan \alpha = \frac{\sum F_y}{\sum F_x} = \frac{-84.108}{120.71} = -0.697$

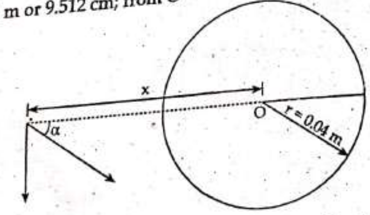
$\therefore$ $\alpha = 34.87°$

Let, resultant at distance x to the left of O as shown in the figure.

Taking moment about O; we have,

$84.108 \times x = 100 \times 0.04 + 100 \times 0.04 + 100 \times 0.04 - 100 \times 0.04$

$\therefore$ $x = 0.0951$ m or 9.512 cm; from O



33. **When $\theta = 0°$ and force $P = 0$, the cable (2 m long) and spring attached each other remain as AB_1C. If $P = 80$ N causes $\theta = 60°$ in the equilibrium position, find unstretched length of the spring. Assume $k = 50$ N/m. Also find tension in the cable.**

Solution:

The given figure is;

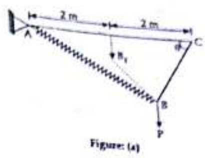


Figure: (a)

Here,

Spring force (F) = $kx = k(L_1 - L_0)$

Drawing free body diagram at joint B as shown in the figure (b) and applying Lami's theorem.

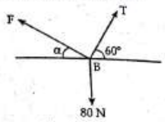


Figure: (b) F.B.D.

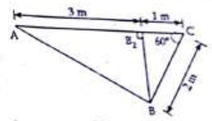


Figure: (c)

Here, angle made by spring with horizontal = α is unknown.

From the geometry of the figure (figure c); we have,

$BB_2 = 2 \sin 60° = 1.732$ m

$B_2C = 2 \cos 60° = 1$ m

$\therefore$ $AB_2 = 3$ m

$AB = \sqrt{(AB_2)^2 + (BB_2)^2} = \sqrt{(3)^2 + (1.732)^2} = 3.464$ m

and, $L_1 = AB = 3.464$ m

$\because$ $\alpha = \tan^{-1}\left(\frac{1.732}{3}\right) = 30°$

$\therefore$ $\frac{T}{\sin 120°} = \frac{F}{\sin 150°} = \frac{80}{\sin 90°}$

or, $\frac{F}{\sin 150°} = \frac{80}{\sin 90°}$

$\therefore$ $F = 40$ N

Now,

$F = k(L_1 - L_0)$

or, $40 = 50(3.464 - L_0)$

$\therefore$ $L_0 = 2.664$ m

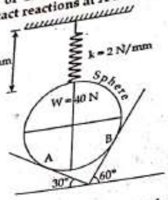
Thus, the required unstretched length of spring $(L_0) = 2.664$ m.

Also,

$\frac{T}{\sin 120°} = \frac{80}{\sin 90°}$

$\therefore$ $T = 69.282$ N

34. The figure shows a sphere resting in a smooth V-shaped groove and is subjected to a spring force. The spring is compressed to a length of 100 mm from its free length of 150 mm. If the stiffness of the spring is 2 N/mm, determine the contact reactions at A and B.



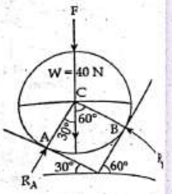
Solution:

Firstly drawing free body diagram of the sphere as shown in the figure.

Here,

Spring force $(F) = kS = 2(150 - 100) = 100\ N$

Considering equilibrium of the sphere; we have,



$\Sigma F_x = 0$

or, $R_A \sin 30° - R_B \sin 60° = 0$

$\therefore$ $R_A = \dfrac{R_B \sin 60°}{\sin 30°} = 1.732 R_B$

Also,

$\sum F_y = 0$

or, $R_A \cos 30° + R_B \cos 60° - W - F = 0$

or, $1.732 R_B \cos 30° + R_B \cos 60° - 40 - 100 = 0$

$\therefore$ $R_B = 70\ N$

and, $R_A = 1.732 \times 70 = 121.24\ N$

35. Two cylinders are placed in a trough as shown in the figure (a). Neglecting friction, find the reactions at all the contact surfaces. Given:
Diameter of the first cylinder = 120 mm
Diameter of the second cylinder = 60 mm
Weight of the first cylinder = 250 N
Weight of the second cylinder = 100 N

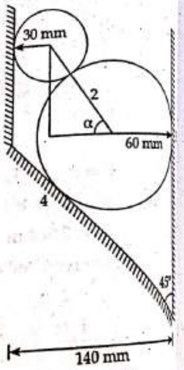


Figure: (a)

Solution:

Referring to the figure (a); we have,

$$\cos\alpha = \frac{140 - 30 - 60}{60 + 30} = \frac{50}{90}$$
$$= 0.556$$

$\therefore$ $\alpha = \cos^{-1}(0.556) = 56.25°$

Now, drawing free body diagram of the figure (a); we get,

Considering equilibrium of the second cylinder; we get,

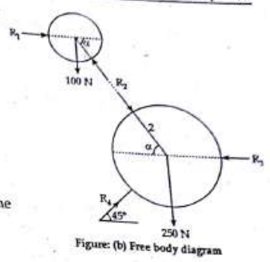


Figure: (b) Free body diagram

$(^+\uparrow) \sum F_y = 0$

or, $R_2 \sin 56.25° - 100 = 0$

$\therefore$ $R_2 = 120.3\ N$

$(^+\rightarrow) \Sigma F_x = 0$

or, $R_1 - R_2 \cos\alpha = 0$

or, $R_1 - 120.3 \times \cos 56.25° = 0$

$\therefore$ $R_1 = 66.82\ N$

Considering equilibrium of the first cylinder; we get,

$(^+\uparrow) \sum F_y = 0$

or, $R_4 \sin 45° - 250 - R_2 \sin\alpha = 0$

$\therefore$ $R_4 = \dfrac{250 + 120.3 \sin 56.25°}{\sin 45°} = 495\ N$

and, $\Sigma F_x = 0$

or, $R_4 \cos 45° + R_2 \cos\alpha - R_3 = 0$

$\therefore$ $R_3 = 495 \cos 45° + 120.3 \cos 56.25° = 416.9\ N$

36. The sphere A, B and C weighing 200 N, 400 N and 200 N respectively and having radii 400 mm, 600 mm and 400 mm respectively are placed in a trench as shown in the figure (a). Treating all contact surfaces as smooth, determine the reactions developed.

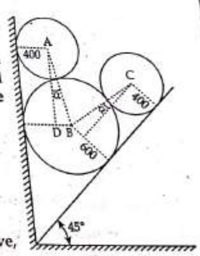


Figure: (a)

Solution:

Referring to the figure (a); we have,

$$\sin\alpha = \frac{BD}{AB} = \frac{600 - 400}{400 + 600} = 0.2$$

$\therefore$ $\alpha = 11.537°$

Drawing free body diagram of figure (a); we have,

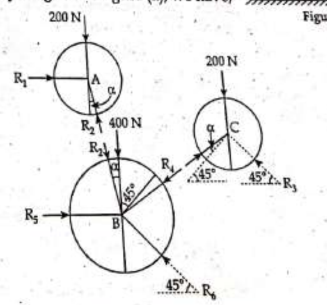


Figure: (b) Free body diagram

Referring to free body diagram of sphere A; we get,

$(^+\uparrow)\sum F_y = 0$

or, $R_2 \cos\alpha - 200 = 0$

$\therefore$ $R_2 = \frac{200}{\cos 11.537^\circ} = 204.1\text{ N}$

and, $\Sigma F_x = 0$

or, $R_1 - R_2 \sin\alpha = 0$

$\therefore$ $R_1 = 204.1 \times \sin 11.537^\circ = 40.8\text{ N}$

Referring to free body diagram of sphere C; we get,

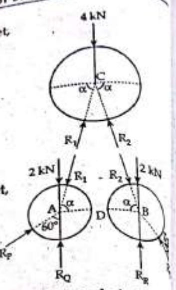


Figure: (b) Free body diagram

Referring to free body diagram of sphere C; we get,

$\sum$ Forces parallel to inclined plane = 0

$(^+\nearrow) R_4 \cos\alpha - 200 \cos 45^\circ = 0$

$\therefore$ $R_4 = \frac{200\cos 45^\circ}{\cos 11.537^\circ} = 144.3\text{ N}$

and, $(^+\rightarrow)\Sigma F_x = 0$

or, $R_4 \cos(45^\circ - \alpha) - R_3 \cos 45^\circ = 0$

$\therefore$ $R_3 = 170.3\text{ N}$

Now, referring to free body diagram of cylinder B; we get,

$(^+\uparrow)\sum F_y = 0$

or, $R_6 \sin 45^\circ - 400 - R_2 \cos\alpha - R_4 \cos(45^\circ + \alpha) = 0$

or, $R_6 \sin 45^\circ = 400 + 204.1 \cos 11.537^\circ + 144.3 \cos 56.537^\circ$

$\therefore$ $R_6 = 961\text{ N}$

and, $(^+\rightarrow)\Sigma F_x = 0$

or, $R_5 - R_2 \sin\alpha - R_4 \sin(45^\circ + \alpha) - R_6 \cos 45^\circ = 0$

$\therefore$ $R_5 = 204.1 \sin 11.537^\circ + 144.3 \sin 56.537^\circ + 961 \cos 45^\circ = 840.7\text{ N}$

37. Three sphere are piled in a trench as shown in the figure (a). Self weight and radii of the cylinders are given below.

Sphere	Weight	Radius
A	2 kN	400 mm
B	2 kN	400 mm
C	4 kN	600 mm

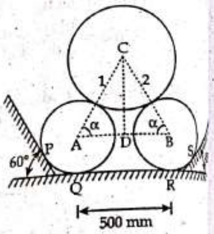


Figure: (a)

Treating all contact surfaces as smooth, determine the reactions developed of the contact surfaces P, Q, R and S. Given centre to centre distance between sphere A and B is 500 mm.

Solution:

Drawing free body diagram of the given figure; we get,

From triangle ABC in the figure (a); we get,

$$\cos\alpha = \frac{AD}{AC} = \frac{250}{400 + 600} = 0.25$$

$\therefore$ $\alpha = \cos^{-1}(0.25) = 75.522^\circ$

Referring to free body diagram of sphere C; we get,

$\Sigma F_x = 0$

or, $R_1 \cos\alpha - R_2 \cos\alpha = 0$

i.e., $R_1 = R_2$

Also,

$\sum F_y = 0$

$R_1 \sin\alpha + R_2 \sin\alpha - 4 = 0$

or, $R_1 \sin\alpha + R_2 \sin\alpha = 4$

$\therefore$ $R_1 = \frac{4}{2\sin(75.522^\circ)} = 2.066\text{ kN} = R_2$

Referring to F.B.D. of sphere A; we get,

$\Sigma F_x = 0$

or, $R_P \sin 60^\circ - R_1 \cos\alpha = 0$

$\therefore$ $R_P = \frac{2.066\cos(75.522^\circ)}{\sin 60^\circ} = 0.596\text{ kN}$

and, $\sum F_y = 0$

or, $R_Q - R_1 \sin\alpha - 2 - R_P \cos 60^\circ = 0$

$\therefore$ $R_Q = 2.066 \sin 75.522^\circ + 2 + 0.596 \cos 60^\circ = 4.298\text{ kN}$

Referring to F.B.D. of sphere B; we get,

$\Sigma F_x = 0$

or, $R_2 \cos\alpha - R_s \sin 45^\circ = 0$

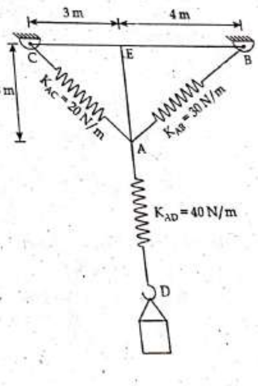
$\therefore$ $R_S = \frac{2.066\cos(75.522^\circ)}{\sin 45^\circ} = 0.730\text{ kN}$

and, $\sum F_y = 0$

or, $R_R + R_S \cos 45^\circ - R_2 \sin\alpha - 2 = 0$

$\therefore$ $R_R = -0.730 \cos 45^\circ + 2.066 \sin 75.522^\circ + 2 = 1.484\text{ kN}$

38. In a system shown in the figure, the spring CA is kept fixed at C and connected to a weightless ring at A. Another spring BA is kept fixed at B and connected to the same weightless ring A and the end D is holding a box with weight. The fixed point C and B are in the same horizontal level. Determine the mass of box held at D. Unstretched length of the spring AB = 2 m. The system is in equilibrium.



Solution:

Here,

Length of AB after stretching $= \sqrt{(4)^2 + (3)^2} = 5$ m

Unstretched length of AB = 2 m

$\therefore$ Stretching of AB = 5 − 2 = 3 m

Force exerted along AB = 3 × 30 = 90 N

Let, the mass of the box at D = m

The force by it is 9.81 mN downward.

Let, the vertical through A meet CB at E.

$\measuredangle ECA = 45°$

and, $\measuredangle EAC = 45°$

$\measuredangle BAE = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$

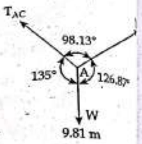
$\therefore$ $\measuredangle CAB = 45° + 53.13° = 98.13°$

Since, AC is inclined to the horizontal by 45°.

$\measuredangle CAD = 45° + 90° = 135°$

$\measuredangle BAD = 180° - 53.13° = 126.87°$

All these angles are marked in the figure aside.



Since, the system is in the equilibrium, applying Lami's theorem; we get,

$$\frac{T_{AB}}{\sin 135°} = \frac{T_{AC}}{\sin 126.87°} = \frac{W}{\sin 98.13°}$$

or, $$\frac{T_{AB}}{\sin 135°} = \frac{9.81m}{\sin 98.13°}$$

$\therefore$ Mass of box (m) $= \frac{90 \sin 98.13°}{9.81 \sin 135°} = 12.844$ kg

39. ABC, a spring having an unstretched length of 6 m and a stiffness of 420 N/m is being pulled by the horizontal force, F as shown in the figure (a). The displacement of the spring to which B (mid-point of the spring) is connected is 1.4 m from the wall on which the fixed positions A and C exist. Determine the force, F.

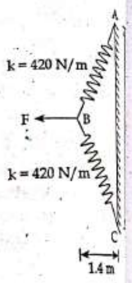


Figure: (a)

Solution:

Spring AB

Elongated length $= \sqrt{(3)^2 + (1.4)^2} = 3.311$ m

$\therefore$ Stretch or elongation of AB = 3.311 − 3

= 0.311 m

$\therefore$ Force exerted by AB (T_{AB}) = 420 × 0.311

= 130.62 N

Due to symmetry, it is taken that the force exerted by BC is;

$T_{BC} = 130.62$ N

From the force triangle shown in the figure (b); we have,

$$\tan\theta = \frac{1.4}{3} = 0.467$$

$\therefore$ $\theta = \tan^{-1}(0.467) = 25.02°$ (angle between AB and vertical wall)

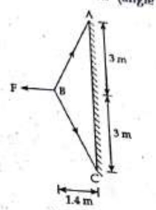


Figure: (b)

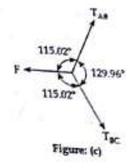


Figure: (c)

Similarly,

Angle between the vertical wall and BC = 25.02°

[due to figure (b) symmetry]

$\therefore$ $\measuredangle ABC = 180° - (2 \times 25.02°) = 129.96°$

Angle between horizontal and AB $= \frac{360° - 129.96°}{2} = 115.02°$

Since, the system is in the equilibrium. From the figure (c) and applying Lami's theorem; we have,

$$\frac{F}{\sin 129.96°} = \frac{T_{AB}}{\sin 115.02°}$$

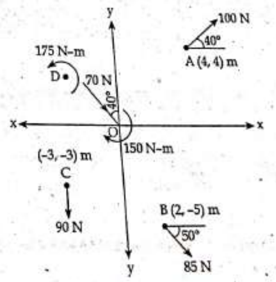
$\therefore$ $$F = \frac{T_{AB} \sin 129.96°}{\sin 115.02°} = \frac{130.62 \sin 129.96°}{\sin 115.02°} = 110.48 \text{ N}$$

Hence, the horizontal force has a magnitude of 110.48 N and it acts from right to left.

40. i) Find the resultant of the forces system shown in the figure.

ii)Replace the given force and couple by a single force and couple system at A

Solution:



Resolving forces into x and y-direction; we get,

$$(^{+}\rightarrow) \sum F_x = 100 \cos 40° + 85 \cos 50° + 70 \sin 40°$$

$\therefore$ $\sum F_x = 176.24$ N($\rightarrow$)

$(+\uparrow) \sum F_y = 100 \sin 40° - 85 \sin 40° - 90 - 70 \cos 40° = -144.46$ N

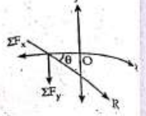
$\therefore \quad \sum F_y = 144.46$ N$(\downarrow)$

Magnitude of resultant (R) $= \sqrt{(176.24)^2 + (144.46)^2} = 227.88$ N

$\therefore$ Inclination of resultant $(\theta) = \tan^{-1}\left(\frac{144.46}{176.24}\right) = 39.34°$

Now, taking moment about O; we get,

$$\Sigma M_O = (-100 \cos 40°) \times 4 + (100 \sin 40°) \times 4 + (85 \cos 50°) \times 5 - (85 \sin 50°) \times 2 + 90 \times 3 + 175 - 130$$



$\therefore \quad \Sigma M_O = 388.65$ Nm$(\circlearrowleft)$

Applying Varignon's theorem at O; we get,

$x \sum F_y = \Sigma M_O$

or, $\quad x \times 144.46 = 388.65$

$\therefore \quad x = 2.69$ m

and, $\quad y \Sigma F_x = \Sigma M_O$

or, $\quad y \times 176.24 = 388.65$

$\therefore \quad y = 2.21$ m

Alternative method (Vector approach)

Resolving forces into x and y-direction; we get,

$\Sigma F_x = (100 \cos 40°)\vec{i} + (85 \cos 50°)\vec{i} + (70 \sin 40°)\vec{i}$

$\therefore \quad \Sigma F_x = 176.24\vec{i}$ N$(\rightarrow)$

and, $\quad \sum F_y = (100 \sin 40°)\vec{j} - (85 \sin 40°)\vec{j} - 30\vec{j} - (70 \cos 40°)\vec{j}$

$\therefore \quad \sum F_y = -144.46\vec{j} = 144.46\vec{j}(\downarrow)$

$\therefore \quad R = 176.24\vec{i} - 144.46\vec{j}$

For resultant taking moment theorem about O; we get,

$$(x\vec{i} + y\vec{j}) \times (176.24\vec{i} - 144.46\vec{j}) = (4\vec{i} + 4\vec{j}) \times (100 \cos 40° \vec{i} - 100 \sin 40°\vec{j}) + (2\vec{i} - 5\vec{j}) \times (85 \cos 50° \vec{i} - 85 \sin 50°\vec{j}) + (-3\vec{i} - 3\vec{j}) \times (-90\vec{j}) + 175\vec{k} - 130\vec{k}$$

or, $\quad -144.46x\vec{k} - 176.24y\vec{k} = 257.12\vec{k} - 306.42\vec{k} + 270\vec{k} - 130.03\vec{k} + 273.1\vec{k} + 175\vec{k} - 130\vec{k}$

or, $\quad \dfrac{x}{-\left(\frac{388.65}{144.46}\right)} + \dfrac{y}{-\left(\frac{388.65}{176.24}\right)} = 1$

or, $\quad \dfrac{x}{-2.69} + \dfrac{y}{-2.21} = 1$

Thus, x-intercept $= -2.69$ m and y-intercept $= -2.21$ m.

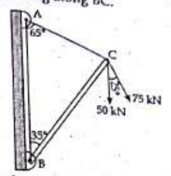
41. **Determine:**

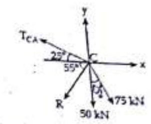
i) **the required tension in the cable AC, knowing that the resultant of three forces exerted at point C of bottom BC must be directed along BC**

ii) **the corresponding magnitude of the resultant.**

Solution:

The forces acting at C will be 50 N, 75 N, T_{CA} directed from C to A and resultant R acting along BC.





Resolving forces in x and y direction; we get,

$R_x = \Sigma F_x$

or, $\quad -R \cos 55° = -T_{CA} \cos 25° + 75 \sin 25°$

$\therefore \quad -R \cos 55° + T_{CA} \cos 25° = 75 \sin 25°$ (1)

and, $\quad R_y = \sum F_y$

or, $\quad -R \sin 25° = T_{CA} \sin 25° - 50 - 75 \cos 25°$

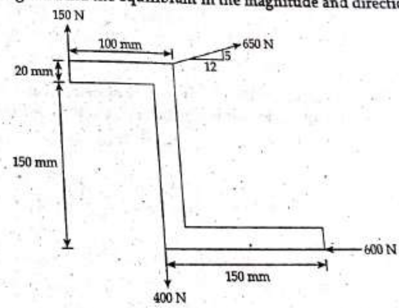
$\therefore \quad R \sin 25° + T_{CA} \sin 25° = 50 + 75 \cos 25°$ (2)

From equations (1) and (2); we get,

$T_{CA} = 95.075$ N

and, $\quad R = 94.97$ N

42. **Z-shaped lamina of uniform width of 20 mm is subjected to four forces as shown in the figure. Find the equilibrant in the magnitude and direction.**



Solution:

To find equilibrant, we have to find the resultant first.

Resolving forces in x and y-direction; we get,

$$\Sigma F_x = \left(650 \times \frac{12}{13}\right)\vec{i} - 600\vec{i} = 0$$

and, $\quad \sum F_y = 150\vec{j} - 400\vec{j} + \left(650 \times \frac{5}{13}\right)\vec{j} = 0$

$\therefore \quad R = \sqrt{(\Sigma F_x)^2 + \left(\sum F_y\right)^2} = 0$

When R = 0, the resultant can be a moment which will be same about a point in the plane. Taking moment about the point of the application of N force; we get,

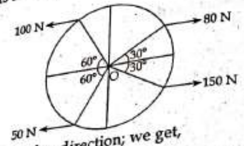
$M = -150 \times 100 + 400 \times 20 - 600 \times 170 = -109000$ N mm

$\therefore \quad M = 109$ Nm(↺)

$\therefore$ Equilibrant is a moment of 109 Nm(↺).

43. **Determine resultant of the following parallel forces and locate w respect to O. Radius is 1 m.**

Solution:

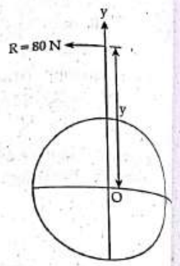


Resolving forces into x and y-direction; we get,

$\Sigma F_x = 100 - 50 - 80 + 150 = -80$ N

and, $\sum F_y = 0$

$\therefore \quad R = 80$ N(←)

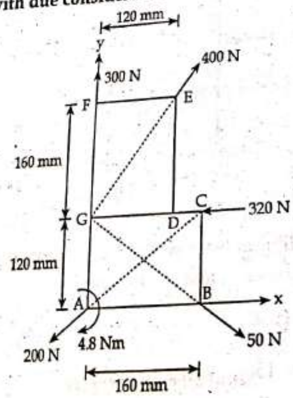


A horizontal force can intersect y-axis only. Let, R cut y-axis at a distance y as shown in the figure. Using Varignon's theorem at O; we get,

$(80 \times 1 \sin 30°) + (150 \times 1 \sin 30°)$
$+ (100 \times 1 \sin 60°) - (50 \times 1 \sin 45°) = 80y$

$\therefore \quad y = 2.078$ m

44. **Find the resultant of co-planar force system given in the figure and l the same on AB with due consideration to the applied moment.**



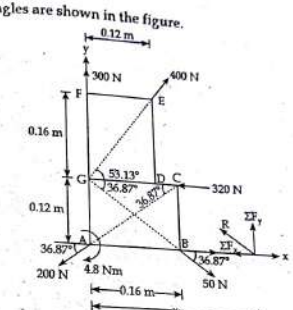
Solution:

The angles for the forces with horizontal are calculated as;

$\theta_1 = \tan^{-1}\left(\frac{160}{120}\right) = 53.13°$

$\theta_2 = \tan^{-1}\left(\frac{120}{160}\right) = 36.87°$

and, different angles are shown in the figure.



Resolving the forces in x and y-direction; we get,

$\Sigma F_x = -200 \cos 36.87° + 400 \cos 53.13° + 50 \cos 36.87° - 320 = -200$ N

and, $\sum F_y = 300 - 200 \sin 36.87° + 400 \sin 53.13° - 50 \sin 36.87° = 470$ N

$\therefore \quad R = \sqrt{(\Sigma F_x)^2 + \left(\sum F_y\right)^2} = \sqrt{(-200)^2 + (470)^2} = 510.784$ N

$\therefore \quad \theta = \tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right) = \tan^{-1}\left(\frac{470}{200}\right) = 66.95°$

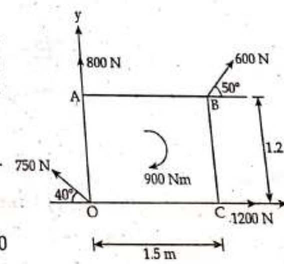
Plotting resultant on x-axis at distance x from A and using Varignon's theorem at A;

$-4.8 - (400 \cos 53.13° \times 0.12) + (320 \times 0.12) - (50 \sin 36.87° \times 0.16) = 470x$

$\therefore \quad x = 1.7 \times 10^{-7}$ m $\simeq 0$

Hence, resultant acts at almost the point A.

45. **Find the resultant of the force system acting on a body OABC as shown in the figure. Also, find the points where the resultant will cut the X and Y axis.**



Solution:

Resolving forces into x and y-direction; we get,

$\Sigma F_x = -750 \cos 40° + 600 \cos 50° + 1200$

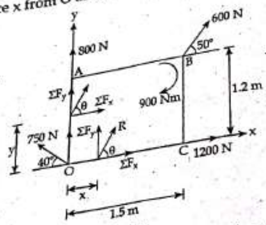
$= 1011.14\ N(\rightarrow)$

and, $\sum F_y = 750\sin 40° + 800 + 600\sin 50° = 1741.72\ N$

$\therefore \quad R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2} = \sqrt{(1011.14)^2 + (1741.72)^2} = 2013.95\ N$

and, $\theta = \tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right) = \tan^{-1}\left(\frac{1741.72}{1011.14}\right) = 59.86°$

The X-intercept of the resultant can be obtained by plotting the resultant on X-axis at a distance x from O as shown in the figure.



Using Varignon's theorem at O; we have,

$$-900 - (600\cos 50° \times 1.2) + (600\sin 50° \times 1.5) = 1741.72x$$

$\therefore \quad x = -0.387\ m$

i.e., resultant acts at a distance of 0.387 m to the left of O.

The Y-intercept of the resultant can be obtained by plotting the resultant on Y-axis at a distance y from O as shown in the figure.

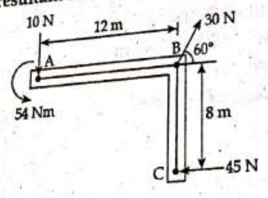
Using Varignon's theorem at O; we have,

$$-900 - (600\cos 50° \times 1.2) + (600\sin 50° \times 1.5) = -1011.14y$$

$\therefore \quad y = 0.666\ m$

i.e., resultant acts at a distance of 0.666 m above O.

46. The three forces and a couple shown are applied to an angle bracket. Find the resultant of the system of forces. (b) Locate the points where line of action of the resultant intersects line AB and BC.



Solution:

Resolving forces into x and y-direction; we get,

$$\sum F_x = (30\cos 60°)\vec{i} - 45\vec{i} = -30\ N\vec{i}$$

and, $\sum F_y = (30\sin 60°)\vec{j} - 10\vec{j} = 15.98\ N\vec{j}$

$\therefore \quad R = -30\ N\vec{i} + 15.98\ N\vec{j}$

First reduce the given forces and couple to an equivalent force couple system $(R \cdot M_B)$ at B. We have,

$$\Sigma M_B = 54\vec{k} - [12\vec{i} \times (-10\vec{j})] - [8\vec{j} \times (-45\vec{i})]$$
$$= 54\vec{k} + 120\vec{k} - 360\vec{k} = -186\ Nm\ \vec{k}$$

Then, with R at D; we get,

$$\Sigma M_B = x\sum F_y$$

or, $\quad 186 = x \times 15.98$

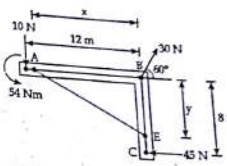
$\therefore \quad x = 11.64\ m$

and, with R at E; we have,

$$\Sigma M_B = y\Sigma F_x$$

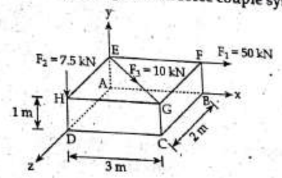
or, $\quad 186 = y \times 30$

$\therefore \quad y = 6.2\ m$



The line of action of R intersects line AB 11.64 m to the left to B and intersects line BC 6.20 m below B.

47. A rectangular block is acted upon by forces as shown in the figure. Replace this set of forces by an equivalent force couple system acting at A.



Solution:

Co-ordinates of various points are A(0, 0, 0), E(0, 1, 0), D(0, 0, 2), F(3, 1, 0), G(3, 1, 2) and H(0, 1, 2).

The different force vectors can be determined as follows:

$$\vec{F}_1 = \vec{F}_{EF} = 5\left[\frac{(3-0)\vec{i} + (1-1)\vec{j} + (0-0)\vec{k}}{\sqrt{(3-0)^2 + (1-1)^2 + (0-0)^2}}\right]$$

$\therefore \quad \vec{F}_1 = 5\vec{i}$

$$\vec{F}_2 = \vec{F}_{HD} = 7.5\left[\frac{(0-0)\vec{i} + (0-1)\vec{j} + (2-2)\vec{k}}{\sqrt{(0-0)^2 + (0-1)^2 + (2-2)^2}}\right]$$

$\therefore \quad \vec{F}_2 = -7.5\vec{j}$

and, $\vec{F}_3 = \vec{F}_{EG} = 10\left[\frac{(3-0)\vec{i} + (1-1)\vec{j} + (2-0)\vec{k}}{\sqrt{(3-0)^2 + (1-1)^2 + (2-0)^2}}\right]$

$\therefore \quad \vec{F}_3 = 8.32\vec{i} + 5.545\vec{k}$

$\therefore$ Resultant force (R) $= \sum F = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 5\vec{i} - 7.5\vec{j} + 8.32\vec{i} + 5.545\vec{k}$

$= 13.32\vec{i} - 7.5\vec{j} + 5.545\vec{k}$

The resultant couple is found by summing the moments of each force (F_1, F_2 and F_3) about A. Position vectors from A to convenient points on forces F_1, F_2 and F_3 respectively being.

$$r_{AE} = (0-0)\vec{i} + (1-0)\vec{j} + (0-0)\vec{k} = 1\vec{j}$$

$$r_{AH} = (0-0)\vec{i} + (1-0)\vec{j} + (2-0)\vec{k} = 1\vec{j} + 2\vec{k}$$

Moments of all the forces about A;

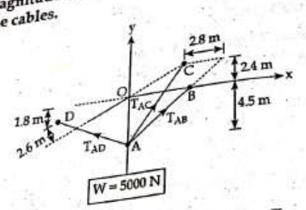
$$(M_R)_A = (r_{AE} \times \vec{F}_{EF}) + (r_{AH} \times \vec{F}_{HD}) + (r_{AE} \times \vec{F}_{EG})$$

$$= (1\vec{j} \times 5\vec{i}) + \left[(1\vec{j} + 2\vec{k}) \times (-7.5\vec{j})\right] + \left[1\vec{j} \times \left(8.32\vec{i} + 5.545\vec{k}\right)\right]$$

$$= -5\vec{k} + 15\vec{i} - 8.32\vec{k} + 5.545\vec{k}$$

$\therefore \quad \vec{R} = 15\vec{i} - 7.775\vec{k}$ kNm

48. A load, W of magnitude 5000 N is supported by three cables. Determine the tension in the cables.



Solution:

This is a problem of concurrent in the space. T_{AB}, T_{AC} and T_{AD} be the tensions in the cables whose directions are known.

The co-ordinates of the various points are A(0, 4.5, 0), B(2.8, 0, 0), C(0, 0, 2.4) and D(−2.6, 0, 1.8).

Tension in the cable AB [directed from A(0, 4.5, 0) to B(2.8, 0, 0)]

$$\therefore \quad \vec{T}_{AB} = \frac{T_{AB}}{\sqrt{(2.8)^2 + (4.5)^2}}\left[(2.8-0)\vec{i} + (0+4.5)\vec{j} + (0-0)\vec{k}\right]$$

$$\therefore \quad \vec{T}_{AB} = T_{AB}(0.528\vec{i} + 0.85\vec{j})$$

Tension in cable AB [directed from A(0, 4.5, 0) to C(0, 0, − 2.4)]

$$\vec{T}_{AC} = \frac{T_{AC}}{\sqrt{(4.5)^2 + (2.4)^2}}\left[(0-0)\vec{i} + (0-4.5)\vec{j} + (-2.4-0)\vec{k}\right]$$

$$\therefore \quad \vec{T}_{AC} = T_{AC}\left(0.88\vec{j} - 0.47\vec{k}\right)$$

Tension in the cable AD [directed from A(0, 4.5, 0) to D(−2.6, 0, 1.8)]

$$\vec{T}_{AD} = \frac{T_{AD}}{\sqrt{(2.6)^2 + (4.5)^2 + (1.8)^2}}\left[(-2.6-0)\vec{i} + (0+4.5)\vec{j} + (1.8-0)\vec{k}\right]$$

$$\therefore \quad \vec{T}_{AD} = T_{AD}\left(-0.473\vec{i} + 0.818\vec{j} + 0.327\vec{k}\right)$$

The weight, W is acting vertically downwards,

$$W = 5000 \times (-1\vec{j}) = -5000\vec{j}$$

Applying equation of the equilibrium; we get,

$$\sum F_x = 0$$

or, $0.528T_{AB} + 0.0T_{AC} - 0.473T_{AD} = 0$ (1)

$$\sum F_y = 0$$

or, $0.85T_{AB} + 0.88T_{AC} + 0.818T_{AD} - 5000 = 0$ (2)

and, $\sum F_z = 0$

or, $0.0T_{AB} - 0.47T_{AC} + 0.327T_{AD} = 0$ (3)

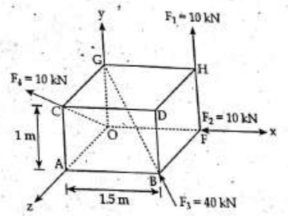
Solving these equations; we get,

$$T_{AD} = 2282 \text{ N}$$

$$T_{AB} = 2044.3 \text{ N}$$

$$T_{AC} = 1587.7 \text{ N}$$

49. A rectangular block is acted upon by the forces shown in the figure. Replace this set of forces by a single resultant force and couple acting at the point O.



Solution:

The co-ordinates of the various points are A(0, 0, 1), B(1.5, 0, 1), C(0, 1, 1), F(1.5, 0, 0), G(0, 1, 0) and H(1.5, 1, 0).

The different forces vectors are determined as follows:

$$\vec{F}_{OC} = F_4\frac{\vec{F}_{OC}}{\sqrt{F_{OC}}} = 10\left[\frac{(0-0)\vec{i} + (1-0)\vec{j} + (1-0)\vec{k}}{\sqrt{(1)^2 + (1)^2}}\right] = 5\sqrt{2}\vec{j} + 5\sqrt{2}\vec{k}$$

$$\vec{F}_{BG} = F_3\frac{\vec{F}_{BG}}{\sqrt{F_{BG}}} = 40\left[\frac{(0-1.5)\vec{i} + (1-0)\vec{j} + (0-1)\vec{k}}{\sqrt{(1.5)^2 + (1)^2 + (-1)^2}}\right] = -29.1\vec{i} + 19.4\vec{j} - 19.4\vec{k}$$

Similarly;

$$\vec{F}_{FH} = 20\vec{j}$$

and, $\vec{F}_{FO} = -10\vec{i}$

$$\therefore \quad \vec{R} = \vec{F}_{OC} + \vec{F}_{BG} + \vec{F}_{FH} + \vec{F}_{OF}$$

$$= 5\sqrt{2}\vec{j} + 5\sqrt{2}\vec{k} - 29.1\vec{i} + 19.4\vec{j} - 19.4\vec{k} + 20\vec{j} - 10\vec{i}$$

$= -39.1\vec{i} + 46.47\vec{j} - 12.33\vec{k}$

Taking moments about O; we get,

$$M_O = (\vec{r}_{OC} \times \vec{F}_{OC}) + (\vec{r}_{OB} \times \vec{F}_{BG}) + (\vec{r}_{OF} \times \vec{F}_{FO}) + (\vec{r}_{FO} \times \vec{F}_{FH})$$

$$= (1\vec{i} + 1\vec{k}) \times (5\sqrt{2}\vec{j} + 5\sqrt{2}\vec{k}) + (1.5\vec{i} + 1\vec{k}) \times (-29.1\vec{i} + 19.4\vec{j} - 19.4\vec{k})$$

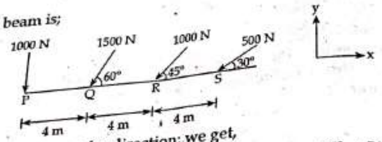
$$+1.5\vec{i} \times (-10\vec{i}) + 1.5\vec{i} \times 2$$

$\therefore \quad M_O = -19.40\vec{i} + 59.1\vec{k}$ kNm

50. **A horizontal beam PQRS is 12 m long where PQ = QR = RS = 4 m. Forces of 1000 N, 1500 N, 1000 N and 500 N act at the points P, Q, R and S respectively with the downward direction. The lines of action of these forces makes angles 90°, 60°, 45° and 30° respectively with PS. Find the magnitude, direction and position of resultant.**

Solution:

The given beam is;



Resolving forces in x and y direction; we get,

$$(^+\leftarrow) \sum F_x = 1000\cos 90° + 1500\cos 60° + 1000\cos 45° + 500\cos 30°$$

$$= 1890 \text{ N}(\leftarrow)$$

$$(^+\downarrow) \sum F_y = 1000\sin 90° + 1500\sin 60° + 1000\sin 45° + 500\sin 30°$$

$$= 3256 \text{ N}$$

$$\therefore \quad \text{Resultant (R)} = \sqrt{(\Sigma F_x)^2 + \left(\sum F_y\right)^2}$$

$$= \sqrt{(1890)^2 + (3256)^2}$$

$$= 3760 \text{ N}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right)$$

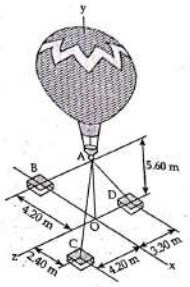
$$= \tan^{-1}\left(\frac{3256}{1890}\right) = 59.87°$$

Let, resultant acts x distance from P. Then, taking moment about P; we get

$$1000 \times 0 + 4 \times 1500\sin 60° + 8 \times 1000\sin 45° + 12 \times 500\sin 30° = x\sum$$

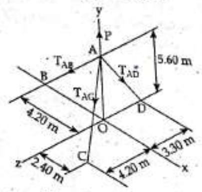
$$\therefore \quad x = \frac{13853}{3256} = 4.25 \text{ m; from P.}$$

51. **Three cables are used to tether a balloon as shown in the figure. Determine the vertical forces P exerted by the balloon at A knowing that the tension in the cable AB is 259 N.**



Solution:

Drawing free body diagram of the given figure



The forces applied at A are T_{AB}, T_{AC}, T_{AD} and P where, $P = P\vec{j}$.

The co-ordinate of the different points are A(0, 5.6, 0), B(−4.2, 0, 0), C(2.4, 0, 4.2) and D(0, 0, −3.3).

To express the other forces in terms of unit vectors $\vec{i}$, $\vec{j}$ and $\vec{k}$; we get,

$$\overrightarrow{AB} = -4.2\vec{i} - 5.6\vec{j}$$

$$\therefore \quad AB = 7 \text{ m}$$

$$\overrightarrow{AC} = 2.4\vec{i} - 5.6\vec{j} + 4.2\vec{k}$$

$$\therefore \quad AC = 7.40 \text{ m}$$

$$\text{and,} \quad \overrightarrow{AD} = -5.6\vec{j} - 3.30\vec{k}$$

$$\therefore \quad AD = 6.50 \text{ m}$$

$$\text{and,} \quad T_{AB} = T_{AB}f_{AB} = T_{AB}\left(\frac{\overrightarrow{AB}}{AB}\right) = (-0.6\vec{i} - 0.8\vec{j})T_{AB}$$

$$T_{AC} = T_{AC}f_{AC} = T_{AC}\left(\frac{\overrightarrow{AC}}{AC}\right) = \left(-0.324\vec{i} - 0.757\vec{j} + 0.568\vec{k}\right)T_{AC}$$

$$T_{AD} = T_{AD}f_{AD} = T_{AD}\left(\frac{\overrightarrow{AD}}{AD}\right) = \left(-0.861\vec{j} - 0.508\vec{k}\right)T_{AD}$$

Equilibrium conditions;

$\Sigma F = 0$

or, $T_{AB} + T_{AC} + T_{AD} + P\vec{j} = 0$

Substituting expression obtained for T_{AB}, T_{AC} and T_{AD} and factoring $\vec{i}, \vec{j}$ and $\vec{k}$; we have,

$$(-0.6T_{AB} + 0.324T_{AC})\vec{i} + (-0.8T_{AB} - 0.757T_{AC} - 0.861T_{AD} + P)\vec{j} + (0.568T_{AC} - 0.508T_{AD})\vec{k} = 0$$

Equating to zero, the co-efficient of $\vec{i}, \vec{j}$ and $\vec{k}$; we get,

$-0.6T_{AB} - 0.324T_{AC} = 0$ (1)

$-0.8T_{AB} - 0.757T_{AC} - 0.861T_{AD} + P = 0$ (2)

$0.568T_{AC} - 0.508T_{AD} = 0$ (3)

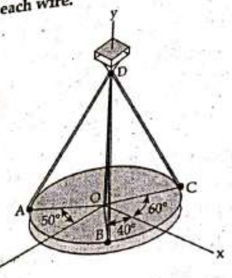
Putting $T_{AB} = 259$ in the equation (1) and (2) and solving; we get,

$T_{AC} = 479.15\ N$

$T_{AD} = 535.66\ N$

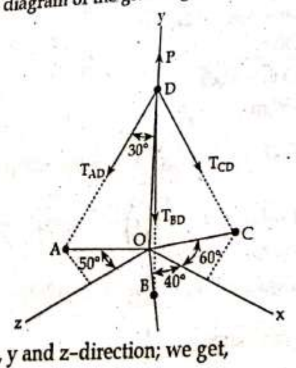
and, $P = 1031\ N(\uparrow)$

52. **A horizontal circular plate weighing 60 N is suspended as shown from three wires that are attached to a support at D and form 30° with vertical. Determine tension in each wire.**



Solution:

Drawing free body diagram of the given figure.



Resolving forces into x, y and z-direction; we get,

$(^+\searrow)\ \Sigma F_x = 0$

or, $-T_{AD} \sin 30° \sin 50° + T_{BD} \sin 30° \cos 40° + T_{CD} \sin 30° \cos 40° = 0$

or, $-0.766T_{AD} + 0.766T_{BD} + 0.5T_{CD} = 0$ (1)

$(^+\uparrow)\ \sum F_y = 0$

or, $-T_{AD} \cos 30° - T_{BD} \cos 30° - T_{CD} + 60 = 0$

or, $T_{AD} + T_{BD} + T_{CD} = 69.28$ (2)

$(^+\swarrow)\ \Sigma F_z = 0$

or, $T_{AD} \sin 30° \cos 30° + T_{BD} \sin 30° \cos 40° - T_{CD} \sin 30° \sin 60° = 0$

or, $0.643T_{AD} + 0.643T_{BD} - 0.866T_{CD} = 0$ (3)

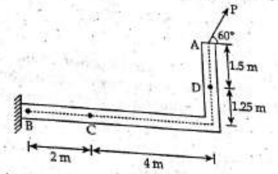
Solving equations (1), (2) and (3); we get,

$T_{AD} = 29.5\ N$

$T_{BD} = 10.25\ N$

and, $T_{CD} = 29.5\ N$

53. **A 160 N force P is applied at point A of a structural member. Replace P with (a) an equivalent force couple system at C (b) an equivalent system consisting of a vertical force at B and a second force at D.**



Solution:

Resolving forces in x and y-direction; we get,

a) $\Sigma F_x = P \cos 60° = 160 \cos 60° = 80\ N$

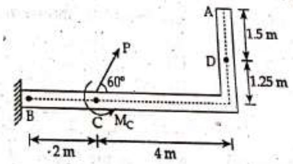
$\sum F_y = P \sin 60° = 160 \sin 60° = 138.56\ N$

$\therefore\ R = P = 160\ N$

and, moment about C is;

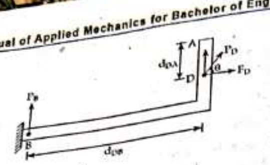
$M_C = -80 \times 2.75 + 138.56 \times 4 = 334.24\ Nm\ (\circlearrowleft)$

Hence, the equivalent force couple system at C as shown in the figure.



b) $(F_d)_x = 80\ N$

Taking moment about D; we get,



$$P\cos 60° \times d_{DA} = P_B \times d_{DB}$$

or, $80 \times 1.5 = P_B \times 6$

$\therefore$ $P_B = 20\,N$

and, $\sum F_y = 0$

or, $P\sin 60° = P_B + (F_d)_y$

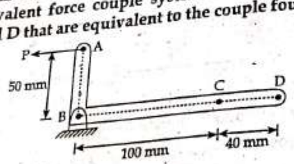
$\therefore$ $138.56 = 20 + (F_d)_y$

$\therefore$ $(F_d)_y = 118.56\,N$

$\therefore$ $P_D = \sqrt{[(F_d)_x]^2 + [(F_d)_y]^2} = \sqrt{(80)^2 + (118.56)^2} = 143.03\,N$

$\therefore$ $\theta = \tan^{-1}\left(\frac{\sum (F_d)_y}{\sum (F_d)_x}\right) = \tan^{-1}\left(\frac{118.56}{80}\right) = 56°$

54. The 80 N horizontal force P acts on a bell crank as shown. (a) Replace with an equivalent force couple system at B. (b) Find the two verti forces at C and D that are equivalent to the couple found in part (a).



Solution:

a) *Resolving forces; we get,*

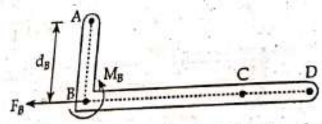
$\sum F: F_B = F = 80\,N$

or, $F_B = 80\,N(\leftarrow)$

Taking moment about B; we get,

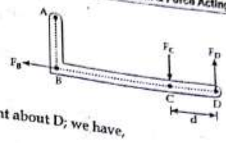
$\therefore$ $M_B = 80 \times 0.05 = 4\,Nm(\circlearrowleft)$

The equivalent force couple system at B as shown in the figure.



b) *If two vertical forces are to be equivalent to M_B, they must be a co Further, the sense of the moment of this couple must be anti-clockwi*

Then, F_C and F_D acting as shown in the figure.



Now, taking moment about D; we have,

$M_D = F_C \times d$

or, $4 = F_C \times 0.04$

$\therefore$ $F_C = 100\,N(\downarrow)$

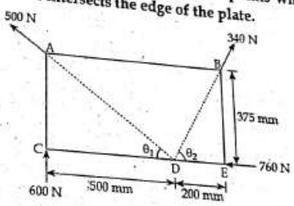
Also,

$\sum F_y = 0$

or, $F_D - F_C = 0$

$\therefore$ $F_D = 100\,N(\uparrow)$

55. Four forces acts on a 700 × 375 mm plate as shown in the figure. (a) Find the resultant of these forces. (b) Locate the two points where the line of action of the resultant intersects the edge of the plate.



Solution:

Taking C as the origin. From the figure; we have,

$\tan\theta_1 = \frac{375}{500} = \frac{3}{4}$

$\therefore$ $\sin\theta_1 = \frac{3}{5} = 0.6$

and, $\cos\theta_1 = \frac{4}{5} = 0.8$

Similarly,

$\tan\theta_2 = \frac{375}{200} = \frac{15}{8}$

$\therefore$ $\sin\theta_2 = \frac{15}{\sqrt{(15)^2 + (8)^2}} = \frac{15}{17}$

and, $\cos\theta_2 = \frac{8}{17}$

Now, resolving the forces in x and y-direction; we get,

$$(^+\rightarrow)\sum F_x = [(-500) \times 0.8]\vec{i} + \left(340 \times \frac{8}{17}\right)\vec{i} - 760\vec{i} = -(1000\,N)\vec{i}$$

and, $(^{+}\uparrow) \sum F_y = (500 \times 0.6)\vec{j} + \left(340 \times \frac{15}{17}\right)\vec{j} + 600\vec{j} = (1200\ N)\vec{j}$

$\therefore \quad R = \sqrt{(\Sigma F_x)^2 + \left(\sum F_y\right)^2} = \sqrt{(-1000)^2 + (1200)^2} = 1562\ N$

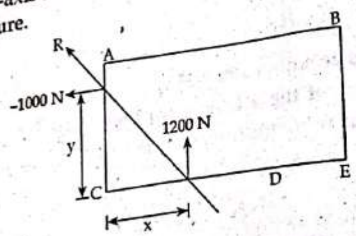
and, $\theta = \tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right) = \tan^{-1}\left(\frac{1200}{-1000}\right) = -50.194°$

Taking moment about C; we get,

$$M_C = 0.5\vec{i} \times \left(500 \times 0.6 + 340 \times \frac{15}{17}\right)\vec{j}$$

$$= (300\ Nm)\vec{k}\ (\circlearrowleft)$$

Let, resultant cut x-axis at distance x from C and y-axis at distance y from C as shown in the figure.



$$M_C = x\vec{i} \times 1200\vec{j}$$

or, $300\vec{k} = 1200x\vec{k}$

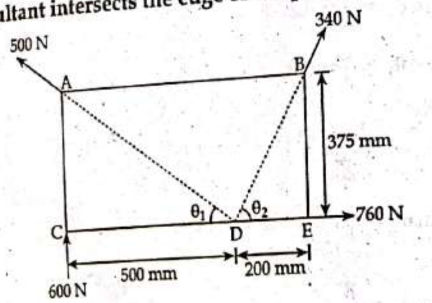
$\therefore \quad x = 0.250\ m = 250\ mm$

and, $300\vec{k} = y\vec{j} \times (-1000\vec{i})$

$\therefore \quad y = 0.30\ m = 300\ mm$

Intersection 250 mm to right of C and 300 mm above C.

56. **Four forces acts on a 700 × 375 mm plate as shown in the figure. (a) Find the resultant of these forces. (b) Locate the two points where the line of action of the resultant intersects the edge of the plate.**



Solution:

Let us consider C as the origin. Then, the plate rests at plane xy such that CE as x-axis and CA as y-axis.

Resolving forces into x and y-direction; we get,

$$(^{+}\rightarrow) \Sigma F_x = (-500) \times \left(\frac{500}{\sqrt{(500)^2 + (375)^2}}\right)\vec{i} + 760\vec{i} + 340 \times \left(\frac{200}{\sqrt{(200)^2 + (375)^2}}\right)\vec{i}$$

$\therefore \quad \Sigma F_x = 520\vec{i}$

and, $\sum F_y = 500 \times \left(\frac{375}{\sqrt{(375)^2 + (500)^2}}\right)\vec{j} + 600\vec{i} + 340 \times \left(\frac{375}{\sqrt{(375)^2 + (200)^2}}\right)\vec{j}$

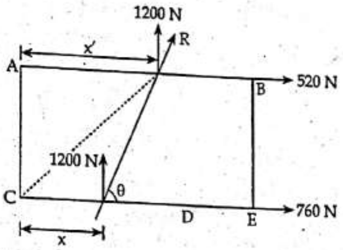
$\therefore \quad \sum F_y = 1200\vec{j}$

$\therefore \quad R = 520\vec{i} + 1200\vec{j}$

and, $\theta = \tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right) = \tan^{-1}\left(\frac{1200}{520}\right) = 66.57°$

$\therefore \quad R = \sqrt{(\Sigma F_x)^2 + \left(\sum F_y\right)^2}$

$$= \sqrt{(520)^2 + (1200)^2} = 1308\ N$$



Since, resultant lies in first quadrant so it cuts plate as shown in the figure.

Taking moment about C; we get,

$$M_C = 0.5\vec{i} \times \left(500 \times 0.6 + 340 \times \frac{15}{17}\right)\vec{j}$$

$$= (300\ Nm)\vec{k}$$

Also,

$$300\vec{k} = x\vec{i} \times (1200\vec{j})$$

$\therefore \quad x = 0.250\ m = 250\ mm$

To find x'

$$CB' = x'\vec{i} + 0.375\vec{j}$$

$\therefore \quad M_C = CB' \times \vec{R}$

or, $300\vec{k} = (x'\vec{i} + 0.375\vec{j}) \times (520\vec{i} + 1200\vec{j})$

or, $300\vec{k} = (1200x' - 195)\vec{k}$

$\therefore \quad x' = 0.4125\ m = 412.5\ mm$

Thus, intersection 412.5 mm right to A and 250 mm to right of C.

57. A force 22 N acts through the point A(4, −1, 7) in the direction of vector $9\vec{i}+6\vec{j}-2\vec{k}$. Find the moment of the force about the point A(1, −3, 2).

Solution:

Unit vector in the direction vector $9\vec{i}+6\vec{j}-2\vec{k}$ is;

$$\frac{9\vec{i}+6\vec{j}-2\vec{k}}{\sqrt{(9)^2+(6)^2+(-2)^2}}=\frac{9\vec{i}+6\vec{j}-2\vec{k}}{11}$$

$$\text{Force (F)} = 22\times\frac{1}{11}\times\left(9\vec{i}+6\vec{j}-2\vec{k}\right)$$

$$\therefore\quad F=\left(18\vec{i}+12\vec{j}-4\vec{k}\right)$$

Position vector, r of the point A on the force with respect to the point O is;

$$r=(4-1)\vec{i}+\{-1-(-3)\}\vec{j}+(7-2)\vec{k}$$

$$\therefore\quad r=\left(3\vec{i}+2\vec{j}+5\vec{k}\right)$$

Now,

$$M=r\times F$$

$$\text{or,}\quad M=\left(3\vec{i}+2\vec{j}+5\vec{k}\right)\times\left(18\vec{i}+12\vec{j}-4\vec{k}\right)$$

$$\therefore\quad M=-68\vec{i}+102\vec{j}$$

CHAPTER 4 Centre of Gravity, Centroid and Moment of Inertia

DEFINITIONS

Center of gravity

The point of application of the resultant of the gravitational forces acting on the particle of a body is called the center of gravity.

Centroid

The centroid or center of area is defined as the point where the whole area of the figure is assured to be concentrated. The centroid can be taken as quite analogous to center of gravity when bodies have only area and not weight.

Centroid of a line

The centroid of a line of entire length 'L' is given by;

$$\bar{x}=\frac{\int x\,dL}{L}$$

$$\text{and,}\quad \bar{y}=\frac{\int y\,dL}{L}$$

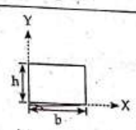
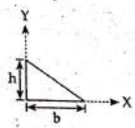
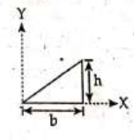
Centroid of an area

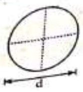
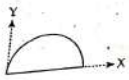
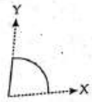
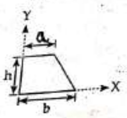
The centroid of a plane lamina of area 'A' is given as;

$$\bar{x}=\frac{\int x\,dA}{A}$$

$$\text{and,}\quad \bar{y}=\frac{\int y\,dA}{A}$$

Positions of centroid of plane geometrical figures

Shape	Figure	Area	$\bar{x}$	$\bar{y}$
Rectangle		bh	$\frac{b}{2}$	$\frac{h}{2}$
Triangle		$\frac{1}{2}bh$	$\frac{b}{3}$	$\frac{h}{3}$
Triangle		$\frac{1}{2}bh$	$\frac{2}{3}b$	$\frac{h}{3}$

Circle		πr^2	$\frac{d}{2}$	$\frac{d}{2}$
Semi-circle		$\frac{\pi r^2}{2}$	$\frac{4r}{3\pi}$	$\frac{d}{2}$
Semi-circle		$\frac{\pi r^2}{2}$	$\frac{d}{2}$	$\frac{4r}{3\pi}$
Quarter-circle		$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Trapezium		$\frac{(a+b)h}{2}$	$\frac{a^2+b^2+ab}{3(a+b)}$	$\frac{(2a+b)h}{3(a+b)}$

Second moment of area or moment of inertia

The second moment of area of a lamina of area 'A' with respect to the x-axis is defined as,

$$I_x = \int y^2 dA$$

Similarly, the second moment of area with respect to the Y-axis is defined as,

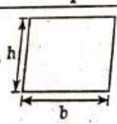
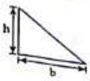
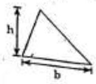
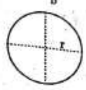
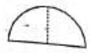
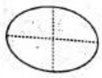
$$I_y = \int x^2 dA$$

Radius of gyration

It is defined as the distance from the axis to a point where the entire area of the lamina could be concentrated into a thin strip and have the same moment of Inertia with respect to given axis.

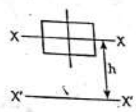
$$K_x = \sqrt{\frac{I_x}{A}},\ K_y = \sqrt{\frac{I_y}{A}}$$

Moment of inertia of regular shapes about centroid axes

Area	Shape	I_x	I_y
Rectangle		$\frac{bh^3}{12}$	$\frac{hb^3}{12}$
Right-Angled triangle		$\frac{bh^3}{36}$	$\frac{hb^3}{36}$
Isosceles triangle		$\frac{bh^3}{36}$	$\frac{hb^3}{48}$
Circle		$\frac{\pi r^4}{4}$	$\frac{\pi r^4}{4}$
Semi-circle		$0.11r^4$	$\frac{\pi r^4}{8}$
Semi-circle		$\frac{\pi r^4}{8}$	$0.11r^4$
Quarter-Circle		$0.055r^4$	$0.055r^4$
Ellipse		$\frac{\pi ab^3}{4}$	$\frac{\pi ba^3}{4}$

Parallel axis theorem

It states that "the moment of inertia of a plane area about any axis parallel to the centroidal axis is equal to the sum of moment of inertia about a parallel centroidal axis and the product of the area and square of the distance between the two axes."



$$I_{x'x'} = I_{xx} + Ah^2$$

Perpendicular axis theorem

It states that "the moment of inertia of a plane area about on axis perpendicular to the plane and passing through the intersection of the other two axes X-X and Y-Y contained by the plane is equal to the sum of the moment of inertia about X-X and Y-Y axes."

$$I_{ZZ} = I_{XX} + I_{YY}$$

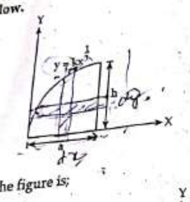
Polar moment of inertia

The point of intersection of the axis 'X' and 'Y' is called pole, and the axis passing through a point and perpendicular to the area is called the polar axis. The moment of inertia about this axis is called polar moment of inertia.

$$I_p = I_X + I_Y$$

EXAM SOLUTION

1. **Determine the moment of inertia of the area about the X and Y axes as shown in figure below.** [2060 Poush]



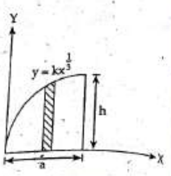
Solution:

The given equation of the figure is;

$$y = kx^{1/3}$$

When $x = a$, $y = h$

$$\therefore \quad k = \frac{h}{a^{1/3}}$$



Therefore, the equation becomes;

$$y = \frac{h}{a^{1/3}}x^{1/3}$$

M.I. about X-axis is given by;

$$\therefore \quad I_x = \frac{1}{3}\int y^3\,dx = \frac{1}{3}\int_0^a \left(\frac{h}{a^{1/3}}x^{1/3}\right)^3 dx \qquad I_x = \int y^2\,dA.$$

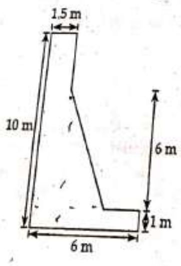
$$= \frac{1}{3}\frac{h^3}{a}\int_0^a x\,dx = \frac{h^3}{3a}\left[\frac{x^2}{2}\right]_0^a = \frac{h^3}{3a}\frac{a^2}{2} = \frac{ah^3}{6}$$

$y^2 dA$

$$\text{and, } I_y = \int x^2\,dA = \int x^2 y\,dx = \int_0^a x^2\left(\frac{h}{a^{1/3}}x^{1/3}\right)dx = \frac{h}{a^{1/3}}\int_0^a x^{2+1/3}$$

$$= \frac{h}{a^{1/3}}\int_0^a x^{7/3} = \frac{h}{a^{1/3}}\left[\frac{3}{10}x^{10/3}\right]_0^a = \frac{h}{a^{1/3}}\times\frac{3}{10}\times a^{10/3} = \frac{3}{10}ha^3$$

2. **Determine the moment of inertia of the section shown in figure about centroidal axes.** [2061 Baishakh]



Solution:

Dividing the whole figure into three geometrical figures; we have,

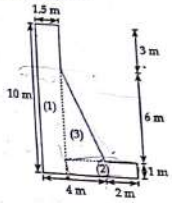
i) Rectangle (1)
ii) Rectangle (2)
iii) Triangle (3)

Now, calculating the area and centroid of each geometrical figure;

$A_1 = 1.5 \times 10 = 15\ m^2$

$A_2 = 4.5 \times 1 = 4.5\ m^2$

$A_3 = \frac{1}{2}\times 2.5 \times 6 = 7.5\ m^2$



$x_1 = 0.75$ m

$y_1 = 5$ m

$x_2 = 1.5 + 2.25 = 3.75$ m

$y_2 = 0.5$ m

$x_3 = 1.5 + \frac{4.5}{3} = 3$ m

$y_3 = 1 + \frac{6}{3} = 3$ m

$$\therefore \quad \bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3} = 1.875\ cm$$

$$\text{and, } \bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3} = 3.69\ cm$$

Now, calculating moment of inertia of the figure;

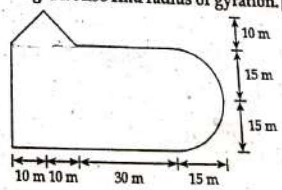
$$\therefore \quad I_X = \left[\frac{1.5\times 10^3}{12} + 15\,(5 - 3.69)^2\right] + \left[\frac{4.5\times 1^3}{12} + 4.5(0.5 - 3.694)^2\right] + \left[\frac{2.5\times 6^3}{36} + 7.5(3 - 3.69)^2\right]$$

$$= 150.74 + 46.17 + 18.57 = 215.48\ m^4$$

$$\text{and, } I_y = \left[\frac{10\times 1.5^3}{12} + 15\,(0.75 - 1.875)^2\right] + \left[\frac{1\times 4.5^3}{12} + 4.5(3.75 - 1.875)^2\right] + \left[\frac{6\times 2.5^3}{36} + 7.5(3 - 1.875)^2\right]$$

$$= 21.80 + 23.41 + 36.47 = 81.68\ m^4$$

3. **Find centre of gravity and moment of inertia about centroidal axes of the plane figure as shown in figure. Also find radius of gyration.** [2062 Baishakh]



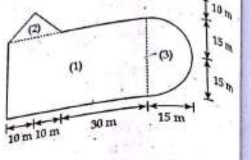
Solution:

Dividing the whole figure into three geometrical figures; we have,

i) Rectangle (1)
i) Isosceles triangle (2)

iii) Semi-circle (3)

Now, calculating the area and centroid of each geometrical figure;



$$A_1 = 50 \times 30 = 1500 \text{ cm}^2$$

$$A_2 = \frac{1}{2} \times 20 \times 10 = 100 \text{ cm}^2$$

$$A_3 = \frac{\pi \times b15^2}{2} = 353.43 \text{ cm}^2$$

$$x_1 = 25 \text{ cm}$$

$$y_1 = 15 \text{ cm}$$

$$x_2 = 10 \text{ cm}$$

$$y_2 = 30 + \frac{10}{3} = 33.33 \text{ cm}$$

$$x_3 = 50 + \frac{4 \times 15}{3\pi} = 56.37 \text{ cm}$$

$$y_3 = 15 \text{ cm}$$

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3} = 29.91 \text{ cm}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3} = 15.94 \text{ cm}$$

Now, calculating moment of inertia about centroidal axes;

$$I_X = \left[\frac{50 \times 30^3}{12} + 1500(15 - 15.94)^2\right] + \left[\frac{20 \times 10^3}{36} + 100(33.33 - 15.94)^2\right] + \left[\frac{\pi \times 15^4}{8} + 353.43\,(15 - 15.94)^2\right]$$

$$= 113825.4 + 30796.76 + 20192.68$$

$$\therefore \quad I_X = 164814.84 \text{ cm}^4$$

$$\text{and, } I_y = \left[\frac{h_1b_1^3}{12} + A_1(x_1 - \bar{x})^2\right] + \left[\frac{h_2b_2^3}{48} + A_2(x_2 - \bar{x})^2\right] + \left[0.11r^4 + A_3(x_3 - \bar{x})^2\right]$$

$$= \left[\frac{30 \times 50^3}{12} + 1500(25 - 29.91)^2\right] + \left[\frac{10 \times 20^3}{48} + 100(10 - 29.91)^2\right] + \left[0.11 \times 15^4 + 353.43(56.37 - 29.91)^2\right]$$

$$= 348662.15 + 41307.48 + 253016.26$$

$$\therefore \quad I_y = 642985.89 \text{ cm}^4$$

Now, radius of gyration is calculated as;

$$\therefore \quad k_x = \sqrt{\frac{I_X}{A}}$$

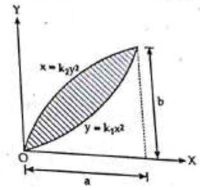
$$= \sqrt{\frac{164814.84}{1953.43}} = 9.18 \text{ cm}$$

$$\text{and, } k_y = \sqrt{\frac{I_y}{A}}$$

$$= \sqrt{\frac{642985.89}{1953.43}} = 18.14 \text{ cm}$$

4. Locate the centroid of the shaded plane area bounded by two curves as shown in figure.

[2063 Baishakh]



Solution:

The given equations of curves are;

$$y = k_1x^2$$

$$\text{and, } x = k_2y^2$$

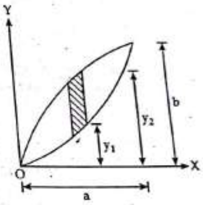
When, $x = a$, $y = b$

$$b = k_1a^2$$

$$\text{and, } a = k_2b^2$$

$$\therefore \quad k_1 = \frac{b}{a^2}$$

$$\therefore \quad k_2 = \frac{a}{b^2}$$



Hence, the equation becomes;

$$y_1 = \frac{b}{a^2}x^2$$

$$\text{and, } y_2 = \frac{b}{a^{1/2}}x^{1/2}$$

Now,

$$A = \int dA = \int_0^a (y_2 - y_1)\,dx = \int_0^a \left(\frac{b}{a^{1/2}}x^{1/2} - \frac{b}{a^2}x^2\right)dx$$

$$= \left[\frac{2}{3}\,\frac{b}{a^{1/2}}x^{3/2} - \frac{b}{a^2}\,\frac{x^3}{3}\right]_0^a = \frac{2}{3}\frac{b}{a^{1/2}}a^{3/2} - \frac{b}{a^2}\frac{a^3}{3} = \frac{2ab}{3} - \frac{ab}{3}$$

$$\therefore \quad A = \frac{ab}{3}$$

Now, taking a vertical strip parallel to the y-axis; then,

$$\bar{x} = \frac{\int x\,dA}{A} = \frac{3}{ab}\int_0^a \left(\frac{b}{a^{1/2}}x^{3/2} - \frac{b}{a^2}x^3\right)dx$$

$$= \frac{3}{ab}\left[\frac{2}{5}\,\frac{b}{a^{1/2}}x^{5/2} - \frac{b}{a^2}\,\frac{x^4}{4}\right]_0^a$$

$$= \frac{3}{ab}\left[\frac{2}{5}\,\frac{b}{a^{1/2}}a^{5/2} - \frac{b}{a^2}\,\frac{a^4}{4}\right]$$

$$= \frac{3}{ab}\left(\frac{2}{5}ba^2 - \frac{ba^2}{4}\right) = \frac{3}{ab}\,\frac{3a^2b}{20}$$

$$\therefore \quad \bar{x} = \frac{9a}{20}$$

Similarly;

$$\bar{y} = \frac{\int y\,dA}{A} = \frac{3}{ab}\int_0^a \left[y_1 + \left(\frac{y_2 - y_1}{2}\right)\right](y_2 - y_1)dx$$

$$= \frac{3}{ab}\int_0^a \left(\frac{y_1 + y_2}{2}\right)(y_2 - y_1)dx = \frac{3}{2ab}\int_0^a (y_2^2 - y_1^2)dx$$

$$= \frac{3}{2ab}\left[\frac{b^2}{a}\cdot\frac{x^2}{2} - \frac{b^2}{a^4}\cdot\frac{x^5}{5}\right]_0^a = \frac{3}{2ab}\left[\frac{b^2}{a}\cdot\frac{a^2}{2} - \frac{b^2}{a^4}\cdot\frac{a^5}{5}\right]$$

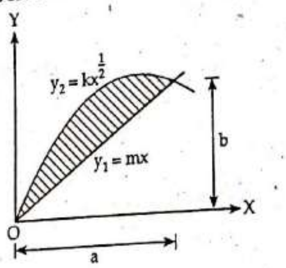
$$= \frac{3}{2ab}\left[\frac{ab^2}{2} - \frac{ab^2}{5}\right] = \frac{3}{2ab}\,\frac{3a^2b^2}{10}$$

$$\therefore \quad \bar{y} = \frac{9b}{20}$$

Hence, the centroid of the area bounded by given two curve is;

$$(\bar{x}, \bar{y}) = \left(\frac{9a}{20}, \frac{9b}{20}\right)$$

5. **Determine the moment of inertia and radius of gyration of the hatched area as shown in figure below about X and Y axis.** [2064 Jestha]



Solution:

Let us consider a small strip parallel to x-axis which is at a distance 'y' from it.

When x = a, y = b

$\therefore \quad b = ma$

and, $b = k\sqrt{a}$

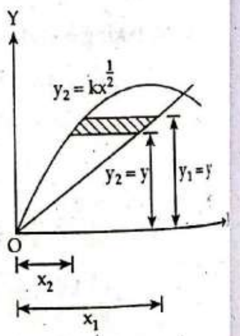
$\therefore \quad m = \frac{b}{a}$

and, $k = \frac{b}{a^{1/2}}$

Hence, the given equation becomes;

$Y_1 = \frac{b}{a}x$

and, $Y_2 = \frac{b}{a^{1/2}}x^{1/2}$



Now, moment of inertia about x-axis is given by;

$$I_x = \int y^2 dA = \int_0^b y^2(x_1 - x_2)\,dy = \int_0^b y^2\left(\frac{y}{m} - \frac{y^2}{k_2}\right)$$

$$= \int_0^b \left(\frac{y^3}{m} - \frac{y^4}{k_2}\right)dy = \left(\frac{y^4}{4m} - \frac{y^5}{5k^2}\right)_0^b = \frac{a}{b}\frac{b^4}{4} - \frac{b^5}{5}\frac{a}{b^2} = \frac{ab^3}{4} - \frac{ab^3}{5}$$

$$\therefore \quad I_x = \frac{1}{20}ab^3$$

Now, to calculate I_y taking a vertical strip parallel to y-axis at a distance of 'x' from it;

$$\therefore \quad I_y = \int x^2 dA = \int_0^a x^2(y_1 - y_2)\,dx$$

$$= \int_0^a x^2\,(kx^{1/2} - mx)\,dx$$

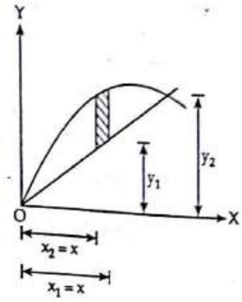
$$= \int_0^a x^2\,(kx^{5/2} - mx^3)\,dx$$

$$= \left[\frac{2}{7}\,\frac{b}{a^{1/2}}x^{7/2} - \frac{b}{a}\,\frac{x^4}{4}\right]_0^a$$

$$= \frac{2}{7}\,\frac{b}{a^{1/2}}a^{7/2} - \frac{b}{a}\,\frac{a^4}{4}$$

$$= \frac{2}{7}ba^3 - \frac{1}{4}ba^3$$

$$\therefore \quad I_y = \frac{a^3b}{28}$$



Now,

$$A = \int_0^a (y_2 - y_1)\,dx$$

$$= \left[\frac{2}{3}\,\frac{b}{a^{1/2}}x^{3/2} - \frac{b}{a}\,\frac{x^2}{2}\right]_0^a$$

$$= \frac{2}{3}ba - \frac{ab}{2}$$

$$\therefore \quad A = \frac{ab}{6}$$

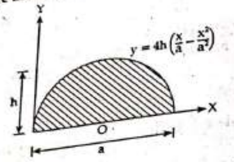
$$\therefore \quad k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{ab^3}{20}\times\frac{6}{ab}}$$

$$\therefore \quad k_x = 0.545\,b$$

$$\text{and,}\quad k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{a^3b}{28}\times\frac{6}{ab}}$$

$$\therefore \quad k_y = 0.463\,a$$

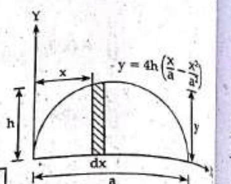
6. **Determine the moment of inertia and radius of gyration of the shaded area as shown in figure about X and Y axes.** [2065 Shrawan]



Solution:

Let us consider a small strip of area dA parallel to Y-axis at a distance of x from Y-axis.

M.I. of the figure about X-axis is given by;



$$I_x = \int \frac{y^3}{3}dx = \frac{1}{3}\int_0^a y^3 dx$$

$$= \frac{1}{3}\int_0^a \left[4h\frac{x}{a} - \frac{x^2}{a^2}\right]^3 dx$$

$$= \frac{1}{3}\times 64h^3 \int_0^a \left[\frac{x^3}{a^3} - 3\frac{x^2}{a^2}\cdot\frac{x^2}{a^2} + 3\frac{x}{a}\cdot\frac{x^4}{a^4} - \frac{x^6}{a^6}\right]$$

$$= \frac{64}{3}h^3\left[\frac{x^4}{4a^3} - \frac{3}{a^4}\cdot\frac{x^5}{5} + \frac{3}{a^5}\cdot\frac{x^6}{6} - \frac{x^7}{7a^6}\right]_0^a$$

$$= \frac{64}{3}h^3\left[\frac{a^4}{4a^3} - \frac{3a^5}{5a^4} + \frac{3a^6}{6a^5} - \frac{a^7}{7a^6}\right] = \frac{64}{3}h^3\left[\frac{a}{4} - \frac{3a}{5} + \frac{3a}{6} - \frac{a}{7}\right]$$

$$\therefore \quad I_X = \frac{16}{105}ah^3$$

Similarly;

$$I_y = \int x^2 dA = \int_0^a x^2 . y\, dx = \int_0^a 4h\left(\frac{x^3}{a} - \frac{x^4}{a^2}\right)dx = 4h\left(\frac{x^4}{4a} - \frac{x^5}{5a^2}\right)_0^a dx$$

$$= 4h\left(\frac{a^4}{4a} - \frac{a^5}{5a^2}\right) = 4h\left(\frac{a^3}{4} - \frac{a^3}{5}\right)$$

$$\therefore \quad I_y = \frac{1}{5}ha^3$$

Now,

$$A = \int y\, dx = \int_0^a 4h\left(\frac{x}{a} - \frac{x^2}{a^2}\right) = 4h\left(\frac{x^2}{2a} - \frac{x^3}{3a^2}\right)_0^a = 4h\left(\frac{a^2}{2a} - \frac{a^3}{3a^2}\right)$$

$$= 4h\left(\frac{a}{2} - \frac{a}{3}\right)$$

$$\therefore \quad A = \frac{2ah}{3}$$

Now, radius of gyration can be calculated as;

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{16}{105}h^3a \times \frac{3}{2ah}}$$

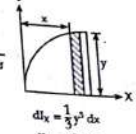
$$\therefore \quad k_x = 0.478\, h$$

$$\text{and,} \quad k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{ha^3}{5}\times\frac{3}{2ah}}$$

$$\therefore \quad k_y = 0.548a$$

Note

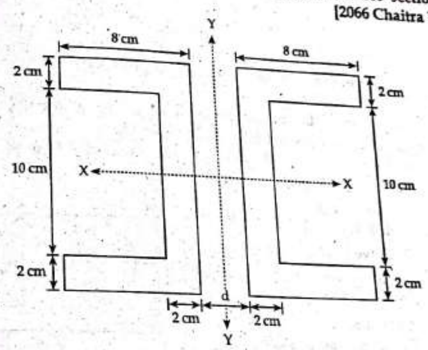
If we are to calculate moment of inertia with a single strip as shown in the figure; use,



$$I_X = \int \frac{1}{3}y^3\, dx$$

and, $I_Y = x^2 dA = x^2 y\, dx$

7. **Two equal channels are kept as shown in figure at a distance 'd' between them, to form the cross-section of a column. Calculate the value of 'd' so that the centroidal moment of inertia I_{x-x} and I_{y-y} of the cross-section are equal.** [2066 Chaitra Back]



Solution:

Here, 'd' is the distance between channel sections $I_x = I_y$ (Given)

Now, let us find out I_x and I_y separately and equate then,

I_x = M.O.I. of flanges (4 nos.) + M.O.I. of webs (2 nos.)

$$= 4\left[\frac{8\times 2^3}{12} + 8\times 2\times(7-1)^2\right] + 2\left[\frac{2\times 10^3}{12}\right]$$

$$= 2325.33 + 333.33 = 2658.66\ cm^4$$

and, I_y = M.O.I. of flanges (4 nos.) + M.O.I. of webs (2 nos.)

$$= 4\left[\frac{2\times 8^3}{12} + 8\times 2\times\left(4+\frac{d}{2}\right)^2\right] + 2\left[\frac{10\times 2^3}{12} + 10\times 2\left(\frac{2}{2}+\frac{d}{2}\right)^2\right]$$

$$= 4\left[85.33 + 16\left(16 + 4d + \frac{d^2}{4}\right)\right] + 2\left[6.67 + 20\left(1 + 2d + \frac{d^2}{4}\right)\right]$$

$$= 341.32 + 1024 + 256d + 16d^2 + 13.34 + 40 + 80d + 10d^2$$

$\therefore \quad I_y = 1418.66 + 336d + 26d^2$

According to the question; we have,

$I_X = I_y$

or, $26d^2 + 336d + 1418.66 = 2658.66$

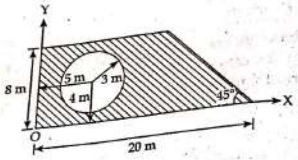
or, $26d^2 + 336d - 1240 = 0$

Solving this equation; we get,

$d = 3$ cm

Hence distance between two channel sections is 3 cm.

8. **Find the polar moment of inertia at point 'O' for the shaded area in the figure.** [2066 Chaitra Back]



Solution:

Since polar moment of inertia about 'O' is given by,

$I_p = I_X + I_y$

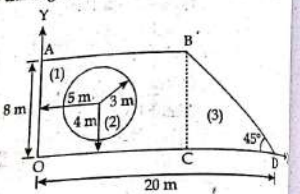
where, I_X and I_y are the moment of inertia about OX and OY axes respectively, we have to first calculate I_X and I_y.

Now, dividing the given figure into three geometrical figures;

i) Rectangle (1) (+)

ii) Circle (2) (−)

iii) Triangle (3) (+)



In Δ BCD; we have,

$CD = \dfrac{BC}{\tan 45^\circ}$

$\therefore \quad CD = 8$ m

$OC = (20 - 8) = 12$ m

Now, calculating the area and centroid of each geometrical figure;

$A_1 = 12 \times 8 = 96\ m^2$

$A_2 = \pi r^2 = \pi \times 3^2 = 28.27\ m^2$

$A_3 = \dfrac{1}{2} \times 8 \times 8 = 32\ m^2$

$x_1 = 6$ m

$y_2 = 4$ m

$x_2 = 5$ m

$y_2 = 4$ m

$x_3 = 12 + \dfrac{1}{3} \times 4 = 14.67$ m

$y_3 = \dfrac{1}{3} \times 8 = 2.67$ m

$\therefore \quad \bar{x} = \dfrac{A_1x_1 - A_2x_2 + A_3x_3}{A_1 - A_2 + A_3} = 9.06$ m

and, $\bar{y} = \dfrac{A_1y_1 - A_2y_2 + A_3y_3}{A_1 - A_2 + A_3} = 3.57$ m

The moment of inertia about OX and OY axes can be calculated as;

$$I_x = \left[\frac{b_1h_1^3}{12} + A_1y_1^2\right] - \left[\frac{\pi r^4}{4} + A_2y_2^2\right] + \left[\frac{b_3h_3^3}{36} + A_3y_3^2\right]$$

$$= \left[\frac{12 \times 8^3}{12} + 96 \times 4^2\right] - \left[\frac{\pi \times 3^4}{4} + 28.27 \times 4^2\right] + \left[\frac{8 \times 8^3}{36} + 32 \times 2.67^2\right]$$

$$= 2048 - 515.94 + 341.90$$

$\therefore \quad I_X = 1873.96\ m^4$

Similarly;

$$I_y = \left[\frac{h_1b_1^3}{12} + A_1x_1^2\right] - \left[\frac{\pi r^4}{4} + A_2x_2^2\right] + \left[\frac{h_3b_3^3}{36} + A_3x_3^2\right]$$

$$= \left[\frac{8 \times 12^3}{12} + 96 \times 6^2\right] - \left[\frac{\pi \times 3^4}{4} + 28.27 \times 5^2\right] + \left[\frac{8 \times 8^3}{36} + 32 \times (14.67)^2\right]$$

$$= 4608 - 770.37 + 583.22$$

$\therefore \quad I_y = 4420.85\ m^4$

Hence, polar moment of inertia $(I_p) = I_X + I_y$

or, $I_p = 1873.96 + 4420.85$

$\therefore \quad I_p = 6294.81\ m^4$

Note

1. Apply parallel axis theorem simply if moment of inertia about centroidal axis is required to be found as,

$(I_X)_i = (\bar{I}_X)_i + A_i(\bar{y}_i - \bar{y})^2$

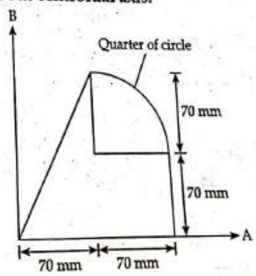
$(I_y)_i = (\bar{I}_y)_i + A_i(\bar{x}_i - \bar{x})^2$

2. If moments of inertia of composite section about non-centroidal reference axes are required apply transfer formula or use,

$I_X = \Sigma(\bar{I}_X)_i + \Sigma A_i(\bar{y}_i)^2$

$I_y = \Sigma(\bar{I}_y)_i + \Sigma A_i(\bar{x}_i)^2$

9. **Determine the moment of inertia and radius of gyration of the given area as shown in figure about centroidal axis.** [2066 Shrawan]



Solution:

The whole area can be divided into following geometrical figures;

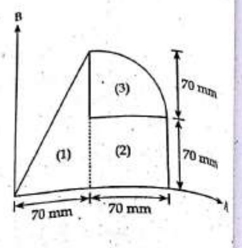
i) Triangle (1)
ii) Rectangle (2)
iii) Quarter – circle (3)

Now, calculating the area and centroid of each geometrical figures;

$$A_1 = \frac{1}{2} \times 70 \times 70 = 4900 \text{ mm}^2$$

$$A_2 = 70 \times 70 = 4900 \text{ mm}^2$$

$$A_3 = \frac{\pi r^2}{4} = \frac{\pi \times 70^2}{4} = 3848.45 \text{ mm}^2$$



$$x_1 = \frac{2 \times 70}{3} = 46.67 \text{ mm}$$

$$y_1 = \frac{140}{3} = 46.67 \text{ mm}$$

$$x_2 = 70 + \frac{70}{2} = 105 \text{ mm}$$

$$y_2 = \frac{70}{2} = 35 \text{ mm}$$

$$x_3 = 70 + \frac{4 \times 70}{3\pi} = 99.71 \text{ mm}$$

$$y_3 = 70 + \frac{4 \times 70}{3\pi} = 99.71 \text{ mm}$$

$$\therefore \quad \bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3} = 82.57 \text{ mm}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3} = 57.44 \text{ mm}$$

Now, moment of inertia about centroidal axis is given by;

$$I_X = \frac{70 \times 140^3}{36} + 4900\,(57.44 - 46.67)^2 + 4900\,(57.44 - 35)^2$$

$$+ \frac{70 \times 70^3}{12} + 0.055 \times 70^4 + 3848.45\,(99.71 - 57.44$$

$$\therefore \quad I_X = 18568945.94 \text{ mm}^4$$

$$\text{and, } I_y = \frac{140 \times 70^3}{36} + 4900\,(82.67 - 46.67)^2 + \frac{70 \times 70^3}{12} \times 4900$$

$$(105 - 82.57)^2 + 0.055 \times 70^4 + 3848.45\,(99.71 - 82.57$$

$$= 14566251.33 \text{ mm}^4$$

Now, radius of gyration is given by;

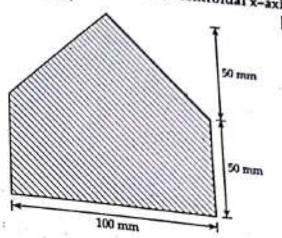
$$k_x = \sqrt{\frac{I_X}{A}}$$

$$= \sqrt{\frac{18568945.94}{13648.45}} = 36.88 \text{ mm}$$

$$k_y = \sqrt{\frac{I_Y}{A}}$$

$$= \sqrt{\frac{14566251.33}{13648.45}} = 32.67 \text{ mm}$$

10. Determine the moment of inertia about the centroidal x-axis of the shaded area shown in figure below. **[2068 Baishakh]**



Solution:

The whole area is divided into two geometrical figures.

i) Rectangle (1) (+)
ii) Triangle (2) (–)

Calculating the area and centroid of each geometrical figure; we have,

$$A_1 = 100 \times 50 = 5000 \text{ mm}^2$$

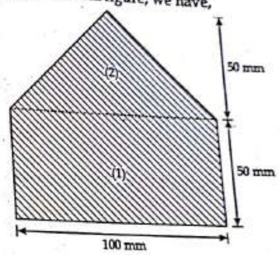
$$A_2 = \frac{1}{2} \times 100 \times 50 = 2500 \text{ mm}^2$$

$$x_1 = 50 \text{ mm}$$

$$y_1 = 25 \text{ mm}$$

$$x_2 = 50 \text{ mm}$$

$$y_2 = 50 + 16.67 = 66.67 \text{ mm}$$



$$\therefore \quad \bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = 50 \text{ mm}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = 38.89 \text{ mm}$$

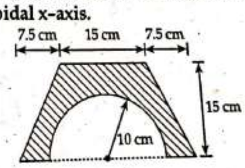
Now, moment of inertia about centroidal x-axis can be calculated as;

$$I_X = \left[\frac{b_1h_1^3}{12} + A_1(y_1 - \bar{y})^2\right] + \left[\frac{b_2h_2^3}{36} + A_2(y_2 - \bar{y})^2\right]$$

$$= \frac{100 \times 50^3}{12} + 5000\,(25 - 38.89)^2 + \frac{100 \times 50^3}{36} + 2500(66.67 - 38.89)^2$$

$$\therefore \quad I_X = 4481845.389 \text{ mm}^4$$

11. Determine the moment of inertia of the shaded area shown in figure below about its centroidal x-axis. **[2068 Chaitra]**



Solution:

Considering the whole rectangle and dividing into three geometrical areas.

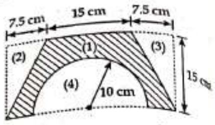
i) Whole rectangle (1) (+)

ii) Triangle (2) (–)

iii) Triangle (3) (–)

iv) Semi-circle (4) (–)

Now, calculating the area and centroid of each geometrical figure; we get,



$$A_1 = 30 \times 15 = 450\ m^2$$

$$A_2 = \frac{1}{2} \times 7.5 \times 15 = 56.25\ m^2$$

$$A_3 = \frac{1}{2} \times 7.5 \times 15 = 56.25\ m^2$$

$$A_4 = \frac{\pi \times 10^2}{2} = 157.08\ m^2$$

$x_1 = 15$ m

$y_1 = 7.5$ m

$x_2 = 2.5$ m

$y_2 = 5$ m

$$x_3 = 22.5 + \frac{2}{3} \times 7.5 = 27.5\ m$$

$y_3 = 5$ m

$x_4 = 5 + 10 = 15$

$$y_4 = \frac{4 \times 10}{3\pi} = 4.24\ m$$

$$\therefore \quad \bar{x} = \frac{A_1x_1 - A_2x_2 - A_3x_3 - A_4x_4}{(A_1 - A_2 - A_3 - A_4)} = 15\ m$$

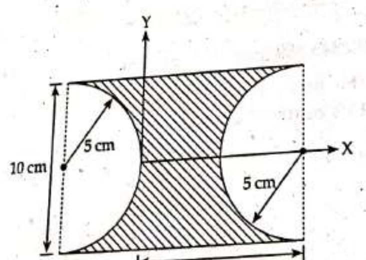
$$\text{and,} \quad \bar{y} = \frac{A_1y_1 - A_2y_2 - A_3y_3 - A_4y_4}{A_1 - A_2 - A_3 - A_4} = 11.90\ m$$

Now, moment of inertia about centroidal x-axis is given by;

$$\therefore \quad I_X = \left[\frac{b_1h_1^3}{12} + A_1(y_1 - \bar{y})^2\right] - \left[\frac{b_2h_2^3}{36} + A_2(y_2 - \bar{y})^2\right] - \left[\frac{b_3h_3^3}{36} + A_3(y_3 - \bar{y})^2\right] - \left[0.11r^4 + A_4(y_4 - \bar{y})^2\right]$$

$$= 17149.5 - 3381.19 - 3381.19 - 10316.76 = 70.365\ m^4$$

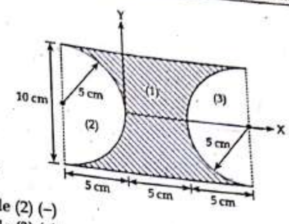
12. **Find the moment of inertia and radius of gyration about x–y axis of the figure below.** **[2068 Baishakh]**



Solution:

The whole area is divided into following three areas;

i) Whole rectangle (1) (+)



ii) Semi-circle (2) (–)

iii) Semi-circle (3) (–)

Now, the moment of inertia about the given x–y axes can be calculated as;

$$I_X = \left[\frac{b_1h_1^3}{12} + A_1y_1^2\right] - \left[\frac{\pi r^4}{8} + A_2y_2^2\right] - \left[\frac{\pi r^4}{8} + A_3y_3^2\right]$$

$$= \left[\frac{15 \times 10^3}{12} + 150 \times 0^2\right] - \left[\frac{\pi \times 5^4}{8} + \frac{\pi \times 5^2}{2} \times 0^2\right] - \left[\frac{\pi \times 5^4}{8} + \frac{\pi \times 5^4}{2} \times 0^2\right]$$

$$= 1250 - 245.44 - 245.44$$

$$\therefore \quad I_X = 759.12\ cm^4$$

$$\text{and,} \quad I_y = \left[\frac{h_1b_1^3}{12} + A_1x_1^2\right] - \left[0.11r^4 + A_2x_2^2\right] - \left[0.11r^4 + A_3x_3^2\right]$$

$$= \left[\frac{10 \times 15^3}{12} + 150 \times 2.5^2\right] - \left[0.11 \times 5^4 + \frac{\pi \times 5^2}{2} \times \left(5 - \frac{4 \times 5}{3\pi}\right)\right] - \left[0.11 \times 5^4 + \frac{\pi \times 5^2}{2} \times \left(5 + 5 - \frac{4 \times 5^2}{3\pi}\right)\right]$$

$$= 3750 - 394 - 2505.912$$

$$\therefore \quad I_y = 850.087\ cm^4$$

Hence, moment of inertia is $(I_X, I_y) = (759.12\ cm^4, 850.087\ cm^4)$

Now, the radius of gyration about x–y axes can be calculated as;

$$K_A = \sqrt{\frac{I_X}{A}} = \sqrt{\frac{759.12}{71.46}} = 3.26\ cm$$

$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{850.087}{71.46}} = 3.45\ cm$$

13. **Determine the centroid of right angle triangle by method of integration.** **[2068 Baishakh]**

Solution:

Let us take a small strip of thickness dx, parallel to y-axis.

From similar triangles;

$$\frac{y}{b - x} = \frac{h}{b}$$

$$\text{or,} \quad y = (b - x)\frac{h}{b}$$

We have,

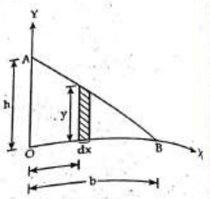
$$A = \int dA = \int y\, dx$$

or, $A = \int_0^b (b-x)\frac{h}{b}dx$

$= \frac{h}{b}\int_0^b (b-x)\,dx = \frac{h}{b}\left[bx - \frac{x^2}{2}\right]_0^b$

$= \frac{h}{b}\left[b^2 - \frac{b^2}{2}\right]$

$= \frac{h}{b}\frac{b^2}{2} = \frac{hb}{2}$



We have,

$$\bar{x} = \frac{\int x\,dA}{A} = \frac{\int xy\,dx}{\int dA} = \frac{2}{hb}\cdot\frac{h}{b}\int_0^b bx - x^2 = \frac{2}{b^2}\left[b\frac{x^2}{2} - \frac{x^2}{3}\right]_0^b = \frac{2}{b^2}\times\frac{b^3}{6}$$

$\therefore \quad \bar{x} = \frac{b}{3}$

Similarly;

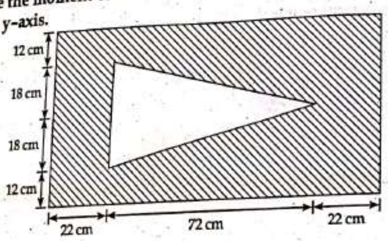
$$\bar{y} = \frac{\int y\,dA}{A} = \frac{\int \frac{y}{2}\times y\,dx}{\int dA} = \frac{2}{hb}\int_0^b (b-x)^2\frac{h^2}{2b^2}dx = \frac{h}{b^3}\int_0^b (b^2 - 2bx + x^2)\,dx$$

or, $\bar{y} = \frac{h}{b^3}\left[b^2x - 2b\frac{x^2}{2} + \frac{x^3}{3}\right]_0^b = \frac{h}{b^3}\times\frac{b^3}{3}$

$\therefore \quad \bar{y} = \frac{h}{3}$

Hence, the centroid of rigid angled triangle is, $(\bar{x}, \bar{y}) = \left(\frac{b}{3}, \frac{h}{3}\right)$

14. Determine the moment of inertia of the shaded area shown with respect to centroidal y-axis. **[2068 Magh]**



Solution:

Considering whole rectangle from which a triangle is taken out so as to from the shaded area.

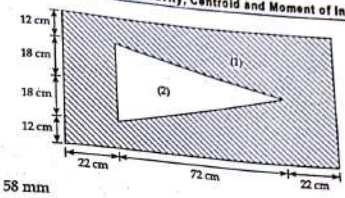
i) Rectangle (1) (+)

ii) Triangle (2) (–)

Now, calculating area and centroid of each area separately;

$A_1 = 116 \times 60 = 6960\ mm^2$

$A_2 = \frac{1}{2}\times 36 \times 76 = 1296\ mm^2$



$x_1 = 58\ mm$

$y_1 = 30\ mm$

$x_2 = 22 + 36 = 58\ mm$

$y_2 = 12 + 18 = 30\ mm$

$\therefore \quad \bar{x} = \frac{A_1x_1 - A_2y_2}{A_1 - A_2} = 58\ mm$

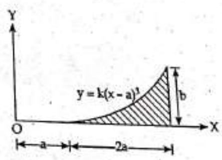
$\bar{y} = \frac{A_1y_1 - A_2x_2}{A_1 - A_2} = 30\ mm$

Now,

$$\therefore \quad I_y = \left[\frac{h_1b_1^3}{12} + A_1(x_1 - \bar{x})^2\right] - \left[\frac{h_2b_2^3}{48} + A_2(x_2 - \bar{x})^2\right] = \frac{60\times116^3}{12} - \frac{36\times72^3}{48}$$

$= 7804480 - 279936 = 7524544\ mm^4$

15. Determine by direct integration method, the centroid of the following shaded area. **[2068 Magh]**



Solution:

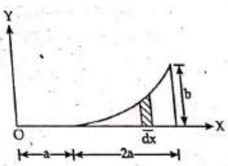
The given equation of the area is;

$y = k(x-a)^3$

When, $x = 2a$, $y = b$

$b = k(2a-a)^3$

$\therefore \quad k = \frac{b}{a^3}$



$\therefore \quad A = \int dA = \int_a^{2a} y\,dx$

$= \int_a^{2a} k(x-a)^3\,dx = k\int_a^{2a}(x-a)^3\,dx = k\left[\frac{(x-a)^4}{4}\right]_a^{2a}$

$= \frac{b}{4a^3}[(2a-a)-(a-a)]^4 = \frac{b}{4a^3}\times a^4$

$\therefore \quad A = \frac{ab}{4}$

To calculate, $\bar{x}$, taking a vertical strip parallel to y-axis;

$$\therefore \quad \bar{x} = \frac{\int x\,dA}{A} = \frac{4}{ab}\int_a^{2a} x\,k\,(x-a)^3 dx = \frac{4k}{ab}\int_a^{2a} x\,(x^3 - 3x^2 a + 3a^2 x - a^3)dx$$

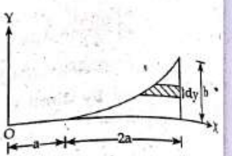
$$= \frac{4k}{ab}\int_a^{2a} (x^4 - 3x^3 a + 3a^2x^2 - a^3 x)dx = \frac{4k}{ab}\left[\frac{x^5}{5} - \frac{3x^4 a}{4} + 3a^2\frac{x^3}{3} - \frac{a^3x^2}{2}\right]_a^{2a}$$

$$= \frac{4k}{ab}\left[\frac{1}{5}(32a^5 - a^5) - \frac{3a}{4}(16a^4 - a^4) + a^2(8a^3 - a^3) - \frac{a^3}{2}(4a^2 - a^2)\right]$$

$$= \frac{4k}{ab}\left[\frac{31a^5}{5} - \frac{45a^5}{4} + 7a^5 - \frac{3a^5}{2}\right] = \frac{4}{ab}\times\frac{b}{a^3}\times\frac{9}{20}a^5$$

$$\therefore \quad \bar{x} = \frac{9}{5}a$$

Now, to calculate $\bar{y}$, taking horizontal strip parallel to x-axis;



$$\therefore \quad \bar{y} = \frac{\Sigma y\,dA}{A} = \frac{4}{ab}\int_0^b y.x\,dy$$

$$= \frac{4}{ab}\int_0^b y\left(a + \frac{y^{1/3}}{k^{1/3}}\right)dy$$

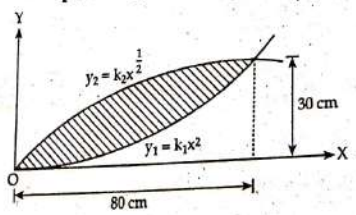
$$= \frac{4}{ab}\int_0^b \left(ya + \frac{y^{4/3}}{k^{1/3}}\right)dy$$

$$= \frac{4}{ab}\left(\frac{ay^2}{2} + \frac{3}{7k^{1/3}}y^{7/3}\right) = \frac{4}{ab}\left[\frac{ab^2}{2} + \frac{3}{7}\frac{a}{b^{1/3}}.b^{7/3}\right] = \frac{4}{ab}\times\frac{13}{14}ab^2$$

$$\therefore \quad \bar{y} = \frac{26}{7}b$$

Hence, the centroid of the area is $(\bar{x}, \bar{y}) = \left(\frac{9}{5}a, \frac{26}{7}b\right)$

16. **Determine the moment of inertia and radius of gyration of the common area as shown in figure below about 'X' and 'Y' axis.**
[2064 Falgun back, 2067 Ashadh, 2069 Ashadh back]



Solution:

First of all calculating the value of constants; we have,

When x = 80 cm, Y = 30 cm

$30 = k_1 (80)^2$

$\therefore \quad k_1 = 0.0047$

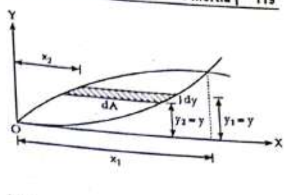
$30 = k_2\sqrt{20}$

$\therefore \quad k_2 = 3.354$

Hence,

$y_1 = 0.0047\,x^2$

$y_2 = 3.354\sqrt{x}$



Now,

$$A = \int dA = \int (x_1 - x_2)\,dy$$

$$= \int_0^{30}\left(\sqrt{\frac{y}{k_1}} - \frac{y^2}{k_2^2}\right)dy = \int_0^{30}\left(\frac{y^{1/2}}{0.068} - \frac{y^2}{11.24}\right)dy$$

$$= \left[\frac{y^{3/2}}{\frac{3}{2}\times 0.068} - \frac{y^3}{3\times 11.24}\right]_0^{30}$$

$$\therefore \quad A = \left(\frac{30^{3/2}}{0.102} - \frac{30^3}{33.74}\right) = 810.71\ cm^2$$

Moment of Inertia about x-axis can be calculated by considering a small strip parallel to x-axis as shown in figure above.

$$I_X = \int y^2 dA = \int y^2(x_1 - x_2)\,dy = \int y^2\left(\sqrt{\frac{y_1}{k_1}} - \frac{y_2^2}{k_2^2}\right)dy$$

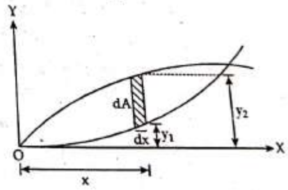
Here, strip is parallel to x-axis so,

$y_1 = y_2 = y$

$$\therefore \quad I_X = \int_0^{30}\left(\frac{y^{5/2}}{0.068} - \frac{y^4}{11.24}\right)dy = \left[\frac{y^{7/2}}{\frac{7}{2}\times 0.068} - \frac{y^5}{5\times 11.24}\right]_0^{30} = \frac{30^{7/2}}{0.238} - \frac{30^5}{56.25}$$

or, $\quad I_X = 1.89\times 10^5\ cm^4$

Now, to calculate moment of inertia about y-axis, considering an elemental strip parallel to y-axis is shown in figure below.



$$I_y = \int x^2 dA = \int x^2(y_2 - y_1)\,dx$$

$$= \int x^2(k_2\sqrt{x} - k_1x^2)\,dx$$

$$= \int_0^{80}(3.354\,x^{5/2} - 0.0047x^4)\,dx = \left[\frac{3.354\,x^{7/2}}{\frac{7}{2}} - \frac{0.0047x^5}{5}\right]_0^{80}$$

$$= 0.958(80)^{7/2} - 0.00094(80)^5$$

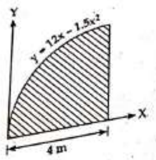
$$\therefore \quad I_y = 1.307\times 10^6\ cm^4$$

Now, radius of gyration can be calculated as;

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{1.89\times 10^5}{810.71}} = 15.28\ cm$$

and, $K_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{1.307\times10^6}{810.71}} = 40.15$ cm

17. Determine centroidal x-coordinate of the shaded area shown in figure below. **[2069 Chaitra]**

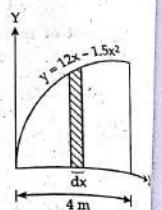


Solution:

The given equation for the curve is,

$$y = 12x - 1.5x^2$$

Taking vertical strip parallel to Y-axis as shown in figure,



$$A = \int_0^4 y\,dx = \int_0^4 (12x - 1.5x^2)dx$$

$$= \left[12\frac{x^2}{2} - 1.5\frac{x^3}{3}\right]_0^4 = 12\times\frac{16}{2} - 1.5\times\frac{64}{3}$$

$\therefore$ $A = 64\ m^2$

Now; we have,

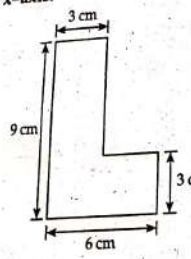
$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{1}{64}\int_0^4 x\cdot y\,dx$$

$$= \frac{1}{64}\int_0^4 (12x^2 - 1.5x^3)\,dx$$

$$= \frac{1}{64}\left[12\times\frac{64}{3} - 1.5\times\frac{256}{4}\right] = \frac{1}{64}[256 - 96]$$

$\therefore$ $\bar{x} = 2.5$ m

18. Determine radius of gyration (r_x) of the angle section shown in figure below about centroidal x-axis. **[2069 Chaitra]**



Solution:

Dividing the angle section into two parts as shown in the figure.

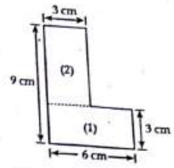
Now, calculating area and centroidal Y-coordinate of each part

$A_1 = 6\times3 = 18\ cm^2$

$A_2 = 6\times3 = 18\ cm^2$

$Y_1 = 1.5$ cm

$Y_2 = 3 + 3 = 6$ cm



$$\therefore\quad \bar{y} = \frac{A_1Y_1 + A_2Y_2}{A_1 + A_2}$$

$$= \frac{18\times1.5 + 18\times6}{18+18} = 3.37\ cm$$

Now, M.I. about centroidal x-axis is given by,

$$I_x = \left[\frac{b_1h_1^3}{12} + A_1(Y_1 - \bar{y})^2\right] + \left[\frac{b_2h_2^3}{12} + A_2(Y_2 - \bar{y})^2\right]$$

$$= \left[\frac{6\times3^3}{12} + 18\times(1.5 - 3.75)^2\right] + \left[\frac{3\times6^3}{12} + 18\times(6 - 3.75)^2\right]$$

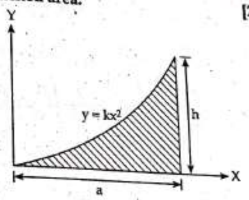
$$= 104.625 + 145.125$$

$\therefore$ $I_x = 249.75\ cm^4$

Radius of gyration about centroidal x-axis can be calculated by,

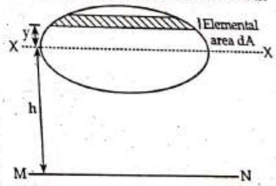
$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{249.75}{36}} = 2.63\ cm$$

19. State and prove parallel axis theorem. Also determine the centroidal X and Y coordinate of the hatched area. **[2070 Ashadh Back]**



Solution:

Parallel axis theorem states that "The moment of inertia of a lamina about any axis in the plane of the lamina equals the sum of moment of inertia about a parallel centroidal axis in the plane of lamina and the product of the area of the lamina with the square of the distance between two axes."



Consider a lamina of area 'A' shown in the figure. Let MN be an axis in the plane of the lamina which is parallel to the centroidal axis xx.

Consider an elemental area dA at a distance 'y' from xx axis.

$$I_{XX} = \text{M.I. of the lamina about xx axis} = \sum dA\,y^2$$

Let h be the distance between the two axes and $(h+y)$ be the distance of elemental area from MN axis.

$\therefore$ I_{MN} = M.I. of the lamina about MN axis or, $I_{MN} = \sum dA\,(h+y)^2 = \sum dA\,(h^2+2hy+y^2)$

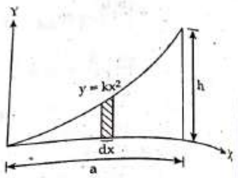
$= \sum dA\,h^2 + \sum dA\,y^2 + 2h\sum dA\,y = Ah^2 + I_{xx} + 2h\sum dA\,y$

As the distance of the total area from xx is zero;

$\sum dA\,y = 0$

∴ $I_{MN} = I_{xx} + Ah^2$

Let us consider a small strip of area dA parallel to Y-axis as shown in figure.



For x = a and y = h

$h = ka^2$

∴ $k = \frac{h}{a^2}$

Hence,

$y = \frac{h}{a^2}x^2$

∴ $A = \int dA = \int_0^a y\,dx$

$= \int_0^a \frac{h}{a^2}x^2\,dx = \frac{h}{a^2}\times\frac{a^3}{3} = \frac{ah}{3}$

Now,

$\bar{x} = \frac{\int x\,dA}{A} = \frac{3}{ah}\int_0^a x\cdot y\,dx = \frac{3}{ah}\int_0^a x\cdot\frac{h}{a^2}\cdot x^2\,dx = \frac{3}{ah}\times\frac{h}{a^2}\times\frac{a^4}{4}$

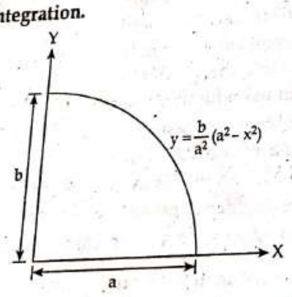
∴ $\bar{x} = \frac{3a}{4}$

and, $\bar{y} = \frac{\int y_C\,dA}{A} = \frac{3}{ah}\int\frac{y}{2}\times y\,dx = \frac{3}{2ah}\int\frac{h^2}{a^4}\times x^4\,dx = \frac{3}{2ah}\times\frac{h^2}{a^4}\times\frac{a^5}{5}$

∴ $\bar{y} = \frac{3h}{10}$

Hence, $(\bar{x},\bar{y}) = \left(\frac{3a}{4},\frac{3h}{10}\right)$

20. Locate the centroid of the area bounded by the curve as shown in figure by the method of integration. [2070 Bhadra]

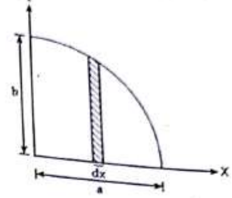


Solution:

The given equation for the provided area is,

$y = \frac{b}{a^2}(a^2 - x^2)$

Taking vertical strip parallel to Y-axis as shown in the figure and calculating centroid as,



$A = \int dA = \int_0^a y\,dx$

$= \int_0^a\left[\frac{b}{a^2}(a^2-x^2)\right]dx$

$= \frac{b}{a^2}\int_0^a(a^2-x^2)dx$

$= \frac{b}{a^2}\left[a^2x - \frac{x^3}{3}\right]_0^a$

$= \frac{b}{a^2}\left[a^3 - \frac{a^3}{3}\right]$

$= \frac{b}{a^2}\times\frac{2a^3}{3}$

∴ $A = \frac{2ab}{3}$

Now; we know,

$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{3}{2ab}\int_0^a x\cdot y\,dx$

$= \frac{3}{2ab}\int_0^a\left[x\cdot\frac{b}{a^2}(a^2-x^2)\right]dx$

$= \frac{3}{2ab}\times\frac{b}{a^2}\int_0^a(a^2x - x^3)dx$

$= \frac{3}{2a^3}\left[a^2\cdot\frac{a^2}{2} - \frac{a^4}{4}\right] = \frac{3}{2a^3}\times\frac{a^4}{4}$

∴ $\bar{x} = \frac{3a}{8}$

and, $\bar{y} = \frac{\int y_C\,dA}{\int dA} = \frac{3}{2ab}\int_0^a\frac{y}{2}\times y\,dx = \frac{3}{4ab}\int_0^a y^2\,dx$

$= \frac{3}{4ab}\int_0^a\frac{b^2}{a^4}(a^2-x^2)^2dx$

$= \frac{3}{4ab}\times\frac{b^2}{a^4}\int_0^a(a^4 - 2a^2x^2 + x^4)\,dx$

$= \frac{3b}{4a^5}\left[a^5 - 2a^2\cdot\frac{a^3}{3} + \frac{a^5}{5}\right]$

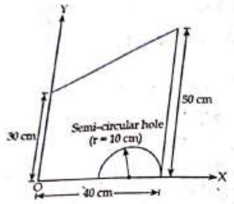
$= \frac{3b}{4a^5}\left[a^5 - \frac{2a^5}{3} + \frac{a^5}{5}\right] = \frac{3b}{4a^5}\times\frac{8}{15}a^5$

∴ $\bar{y} = \frac{2b}{5}$

Hence,

$(\bar{x},\bar{y}) = \left(\frac{3a}{8},\frac{2b}{5}\right)$

21. Calculate the moment of inertia of the composite area as shown in figure about X-axis. [2070 Bhadra]

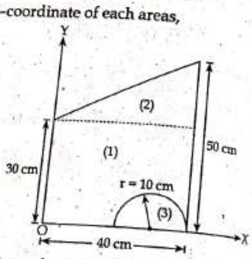


Solution:

The whole area can be divided into following geometrical areas;

i) Rectangle (1) (+)
ii) Triangle (2) (+)
iii) Semi circle (3) (-)

Calculating the areas and centroidal Y-coordinate of each areas,



$A_1 = 40 \times 30 = 1200\ \text{cm}^2$

$A_2 = \frac{1}{2} \times 40 \times 20 = 400\ \text{cm}^2$

$A_3 = \frac{\pi \times 10^2}{2} = 157.08\ \text{cm}^2$

$Y_1 = 15\ \text{cm}$

$Y_2 = 30 \times \frac{1}{3} \times 20 = 36.67\ \text{cm}$

$Y_3 = \frac{4 \times 10}{2\pi} = 4.24\ \text{cm}$

Now, moment of inertia about given X-axis can be calculated as,

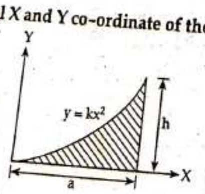
$$I_x = \left[\frac{b_1 h_1^3}{12} + A_1 Y_1^2\right] + \left[\frac{b_2 h_2^3}{36} + A_2 Y_2^2\right] - \left[0.11\, r^4 + A_3 Y_3^2\right]$$

$$= \left[\frac{40 \times (30)^3}{12} + 1200 \times (15)^2\right] + \left[\frac{40 \times (20)^3}{36} + 400 \times (36.67)^2\right] - \left[0.11 \times (10)^4 + 157.08 \times (4.24)^2\right]$$

$$= 36000 + 546764.45 - 3923.92$$

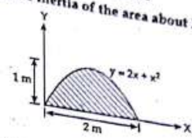
$\therefore\ I_x = 902840.53\ \text{cm}^4$

22. **Determine centroidal X and Y co-ordinate of the shaded area. [2070 Magh]**



Solution: See the solution of Q. no. 19 on page no. 121

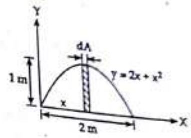
23. **Determine the moment of inertia of the area about X-axis. [2070 Magh]**



Solution:

Let's consider small strip dA parallel to Y-axis at a distance of x from Y-axis.

Moment of inertia of the figure about x-axis is given by;



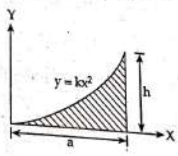
$$I_x = \int \frac{y^3}{3}\,dx = \frac{1}{3}\int_0^2 y^3\,dx$$

$$= \frac{1}{3}\int_0^2 (2x + x^2)\,dx = \frac{1}{3}\left[\frac{2x^2}{2} + \frac{x^3}{3}\right]_0^2$$

$$= \frac{1}{3}\left[(2)^2 + \frac{(2)^3}{3}\right]_0^2$$

$\therefore\ I_x = \frac{20}{9}$

24. **Determine centroid of the given plane figure. State and prove parallel axes theorem for the moment of inertia. Define centroid and centre of gravity. [2070 Chaitra]**



Solution:

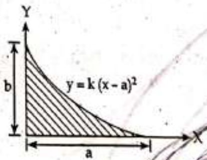
For the first part

See the solution of Q. no. 19 on page no. 121

Centroid and centre of gravity

See the definition part on page no. 99

25. **Determine by direct integration method, the centroid of the area shown in the figure. [2071 Shrawan]**



Solution:

The given equation of the area is;

$y = k(x-a)^2$

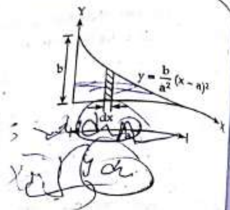
When $x = 0$; then,

$y = b$

$\therefore \quad b = k(0-a)^2$

or, $\quad b = ka^2$

$\therefore \quad k = \frac{b}{a^2}$



Hence, the equation becomes;

$$y = \frac{b}{a^2}(x-a)^2$$

$$A = \int dA = \int_0^a y\,dx = \int_0^a \frac{b}{a^2}(x-a)^2\,dx = \frac{b}{a^2}\int_0^a (x^2 - 2ax + a^2)\,dx$$

$$= \frac{b}{a^2}\left[\frac{x^3}{3} - 2a\left(\frac{x^2}{2}\right) + a^2x\right]_0^a = \frac{b}{a^2}\left[\frac{a^3}{3} - a^3 + a^3\right] = \frac{b}{a^2}\times\frac{a^3}{3}$$

$$\therefore \quad A = \frac{ab}{3}$$

To calculate $\bar{x}$, taking a vertical strip parallel to y-axis;

$$\bar{x} = \frac{\int x\,dA}{A} = \frac{3}{ab}\int_0^a xy\,dx = \frac{3}{ab}\int_0^a x\left\{\frac{b}{a^2}(x-a)^2\right\}dx$$

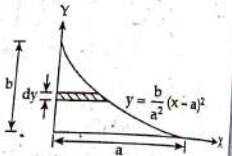
$$= \frac{3}{ab}\times\frac{b}{a^2}\int_0^a x(x-a)^2\,dx = \frac{3}{a^3}\int_0^a (x^3 - 2ax^2 + a^2x)\,dx$$

$$= \frac{3}{a^3}\left[\frac{x^4}{4} - 2a\left(\frac{x^3}{3}\right) + \frac{a^2x^2}{2}\right]_0^a = \frac{3}{a^3}\left(\frac{a^4}{4} - \frac{2a^4}{3} + \frac{a^4}{2}\right) = \frac{3}{a^3}\times\frac{a^4}{12}$$

$$\therefore \quad \bar{x} = \frac{a}{4}$$

Now, to calculate $\bar{y}$, taking horizontal strip parallel to x-axis;

$$\bar{y} = \frac{\sum y\,dA}{A} = \frac{3}{ab}\int xy\,dy$$



$$= \frac{3}{ab}\int_0^b y\left(a + a\frac{y^{\frac{1}{2}}}{b^{\frac{1}{2}}}\right)dy$$

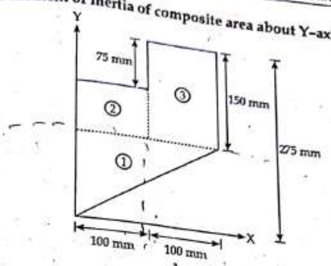
$$= \frac{3}{ab}\int_0^b y\cdot a\left(1 + \frac{y^{\frac{1}{2}}}{b^{\frac{1}{2}}}\right)dy = \frac{3}{b}\int_0^b \left(y + \frac{y^{\frac{3}{2}}}{b^{\frac{1}{2}}}\right)dy$$

$$= \frac{3}{b}\left[\frac{y^2}{2} + \frac{2y^{\frac{5}{2}}}{5b^{\frac{1}{2}}}\right]_0^b = \frac{3}{b}\left(\frac{b^2}{2} + \frac{2b^2}{5}\right) = \frac{3}{b}\times\frac{9b^2}{10}$$

$$\therefore \quad \bar{y} = \frac{27b}{10}$$

Hence, the centroid of the given area is $(\bar{x}, \bar{y}) = \left(\frac{a}{4}, \frac{27b}{10}\right)$.

26. Calculate moment of inertia of composite area about Y-axis. **[2071 Shrawan]**



Solution:

Dividing the given diagrams into three geometrical area;

i) Triangle (1)(+)

ii) Rectangle (2)(+)

iii) Rectangle (3)(+)

Now, calculating the area and centroid of each geometrical figures.

$A_1 = \frac{1}{2}\times 200\times 125 = 12500\ \text{mm}^2$

$A_2 = 75\times 100 = 7500\ \text{mm}^2$

$A_3 = 150\times 100 = 15000\ \text{mm}^2$

$x_1 = \frac{200}{3}\ \text{mm}$

$x_2 = 50\ \text{mm}$

$x_3 = 100 + \frac{100}{2} = 150\ \text{mm}$

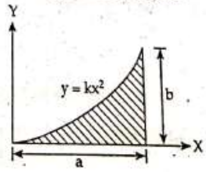
Moment of inertia about y-axis is given by;

$$I_y = \left[\frac{h_1b_1^3}{36} + A_1x_1^2\right] + \left[\frac{h_2b_2^3}{12} + A_2x_2^2\right] + \left[\frac{h_3b_3^3}{12} + A_3x_3^2\right]$$

$$= \left[\frac{125\times(200)^3}{36} + 12500\times\left(\frac{200}{3}\right)^2\right] + \left[\frac{75\times(100)^3}{12} + 7500\times(50)^2\right] + \left[\frac{150\times(100)^3}{12} + 15000\times(150)^2\right]$$

$$\therefore \quad I_y = 458333333.33\ \text{mm}^4$$

27. Determine the moment of inertia about centroidal axis of the shaded plane area by using direct integration method. **[2071 Bhadra]**

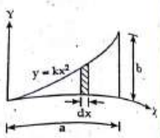


Solution:

The given equation of the curve is;

$$y = kx^2$$

When $x = 0$ and $y = b$;



$$b = ka^2$$

$$\therefore \quad k = \frac{b}{a^2}$$

$$\therefore \quad y = \left(\frac{b}{a^2}\right)x^2$$

Considering a vertical strip parallel to y-axis; we have,

Moment of inertia about x-axis is given by;

$$I_x = \frac{1}{3}\int y^3 dx = \frac{1}{3}\int_0^a \left\{\left(\frac{b}{a^2}\right)x^2\right\}^3 dx = \frac{1}{3} \times \frac{b^3}{a^6}\int_0^a x^6 dx$$

$$= \frac{b^3}{3a^6}\left[\frac{x^7}{7}\right]_0^a = \frac{b^3}{3a^6} \times \frac{a^7}{7}$$

$$\therefore \quad I_x = \frac{ab^3}{21}$$

Using parallel axis theorem to calculate M.I. about centroidal axis;

$$I_{G(x\text{-axis})} = I_x - A\bar{y}^2$$

Now, we have to calculate A and $(\bar{x}, \bar{y})$;

$$A = \int_0^a y\,dx = \int_0^a \left\{\left(\frac{b}{a^2}\right)x^2\right\} dx = \frac{b}{a^2}\left[\frac{x^3}{3}\right]_0^a = \frac{b}{a^2} \times \frac{a^3}{3}$$

$$\therefore \quad A = \frac{ab}{3}$$

$$\therefore \quad \bar{x} = \frac{\int x\,dA}{A} = \frac{3}{ab}\int_0^a xy\,dx = \frac{3}{ab}\int_0^a x\left\{\left(\frac{b}{a^2}\right)x^2\right\} dx = \frac{3}{ab} \times \frac{b}{a^2}\int_0^a x^3\,dx$$

$$= \frac{3}{a^3} \times \frac{a^4}{4} = \frac{3a}{4}$$

$$\therefore \quad \bar{y} = \frac{\int y\,dA}{A} = \frac{3}{ab}\int_0^a \left(\frac{y}{2}\right) y\,dx = \frac{3}{2ab}\int_0^a \left(\frac{b^2}{a^4}\right)x^4\,dx = \frac{3b}{10}$$

$$\therefore \quad I_{G(x\text{-axis})} = I_x - A\bar{y}^2 = \frac{ab^3}{21} - \left[\frac{ab}{3} \times \left(\frac{3b}{10}\right)^2\right] = \frac{37ab^3}{2100}$$

Now, moment of inertia about y-axis is given by;

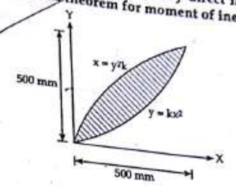
$$I_y = \int x^2\,dx = \int x^2 y\,dx = \int_0^a x^2\left\{\left(\frac{b}{a^2}\right)x^2\right\} dx = \frac{b}{a^2}\int_0^a x^4\,dx = \frac{b}{a^2} \times \frac{a^5}{5}$$

$$= \frac{a^3 b}{5}$$

Using parallel axis theorem to calculate M.I. about centroidal axis;

$$I_{G(y\text{-axis})} = I_y - A\bar{x}^2 = \frac{a^3 b}{5} - \left[\frac{ab}{3} \times \left(\frac{3a}{4}\right)^2\right] = \frac{a^3 b}{80}$$

28. **Determine the centroid of the hatched area by direct integration method. State and prove parallel axis theorem for moment of inertia.** [2071 Magh]



Solution:

The given equations of the curves are;

$$x = ky^2$$

$$y = kx^2$$

When $x = 500$ mm, $y = 500$ mm;

$$500 = k \times (500)^2$$

$$\therefore \quad k = \frac{1}{500}$$

$$\therefore \quad x = \left(\frac{1}{500}\right)y^2$$

$$y = \frac{x^2}{500} \qquad (1)$$

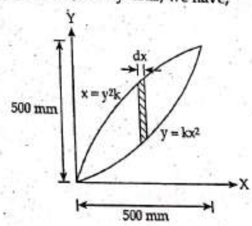
Now, (2)

$$A = \int dA = \int_0^{500}(y_2 - y_1)\,dx = \int_0^{500}\left[(500x)^{\frac{1}{2}} - \frac{x^2}{500}\right] dx$$

$$\text{or,} \quad A = \int_0^{500}\left(22.36x^{\frac{1}{2}} - \frac{x^2}{500}\right) dx = \left[\frac{2}{3} \times 22.36 - \frac{x^3}{3} \times \frac{1}{500}\right]_0^{500}$$

$$\therefore \quad A = 83333.34$$

Taking a vertical strip parallel to the y-axis; we have,



$$\bar{x} = \frac{\int x\,dA}{A} = \frac{1}{83333.34}\int_0^{500} x\left[(500x)^{\frac{1}{2}} - \frac{x^2}{500}\right] dx$$

$$\text{or,} \quad \bar{x} = \frac{1}{83333.34}\int_0^{500}\left(22.36x^{\frac{3}{2}} - \frac{x^3}{500}\right) dx$$

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or, $\bar{x} = \frac{1}{83333.34}\left[22.36 \times (500)^{\frac{5}{2}} - \frac{2}{5} \times \frac{(500)^4}{4 \times 500}\right]$

$\therefore \quad \bar{x} = 225$ mm

and, $\bar{y} = \frac{\int y\, dA}{A} = \frac{1}{83333.34}\int_0^{500}\left[y_1 + \left(\frac{y_2 - y_1}{2}\right)\right](y_2 - y_1)\, dx$

or, $\bar{y} = \frac{1}{83333.34}\int_0^{500}\left(\frac{y_2 - y_1}{2}\right)(y_2 - y_1)\, dx$

or, $\bar{y} = \frac{1}{2 \times 83333.34}\int_0^{500}(y_2^2 - y_1^2)\, dx$

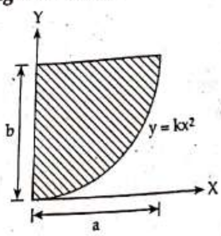
or, $\bar{y} = \frac{1}{2 \times 83333.34}\int_0^{500}\left(500x - \frac{x^4}{(500)^2}\right) dx$

or, $\bar{y} = \frac{1}{2 \times 83333.34}\left[500 \times \frac{(500)^2}{2} - \frac{(500)^5}{5 \times (500)^2}\right]$

$\therefore \quad \bar{y} = 225$ mm

$\therefore \quad (\bar{x}, \bar{y}) = (225 \text{ mm}, 225 \text{ mm})$

29. **Explain radius of gyration. Determine the centroid of the shaded area shown in the figure using direct integration method.** **[2071 Chaitra]**



Solution:

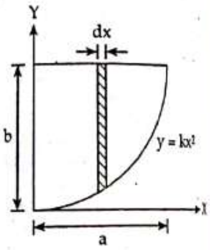
The given equation of the curve is;

$y = kx^2$

When x = a, y = b;

$b = ka^2$

$\therefore \quad k = \frac{b}{a^2}$



Now,

$A = \int dA = \int_0^a (y_2 - y_1)\, dx$

$= \left[bx - k\frac{x^3}{3}\right]_0^a = ab - \frac{b}{a^2} \cdot \frac{a^3}{3} = \frac{2ab}{3}$

We have,

$\bar{x} = \frac{\int x\, dA}{A} = \frac{3}{2ab}\int_0^a (bx - kx^3)\, dx = \frac{3}{2ab}\left[\frac{bx^2}{2} - \frac{b}{a^2} \cdot \frac{x^4}{4}\right]_0^a$

$\therefore \quad \bar{x} = \frac{3}{2ab}\left(\frac{ba^2}{2} - \frac{b}{a^2} \times \frac{a^4}{4}\right) = \frac{3}{2ab}\left(\frac{ba^2}{2} - \frac{ba^2}{4}\right) = \frac{3a}{8}$

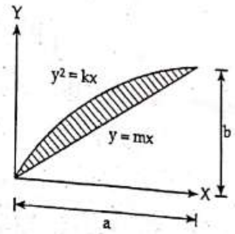
and, $\bar{y} = \frac{\int y\, dA}{A} = \frac{3}{2ab}\int_0^a \left[y_1 + \left(\frac{y_2 - y_1}{2}\right)\right](y_2 - y_1)\, dx$

or, $\bar{y} = \frac{3}{2 \times 2ab}\int_0^a (y_2^2 - y_1^2)\, dx = \frac{3}{4ab}\int_0^a (b^2 - k^2x^4)\, dx$

$\therefore \quad \bar{y} = \frac{3}{4ab}\left(ab^2 - \frac{b^2}{a^4} \times \frac{a^5}{5}\right) = \frac{3}{4ab}\left(ab^2 - \frac{ab^2}{5}\right) = \frac{3}{4ab} \times \frac{4ab^2}{5} = \frac{3b}{5}$

Hence, the centroid of the area is $(\bar{x}, \bar{y}) = \left(\frac{3a}{8}, \frac{3b}{5}\right)$.

30. **Locate the centroid of the plane area as shown in the figure by the method of integration.** **[2072 Ashwin]**



Solution:

When x = a, y = b;

$b^2 = ka$

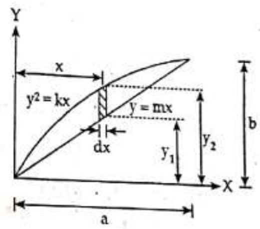
$\therefore \quad k = \frac{b^2}{a}$

and, $b = ma$

$\therefore \quad m = \frac{b}{a}$

$\therefore \quad y_2 = \left(\frac{b}{a^{\frac{1}{2}}}\right)x^{\frac{1}{2}}$

and, $y_1 = \left(\frac{b}{a}\right)x$



We have,

$A = \int dA = \int_0^a (y_2 - y_1)\, dx = \int_0^a \left[\left(\frac{b}{a^{\frac{1}{2}}}\right)x^{\frac{1}{2}} - \left(\frac{b}{a}\right)x\right] dx$

$\therefore \quad A = \left[\frac{b}{a^{\frac{1}{2}}} \times \frac{a^{\frac{3}{2}} \times 2}{3} - \frac{b}{a} \times \frac{a^2}{2}\right] = \frac{2ab}{3} - \frac{ab}{2} = \frac{ab}{6}$

Now,

$\bar{x} = \frac{\int x\, dA}{A} = \frac{6}{ab}\int_0^a x(y_2 - y_1)\, dx = \frac{6}{ab}\int_0^a \left[\left(\frac{b}{a^{\frac{1}{2}}}\right)x^{\frac{3}{2}} - \left(\frac{b}{a}\right)x^2\right] dx$

$$\therefore \quad \bar{x} = \frac{6}{ab}\left(\frac{b}{a^{\frac{1}{2}}} \times \frac{2a^{\frac{5}{2}}}{5} - \frac{b}{a} \times \frac{a^3}{3}\right) = \frac{6}{ab}\left(\frac{2a^2b}{5} - \frac{a^2b}{3}\right) = \frac{6}{ab} \times \frac{a^2b}{15} = \frac{2a}{5}$$

Similarly,

$$\bar{y} = \frac{\int y\,dA}{A} = \frac{6}{ab}\int_0^a \left[y_1 + \left(\frac{y_2 - y_1}{2}\right)\right](y_2 - y_1)\,dx$$

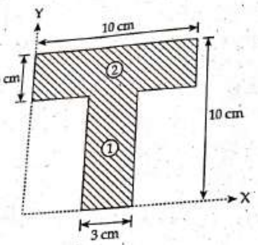
$$= \frac{6}{ab}\int_0^a \left(\frac{y_2 + y_1}{2}\right)(y_2 - y_1)\,dx = \frac{6}{2ab}\int_0^a (y_2^2 - y_1^2)\,dx$$

$$= \frac{3}{ab}\left(\frac{b^2}{a} \times \frac{a^2}{2} - \frac{b^2}{a^2} \times \frac{a^3}{3}\right) = \frac{3}{ab}\left(\frac{ab^2}{2} - \frac{ab^2}{3}\right) = \frac{3}{ab} \times \frac{ab^2}{6}$$

$$\therefore \quad \bar{y} = \frac{b}{2}$$

$\therefore$ Centroid of the given area $(\bar{x}, \bar{y}) = \left(\frac{2a}{5}, \frac{b}{2}\right)$.

31. Determine the moment of inertia of the given section about its centroidal axes. [2072 Ashwin]



Solution:

The whole area is divided into two geometrical figures.

i) Rectangle (1)(+)

ii) Rectangle (2)(−)

Calculating the area and centroid of each geometrical figures;

$A_1 = 3 \times 7 = 21\ \text{cm}^2$

$A_2 = 3 \times 10 = 30\ \text{cm}^2$

$x_1 = 3.5 + \frac{3}{2} = 5\ \text{cm}$

$x_2 = 5\ \text{cm}$

$y_1 = 3.5\ \text{cm}$

$y_2 = 7 + \frac{3}{2} = 8.5\ \text{cm}$

$$\therefore \quad \bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{21 \times 5 + 30 \times 5}{21 + 30} = 5\ \text{cm}$$

$$\therefore \quad \bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{21 \times 3.5 + 30 \times 8.5}{21 + 30} = 6.44\ \text{cm}$$

Now,

$$I_x = \left[\frac{b_1h_1^3}{12} + A_1(y_1 - \bar{y})^2\right] + \left[\frac{b_2h_2^3}{12} + A_2(y_2 - \bar{y})^2\right]$$

$$\text{or,} \quad I_x = \left[\frac{3 \times (7)^3}{12} + 21(3.5 - 6.44)^2\right] + \left[\frac{10 \times (3)^3}{12} + 30(8.5 - 6.44)^2\right]$$

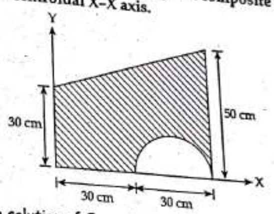
$$\therefore \quad I_x = 417.08$$

$$\text{and,} \quad I_y = \left[\frac{h_1b_1^3}{12} + A_1(x_1 - \bar{x})^2\right] + \left[\frac{h_2b_2^3}{12} + A_2(x_2 - \bar{x})^2\right]$$

$$\text{or,} \quad I_y = \left[\frac{7 \times (3)^3}{12} + 21(5 - 5)^2\right] + \left[\frac{3 \times (10)^3}{12} + 30(5 - 5)^2\right]$$

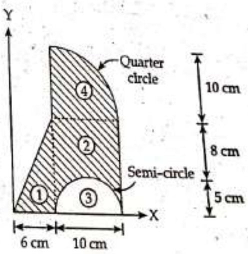
$$\therefore \quad I_y = 265.75$$

32. State and prove the parallel area theorem for moment of inertia. Determine the moment of inertia of the given composite area as shown in the figure about its centroidal X-X axis. [2072 Kartik]



Solution: Proceed as the solution of Q. no. 21 on page no. 124

33. State and prove the parallel axis theorem for moment of inertia. Find the moment of inertia about the axes through centroid of the given shaded area. [2072 Magh]



Solution:

For the first part

See the solution of Q. no. 19 on page no. 121

For the second part

The whole area can be divided into the following geometrical areas;

i) Triangle (1)(+)
ii) Rectangle (2)(+)
iii) Semi circle (3)(−)
iv) Quarter circle (4)(+)

Calculating the areas and centroidal x and y co-ordinate of each areas.

$$A_1 = \frac{1}{2} \times 6 \times 13 = 39 \text{ cm}^2$$

$$A_2 = 10 \times 13 = 130 \text{ cm}^2$$

$$A_3 = \frac{\pi r^2}{2} = \frac{\pi \times (5)^2}{2} = 39.27 \text{ cm}^2$$

$$\text{and, } A_4 = \frac{\pi r^2}{4} = \frac{\pi \times (10)^2}{4} = 78.54 \text{ cm}^2$$

Now,

$$x_1 = \frac{2b}{3} = \frac{2 \times 6}{3} = 3 \text{ cm}$$

$$x_2 = 6 + 5 = 11 \text{ cm}$$

$$x_3 = 6 + 5 = 11 \text{ cm}$$

$$\text{and, } x_4 = 6 + \frac{4r}{3\pi} = 6 + \frac{4 \times 10}{3\pi} = 10.24 \text{ cm}$$

Again,

$$y_1 = \frac{h}{3} = \frac{13}{3} = 4.33 \text{ cm}$$

$$y_2 = \frac{h}{2} = \frac{13}{2} = 6.5 \text{ cm}$$

$$y_3 = \frac{4r}{3\pi} = \frac{4 \times 5}{3\pi} = 2.12 \text{ cm}$$

$$\text{and, } y_4 = 13 + \frac{4r}{3\pi} = 13 + \frac{4 \times 10}{3\pi} = 17.24 \text{ cm}$$

$$\therefore \quad \bar{x} = \frac{A_1x_1 + A_2x_2 - A_3x_3 + A_4x_4}{A_1 + A_2 - A_3 + A_4}$$

$$= \frac{(39 \times 3) + (130 \times 11) - (39.27 \times 11) + (78.54 \times 10.24)}{39 + 130 - 39.27 + 78.54}$$

$$= 9.4 \text{ cm}$$

$$\text{and, } \bar{y} = \frac{A_1y_1 + A_2y_2 - A_3y_3 + A_4y_4}{A_1 + A_2 - A_3 + A_4}$$

$$= \frac{(39 \times 4.33) + (130 \times 6.5) - (39.27 \times 2.12) + (78.54 \times 17.24)}{39 + 130 - 39.27 + 78.54}$$

$$= 10.97 \text{ cm}$$

Now, moment of inertia about centroidal x and y axis is given by;

$$I_x = \left[\frac{b_1h_1^3}{36} + A_1(y_1 - \bar{y})^2\right] + \left[\frac{b_2h_2^3}{12} + A_2(y_2 - \bar{y})^2\right] - \left[0.11r^4 + A_3(y_3 - \bar{y})^2\right] + \left[0.055r^4 + A_4(y_4 - \bar{y})^2\right]$$

$$= \left[\frac{6 \times (13)^3}{36} + 39(4.33 - 10.97)^2\right] + \left[\frac{10 \times (13)^3}{12} + 130(6.5 - 10.97)^2\right] - \left[0.11 \times (5)^4 + 39.27(2.12 - 10.97)^2\right] + \left[0.055 \times (10)^4 + 78.54(17.24 - 10.97)^2\right]$$

$$= 2085.66 + 4428.35 - 3144.47 + 3637.64$$

$$\therefore \quad I_x = 7007.18 \text{ cm}^4$$

Similarly,

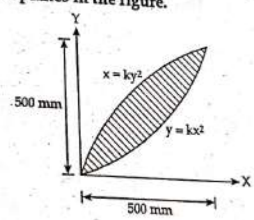
$$I_y = \left[\frac{h_1b_1^3}{36} + A_1(x_1 - \bar{x})^2\right] + \left[\frac{h_2b_2^3}{12} + A_2(x_2 - \bar{x})^2\right] - \left[\frac{\pi r^4}{8} + A_3(x_2 - \bar{x})^2\right] + \left[0.055r^4 + A_4(x_4 - \bar{x})^2\right]$$

$$= \left[\frac{13 \times (6)^3}{36} + 39(4 - 9.4)^2\right] + \left[\frac{13 \times (10)^3}{12} + 130(11 - 9.4)^2\right] - \left[\frac{\pi \times (5)^4}{8} + 39.27(11 - 9.4)^2\right] + \left[0.055 \times (10)^4 + 78.54(10.24 - 9.4)^2\right]$$

$$= 1215.24 + 1416.13 - 345.97 + 605.42$$

$$\therefore \quad I_y = 2890.82 \text{ cm}^4$$

34. State and prove parallel axes theorem for moment of inertia. Determine centroid of the given planes in the figure. [2072 Chaitra]



Solution:

For the first part

See the solution of Q. no. 19 on page no. 121

For the second part

The given equation of the curves are;

$$x = ky^2$$

$$\text{and, } y = kx^2$$

When x = 500 mm and y = 500 mm; then,

$$500 = k \times (500)^2$$

$$\text{or, } \quad k = \frac{500}{(500)^2} = \frac{1}{500}$$

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$\therefore \quad x = \frac{y^2}{500}$

and, $y = \frac{x^2}{500}$

Now,

$$A = \int dA = \int_0^{500} (y_2 - y_1)\, dx$$

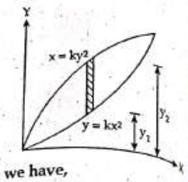
$$= \int_0^{500} \left[(500x)^{\frac{1}{2}} - \frac{x^2}{500}\right] dx$$

$$= \left[10\sqrt{5} \times \frac{2}{3} x^{\frac{3}{2}} - \frac{1}{500} \times \frac{1}{3} x^3\right]_0^{500}$$

$$= 14.91 \times (500)^{\frac{3}{2}} - \frac{1}{1500} \times (500)^3$$

$$= 166666.67 - 83333.33$$

$\therefore \quad A = 83333.34$



Now, taking a vertical strip parallel to the y-axis; we have,

$$\bar{x} = \frac{\int x\, dA}{A} = \frac{1}{83333.34} \int_0^{500} x (y_2 - y_1)\, dx$$

$$= \frac{1}{83333.34} \int_0^{500} \left(10\sqrt{5} x^{\frac{3}{2}} - \frac{1}{500} x^3\right) dx$$

$$= \frac{1}{83333.34} \int_0^{500} \left[10\sqrt{5} \times \frac{2}{5} \times (500)^{\frac{5}{2}} - \frac{1}{500} \times \frac{1}{4} (500)^4\right] dx$$

$\therefore \quad \bar{x} = 225 \text{ mm}$

Similarly,

$$\bar{y} = \frac{\int y\, dA}{A} = \frac{1}{83333.34} \int_0^{500} \left(y_1 + \frac{y_2 - y_1}{2}\right)(y_2 - y_1)\, dx$$

$$= \frac{1}{83333.34} \int_0^{500} \left[\frac{(y_2 + y_1)(y_2 - y_1)}{2}\right] dx$$

$$= \frac{1}{166666.68} \int_0^{500} (y_2^2 - y_1^2)\, dx$$

$$= \frac{1}{166666.68} \int_0^{500} \left[500x - \frac{x^4}{(500)^2}\right] dx$$

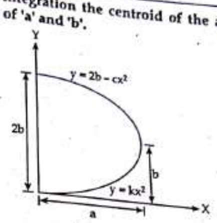
$$= \frac{1}{166666.68} \left[\frac{500x^2}{2} - \frac{x^5}{(500)^2 \times 5}\right]_0^{500}$$

$\therefore \quad \bar{y} = 225 \text{ mm}$

$\therefore \quad (\bar{x}, \bar{y}) = (225 \text{ mm}, 225 \text{ mm})$

ADDITIONAL PROBLEMS

1. **Determine by direct integration the centroid of the area shown. Express your answer in terms of 'a' and 'b'.**



Solution:

The given equations are;

$y = kx^2$

and, $y = 2b - cx^2$

When $x = a$, $y = b$ (for both curves as the upper curve is as well at a distance b from its starting point)

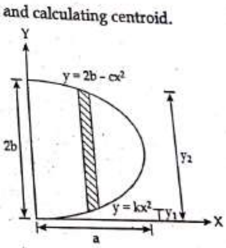
$b = ka^2$

and, $b = 2b - cx^2$

$\therefore \quad k = \frac{b}{a^2}$

$\therefore \quad c = \frac{b}{a^2}$

Now, taking the vertical strip parallel to Y-axis and calculating centroid.



$$A = \int dA = \int_0^a (y_2 - y_1)\, dx$$

$$= \int_0^a (2b - cx^2 - kx^2)\, dx$$

$$= \left[2bx - \frac{cx^3}{3} - \frac{kx^3}{3}\right]_0^a$$

$$= 2ab - \frac{ca^3}{3} - \frac{ka^3}{3}$$

$$= 2ab - \frac{b}{a^2} \times \frac{a^3}{3} - \frac{b}{a^2} \times \frac{a^3}{3} = 2ab - \frac{ab}{3} - \frac{ab}{3}$$

$\therefore \quad A = \frac{4ab}{3}$

Now; we know,

$$\bar{x} = \frac{\int x\, dA}{A} = \frac{3}{4ab} \int_0^a x (y_2 - y_1)\, dx = \frac{3}{4ab} \int_0^a (2bx - ax^3 - kx^3)\, dx$$

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$$= \frac{3}{4ab}\left(ba^2 - \frac{b}{a^2}\times\frac{a^4}{4} - \frac{b}{a^2}\times\frac{a^4}{4}\right) = \frac{3}{4ab} - \frac{a^2b}{4} - \frac{a^2b}{4}$$

$$\therefore \quad \bar{x} = \frac{3a}{8}$$

Similarly;

$$\bar{y} = \frac{\int y\,dA}{A} = \frac{3}{4ab}\int_0^a \left[y_1 + \left(\frac{y_2 - y_1}{2}\right)\right](y_2 - y_1)\,dx$$

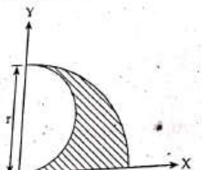
$$= \frac{3}{8ab}\int_0^a (y_2^2 - y_1^2)\,dx = \frac{3}{8ab}\int_0^a (4b^2 - 4bcx^2 + c^2x^4 - k^2x^4)\,dx$$

$$= \frac{3}{8ab}\left[4ab^2 - \frac{4}{3}ab^2 + \frac{1}{5}ab^2 - \frac{1}{5}ab^2\right] = \frac{3}{8ab}\times\frac{8}{3}ab^2$$

$$\therefore \quad \bar{y} = b$$

Hence the centroid of the given area is $(\bar{x}, \bar{y}) = \left(\frac{3a}{8}, b\right)$

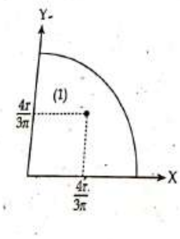
2. **Determine the centroid of the shaded area formed by removing a semi-circle of diameter 'r' from the quarter-circle of radius 'r'.**

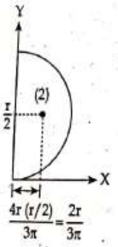


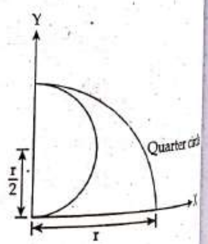
Solution:

The given area can be considered a quarter circular area of radius 'r', from which a semi-circular area of radius $\frac{r}{2}$ is removed.

Now, dividing the area into two part such that first part represents full quarter circle of radius 'r' and second part represents the semi-circle of radius $\frac{r}{2}$.







Now, calculating area and centroid of each figure;

$$A_1 = \frac{\pi r^2}{4}\ (+)$$

$$A_2 = \frac{\pi\cdot\left(\frac{r}{2}\right)^2}{2} = \frac{\pi r^2}{8}\ (-)$$

$$x_1 = \frac{4r}{3\pi}$$

$$y_1 = \frac{4r}{3\pi}$$

$$x_2 = \frac{2r}{3\pi}$$

$$y_2 = \frac{r}{2}$$

We have,

$$\bar{x} = \frac{\sum Ax}{\sum A} = \frac{\frac{\pi r^2}{4}\times\frac{4r}{3\pi} - \frac{\pi r^2}{8}\times\frac{2r}{3\pi}}{\frac{\pi r^2}{4} - \frac{\pi r^2}{8}} = \frac{r^3}{4}\times\frac{8}{\pi r^2} = \frac{2r}{\pi}$$

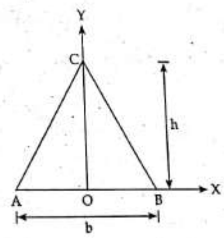
$$\therefore \quad \bar{x} = 0.637\,r$$

$$\text{and,}\ \bar{y} = \frac{\sum Ay}{\sum A} = \frac{\frac{\pi r^2}{4}\times\frac{4r}{3\pi} - \frac{\pi r^2}{8}\times\frac{r}{3}}{\frac{\pi r^2}{4} - \frac{\pi r^2}{8}} = \frac{6.58^3}{48}\times\frac{8}{\pi r^2}$$

$$\therefore \quad \bar{y} = 0.349\,r$$

Hence, the required centroid of the given area is $(\bar{x}, \bar{y}) = (0.637r, 0.349r)$.

3. **Determine the moments of inertia of an isosceles triangle about its centroidal axes.**



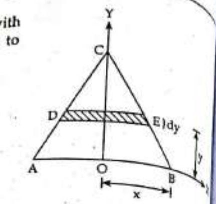
Solution:

From similar triangles ACB and DCE; we get,

$$\frac{2x}{h - y} = \frac{b}{h}$$

Therefore, $dA = 2x\,dy = \frac{b}{h}(h - y)\,dy$

For, moment of inertia about x-axis with regards to its base, taking a strip parallel to x-axis.



$$\therefore \quad I_X = \int_0^h y^2(h-y)\,dy$$

$$= \frac{b}{h}\left[h\frac{y^3}{3} - \frac{y^4}{4}\right]_0^h = \frac{b}{h}\left[h\frac{h^3}{3} - \frac{h^4}{4}\right]$$

$$= \frac{bh^3}{12}$$

Hence moment of inertia of the isosceles triangle about its horizon[tal] centroidal axis is obtained as;

$$\bar{I}_X = I_X - A\left(\frac{h}{3}\right)^2$$

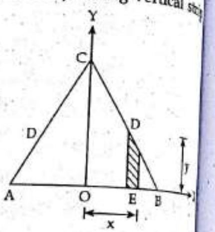
$$= \frac{bh^3}{12}\frac{1}{2}bh\left[\frac{h}{3}\right]^2$$

$$\therefore \quad \bar{I}_X = \frac{bh^3}{36}$$

Now, to calculate moment of inertia about base for I_y, taking vertical st[rip] parallel to y-axis;

$$\therefore \quad dA = y\,dx$$

From similar triangles BED and Bx; we get,



$$\frac{\frac{b}{2}}{h} = \frac{\frac{b}{2}-x}{y}$$

$$\text{or,} \quad y = \frac{2h}{b}\left(\frac{b}{2}-x\right)$$

$$\therefore \quad dA = y\,dx = \frac{2h}{b}\left(\frac{b}{2}-x\right)dx$$

Taking second moment of area of the strip about the y-axis,

$$I_y = \int x^2\,dA = \int \frac{2h}{b}x^2\left(\frac{b}{2}-x\right)dx$$

Note

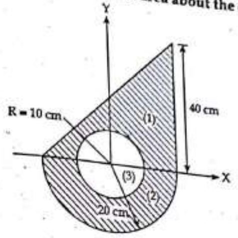
The y-axis passes through the centroid and hence, a bar sign is use[d]. Therefore, the moment of inertia of the triangle about y-axis (or centroi[dal] here) is given as,

$$\bar{I}_y = \int_{-b/2}^{b/2} \frac{2h}{b}x^2\left(\frac{b}{2}-x\right)dx$$

$$= \frac{4h}{b}\left[\frac{b}{2}\times\frac{x^3}{3} - \frac{x^4}{4}\right]_0^{b/2} = \frac{4h}{b}\left[\frac{b}{2}\times\frac{b^3}{3} - \frac{b^4}{4}\right]$$

$$\therefore \quad \bar{I}_y = \frac{hb^3}{48}$$

4. Find the moments of inertia of the shaded area about the centroidal axes.



Solution:

The given area can be divided into three geometrical areas as follows.

i) Triangle (1) (+)
ii) Semi-circle (2) (+)
iii) Circle (3) (–)

Now, calculating the area and centroid of each figure taking x-y axes as reference axes;

$$A_1 = \frac{1}{2}\times 40\times 40 = 800\text{ cm}^2$$

$$A_2 = \frac{\pi\times 20^2}{2} = 628.32\text{ cm}^2$$

$$A_3 = \pi\times 10^2 = 314.16\text{ cm}^2$$

$$x_1 = \frac{2}{3}\times 40 = 26.67\text{ cm}$$

$$x_2 = 20\text{ cm}$$

$$x_3 = 20\text{ cm}$$

$$y_1 = \frac{1}{3}\times 40 = 13.33\text{ cm}$$

$$y_2 = \frac{-4\times 20}{3\pi} = -8.49\text{ cm}$$

$$y_3 = 0$$

$$\therefore \quad \bar{x} = \frac{A_1x_1 + A_2x_2 - A_3x_3}{A_1 + A_2 + A_3} = 24.79\text{ cm}$$

$$\text{and,} \quad \bar{y} = \frac{A_1y_1 + A_2y_2 - A_3y_3}{A_1 + A_2 + A_3} = 4.78\text{ cm}$$

Now, calculating moment of inertia about centroidal axis;

$$I_X = \left[\frac{40\times 40^3}{36} + 800(13.33-4.78)^2\right] + [0.11\times(201^4 + 628.32(-8.49-4.78)^2] + \left[\frac{\pi\times 10^4}{4} + 314.16(0-4.78)^2\right]$$

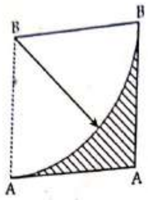
$$\therefore \quad I_X = 242803.77\text{ cm}^4$$

$$\text{and,} \quad I_y = \left[\frac{40\times 40^3}{36} + 800(26.67-24.79)^2\right] + \left[\frac{\pi\times 20^4}{8} + 628.32(20-24.79)^2\right]$$

$$+\left[\frac{\pi \times 10^4}{4} + 314.16(20 - 24.7\ldots\right.$$

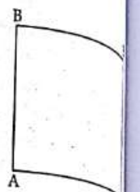
$\therefore \quad I_y = 136124.62\ \text{cm}^4$

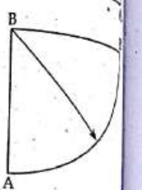
5. **Determine the moment of inertia of the quarter circular spandrel as shown about AA axis.**



Solution:

The shaded area can be obtained by removing a quarter-circular area of radius 'r' from a square of side 'r' as shown in the figure.





We know that; moment of inertia of a square about its base is;

$$I_1 = \frac{1}{3} r \times r^3 = 0.33 r^4$$

The moment of inertia of a quarter-circular area about its centroidal axis is;

$$\bar{I}_2 = 0.055 r^4$$

Therefore, its moment of inertia about A-A axis is;

$$I_2 = 0.055\ r^4 + \frac{\pi}{4} r^2 \left(r - \frac{4r}{3\pi}\right)^2$$

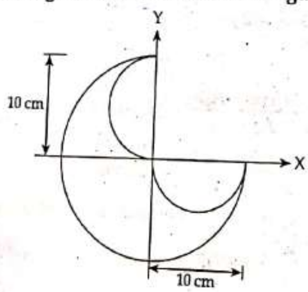
$\therefore \quad I_2 = 0.315\ r^4$

Hence, the moment of inertia of the spandrel about A-A axis is obtained as;

$$I_{AA} = I_1 - I_2 = 0.33 r^4 - 0.315 r^4$$

$\therefore \quad I_{AA} = 0.015 r^4$

6. **Find the centroid of the given area as shown in the figure.**



Solution:

Considering the whole circle and dividing it into numbers of parts as;

i) Circle with radius 10 cm (1)
ii) Quadrant with radius 10 cm (2)
iii) Semi circle with radius 5 cm (3)

iv) Semi-circle with radius 5 cm (4)

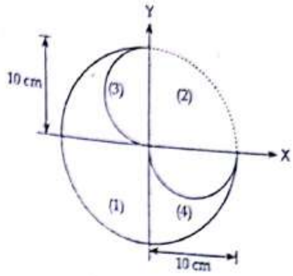
Now, calculating the area and centroid of each part;

$$A_1 = \pi \times 10^2 = 314.16\ \text{cm}^2$$

$$A_2 = \frac{\pi r^2}{4} = \frac{\pi \times 10^2}{4} = 78.54\ \text{cm}^2$$

$$A_3 = \frac{\pi r^2}{2} = \frac{\pi \times 5^2}{2} = 39.27\ \text{cm}^2$$

$$A_4 = \frac{\pi r^2}{2} = \frac{\pi \times 5^2}{2} = 39.27\ \text{cm}^2$$



$$x_1 = 0$$

$$x_2 = \frac{4r}{3\pi} = \frac{4 \times 10}{3\pi} = 4.24\ \text{cm}$$

$$x_3 = \frac{-4r}{3\pi} = \frac{-4 \times 5}{3\pi} = -2.12\ \text{cm}$$

$$x_4 = 5\ \text{cm}$$

$$y_1 = 0$$

$$y_2 = \frac{4 \times 10}{3\pi} = 4.24\ \text{cm}$$

$$y_3 = 5\ \text{cm}$$

$$y_4 = \frac{-4 \times 5}{3\pi} = -2.12\ \text{cm}$$

We have,

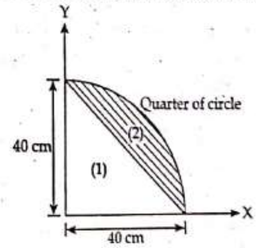
$$\bar{x} = \frac{\sum Ax}{\sum A} = \frac{A_1x_1 - A_2x_2 - A_3x_3 - A_4x_4}{A_1 - A_2 - A_3 - A_4} = -2.84\ \text{cm}$$

and, $$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{A_1y_1 + A_2y_2 - A_3y_3 - A_4x_4}{A_1 - A_2 - A_3 - A_4} = -2.84\ \text{cm}$$

Hence, the centroid of the given area about given axis is;

$$(\bar{x}, \bar{y}) = (-2.84\ \text{cm}, -2.84\ \text{cm})$$

7. **Determine the moment of inertia of the section shown about centroidal axes.**



Solution:

The given area can be divided in two parts as;

i) Triangle (1) (–)
ii) Quarter circle (2)

The above figure is the outcome of removing of triangle from the quarter-circle so now calculating the area and centroid of each of them.

$$A_1 = \frac{1}{2} \times 40 \times 40 = 800 \text{ mm}^2$$

$$A_2 = \frac{\pi \times 40^2}{4} = 1256.64 \text{ mm}^2$$

$$x_1 = \frac{40}{3} = 13.33 \text{ mm}$$

$$y_1 = \frac{40}{3} = 13.33 \text{ mm}$$

$$x_2 = \frac{4r}{3\pi} = 16.97 \text{ mm}$$

$$y_2 = \frac{4r}{3\pi} = 16.97 \text{ mm}$$

We have,

$$\bar{x} = \frac{A_2x_2 - A_1x_1}{A_2 - A_1} = 23.35 \text{ mm}$$

and, $$\bar{y} = \frac{A_2y_2 - A_1y_1}{A_2 - A_1} = 23.35 \text{ mm}$$

Hence, the centroid of the shaded portion is $(\bar{x}, \bar{y}) = (23.35 \text{ mm}, 23.35 \text{ mm})$

Now, moment of inertia can be calculated as;

$$I_X = [0.055 \times 40^4 + 1256.64(23.35 - 16.97)^2] - \left[\frac{40 \times 40^3}{36} + 800(23.35 - 13.33)^2\right]$$

$$= 191950.78 - 151431.43$$

$\therefore \quad I_X = 40519.35 \text{ mm}^4$

and, $$I_y = [0.055 \times 40^4 + 1256.64(23.35 - 16.97)^2] - \left[\frac{40 \times 40^3}{36} + 800(23.35 - 13.33)^2\right]$$

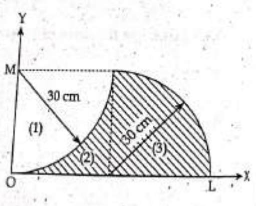
$$= 191950.78 - 151431.43$$

$\therefore \quad I_y = 40519.35 \text{ mm}^4$

8. For the area given below;

i) With reference to the shaded area, locate the centroid with respect to OL and OM.

ii) Determine the moment of inertia of the area about the horizontal axis passing through the centroid.



Solution:

Dividing the area in to three areas as;

i) Quarter circle (1) (–)

ii) Square (2) (+)

iii) Quarter circle (3) (+)

Now, calculating the area and centroid for each area;

$$A_1 = \frac{\pi \times 30^2}{4} = 706.85 \text{ mm}^2$$

$$A_2 = 30 \times 30 = 900 \text{ mm}^2$$

$$A_3 = \frac{\pi \times 30^2}{4} = 706.85 \text{ mm}^2$$

$$x_1 = \frac{4 \times 30}{3\pi} = 12.73 \text{mm}$$

$$y_1 = 30 - \frac{4 \times 30}{3\pi}$$

$$x_2 = \frac{30}{2} = 15 \text{ mm}$$

$$y_2 = \frac{30}{2} = 15 \text{ mm}$$

$$x_3 = \frac{4 \times 30}{3\pi} = 12.73 \text{mm}$$

$$y_3 = \frac{4 \times 30}{3\pi} = 12.73 \text{ mm}$$

$$\therefore \quad \bar{x} = \frac{A_2x_2 + A_3x_3 - A_1x_1}{A_2 + A_3 - A_1} = 38.56 \text{ mm}$$

and, $$\bar{y} = \frac{A_2y_2 + A_3y_3 - A_1x_1}{A_2 + A_3 - A_1} = 11.43 \text{ mm}$$

Now, moment of inertia about centroidal x-axis can be calculated as,

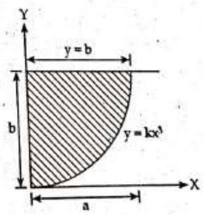
$$I_X = -I_{X1} + I_{X2} + I_{X3}$$

$$= -\left[0.055 \times 30^4 + \frac{\pi \times 30^2}{4} \times \left(18.57 - \frac{4 \times 30}{3\pi}\right)^2\right] + \left[\frac{30 \times 30^3}{12} + 30 \times 30\,(15 - 11.43)^2\right] + \left[0.055 \times 30^4 + \frac{\pi \times 30^2}{4} \times \left(\frac{4 \times 30}{3\pi} - 11.43\right)^2\right]$$

$$= -68641 + 78970 + 45748$$

$\therefore \quad I_X = 56077 \text{ mm}^4$

9. Determine by direct integration the centroid of the shaded area as shown in figure below.

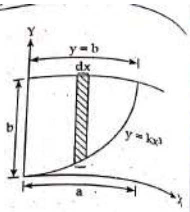


Solution:

The given equation of curve is;

$$y = kx^3$$

When, $x = a$, $y = b$



$b = ka^3$

$\therefore \quad k = \frac{b}{a^3}$

Now,

$$A = \int dA = \int_0^a (y_2 - y_1)\,dx$$

$$= \int_0^a (b - kx^3)\,dx = \left[bx - \frac{kx^4}{4}\right]_0^a$$

$$= ab - \frac{b}{a^3}\frac{a^4}{4} = ab - \frac{ab}{4} = \frac{3}{4}ab$$

We know,

$$\bar{x} = \frac{\int x\,dA}{A} = \frac{4}{3ab}\int_0^a (bx - kx^4)\,dx = \frac{4}{3ab}\left[\frac{ab^2}{2} - \frac{b}{a^3}\frac{a^5}{5}\right]$$

$$= \frac{4}{3ab}\left[\frac{a^2b}{2} - \frac{a^2b}{5}\right] = \frac{4}{3ab} \times \frac{3}{10}a^2b$$

$$\therefore \quad \bar{x} = \frac{2}{5}a$$

$$\text{and, } \bar{y} = \frac{\int y\,dA}{A} = \frac{4}{3ab}\int_0^a \left[y_1 + \left(\frac{y_2 - y_1}{2}\right)\right](y_2 - y_1)\,dx$$

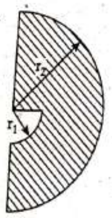
$$= \frac{4}{6ab}\int_0^a (y_2^2 - y_1^2)\,dx = \frac{2}{3ab}\int_0^a (b^2 - k^2x^6)\,dx$$

$$= \frac{2}{3ab}\left[ab^2 - \frac{b^2}{a^6} \times \frac{a^7}{7}\right] = \frac{2}{3ab}\left[ab^2 - \frac{ab^2}{7}\right] = \frac{2}{3ab} \times \frac{6}{7}ab^2$$

$$\therefore \quad \bar{y} = \frac{4}{7}b$$

Hence, the centroid of the area is $(\bar{x}, \bar{y}) = \left(\frac{2a}{5}, \frac{4b}{7}\right)$

10. For the shaded area shown below, determine the ratio $\frac{r_2}{r_1}$ so that $\bar{x} = \frac{4r_1}{3}$.

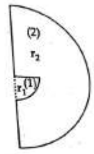


Solution:

Considering whole semi-circle and then a quarter circle of different radius inside it; we have,

i) Semi-circle of radius $r_2(+)$

ii) Quarter circle of radius $r_1(-)$



$$A_1 = \frac{\pi}{4}r_1^2$$

$$A_2 = \frac{\pi}{2}r_2^2$$

$$x_1 = \frac{4r_1}{3\pi}$$

$$x_2 = \frac{4r_2}{3\pi}.$$

$$\therefore \quad \bar{x} = \frac{A_2x_2 - A_1x_1}{A_2 - A_1} = \frac{\frac{\pi}{2}r_2^2 \times \frac{4r_2}{3\pi} - \frac{\pi}{4}r_1^2 \times \frac{4r_1}{3\pi}}{\frac{\pi}{2}r_2^2 - \frac{\pi}{4}r_1^2} = \frac{\frac{2r_2^3}{3} - \frac{r_1^3}{3}}{\frac{\pi}{4}(2r_2^2 - r_1^2)}$$

Since,

$$\bar{x} = \frac{4r_1}{3}$$

$$\text{or,} \quad \frac{4r_1}{3} = \frac{\frac{1}{3}(2r_2^3 - r_1^3)}{\frac{\pi}{4}(2r_2^2 - r_1^2)}$$

$$\text{or,} \quad \frac{4r_1}{3} \times \frac{\pi}{4}(2r_2^2 - r_1^2) = \frac{1}{3}(2r_2^3 - r_1^3)$$

$$\text{or,} \quad \frac{r_1}{3}\pi(2r_2^2 - r_1^2) = \frac{1}{3}(2r_2^3 - r_1^3)$$

$$\text{or,} \quad r_1\pi(2r_2^2 - r_1^2) = (2r_2^3 - r_1^3)$$

Dividing both side by r_1^3; we get,

$$\frac{r_1}{r_1}\pi\left[2\left(\frac{r_2}{r_1}\right)^2 - 1\right] = \left[2\left(\frac{r_2}{r_1}\right)^3 - 1\right]$$

Let, $\frac{r_2}{r_1} = x$

$$\therefore \quad \pi(2x^2 - 1) = 2x^3 - 1$$

$$\text{or,} \quad 2x^3 - 2\pi x^2 + (\pi - 1) = 0$$

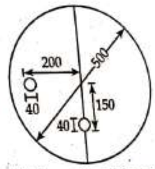
$$\text{or,} \quad 2x^3 - 6.28x^2 + 2.14 = 0$$

Solving; we get,

$$x = 3.02$$

$$\therefore \quad \frac{r_2}{r_1} = 3.02$$

11. A circular plate of uniform thickness and of diameter 500 mm as shown in the figure has two circular holes of 40 mm diameter each. Where should an 80 mm diameter hole be drilled so that center of gravity of the plate will be at geometric centre?



(All dimensions are in mm)

Solution:

Let a hole of 80 mm diameter be drilled at a distance x and y from OY and OX axis as shown in the figure.

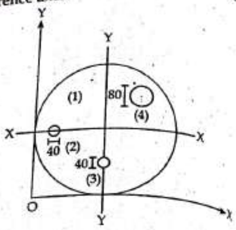
a_1 = Area of circle of 500 mm diameter $= \frac{\pi}{4} \times (500)^2 = 196349.54\ mm^2$

$a_2 = a_3$ = Area of circle of 40 mm diameter $= \frac{\pi}{4} \times (40)^2$

$= 1256.64\ mm^2(-)$

a_4 = Area of circle of 80 mm diameter $= \frac{\pi}{4} \times (80)^2 = 5026.55\ mm^2(-)$

Let (x_1, y_1), (x_2, y_2), (x_3, y_3) and (x_4, y_4) be the co-ordinates of the C.G. of the circle (1), (2), (3) and (4) from the reference axes.



$x_1 = \frac{500}{2} = 250$ mm

$x_2 = 250 - 200 = 50$ mm

$x_3 = 250$ mm

$x_4 = x$ mm

$y_1 = \frac{500}{2} = 250$ mm

$y_2 = 250$ mm

$y_3 = 250 - 150 = 100$ mm

$x_4 = y$ mm

$\overline{X}$ = Distance of C.G. of the plate from OY-axis = 250 mm (given)

$\overline{Y}$ = Distance of C.G. of the plate from OX-axis = 250 mm (given)

$$\overline{X} = \frac{a_1x_1 - a_2x_2 - a_3x_3 - a_4x_4}{a_1 - a_2 - a_3 - a_4}$$

or, $$250 = \frac{196349.54 \times 250 - 1256.44 \times 50 - 1256.64 \times 250 - 5026.44 \times x}{196349.54 - 1256.64 - 1256.64 - 5026.44}$$

or, $250 \times 188809.71 = 48710393 - 5026.55x$

$\therefore$ $x = 300$ mm from OX-axis

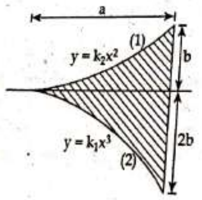
$$\overline{Y} = \frac{a_1y_1 - a_2y_2 - a_3y_3 - a_4y_4}{a_1 - a_2 - a_3 - a_4}$$

or, $$250 = \frac{48647561 - 5026.55y}{188809.71}$$

or, $5026.55y = 1445133.5$

$\therefore$ $y = 287.5$ mm from OY-axis.

12. **Determine by direct integration the centroid of the area shown. Express your answer in terms of 'a' and 'b'.**



Solution:

Taking an elemental strip parallel to Y-axis; we have,

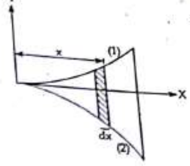
For the element shown on line 1 at,

$x = a$

and, $b = k_2a^2$

$\therefore$ $k_2 = \frac{b}{a^2}$

$\therefore$ $y = \frac{b}{a^2}x^2$



On line (2); we have,

$x = a,\ -2b = k_1a^3$

$\therefore$ $k_1 = \frac{-2b}{a^3}$

$\therefore$ $y = \frac{-2b}{a^3}x^3$

$$dA = \left(\frac{b}{a^2}x^2 + \frac{2b}{a^3}x^3\right) dx$$

Then,

$$A = \int dA$$

$$= \int_0^a \frac{b}{a^2}\left(x^2 + \frac{2x^3}{x}\right) dx = \frac{b}{a^2}\left[\frac{x^3}{3} - \frac{2x^4}{4a}\right]_0^a$$

$$= ab\left(\frac{1}{3} + \frac{1}{2}\right)$$

$\therefore$ $A = \frac{5}{6}ab$

Now,

$$\int x\, dA = \int_0^a x\left(\frac{b}{a^2}x^2 + \frac{2b}{a^3}x^3\right) dx$$

$$= \frac{b}{a^2}\left[\frac{x^4}{4} + \frac{2x^5}{5a}\right]_0^a = \frac{13}{20}a^2b$$

and, $$\int y\, dA = \int_0^a \frac{1}{2}\left(\frac{b}{a^2}x^2 - \frac{2b}{a^3}x^3\right)\left(\frac{b}{a^2}x^2 + \frac{2b}{a^3}x^3\right) dx$$

$$= \int_0^a \frac{1}{2}\left[\left(\frac{b}{a^2}x^2\right)^2 - \left(\frac{2b}{a^3}x^3\right)^2\right] dx$$

$$= \frac{b^2}{2a^4}\left[\frac{x^5}{5} - \frac{2x^7}{7a^2}\right]_0^a = \frac{-13}{70}ab^2$$

We know,

$$\overline{x} = \frac{\int x\, dA}{A} = \frac{13}{20}a^2b \times \frac{6}{5ab} = \frac{39}{50}a$$

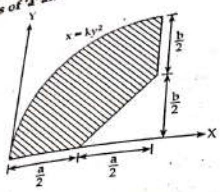
and, $$\overline{y} = \frac{\int y\, dA}{A} = \frac{-13}{20}ab^2 \times \frac{6}{5ab}$$

$\therefore$ $\overline{y} = \frac{-39}{175}b$

Hence,

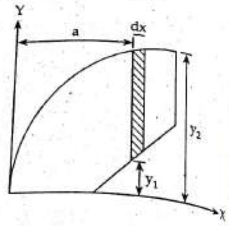
$$(\overline{x}, \overline{y}) = \left(\frac{39}{50}a, \frac{-39}{175}b\right)$$

13. **Determine by direct integration, the centroid of the area shown. Express your answer terms of 'a' and 'b'.**



Solution:

Taking an elemental strip parallel to Y–axis as shown in the figure;



For y_2;

At $x = a$, $y = b$;

$$a = kb^2$$

$$\therefore \quad k_2 = \frac{a}{b^2}$$

Then,

$$y_2 = \frac{b}{\sqrt{a}} x^{\frac{1}{2}}$$

For $0 \le x \le \frac{a}{2}$;

$$\bar{y} = \frac{y_2}{2} = \frac{b}{2}\frac{x^{\frac{1}{2}}}{\sqrt{a}} dA = y_2 dx = \frac{bx^{\frac{1}{2}}}{\sqrt{a}} dx$$

For $\frac{a}{2} \le x \le a$;

$$\bar{y} = \frac{1}{2}(y_1 + y_2) = \frac{b}{2}\left(\frac{x}{a} - \frac{1}{2} + \frac{x^{\frac{1}{2}}}{\sqrt{a}}\right)$$

$$dA = (y_2 - y_1)dx = b\left(\frac{x^{\frac{1}{2}}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2}\right)dx$$

Then,

$$A = \int dA = \int_0^{\frac{a}{2}} b\frac{x^{\frac{1}{2}}}{\sqrt{a}} dx + \int_{\frac{a}{2}}^{a} b\left(\frac{x^{\frac{1}{2}}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2}\right)dx$$

$$= \frac{b}{\sqrt{a}}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^{\frac{a}{2}} + b\left[\frac{2}{3}\frac{x^{\frac{3}{2}}}{\sqrt{a}} - \frac{x^2}{2a} + \frac{1}{2}x\right]_{\frac{a}{2}}^{a}$$

$$= \frac{2}{3}\frac{b}{\sqrt{a}}\left[\left(\frac{a}{2}\right)^{\frac{3}{2}} + (a)^{\frac{3}{2}} - \left(\frac{a}{2}\right)^{\frac{3}{2}}\right] + b\left[\frac{-1}{2a}\left\{(a)^2 - \left(\frac{a}{2}\right)^2\right\} + \frac{1}{2}\left\{(a) - \left(\frac{a}{2}\right)\right\}\right]$$

$$= \frac{13}{24}ab$$

Now,

$$\int x\, dA = \int_0^{\frac{a}{2}} x\left(b\frac{x^{\frac{1}{2}}}{\sqrt{a}}\right)dx + \int_{\frac{a}{2}}^{a}\left[x\left(\frac{x^{\frac{1}{2}}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2}\right)\right]dx$$

$$= \frac{b}{\sqrt{a}}\left[\frac{2}{5}x^{\frac{5}{2}}\right]_0^{\frac{a}{2}} + b\left[\frac{2}{5}\frac{x^{\frac{5}{2}}}{\sqrt{a}} - \frac{x^3}{3a} + \frac{x^4}{4}\right]_{\frac{a}{2}}^{a}$$

$$= \frac{2}{5}\frac{b}{\sqrt{a}}\left[\left(\frac{a}{2}\right)^{\frac{5}{2}} + (a)^{\frac{5}{2}} - \left(\frac{a}{2}\right)^{\frac{5}{2}}\right] + b\left[\frac{-1}{3a}\left\{(a)^3 - \left(\frac{a}{2}\right)^3\right\} + \frac{1}{4}\left\{(a)^2 - \left(\frac{a}{2}\right)^2\right\}\right]$$

$$= \frac{71}{240}a^2b$$

Similarly;

$$\int y\, dA = \int_0^{\frac{a}{2}} \frac{b}{2}\frac{x^{\frac{1}{2}}}{\sqrt{a}}\left(b\frac{x^{\frac{1}{2}}}{\sqrt{a}}\right)dx + \int_{\frac{a}{2}}^{a}\frac{b}{2}\left(\frac{x}{a} - \frac{1}{2} + \frac{x^{\frac{1}{2}}}{\sqrt{a}}\right)\left[b\left(\frac{x^{\frac{1}{2}}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2}\right)\right]dx$$

$$= \frac{b^2}{2a}\left[\frac{1}{2}x^2\right]_0^{\frac{a}{2}} + \frac{b^2}{2}\left[\frac{x^2}{2a} - \frac{1}{3a}\left(\frac{x}{a} - \frac{1}{2}\right)^3\right]_{\frac{a}{2}}^{a}$$

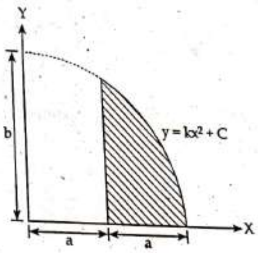
$$= \frac{b}{4a}\left[\left(\frac{a}{2}\right)^2 + (a)^2 - \left(\frac{a}{2}\right)^2\right] - \frac{b^2}{6a}\left(\frac{a}{2} - \frac{1}{2}\right)^3 = \frac{11}{48}ab^2$$

We know,

$$\bar{x} = \frac{\int x\, dA}{A} = \frac{71}{130}a = 0.546a$$

and, $\bar{y} = \frac{\int y\, dA}{A} = \frac{11}{26}b = 0.423b$

14. **Determine by direct integration the moment of inertia of the shaded area shown in figure.**



Solution:

We have,

$$y = kx^2 + C$$

At $x = 0$, $y = b$; $b = k(0) + C$

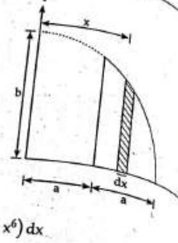
$\therefore \quad C = b$

At $x = 2a$, $y = 0$; $0 = k(2a)^2 + b$

$$\therefore \quad k = \frac{-b}{4a^2}$$

Then,

$$y = \frac{b}{4a^2}(4a^2 - x^2)$$



Now,

$$dI_x = \frac{1}{3}y^3dx = \frac{1}{3}\frac{b^3}{64a^6}(4a^2 - x^2)^3dx$$

Then,

$$I_x = \int dI_x$$

$$= \frac{1}{3}\frac{b^3}{64a^6}\int_a^{2a}(4a^2 - x^2)^3 dx$$

$$= \frac{b^3}{192a^6}\int_a^{2a}(64a^6 - 48a^4x^2 + 12a^2x^4 - x^6)\,dx$$

$$= \frac{b^3}{192a^6}\left[64a^2x - 16a^4x^3 + \frac{12}{5}a^2x^5 - \frac{x^7}{7}\right]_a^{2a}$$

$$= \frac{b^3}{192a^6}\left[64a^7(2-1) - 16a^7(8-1) + \frac{12}{5}a^7(32-1) - \frac{1}{7}(128-1)\right]$$

$$= \frac{ab^3}{192}\left[64 - 112 + \frac{372}{5} - \frac{127}{7}\right]$$

$\therefore \quad I_x = 0.043ab^3$

Now,

$$I_y = \int x^2 dA$$

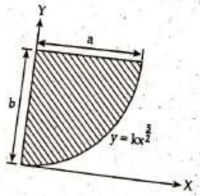
$$dA = y\,dx = \frac{b}{4a^2}(4a^2 - x^2)dx$$

$$\therefore \quad I_y = \int_a^{2a} x^2 dA = \frac{b}{4a^2}\int_a^{2a} x^2(4a^2 - x^2)\,dx = \frac{b}{4a^2}\left[4a^2\frac{x^3}{3} - \frac{x^5}{5}\right]_a^{2a}$$

$$= \frac{b}{3}(8a^3 - a^3) - \frac{b}{20a^2}(32a^5 - a^5) = \frac{7a^3b}{3} - \frac{31a^3b}{20}$$

$$\therefore \quad I_y = \frac{47}{60}a^3b$$

15. Determine by direct integration the moment of inertia of the shaded area shown in figure.



Solution:

Given that,

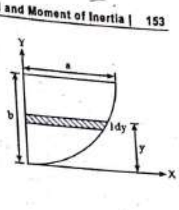
$$y = kx^{\frac{5}{2}}$$

At, $x = a, y = b; b = ka^{\frac{5}{2}}$

$$\therefore \quad k = \frac{b}{a^{\frac{5}{2}}}$$

$$\therefore \quad y = \frac{b}{a^{\frac{5}{2}}}x^{\frac{5}{2}}$$

and, $x = \dfrac{a}{b^{\frac{2}{5}}}y^{\frac{2}{5}}$



$$I_x = \int y^2 dA,\ dA = x\,dy$$

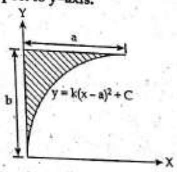
$$\therefore \quad I_x = \int_0^b y^2\left[\frac{a}{b^{\frac{2}{5}}}y^{\frac{2}{5}}\right]dy = \frac{a}{b^{\frac{2}{5}}}\times\frac{5}{17}\left[y^{\frac{17}{5}}\right]_0^b = \frac{5a}{17}\times\frac{b^{\frac{17}{5}}}{b^{\frac{2}{5}}} = \frac{5}{17}ab^3$$

Now,

$$dI_y = \frac{1}{3}x^3dy = \frac{1}{3}\frac{a^3}{b^{\frac{6}{5}}}y^{\frac{6}{5}}dy$$

$$\therefore \quad I_y = \frac{1}{3}\frac{a^3}{b^{\frac{6}{5}}}\int_0^b y^{\frac{6}{5}}dy = \frac{1}{3}\times\frac{5}{11}\frac{a^3}{b^{\frac{6}{5}}}\left[y^{\frac{11}{5}}\right]_0^b = \frac{5}{33}\frac{a^3}{b^{\frac{6}{5}}}b^{\frac{11}{5}} = \frac{5}{33}a^3b$$

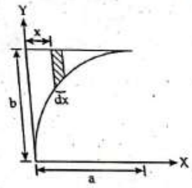
16. Determine by direct integration the moment of inertia of the shaded area shown in figure with respect to y-axis.



Solution:

Given that;

$$y = k(x-a)^2 + C$$



At, $x = 0, y = 0;\ 0 = ka^2 + C$

$$\therefore \quad k = \frac{-C}{a^2}$$

At, $x = a, y = b;\ b = C$

$$\therefore \quad k = \frac{-b}{a^2}$$

Then,

$$y = \frac{-b}{a^2}(x-a)^2 + b = \frac{-b}{a^2}(x^2 - 2ax + a^2) + b$$

Now,

$$dI_y = x^3dA = (y_2 - y_1)x^2dx$$

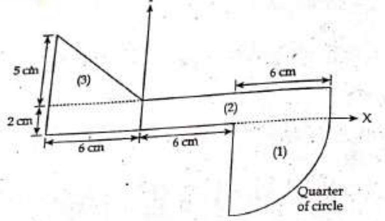
$$= x^2\left(b + \frac{b}{a^2}x^2 - \frac{2b}{a}x + b - b\right)dx = \left(\frac{b}{a^2}x^4 - \frac{2b}{a}x^3 + bx^2\right)dx$$

$$I_y = \int_0^a \left(\frac{b}{a^2}x^4 - \frac{2b}{a}x^3 + bx^2\right) dx = \left[\frac{1}{5}\frac{b}{a^2}x^5 - \frac{b}{2a}x^4 + \frac{b}{3}x^3\right]_0^a$$

$$= a^3b\left(\frac{1}{5} - \frac{1}{2} + \frac{1}{3}\right)$$

$$\therefore \quad I_y = \frac{1}{30}a^3b$$

17. Determine the moments of inertia of the area shown in the figure about given 'X' and 'Y' axes.



Solution:

The whole area can be divided into following three areas;

i) Quarter circle (1) (+)
ii) Rectangle (2) (+)
iii) Triangle (3) (+)

$$A_1 = \frac{\pi r^2}{4} = \frac{\pi \times (6)^2}{4} = 28.27\ cm^2$$

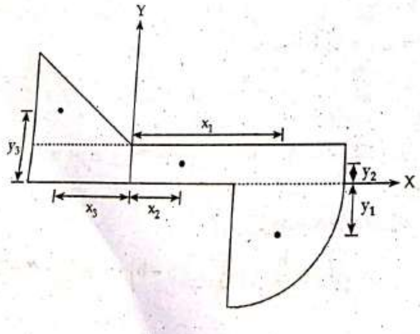
$$A_2 = 18 \times 2 = 36\ cm^2$$

$$A_3 = \frac{1}{2} \times 5 \times 6 = 15\ cm^2$$

Now, distance of centroid of each geometrical area from given reference XY axes are;

$$x_1 = 6 + \frac{4 \times 6}{3\pi} = 8.55\ cm$$

$$x_2 = 9 - 6 = 3\ cm$$



$$x_3 = \frac{2}{3} \times 6 = 4\ cm$$

$$y_1 = \frac{4 \times 6}{3\pi} = 2.55\ cm$$

$$y_2 = 1\ cm$$

$$y_3 = \frac{1}{3} \times 5 + 2 = 3.67\ cm$$

Now,

$$I_x = [0.055r^4 + A_1y_1^2] + \left[\frac{b_2h_2^3}{12} + A_2y_2^2\right] + \left[\frac{b_3h_3^3}{36} + A_3y_3^2\right]$$

$$= [0.055 \times (6)^4 + 28.27 \times (2.55)^2] + \left[\frac{18 \times (2)^3}{12} + 36 \times (1)^2\right] + \left[\frac{6 \times (5)^3}{36} + 15 \times (3.67)^2\right]$$

$$= 255.10 + 48 + 222.87$$

$$\therefore \quad I_x = 325.97 cm^4$$

and,

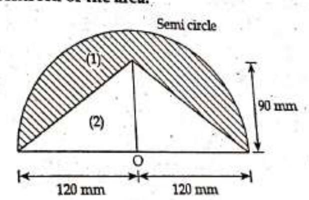
$$I_y = [0.055r^4 + A_1x_1^2] + \left[\frac{h_2b_2^3}{12} + A_2x_2^2\right] + \left[\frac{h_3b_3^3}{36} + A_3x_3^2\right]$$

$$= [0.055 \times (6)^4 + 28.27 \times (8.55)^2] + \left[\frac{2 \times (18)^3}{12} + 36 \times (3)^2\right] + \left[\frac{5 \times (6)^3}{36} + 15 \times (4)^2\right]$$

$$= 2137.89 + 1296 + 270$$

$$\therefore \quad I_y = 3703.89\ cm^4$$

18. Determine the polar moment of inertia of area shown with respect to (a) point 'O', (b) the centroid of the area.



Solution:

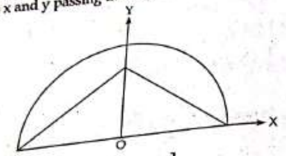
Dividing the whole area into two parts so as to extract the shaded area by subtracting triangle from semi-circle; we have,

i) Semi-circle (1) (+)
ii) Triangle (2) (−)

$$A_1 = \frac{\pi}{2} \times 120^2 = 7200\pi\ mm^2$$

$$A_2 = \frac{1}{2} \times 240 \times 90 = 10800\ mm^2$$

a) To calculate polar moment of inertia about 'O', calculating I_x and I_Y taking reference axes x and y passing through 'O' as shown in figure. Now,



$$I_x = [0.11r^4 + A_1 y_1^2] - \left[\frac{b_2 h_2^3}{36} + A_2 y_2^2\right]$$

$$= \left[0.11 \times 120^4 + 7200\pi\left(\frac{4\times 120}{3\pi}\right)^2\right] - \left[\frac{240\times(90)^3}{36} + 10800\times(30)^2\right]$$

$$= 81480478.22 - 14580000$$

$$\therefore \quad I_x = 66.9 \times 10^6 \text{mm}^4$$

$$\text{and, } I_y = \left[\frac{\pi}{8} r^4 + A_1 x_1^2\right] - \left[\frac{h_2 b_2^3}{48} + A_2 x_2^2\right]$$

$$= \left[\frac{\pi}{8}\times(120)^4 + 7200\pi\times 0\right] - \left[\frac{90\times(240)^3}{48} + 10800\times 0\right]$$

$$= 81430081.58 - 25920000$$

$$\therefore \quad I_y = 55.51 \times 10^6 \text{mm}^4$$

Hence, polar moment of inertia about O,

$$\therefore \quad I_O = I_x + I_y = (66.9 + 55.51)\times 10^6 = 122.4 \times 10^6 \text{mm}^4$$

b) To calculate polar moment of inertia about centroidal axis, simply calculating M.I. about centroidal axis,

$$A_1 = 7200\pi \text{ mm}^2$$
$$A_2 = 10800 \text{ mm}^2$$
$$x_1 = 120 \text{ mm}$$
$$x_2 = 120 \text{ mm}$$
$$y_1 = \frac{4\times 120}{3\pi} = 50.93 \text{ mm}$$
$$y_2 = \frac{1}{3}\times 90 = 30 \text{ mm}$$

$$\therefore \quad \bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2} = 120 \text{ mm}$$

$$\therefore \quad \bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = 70.054 \text{ mm}$$

$$I_x = [0.11\times(120)^4 + 7200\pi\times(50.93 - 70.054)^2] - \left[\frac{240\times(90)^3}{36} + 10800\times(30 - 70.054)^2\right]$$

$$= 31.08\times 10^6 - 22.19\times 10^6$$

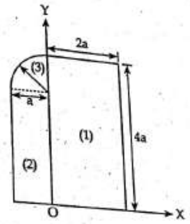
$$\therefore \quad I_x = 8.89\times 10^6 \text{mm}^4$$

$$\therefore \quad I_y = \left[\frac{\pi}{8}\times 120^4\right] - \left[\frac{90\times 240^3}{48}\right] = 55.51\times 10^6 \text{mm}^4$$

Now, polar moment of inertia about centroidal axis is given by;

$$I_C = I_x + I_y$$
$$= 8.89\times 10^6 + 55.51\times 10^6$$
$$= 64.4\times 10^6 \text{mm}^4$$

19. **Determine the moments of inertia of the shaded area shown with respect to X and Y axes.**



Solution:

Dividing the whole area into three geometrical areas; we have,

i) Rectangle (1) (+)
ii) Rectangle (2) (+)
iii) Quarter-circle (3) (+)

It is required to calculate M.I. about given X–Y axes.

$$\therefore \quad I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

$$= \left[\frac{2a\times(4a)^3}{12} + 2a\times 4a\times(2a)^2\right] + \left[\frac{a\times(3a)^3}{12} + a\times 3a\times(1.5a)^2\right] + \left[0.055\times a^4 + \frac{\pi}{4}\times a^2\times\left(3a + \frac{4a}{3\pi}\right)^2\right]$$

$$= \left[\frac{32}{3}a^4 + 32a^4\right] + \left[\frac{9}{4}a^4 + \frac{27}{4}a^4\right] + [0.055a^4 + 9.21a^4]$$

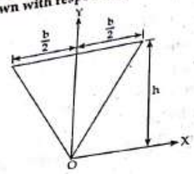
$$= 60.92a^4$$

$$\text{and, } I_y = \left[\frac{4a\times(2a)^3}{12} + 2a\times 4a\times(a)^2\right] + \left[\frac{3a\times(a)^3}{12} + a\times 3a\times(0.5a)^2\right] + \left[0.055\times a^4 + \frac{\pi}{4}\times a^2\times\left(\frac{4a}{3\pi}\right)^2\right]$$

$$= \left[\frac{8}{3}a^4 + 8a^4\right] + \left[\frac{3}{12}a^4 + \frac{3}{4}a^4\right] + [0.055a^4 + 0.14a^4]$$

$$= 11.86a^4$$

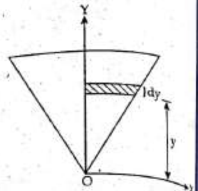
20. Determine the polar moment of inertia and the radius of gyration of the isosceles triangle shown with respect to point 'O'.



Solution:

From similar triangle, after taking horizontal strip;

$$\frac{h}{\frac{b}{2}} = \frac{y}{x}$$



$$\therefore \quad y = x\frac{h}{\frac{b}{2}}$$

and, $x = \frac{b}{2h}y$

Now,

$$dA = x\,dy = \left(\frac{b}{2h}y\right) dy$$

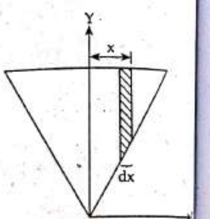
and, $dI_x = y^2 dA = \frac{b}{2h}y^3 dx$

$$\therefore \quad I_x = \int dI_x = 2\int_0^h \frac{b}{2h}y^3\,dy = \frac{b}{h}\left[\frac{y^4}{4}\right]_0^h = \frac{1}{4}bh^3$$

Now, to calculate I_y, lets us consider a vertical strip as shown in the figure;

From above;

$$y = \frac{2h}{b}x$$



Now,

$$dA = (h-y)\,dx = \left(h - \frac{2h}{b}x\right) dx$$

$$= \frac{h}{b}(b-2x)dx$$

and, $dI_y = x^2 dA = x^2\frac{h}{b}(b-2x)dx$

Then,

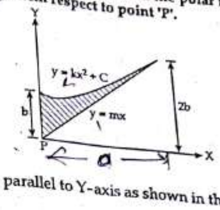
$$\therefore \quad I_y = \int dI_y$$

$$= 2\int_0^{\frac{b}{2}} \frac{h}{b}x^2(b-2x)\,dx = 2\frac{h}{b}\left[\frac{1}{3}bx^3 - \frac{1}{2}x^4\right]_0^{\frac{b}{2}}$$

$$= 2\frac{h}{b}\left[\frac{b}{3}\left(\frac{b}{2}\right)^3 - \frac{1}{2}\left(\frac{b}{2}\right)^4\right] = \frac{1}{48}b^3h$$

21. Determine the polar moment of inertia and the polar radius of gyration of the shaded area shown with respect to point 'P'.

Imp



Solution:

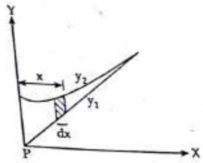
Taking vertical strip parallel to Y-axis as shown in the figure;

For y_1;

At, $x = a$, $y = 2b$;

$$2b = ma$$

$$\therefore \quad m = \frac{2b}{a}$$



Then,

$$y_1 = \frac{2b}{a}x$$

For y_2;

At, $x = 0$, $y = b$;

$$b = k(0) + C$$

$$\therefore \quad C = b$$

At, $x = a$, $y = 2b$;

$$2b = ka^2 + b$$

$$\therefore \quad k = \frac{b}{a^2}$$

Then,

$$y_2 = \frac{b}{a^2}x^2 + b = \frac{b}{a^2}(x^2 + a^2)$$

Now,

$$\therefore \quad A = \int (y_2 - y_1)dx = \int_0^a \left[\frac{b}{a^2}(x^2+a^2) - \frac{2b}{a}x\right] dx$$

$$= \left[\frac{b}{a^2}\left(\frac{x^3}{3} + a^2x\right) - \frac{b}{a}x^2\right]_0^a = \frac{b}{a^2}\left(\frac{a^3}{3} + a^3\right) - \frac{b}{a}a^2 = \frac{ab}{3}$$

$$\therefore \quad I_y = \int x^2 dA = \int_0^a x^2\left[\frac{b}{a^2}(x^2+a^2) - \frac{2b}{a}x\right] dx$$

$$= \left[\frac{b}{a^2}\left(\frac{1}{5}x^5 + \frac{1}{3}a^2x^3\right) - \frac{2b}{a}\frac{x^4}{4}\right]_0^a = \frac{b}{a^2}\left[\frac{a^5}{5} + \frac{1}{3}a^5\right] - \frac{2b}{a}\frac{a^4}{4} = \frac{1}{30}a^3b$$

and, $I_x = \int dI_x = \int \left(\frac{1}{3}y_2^3 - \frac{1}{3}y_1^3\right) dx$

$$= \frac{1}{3}\int_0^a \left[\frac{b^3}{a^6}(x^2+a^2)^3 - \frac{8b^3}{a^3}x^3\right] dx$$

$$= \frac{1}{3}\frac{b^3}{a^3}\int_0^a \left[\frac{1}{a^3}\left(x^6 + 3x^4a^2 + 3x^2a^4 + a^6\right) - 8x^3\right] dx$$

$$= \frac{1}{3}\frac{b^3}{a^3}\left[\frac{1}{a^3}\left(\frac{a^7}{7} + \frac{3}{5}a^7 + a^7 + a^7\right) - 2a^4\right]$$

$$\therefore \quad I_x = \frac{26}{105}ab^3$$

Finally, polar moment of inertia can be obtained by;

$$J_P = I_x + I_y = \frac{26}{105}ab^3 + \frac{1}{30}a^3b$$

$$\therefore \quad J_P = \frac{ab}{210}\left(7a^2 + 52b^2\right)$$

and, polar radius of gyration is given by;

$$k_p = \sqrt{\frac{J_P}{A}} = \sqrt{\frac{\frac{ab}{210}\left(7a^2 + 52b^2\right)}{\frac{1}{3}ab}}$$

$$\therefore \quad k_p = \sqrt{\frac{7a^2 + 52b^2}{70}}$$

CHAPTER 5 Friction

DEFINITIONS

Friction and Frictional force

A special type of force that acts on the body which opposes its motion is called frictional force and this phenomenon of opposition is called friction.

Types of Friction

a) Static friction

The frictional force experienced by a body at rest. In fact it is the frictional force acting on a body which tends to move.

b) Dynamic friction

The frictional force experiences by a body in motion. It is also called kinetic friction.

Types of Dynamic Friction

i) Sliding friction

The frictional force experienced by a body which slides on another body.

ii) Rolling friction

The frictional force experienced by a body that rolls over another body.

iii) Fluid friction

iv) Columb or Dry friction

Laws of Friction

1. Laws of static friction

- Friction force always acts in opposite direction to the direction in which body tends to move.
- The magnitude of frictional force is equal to the force applied to move the body.
- The ratio of limiting friction (F_s) to normal reaction (R) is always constant.

  i.e., $\frac{F_s}{R} = \mu_s$ (constant)
- Frictional force is independent of the area of contact.
- Frictional force depends on roughness of surface.

2. Laws of dynamic or kinetic friction

- Frictional force always acts in opposite direction to the direction of motion of body.
- The ratio of kinetic friction (F_d) to normal reaction (R) is always constant but slightly less than in case of limiting friction.

  $$\frac{F_d}{R} = \mu_d \text{ (Constant)}$$
- The force of kinetic friction is constant for moderate speed and decreases slightly with increase in speed.

The maximum value of frictional force which acts on the body when it just starts to slide over another body is termed as limiting friction.

Limiting friction

Normal reaction

The weight of a body acts vertically downward which is balanced by the normal force of reaction offered by the surface of contact. This reaction force is called normal reaction.

Static co-efficient of friction

The ratio of limiting friction to normal reaction

$$\mu_s = \frac{F_s}{R}$$

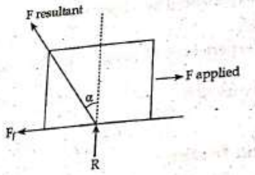
Dynamic or kinetic co-efficient of friction

The ratio of force of kinetic friction to normal reaction is called dynamic or kinetic coefficient of friction.

$$\mu d = \frac{F_d}{R}$$

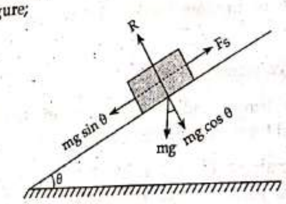
Angle of friction (α)

An angle made by resultant of normal reaction and frictional force with normal reaction.



Angle of repose (θ)

The minimum angle of inclination for which the body on inclined surface is in the verge of motion without application of any external force is known as angle of repose. In the given figure;



$$F_s = mg \sin\theta$$
$$R = mg \cos\theta$$
$$\tan\theta = \frac{F_S}{R} = \mu$$

We have,

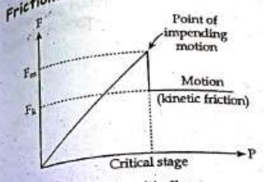
$$\tan\alpha = \frac{F_S}{R} = \mu$$

From (1) and (2); we get,

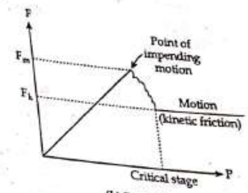
$$\theta = \alpha$$

Angle of repose = Angle of friction

Frictional force (F) Vs Applied force (P)



(a) Theoritically



(b) Practically

Impending motion

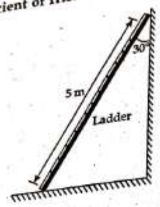
When the body is on the verge of motion (or sliding), this instant is known as point of Impending motion and the motion generated is Impending motion.

Tricks for solving numerical

1. When force is applied to the body perpendicular to surface of contact, no frictional force is developed.
2. If the applied force tend to move the body along the surface of contact but are not large enough to set in motion we cannot use, $F_s = \mu_s N$. We can only use equation of equilibrium.
3. If the force applied is such that body is just about to move (or slide), both equation of equilibrium and $F_s = \mu_s N$ can be used.
4. If the force applied is such that the body is in motion the equation of equilibrium can't be used. We use $F_k = \mu_k N$.

EXAM SOLUTION

1. **A uniform ladder of weight 250 N and length 5 m is placed against a vertical wall in a position where its inclination to the vertical is 30°. A man weighing 800 N climbs the ladder. At what position will be induce slipping? Take coefficient of friction $\mu = 0.2$ at both the contact surface of the ladder.** [2066 Chaitra]



Solution:

Given that;

$W = 250\text{ N}$

$W' = 800\text{ N}$

$\Sigma F_y = 0$

$F_B + R_A = W + W'$

or, $F_B + R_A = 800 + 250$

or, $F_B + R_A = 1050$

or, $0.2 \times R_B + R_A = 1050$

$\therefore$ $R_A + 0.2\,R_B = 1050$ (1)

$\Sigma F_X = 0$

$R_B = F_A$ (2)

or, $R_B = 0.2\,R_A$

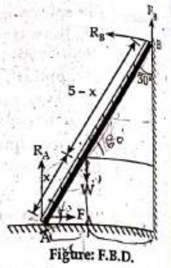


Figure: F.B.D.

Now, from equation (1) and (2); we get,

$R_A + 0.2 \times 0.2\,R_A = 1050$

or, $1.04\,R_A = 1050$

$\therefore$ $R_A = 1009.61\text{ N}$

Then, from equation (2); we get,

$R_A = \dfrac{R_B}{0.2}$

or, $R_B = 0.2 \times R_A$

or, $R_B = 0.2 \times 1009.61$

$\therefore$ $R_B = 201.922\text{ N}$

Now, taking moment about 'B'; we get,

$(^{+}\circlearrowleft)\ \Sigma M = 0$

$[W \times 2.5\cos 60°] + [W'(5-x)\cos 60°] - [R_A \times 5\cos 60°] + [F_A \times 5\sin 60°] = 0$

or, $[250 \times 2.5 \times \cos 60°] + [800(5-x)\cos 60°] + [0.2 \times 1009.61 \times 5 \times \sin 60°]$
$= [1009.61 \times 5 \times \cos 60°]$

or, $312.5 + 400(5-x) + 874.34 = 2524.025$

or, $400(5-x) = 1337.185$

or, $5 - x = 3.35$

$\therefore$ $x = 1.65\text{ m}$

Hence, at 1.65 m from base the man will induce slipping.

2. **Define limiting friction and impending motion. Justify why coefficient of static friction is greater than coefficient of kinetic friction.** [2062 Baishakh, 2064 Falgun, 2067 Ashadh]

Solution:

The maximum value of frictional force which acts on the body when it just starts to slide over another body is caused limiting friction.

When the force applied on a body becomes equal to maximum frictional force acting on it, the body just comes into motion. This stage of motion is caused impending motion.

As the body at rest comes into motion, the interlocking between the surfaces gets disturbed due to which the value of frictional force decrease.

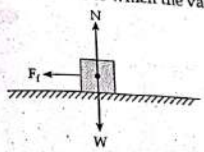


Figure: (i) Body at rest

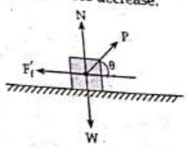


Figure: (ii) Body in motion

Case I

$N = w$

so, $\mu_s = \dfrac{F_f}{N} = \dfrac{F_f}{w}$ (1)

Case II

$F'_f = F_f - P\cos\theta$

so, $\mu_k = \dfrac{F'_f}{N} = \dfrac{F_f - P\cos\theta}{w}$ (2)

Now, from equation (1) and (2); we get,

$\mu_S > \mu_K$

Hence, due to this reason, coefficient of static friction is greater than coefficient of kinetic friction.

3. **Define limiting friction, angle of friction and coefficient of static and dynamic friction.** [2067 Mangshir, 2062 Poush, 2063 Baishakh]

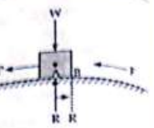
Ans: See the definition part on page no. 162

4. **What is the angle of friction? Explain about tipping and sliding of block.** [2067 Baishakh]

Solution:

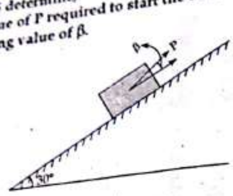
The angle made by the resultant of normal reaction and frictional force with normal reaction is called angle of friction.

As the magnitude 'F' of friction force increases from zero to 'F_{max}', the point of application 'A' of the normal reaction 'R' moves to right, so that the couple



formed by 'P' and 'F' and 'W' and 'R' respectively remain balanced. If 'R' reaches 'B' before 'F' reaches 'F_{max}' the block will tip about 'B' before it start sliding which is called tipping of block. When the applied force 'P' is such that the frictional force 'F' is quite greater than its maximum value 'F_{max}', the body starts to slide along the surface of contact which is called sliding of block.

5. **Knowing that the coefficient of friction between 25 kg block and the incline is μ_s = 0.25 determine;**
   i) **Smallest value of P required to start the block moving up the incline.**
   ii) **Corresponding value of β.** [2068 Bhadra]



Solution:

Here,

Mass of block (m) = 25 kg

Coefficient of static friction (μ_s) = 0.25

Given force is 'P' and angle is β

Resolving perpendicular to inclined surface;

$P\sin\beta + R - mg\cos 30° = 0$

or, $R = 25 \times 9.8 \times \cos 30° - P\sin\beta$

or, $R = 212.18 - P\sin\beta$ (1)

Resolving along the inclined surface;

$P\cos\beta - mg\sin 30° - F_f = 0$

or, $P\cos\beta - 25 \times 9.8 \times \sin 30° - 0.25 \times R = 0$

or, $P\cos\beta - 122.5 - 0.25\,(212.18 - P\sin\beta) = 0$ [From equation (1)]

or, $P\cos\beta - 122.5 - 53.045 + 0.25P\sin\beta = 0$

or, $P\cos\beta + 0.25P\sin\beta - 175.545 = 0$ (2)

For the value of P to be smallest;

$\frac{dP}{d\beta} = 0$

or, $-P\sin\beta + 0.25\,P\cos\beta = 0$

or, $\sin\beta = 0.25\cos\beta$

or, $\tan\beta = 0.25$

$\therefore\ \beta = 14.04°$

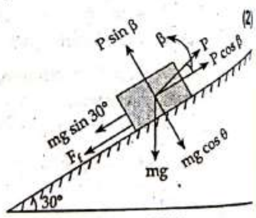


Figure: F.B.D.

Now, from equation (2); we have,

$P\cos 14.04° + 0.25P\sin 14.04° - 175.545 = 0$

$\therefore\ P = 170.3$ N

6. **The coefficient of friction between blocks A and B and the horizontal surfaces are μ_s = 0.24 and μ_k = 0.20. Knowing that m_A = 5 kg, m_B = 10kg and m_C = 10kg, determine;**
   i) **the tension in the chord and**
   ii) **acceleration of each block** [2068 Magh]

Solution:

Given that;

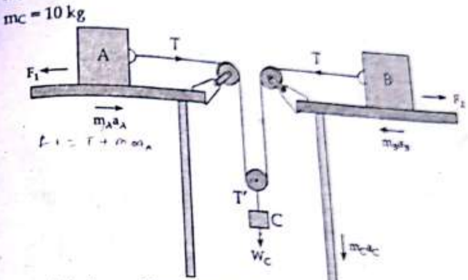
$\mu_s = 0.24$

$\mu_k = 0.20$

$m_A = 5$ kg

$m_B = 10$ kg

$m_C = 10$ kg



Let the accelerations of the blocks A, B and C be a_A, a_B and a_C respectively.

In this pulley mass system;

$2a_C = a_A + a_B$ (1)

Let, tension on the continuous string be 'T' and that of string connecting block 'C' be ' T' '. As it is the cause of impending motion, so use μ_s

Here,

$F_1 = m_A \times 9.8 \times \mu_S$

or, $F_1 = 5 \times 9.8 \times 0.24$

$\therefore\ F_1 = 11.76$ N

and, $w_C = m_C \times 9.8$

or, $w_C = 10 \times 9.8$

$\therefore\ w_C = 98$ N

Also,

$F_2 = m_B \times 9.8 \times \mu_S$

or, $F_2 = 10 \times 9.8 \times 0.24$

$\therefore\ F_2 = 23.52$ N

For block A

$m_Aa_A = T - F_1$

or, $T = 50\,a_A + 11.76$ (2)

For block B

$m_B a_B = T - F_2$

$\therefore \quad T = 10\,a_B + 23.52$ (3)

From equation (2) and (3); we get

$5a_A + 11.76 = 10a_B + 23.52$

or, $5a_A - 10a_B = 11.76$

$\therefore \quad a_A - 2a_B = 2.352$ (4)

For Block C

Since, $T' = 2T$

so, $m_C a_C = w_C - T'$

or, $m_C a_C = w - 2T$

or, $10\,a_C = 98 - 2\,(5a_A + 11.76)$

or, $10\left(\frac{a_A + a_B}{2}\right) = 98 - 10\,a_A - 23.52$

or, $5a_A + 5a_B + 10a_A = 74.48$

or, $15a_A + 5a_B = 74.48$

$\therefore \quad 3a_A + a_B = 14.896$ (5)

Now, multiplying equation (5) by 2 and adding them; we get,

$$\begin{array}{l} a_A - 2a_B = 2.352 \\ \underline{2 \times (3a_A + a_B) = 14.896} \\ 7a_A = 32.144 \end{array}$$ (6)

$\therefore \quad a_A = 4.592\ ms^{-2}$

Now, putting value of a_A in equation (5); we get,

$(3 \times 4.592) + a_B = 14.896$

$\therefore \quad a_B = 1.12\ ms^{-2}$

Also,

Since, $a_C = \frac{a_A + a_B}{2}$

or, $a_C = \frac{4.592 + 1.120}{2}$

$\therefore \quad a_C = 2.856\ ms^{-2}$

Now, from equation (2); we get,

$T = (5 \times 4.592) + 11.76$

$\therefore \quad T = 34.72\ N$

Also,

$T' = 2T = 69.44\ N$

Hence,

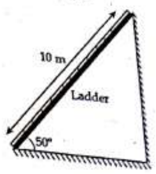
i) Tension in the cord is 34.72 N

ii) $a_A = 4.592\ ms^{-2}$

$a_B = 1.12\ ms^{-2}$

$a_C = 2.856\ ms^{-2}$

7. A 10 m ladder is leaning against a smooth vertical wall and the floor with the friction coefficient 0.4. Determine the normal reactions and the friction force at the top and bottom of the ladder.
Note: Assume weight of ladder be 250 N. [2068 Chaitra]



Solution:

Here,

$+\uparrow \Sigma F_y = 0$

$R_A + F_B - w = 0$

or, $R_A + F_B = w$

or, $R_A + \mu R_B = w$

$\therefore \quad R_A + 0.4\,R_B = 250.$ (1)

Also,

$+\rightarrow \Sigma F_x = 0$

$F_A - R_B = 0$

or, $R_B = F_A$

or, $R_B = \mu R_A$

$\therefore \quad R_B = 0.4\,R_A$ (2)

Now, from equation (1) and (2); we get,

$R_A + 0.4\,R_B = 250$

$R_A + 0.16\,R_A = 250$

or, $1.16\,R_A = 250$

$\therefore \quad R_A = 215.51\ N$, so, $F_A = 0.4 \times R_A$

$\therefore \quad F_A = 86.204\ N$

Putting R_A in equation (ii); we get,

$R_B = 0.4\,R_A$

or, $R_B = 0.4 \times 215.51,$

$\therefore \quad R_B = 86.204\ N$

so, $F_B = 0.4\,R_B$

$\therefore \quad F_B = 34.48\ N$

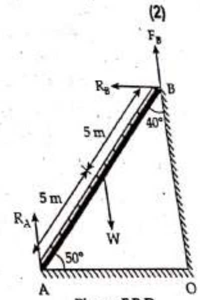


Figure: F.B.D.

Hence, the required normal reactions and frictions at A and B are 215. 51 N, 86.204 N and 86.20 N, 34.48 N, respectively.

8. Explain the laws of friction. Also give two examples of engineering usage of friction. [2069 Ashadh, 2059 Chaitra]

Solution:

The laws of frictions are as follows;

i) The force of friction acts in opposite direction of the motion of body.

ii) The magnitude of frictional force is equal to force applied to move the body.

iii) The ratio of limiting friction to normal reaction is always constant, called co-efficient of friction.

$$\frac{F}{R} = \text{constant} = \mu$$

iv) Frictional force is independent of area of contact.

v) Frictional force depends upon the roughness of the surface.

vi) The force of kinetic friction is constant for moderate speed and decreases slightly with increase in speed.

The two examples of the engineering usage of friction are as follows;

- Application of friction in nut-bolt system
- Application of friction in mechanism of vehicle engine

9. Justify why coefficient of static friction is greater than coefficient of kinetic friction. [2069 Bhadra]

Solution: See the solution of Q. no. 8 on page no. 169

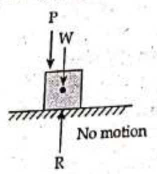
10. Illustrate impending motion state of friction and demonstrate the change in frictional force for different motion stages using relevant figure. [2069 Chaitra]

Solution:

When a force is applied on body placed on a surface is gradually increased, the frictional force increases correspondingly to maintain equilibrium. However, the frictional force can increase to its maximum limiting value $F_{s,max}$. At this stage, the body is on the verge of sliding and this is referred as impending motion state.

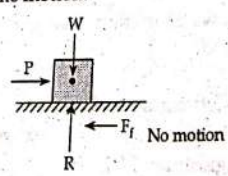
The different motion stage and their corresponding friction force are as follows;

i) The force applied to the body perpendicular to the surface of contact



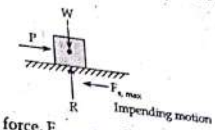
Here, frictional force, $F_f = 0$

ii) The force applied tends to move the body along the surface but are not large enough to set in the motion.



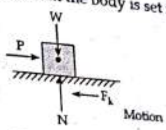
Here, frictional force, $F_f = P$

iii) The applied force is such that the body is about to slide. (Point of impending motion)



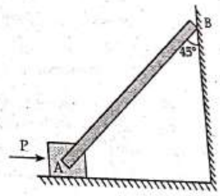
Here, frictional force, $F_{s,max} = \mu_s R$

iv) The applied force is such that the body is set into motion.



Here, frictional force, $F_k = \mu_k R$

11. A uniform bar AB, weighing 424 N, is fastened by a frictionless pin to a block weighing 200 N as shown in figure. At the vertical wall, μ = 0.268 while under the block, μ = 0.2. Determine the force 'P' needed to start motion to the right. [2070 Ashadh]



Solution:

Given that;

Weight of bar AB = 424 N

Weight of block = 200 N

$\mu_1 = 0.268$

$\mu_2 = 0.2$

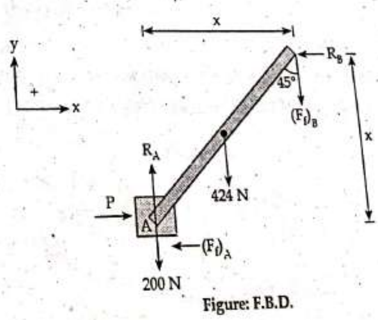


Figure: F.B.D.

Now,

$$+\uparrow \sum F_y = 0$$

or, $R_A - 200 - 424 - (F_f)_B = 0$ $[\because (F_f)_B = \mu_1 \times R_B]$

or, $R_A - 624 - \mu_1 \times R_B = 0$

or, $R_A - 0.268R_B = 624$ (1)

and, $\overset{+}{\rightarrow} \sum F_x = 0$ $[\because (F_f)_A = \mu_2 \times R_A]$

or, $P - (F_f)_A - R_B = 0$

or, $P - \mu_2 \times R_A - R_B = 0$

or, $P = 0.2R_A + R_B$ (2)

Taking moment about point 'A' and applying condition of equilibrium;

$$+\circlearrowleft \sum M_A = 0$$

or, $424 \times \frac{x}{2} + (F_f)_B \times x - R_B \times x = 0$

or, $212x + 0.268R_B - xR_B = 0$

or, $212 + 0.268R_B - R_B = 0$

$\therefore$ $R_B = 289.62\ N$

From equation (1); we have,

$R_A - 0.268 \times 289.62 = 624$

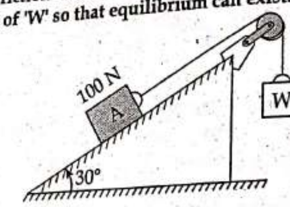
$\therefore$ $R_A = 701.62\ N$

From equation (2); we have,

$P = 0.2 \times 701.62 + 289.62$

$\therefore$ $P = 429.94\ N$

12. **A block 'A' of weight 100 N rests on an inclined plane and another weight W is attached to the first weight through a string as shown in figure. If the co-efficient of friction between the block and plane is 0.3, determine the maximum value of 'W' so that equilibrium can exist.** **[2070 Bhadra]**



Solution:

Given that;

Weight of block A, $W_A = 100\ N$

Co-efficient of friction, $\mu_S = 0.3$

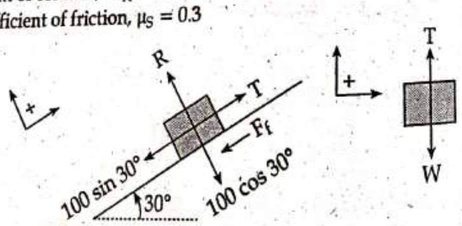


Figure: F.B.D.

Now, for block A;

Applying condition of equilibrium perpendicular to the inclined plane;

$R - 100\cos 30° = 0$

$\therefore$ $R = 86.6\ N$

Now, applying condition of equilibrium parallel to the inclined plane;

$T - 100\sin 30° - F_f = 0$

We have,

$F_f = \mu_S \times R = 0.3 \times 86.6 = 25.98\ N$

or, $T - 50 - 25.98 = 0$

$\therefore$ $T = 75.98\ N$

For weight W;

$$+\uparrow \sum F_y = 0$$

or, $T - W = 0$

$\therefore$ $W = T = 75.98\ N$

13. **Define angle of friction and also write laws of static friction. [2070 Chaitra]**

Solution: See the definition part on page no. 161

14. **State laws of dry friction. How can we assume the condition of overturning and sliding of a block? Explain with suitable example.** **[2071 Shrawan]**

Solution:

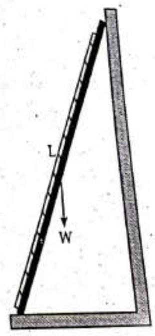
Laws of dry friction

See the definition part on page no. 161

For the second part

See the solution of Q. no. 4 on page no. 165

15. **Determine the minimum angle θ (made by the ladder AB of length, L with the floor) at which a uniform ladder can be placed against a wall without slipping under its own weight, W. The coefficient of friction for all surfaces is 0.2.** **[2071 Bhadra]**



Solution:

Here,

$$\sum F_y = 0$$

or, $F_B + R_A = W$

or, $0.2R_B + R_A = W$ (1)

∴ $R_A + 0.2R_B = W$

$\Sigma F_x = 0$

or, $F_A = R_B$

∴ $R_B = 0.2R_A$ (2)

From equation (1) and (2); we have,

$R_A + (0.2 \times 0.2R_A) = W$

or, $1.04R_A = W$

∴ $R_A = 0.962W$

From equation (2); we have,

$R_B = 0.2R_A = 0.2 \times 0.962W = 0.192W$

Now, taking moment about B; we have,

$(^{+}\circlearrowleft) \Sigma M_B = 0$

$W \times \frac{L}{2}\cos\theta - R_A \times L\cos\theta + F_A \times L\sin\theta = 0$

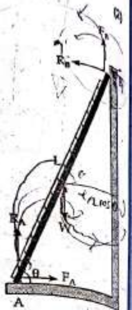
or, $0.5WL\cos\theta - 0.962WL\cos\theta + 0.192WL\sin\theta = 0$

or, $0.5\cos\theta - 0.962\cos\theta + 0.192\sin\theta = 0$

or, $0.192\sin\theta = 0.462\cos\theta$

or, $\frac{\sin\theta}{\cos\theta} = \frac{0.462}{0.192} = 2.406$

∴ $\theta = 67.43°$



Hence, the minimum angle at which the ladder can be placed against the wall without slipping under its own weight is 67.43°.

16. Explain the laws of static friction. Also define the limiting friction and angle of friction with suitable example. [2071 Magh]

Solution: *See the definition part on page no. 161*

17. What are the advantages and disadvantages of friction? Also explain the working principles of high tension friction grip bolts. [2071 Chaitra]

Solution:

The following are the advantages of the friction:

i) Friction prevents us from slipping when walking or running.
ii) Friction helps us to write on the blackboard or on a paper.
iii) Brakes make use of friction to stop the vehicles.
iv) Friction helps to transmit power from the motors and engine to other machines by making use of belts and clutches.

The following are the disadvantages of friction:

i) Friction causes unnecessary wear and tear of the machinery.
ii) A part of useful energy is dissipated in overcoming the friction.
iii) Due to the friction between the moving parts of a machine heat is produced, which effects the working as well as the life of machine.
iv) It opposes the motion.

The strength of the joint fabricated by means of high tension friction grip bolts is obtained by bearing or friction (grip) developed as a result of very high initial tension in the bolts produced by tightening the nuts to the specified bolt tension.

18. Define friction force and explain the condition of tipping and sliding of a block. [2072 Kartik]

Solution:

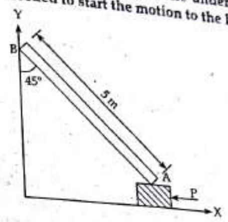
Friction force

See the definition part on page no. 161

Condition of tipping and sliding of a block

See the solution of Q. no. 4 on page no. 165

19. A uniform bar AB having length 5 m and weighing 500 N is fastened by a frictionless pin to O block, weighing 200 N as shown in the figure. At the vertical wall, coefficient of friction is 0.30 while under the block is 0.20. Determine the force P needed to start the motion to the left. [2072 Magh]



Solution:

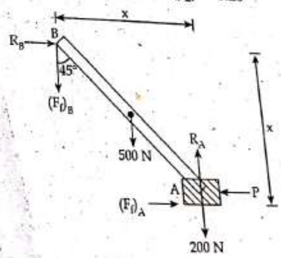
Given that;

Weight of bar AB = 500 N

Weight of block = 200 N

Coefficient of friction at vertical wall $(\mu_1) = 0.30$

Coefficient of friction under the block $(\mu_2) = 0.20$



Now,

$\sum F_y = 0$

or, $R_A - 200 - 500 - (F_f)_B = 0$

or, $R_A - 700 - (\mu_1 \times R_B) = 0$

or, $R_A - 0.3R_B = 700$ (1)

$\Sigma F_x = 0$

or, $(F_f)_A - P + R_B = 0$

or, $0.2R_A + R_B = P$ (2)

Taking moment about point A and applying condition of equilibrium; we have,

$(^{+}\circlearrowleft)\ \Sigma M_A = 0$

or, $-500 \times \frac{x}{2} - (F_f)_B \times x + R_B \times x = 0$

or, $-225 - 0.3R_B + R_B = 0$

or, $0.7R_B = 225$

$\therefore$ $R_B = 321.43$ N

From equation (1); we have,

$R_A - 0.3R_B = 700$

or, $R_A - (0.3 \times 321.43) = 700$

$\therefore$ $R_A = 796.43$ N

Again from equation (2); we have,

$0.2R_A + R_B = P$

or, $(0.2 \times 796.43) + 321.43 = P$

$\therefore$ $P = 480.72$ N

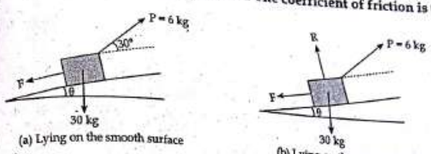
20. **Determine the condition illustrating no friction, no motion, impending motion and motion with proper sketch. How can we assure condition of sliding and overturning of a block? Explain with proper sketch.[2072 Chaitra]**

Solution: See the solution of Q. no. 10 and 4 on page no. 170 and 65 respectively

ADDITIONAL PROBLEMS

1. **Find the force required to move a load of 30 kg up a rough plane, the force being applied parallel to the plane. The inclination of the plane is such that a force of 6 kg inclined at 30° to a similar smooth inclined plane, would keep the same load in equilibrium. The coefficient of friction is 0.3.**

Solution:



(a) Lying on the smooth surface

(b) Lying on the rough surface

Figure: F.B.D.

a) Force, $P = 6$ kg

Inclination of the force with the plane;

$\alpha = 30°$

Force of friction, $F = 0$ [for smooth plane surface]

Let, θ = Angle of inclination

Resolving the forces along the plane; we get,

$6 \cos 30° = 30 \sin \theta$

or, $\sin \theta = \frac{6 \cos 30°}{30}$

$\therefore$ $\theta = 9.97°$

b) Inclination of plane $\theta = 9.97°$

Let, P = Force required to move

R = Normal reaction

F = Force of friction

Resolving the forces along the inclined plane; we have,

$P = F + 30 \sin \theta$

or, $P = 0.3R + 30 \sin (9.97°)$ (1)

Resolving the forces at right angle to the plane;

$R = 30 \cos (9.97°)$ kg

Putting the value of 'R' in equation (1); we get,

$P = 0.3 \times 30$ ws $(9.97°) + 30 \sin (9.97°)$ kg

$\therefore$ $P = 14.06$ kg

2. **A ski is dropped by a skier ascending a ski slope. The ski begins to slide from the down the slope which is inclined at 15° with horizontal. Upon that the ski has a mass 'm' kg and has a acceleration of 1.2 ms^{-2} what is the coefficient of sliding friction between ski and the slope?**

Solution:

Since the acceleration down the plane is 1.2ms^{-2} so, force down the plane is

$F = ma = 1.2$ m (1)

Now,

$mg \sin 15° - F_f = 1.2\ m$ (2)

or, $mg \sin 15° - \mu N = 1.2\ m$

and, $N = mg \cos 15°$

From equation (1); we get,

$mg \sin 15° - \mu\, mg \cos 15° = 1.2\ m$

or, $\mu = \dfrac{g \sin 15° - 1.2}{g \cos 15°}$

$\therefore \quad \mu = 0.141$

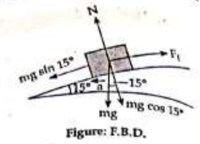
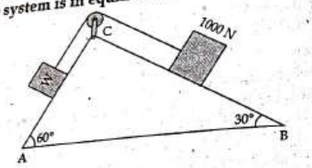


Figure: F.B.D.

3. **Two inclined planes AC and BC inclined at 60° and 30° to the horizontal meet at 'A'. Load of 1000 N rests on the inclined plane BC and is tied to a rope which passes over a smooth pulley at a ridge. The other end of the rope being connected to a block weighing 'W' and resting on plane AC. If the coefficient of friction between the load and plane BC be 0.28 and that between the plane AC be 0.2. Determine least and greatest value of 'W' for which the whole system is in equilibrium.**



Solution:

a) When the value of 'W' is least, the load of 1000 N moves down and 'W' moves up.

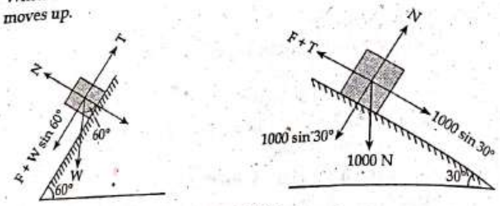


Figure: F.B.D.

For the plane BC, let 'T' be the tension in the string.

$+\uparrow \Sigma F_y = 0$

or, $N - 1000 \cos 30° = 0$

or, $N = 1000 \cos 30° = 866.025\ N$

$+\rightarrow \Sigma F_X = 0$

or, $-F - T + 1000 \sin 30° = 0$

or, $\mu N + T = 1000 \sin 30°$

or, $0.28 \times 866.025 + T = 1000 \sin 30°$

$\therefore \quad T = 257.5\ N$

For plane AC;

$+\uparrow \Sigma F_Y = 0$

or, $N - W \cos 60° = 0$

or, $N = W \cos 60°$

$+\rightarrow \Sigma F_X = 0$

or, $T - F - W \sin 60° = 0$ (1)

or, $T - \mu N - W \sin 60° = 0$

or, $T - \mu W \cos 60° - W \sin 60° = 0$

or, $W = \dfrac{T}{\mu \cos 60° + \sin 60°}$ [From equation (1)]

or, $W = \dfrac{257.5}{0.2 \cos 60° + \sin 60°}$

$\therefore \quad W = 266.5\ N$

b) When the value of 'W' is greatest, at this condition, the weight 'W' just move down and 1000 N load moves up for plane BC.

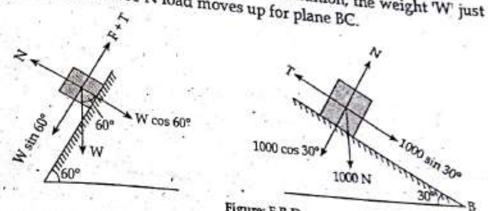


Figure: F.B.D.

$+\rightarrow \Sigma F_X = 0$

or, $-T + 1000 \sin 30° + F = 0$

$\therefore \quad T = 1000 \sin 30° + \mu N$ (1)

$+\uparrow \Sigma F_y = 0$

or, $N - 1000 \cos 30° = 0$

or, $N = 1000 \cos 30° = 866.025\ N$

Now, from equation (1); we get,

$T = 1000 \sin 30° + 0.28 \times 866.025$

$\therefore \quad T = 742.5N$

For plane AC

$+\uparrow \Sigma F_y = 0$

or, $N - W \cos 60° = 0$

or, $N = W \cos 60°$ (2)

$+\rightarrow \Sigma F_X = 0$

or, $F + T - W \sin 60° = 0$

or, $\mu N + 742.5 - W \sin 60° = 0$

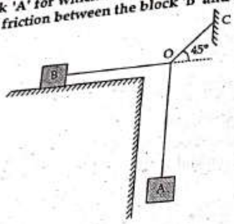
or, $\mu W \cos 60° + 742.5 - W \sin 60° = 0$

or, $W = \dfrac{742.5}{\sin 60° - \mu \cos 60°}$

or, $W = \dfrac{742.5}{\sin 60° - 0.2 \cos 60°}$

∴ $W = 969.3N$

4. **Block 'B' in the figure below weighs 100 N. Determine the maximum weight of the block 'A' for which the system will be in equilibrium. The coefficient of static friction between the block 'B' and the table is 0.02.**



Solution:

The free body diagram of the above figure is given below.

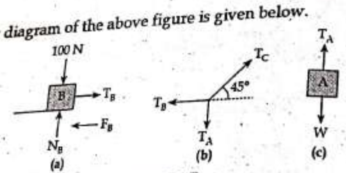


Figure: F.B.D.

Equilibrium of block B

$\Sigma F_y = 0$

or, $N_B - 100 = 0$

∴ $N_B = 100\ N$

$\Sigma F_X = 0$

$T_B - F_B = 0$

Also,

$T_B = F_B = \mu_B N_B$

or, $T_B = 0.2 \times 100$

∴ $T_B = 20\ N$ (1)

Equilibrium of point O

$\Sigma F_x = 0$

$T_C \cos 45° - T_B = 0$

$T_C \cos 45° = T_B$ (2)

Also,

$\Sigma F_y = 0$

$T_C \sin 45° - T_A = 0$

or, $T_C \sin 45° = T_A$

Dividing equations (2) and (3); we get,

$\tan 45° = \dfrac{T_A}{T_B}$ (3)

or, $T_A = T_B$ [Since tan 45° = 1]

∴ $T_A = 20\ N$ [from equation (1)]

Equilibrium of block A

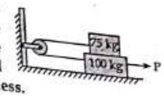
$\Sigma F_y = 0$

or, $T_A - W = 0$

or, $w = T_A$

∴ $T_A = 20\ N$

5. **In figure shown determine the horizontal force 'P' applied to the lower block to just pull it to the right. The coefficient of friction between the blocks is 0.2 and that between the lower block and the plane is 2.25. Assume the pulley to be friction less.**



Solution:

For upper block

$\Sigma F_y = 0$

$N_2 - (75 \times g) = 0$

∴ $N_2 = (75 \times g) = 735.75\ N$

$\Sigma F_X = 0$

$F_2 - T = 0$

or, $T = F_2 = \mu_2 N_2$

∴ $T = (0.2) \times (735.75) = 147.15\ N$

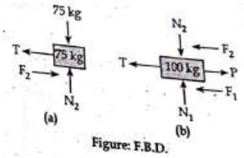


Figure: F.B.D.

Note

At the point of impending motion $F_2 = \mu_2 N_2$

Lower block

At point of impending motion, applying the conditions of equilibrium along X and Y directions;

$\Sigma F_y = 0$

$N_1 - N_2 - 100\ g = 0$

or, $N_1 = N_2 + (100 \times g)$

or, $N_1 = (735.75) + (100 \times 9.81)$

∴ $N_1 = 1716.75\ N$

$\Sigma F_x = 0$

$P - F_1 - F_2 - T = 0$

or, $P = F_1 + F_2 + T$

or, $P = (\mu_1, N_1) + (\mu_2 N_2) + T$

or, $P = (0.25 \times 1716.75) + (0.2) \times 735.75) + (147.15)$

∴ $P = 723.5\ N$

6. **A 6 m long ladder AB leans against a wall as shown in figure. If the coefficient of friction between the ladder and the wall and that between the ladder and the floor are same and equal to 0.3, determine the**

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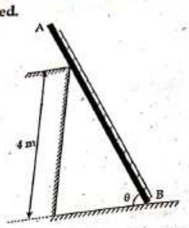
minimum inclination of the ladder with respect to the floor for which equilibrium is maintained.



Solution:

The free body diagram of above figure is given below.

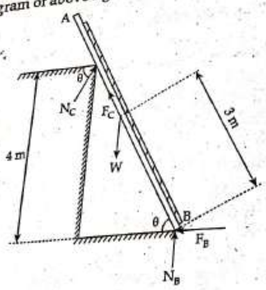


Figure: F.B.D.

In the free body diagram symbol denote their respective notations.

Now,

$$\Sigma F_X = 0$$

or, $N_C \sin\theta - F_C \cos\theta - F_B = 0$

or, $N_C \sin\theta - \mu N_C \cos\theta - \mu N_B = 0$

or, $N_C[\sin\theta - \mu\cos\theta] - \mu N_B = 0$ (1)

Also,

$$\Sigma F_y = 0$$

or, $N_C \cos\theta + F_C \sin\theta + N_B - W = 0$

or, $N_C \cos\theta + \mu N_C \sin\theta + N_B - W = 0$

or, $N_C[\cos\theta + \mu\sin\theta] + N_B = W$ (2)

Again,

$$\Sigma M_B = 0$$

or, $-N_C \dfrac{4}{\sin\theta} + W\,3\cos\theta = 0$

or, $N_C = \dfrac{3}{4} W \sin\theta\cos\theta$ (3)

Multiplying equation (2) by μ and adding it to equation (1); we get,

$N_C(\sin\theta - \mu\cos\theta) + \mu N_C(\cos\theta + \mu\sin\theta) = \mu W$

or, $N_C = (\sin\theta - \mu\cos\theta + \mu\cos\theta + \mu^2\sin\theta) = \mu W$

or, $N_C \sin\theta(1 + \mu^2) = \mu w$

Now, putting the value of N_C from equation (3); we get,

$$\frac{3}{4} W \sin^2\theta\cos\theta[1 + \mu^2] = \mu W$$

or, $\sin^2\theta\cos\theta = \dfrac{4}{3}\dfrac{\mu}{1+\mu^2}$

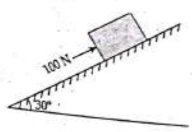
or, $\sin^2\theta \cdot \cos\theta = \dfrac{4}{3}\dfrac{0.3}{[1 + (0.3)^2]}$

or, $\sin^2\theta\cos\theta = 0.367$

On solving; we get,

$$\theta = 61.8°$$

7. **A block of 50 kg mass is pushed upon inclined by a force 100 N acting parallel to the inline as shown in figure. Determine whether the block is in equilibrium or not. Also find the magnitude and direction of the frictional force. $\mu_s = 0.25$ and $\mu_k = 0.2$.**



Solution:

Given that;

Mass of block = 50 kg

Force applied = 100 N

$\mu_s = 0.25$

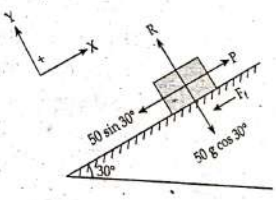
$\mu_k = 0.2$

Suppose, the block is at the point of moving upwards;

$$+\uparrow \sum F_y = 0$$

or, $R - 50g\cos 30° = 0$

∴ $R = 50 \times 9.81 \times \cos 30° = 424.79$ N



The maximum static friction is;

$F_{s,max} = \mu_s R$

$= 0.25 \times 424.79$

$= 106.2$N

$$+\!\!\rightarrow \sum F_x = 0$$

or, $100 - 50g\sin 30° - F_f = 0$

∴ $F_f = -145.25$ N

Since, the above value is negative, the block slides downward.

Since, the $F_f > F_{s,max}$, maximum, the block is in motion and actual frictional force is;

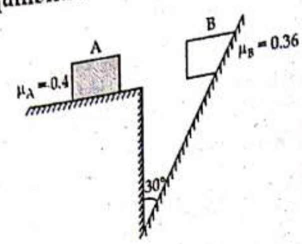
$F_f = F_k$

$= \mu_K R$

$= 0.2 \times 424.79$

$= 84.96$ N

8. Two blocks connected by a horizontal link AB are supported on two rough planes as shown in figure. What is the smallest weight 'W' of the block 'A' for which equilibrium of the system can exist, if weight of block B is 5 kN?



Solution:

Given that;

Weight of block B, $W_B = 5$ KN

$\mu_A = 0.4$

$\mu_B = 0.36$

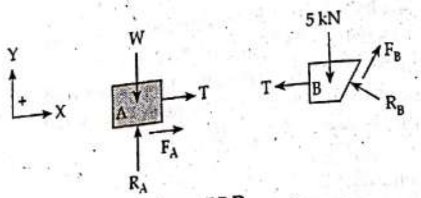


Figure: F.B.D.

Block B

$\overset{+}{\rightarrow} \sum F_x = 0$

or, $-R_B \cos 30° - T + F_B \sin 30° = 0$

or, $-R_B \cos 30° - T + 0.36 \sin 30° = 0$

or, $T = -0.686\, R_B$ (1)

$+\uparrow \sum F_y = 0$

or, $-5 + F_B \cos 30° + R_B \sin 30° = 0$

or, $-5 + 0.36\, R_B \cos 30° + R_B \sin 30° = 0$

$0.81\, R_B = 5$

$\therefore$ $R_B = 6.17$ KN

From equation (1)

$T = -0.686 \times 6.17 = -4.23 \text{ KN} = 4.23 \text{ KN}(\rightarrow)$

Block A

$\overset{+}{\rightarrow} \sum F_x = 0$

or, $T + F_A = 0$

or, $-4.23 + F_A = 0$

$\therefore$ $F_A = 4.23$

$+\uparrow \sum F_y = 0$

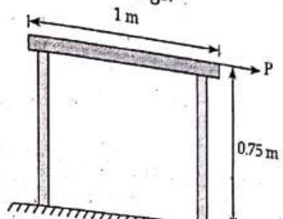
or, $W - R_A = 0$

or, $W - \dfrac{R_A}{\mu_A} = 0$

or, $W - \dfrac{4.23}{0.4} = 0$

$\therefore$ $W = 10.57$ KN

9. A table of 1 m length, 0.75 m height and 25 kg weight is to be pulled on a rough surface ($\mu_S = 0.3$) as shown in figure. Determine; (i) the horizontal force required to just pull it to the right, (ii) the normal reactions at the front and rear legs. Suppose the table is pushed with a force of 200 N, determine the maximum height at which the force can be applied without causing the table to tip. Assume the centre of gravity of the table to be midway between the front and rear legs.



Solution:

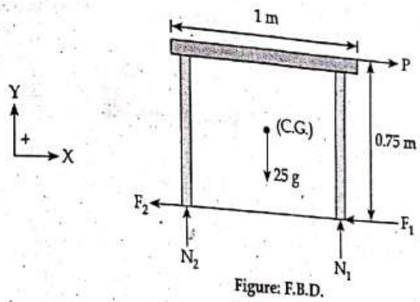


Figure: F.B.D.

i) $+\uparrow \sum F_y = 0$

or, $N_1 + N_2 - 25 \times 9.81 = 0$

or, $N_1 + N_2 = 245.25$ N

and, $\overset{+}{\rightarrow} \sum F_x = 0$

or, $P - F_1 - F_2 = 0$

or, $P - \mu_S N_1 - \mu_S N_2 = 0$

or, $P = \mu_S(N_1 + N_2) = 0.3(245.25)$

or, $P = 73.52$ N

ii) Taking moment about the front leg and applying equilibrium condition;

$+\circlearrowleft \sum M_{\text{front leg}} = 0$

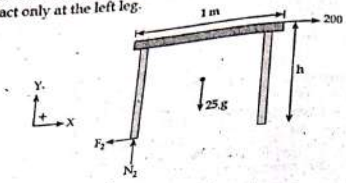
or, $-P \times 0.75 - N_2 \times 1 + 25 \times 9.81 \times 0.5 = 0$

or, $-73.52 \times 0.75 - N_2 + 122.625 = 0$

$\therefore$ $N_2 = 67.44$ N

$N_1 = 245.25 - 67.44 = 177.81$ N

iii) When the table is pushed at the point of tipping, the table will have contact only at the left leg.



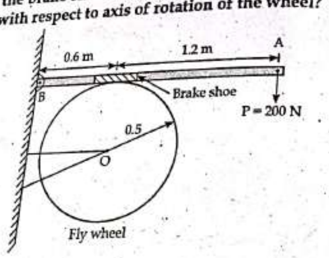
Taking moment about the left leg;

$$\sum M_{left\ leg} = 0$$

or, $200 \times h - 25 \times 9.81 \times 0.5 = 0$

$\therefore$ $h = 0.613$ m

10. **A flywheel of radius 0.5 m supported on a bracket and rotating about the axis 'O' is braked by means f a lever as shown in the figure. The force applied at A is 200 N. If the co-efficient of kinetic friction between the flywheel and the brake shoe is 0.35, what braking moment will be applied on the wheel with respect to axis of rotation of the wheel?**



Solution:

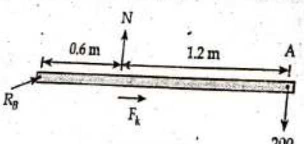
Given that;

Radius of flywheel, $r = 0.5$ m

Applied force, $P = 200$ N

$\mu_K = 0.35$

Now,



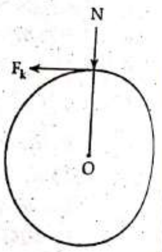


Figure: F.B.D.

Lever AB

Taking moment about B; we have,

$-N \times 0.6 + 200 \times 1.8 = 0$

$\therefore$ $N = 600$ N

Frictional force $(F_K) = \mu_K \times N = 0.35 \times 600 = 210$ N

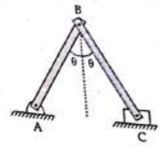
Flywheel

Taking moment about O; we have,

$M_O = F_K \times 0.5 = 210 \times 0.5 = 105$ N (anticlockwise)

$\therefore$ Braking moment of 105N is required to stop the wheel.

11. **Determine the vertical force 'P' to be applied at the pin joint 'B' of a link mechanism shown in figure to cause motion to impend in 20 kg block at 'C'. The co-efficient of friction between the block and the horizontal plane is 0.2, θ = 30°. If the 'P' is applied horizontally, determine its value to cause motion to impend in block 'C'.**



Solution:

When 'P' is applied vertically;

Due to symmetry the forces in each member AB and BC will be equal and let that force be 'S'.

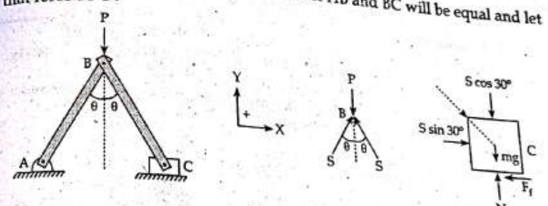


Figure: F.B.D.

Taking F.B.D. of block C;

$$+\uparrow \sum F_y = 0$$

$$N - mg - S\cos 30^\circ = 0$$

$\therefore$ $N = mg + S\cos 30^\circ$ (1)

$$\xrightarrow{+} \sum F_y = 0$$

$$S\sin 30^\circ - F_f = 0$$

or, $S\sin 30^\circ - \mu N = 0$

or, $S\sin 30^\circ - \mu(mg + S\cos 30^\circ) = 0$

or, $S = \dfrac{\mu mg}{\sin 30^\circ - \mu\cos 30^\circ}$

$$= \frac{0.2 \times 20 \times 9.81}{\sin 30^\circ - 0.2\cos 30^\circ} = 120.08 \text{ N}$$

Point B.

$$+\uparrow \sum F_y = 0$$

or, $P = 2S\cos 30° = 2 \times 120.08\cos 30°\ P = 208\ N$

When P is applied horizontally;

Here, the forces in members AB and BC are not equal. Let the force in member AB be S_1 and that in BC be S_2.

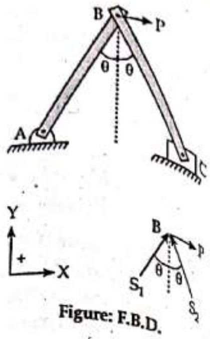


Figure: F.B.D.

From above case;

$S_2 = 120.08\ N$

Point B

$+\uparrow \sum F_y = 0$

or, $S_1\cos 30° + S_2\cos 30° = 0$

or, $S_1 = -S_2$

$\therefore$ $S_1 = -120.08\ N$

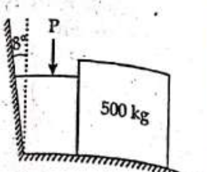
$+\rightarrow \sum F_x = 0$

or, $P + S_1\sin 30° - S_2\sin 30° = 0$

or, $P - 120.08\sin 30° - 120.08\sin 30° = 0$

$\therefore$ $P = 120.08\ N$

12. A heavy block of mass 500 kg is to be adjusted horizontally using an 8 wedge by applying a vertical force 'P'. If the co-efficient of static friction for both the contact surface of the wedge is 0.25 and that between the block and the horizontal surface is 0.5, determine the least force 'P' required to move the block.



Solution:

Let the angle of friction for the wedge be ϕ_1.

For wedge surface;

$\mu = 0.25$

We have,

$\tan\phi_1 = \mu$

or, $\tan\phi_1 = 0.25$

$\therefore$ $\phi_1 = 14.04°$

Let the angle of friction for the horizontal surface be ϕ_2.

For horizontal surface;

$\mu = 0.5$

$\tan\phi_2 = 0.5$

$\therefore$ $\phi_2 = 26.57°$

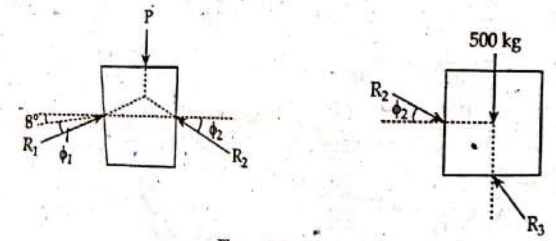


Figure: F.B.D.

For the block, the forces, R_2, R_3 and 500 g are concurrent.

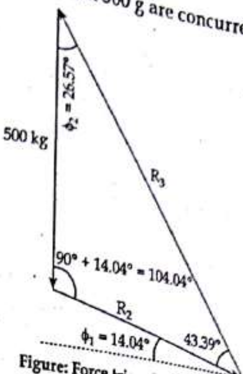


Figure: Force triangle for block

From sine law; we have,

$$\frac{500 \times 9.81}{\sin(43.39°)} = \frac{R_2}{\sin(26.57°)}$$

$\therefore$ $R_2 = 2890\ N$

Similarly; the forces, R_1, R_2 and 'P' are concurrent.

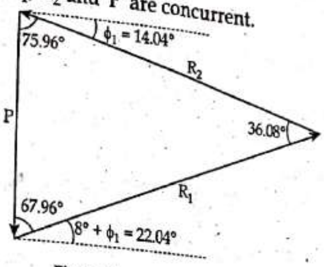


Figure: Force triangle for wedge

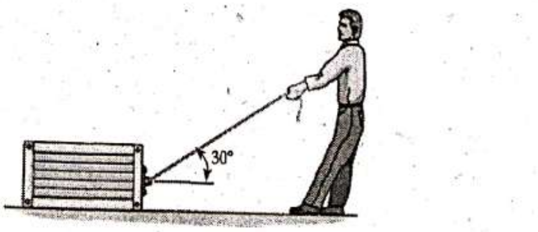
Again, from sine law,

$$\frac{P}{\sin(36.08°)} = \frac{R_2}{\sin(67.96°)}$$

or, $$\frac{P}{\sin(36.08°)} = \frac{2890}{\sin(67.96°)}$$

$\therefore$ $P = 1836.14\ N$

13. **The coefficient of static friction between the 150 kg crate and the ground is $\mu_s = 0.3$, while the coefficient of static friction between the 80 kg man's shoes and the ground is $\mu_s' = 0.4$. Determine if the man can move the crate.**



Solution:

Equation of equilibrium

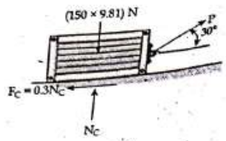


Figure: (a)

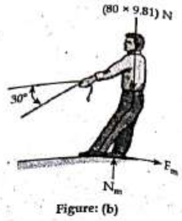


Figure: (b)

Referring to figure (a); we have,

$$+\uparrow \sum F_y = 0$$

or, $N_C + P\sin 30° - 150 \times 9.81 = 0$ (1)

and, $\overset{+}{\rightarrow} \sum F_x = 0$

or, $P\cos 30° - 0.3N_C = 0$ (2)

Solving above equations; we have,

$P = 434.49\ N$

and, $N_C = 1254.26\ N$

Using the result of P and referring to figure (a); we have,

$$+\uparrow \sum F_y = 0$$

or, $N_m - 434.49\sin 30° - 80 \times 9.81 = 0$

$\therefore$ $N_m = 1002.04\ N$

and, $\overset{+}{\rightarrow} \sum F_x = 0$

or, $F_m - 434.49\cos 30° = 0$

$\therefore$ $F_m = 376.28\ N$

Since, $F_m < F_{max} = \mu_s' N_m = 0.4 \times 1002.04 = 400.82$ N, the man does not slip. Thus, man can move the crate easily.

CHAPTER 6 Analysis of Beam and Frame

DEFINITIONS

Continuum

The body is assumed to be composed of continuous distribution of matter.

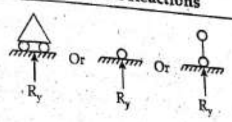
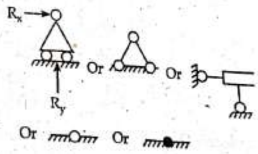
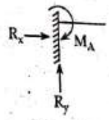
Discrete

The body is assumed to be composed of finite elements.

Types of loads

1. According to nature
   - Static/dead load
   - Live/imposed load
   - Dynamic/earthquake load
   - Temperature/settlement effects
2. According to intensity/acting mechanism
   - Point load/concentrated load
   - Distributed loads
3. According to load resisting mechanism
   - Axial Load (Tensile or Compressive)
   - Bending load
   - Shear/Traverse load
   - Twisting loads

Types of supports

Types of supports	Symbol and Reactions	No. of unknowns
Roller and rocker DOF = 2	 Or Or (R_y)	1
Hinge or pin DOF = 1	 Or Or Or Or (R_x, R_y)	2
Fixed support DOF = 0	 (R_x, M_A, R_y)	3
Cable DOF = 2 	 T Tension acting along the direction of the cable	1

Beam

Beam is a structural element (horizontal) capable of withstanding load by resisting the bending moment.

Frame

Frame is a network of straight members (*i.e.*, beam and column which are connected or joined together to resist the load and moment.

Types

Rigid frame

Angles made by the members of frame remain constant (*i.e.*, 90°) before and after the application of load and moment.

Non-rigid frame

Angles do not remain constant.

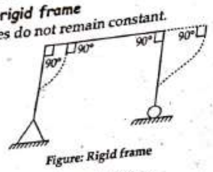


Figure: Rigid frame

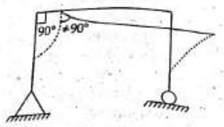


Figure: Non-rigid frame

Equation of static equilibrium

$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_z = 0$

Equation of condition

Equations obtained due to condition (*i.e.*, due to construction). Hinge provides one whereas roller and link provides two equations of condition.

Determinacy and indeterminacy of structure

The structure is statically determinant if all the external as well as internal unknown (forces, reactions and moments) developed in the member can be completely determined by using three equations of static equilibrium and conditional equations, otherwise it is statically Indeterminate.

Frame

Degree of static indeterminacy

$D_S = 3m + r - (3j + c)$

Degree of external static indeterminacy

$D_{SE} = r - 3 - c$

Degree of internal static indeterminacy

$D_{SI} = D_S - D_{SE}$

Truss

Degree of static indeterminacy,

$D_S = m + r - 2j$

Degree of external static indeterminacy

$D_{SE} = r - 3$

Degree of internal static indeterminacy,

$D_{SI} = D_S - D_{SE}$

where, r is the number of reaction.

j is the number of joints

m is the number of members.

C is the number of conditional equation.

Kinetic indeterminacy

If the displacement components of its joints cannot be determined by the compatibility equations, the structure is kinetically indeterminate.

Degree of kinetic indeterminacy,

$D_K = 3j - e$

where 'e' is the number of compatibility conditions known.

Axial force (A.F.)

It is the algebraic sum of all the forces acting along the longitudinal axis of the member on either side of the considered section.

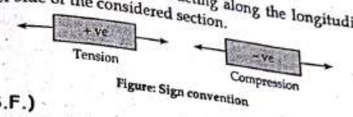


Figure: Sign convention

Shear force (S.F.)

It is the algebraic sum of all the forces on the left hand side or to the right hand side (perpendicular to the axis of member) of the considered section.

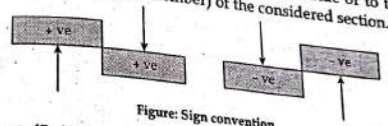


Figure: Sign convention

Bending moment (B.M.)

It is the algebraic sum of moment of all forces on either side of considered section.

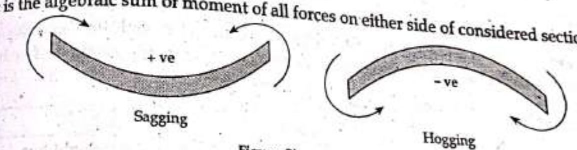
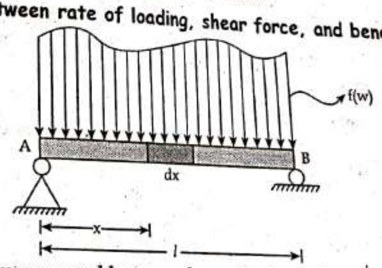


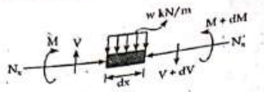
Figure: Sign convention

Relationship between rate of loading, shear force, and bending moment



Consider a simply supposed beam as shown in above figure. Taking a strip 'dx' 'x' distance from 'A'. 'V' be shear force and M be bending moment at left end and V + dV and M + dM at right end.

Since, beam is in equilibrium, so 'dx' is also in equilibrium.



$^{+}\uparrow \Sigma F_y = 0$

or, $V - w\,dx - (V + dV) = 0$

or, $\frac{dV}{dx} = -w$

The rate of change of shear force with respect to x is equal to intensity of loading.

$\therefore \quad V = -\int w\,dx + c$

Area under the load curve with negative sign gives change in shear force.

Also,

$^{+}\circlearrowleft \Sigma M_0 = 0$

or, $M + V\,dx - w\,dx\,\frac{dx}{2} - (M - dM) = 0$

or, $M + V\,dx - M - dM = 0 \left[\text{Neglecting } w\frac{(dx)^2}{2}\right]$

$\therefore \quad V = \frac{dM}{dx}$

The rate of change of bending moment with respect to x is equal to shear force.

$\therefore \quad M = \int V\,dx + c_1$

Area under shear curve gives change in bending moment.

Axial force diagram (A.F.D.) shear force diagram (S.F.D.) and Bending Moment Diagram (B.M.D.) are the graphical representation of AF, SF and BM respectively such that the values are plotted along y-axis with corresponding length on X-axis.

Properties of S.F.D. and B.M.D.

Loading	S.F.D.	B.M.D.
1. Between pt. load	Horizontal straight line	Inclined straight line
2. U.D.L.	Inclined	Parabolic curve
3. U.V.L.	Parabolic	Cubic curve

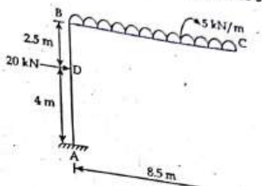
4. Maximum BM occurs when SF = 0.

Point of contraflexure or inflection

The point where BM changes its sign is called point of contraflexure or inflection. At this point BM is zero.

EXAM SOLUTION

1. Draw A.F., S. F. and B.M. diagrams of the frame structure shown in figure below. [2059 Chaitra]



Solution:

Calculation of reactions

$^{+}\rightarrow \Sigma F_x = 0$

$20 + R_{Ax} = 0$

$R_{Ax} = 20 \text{ kN } (\leftarrow)$

$^{+}\uparrow \Sigma F_y = 0$

$R_{Ay} - 5 \times 8.5 = 0$

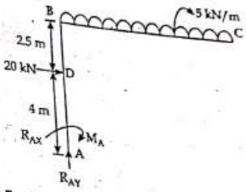
$R_{Ay} = 42.5 \text{ kN } (\uparrow)$

$^{+}\circlearrowleft \Sigma F_C = 0$

$R_{Ay} \times 8.5 + R_{Ax} \times 6.5 + M_A - 20 \times 2.5 - 5 \times 8.5 \times 4.25 = 0$

$M_A = -260.625 \text{ kNm}$

$M_A = 260.625 \text{ kNm } (^{+}\circlearrowright)$



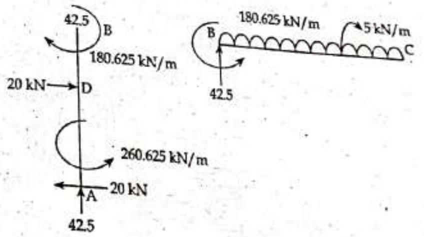


Figure: F.B.D.

Axial Force

$F_{AB} = 42.5 \text{ kN (C)}$

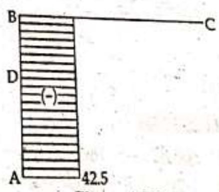


Figure: A.F.D.

Shear Force

$(V_A)_L = 0$

$(V_A)_R = 20$ kN

$(V_D)_L$ 20 kN

$(V_D)_R = 20 - 20 = 0$

$(V_B) = 0$

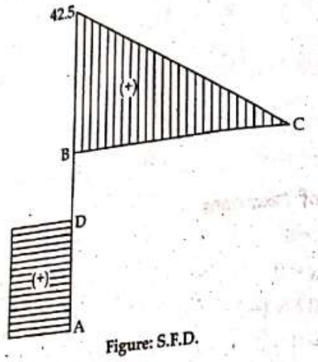


Figure: S.F.D.

Portion BC

$^{+}\uparrow \Sigma F_y = 0$

$^{+}\uparrow \Sigma F_y = 0$

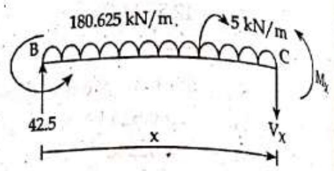
$V_X = 42.5 - 5x$

$(V_B)_L = 0$

$(V_B)_R = 42.5$ kN

$(V_C)\ 42.5 - 5 \times 8.5 = 0$

$(V_x) = 0$ at $x = 8.5$ m



Bending moment

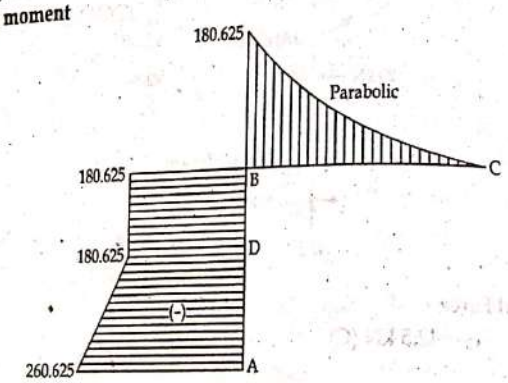


Figure: B.M.D.

Portion AB

$(M_A)_L = 0$

$(M_A)_R = -260.625$ kNm

$(M_D) = 20 \times 4 - 260.625 = -180.625$ kNm

$(M_B)_L = 20 \times 6.5 - 20 \times 2.5 - 260.625 = -180.65$ kNm

$(M_B)_R = -180.625 - 1180.625 = 0$

Portion BC

$M_x = 42.5x - 5 \times x \times \frac{x}{2} - 180.625$

$M_x = 42.5x - 2.5x^2 - 180.625$

$M_{mid} = M_{4.25} = -45.156$ kNm

$(M_B)_L = 0$

$(M_B)_R = -180.625$ kNm

$M_c = 42.5 \times 8.5 - 180.625 - 5 \times 8.5 \times 4.25 = 0$

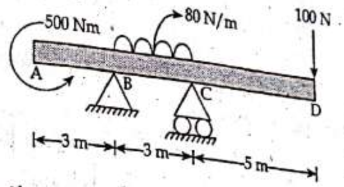
2. **Write short notes on:**
   a) **Statical determinacy**

Solution: See the definition part on page no. 120

3. a) **Explain different types of supports.**

Solution: See the table on page no. 119

b) **Draw shear force, bending moment diagrams for the beam shown in figure.** **[2061 Baishakh]**

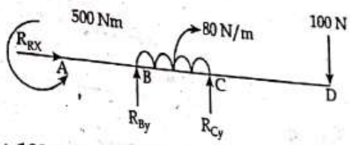


Solution:

Calculation of reactions

$^{+}\rightarrow \Sigma F_X = 0$

$R_{Bx} = 0$



$^{+}\circlearrowright M_B = 0$

$-500 + 80 \times 3 \times 1.5 - R_{Cy} \times 3 + 100 \times 8 = 0$

$R_{Cy} = 220$ N

$^{+}\uparrow \Sigma F_y = 0$

$R_{By} + R_{Cy} - 80 \times 3 - 100 = 0$

$R_{By} = 120$ N

Shear Force

$V_A = 0$

$(V_B)_L = 0$

$(V_B)_R = 120$ N

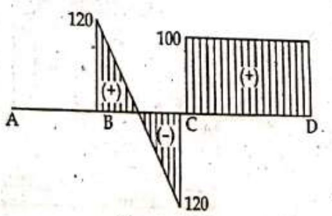
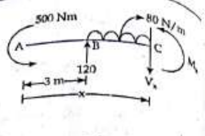


Figure: S.F.D.



Portion BC

$V_x = 120 - 80x$

$(V_x) = 0$ at $x = 1.5$ m

$(V_C)_L = 120 - 80 \times 3 = -120$ N

$(V_c)_R = -120 + 220 = 100$ N

$(V_D)_L = 100$ N

$(V_D)_R = 100 - 100 = 0$

Bending moment

$(M_A)_L = 0$

$(M_A)_R = -500$ Nm

$(M_B) = -500$ Nm

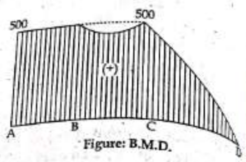


Figure: B.M.D.

Portion BC

$M_x = 120x - 80x \times \frac{x}{2} - 500$

$M_x = 120x - 40x^2 - 500$

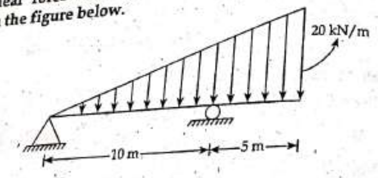
$M_{x=1.5} = -410$ Nm

$M_{mid} = -410$ Nm

$M_C = -500 + 120 \times 3 - 80 \times 3 \times 1.5 = -500$ Nm

$M_D = -500 + 120 \times 8 + 220 \times 5 - 80 \times 3 \times 6.5 = 0$

4. Draw shear force bending moment of the following beam loaded as shown in the figure below. [2062 Poush]



Solution:

Calculation of reactions

$\xrightarrow{+} \Sigma F_x = 0$

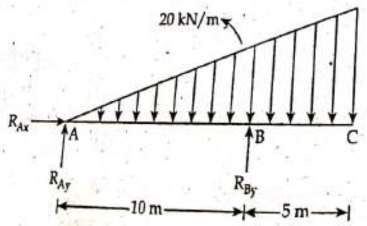
$R_{Ax} = 0$

$^+\circlearrowleft \Sigma M_A = 0$

$\frac{1}{2} \times 20 \times 15 \times \frac{2}{3} \times 15 - R_{By} \times 10 = 0$

$\therefore \quad R_{By} = 150$ kN ($\uparrow$)

$^+\uparrow \Sigma F_y = 0$



$R_{Ay} + R_{By} = \frac{1}{2} \times 15 \times 20$

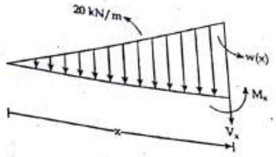
$\therefore \quad R_{Ay} = 0$

Shear Force

Portion AB $(0 \le x \le 10)$, till just left of B

$\frac{w(x)}{x} = \frac{13.333}{10}$

$w(x) = 1.333\,x$ kN/m.



Since, 'w' at point B = 13.33 kN/m, by similar triangles.

$^+\uparrow \Sigma F_y = 0$

$V_x = -\frac{1}{2} \times x \times 1.333\,x$

$\therefore \quad V_x = -0.6665x^2$

$V_{mid} = V_{x=5} = -16.6625$ kN

$(V_A) = 0$

$(V_B)_L = -\frac{1}{2} \times 10 \times 13.33 = -66.65$ kN

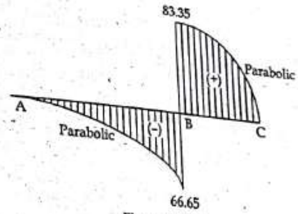


Figure: S.F.D.

Portion BC $(10 \le x \le 15)$, after just right of B

$\frac{w(x)}{x} = \frac{20}{15}$

$w(x) = 1.333\,x$

$V_x = 150 - \frac{1}{2} \times x \times 1.333x$

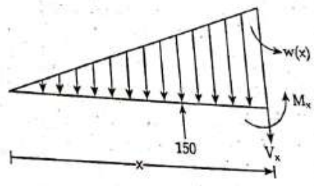
$V_x = 150 - 0.66665\,x^2$

$(V_x) = 0$ at $x = 15$ m

$V_{x=12.5} = V_{mid} = 45.86$ kNm

$(V_B)_R - 66.65 + 150 = 83.35$ kN

$V_c = \frac{1}{2} \times 15 \times 20 - 150 = 0$



Bending Moment

Portion AB $(0 \le x \le 10)$, till just left of B

$M_x = -\frac{1}{2} \times x \times 1.333\,x \times \frac{x}{3} = -0.2222x^3$

$M_A = 0$

$(M_B)_L = -\frac{1}{2} \times 10 \times 13.33 \times \frac{10}{3} = -222.2$ kN m

Portion BC ($10 \le x \le 15$), from just right of C

$M_x = 150\,(x - 10) - \frac{1}{2} \times x \times 1.333x \times \frac{x}{3}$

$M_x = 150x - 0.2222x^3$

$(M_B)_R = -222.2$ kN m

$M_C = -\frac{1}{2} \times 15 \times 20 \times \frac{15}{3} + 150 \times 5 = 0$ kNm

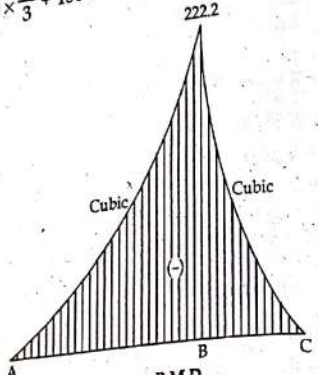
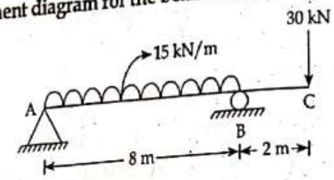


Figure: B.M.D.

5. Draw bending moment diagram for the beam shown in figure. [2062 Baishakh]

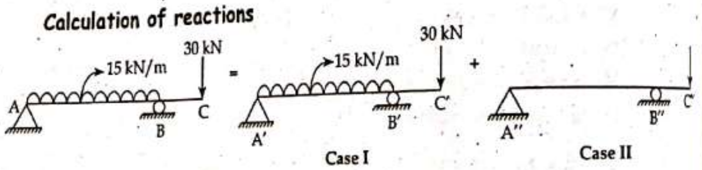


Solution:

This beam is solved by using super-position method. It can also be solved without it.

The two loading w = 15 kN/m and 30 kN are considered in two separate cases and finally summed up.

Calculation of reactions



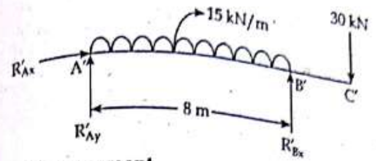
Case I

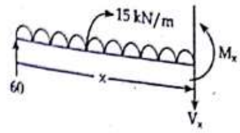
By symmetric;

$R_{A'y} = R_{B'y} = \frac{15 \times 8}{2} = 60$ kN

$^{+}\!\!\rightarrow \Sigma F_x = 0$

$R_{A'x} = 0$ kN





Bending moment

$M_x = 60\,x - 15\,\frac{x^2}{2}$

$(M_{A'})_L = 0$

$(M_{B'}) = 60 \times 8 - 15 \times 8 \times 4 = 0$

$M_{mid} = M_4 = 60 \times 4 - 15 \times 4 \times 2$

$= 120$ kNm

$M_C = 0$

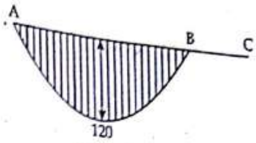


Figure: B.M.D.-I

Case II

$^{+}\!\!\rightarrow \Sigma F_x = 0$

$R_{A''x} = 0$

$^{+}\circlearrowright \Sigma M_{A''} = 0$

$30 \times 10 - R_{B''y} \times 8 = 0$

$\therefore\ R_{B''y} = 37.5$ kN (↑)

$^{+}\!\uparrow \Sigma F_y = 0$

$R_A + R_C = 30$

$R_A = -7.5$ kN

$R_A = 7.5$ kN (↓)

Bending moment

$M_{A''} = 0$

$M_{B''} = -7.5 \times 8 = -60$ kNm

$M_{C''} = -7.5 \times 10 + 37.5 \times 2 = 0$ kNm

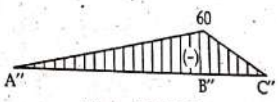


Figure: B.M.D.-II

So, the B_{MD} for the beam will be;

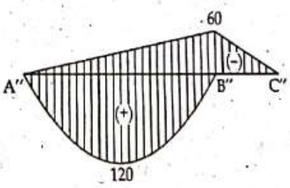


Figure: B.M.D.

If the above beam is solved without superposition theorem, the equivalent B_{MD} will be as below.

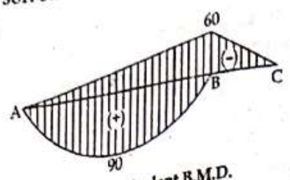
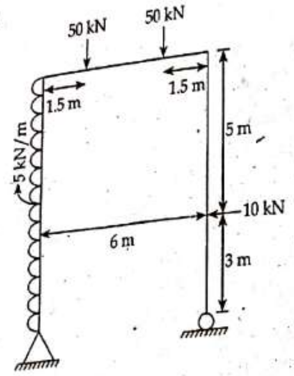


Figure: Equivalent B.M.D.

6. Draw axial force, shear force and bending moment diagrams for the following frame. [2063 Baishakh]



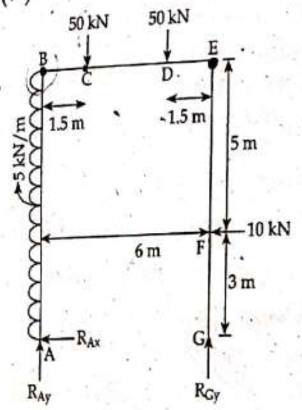
Solution:

Calculation of reactions

$\overset{+}{\rightarrow} \Sigma F_x = 0$

or, $5 \times 8 - 10 - R_{Ax} = 0$

$\therefore$ $R_{Ax} = 30\ kN\ (\leftarrow)$



$\overset{+}{\circlearrowleft} \Sigma M_A = 0$

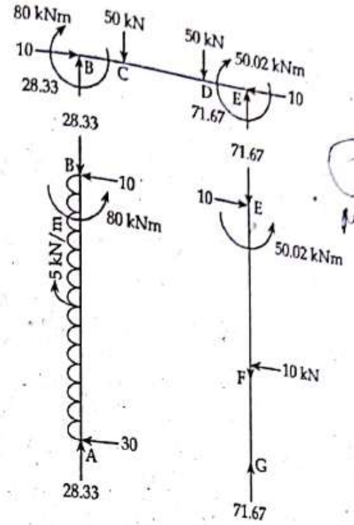
or, $5 \times 8 \times 4 + 50 \times 1.5 + 50 \times 4.5 - 10 \times 3 - R_{Ey} \times 6 = 0$

$\therefore$ $R_{Ey} = 71.67\ kN\ (\uparrow)$

$\overset{+}{\uparrow} \Sigma F_y = 0$

or, $50 + 50 = R_{Ay} + 71.67$

$\therefore$ $R_{Ay} = 28.33\ kN\ (\uparrow)$



$M_C = 80 + 5x^2/2 + 50 \times 1.5 - 28.3x$

Axial force

$F_{AB} = 28.33\ kN\ (C)$

$F_{BE} = 10\ kN\ (C)$

$F_{EG} = 71.67\ kN\ (C)$

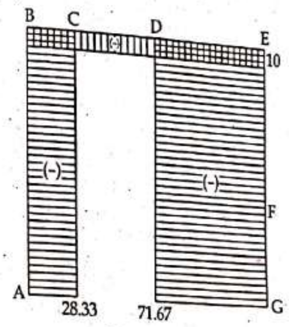


Figure: A.F.D.

Shear force

Member AB

$\overset{+}{\uparrow} \Sigma F_y = 0$

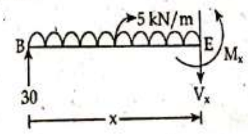
$V_x = 30 - 5x$

$(V_x) = 0$ at $x = 6\ m$

$(V_A)_L = 0$

$(V_A)_R = 30\ kN$

$(V_B)_L = 30 - 5 \times 8 = -10$



$(V_B)_R = -10 + 10 = 0$

Member BE

$(V_B)_L = 0$

$(V_B)_R = 28.33$ kN

$(V_C)_L = 28.33$ kN

$(V_C)_R = 28.33 - 50 = -21.67$ kN

$(V_D)_L = -21.67$ kN

$(V_D)_R = -21.67 - 50 = -71.67$ kN

$(V_E)_L = -71.67$ kN

$(V_E)_R = -71.67 + 71.67 = 0$

Member EG

$(V_E)_L = 0$

$(V_E)_R = 10$

$(V_F)_L = 10$

$(V_F)_R = 10 - 10 = 0$

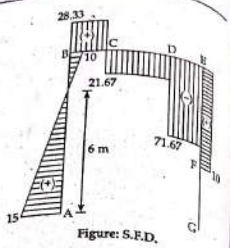


Figure: S.F.D.

Bending moment

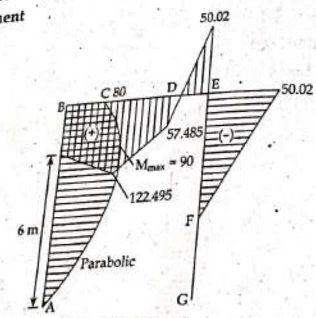


Figure: B.M.D.

Member AB

$\circlearrowleft^+ \Sigma M_F' = 0$

$M_x = 30x - \frac{5x^2}{2}$

$M_{mid} = M_{x=3} = 80$ kNm

$(M_A) = 0$

$(M_B)_L = 30 \times 8 - 5 \times 8 \times 4 = 80$ kNm $(M_E)_R = 0$

$(M_B)_R = 80 - 80 = 0$

$M_{x=6} = M_{max} = 90$ kNm

Member BE

$(M_B)_L = 0$

$(M_B)_R = 80$ kNm

$(M_C) = 122.495$ kNm

$(M_D) = 57.485$ kNm

$(M_E)_L = -50.02$ kNm

Member EG

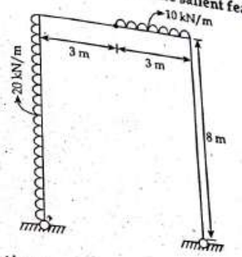
$(M_E)_L = 0$

$(M_E)_R = -50.02$ kNm

$M_F = 0$

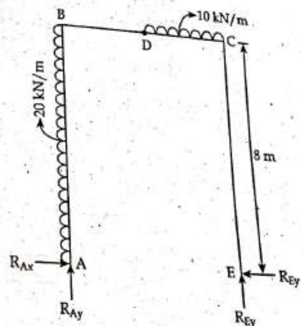
$M_G = 0$

7. **Draw B.M.D., S.F.D. and A.F.D. Also indicate salient features. [2064 Jestha]**



Solution:

Calculation of reactions



$\circlearrowleft^+ \Sigma(M_c)_{right} = 0$

$10 \times 3 \times 1.5 + R_{Ex} \times 8 - R_{Ey} \times 3 = 0$

$3 R_{Ey} - 8R_{Ex} = 45$ (1)

$\circlearrowleft^+ \Sigma M_A = 0$

$20 \times 8 \times 4 + 10 \times 3 \times 4.5 - R_{Ey} \times 6 = 0$

$R_{Ey} = 129.167$ kN (↑)

From equation (1); we get,

$R_{Ex} = 42.81$ kN (←)

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$R_{Ax} - R_{Ex} + 20 \times 8 = 0$

$R_{Ax} = 117.19$ kN (←)

$+\uparrow \Sigma F_y = 0$

$R_{Ay} + R_{Ey} = 10 \times 3$

$R_{Ay} = -99.167$ kN $= 99.167$ kN (↓)

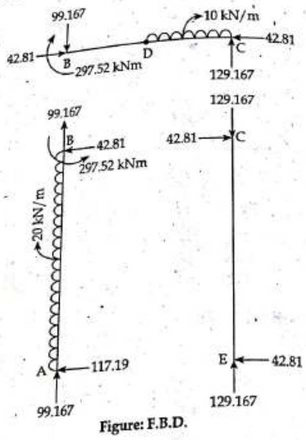


Figure: F.B.D.

Axial force

$F_{AB} = 99.167$ kN (T)

$F_{BC} = 42.81$ kN (C)

$F_{CE} = 123.167$ kN (C)

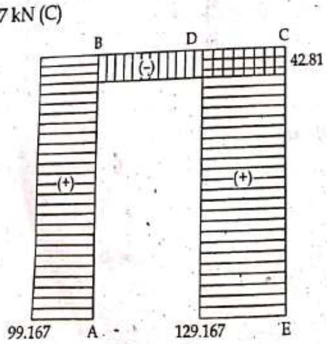
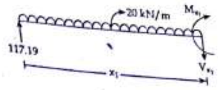


Figure: A.F.D.

Shear force

Member AB

$+\uparrow \Sigma F_y = 0$



$V_{x_1} = 117.19 - 20x_1$

$(V_{x_1}) = 0$ at $x_1 = 5.86$ m

$(V_A)_L = 0$

$(V_A)_R = 117.19$ kN

$(V_B)_L = 117.19 - 160 = -42.81$ kN

$(V_B)_R = -42.81 + 42.81 = 0$

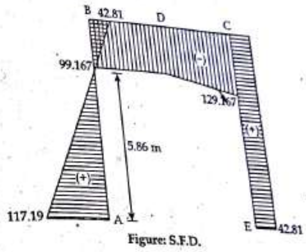


Figure: S.F.D.

Member BD

$(V_B)_L = 0$

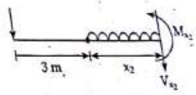
$(V_B)_R = -99.167$ kN

$V_D = -99.167$ kN

Portion DC

$V_{x_2} = -99.67 - 10\,x_2$

where, x_2 is measured from D.



$V_D = -99.167$ kN

$(V_C)_L = -99.167 - 30 = -129.167$ kN

$(V_C)_R = -129.167 + 129.167 = 0$

Member CE

$(V_C)_L = 0$

$(V_C)_R = 42.81$

$(V_E)_L = 42.81$

$(V_E)_R = 42.81 - 42.81 = 0$

Bending moment

Member AB

$M_{x_1} = 117.19x - 10x^2$

$M_{mid} = 308.76$ kNm

$M_{x_1 = 5.86} = 343.33$ kNm

$M_A = 0$

$(M_B)_L = 297.52$ kNm

$(M_B)_R = 0$

Member BC

$(M_B)_L = 0$

$(M_B)_R = 297.52$ kNm

$(M_D) = 297.52 - 297.52 = 0$

Portion DC

$M_{x_2} = -99.167\,(x+3) - 10 \times x_2 \times \frac{x_2}{2} + 297.52$

$M_{x_2} = -99.167x_2 - 5x_2^2$

where, x_2 is measured from D.

$M_{mid} = -160$ kNm $= M_{x_2 = 1.5}$

$(M_D)_L = 0$

$(M_C)_L = -342.482$ kNm

$(M_C)_R = 0$

Member CE

$(M_C)_L = 0$

$(M_C)_R = -342.482$ kNm

$(M_E)_L = 0$

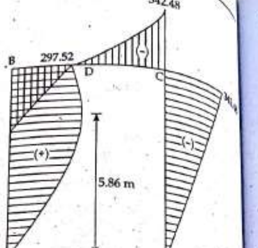


Figure: B.M.D.

Salient features

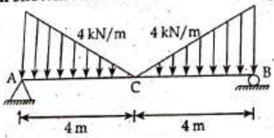
Member AB

$V_{x_1} = 0$ at $x_1 = 5.86$ m (Member AB)

$M_{max_1} = M_{x_1 = 5.86} = 343.33$ kNm (Member AB)

$M_{x_2} = 0$ at $x_2 = 3$ m (Member BC)

8. Draw bending moment diagram, shear force diagram and axial force diagram for the beam shown. Also indicate the salient points. [2064 Falgun]



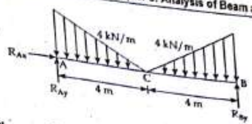
Solution:

Calculation of reactions

$\circlearrowleft^{+} \Sigma M_A = 0$

$$\frac{1}{2} \times 4 \times 4\left(\frac{1}{3} \times 4\right) + \frac{1}{2} \times 4 \times 4 \times \left(4 + \frac{2}{3} \times 4\right) - R_{By} \times 8 = 0$$

$\therefore \quad R_{By} = 8$ kN (↑)



$+\uparrow \Sigma F_y = 0$

$R_{Ay} + R_{By} = \left(\frac{1}{2} \times 4 \times 4\right) \times 2$

$R_{Ay} = 8$ kN (↑)

Axial Force

$F = 0$

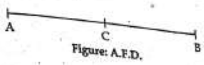


Figure: A.F.D.

Shear Force

Portion AC

$\frac{w(x)}{x_1} = \frac{4}{4}$

$w(x) = x_1$

$+\uparrow \Sigma F_y = 0$

$V_{x_1} = -8 + \frac{1}{2} \times 4 \times 4 + \frac{1}{2} \times x_1 \times x_1$

$V_{x_1} = \frac{x_1^2}{2}$

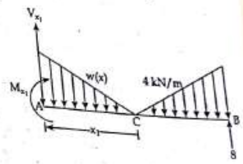
$(V_{x_1 = 0})$ at $x_1 = 0$ m

$V_{mid} = V_2 = 2$ kN

$V_{x_1 = 0} = V_c = 0$

$V_{x_1 = 4} = (V_A)_R = \frac{4^2}{2} = 8$ kN

$(V_A)_L = 8 - 8 = 0$



Portion CB

$\frac{w(x)}{x_2} = \frac{4}{4}$

$w(x) = x_2$

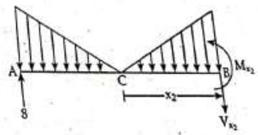
$+\uparrow \Sigma F_y = 0$

$V_{x_2} = 8 - \frac{1}{2} \times 4 \times 4 - \frac{1}{2} \times x_2 \times x_2$

$V_{x_2} = -\frac{x_2^2}{2}$, $V_{x_2} = 0$ at $x_2 = 0$

$V_{mid} = V_2 = -2$KN

$V_{x_2 = 0} = V_c = 0$



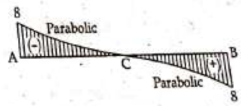


Figure: S.F.D.

$V_{x_2=4} = (V_B)_L = -8KN$

$(V_B)_R = -8 + 8 = 0$

Bending Moment

Portion AC

$$M_{x_1} = -\frac{1}{2} \times x_1 \times x_1 \times \frac{x_1}{3} - \frac{1}{2} \times 4 \times 4 \times \left(x_1 + \frac{2}{3} \times 4\right) + 8 \times (x_1 + 4)$$

$$M_{x_1} = 10.667 - \frac{x_1^3}{6}$$

$(M_{x_1}) = 0$ at $x_1 = 4$ m from C (left ward)

$M_{x_1=0} = (M_C) = 10.667$ kNm

$M_{x_1=4} = (M_A) = 0$

$M_{mid} = M_{x_1=2} = 9.33$ kNm

Portion CB

$$M_{x_2} = 8(x_2 + 4) - \frac{1}{2} \times 4 \times 4 \times \left(x_2 + \frac{2}{3} \times 4\right) - \frac{1}{2} \times x_2 \times x_2 \times \frac{1}{3} x_2$$

$$M_{x_2} = 10.667 - \frac{x_2^3}{6}$$

$(M_{x_2}) = 0$ at $x_2 = 4$ m from C (rightwards)

$M_{x_2=0} = M_c = 10.667$ kNm

$M_D = M_{x_2=4} = 0$

$M_{mid} = M_2 = 9.33$ kNm

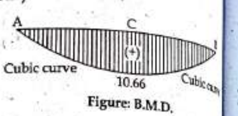


Figure: B.M.D.

Salient Features

$V_{x_2} = 0$ at $x_1 = 0$ (Portion AC)

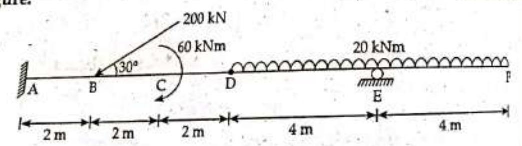
$V_{x_2} = 0$ at $x_1 = 0$ (Portion CB)

$M_{x_2} = 0$ at $x_2 = 4$ m from C towards left (Portion AC)

$M_{x_2} = 0$ at $x_2 = 4$ m from C towards right (Portion CB)

$M_{max} = 10.667$ kNm

9. **Draw bending moment, shear force and axial force diagrams for the figure.** [2060 Shrawan]



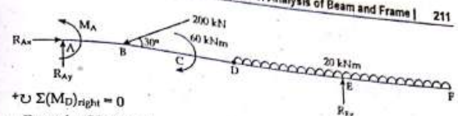
Solution:

Calculation of reactions

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$R_{Ax} - 200 \cos 30° = 0$

$R_{Ax} = 100$ kN $(\rightarrow)$



$\overset{+}{\circlearrowright} \Sigma(M_D)_{right} = 0$

$-R_{Ey} \times 4 + 20 \times 8 \times 4 = 0$

$R_{Ey} = 160$ kN (↑)

$\overset{+}{\circlearrowright} \Sigma M_F = 0$

$R_{Ay} \times 12 - M_A - 100 \times 10 + 60 - 20 \times 8 \times 4 + R_{Ey} \times 4 = 0$

$\therefore$ $M_A = 260$ kNm (↺)

$\overset{+}{\uparrow} \Sigma F_y = 0$

$R_{Ay} - 200 \sin 30° - 20 \times 8 + R_{Ey} = 0$

$\therefore$ $R_{Ay} = 100$ kN (↑)

Axial force

Portion AB

$F_{AB} = 173.20$ (C)

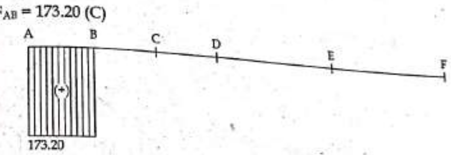


Figure: A.F.D.

Shear force

$(V_A)_L = 0$

$(V_A)_R = 100$

$(V_B)_L = 100 - 100 = 0$

$(V_C) = 0$

$(V_D) = 0$

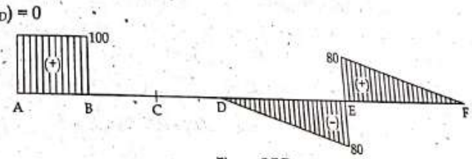
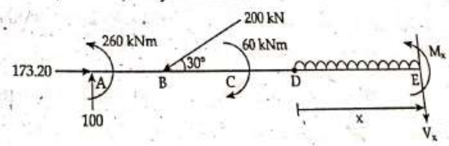


Figure: S.F.D.

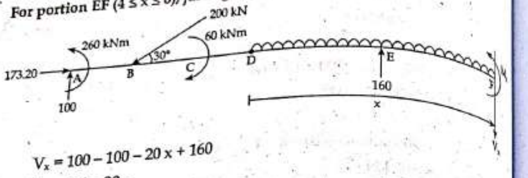
For portion DE $(0 \le x \le 4)$, **till just left of E**



Scanned by CamScanner

$$^{+}\uparrow \Sigma F_y = 0$$
$$V_x = 100 - 100 - 20\,x$$
$$V_x = -20\,x$$
$$(V_D) = 0$$
$$(V_E)_L = 100 - 100 - 20 \times 4 = -80 \text{ kN}$$

For portion EF $(4 \le x \le 8)$, just right of E



$$V_x = 100 - 100 - 20\,x + 160$$
$$V_x = 160 - 20x$$
$$(V_E)_R = 160 - 20 \times 4 = 80 \text{ kN}$$
$$V_F = +160 - 20 \times 8 = 0$$

Bending moment

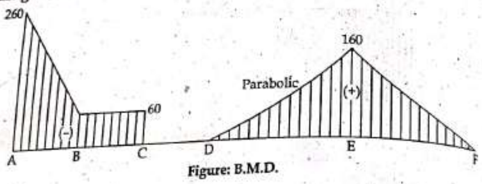


Figure: B.M.D.

$$(M_A)_L = 0$$
$$(M_A)_R = -260 \text{ kNm}$$
$$(M_B) = 100 \times 2 - 260 = -60 \text{ kNm}$$
$$(M_c)_L = 100 \times 4 - 260 - 100x = -60 \text{ kNm}$$
$$(M_C)_R = -60 + 60 = 0$$
$$M_D = 100 \times 6 - 260 - 100 \times 4 + 60 = 0$$

For portion DE $(0 \le x \le 4)$, till just left of E

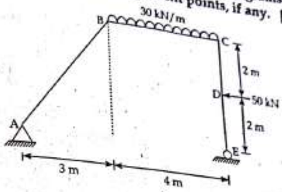
$$M_x = 100(x + 6) - 260 - 100\,(x + 4) + 60 - 20 \times x \times \frac{x}{2}$$
$$M_x = -10x^2$$
$$M_{mid} = M_2 = -40 \text{ kNm}$$
$$M_E = 100 \times 10 - 260 - 100 \times 8 + 60 - 20 \times 4 \times 2 = -160 \text{ kNm}$$

Portion EF $(4 \le x \le 8)$, after just right of E

$$M_x = 100 \times (6 + x) - 260 - 100(4 + x) + 60 - 20 \times x \times \frac{x}{2} + 160\,(x - 4)$$
$$M_x = -10x^2 + 160\,x - 640$$

$$M_x = 160x - 10x^2 - 640$$
$$M_{mid} = M_6 = -40 \text{ kNm}$$
$$M_E = -160 \text{ kNm}$$
$$M_F = 0$$

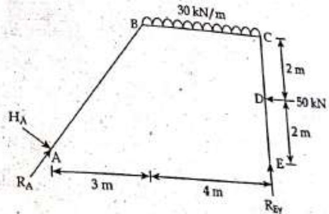
10. **Draw bending moment, shear force and axial force diagrams for the given figure. And also give ordinates of the salient points, if any. [2067 Ashadh]**



Solution:

Calculation of reactions

$$^{+}\circlearrowleft \Sigma M_A = 0$$
$$30 \times 4 \times 5 - 50 \times 2 - R_{Ey} \times 7 = 0$$
$$\therefore \quad R_{Ey} = 71.43 \text{ kN } (\uparrow)$$



$$^{+}\rightarrow \Sigma F_x = 0$$

or, $H_A \sin\theta + R_A \cos\theta - 50 = 0$

or, $\theta = \tan^{-1}\frac{4}{3} = 53.13°$

$$0.8H_A + 0.6R_A = 50 \qquad (1)$$
$$^{+}\uparrow \Sigma F_y = 0$$

or, $-H_A \cos\theta + R_{A\,\sin}\theta - 120 + 71.43 = 0$

or, $-0.6\,H_A + 0.8\,R_A = 48.57 \qquad (2)$

Solving equation (1) and (2); we get,

$$H_A = 10.858 \text{ kN } (\searrow)$$
$$R_A = 68.856 \text{ kN } (\nearrow)$$

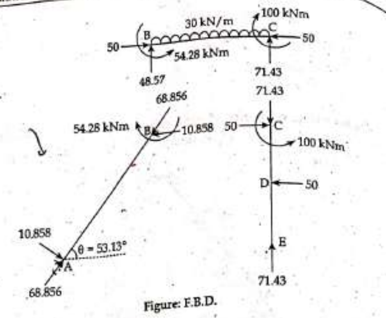


Figure: F.B.D.

Axial force

$F_{AB} = 68.856$ kN (C)

$F_{BC} = 50$ kN (C)

$F_{CE} = 71.43$KN (C)

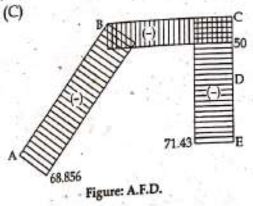


Figure: A.F.D.

Shear force

Member AB

$(V_A)_L = 0$

$(V_A)_R = -10.858$ kN

$(V_B)_L = -10.858$ kN

$(V_B)_R = -10.858 + 10.858 = 0$

Member BC

$+\uparrow \Sigma F_y = 0$

$V_x = 48.57 - 30x$

$(V_x) = 0$ at $x = 1.619$ m

$(V_B)_L = 0$

$(V_B)_R = 48.57$ kN

$(V_C)_L = -71.43$ kN

$(V_C)_R = 0$

Member CE

$(V_C)_L = 0$

... $= 50$ kN

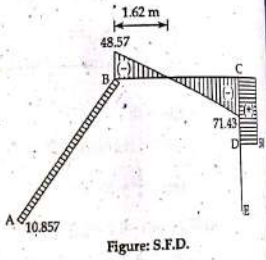
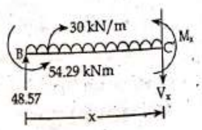


Figure: S.F.D.



$(V_D)_L = 50$ kN

$(V_D)_R = 50 - 50 = 0$

Bending moment

Member AB

$M_A = 0$

$(M_B)_L = -54.29$ kNm

$(M_B)_R = -54.29 + 54.29 = 0$

Member BC

$+\circlearrowleft \Sigma M_{C'} = 0$

$M_x = 48.57x - 15x^2 - 54.29$

$M_{mid} = M_2 = -17.15$ kNm

$M_{1.619} = M_{max} = -14.97$ kNm

$(M_x) = 0$ at $x = 0$ and $x = 4$ m [$\because$ other not possible]

$(M_B)_L = 0$

$(M_B)_R = -54.29$ kNm

$(M_C)_L = -100.01$ kNm

$(M_C)_R = -100.01 + 100.01 = 0$ kNm

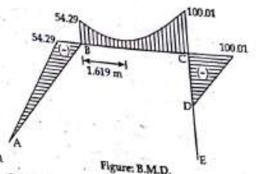


Figure: B.M.D.

Member CE

$(M_C)_L = 0$

$(M_C)_R = -100.01$ kNm

$(M_D) = -100.01 + 50 \times 2 = 0.01 \approx 0$

Salient features

$V_x = 0$ at $x = 1.619$ m

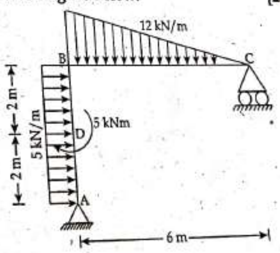
$M_x = 0$ at $x = 0$ and $x = 4$ m

$M_{max} = -14.97$ kNm

11. **Derive the relationship between load, shear force and bending moment.** **[2068 Baishakh]**

Solution: See the definition part on page no. 193

12. **Draw axial force, shear force and bending moment diagram for the given loaded frame as shown in figure below.** **[2068 Baishakh]**



Solution:

Calculation of reactions

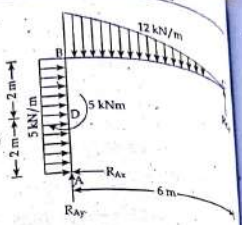
$\xrightarrow{+} \Sigma F_x = 0$

$-R_{AX} + 5 \times 4 = 0$

$\therefore \quad R_{AX} = 20\ kN\ (\leftarrow)$

$+\circlearrowleft \Sigma M_C = 0$

$\frac{1}{2} \times 6 \times 12 \times 4 - 5 \times 4 \times 2 + R_{Ay} \times 6 + 5 + 20 \times 4 = 0$



$\therefore \quad R_{Ay} = 16.5\ kN\ (\uparrow)$

$+\uparrow \Sigma F_y = 0$

$R_{Ay} + R_{Cy} = \frac{1}{2} \times 12 \times 6$

$\therefore \quad R_{Cy} = 19.5\ kN\ (\uparrow)$

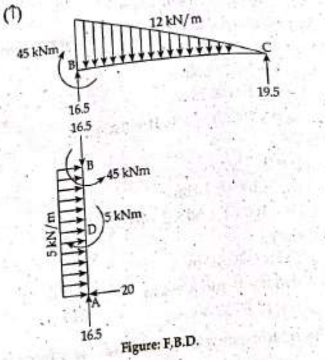


Figure: F.B.D.

Axial force

$F_{AB} = 16.5\ kN\ (C)$

$F_{BC} = 0$

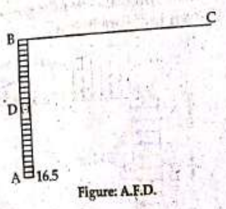


Figure: A.F.D.

Shear force

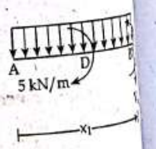
Member AB

$+\uparrow \Sigma F_y = 0$

$V_{x_1} = 20 - 5x_1$

$(V_{x_1}) = 0$ at $x_1 = 4\ m$

$(V_A)_L = 0$



$(V_A)_R = 20\ kN$

$(V_B)_L = 0$

$(V_B)_R = 0$

Member BC

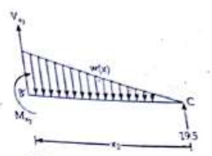
$+\uparrow \Sigma F_y = 0$

$\frac{w(x)}{x} = \frac{12}{6}$

$w(x) = 2x$

$+\uparrow \Sigma F_y = 0$

$V_{x_2} = \frac{1}{2} \times 2x_2 \times x_2 - 19.5$



$V_{x_2} = x_2^2 - 19.5$

$(V_{x_2}) = 0$ at $x_2 = 4.416\ m$

$V_{mid} = V_{x_2 = 3} = -10.5\ kN$

$(V_C)_R = 0$

$(V_C)_L = -19.5\ kN$

$(V_B)_R = -19.5 + \frac{1}{2} \times 12 \times 6 = 16.5\ kN$

$(V_B)_L = 16.5 - 16.5 = 0$

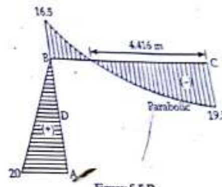


Figure: S.F.D.

Bending moment

Member AB

$M_{x_1} = 20x_1 - 2.5x_1^2;\ 0 \le x_1 \le 2$ till just left of D

$M_{x_1} = 20x_1 - 2.5x_1^2 + 5;\ 2 \le x_1 \le 4$ from just right of D

$M_{mid} = M_{x_1 = 1} = 17.5\ kNm$

$M_{mid} = M_{x_1 = 3} = 42.5\ kNm$

$(M_A) = 0$

$(M_D)_L = 20 \times 2 - 5 \times 2 \times 1 = 30\ kNm$

$(M_D)_R = 40 + 5 - 5 \times 2 = 35\ kNm$

$(M_B)_L = 20 \times 4 + 5 - 20 \times 2 = 45\ kNm$

$(M_B)_R = 45 - 45 = 0$

Member BC

$+\circlearrowleft \Sigma M_{B'} = 0$

$M_{x_2} = 19.5 \times x_2 - \frac{1}{2} x_2 \times 2x_2 \times \frac{x_2}{3}$

$M_{x_2} = 19.5x_2 - \frac{x_2^3}{3}$, x_2 start from C

$M_{mid} = M_{x_2 = 3} = 49.5\ kNm$

$M_{4.416} = 57.41\ kNm = M_{max}$

$(M_C) = 0$

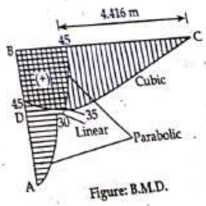
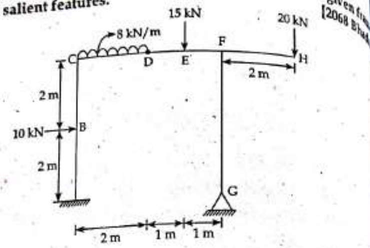


Figure: B.M.D.

$(M_E)_R = 19.5\times6-\frac{1}{2}\times6\times12\times2 = 45$ kNm

$(M_E)_L = 45-45 = 0$

13. Draw the axial force, shear force and bending moment of the given frame. Also show the salient features. [2068 Bhadra]



Solution:

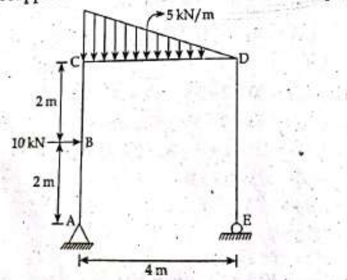
The frame given here is indeterminate as;

r = 5
j = 6
m = 5
c = 1

so, Degree of static indeterminacy (D_S) = $3m + r - (3j + c)$
$= 3\times5+5-(3\times6+1) = 20-19$
$= 1$

As the frame is indeterminate, it cannot be solved by the equation of statics.

14. Calculate reactions at supports and draw A.F.D., S.F.D. and B.M.D. [2068 Magh]



Solution:

Calculation of reactions

$\xrightarrow{+} \Sigma F_x = 0$

$R_{AX} + 10 = 0$

$R_{AX} = -10$

$R_{AX} = 10$ kN (←)

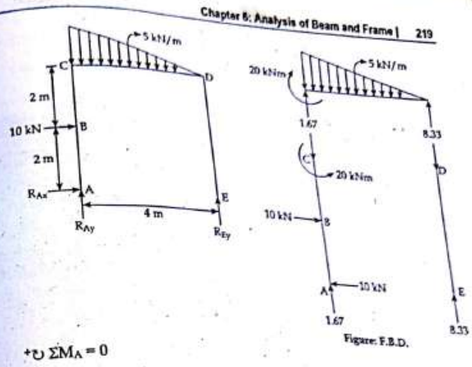


Figure: F.B.D.

$+\circlearrowleft \Sigma M_A = 0$

$10\times2+\frac{1}{2}\times4\times5\times\frac{1}{3}\times4-R_{Ey}\times4 = 0$

$R_{Ey} = 8.33$ kN (↑)

$+\uparrow \Sigma F_y = 0$

$R_{Ey} + R_{Ay} - \frac{1}{2}\times4\times5 = 0$

$R_{Ay} = 1.67$ kN (↑)

Axial force

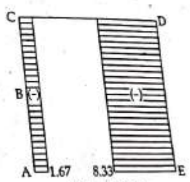


Figure: A.F.D.

$F_{AC} = 1.67$ kN (C)

$F_{CD} = 0$

$F_{DE} = 8.33$ kN (C)

Shear force

Member AC

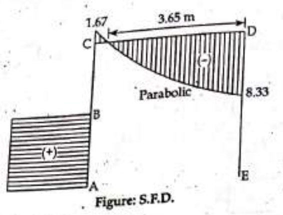
$(V_A)_L = 0$

$(V_A)_R = 10$ kN

$(V_B)_L = 10$ kN

$(V_B)_R = 10-10 = 0$

$V_C = 0$



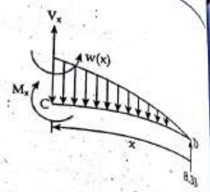


Figure: S.F.D.

Member CD

$\frac{W(x)}{x} = \frac{5}{4}$

$w(x) = 1.25x$

$+\uparrow \Sigma F_y = 0$

$V_x = -8.33 + \frac{1}{2} \times x \times 1.25\,x$

$V_X = 0.625\,x^2 - 8.33$

$(V_X) = 0$ at $x = 3.65$ m from D

$V_{mid} = V_2 = -5.83$ kN

$(V_D)_R = 0$

$(V_D)_L = -8.33$ kN

$(V_C)_R = -8.33 + \frac{1}{2} \times 5 \times 4 = 1.67$ kN

$(V_C)_L = 1.67 - 1.67 = 0$

Member DE

$V = 0$

Bending moment

Member AC

$M_A = 0$

$M_B = 10 \times 2 = 20$ kNm

$(M_C)_L = 10 \times 4 - 10 \times 2 = 20$ kNm

$(M_C)_R = 20 - 20 = 0$

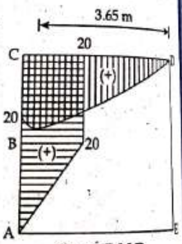


Figure: B.M.D.

Member CD

$M_x = 8.33x - \frac{1}{2} \times x \times 1.25\,x \times \frac{x}{3}$

$M_x = 8.33x - 0.2083\,x^3$

$M_x = 0$ at $x = 0$ m

Note

Here M_x is the minus integration of V_x as direction of 'x' is from right to left.

$M_{mid} = M_2 = 14.99$ kNm

$M_{3.65} = 20.275$ KNm

$(M_D) = 0$

$(M_C)_R = 8.33 \times 4 - \frac{1}{2} \times 5 \times 4 \times \frac{4}{3} = 19.98 = 20$ kNm

$(M_C)_L = 20 - 20 = 0$

Member DE

$M = 0$

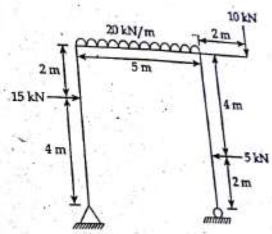
Salient features

$M_x = 0$ at $x = 3.65$ m from D

$M_x = 0$ at $x = 0$

$M_{max} = 20.275$ kNm

15. **What are statically determinant and indeterminate structures? Draw A.F.D., S.F.D. and B.M.D.** [2068 Chaitra]



Solution:

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$R_{AX} + 15 - 5 = 0$

$R_{AX} = -10$

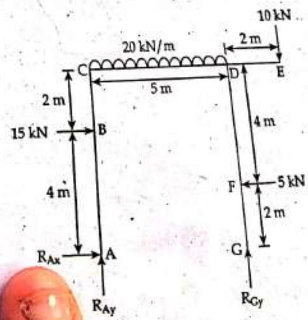
$R_{AX} = 10$KN $(\leftarrow)$

$\overset{+}{\circlearrowright} \Sigma M_A = 0$

$15 \times 4 + 20 \times 5 \times 2.5 + 10 \times 7 - 5 \times 2 - R_{Gy} \times 5 = 0$

$\therefore \quad R_{Gy} = 74$KN $(\uparrow)$

$+\uparrow \Sigma F_y = 0$



$R_{AY} - 20 \times 5 - 10 + 74 = 0$

$R_{AY} = 36$ kN (↑)

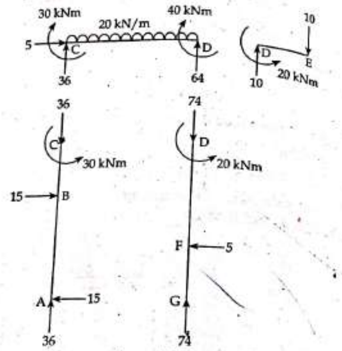


Figure: F.B.D.

Axial force

$F_{AC} = 36$ kN (C)

$F_{CD} = 5$ kN (C)

$F_{DE} = 0$

$F_{DG} = 74$ kN (C)

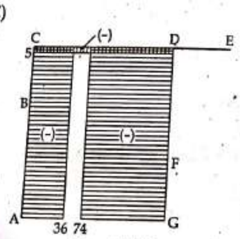


Figure: A.F.D.

Shear force

Member AC

$(V_A)_L = 0$

$(V_A)_R = 10$ kN

$(V_B)_L = 10$ kN

$(V_B)_R = 10 - 15 = -5$ kN

$(V_C)_L = -5$ kN

$(V_C)_R = -5 + 5 = 0$

Member CD

$+\uparrow \Sigma F_y = 0$

$V_x = 36 - 20x$

$(V_x) = 0$ at $x = 1.8$ m

$(V_C)_L = 0$

$(V_C)_R = 36$ kN

$(V_D)_L = 36 - 20 \times 5 = -64$ kN

$(V_D)_R = -64 + 64 = 0$

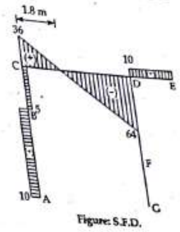


Figure: S.F.D.

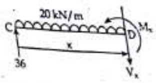
Member DE

$(V_D)_L = 0$

$(V_D)_R = 10$ kN

$(V_E)_L = 10$ kN

$(V_E)_R = 10 - 10 = 0$



Member DG

$(V_D)_L = 0$

$(V_D)_R = 5$ kN

$(V_F)_L = 5$ kN

$(V_F)_R = 5 - 5 = 0$

$V_G = 0$

Bending moment

Member AC

$M_A = 0$

$M_B = 10 \times 4 = 40$ kNm

$(M_C)_L = 20 \times 6 - 15 \times 2 = 50$ kNm

$(M_C)_R = 30 - 30 = 0$

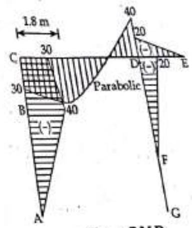


Figure: B.M.D.

Member CD

$M_X = 36x - 20 \times \frac{x}{2}$

$M_X = 36x - 10x^2$

$M_{mid} = M_{2.5} = 27.5$KNm

$M_{x=1.8} = 32.4$ kNm

$(M_X) = 0$ at $x = 3.6$ m

$(M_C)_L = 0$

$(M_C)_R = 30$ kNm

$(M_D)_L = 36 \times 5 - 20 \times 5 \times 2.5 + 30 = -40$

$(M_D)_R = -40 + 40 = 0$

Member DE

$(M_D)_L = 0$

$(M_D)_R = -20$KNm

$(M_E) = -20 + 10 \times 2 = 0$

Member DG

$(M_D)_L = 0$

$(M_D)_R = -20 KNm$

$(M_F) = 5 \times 4 - 20 = 0$

$M_G = 0$

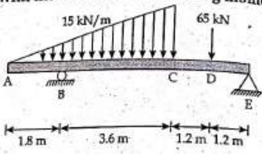
Salient features

$V_x = 0$ at $x = 1.8$ m

$M_x = 0$ at $x = 3.6$ m

$M_{max} = 32.4$ kNm

16. **What do you mean by statically determinate and indeterminate stru... Explain with an example for each. Write the equations for shear force a... bending moment for the beam shown in figure below. Plot the varia... of the shear force and bending moment in the beam. Also indicate sali... features associated with the sear force and bending moment.[2069 Ash...**



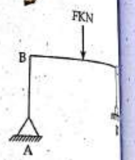
Solution:

Statically determinant and indeterminant structure

See the definition part on page no. 192

Statically determinant

The frame shown in the figure is a statically determinant frame as;



$r = 3$

$m = 3$

$J = 4$

so, $D_s = 3m + r - (3j + c)$

$= 3 \times 3 + 3 - (3 \times 4)$

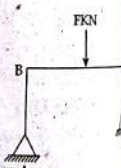
$= 9 + 3 - 12$

$= 0$

$\therefore$ Frame is determinant.

Statically indeterminate

The frame shown here is indeterminate as;



$r = 4$

$m = 3$

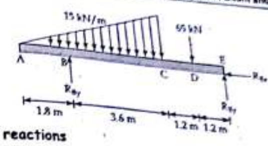
$j = 4$

$D_s = 3m + r - (3j + c)$

$= 3 \times 3 + 4 - (3 \times 4)$

$= 13 - 12 = 1 > 0$

so, frame is indeterminate.



Calculation of reactions

$\xrightarrow{+} \Sigma F_X = 0$

$\therefore$ $R_{EX} = 0$

$+\circlearrowleft \Sigma M_E = 0$

$$R_{By} \times 6 - \frac{1}{2} \times 5.4 \times 15 \times (2.4 + 1.8) - 65 \times 12 = 0$$

$\therefore$ $R_{By} = 41.35$ kN (↑)

$+\uparrow \Sigma F_y = 0$

$$R_{Ey} + R_{By} - \frac{1}{2} \times 5.4 \times 15 - 65 = 0$$

$R_{Ey} = 64.15$ kN (↑)

Shear force

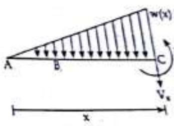
Portion AB ($0 \le x \le 1.8$), up to just left of B

$$\frac{w(x)}{x} = \frac{5}{1.8}$$

Since, w at B = 5 kN/m

$\therefore$ $w(x) = 2.78\,x$

$+\uparrow \Sigma F_y = 0$



$$V_x = -\frac{1}{2} \times x \times 2.18\,x$$

$V_x = -1.39\,x^2$

$V_{mid} = V_{0.9} = -1.26$ kN

$V_x = 0$ at $x = 0$ m

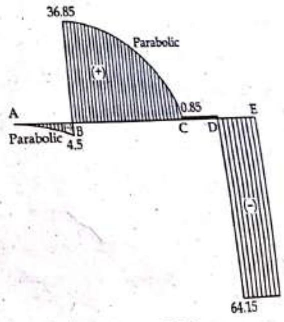


Figure: S.F.D.

$V_0\ (V_A) = 0$

$V_{1.8} = (V_B)_L = -\frac{1}{2} \times 1.8 \times 5 = -4.5$ kN

For portion BC ($1.8 \le x \le 5.4$) from just right of B

$\frac{w(x)}{x} = \frac{15}{5.4}$

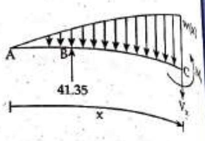
$w(x) = 2.78\ x$

$V_x = 41.35 - \frac{1}{2} \times x \times x \times 2.78 \times$

$= 41.35 - 1.39\ x^2$

$V_x = 0$ at $x = 5.45$ m

$V_{mid} = V_{3.6} = 23.335$ kN



Note

x varies from 1.8 to 5.4

$V_{1.8} = (V_B)_R = -4.5 + 41.35 = 36.85$ kN

$V_{5.4} = (V_c) = 41.35 - \frac{1}{2} \times 5.4 \times 15 = 0.85$ kN

Portion CE

$(V_C) = 0.85$ kN

$(V_D)_L = 0.85$ kN

$(V_D)_R = 0.85 - 65 - 64.15$ kN

$(V_E)_L = -64.15$ kN

$(V_E)_R = -64.15 + 64.15 = 0$

Bending moment

Portion AB ($0 \le x \le 1.8$)

$M_x = -\frac{1}{2} \times x \times 2.78\ x \times \frac{x}{3}$

$M_x = -0.463\ x^3$

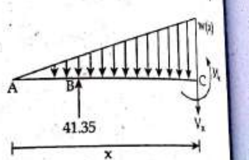
$M_{mid} = M_{3.6} = 52.83$ kNm

$M_0 = (M_A) = 0$

$M_{1.8} = (M_B)_L$

$= -\frac{1}{2} \times 1.8 \times 5 \times \frac{1}{3} \times 1.8$

$= -2.7$ kNm



Portion BC

$M_x = 41.35(x - 1.8) - \frac{1}{2} \times x \times 2.78x \times \frac{x}{3}$

$M_x = 41.35x - 74.43 - 0.463\ x^3$

$(M_X) = 0$ at $x = 1.89$ m

$M_{1.8} = (M_B)_R = -2.7$ kNm

$M_{5.4} = (M_C) = 75.95$ kNm

Portion CE

$(M_C) = 75.95$ kNm

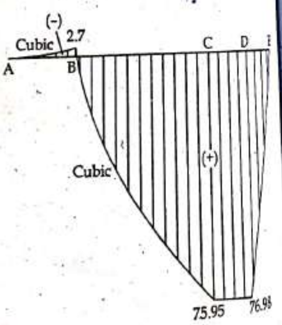


Figure: B.M.D.

$(M_D) = 41.35 \times 4.8 - \frac{1}{2} \times 5.4 \times 15 \times (1.2 + 1.8) = 76.98$ kNm

$M_E = 41.35 \times 6 - \frac{1}{2} \times 5.4 \times 15 \times (2.4 + 1.8) - 65 \times 1.2$

$M_E = 0$

Salient features

$(V_X) = 0$ at $x = 0$ and $x = 5.45$ m

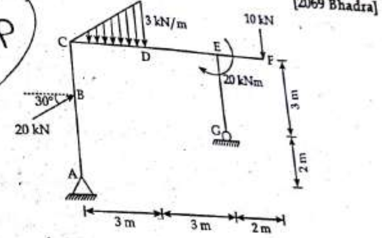
$(M_x) = 0$ at $x = 0$ and $x = 1.89$ m

Note

Above question can be solved by super position theorem.

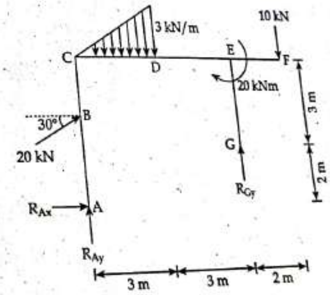
17. **Calculate and draw the A.F.D., S.F.D. and B.M.D. with its salient features for the given frame.** [2069 Bhadra]

amp



Solution:

Calculation of reactions



$^{+}\!\rightarrow \Sigma F_X = 0$

$R_{AX} + 20 \cos 30° = 0$

$\therefore \quad R_{AX} = -17.32$ kN $= 17.32$KN $(\leftarrow)$

$^{+}\circlearrowright \Sigma M_A = 0$

$(20 \cos 30°) \times 3 + \frac{1}{2} \times 3 \times 3 \times 2 + 10 \times 8 + 20 - R_G \times 6 = 0$

$\therefore \quad R_G = 26.83$ kN $(\uparrow)$

Scanned by CamScanner

$+\uparrow \Sigma F_y = 0$

$R_A + 20 \sin 30° - \frac{1}{2} \times 3 \times 3 - 10 + R_G = 0$

$\therefore \quad R_A = -22.33 \text{ kN} = 22.33 \text{ kN} (\downarrow)$

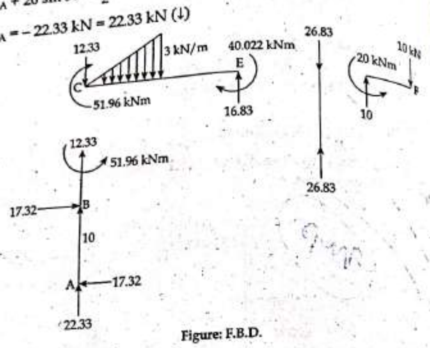


Figure: F.B.D.

Axial force

Member AC

$F_{AB} = 22.33 \text{ kN (T)}$

$F_{BC} = 12.33 \text{ kN (T)}$

Member CE

$F_{CE} = 0$

Member EF

$F_{EF} = 0$

Member EG

$F_{EG} = 26.83 \text{ kN (C)}$

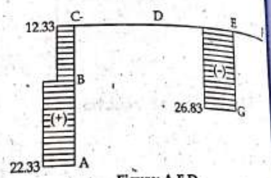


Figure: A.F.D.

Shear force

Member AC

$(V_A)_L = 0$

$(V_A)_R = 17.32 \text{ kN}$

$(V_B)_L = 17.32 \text{ kN}$

$(V_B)_R = 17.32 - 17.32 = 0$

$V_C = 0$

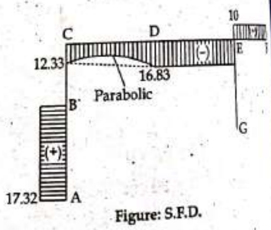


Figure: S.F.D.

Member CE

Portion CD

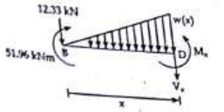
From similar triangles; we have,

$$\frac{W(x)}{x} = \frac{3}{3}$$

$\therefore \quad W(x) = x \text{ kN/m}$

$+\uparrow \Sigma F_y = 0$

$$V_x = -12.33 - \frac{1}{2}x^2 \qquad \text{(a)}$$



$V_{mid} = V_{1.5} = -13.455 \text{ kN}$

$V_x = 0$ at $x = 0$ m and $x = 6$ m

Since, other cases not possible.

$(V_C)_L = 0$

$(V_C)_R = -12.33 \text{ kN}$

$V_D = -12.33 - \frac{1}{2} \times 3 \times 3 = -16.83 \text{ kN}$

Note

Here, the above value can be obtained from equation (a) but to have a check on these values these are calculated separately. Later, one can check these values by substituting value of 'x' in equation (a). (The same process is adopted for all the problems)

Portion DE

$(V_E)_L = -16.83 \text{ kN}$

$(V_E)_R = -16.83 + 16.83 = 0$

Member EF

$(V_E)_L = 0$

$(V_E)_R = 10$

$(V_F)_L = 10$

$(V_F)_R = 10 - 10 = 0$

Member EG

$V = 0$

Bending moment

Member AC

$M_A = 0$

$M_B = 17.32 \times 3 = 51.96 \text{ kNm}$

$(M_C)_L = 17.32 \times 5 - 17.32 \times 2$

$= 51.96 \text{ kNm}$

$(M_C)_R = 51.96 - 51.96 = 0$

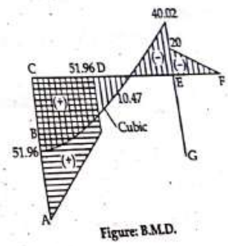


Figure: B.M.D.

Member CE

Portion CD

$+\circlearrowright \Sigma M_D = 0$

$$M_x = 51.96 - 12.33x - \frac{x^3}{6}$$

$M_x = 0$ at 3.58 m

$M_{mid} = M_{1.5} = 32.90 \text{ kNm}$

$(M_C)_L = 0$

$(M_C)_R = 51.96$

$(M_D) = 51.96 - 12.33 \times 3 - \frac{1}{2} \times 3 \times 3 \times 1$

$= 10.47 \text{ kNm}$

Portion DE

$(M_E)_L = 51.96 - 12.33 \times 6 - \frac{1}{2} \times 3 \times 3 \times 4 = -40.02$ kNm

$(M_E)_R = -40.02 + 40.02 = 0$

Member EF

$(M_E)_L = 0$

$(M_E)R = -20$ kNm

$M_F = 10 \times 2 - 20 = 0$

Member EG

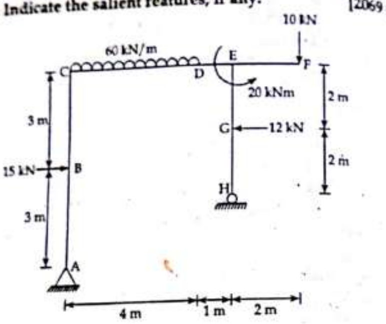
$M = 0$

Salient features

$M_x = 0$ at $x = 3.58$ m

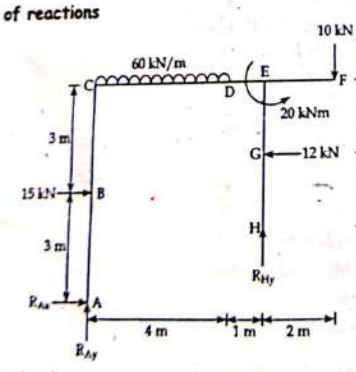
$V_x = 0$ at $x = 0$

18. Draw A.F.D., S.F.D. and B.M.D. of the given frame loaded as shown in figure below. Indicate the salient features, if any. [2069 Chaitra]



Solution:

Calculation of reactions



$\overset{+}{\rightarrow} \sum F_x = 0$

$R_{Ax} + 15 - 12 = 0$

or, $R_{Ax} = -3$ kN

$\therefore$ $R_{Ax} = 3$ kN(←)

Taking moment about A;

$+\circlearrowleft \sum M_A = 0$

or, $15 \times 3 + 60 \times 4 \times 2 - 20 + 10 \times 7 - 12 \times 4 - R_{Hy} \times 5 = 0$

$\therefore$ $R_{Hy} = 105.4$ kN(↑)

$+\uparrow \sum F_y = 0$

or, $R_{Ay} - 60 \times 4 + R_{Hy} - 10 = 0$

or, $R_{Ay} - 250 + 105.4 = 0$

$\therefore$ $R_{Ay} = 144.6$ kN(↑)

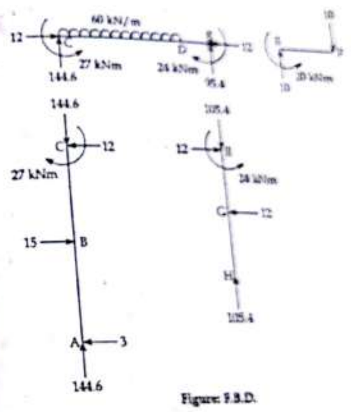


Figure: F.B.D.

Axial force

$F_{AC} = 144.6$ kN(C)

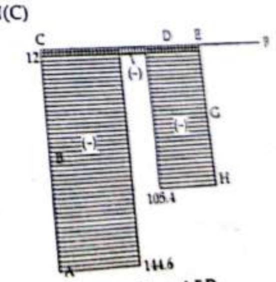
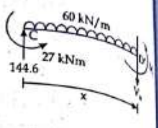


Figure: A.F.D.

$F_{CE} = 12$ kN(C)
$F_{EF} = 0$
$F_{EH} = 105.4$ kN(C)

Shear force
Member AC

$(V_A)_L = 0$
$(V_A)_R = 3$ kN
$(V_B)_L = 3$ kN
$(V_B)_R = 3 - 15 = -12$ kN
$(V_C)_L = -12$ kN
$(V_C)_R = -12 + 12 = 0$



Member CE
Portion CD

$+\uparrow \sum F_y = 0$
$V_x = 144.6 - 60x$
$V_x = 0$ at $x = 2.41$ m
$(V_C)_L = 0$
$(V_C)_R = 144.6 - 60 \times 0 = 144.6$ kN
$(V_D)_L = 144.6 - 60 \times 4 = -95.4$ kN

Portion DE

$(V_D)_R = -95.4$ kN
$(V_E)_L = -95.4$ kN
$(V_E)_R = -95.4 + 95.4 = 0$

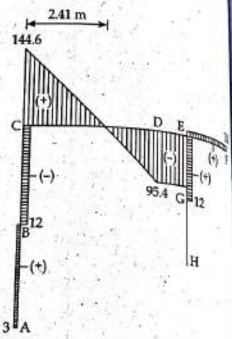


Figure: S.F.D.

Member EF

$(V_E)_L = 0$
$(V_E)_R = 10$ kN
$(V_F)_L = 10$ kN
$(V_F)_R = 10 - 10 = 0$

Member EF

$(V_E)_L = 0$
$(V_E)_R = 12$ kN
$(V_F)_L = 12$ kN
$(V_F)_R = 12 - 12 = 0$
$V_H = 0$

Bending moment
Member AC

$M_A = 0$
$M_B = 3 \times 3 = 9$ kNm
$(M_C)_L = 3 \times 6 - 15 \times 3 = -27$ kNm
$(M_C)_R = -27 + 27 = 0$

Member CE
Portion CD

$\circlearrowleft^{+} \sum M_{D'} = 0$
$M_x = 144.6x - 60 \times x \times \frac{x}{2} - 27 = 144.6x - 30x^2 - 27$
$M_x = 144.6x - 30x^2 - 27$
$\therefore \quad M_x = 0$ at $x = 0.19$ m
$M_{max} = M_{x=2.41} = 144.6 \times 2.41 - 30 \times 2.41^2 - 27 = 147.243$ kNm [$\because$ Other values of x not possible]
$M_{mid} = M_{x=2} = 144.6 \times 2 - 30 \times 2^2 - 27 = 142.2$ kNm
$(M_C)_L = 0$
$(M_C)_R = -27$ kNm
$(M_D)_L = 144.6 \times 4 - 30 \times 4^2 - 27 = 71.4$ kNm

Portion DE

$(M_D)_R = 144.6 \times 4 - 60 \times 4 \times 2 - 27 = 71.4$ kNm
$(M_E)_L = 144.6 \times 5 - 60 \times 4 \times (2 + 1) - 27 = -24$ kNm
$(M_E)_R = -24 + 24 = 0$

Member EF

$(M_E)_L = 0$
$(M_E)_R = -20$ kNm
$M_F = 10 \times 2 - 20 = 0$

Member EH

$(M_E)_L = 0$
$(M_E)_R = -24$ kNm
$M_G = 12 \times 2 + 24 = 0$
$M_H = 12 \times 4 - 12 \times 4 - 24 = 0$

Salient features

$(V_x) = 0$ at $x = 2.41$ m from C
$(M_x) = 0$ at $x = 0.19$ m from C
$M_{max} = 147.243$ kNm

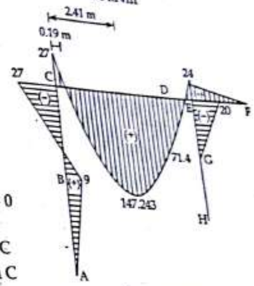
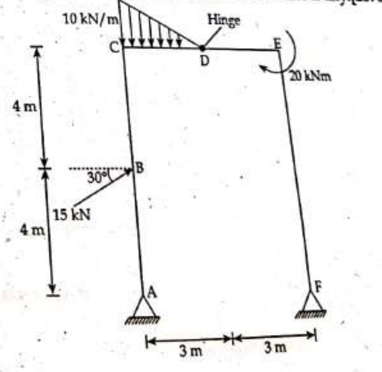


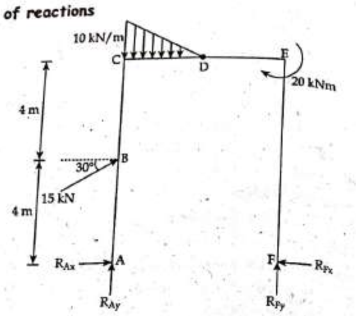
Figure: B.M.D.

19. **Draw axial force, shear force, bending moment diagram for the loaded frame shown in figure. Indicate also the salient features if any.[2070 Bhadra]**



Solution:

Calculation of reactions



$^{+}\rightarrow \sum F_x = 0$

$R_{Ax} - R_{Fx} + 15\cos 30° = 0$

$R_{Ax} - R_{Fx} + 12.99 = 0$ (1)

$^{+}\uparrow \sum F_y = 0$

$R_{Ay} - R_{Fy} + 15\sin 30° - \frac{1}{2} \times 10 \times 3 = 0$

$R_{Ay} - R_{Fy} - 7.5 = 0$ (2)

Taking moment about A;

$^{+}\circlearrowleft \sum M_A = 0$

or, $15\cos 30° \times 4 + \frac{1}{2} \times 10 \times \frac{3}{3}$

$+20 - R_{Fy} \times 6 = 0$

$\therefore \quad R_{Fy} = 14.49 \text{ KN}(\uparrow)$

From equation (2); we have,

$R_{Ay} = 7.5 - 14.49$

$= -6.99$

$\therefore \quad R_{Ay} = 6.99 \text{ kN}(\downarrow)$

Taking right side moment of point D;

$^{+}\circlearrowleft \sum (M_D)_{right} = 0$

or, $20 + R_{fx} \times 8 - R_{Fy} \times 3 = 0$

or, $20 + 8R_{fx} - 14.49 \times 3 = 0$

$\therefore \quad R_{fx} = 2.93 \text{ kN}(\rightarrow)$

From equation (1); we have,

$R_{Ax} - R_{Fx} + 15\cos 30° = 0$

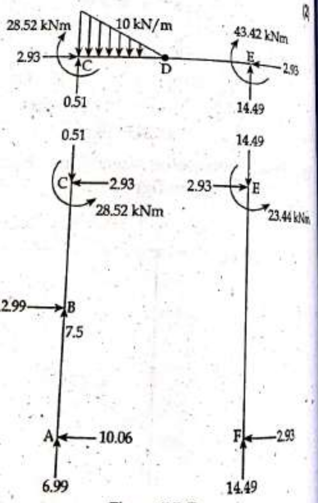


Figure: F.B.D.

or, $R_{Ax} = -10.06 \text{ kN}$

$\therefore \quad R_{Ax} = 10.06 \text{ kN}(\leftarrow)$

Axial force

Member AB

$F_{AB} = 6.99 \text{ kN(T)}$

$F_{BC} = 0.51 \text{ kN(C)}$

Member CE

$F_{CE} = 2.93 \text{ kN(C)}$

Member EF

$F_{EF} = 14.49 \text{ kN(C)}$

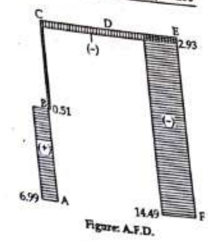


Figure: A.F.D.

Shear force

Member AC

$(V_A)_L = 0$

$(V_A)_R = 10.06 \text{ kN}$

$(V_B)_L = 10.06 \text{ kN}$

$(V_B)_R = 10.06 - 12.99 = -2.93 \text{ kN}$

$(V_C)_L = -2.93 \text{ kN}$

$(V_C)_R = -2.93 + 2.93 = 0$

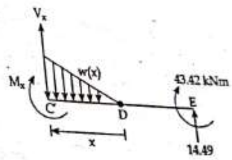
Member CE

Portion CD

By similar triangles; we have,

$\frac{3}{10} \times \frac{x}{w(x)}$

$\therefore \quad w(x) = 3.33x$



$^{+}\uparrow \sum F_y = 0$

$V_x = \frac{1}{2} x \times w(x) - 14.49 = \frac{1}{2} x \times 3.33x - 14.49$

$(V_x) = 1.667x^2 - 14.49$

$V_x = 0$ at $x = 2.95$ m from D

$V_{mid} = V_{x=1.5} = 1.667 \times (1.5)^2 - 14.49 = -10.74 \text{ kN}$

$(V_D)_L = -14.49 \text{ kN}$

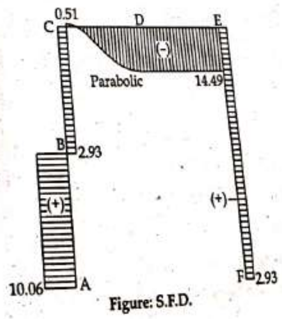


Figure: S.F.D.

$(V_C)_R = 1.667 \times 3^2 - 14.49 = 0.51$ kN

$(V_C)_L = 0.51 - 0.51 = 0$

Portion DE

$(V_E)_R = 0$

$(V_E)_L = -14.49$ kN

$(V_D)_R = -14.49$ kN

Member EF

$(V_E)_L = 0$

$(V_E)_R = 2.93$ kN

$(V_F)_L = 2.93$ kN

$(V_F)_R = 2.93 - 2.93 = 0$

Bending moment

Member AC

$M_A = 0$

$M_B = 10.06 \times 4 = 40.24$ kNm

$(M_C)_L = 10.06 \times 8 - 12.99 \times 4 = 28.52$ kNm

$(M_C)_R = 28.52 - 28.52 = 0$

Member CE

Portion CD

$\circlearrowleft^{+} \sum M_{C'} = 0$

$M_x = 14.49(x + 3) - 43.42 - \frac{1}{2} \times x \times w(x) \times \frac{x}{3}$

$= 14.49x + 43.47 - 43.42 - 0.167\, x^2 \times 3.33x$

$= 14.49x + 0.05 - 0.555\, x^3$

$\therefore \quad M_x = 14.49x - 0.555\, x^3$ [Neglecting 0.05]

$M_x = 0$ at $x = 0$ [∵ Other values of x not possible]

$(M_x)_{max} = M_{x=2.95} = 14.49 \times 2.95 - 0.555 \times (2.95)^3 = 28.5$ kNm

$M_{mid} = M_{x=1.5} = 14.49 \times 1.5 - 0.555 \times 1.5^3 = 19.86$ kNm

$(M_D)_L = 0$

$(M_C)_L = 14.49 \times 3 - 0.555 \times (3)^3 = 28.485 \approx 28.5$ kNm

$(M_C)_R = 28.5 - 28.5 = -0.02 \approx 0$

Portion DE

$(M_E)_R = 0$

$(M_E)_L = -43.42$ kNm

$(M_E)_L = 43.42 + 14.49 \times 3 = 0.05 \approx 0$

Member EF

$(M_E)_L = 0$

$(M_E)_R = -23.44$ kNm

$M_F = -23.44 + 2.93 \times 8 = 0$

Salient features

$(V_x) = 0$ at $x = 2.95$ m from D

$(M_x) = 0$ at $x = 0$, *i.e.*, at D

$M_{max} = 29.5$ kNm

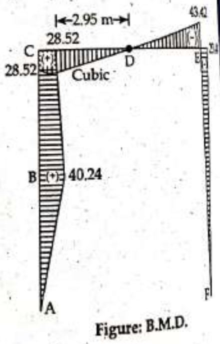
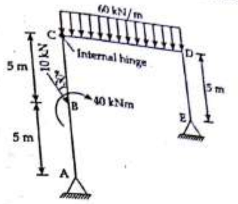


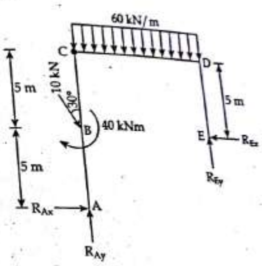
Figure: B.M.D.

20. Draw the axial force, shear force and bending moment diagram of the given frame. [2070 Ashadh]



Solution:

Calculation of reactions



Taking left side moment of C;

$\circlearrowleft^{+} \sum (M_C)_{left} = 0$

or, $-R_{Ax} \times 10 + 40 - 10 \sin 30° \times 5 = 0$

$\therefore \quad R_{Ax} = 1.5$ kN(→)

Taking moment about E;

$\circlearrowleft^{+} \sum M_E = 0$

or, $R_{Ay} \times 8 - R_{Ax} \times 5 - 10 \cos 30° \times 8 - 60 \times 8 \times 4 + 40 = 0$

$\therefore \quad R_{Ay} = 244.6$ kN(↑)

$+\uparrow \sum F_y = 0$

or, $R_{Ay} - 10 \cos 30° - 60 \times 8 + R_{Ey} = 0$

or, $244.6 - 488.66 + R_{Ey} = 0$

$\therefore \quad R_{Ey} = 244.06$ kN(↑)

$+\rightarrow \sum F_x = 0$

or, $R_{Ax} + 10 \sin 30° - R_{Ex} = 0$

Scanned by CamScanner

or, $1.5 - 5 + R_{Ex} = 0$

$\therefore$ $R_{Ex} = 6.5$ kN (←)

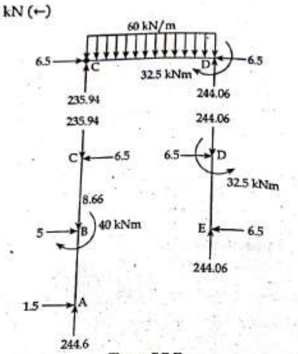


Figure: F.B.D.

Axial force
Member AC

$F_{AB} = 244.6$ kN(C)

$F_{BC} = 235.94$ kN(C)

Member CD

$F_{CD} = 6.5$ kN(C)

Member DE

$F_{DE} = 244.06$ kN(C)

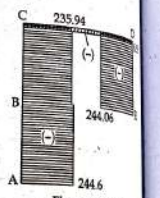


Figure: A.F.D.

Shear force
Member AC

$(V_A)_L = 0$

$(V_A)_R = -1.5$ kN

$(V_B)_L = -1.5$ kN

$(V_B)_R = -1.5 \text{ kN} - 5 = -6.5$ kN

$(V_C)_L = -6.5$ kN

$(V_C)_R = -6.5 + 6.5 = 0$

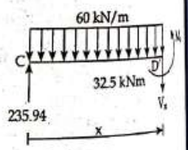
Member CD

$+\uparrow \sum F_y = 0$

$V_x = 235.94 - 60x$

$V_x = 0$ at $x = 3.93$ m

$(V_C)_L = 0$



$(V_C)_R = 235.94$ kN

$(V_D)_L = 235.94 - 60 \times 8 = -244.06$ kN

$(V_D)_R = -244.06 + 244.06 = 0$

Member DE

$(V_D)_L = 0$

$(V_D)_R = 6.5$ kN

$(V_E)_L = 6.5$ kN

$(V_E)_R = 6.5 - 6.5 = 0$

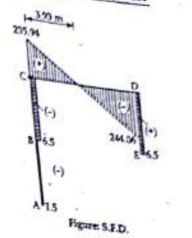


Figure: S.F.D.

Bending moment
Member AC

$M_A = 0$

$(M_B)_L = -1.5 \times 5 = -7.5$ kNm

$(M_B)_R = 40 - 7.5 = 32.5$ kNm

$M_C = -1.5 \times 10 - 5 \times 5 + 40 = 0$

Member CD

$+\circlearrowleft \sum M_{D'} = 0$

$M_x = 235.94x - 60 \times x \times \frac{x}{2} = 235.94x - 30x^2$

$\therefore$ $M_x = 235.94x - 30x^2$

$M_x = 0$ at $x = 7.86$ m and $x = 0$

$M_{max} = M_{x=3.93} = 235.94 \times 3.93 - 30 \times 3.93^2 = 463.9$ kNm

$M_{mid} = M_{x=4} = 235.94 \times 4 - 30 \times 4^2 = 463.76$ kNm

$M_C = 0$

$(M_D)_L = 235.94 \times 8 - 30 \times 8^2$

$= -32.5$ kNm

$(M_D)_R = -32.5 + 32.5 = 0$

Member DE

$(M_D)_L = 0$

$(M_D)_R = -32.5$ kNm

$M_E = -32.5 + 6.5 \times 5 = 0$

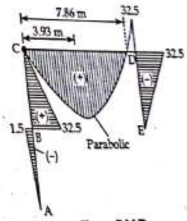


Figure: B.M.D.

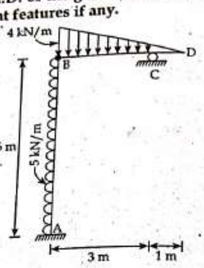
Salient features

$(V_x) = 0$ at $x = 3.93$ m from C

$(M_x) = 0$ at $x = 0$ and $x = 7.86$ m from C

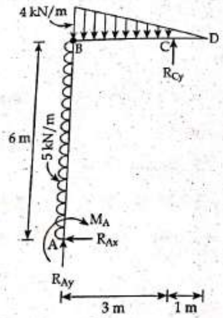
$M_{max} = 463.9$ kNm

21. **Deduce the relationship between load, shear force and bending moment for a beam section loaded uniformly with intensity of load, W. Draw A.F.D., S.F.D. and B.M.D. of the given frame load as shown in the figure. Also indicate the salient features if any.** [2070 Magh]



Solution:

Calculation of reactions



Taking moment about right of B; we have,

$$(^{+}\circlearrowleft)\sum(M_B)_{right} = 0$$

or, $\frac{1}{2}\times 4\times 4\times\frac{1}{3}\times 4 - R_{Cy}\times 3 = 0$

$\therefore$ $R_{Cy} = \frac{32}{9} = 3.56\text{ kN}\ (\uparrow)$

$(^{+}\rightarrow)\sum F_x = 0$

or, $5\times 6 - R_{Ax} = 0$

$\therefore$ $R_{Ax} = 30\text{ kN}\ (\leftarrow)$

Also, taking moment about left of B; we have,

$$(^{+}\circlearrowleft)\sum(M_B)_{left} = 0$$

or, $M_A - 5\times 6\times\frac{1}{2}\times 6 + R_{Ax}\times 6 = 0$

or, $M_A - 90 + 30\times 6 = 0$

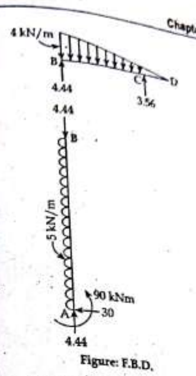


Figure: F.B.D.

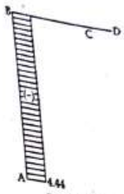


Figure: A.F.D.

$\therefore$ $M_A = -90\text{ kNm} = 90\text{ kNm}\ (\circlearrowright)$

and, $(^{+}\uparrow)\sum F_y = 0$

or, $R_{Ay} + R_{Cy} - \frac{1}{2}\times 4\times 4 = 0$

or, $\frac{32}{9} + R_{Ay} - 8 = 0$

$\therefore$ $R_{Ay} = \frac{40}{9} = 4.44\text{ kN}\ (\uparrow)$

Axial force

Member AB

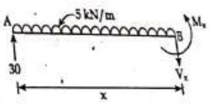
$F_{AB} = 4.44\text{ kN (C)}$

Member BD

$F_{BD} = 0\text{ kN}$

Shear force

Member AB



$$(^{+}\uparrow)\sum F_y = 0$$

or, $V_x = 30 - 5x$

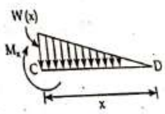
$V_x = 0$ at $x = 6\text{ m}$

$(V_A)_L = 0$

$(V_A)_R = 30\text{ kN}$

$V_B = 30 - 6\times 5 = 0\text{ kN}$

Member DC ($0 \le x \le 1$)



Note: 4 times than original

$\frac{w(C)}{1} = \frac{4}{4}$

or, $w(C) = 1\text{ kN/m}$

and, $\frac{w(x)}{x} = \frac{1}{1} = x$ kN/m

Since, w at point C = 1 kN/m

$$(^+\uparrow)\sum F_y = 0$$

or, $V_x = \frac{1}{2} \times x \times x = \frac{1}{2}x^2$

$\therefore$ $V = 0$ at $x = 0$

$$(V_C)_R = \frac{1}{2} \times (1)^2 = 0.5 \text{ kN}$$

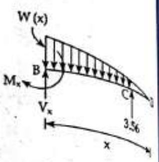
Member CB ($1 \le x \le 4$)

$$\frac{w(x)}{x} = \frac{4}{4}$$

$\therefore$ $w(x) = x$ kN/m

$$(^+\uparrow)\sum F_y = 0$$

$$V_x = \frac{1}{2} \times x \times x - 3.56 = -3.56 + 0.5x^2$$



Shear force is zero at;

$V_x = 0$

or, $-3.56 + 0.5x^2 = 0$

$\therefore$ $x = 2.67$ m

i.e., $V_x = 0$ at $x = 2.67$ m

$$(V_C)_L = -3.56 + \frac{1}{2} \times (1)^2 = -3.08 \text{ kN}$$

$$(V_B)_R = -3.56 + \frac{1}{2} \times (4)^2 = 4.44 \text{ kN}$$

$$(V_B)_L = 4.44 - 4.44 = 0 \text{ kN}$$

Bending moment

Member AB

$$M_x = -90 + 30x - 5x \times \frac{x}{2}$$
$$= 30x - 2.5x^2 - 90$$

Moment is zero at;

$M_x = 0$

or, $30x - 2.5x^2 - 90 = 0$

$\therefore$ $x = 6$ m and $x = -2.5$ m

$\therefore$ $M_x = 0$ at $x = 6$ m

$(M_A)_L = 0$

$(M_A)_R = -90$ kNm

$M_B = -90 + 30 \times 6 - 5 \times 6 \times 3 = 0$ kNm

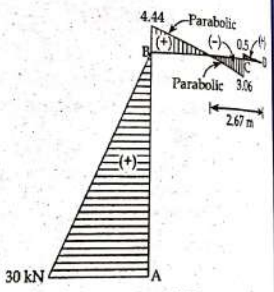


Figure: S.F.D.

Member DC

$$M_x = -\frac{1}{2} \times x \times x \times \frac{x}{3} = -\frac{1}{6}x^3$$

$$M_C = -\frac{1}{6} \times (1)^3 = -0.167 \text{ kNm}$$

For DC

$$M_x = -\frac{1}{2} \times x \times x \times \frac{x}{3} + 3.56(x-1)$$

$\therefore$ $M_x = -\frac{x^3}{6} + 3.56(x-1)$

Moment is zero at;

$M_x = 0$

or, $-\frac{x^3}{6} + 3.56x - 3.56 = 0$

$\therefore$ $x = 1.05$ m, 4 m and $x = -5.06$ m

Thus, $M_x = 0$ at $x = 1.05$ m and $x = 4$ m.

$$(M_C)_L = -\frac{(1)^3}{6} + 3.56(1-1) = -0.167 \text{ kNm}$$

$$(M_B)_R = -\frac{(4)^3}{6} + 3.56(4-1) = 0 \text{ kNm}$$

$$M_{max} = M_{x=2.67} = -\frac{(2.67)^3}{6} + 3.56(2.67-1) = 2.77 \text{ kNm}$$

$$M_{mid} = M_{x=2} = -\frac{(2)^3}{6} + 3.56(2-1) = 2.23 \text{ kNm}$$

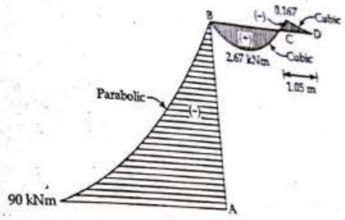
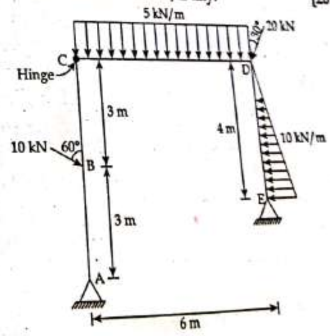


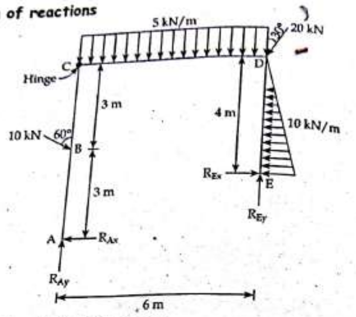
Figure: B.M.D.

22. **Draw axial force, shear force and bending moment diagram for the given frame. Also, indicate salient features, if any.** [2071 Shrawan]



Solution:

Calculation of reactions



Taking moment about left of C; we get,

$(^{+}\circlearrowleft)\ \sum(M_C)_{left} = 0$

or, $R_{Ax} \times 6 - (10\sin 60°) \times 3 = 0$

$\therefore$ $R_{Ax} = 4.33$ kN ($\leftarrow$)

Also,

$(^{+}\rightarrow) \sum F_x = 0$

or, $-R_{Ax} + 10\sin 60° - \left(\frac{1}{2} \times 10 \times 4\right) - 20\sin 30° + R_{Ex} = 0$

or, $-4.33 + 10\sin 60° - 20 - 20\sin 30° + R_{Ex} = 0$

$\therefore$ $R_{Ex} = 25.67$ kN ($\rightarrow$)

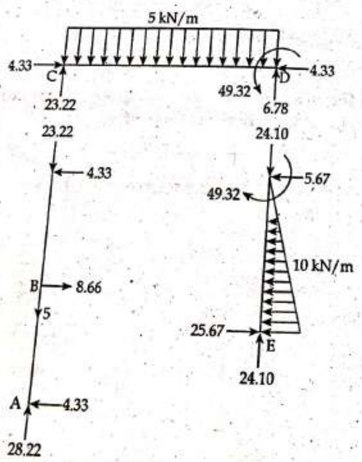


Figure: F.B.D.

$(^{+}\uparrow) \sum F_y = 0$

or, $R_{Ay} + R_{Ey} - 10\cos 60° - (5 \times 6) - 20\cos 30° = 0$

$\therefore$ $R_{Ay} + R_{Ey} = 52.32$ kN

Taking moment about right of C; we get,

$(^{+}\circlearrowleft) \sum(M_C)_{right} = 0$

or, $(5 \times 6 \times 3) + (20\cos 30°) \times 6 + \left(\frac{1}{2} \times 10 \times 4 \times \frac{2}{3} \times 4\right) - (R_{Ey} \times 6) - (R_{Ex} \times 4) = 0$ (1)

or, $R_{Ey} \times 6 = 247.26 - (4 \times 25.67)$

$\therefore$ $R_{Ey} = 24.10$ kN ($\uparrow$)

From equation (1); we get,

$R_{Ay} + 24.10 = 52.32$

$\therefore$ $R_{Ay} = 28.22$ kN ($\uparrow$)

Axial force

Member AB

$F_{AB} = 28.22$ (C)

$F_{BC} = 23.22$ (C)

Member CD

$F_{CD} = 4.33$ (C)

Member DE

$F_{DE} = 24.10$ (C)

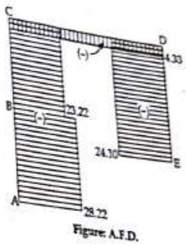


Figure: A.F.D.

Shear force

Member AC

$(V_A)_L = 0$

$(V_A)_R = 4.33$ kN

$(V_B)_L = 4.33$ kN

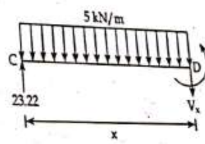
$(V_B)_R = 4.33 - 8.66 = -4.33$ kN

$(V_C)_L = -4.33$ kN

$(V_C)_R = -4.33 + 4.33 = 0$ kN

Member CD

$V_x = 23.22 - 5x$



Shear force is zero at;

$V_x = 0$

or, $23.22 - 5x = 0$

$\therefore$ $x = 4.64$ m

$\therefore$ $V_x = 0$ at $x = 4.64$ m

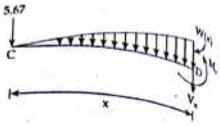
$(V_C)_L = 0$

$(V_C)_R = 23.22$ kN

$(V_D)_L = 23.22 - 30 = -6.78$ kN

$$(V_D)_R = -6.78 + 6.78 = 0\ \text{kN}$$

Member DE



$$\frac{w(x)}{x} = \frac{10}{4}$$

$\therefore$ $w(x) = 2.5x$

$$V_x = -5.67 - \frac{1}{2} \times x \times 2.5x$$
$$= -5.67 - 1.25x^2$$

$$V_x = 0$$

or, $-5.67 - 1.25x^2 = 0$

$\therefore$ x = No case possible since imaginary number

$$(V_D)_L = 0$$
$$(V_D)_R = 4.33 - 20\sin 30° = -5.67\ \text{kN}$$
$$(V_E)_L = -5.67 - \frac{1}{2} \times 4 \times 10 = -25.67\ \text{kN}$$
$$(V_E)_R = -25.67 + 25.67 = 0\ \text{kN}$$

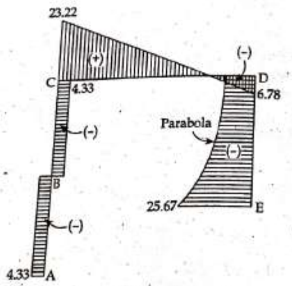


Figure: S.F.D.

Bending moment

Member AC

$$M_A = 0$$
$$M_B = 4.33 \times 3 = 12.99\ \text{kNm}$$
$$M_C = (4.33 \times 6) - (8.66 \times 3) = 0$$

Member CD

$$M_x = 23.22x - 2.5x^2$$

$M_x = 0$ at;

or, $23.22x - 2.5x^2 = 0$

$\therefore$ $x = 0$ and $x = 9.28$ m

(Since 9.28 is greater than span 6 m so no zero moment in CD)

$$M_{max} = M_{x=4.64} = (23.22 \times 4.64) - [2.5 \times (4.64)^2] = 53.92\ \text{m}$$

$$M_C = 0$$

$$(M_D)_L = (23.22 \times 6) - \left(5 \times 6 \times \frac{6}{2}\right) = 49.32\ \text{kNm}$$
$$(M_D)_R = 49.32 - 49.32 = 0$$

Member DE

$$M_x = -5.67x - \left(1.25x^2 \times \frac{2x}{3}\right)$$
$$= -5.67x - 0.83x^3$$

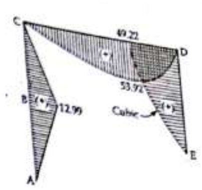


Figure: B.M.D.

Moment is zero at;

$$M_x = 0$$

or, $-5.67x - 0.83x^3 = 0$

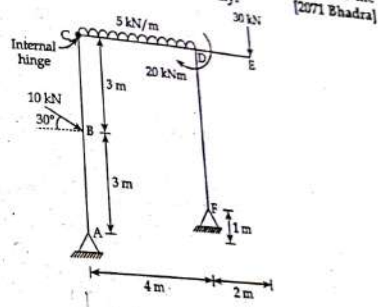
$\therefore$ $x = -6.83$ m

and, $x = -0.83$ m

Thus, moment is not zero in span DE.

$$(M_D)_L = 0\ \text{kNm}$$
$$M_R = 49.32\ \text{kNm}$$
$$M_E = 49.32 - \left(\frac{1}{2} \times 4 \times 10 \times \frac{1}{3} \times 4\right) - (5.67 \times 4) = 0\ \text{kNm}$$

23. **Draw the axial force, shear force and bending moment diagram of the given frame. Indicate also the salient feature, if any.** [2071 Bhadra]



Solution:

Calculation of reactions

Taking moment about left to C; we get,

$$(^+\circlearrowleft) \sum (M_C)_{left} = 0$$

or, $R_{Ax} \times 6 - (10\cos 30°) = 0$

$\therefore$ $R_{Ax} = 4.33$ kN (←)

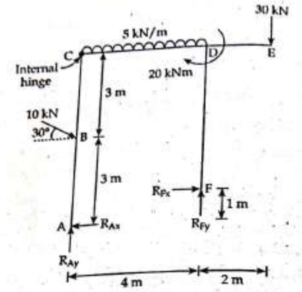
Also,

$$(^+\rightarrow) \sum F_x = 0$$

or, $-R_{Ax} + 10\cos 30° + R_{Fx} = 0$

or, $-4.33 + 8.66 + R_{Fx} = 0$

$\therefore$ $R_{Fx} = 4.33$ kN (←)



and, $(^{+}\uparrow)\sum F_y = 0$

or, $R_{Ay} + R_{Fy} - (5 \times 4) - 30 - 10\sin 30° = 0$

$\therefore$ $R_{Ay} + R_{Fy} = 55$ (1)

Again, taking moment about right of C; we get,

$(^{+}\circlearrowleft)\sum (M_C)_{right} = 0$

or, $(5 \times 4 \times 2) + 20 + (30 \times 6) + (4.33 \times 5) - (R_{Fy} \times 4) = 0$

$\therefore$ $R_{Fy} = 65.41$ kN (↑)

Putting the values of R_{Fy} in the equation (1); we get,

$R_{Ay} + 65.41 = 55$

$\therefore$ $R_{Ay} = -10.41 = 10.41$ kN (↓)

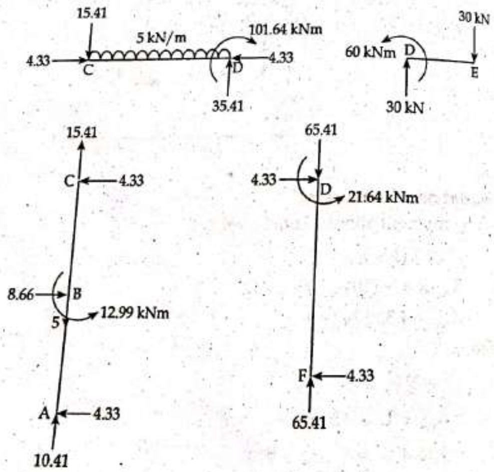


Figure: F.B.D.

Axial force

Member AB

$F_{AB} = 10.41$ kN (T)

$F_{BC} = 5 + 10.41 = 15.41$ kN (T)

Member CD

$F_{CD} = 4.33$ kN (C)

Member DF

$F_{DF} = 65.41$ kN (C)

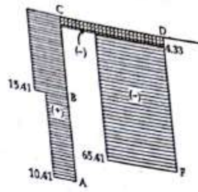


Figure: A.F.D.

Shear force

Member AC

$(V_A)_L = 0$

$(V_A)_R = 4.33$ kN

$(V_B)_L = 4.33$ kN

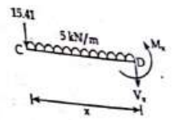
$(V_B)_R = 4.33 - 10\cos 30° = -4.33$ kN

$(V_C)_L = -4.33$ kN

$(V_C)_R = -4.33 + 4.33 = 0$ kN

Member CD

$V_x = -15.41 - 5x$



Shear force is zero at;

$V_x = 0$

or, $-15.41 - 5x = 0$

$\therefore$ $x = -3.08$ m

$\therefore$ $V_x = 0$ at $x = -3.08$ m

$(V_C)_L = 0$

$(V_C)_R = -15.41$ kN

$(V_D)_L = -15.41 - (5 \times 4) = -35.41$ kN

$(V_D)_R = -35.41 + 35.41 = 0$ kN

Member DF

$(V_D)_L = 0$

$(V_D)_R = 4.33$ kN

$(V_E)_L = 4.33$ kN

$(V_E)_R = 4.33 - 4.33 = 0$ kN

Member ED

$(V_E)_R = 0$

$(V_E)_L = 30$ kN

$(V_D)_R = 30$ kN

$(V_D)_L = 30 - 30 = 0$ kN

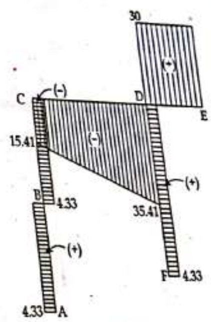


Figure: S.F.D.

Bending moment

Member AC

$M_A = 0$

$M_B = 4.33 \times 3 = 12.99$ kNm

$$M_C = (4.33 \times 6) - (8.66 \times 3) = 0 \text{ kNm}$$

Member CD

$$M_x = -15.41x - \left(5x \times \frac{x}{2}\right) = -15.41x - 2.5x^2$$

Moment is zero at;

$$M_x = 0$$

or, $-15.41x - 2.5x^2 = 0$

or, $x(15.41 + 2.5x) = 0$

$\therefore$ $x = 0$ and $x = -6.16$ m

Thus, moment is not zero in span CD.

$$\begin{aligned} M_{mid} &= M_{x=2} \\ &= [(-15.41) \times 2] - [2.5 \times (2)^2] \\ &= -40.82 \text{ kNm} \end{aligned}$$

$$\begin{aligned} (M_D)_L &= (15.41 \times 4) - (5 \times 4 \times 2) \\ &= -101.64 \text{ kNm} \end{aligned}$$

$$(M_D)_R = -101.64 + 101.64 = 0 \text{ kNm}$$

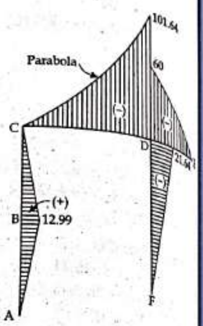


Figure: B.M.D.

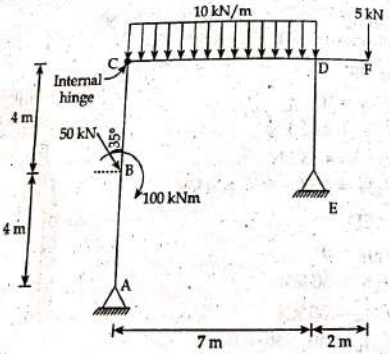
Member DE

$$M_E = 0 \text{ kNm}$$
$$(M_D)_L = -30 \times 2 = -60 \text{ kNm}$$
$$(M_D)_R = -60 + 60 = 0 \text{ kNm}$$

Member DE

$$(M_D)_L = 0 \text{ kNm}$$
$$(M_D)_R = -101.64 + 60 + 20 = -21.64 \text{ kNm}$$
$$M_F = -21.64 + (4.33 \times 5) = 0 \text{ kNm}$$

24. **Draw the axial force, shear force and bending moment diagram of the given frame. Also show the salient feature.** [2071 Magh]

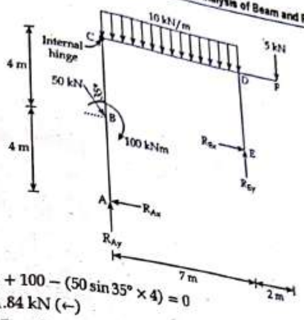


Solution:

Calculation of reactions

Taking moment about left of C; we get,

$$(^+\circlearrowleft) \sum (M_C)_{left} = 0$$



or, $R_{Ax} \times 8 + 100 - (50 \sin 35° \times 4) = 0$

$\therefore$ $R_{Ax} = 1.84$ kN $(\leftarrow)$

$$(^+\rightarrow) \sum F_x = 0$$

or, $-R_{Ax} + 50 \sin 35° + R_{Ex} = 0$

$\therefore$ $R_{Ex} = -50 \sin 35° + 1.84 = -26.84$ kN

i.e., $R_{Ex} = 26.84$ kN $(\leftarrow)$

Also, taking moment about right of C; we get,

$$(^+\circlearrowleft) \sum (M_C)_{right} = 0$$

or, $(R_{Ex} \times 4) - (R_{Ey} \times 7) + (5 \times 9) + (10 \times 7 \times 3.5) = 0$

or, $(26.84 \times 4) - (R_{Ey} \times 7) + 45 + 245 = 0$

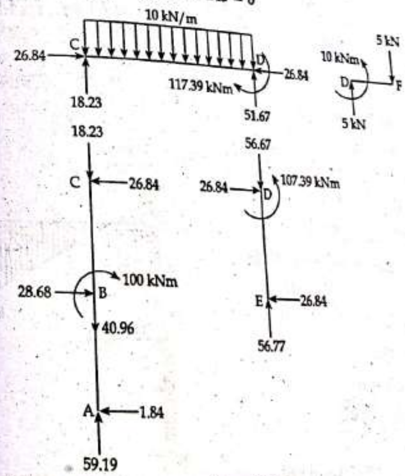


Figure: F.B.D.

$\therefore\quad R_{Ey} = 56.77\text{ kN}(\uparrow)$

and, $(^{+}\uparrow)\sum F_y = 0$

or, $\quad R_{Ay} + R_{Ey} - 50\cos 35° - (10 \times 7) - 5 = 0$

$\therefore\quad R_{Ay} = 59.19\text{ kN}(\uparrow)$

Axial force

Member AC

$F_{AB} = 59.19\text{ kN (C)}$

$F_{BC} = 59.19 - 40.96 = 18.23\text{ kN (C)}$

Member CD

$F_{CD} = 26.84\text{ kN (C)}$

Member DE

$F_{DE} = 51.67 + 5 = 56.67\text{ kN (C)}$

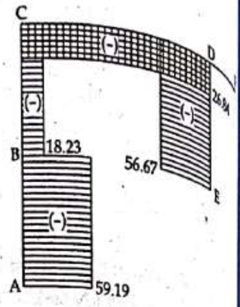


Figure: A.F.D.

Shear force

Member AC

$(V_A)_L = 0$

$(V_A)_R = 1.84\text{ kN}$

$(V_B)_L = 1.84\text{ kN}$

$(V_B)_R = 1.84 - 28.68 = -26.84\text{ kN}$

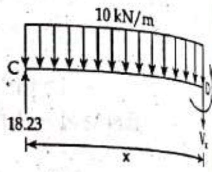
$(V_C)_L = -26.84\text{ kN}$

$(V_C)_R = -26.84 + 26.84 = 0\text{ kN}$

Member CD

$(^{+}\uparrow)\sum F_y = 0$

or, $\quad V_x = 18.23 - 10x$



Shear force is zero at;

$V_x = 0$

or, $\quad 18.23 - 10x = 0$

$\therefore\quad x = 1.82\text{ m}$

Thus, $V_x = 0$ at $x = 1.82$ m.

$(V_C)_L = 0$

$(V_C)_R = 18.23\text{ kN}$

$(V_D)_L = 18.23 - (7 \times 10)$

$= -51.67\text{ kN}$

$(V_D)_R = -51.67 + 51.67 = 0\text{ kN}$

Member DF

$(V_F)_R = 0$

$(V_F)_L = 5\text{ kN}$

$(V_D)_R = 5\text{ kN}$

$(V_D)_L = 5 - 5 = 0\text{ kN}$

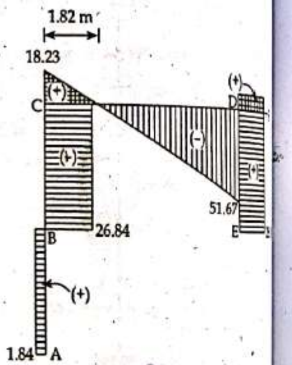


Figure: S.F.D.

Member DE

$(V_D)_L = 0$

$(V_D)_R = 26.84\text{ kN}$

$(V_E)_L = 26.84\text{ kN}$

$(V_E)_R = 26.84 - 26.84 = 0\text{ kN}$

Bending moment

Member AC

$M_A = 0$

$(M_B)_L = 1.84 \times 4 = 7.36\text{ kNm}$

$(M_B)_R = (1.84 \times 4) + 100 = 107.36\text{ kNm}$

$M_C = (1.84 \times 8) + 100 - (28.68 \times 4) = 0\text{ kNm}$

Member CD

$M_x = 18.23x - \left(10x \times \frac{x}{2}\right) = 18.23x - 5x^2$

Moment is zero at;

$M_x = 0$

or, $\quad 18.23x - 5x^2 = 0$

$\therefore\quad x = 0$

and, $\quad x = 3.65\text{ m}$

$\therefore\quad M_x = 0$ at $x = 0$ and $x = 3.65$ m

$M_{max} = M_{x=1.82} = (18.23 \times 1.82) - [5 \times (1.82)^2] = 16.62\text{ kNm}$

$M_{mid} = M_{x=3.5} = (18.23 \times 3.5) - [5 \times (3.5)^2] = 2.56\text{ kNm}$

$(M_D)_L = (18.23 \times 7) - (70 \times 3.5) = -117.39\text{ kNm}$

$(M_D)_R = -117.39 + 117.39 = 0\text{ kNm}$

Member DF

$M_F = 0$

$(M_D)_L = (-5) \times 2 = -10\text{ kNm}$

$(M_D)_R = -10 + 10 = 0\text{ kNm}$

Member DE

$(M_D)_L = 0\text{ kNm}$

$(M_D)_R = -117.39 + 10 = -107.39\text{ kNm}$

$M_E = -107.39 + 107.39 = 0\text{ kNm}$

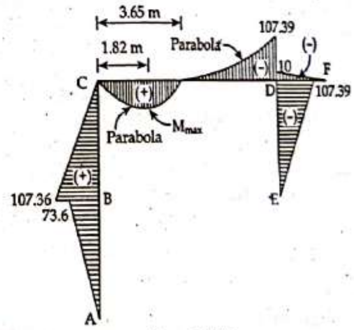
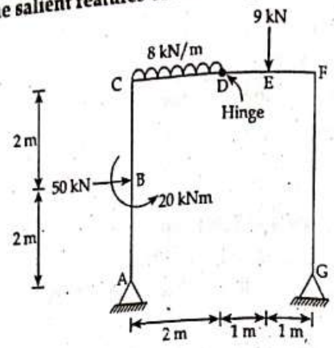


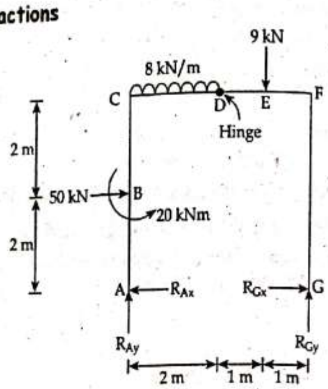
Figure: B.M.D.

25. A frame is loaded as shown in the figure. Draw A.F.D., S.F.D. and B.M.D. and also show the salient features of each diagram. [2071 Chaitra]



Solution:

Calculation of reactions



Taking moment about G; we get,

$(^+\circlearrowleft)\ \Sigma(M_G) = 0$

or, $(R_{Ay} \times 4) + (50 \times 2) - 20 - (8 \times 2 \times 3) - (9 \times 1) = 0$

$\therefore$ $R_{Ay} = -5.75\ kN = 5.75\ kN\ (\downarrow)$

$(^+\uparrow) \sum F_y = 0$

or, $R_{Ay} + R_{Gy} - 9 - (8 \times 2) = 0$

$\therefore$ $R_{Gy} = 30.75\ kN(\uparrow)$

Also, taking moment about right of D; we get,

$(^+\circlearrowleft) \sum (M_D)_{right} = 0$

or, $R_{Gy} \times 2 + (9 \times 1) - (R_{Gx} \times 4) = 0$

or, $-30.75 \times 2 + 9 - (R_{Gx} \times 4) = 0$

$\therefore$ $R_{Gx} = 13.125\ kN\ (\leftarrow)$

and, $(^+\rightarrow) \sum F_x = 0$

or, $-R_{Ax} + R_{Gx} + 50 = 0$

$-R_{Ax} - 13.125 + 50 = 0$

or, $R_{Ax} = 36.875\ kN\ (\leftarrow)$

$\therefore$

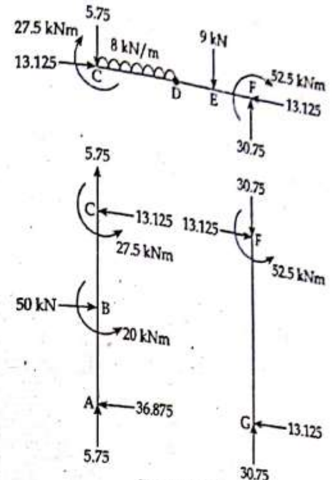


Figure: F.B.D.

Axial force

Member AC

$F_{AC} = 5.75\ kN\ (T)$

Member CF

$F_{CF} = 13.25\ kN\ (C)$

Member FG

$F_{FG} = 30.75\ kN\ (C)$

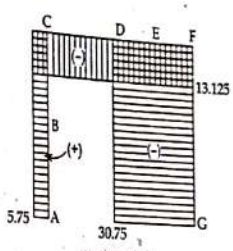


Figure: A.F.D.

Shear force

Member AC

$(V_A)_L = 0$

$(V_A)_R = 36.875\ kN$

$(V_B)_L = 36.875 - 50 = -13.125\ kN$

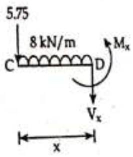
$(V_C)_L = -13.125\ kN$

$(V_C)_R = -13.125 + 13.125 = 0\ kN$

Member CF

Portion CD

$(^+\uparrow) \sum F_y = 0$



or, $V_x = -5.75 - 8x$

Shear force is zero at;

$V_x = 0$

or, $-5.75 - 8x = 0$

$\therefore$ $x = -0.72\ m$

i.e., $V_x = 0$ at $x = -0.72\ m$

$(V_C)_L = 0$
$(V_C)_R = -5.75$ kN
$V_D = -5.75 - (8 \times 2) = -21.75$ kN
$(V_E)_L = -21.75$ kN
$(V_E)_R = -21.75 - 9 = -30.75$ kN
$(V_F)_L = -30.75$ kN
$(V_F)_R = -30.75 + 30.75 = 0$ kN

Member FG

$(V_F)_L = 0$ kN
$(V_F)_R = 13.25$ kN
$(V_G)_L = 13.25$ kN
$(V_G)_R = 13.25 - 13.25 = 0$ kN

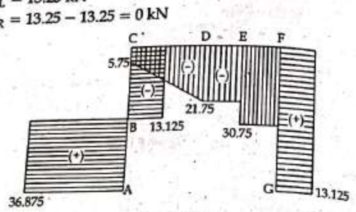


Figure: S.F.D.

Bending moment

Member AC

$M_A = 0$
$(M_B)_L = 36.875 \times 2 = 73.75$ kNm
$(M_B)_R = 73.75 - 20 = 53.75$ kNm
$(M_C)_L = (36.875 \times 4) - 20 - (50 \times 2) = 27.5$ kNm
$(M_C)_R = 27.5 - 27.5 = 0$ kNm

Member CF

$(M_C)_L = 0$ kNm

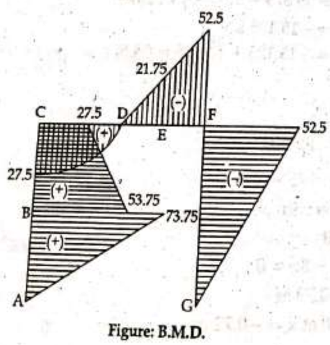


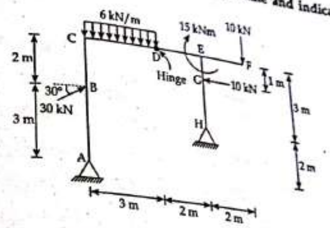
Figure: B.M.D.

$(M_C)_R = 27.5$ kNm
$M_D = [(-5.75) \times 2] - (8 \times 2 \times 1) + 27.5 = 0$ kNm
$M_E = [(-5.75) \times 3] - (8 \times 2 \times 2) + 27.5 = -21.75$ kNm
$(M_F)_L = [(-5.75) \times 4] - (8 \times 2 \times 3) - (9 \times 1) + 27.5 = -52.5$ kNm
$(M_F)_R = -52.5 + 52.5 = 0$ kNm

Member FG

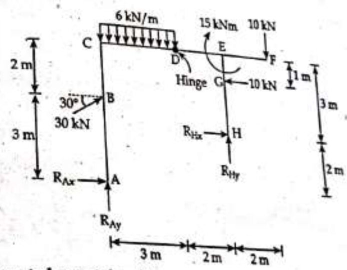
$(M_F)_L = 0$ kNm
$(M_F)_R = -52.5$ kNm
$M_G = -52.5 + (13.125 \times 4) = 0$ kNm

26. **Draw A.F.D., S.F.D. and B.M.D. of the given frame and indicate salient features.**



Solution:

Calculation of reactions



Taking moment about right of D; we get,

$$(^+\circlearrowleft) \sum (M_D)_{right} = 0$$

or, $(10 \times 4) + 15 + (10 \times 1) - (R_{Hx} \times 3) - (R_{Hy} \times 2) = 0$

or, $3R_{Hx} + 3R_{Hy} = 65$ (1)

Also, taking moment about A; we get,

$$(^+\circlearrowleft) \sum (M_A) = 0$$

or, $(R_{Hx} \times 2) - (R_{Hy} \times 5) + (30 \cos 30° \times 3) + (6 \times 3 \times 1.5) + 15 + (10 \times 7) - (10 \times 4) = 0$

or, $2R_{Hx} - 5R_{Hy} = -149.94$

Solving equation (1) and (2); we get,

$R_{Hx} = 1.32\ \text{kN}\ (\rightarrow)$

$R_{Hy} = 30.52\ \text{kN}\ (\uparrow)$

$(^{+}\rightarrow) \sum F_x = 0$

or, $R_{Hx} + 30\cos 30^\circ - 10 - R_{Ax} = 0$

or, $1.32 + 30\cos 30^\circ - 10 - R_{Ax} = 0$

$\therefore$ $R_{Ax} = 17.3\ \text{kN}\ (\leftarrow)$

and, $(^{+}\uparrow) \sum F_y = 0$

or, $R_{Ay} + R_{Hy} - 30\cos 30^\circ - (6 \times 3) - 10 = 0$

or, $R_{Ay} + 30.52 - 30\cos 30^\circ - 18 - 10 = 0$

$\therefore$ $R_{Ay} = 12.48\ \text{kN}(\uparrow)$

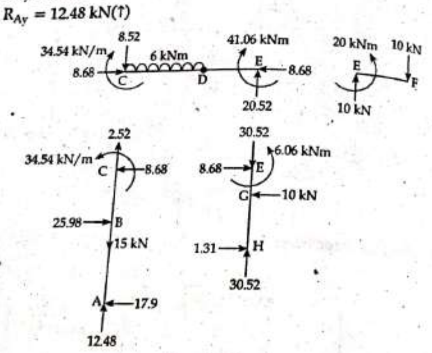


Figure: F.B.D.

Axial force

Member AC

$F_{AB} = 12.48\ \text{kN (C)}$

$F_{BC} = -12.48\ \text{kN} + 15 = 2.52\ \text{(T)}$

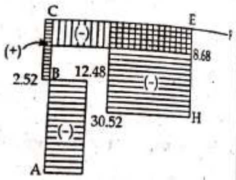


Figure: A.F.D.

Member CE

$F_{CE} = 8.68\ \text{kN (C)}$

Member EH

$F_{EH} = 20.52 + 10 = 30.52\ \text{kN (C)}$

Shear force

Member AC

$(V_A)_L = 0$

$(V_A)_R = 17.3\ \text{kN}$

$(V_B)_L = 17.3\ \text{kN}$

$(V_B)_R = 17.3 - 30\cos 30^\circ = -8.68\ \text{kN}$

$(V_C)_L = -8.68\ \text{kN}$

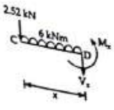
$(V_C)_R = -8.68 + 8.68 = 0\ \text{kN}$

Member CF

Portion CD

$(^{+}\uparrow) \sum F_y = 0$

or, $V_x = -2.52 - 6x$



Shear force is zero at;

$V_x = 0$

or, $-2.52 - 6x = 0$

$\therefore$ $x = -0.42\ \text{m}$

$\therefore$ $V_x = 0$ at $x = -0.42$ m away from C

$(V_C)_L = 0$

$(V_C)_R = -2.52\ \text{kN}$

$V_D = -2.52 - (6 \times 3) = -20.52\ \text{kN}$

$(V_D)_L = -20.52\ \text{kN}$

$(V_D)_R = -20.52 + 20.52 = 0\ \text{kN}$

Member EF

$(V_F)_R = 0\ \text{kN}$

$(V_F)_L = 10\ \text{kN}$

$(V_E)_R = 10\ \text{kN}$

$(V_E)_L = 10 - 10 = 0\ \text{kN}$

Member EH

$(V_E)_L = 0\ \text{kN}$

$(V_E)_R = 8.68\ \text{kN}$

$(V_G)_L = 8.68\ \text{kN}$

$(V_G)_R = 8.68 - 10 = -1.32\ \text{kN}$

$(V_H)_L = -1.32\ \text{kN}$

$(V_H)_R = -1.32 + 1.32 = 0\ \text{kN}$

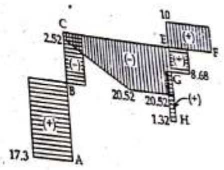


Figure: S.F.D.

Bending moment

Member AC

$M_A = 0\ \text{kNm}$

$M_B = 17.3 \times 3 = 51.3\ \text{kNm}$

$(M_C)_L = 17.3 \times 5 - (25.98 \times 2) = 34.54\ \text{kNm}$

$(M_C)_R = 34.54 - 34.54 = 0\ \text{kNm}$

Member CE

Portion CD

$M_x = 34.54 - 2.52x - 3x^2$

Moment is zero at;

$M_x = 0$

or, $34.54 - 2.52x - 3x^2 = 0$

$\therefore$ $x = 3\ \text{m}$

$\therefore \quad M_x = 0$ at $x = 3$ m

$(M_A)_L = 0$ kNm

$(M_A)_R = 34.54$ kNm

$M_{mid} = M_{x=1.5} = 34.54 - (2.52 \times 1.5) - [3 \times (1.5)^2] = 24.04$ kNm

$= 34.54 - (2.52 \times 3) - [3 \times (3)^2] = 0$ kNm

$M_D = 34.54 - (2.52 \times 5) - (6 \times 3 \times 3.5) = -41.06$ kNm

$(M_E)_L = 34.54 - (2.52 \times 5) - (6 \times 3 \times 3.5) = -41.06$ kNm

$(M_E)_R = -41.06 + 41.06 = 0$ kNm

Member EF

$M_F = 0$

$(M_E)_L = -10 \times 2 = -20$ kNm

$(M_E)_R = -20 + 20 = 0$ kNm

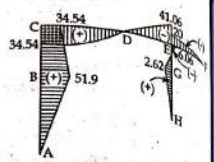


Figure: B.M.D.

Member EH

$(M_F)_L = 0$ kNm

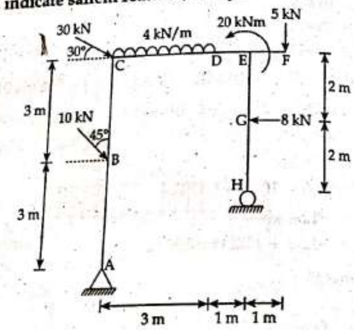
$(M_F)_R = -41.06 + 20 + 15$

$= -6.06$ kNm

$M_G = -6.06 + (8.68 \times 1) = 2.62$ kNm

$M_H = -6.06 + (8.68 \times 3) - (10 \times 2) = 0$ kNm

27. **Draw A.F.D., S.F.D. and B.M.D. of the given frame loaded as shown in the figure. Also, indicate salient features, if any.** [2072 Kartik]



Solution:

Calculation of reactions

$(^+\rightarrow) \sum F_x = 0$

or, $10 \sin 45° - 8 + 30 \cos 30° - R_{Ax} = 0$

$\therefore \quad R_{Ax} = 25.05$ kN $(\leftarrow)$

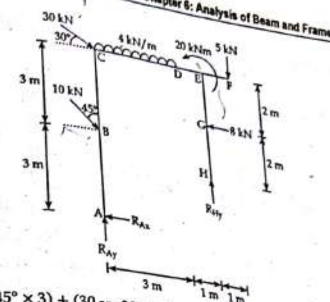
$(^+\uparrow) \sum F_y = 0$

or, $R_{Ay} + R_{Hy} - 10 \cos 45° - 30 \sin 30° - (4 \times 3) - 5 = 0$

or, $R_{Ay} + R_{Hy} = 39.07$ kN $(\uparrow)$

Also, taking moment about A; we get,

$(^+\circlearrowleft) \sum(M_A) = 0$



or, $(10 \sin 45° \times 3) + (30 \cos 30° \times 6) + (4 \times 3 \times 1.5) + (5 \times 5) - (8 \times 4) - 20 - (R_{Hy} \times 4) = 0$

or, $R_{Hy} \times 4 = 168.097$

$\therefore \quad R_{Hy} = 42.02$ kN

Also,

$R_{Ay} + R_{Hy} = 39.07$

or, $R_{Ay} + 42.02 = 39.07$

$\therefore \quad R_{Ay} = -2.95$ kN

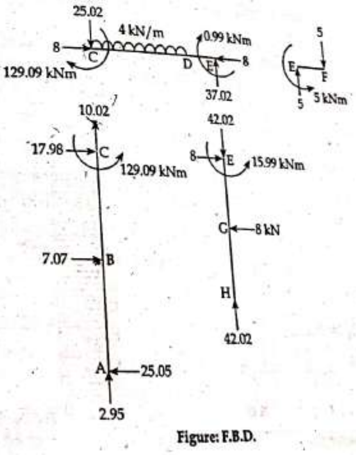


Figure: F.B.D.

Axial force

Member AC

$F_{AB} = 2.95$ kN (T)

$F_{BC} = 2.95 + 7.07 = 10.02$ kN (T)

Member CE

$F_{CE} = 17.98 - 30\cos 30° = -8\text{ kN (C)}$

Member EH

$F_{EH} = -37.02 - 5 = -42.02\text{ kN (C)}$

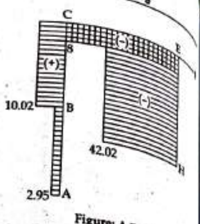


Figure: A.F.D.

Shear force

Member AC

$(V_A)_L = 0\text{ kN}$

$(V_A)_R = 25.05\text{ kN}$

$(V_B)_L = 25.05\text{ kN}$

$(V_b)_R = 25.05 - 7.07 = 17.98\text{ kN}$

$(V_C)_L = 17.98\text{ kN}$

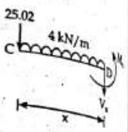
$(V_C)_R = -17.98 + 17.98 = 0\text{ kN}$

Member CE

Portion CD

$(^+\uparrow)\sum F_y = 0$

or, $V_x = -25.05 - 4x$



Shear force is zero at;

$V_x = 0$

or, $-25.05 - 4x = 0$

$\therefore\ x = -6.26\text{ m}$

$\therefore\ V_x = 0$ at $x = -6.26$ m (*i.e.,* 6.26 m away from C)

$(V_C)_L = 0$

$(V_C)_R = -10.02 - 30\sin 30° = -25.02\text{ kN}$

$V_D = -25.02 - (6 \times 2) = -37.02\text{ kN}$

$(V_E)_L = -37.02\text{ kN}$

$(V_E)_R = -37.02 + 37.02 = 0\text{ kN}$

Member EF

$(V_F)_R = 0$

$(V_F)_L = 5\text{ kN}$

$(V_E)_R = 5\text{ kN}$

$(V_E)_L = 5 - 5 = 0\text{ kN}$

Member EH

$(V_E)_L = 0\text{ kN}$

$(V_E)_R = 8\text{ kN}$

$(V_G)_L = 8\text{ kN}$

$(V_G)_R = 8 - 8 = 0\text{ kN}$

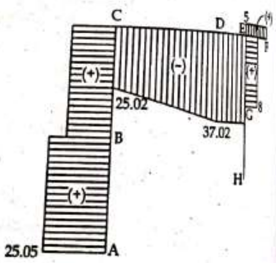


Figure: S.F.D.

Bending moment

Member AC

$M_A = 0\text{ kNm}$

$M_B = 25.05 \times 3 = 75.15\text{ kNm}$

$(M_C)_L = 25.05 \times 6 - (7.07 \times 3) = 129.09\text{ kNm}$

$(M_C)_R = 129.09 - 129.09 = 0\text{ kNm}$

Member CE

Portion CD

$M_x = 129.09 - 25.02x - 2x^2$

Moment is zero at;

$M_x = 0$

or, $-2x^2 - 25.02x + 129.09 = 0$

$\therefore\ x = -3.92\text{ m}$

and, $x = 16.45\text{ m}$

Thus, $M_x = 0$ at $x = -3.92$ m and $x = 16.45$ m.

$(M_C)_L = 0\text{ kNm}$

$(M_C)_R = 129.09\text{ kNm}$

$M_D = 129.09 - (25.02 \times 3) - (4 \times 3 \times 1.5) = 36.03\text{ kNm}$

$(M_E)_L = 129.09 - (25.02 \times 4) - (4 \times 3 \times 2.5) = -0.99\text{ kNm}$

$(M_E)_R = -0.99 + 0.99$

$= 0\text{ kNm}$

Member EF

$M_F = 0\text{ kNm}$

$(M_E)_L = (-5) \times 1 = -5\text{ kNm}$

$(M_E)_R = -5 + 5 = 0\text{ kNm}$

Member EH

$(M_E)_L = 0\text{ kNm}$

$(M_E)_R = 5 - 0.99 - 20 = -15.99\text{ kNm}$

$M_G = -15.99 + (8 \times 2) = 0\text{ kNm}$

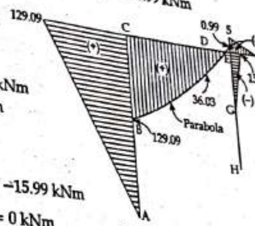
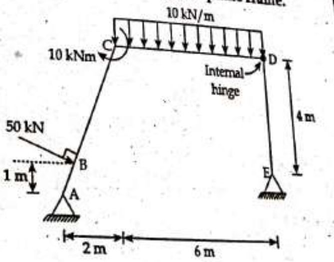


Figure: B.M.D.

28. **Calculate and draw the axial force, shear force and bending moment diagram with salient feature for the given plane frame.** [2072 Magh]



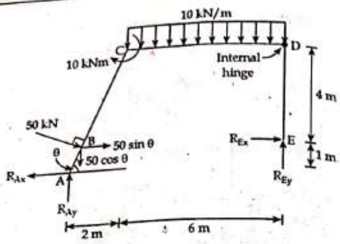
Solution:

Calculation of reactions

Here,

$\tan\theta = \frac{5}{2} = 2.5$

$\therefore\ \theta = \tan^{-1}(2.5) = 68.19°$



and, $AC = \sqrt{(2)^2 + (5)^2} = 5.38$ m

Also,

$$\sin\theta = \frac{1}{AB}$$

$$\therefore \quad AB = \frac{1}{\sin\theta} = \frac{1}{\sin(68.19°)} = 1.08 \text{ m}$$

Taking moment about right of D; we get,

$$(^+\circlearrowleft)\sum (M_D)_{right} = 0$$

or, $(R_{Ey} \times 0) - (R_{Ex} \times 4) = 0$

$\therefore \quad R_{Ex} = 0$

Now,

$$(^+\rightarrow)\sum F_x = 0$$

or, $R_{Ex} - R_{Ax} + 50\sin\theta = 0$

or, $0 - R_{Ax} + 50\sin(68.19°) = 0$

$\therefore \quad R_{Ax} = 46.42$ kN (←)

Also, taking moment about A; we get,

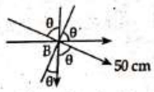


Figure: Load at B

$$(^+\circlearrowleft)\sum M_A = 0$$

or, $[(-R_{Ey}) \times 8] + (R_{Ex} \times 1) + [10 \times 6 \times (3 + 2)] + 10 + (50 \times 1.08) = 0$

or, $-8R_{Ey} + 0 + 300 + 10 + 54 = 0$

$\therefore \quad R_{Ey} = 45.5$ kN (↑)

and, $(^+\uparrow)\sum F_y = 0$

or, $R_{Ay} + R_{Ey} - 10 \times 6 - 50\cos(68.19°) = 0$

or, $R_{Ay} + 45.5 - 60 - 50\cos(68.19°) = 0$

$\therefore \quad R_{Ay} = 33.08$ kN

Drawing free body diagram of the given frame

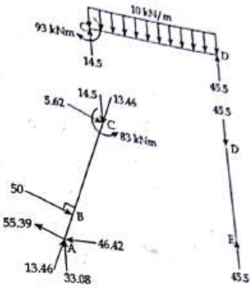


Figure: F.B.D. of frame

Axial force

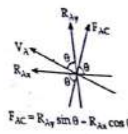
$F_{AC} = 33.08\sin(68.19°) - 46.42\cos(68.19°) = 13.46$ kN (C)

$F_{CD} = 0$ kN

$F_{DE} = 45.5$ kN (C)

Shear force

Member AC



$F_{AC} = R_{Ay}\sin\theta - R_{Ax}\cos\theta$

$(V_A)_L = 0$ kN

$(V_A)_R = R_{Ay}\cos\theta + R_{Ax}\sin\theta$

$= 33.08\cos(68.19°) + 46.42\sin(68.19°) = 55.39$ kN

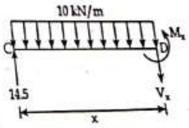
$(V_B)_L = 55.39$ kN

$(V_B)_R = 55.39 - 50 = 5.39$ kN

$(V_C)_L = 5.39$ kN

$(V_C)_R = 5.39 - 5.39 = 0$ kN

Member CD



$$(^+\uparrow)\sum F_y = 0$$

or, $V_x = 14.5 - 10x$

Shear force is zero at;

$V_x = 0$

or, $14.5 - 10x = 0$

or, $x = 1.45$ m

Thus, shear force is zero at x = 1.45 m. Now,

$(V_C)_L = 0$ kN

$(V_C)_R = 14.5$ kN

$(V_D)_L = 14.5 - (10 \times 6) = -45.5$ kN

$(V_D)_R = -45.5 + 45.5 = 0$ kN

Bending moment

Member AC

$M_A = 0$ kNm

$M_B = 55.39 \times 1.08 = 59.82$ kNm

$(M_C)_L = (55.39 \times 5.38) - [50 \times (5.37 - 1.08)] = 82.99 \approx 83$ kNm

$(M_C)_R = 83 - 83 = 0$ kNm

Member CD

$M_x = 93 + 14.5x - 5x^2$

Moment is zero at;

$M_x = 0$

or, $93 + 14.5x - 5x^2 = 0$

$\therefore$ $x = 6$ m or $x = -3.2$ m

$M_x = 0$ at $x = 6$ m or $x = -3.2$ m

Now,

$(M_{max})_{x=1.45} = 93 + (14.5 \times 1.45) - (5 \times (1.45)^2) = 103.51$ kNm

Again,

$(M_C)_L = 0$ kNm

$(M_C)_R = 83 + 10 = 93$ kNm

$M_D = 93 + (14.5 \times 6) - (10 \times 6 \times 3) = 0$ kNm

Salient features

i) Shear force is zero at $x = 1.45$ m in member CD.

ii) Bending moment is zero at $x = 6$ m in the member CD.

iii) Maximum bending moment is 103.51 in the member CD which occurs at $x = 1.45$ m right to C.

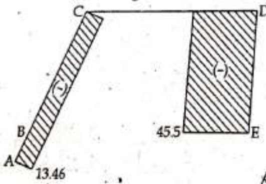


Figure: A.F.D

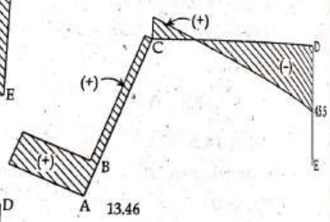


Figure: S.F.D

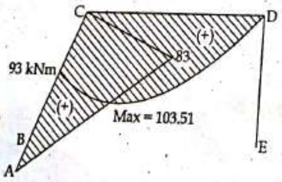
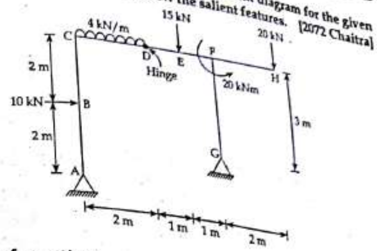


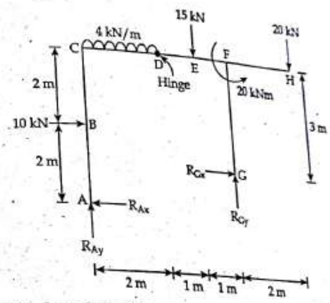
Figure: B.M.D

29. Draw the axial force, shear force and bending moment diagram for the given frame as shown in the figure. Also, show the salient features. [2072 Chaitra]



Solution:

Calculation of reactions



Taking moment about left of D; we get,

$(^+\circlearrowleft) \sum (M_D)_{left} = 0$

or, $(R_{Ay} \times 2) + (R_{Ax} \times 4) - (10 \times 2) - (4 \times 2 \times 1) = 0$

or, $2R_{Ay} + 4R_{Ax} = 28$

$\therefore$ $R_{Ay} + 2R_{Ax} = 14$ (1)

Also, taking moment about G; we get,

$(^+\circlearrowleft) \sum (M_G) = 0$

or, $(R_{Ay} \times 4) + (R_{Ax} \times 1) + (10 \times 1) - (4 \times 2 \times 3) - (15 \times 1) - 20 + (20 \times 2) = 0$

or, $4R_{Ay} + R_{Ax} = 9$ (2)

Solving equation (1) and (2); we get,

$R_{Ax} = \frac{47}{7} = 6.71$ kN

and, $R_{Ay} = \frac{4}{7} = 0.57$ kN

Now,

$(^{+}\rightarrow) \sum F_x = 0$

or, $-R_{Ax} + R_{Gx} + 10 = 0$

or, $-6.71 + R_{Gx} + 10 = 0$

$\therefore$ $R_{Gx} = -3.29 \text{ kN} = 3.29 \text{ kN} (\leftarrow)$

and, $(^{+}\uparrow) \sum F_y = 0$

or, $R_{Ay} + R_{Gy} - (4 \times 2) - 15 - 20 = 0$

or, $0.57 + R_{Gy} = 43$

$\therefore$ $R_{Gy} = 42.43 \text{ kN} (\uparrow)$

Drawing free body diagram of the frame

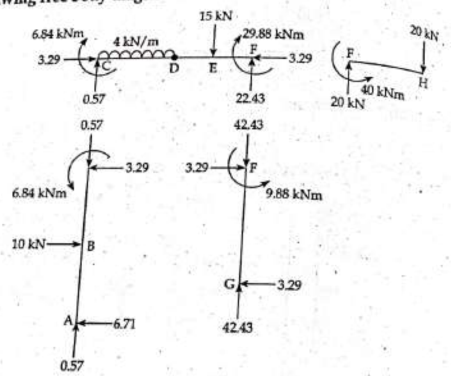


Figure: F.B.D.

Axial force

$F_{AC} = 0.57 \text{ kN (C)}$

$F_{CF} = 3.29 \text{ kN (C)}$

$F_{FG} = 22.43 + 20 = 42.43 \text{ kN (C)}$

Shear force

Member AC

$(v_A)_L = 0 \text{ kN}$

$(v_A)_R = 6.71 \text{ kN}$

$(v_B)_L = 6.71 \text{ kN}$

$(v_B)_R = 6.71 - 10 = -3.29 \text{ kN}$

$(v_C)_L = -3.29 \text{ kN}$

$(v_C)_R = -3.29 + 3.29 = 0 \text{ kN}$

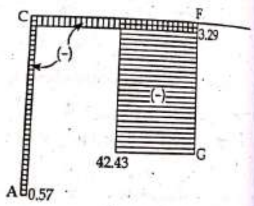


Figure: A.F.D.

Member CF

Portion CD

$v_x = 0.57 - 4x$

Shear force is zero at;

$v_x = 0$

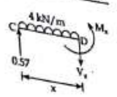
$0.57 - 4x = 0$

or, $x = 0.14 \text{ m}$

$\therefore$ $v_x = 0$ at $x = 0.14$ m (i.e., 0.14 m right from C)

$\therefore$ $(v_A)_L = 0 \text{ kN}$

$(v_A)_R = 0.57 \text{ kN}$



$v_D = 0.57 - (4 \times 2) = -7.43 \text{ kN}$

$(v_E)_L = -7.43 \text{ kN}$

$(v_E)_R = -7.43 - 15 = -22.43 \text{ kN}$

$(v_F)_L = -22.43 \text{ kN}$

$(v_F)_R = -22.43 + 22.43 = 0 \text{ kN}$

Member FH

$(v_F)_R = 0 \text{ kN}$

$(v_F)_L = 20 \text{ kN}$

$(v_H)_R = 20 \text{ kN}$

$(v_H)_L = 20 - 20 = 0 \text{ kN}$

Member FG

$(v_F)_L = 0 \text{ kN}$

$(v_F)_R = 3.29 \text{ kN}$

$(v_G)_L = 3.29 \text{ kN}$

$(v_G)_R = 3.29 - 3.29 = 0 \text{ kN}$

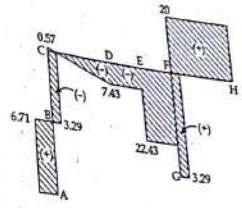


Figure: S.F.D.

Bending moment

Member AC

$M_A = 0 \text{ kNm}$

$M_B = 6.71 \times 2 = 13.42 \text{ kNm}$

$(M_C)_L = 6.71 \times 4 - (10 \times 2) = 6.84 \text{ kNm}$

$(M_C)_R = 6.84 - 6.84 = 0 \text{ kNm}$

Member CF

Portion CD

$M_x = 0.57x - \left(4x \times \frac{x}{2}\right) + 6.84 = 6.84 + 0.57x - 2x^2$

Moment is zero at;

$M_x = 0$

or, $6.84 + 0.57x - 2x^2 = 0$

$\therefore$ $x = 2 \text{ m}$

and, $x = -1.71 \text{ m}$

Thus, $M_x = 0$ at $x = 2$ m and $x = -1.71$ m.

Now,

$(M_C)_L = 0 \text{ kNm}$

$(M_C)_R = 6.84 \text{ kNm}$

$M_D = 6.84 + (0.57 \times 2) - (4 \times 2 \times 1) = 0 \text{ kNm}$

$(M_D)_{mid} = 6.84 + (0.57 \times 1) - (2 \times 1) = 5.41 \text{ kNm}$

$(M_{max})_{v_x=0} = 6.84 + (0.57 \times 0.14) - \{2 \times (0.14)^2\} = 6.88$ kNm

$M_E = 6.84 - (4 \times 2 \times 2) - (0.57 \times 3) = -7.45$ kNm

$(M_F)_L = 6.84 - (4 \times 2 \times 3) + (0.57 \times 4) - (15 \times 1) = -29.88$ kNm

$(M_F)_R = -29.88 + 29.88 = 0$ kNm

Member HF

$M_H = 0$ kNm

$(M_F)_L = (-20) \times 2 = -40$ kNm

$(M_F)_R = -40 + 40 = 0$ kNm

Member FG

$(M_F)_L = 0$ kNm

$(M_F)_R = 40 - 29.88 - 20 = -9.88$ kNm

Here, 20 kNm is given loading couple.

$M_G = -9.88 + (3.29 \times 3) \approx 0$ kNm

Salient features

i) Shear force $v_x = 0$ at $x = 0.14$ m in the member CD.

ii) Bending moment is zero at $x = 2$ m in the member CD.

iii) Maximum bending moment in portion CD is 6.88 kNm at $x = 0.14$ m.

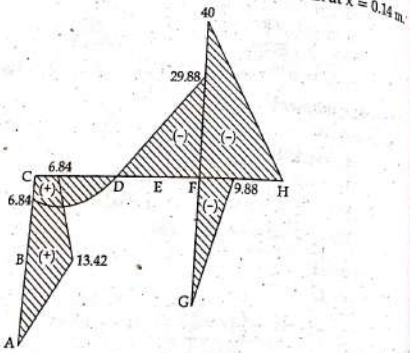
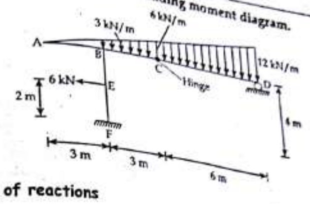


Figure: B.M.D.

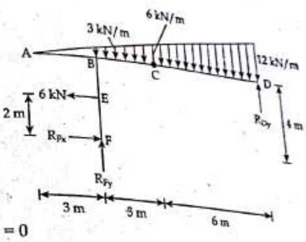
ADDITIONAL PROBLEMS

1. Draw axial force, shear force and bending moment diagram.



Solution:

Calculation of reactions



$\circlearrowleft^{+} \Sigma M_F = 0$

$M_F - 6 \times 2 - \frac{1}{2} \times 3 \times 3 \frac{1}{3} \times 3 + 3 \times 9 \times \frac{3}{2} + \frac{1}{2} \times 9 \times 9 \times \frac{2}{3} \times 9 - R_{Dy} \times 9 = 0$

or, $M_F - 12 - 4.5 + 121.5 + 243 - 9 R_{Dy} = 0$

$\therefore$ $M_F - 9R_{Dy} + 348 = 0$

Again, taking right hand side moment of C

$\circlearrowleft^{+} \Sigma(M_C)_{right} = 0$

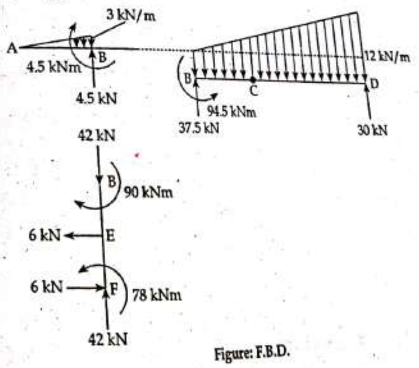


Figure: F.B.D.

$$6\times6\times\frac{6}{2}+\frac{1}{2}\times6\times6\times\frac{2}{3}\times6-R_{Dy}\times6=0$$

or, $108+72=6R_{Dy}$

$\therefore$ $R_{Dy}=30$ kN (↑)

From equation (1); we get,

$M_F=-78$ kNm $=78$ kNm (anticlockwise)

$+\uparrow\Sigma F_y=0$

$$R_{Fy}-\frac{1}{2}\times12\times12+30=0$$

$\therefore$ $R_{Fy}=42$ kN (↑)

Axial force

$E_{AB}=0$

$F_{BD}=0$

$N_{BF}=42$ kN (C)

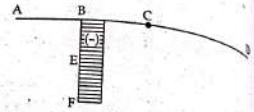


Figure: A.F.D.

Shear force

Member BF

$(V_F)_L=0$

$(V_F)_R=6$ kN

$(V_E)_L=6$ kN

$(V_E)_R=0$

$(V_B)_R=0$

Member AB

$$\frac{w(x_1)}{x_1}=\frac{3}{3}$$

$w(x)=x_1$ kN/m

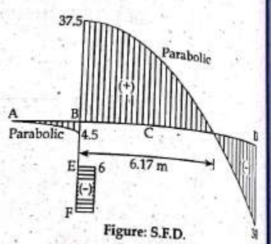
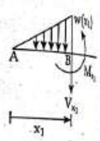


Figure: S.F.D.

$+\uparrow\Sigma F_y=0$

$$V_{x_1}=-\frac{1}{2}\times x_1\times x_1=-\frac{x_1^2}{2}$$



$(V_{x_1})=0$ at $x_1=0$

$(V_A)_L=0$

$(V_B)_L=-4.5$ kN

$(V_B)_R=0$

$V_{mid}=V_{1.5}=-1.125$ kN

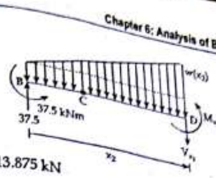
Member BD

$+\uparrow\Sigma F_y=0$

$$V_{x_2}=37.5-3\times x_2-\frac{1}{2}\times x_2\times x_2$$

$$\therefore\quad V_{x_2}=37.5-3x_2-\frac{1}{2}x_2^2$$

$(V_{x_2})=0$ at $x_2=6.17$ m



$V_{mid}=V_{4.5}=13.875$ kN

$(V_B)_L=0$

$(V_B)_R=0$

$(V_C)_L=24$ kN

$(V_C)_R=24$ kN

$(V_D)_L=-30$ KN

$(V_D)_R=0$

Bending moment

Member AB

$(M_A)_L=0$

$(M_A)_R=4.5$

$(M_B)_L=0$

$(M_B)_R=0$

$$M_{x_1}=\frac{x_1^3}{6}-4.5$$

$M_{mid}=-3.94$ kNm

$(M_{x_1})=0$ at $x_1=3$m

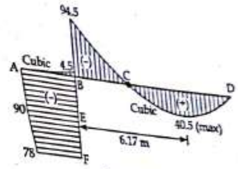


Figure: B.M.D.

Member BF

$(M_F)_L=0$

$(M_F)_R=-78$ kNm

$(M_E)_L=-90$ kNm

$(M_E)_R=-90$ kNm

$(M_B)_L=-90$ kNm

$(M_B)_R=0$

Member BD

$$M_{x_2}=37.5x_2-94.5-\frac{3}{2}x_2^2-\frac{x_2^3}{6}$$

$(M_B)_L=0$

$(M_B)_R=-94.5$ kNm

$(M_C)_L=0$

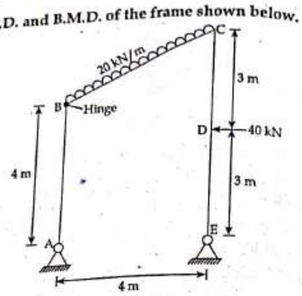
$(M_C)_R=0$

$(M_D)_L=(M_D)_R=0$

$Mx_2=6.17=40.62$ kNm

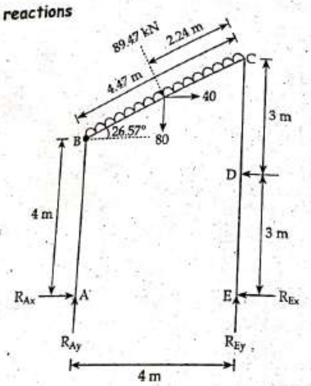
$(Mx_2)_{mid}=28.69$ kNm

2. Draw A.F.D., S.F.D. and B.M.D. of the frame shown below.



Solution:

Calculation of reactions



$\overset{+}{\circlearrowright} \Sigma(M_B)_{right} = 0$

$80 \times 2 + 40 \times 1 + 40 \times 1 - R_{Ex} \times 4 - 60 \times 4 = 0$

$\therefore \quad R_{Ex} = 0 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$R_{Ax} + 40 - 40 = 0$

$R_{Ax} = 0$

$+\uparrow \Sigma F_y = 0$

$R_{Ay} - 80 + 60 = 0$

$R_{Ay} = 20 \text{ kN } (\uparrow)$

Axial force

$F_{AB} = 20 \text{ kN (C)}$

$F_{BC} = 8.95 \text{ kN (C)}$

$N_{CE} = 60 \text{ kN (C)}$

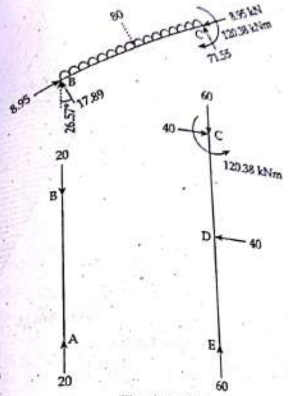


Figure: F.B.D.

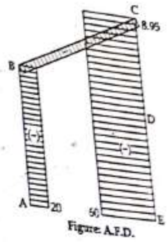
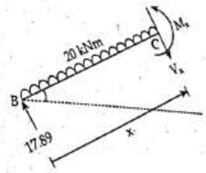


Figure: A.F.D.

Shear force

$(V_A) = 0$

$(V_B) = 0$



Member BC

$17.89 - 20\,x = V_x$

$(V_x) = 0 \text{ at } x = 0\,0.89 \text{ m}$

$(V_B)_L = 0$

$(V_B)_R = 17.89 \text{ kN}$

$(V_C)_L = -71.55 \text{ kN}$

$(V_C)_R = 0$

Member CE

$(V_C)_L = 0$

$(V_C)_R = 40 \text{ kN}$

$(V_D)_L = 40 \text{ kN}$

$(V_D)_R = 0$

$(V_E) = 0$

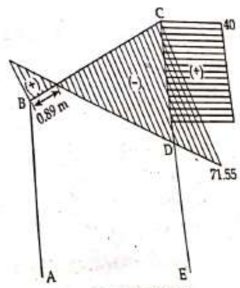


Figure: S.F.D.

Bending moment

$(M_A) = 0$

Member BC

$M_x = 17.89x - 10x^2$

$M_{mid} = M_{2.24} = -10.1024$ kNm

$M_{x=0.89} = M_{max} = 8$ kNm

$(M_B)_L = 0$

$(M_B)_R = 0$

$(M_C)_L = -120.38$ kNm

$(M_c)_R = -120.38 + 120.38 = 0$ kNm

$(M_x) = 0$ at $x = 1.789$m

Member CE

$(M_C)_L = 0$ kNm

$(M_C)_R = 120.38$ kNm

$(M_E)_L = 0.38 \approx 0$ kNm

$(M_E)_R = 0$ kNm

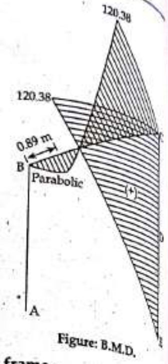
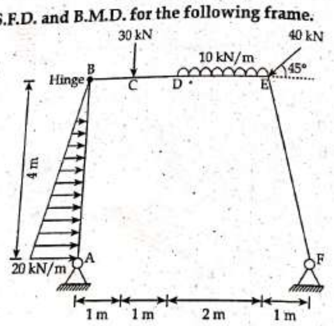


Figure: B.M.D.

3. Draw A.F.D., S.F.D. and B.M.D. for the following frame.



Solution:

Calculation of reactions

$^{+}\circlearrowleft \Sigma M_A = 0$

$$\frac{1}{2}\times 20\times 4\times 1.33 + 30\times 1 + 20\times 3 + 28.28\times 4 - 28.28\times 4 - R_{Fy}\times 5 = 0$$

$\therefore \quad R_{Fy} = 28.67$ kN (↑)

$^{+}\circlearrowleft \Sigma(M_B)_{left} = 0$

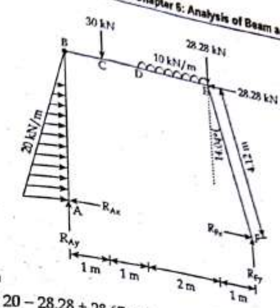
$$-R_A\times 4 + \frac{1}{2}\times 20\times 4\times\frac{2}{3}\times 4 = 0$$

$\therefore \quad R_{AX} = 26.67$ kN (←)

$\xrightarrow{+} \Sigma F_x = 0$

$-26.67 + 40 - 28.28 + R_{Fx} = 0$

$R_{Fx} = 14.95$ kN (→)



$+\uparrow \Sigma F_y = 0$

$R_{Ay} - 30 - 20 - 28.28 + 28.67 = 0$

$\therefore \quad R_{Ay} = 49.61$ kN (↑)

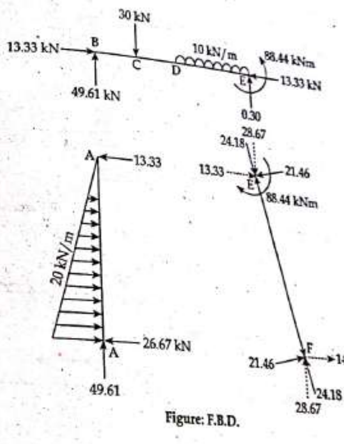


Figure: F.B.D.

Axial force

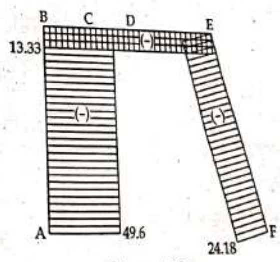
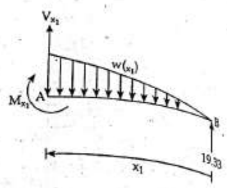


Figure: A.F.D.

F_{AB} 49.61 kN (C)
F_{BE} = 13.33 kN (C)
F_{EF} = 24.18 kN(C)

Shear force

Member AB



$$\frac{w(x_1)}{x_1} = \frac{20}{4}$$

$$w(x_1) = 5x_1$$

$$V_{x_1} = -13.33 + \frac{1}{2} \times x_1 \times 5x_1$$

$$\therefore \quad V_{x_1} = \frac{1}{2} 5x_1^2 - 13.33$$

$(V_{x_1}) = 0$ at $x_1 = 2.31$ m

$V_{mid} = V_{x_1=2} = -3.33$ kN

$(V_B)_R = 0$

$(V_B)_L = -13.33$ kN

$(V_A)_R = 26.67$ kN

$(V_A)_L = 0$

Member BE

Portion BD

$(V_B)_L = 0$

$(V_B)_R = 49.61$ kN

$(V_D)_L = 19.61$ kN

$(V_C)_L = 49.61$ kN

$(V_C)_R = 19.61$ kN

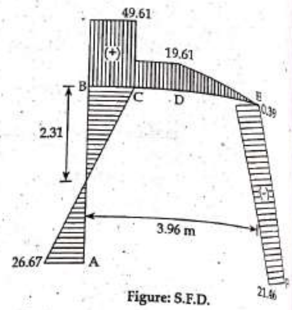
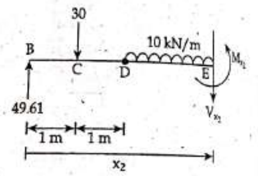


Figure: S.F.D.

Portion DE



$V_{x_2} = 39.61 - 10x_2$

$(V_{x_2}) = 0$ at $x_2 = 3.96\text{ m} \approx 4$ m

$(V_D)_R = 19.61$ kN

$(V_E)_L = -0.39\text{ kN} \approx 0$

$(V_E)_R = 0$ kN

Member EF

$(V_E)_L = 0$

$(V_E)_R = -21.46$ kN

$(V_F)_L = -21.46$ kN

$(V_F)_R = 0$

Bending moment

Member AB

$$M_{x_1} = 13.33x_1 - \frac{5}{6}x_1^3$$

$(M_{x_1}) = 0$ at $x_1 = 4$ m

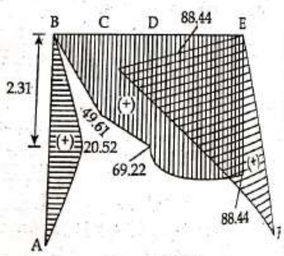


Figure: B.M.D.

$M_{mid} = 19.99$ kNm

$(M_B)_R = 0$

$(M_B)_L = 0$

$(M_A)_R = 13.33 \times 4 - 40 \times 1.33 = 0.12 \approx$ kNm

$(M_A)_L = 0$

$M_{x_1=2.31} = 20.52$ kNm

Member BE

Portion BD

$M_B = 0$

$M_C = 49.61 \times 1 = 49.61$ kNm

$(M_D) = 49.61 \times 2 - 30 \times 1 = 69.22$ kNm

Portion DE

$$M_{x_2} = 49.61x_2 - 30(x_2 - 1) - 10\left(\frac{x_2-2}{2}\right)\left(\frac{x_2-2}{2}\right)$$

$$M_{x_2} = 39.61x_2 - 5x_2^2 + 10$$

$M_D = 69.22$ kNm

$(M_E)_L = 88.44$ kNm

$(M_E)_R = 0$

$(Mx_2) = 0$ at $x_2 = 6.16$m

$M_{x_2=4} = 88.44$ kNm

Member EF

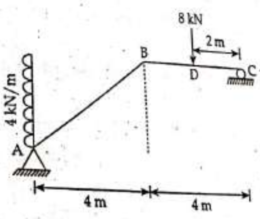
$(M_E)_L = 0$

$(M_E)_R = 88.44$ kNm

$(M_F)_L = 0$

$(M_F)_R = 0$

4. **Draw axial force diagram, shear force diagram and bending moment diagram for the given frame.**



Solution:

Calculation of reactions

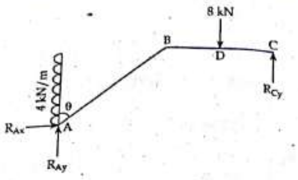
$$\overset{+}{\rightarrow} \Sigma F_x = 0$$

$$4 \times 3 + R_{Ax} = 0$$

$$\therefore \quad R_{AX} = -12\text{ kN} = 12\text{ kN} (\leftarrow)$$

$$^{+}\circlearrowleft \Sigma M_A = 0$$

$$8 \times (4 + 2) - R_{Cy} \times 8 + 4 \times 3 \times 1.5 = 0$$



$\therefore \quad R_{Cy} = 8.25$ kN (↑)

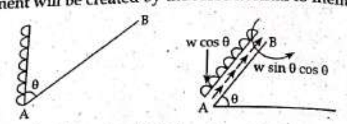
$+\uparrow \Sigma F_y = 0$

$R_{Ay} + R_{Cy} = 8$

$\therefore \quad R_{Ay} = -0.25$ kN

$R_{Ay} = 0.25$ kN (↓)

Here, U.D.L. is horizontal and member AB is inclined so shear force and bending moment will be created by the force normal to member AB.



Vertical length of member AB = l

Inclined length of member AB = $l' = \dfrac{l}{\cos\theta}$

or, $\quad l = l' \cos\theta$

Load acting over vertical length = wl

Force perpendicular to the axis of AB = $wl \cos\theta = wl' \cos^2\theta$.

Load perpendicular to the per unit inclined length = $w \cos^2\theta$

Force along the inclined axis AB = $wl \sin\theta = wl' \cos\theta \sin\theta$

Force along the inclined axis per unit length = $w \sin\theta \cos\theta$

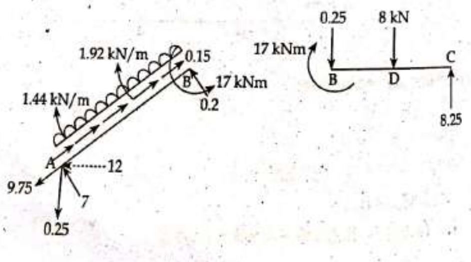
Here,

Load perpendicular to axis per unit length $= 4 \times \cos^2 53.13°$

$= 1.44$ kN/m

Force along the axis per unit length $= 4 \sin 53.13° \cos 53.13°$

$= 1.92$ kN/m



Axial force

Member AB

$\overset{+}{\leftarrow} \Sigma F_x = 0$

$F_x = 9.75 - 1.92\,x$

$(F_x) = 0$ at $x = 5$ m

$(F_A)_L = 0$

$(F_A)_R = 9.75$ kN

$(F_A)_L = 0.15$ kN

$(F_B)_R = 0.15 - 0.15 = 0$

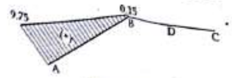


Figure: A.F.D.

Note

Axial force sign convention: $\overset{-}{\rightarrow}\ \overset{+}{\leftarrow}$

Member BC

$F_{BC} = 0$

Shear force

Member AB

$V_x = 7 - 1.44x$

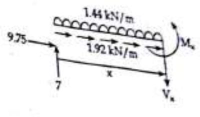
$(V_x) = 0$ at $x = 4.86$ m

$(V_A)_L = 0$

$(V_B)_R = 7$

$(V_B)_L = -0.2$

$(V_B)_R = -0.2 + 0.2 = 0$



Member BC

$(V_B)_L = 0$

$(V_B)_R = -0.25$ kN

$(V_D)_L = -0.25$ kN

$(V_D)_R = -0.25 - 8 = -8.25$ kN

$(V_C)_L = -8.25$ kN

$(V_C)_R = -8.25 + 8.25 = 0$

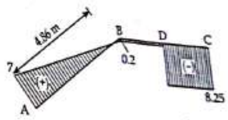


Figure: S.F.D.

Bending moment

Member AB

$M_x = 7x - 1.44 \times x \times \dfrac{x}{2}$

$M_x = 7x - 0.72\,x^2$

$M_x = 0$ at $x = 0$ m

$(M_A) = 0$

$(M_B)_L = 17$ kNm

$(M_B)_L = 17 - 17 = 0$

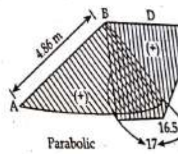


Figure: B.M.D.

Member BC

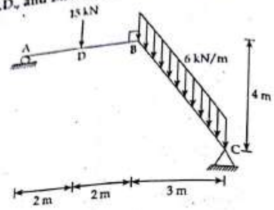
$(M_B)_L = 0$

$(M_B)_R = 17$ kNm

$M_D = 17 - 0.25 \times 2 = 16.5$

$M_C = 0.25 \times 4 - 8 \times 2 + 17 = 0$

5. Draw A.F.D., S.F.D., and B.M.D. for the given form.



Solution:

Calculation of reactions

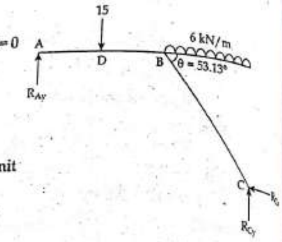
$^{+}\circlearrowleft \Sigma M_c = 0$

$R_{Ay} \times 7 - 15 \times 5 - 6 \times 3 \times 1.5 = 0$

$R_{Ay} = 14.57$ kN (↑)

$^{+}\uparrow \Sigma F_y = 0$

$R_{Ay} = 18.43.$ kN (↑)



Here, for portion BC,

Load perpendicular to BC per unit inclined length $= w \cos^2\theta$

$= 6 \times \cos^2 53.13°$

$= 2.16$ kN/m

Load along the axial the of BC per unit inclined $= w \cos\theta \sin\theta$

$= 2.88$ kN/m

See additional question no. 4

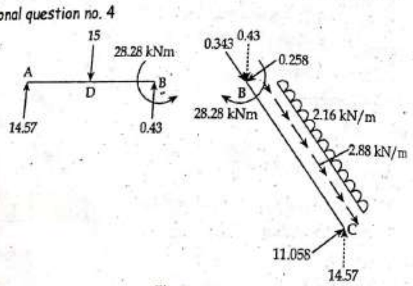


Figure: F.B.D.

Axial force

Member AB

$F_{AB} = 0$

Member BC

$^{+}\leftarrow \Sigma F_x = 0$

$F_x = -0.343 - 2.88x$

$(F_x) = 0$ at $x = 0.11$ m $= 0$

$(F_B)_L = 0$

$(F_B)_R = -0.343$ kN

$(F_c)_L = -6.343 - 14.4 = -14.743$ kN

$(F_C)_R = -14.743 + 14.14.743 = 0$

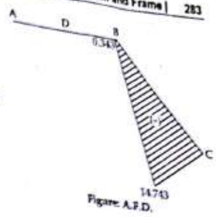


Figure: A.F.D.

Shear force

Member AB

$(V_A)_L = 0$

$(V_A)_R = 14.57$ kN

$(V_D)_L = 14.57$ kN

$(V_D)_R = 14.57 - 15 = -0.43$ kN

$(V_B)_E = -0.43$ kN

$(V_B)_R = -0.43 + 0.43 = 0$

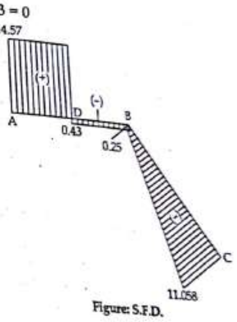


Figure: S.F.D.

Member BC

$^{+}\uparrow \Sigma F_y = 0$

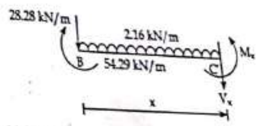
$V_x = -0.250 - 2.16x$

$(V_B)_L = 0$

$(V_B)_R = -0.258$

$(V_C)_L = -0.258 - 2 \times 16 \times 5 = -11.058$ kN

$(V_C)_R = -11.0.58 + 11.058 = 0$



Bending moment

Member AB

$M_A = 0$

$(M_D) = 14.57 \times 2 = 29.14$ kNm

$(M_B)_L = 14.57 \times 4 - 15 \times 2 = 28.28$ kNm

$(M_B)_R = 28.28 - 28.28 = 0$

Member BC

$^{+}\circlearrowleft \Sigma M_c' = 0$

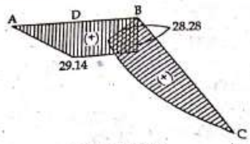


Figure: B.M.D.

$$M_x = -0.258\,x - 2.16 \times x \times \frac{x}{2} + 28.28$$

$$= -0.258\,x - 1.08x^2 + 28.28$$

$(M_x) = 0$ at $x = 5$ m

$M_{mid} = M_{2.5} = 20.885$ kNm

$(M_B)_L = 0$

$(M_B)_R = 28.28$ kNm

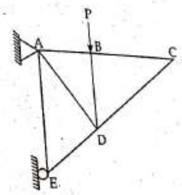
$(M_C) = 28.28 - 0.258 \times 5 - 2.16 \times 5 \times 2.5 = 0.01 \approx 0$ kNm

CHAPTER 7 Analysis of Plane Trusses

DEFINITIONS

1. Truss is the assembly of straight structural member connected such that no member is continuous through the joint. They are subjected to axial force only. Triangle is the simplest form of truss.
2. Generally two analytical methods are used to calculate the force in the members of the truss. They are joint method and section method.
3. In method of joint, each and every joint is treated separately as a free body in equilibrium. Unknown forces are determined by two equilibrium equations, $\Sigma F_x = 0$ and $\Sigma F_y = 0$ and it is applicable only when joint contains no more than two unknown members.
4. Method of section is used when forces in few members of a truss are required. All three equilibrium conditions $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M_Z = 0$ are used to calculate unknown forces and it is applicable only when section contains no more than three unknown members.
5. It is often certain members of truss carry no load such members are called zero force member.
6. The member is zero force member if;
   i) Only two members form a non-collinear joint and no external load or support reaction is applied to the joint.

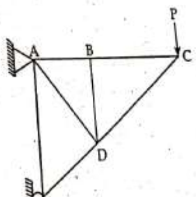
For example;



Here, CD and BC are zero force members.

   ii) When three members form a truss joint for which two of the members are collinear and the third forms an angle with the first two, then the non-collinear member is zero force member.

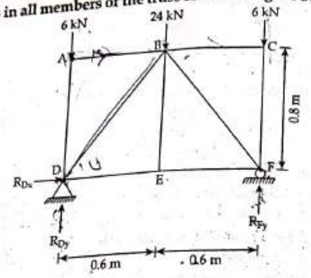
For example;



Here, BD and AD are zero force members.

EXAM SOLUTION

1. Find the forces in all members of the truss shown in figure below. [2060 Poush]



Solution:

Let us first calculate the unknown reaction components.

$^{+}\uparrow \Sigma F_y = 0$

$R_{Dy} + R_{Fy} = 36$ ... (1)

$^{+}\circlearrowleft \Sigma M_F = 0$

$R_{Dy} \times 1.2 - 6 \times 1.2 - 24 \times 0.6 = 0$

$\therefore \quad R_{Dy} = 18\ kN\ (\uparrow)$

From equation (1); we get,

$R_{Fy} = 18\ kN\ (\uparrow)$

Now, using joint method for finding the forces in all the members,

Joint A

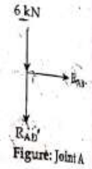


Figure: Joint A

$\Sigma F_x = 0$

$\therefore \quad R_{AB} = 0$

$\Sigma F_y = 0$

$-R_{AD} - 6 = 0$

$\therefore \quad R_{AD} = -6\ kN = 6\ kN\ (C)$

Joint D

In ΔDEB; we have,

$$\tan\theta = \frac{BE}{DE}$$

$\therefore \quad \theta = 53.13°$

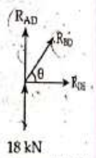


Figure: Joint D

$\Sigma F_y = 0$

$R_{AD} + R_{BD} \sin 53.13° = -18$

$\therefore \quad R_{BD} = -15\ kN = 15\ kN\ (C)$

$\Sigma F_x = 0$

$R_{DE} + R_{BD} \cos 53.13° = 0$

$\therefore \quad R_{DE} = 9\ kN\ (T)$

Joint E

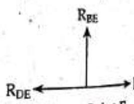


Figure: Joint E

$\Sigma F_x = 0$

$R_{DE} = R_{EF} = 9\ kN\ (T)$

$\Sigma F_y = 0$

$R_{BE} = 0$

Joint B

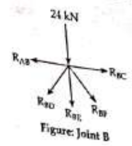


Figure: Joint B

$\Sigma F_y = 0$

$-R_{BE} - R_{BD} \sin 53.13° - R_{BF} \sin 53.13° - 24 = 0$

$\therefore \quad R_{BF} = -15\ kN = 15\ kN\ (C)$

$\Sigma F_x = 0$

$-R_{AB} + R_{BC} - R_{BD} \cos 53.13° + R_{BF} \cos 53.13° = 0$

$\therefore \quad R_{BC} = 0$

Joint C

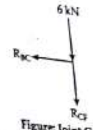


Figure: Joint C

$\Sigma F_x = 0$

$R_{BC} = 0$

$\Sigma F_y = 0$

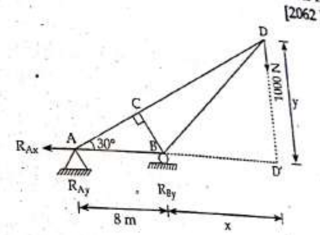
$R_{CF} = -6\ kN = 6\ kN\ (C)$

Hence,

Member	AB	AD	BD	BE	DE	BC	BF	CF	EF
Member force (kN)	0	6	15	0	9	0	15	6	9
Nature (T or C)	–	C	C	–	T	–	C	C	T

2. **Determine the forces in all the members of the truss shown in the figure given below.**

[2062 Poush Back]



Solution:

In $\Delta ADD'$; we have,

$$\tan 30° = \frac{y}{x+8}$$

or, $(x+8)\tan 30° = y$ ... (1)

In $\Delta BDD'$; we have,

$$\tan 60° = \frac{y}{x}$$

or, $y = x \tan 60°$ ... (2)

From equation (1) and (2); we get,

$$(x+8)\tan 30° = x \tan 60°$$

or, $x + 8 = \dfrac{x \tan 60°}{\tan 30°}$

or, $x + 8 = 3x$

$\therefore \quad x = 4\ m$

From equation (2); we have,

$y = 4 \tan 60° = 6.93$ m

Now, finding the support reactions;

$(^{+}\uparrow) \sum F_y = 0$

or, $R_{Ay} + R_{By} = 1$

$(^{+}\circlearrowleft) \sum M_B = 0$

or, $R_{Ay} \times 8 + 1 \times 4 = 0$

$\therefore$ $R_{Ay} = -0.5$ kN $= 0.5$ kN $(\downarrow)$

and, $R_{By} = 1 - (-0.5) = 1.5$ kN $(\uparrow)$

Now, using joint method to calculate the forces in each member;

Joint A

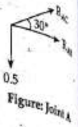


Figure: Joint A

$\sum F_y = 0$

or, $R_{AC} \sin 30° - 0.5 = 0$

$\therefore$ $R_{AC} = 1$ kN (T)

$\sum F_x = 0$

or, $R_{AB} + R_{AC} \cos 30° = 0$

$\therefore$ $R_{AB} = -0.87$ kN $= 0.87$ kN (C)

Joint C

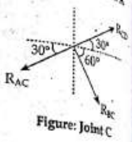


Figure: Joint C

$\sum F_x = 0$

or, $R_{CD} \cos 30° - R_{AC} \cos 30° + R_{BC} \cos 30° = 0$

$\therefore$ $R_{CD} \cos 30° + R_{BC} \cos 30° = 0.87$ (3)

$\sum F_y = 0$

or, $R_{CD} \sin 30° - R_{AC} \sin 30° - R_{BC} \sin 30° = 0.87$

or, $R_{CD} \sin 30° - R_{BC} \sin 30° = 0.5$ (4)

Solving equation (3) and (4); we get,

$R_{CD} = 1$ kN (T)

$R_{BC} = 0$

Joint D

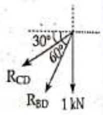


Figure: Joint D

$\sum F_x = 0$

or, $-R_{CD} \cos 30° - R_{BD} \cos 30° = 0$

$\therefore$ $R_{BD} = 1.75$ kN (C)

Check

$\sum F_y = 0$

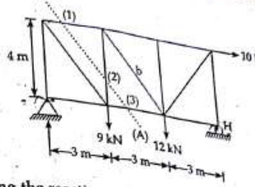
or, $-1 - R_{CD} \sin 30° - R_{BD} \sin 60° = 0$

$\therefore$ $R_{BD} = 1.73$ kN (C)

Hence,

Member	AB	AC	BC	BD	CD
Member force (kN)	0.87	1	0	1.73	1
Nature (T or C)	C	T	-	C	T

3. **Find the reactions at supports and forces in members marked 1, 2 and 3 of the truss.** [2063 Baishakh]



Solution:

First of all, finding the reaction at supports;

$^{+}\uparrow \Sigma F_y = 0$

$R_{Ay} + R_{Hy} = 21$ (1)

$^{+}\rightarrow \Sigma F_x = 0$

$R_{Ax} = -10$ kN $= 10$ kN $(\leftarrow)$

$^{+}\circlearrowleft \Sigma M_H = 0$

$R_{Ay} \times 9 - 9 \times 6 - 12 \times 3 + 10 \times 4 = 0$

$\therefore$ $R_{Ay} = 5.56$ kN $(\uparrow)$

From equation (1); we get,

$R_{Hy} = 15.44$ kN $(\uparrow)$

Now, taking section (A)-(A) and considering left part of the truss under equilibrium; we get,

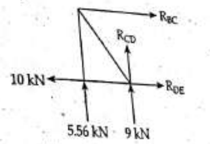


Figure: Section (A)-(A)

$\Sigma F_y = 0$

$R_{CD} + 5.56 - 9 = 0$

$\therefore$ $R_{CD} = 3.34$ kN (T)

$\Sigma F_x = 0$

$R_{BC} + R_{DF} = 10$ (2)

$^{+}\circlearrowleft \Sigma F_D = 0$

$5.56 \times 3 + 10 \times 4 + R_{BC} \times 4 = 0$

$\therefore$ $R_{BC} = -14.17$ kN $= 14.17$ kN (C)

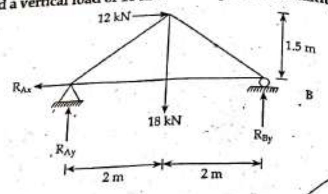
From equation (2); we get,

$R_{DF} = 24.17$ kN (T)

Hence,

Member	BC or 1	CD or 2	DF or 3
Member force (kN)	14.17	3.44	24.17
Nature (T or c)	C	T	T

4. Determine the member forces in truss shown in figure which carries a horizontal load of 12 kN and a vertical load of 18 kN. [2062 Baishakh, 2064 Falgun]



Solution:

Let us first calculate the reactions at supports.

$R_{Ay} = R_{By} = 9$ kN (↑)

$R_{Ax} = 12$ kN (←)

Now, using joint method to calculate member forces;

Joint A

$\Sigma F_y = 0$

$R_{AD} \sin 36.87° + 9 = 0$

$\therefore$ $R_{AD} = -15$ kN = 15 kN (C)

$\Sigma F_x = 0$

$R_{Ac} + R_{AD} \cos 36.87° - 12 = 0$

$\therefore$ $R_{Ac} = 24$ kN (T)

Joint C

$\Sigma F_y = 0$

$R_{CD} = 18$ kN (T)

$\Sigma F_x = 0$

$R_{Ac} = R_{AC} = 24$ kN (T)

Joint B

$\Sigma F_y = 0$

$R_{BD} \sin 36.87° + 9 = 0$

$\therefore$ $R_{BD} = -15$ kN = 15 kN (C)

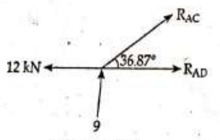


Figure: Joint A

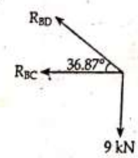


Figure: Joint B

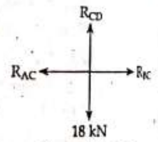
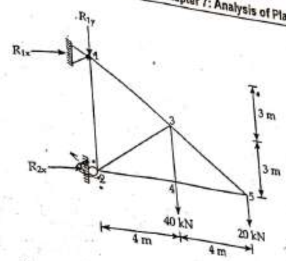


Figure: Joint C

Hence,

Member	AD	AC	CD	BC	BD
Member force (KN)	15	24	18	24	15
Nature (T or C)	C	T	T	T	C

5. **Determine the member forces in the pin joined truss shown in figure below.** [2065 Shrawan]



Solution:

First calculating the reactions;

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$R_{1x} + R_{2x} = 0$

$+\circlearrowleft \Sigma M_2 = 0$

$R_{1x} \times 6 + 40 \times 4 + 20 \times 8 = 0$

$\therefore$ $R_{1x} = -53.33$ kN = 53.33 kN (←)

$\therefore$ $R_{2x} = 53.33$ kN (→)

Now, using joint method to calculate forces in each member

Joint 5

$\Sigma F_y = 0$

$R_{35} \sin 36.87° - 20 = 0$

$\therefore$ $R_{35} = 33.33$KN (T)

$\Sigma F_x = 0$

$- R_{45} - R_{35} \cos 36.87° = 0$

$\therefore$ $R_{45} = -26.66$ kN = 26.66 kN (C)

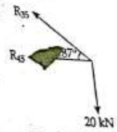


Figure: Joint 5

Joint 4

$\Sigma F_y = 0$

$R_{34} - 40 = 0$

$\therefore$ $R_{34} = 40$KN (T)

$\Sigma F_x = 0$

$- R_{45} - R_{24} = 0$

$\therefore$ $R_{24} = R_{45} = -26.66$ kN = 26.66 kN (C)

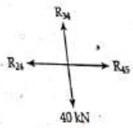


Figure: Joint 4

Joint 3

$\Sigma F_y = 0$

$R_{13} \sin 36.87° - R_{23} \sin 36.87° - R_{34} = 0$

$\therefore$ $R_{13} - R_{23} = 66.67$ (1)

$\Sigma F_x = 0$

$- R_{13} \cos 36.87° - R_{23} \cos 36.87° = 0$

$\therefore$ $R_{13} + R_{23} = 0$ (2)

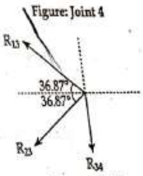


Figure: Joint 3

Solving equation (1) and (2); we get,

$R_{13} = 33.34$ kN (T)

$R_{23} = -33.34$ kN = 33.34 kN (C)

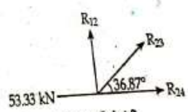


Figure: Joint 2

Joint 2

$\Sigma F_y = 0$

$R_{12} = R_{23} \sin 36.87° = 0$

$\therefore \quad R_{12} = 20 KN\ (T)$

Check

$\Sigma F_x = 0$

$53.33 + R_{23} \cos 36.87° + R_{24} = 0$

$\therefore \quad R_{24} = -26.66\ kN = 26.66\ kN\ (C)$

Hence,

Member	12	13	23	24	34	35	45
Member force (KN)	20	33.34	33.34	26.66	40	33.33	26.66
Nature (T or C)	T	T	C	C	T	T	C

Note

Reactions developed at hinge, roller, etc. are always perpendicular to their surface for vertical reaction and parallel for horizontal reaction as shown in the problem-above.

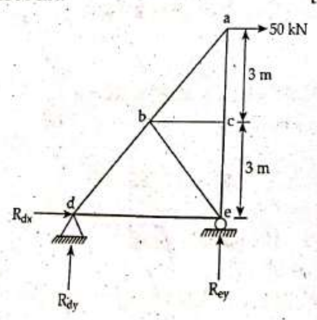
6. **What are the advantages of method of sections over method of joints for the analysis of member force in truss?** **[2066 Chaitra Back]**

Solution:

Followings can be considered as advantage of method of section over method of joint for analysis of member force in truss.

i) Method of section is more convenient if force in few members of truss are only required.
ii) All three equilibrium equations, $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M_z = 0$ can be used in method of section while only two equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$ can be used in joint method.
iii) Method of section can be used for up to three unknown members in a section while in joint method only two unknown members at a joint can be analyzed.
iv) Method of section can be used to check few member force calculated by joint method.
v) Method of section is easy to analyze compound truss.

7. **Determine the forces in the truss shown in the figure which carries a horizontal load of 50 kN.** **[2066 Chaitra Back]**



Solution:

To find the support reactions;

$+\uparrow \Sigma F_y = 0$

$R_{dy} + R_{ey} = 0 \qquad (1)$

$+\circlearrowleft \Sigma M_e = 0$

$R_{dy} \times 5 + 50 \times 6 = 0$

$\therefore \quad R_{dy} = -60\ kN$

or, $\quad R_{dy} = 60\ kN\ (\downarrow)$

$\therefore$ From equation (1); we get,

$R_{ey} = 60\ kN\ (\uparrow)$

and, $\quad \overset{+}{\rightarrow} \Sigma F_x = 0$

$\therefore \quad R_{dx} = 50\ kN\ (\leftarrow)$

Now, using joint method to calculate force in each member; we have,

Joint d

$\Sigma F_y = 0$

$R_{bd} \sin 50.19° = 60$

$\therefore \quad R_{bd} = 78.11 KN\ (T)$

$\Sigma F_x = 0$

$R_{de} + R_{db} \cos 50.19° - 50 = 0$

$\therefore \quad R_{de} = 1\ kN\ (T)$

Joint e

$\Sigma F_x = 0$

$-R_{de} - R_{be} \cos 50.19° = 0$

$\therefore \quad R_{be} = -1.56\ kN = 1.56\ kN\ (C)$

$\therefore \quad \Sigma F_y = 0$

$R_{ce} + R_{be} \sin 50.19° + 60 = 0$

$\therefore \quad R_{ce} = -58.80\ kN = 58.80\ kN\ (C)$

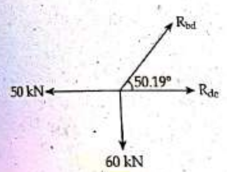


Figure: Joint d

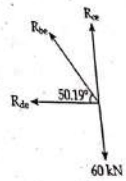


Figure: Joint e

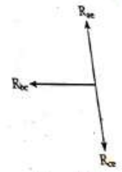


Figure: Joint c

Joint c

$\Sigma F_x = 0$

$\therefore \quad R_{bc} = 0$

$\Sigma F_y = 0$

$\therefore \quad R_{ae} - R_{ce} = 0$

$\therefore \quad R_{ae} = R_{ce} = -58.80\ kN = 58.80\ kN\ (C)$

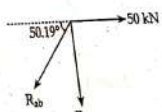


Figure: Joint a

Joint a

$\Sigma F_x = 0$

$-R_{ab} \cos 50.19° + 50 = 0$

$\therefore \quad R_{ab} = 78.10$ kN (T)

$\Sigma F_y = 0$

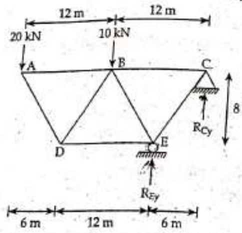
$-R_{ae} - R_{ab} \sin 50.19° = 0$

$\therefore \quad R_{ae} = -60$ kN $= 60$ kN (C)

Hence,

Member	bd	de	be	bc	ce	ab	ae
Member force (kN)	78.1	1	1.56	0	58.8	78.1	60
Nature (T or C)	T	T	C	–	C	T	C

8. Calculate the member forces of the given truss shown in the figure below. [2067 Ashadh, similar in 2064 Jestha]



Solution:

First finding the support reaction;

$+\uparrow \Sigma F_y = 0$

$R_{Cy} + R_{Ey} = 30 \quad (1)$

$+\circlearrowleft \Sigma M_C = 0$

$R_{Ey} \times 6 - 20 \times 24 - 10 \times 12 = 0$

$\therefore \quad R_{Ey} = 100$ kN $(\uparrow)$

From equation (1); we get,

$R_{Cy} = 30 - 100 = -70$ N $= 70$ N $(\downarrow)$

Since, here is no horizontal load, $R_{CX} = 0$.

Now, using joint method to calculate each member force;

Joint A

$\tan \theta = \frac{8}{6}$

$\therefore \quad \theta = 53.13°$

$\Sigma F_y = 0$

$-20 - R_{AD} \sin 53.13° = 0$

$\therefore \quad R_{AD} = -25$ N $= 25$ kN (C)

$\Sigma F_x = 0$

$R_{AB} + R_{AD} \cos 53.13° = 0$

$\therefore \quad R_{AB} = 15$ kN (T)

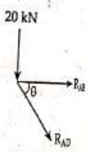


Figure: Joint A

Joint D

$\Sigma F_y = 0$

$R_{AD} \sin 53.13° + R_{BD} \sin 53.13° = 0$

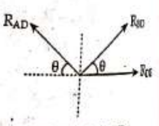


Figure: Joint D

$\therefore \quad R_{BD} = R_{AD} = -(-25$ kN$) = 25$ kN (T)

$\Sigma F_x = 0$

$-R_{AD} \cos 53.13° + R_{BD} \cos 53.13° + R_{DE} = 0$

$\therefore \quad R_{DE} = -30$ kN $= 30$ kN (C)

Joint E

$\Sigma F_x = 0$

$-R_{BE} \cos 53.13° + R_{CE} \cos 53.13° = 30 \quad (2)$

$\Sigma F_y = 0$

$R_{BE} \sin 53.13° + R_{CE} \sin 53.13° = -100 \quad (3)$

Solving equation (2) and (3); we get,

$R_{BE} = -37.5$ kN $= 37.5$ kN (C)

$R_{CE} = -87.5$ kN $= 87.5$ kN (C)

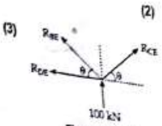


Figure: Joint E

Joint C

$\Sigma F_x = 0$

$-R_{CE} \cos 53.13° - R_{BC} = 0$

$\therefore \quad R_{BC} = 52.5$ kN (T)

Check

$\Sigma F_y = 0$

$-R_{CE} \sin 53.13° - 70 = 0$

$\therefore \quad R_{CE} = 87.5$ kN (C)

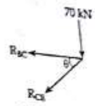
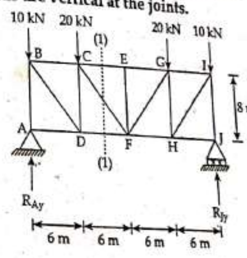


Figure: Joint C

Hence,

Member	AB	AD	DE	BE	BC	CE
Member force (kN)	15	25	30	37.5	52.5	87.5
Nature (T or C)	T	C	C	C	T	C

9. **Find bar forces in members a, b and c in the truss as individual in figure below. Shown loads are vertical at the joints.**



Solution:

Let us first calculate reactions;

$+\uparrow \Sigma F_y = 0$

$R_{Ay} + R_{Jy} = 60 \quad (1)$

$+\circlearrowleft \Sigma M_J = 0$

$R_{Ay} \times 24 - 10 \times 24 - 20 \times 18 - 20 \times 6 = 0$

$\therefore \quad R_{Ay} = 30$ kN $(\uparrow)$

From equation (1); we get,

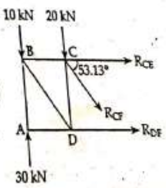


Figure: Section (1)–(1)

$R_{Iy} = 30$ kN (↑)

Now, taking section (1)–(1) and considering left part;

$\Sigma F_y = 0$

$-R_{CF} \sin 53.13° - 20 - 10 + 30 = 0$

∴ $R_{CF} = 0$

$\Sigma F_x = 0$

$R_{CE} + R_{CF} = 0$ (2)

$^{+}\circlearrowleft M_D = 0$

$30 \times 6 - 10 \times 6 + R_{CE} = 0$

∴ $R_{CE} = -120$ kN $= 120$ kN (C)

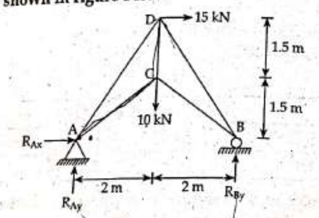
From equation (2); we get,

$R_{DF} = 120$ kN (T)

Hence,

Member	CE or a	CF or b	DF or c
Member force (KN)	120	0	120
Nature (T or C)	C	–	T

10. **Determine the support reactions and forces in all member in the given pin joined truss as shown in figure below.** [2068 Baishakh]



Solution:

Let us first calculate reactions.

$^{+}\uparrow \Sigma F_y = 0$

$R_{Ay} + R_{BY} = 10$ (1)

$^{+}\rightarrow \Sigma F_x = 0$

$R_{Ax} + 15 = 0$

∴ $R_{Ax} = -15$ kN $= 15$ kN (←)

$^{+}\circlearrowleft \Sigma M_A = 0$

$-R_{By} \times 4 + 10 \times 2 + 15 \times 3 = 0$

∴ $R_{By} = 16.25$ kN (↑)

From equation (1); we get,

∴ $R_{Ay} = 10 - 16.25 = 6.25$ kN (↓)

Now, using joint method to calculate force in all members;

Joint A

$\tan\theta = \frac{1.5}{2}$

∴ $\theta = 36.87°$

and, $\tan\beta = \frac{3}{2}$

∴ $\beta = 56.31°$

$\Sigma F_x = 0$

$R_{AC} \cos 36.87° + R_{AD} \cos 56.31° = 15$ (2)

$\Sigma F_y = 0$

$R_{AC} \sin 36.87° + R_{AD} \sin 56.31° = 6.25$ (3)

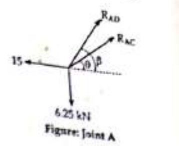


Figure: Joint A

Solving equation (2) and (3); we get

$R_{AC} = 27.08$ kN (T)

$R_{AD} = -12.02$KN $= 12.02$ kN (C)

Joint C

$\Sigma F_x = 0$

$-R_{AC} \cos 36.87° + R_{CB} \cos 36.87° = 0$

∴ $R_{CB} = R_{AC} = 27.08$ kN (T)

$\Sigma F_y = 0$

$R_{CD} - R_{AC} \sin 36.87° - R_{BC} \sin 36.87° - 10 = 0$

∴ $R_{CD} = 42.5$ kN (T)

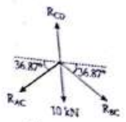


Figure: Joint C

Joint B

$\Sigma F_x = 0$

$-R_{BC} \cos 36.87° - R_{BD} \cos 56.31°$

∴ $R_{BD} = -39.05$ kN

$R_{BD} = 39.05$ kN (C)

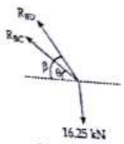


Figure: Joint B

Check

$\Sigma F_y = 0$

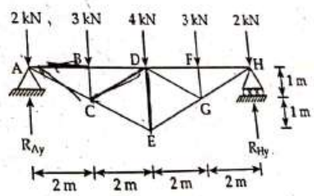
$R_{BC} \sin 36.87° + R_{BD} \sin 56.31° + 16.25 = 0$

∴ $R_{BD} = 39.05$ kN (C)

Hence,

Member	AD	AC	BC	CD	BD
Member force	12.02	27.08	27.08	42.5	39.05
Natural (T or C)	C	T	T	T	C

11. **Determine the force in members DE, CD, AB and AC for the inverted roof truss shown in figure below, state weather each member is in tension or compression.** [2068 Magh]



Solution:

The truss is supported by on hinge at 'A' and on rollers at 'H'. Since, all the external loads are vertical; there will be no horizontal reaction.

$^{+}\uparrow \Sigma F_y = 0$

$R_{Ay} + R_{Hy} = 14$ (1)

$+\circlearrowleft \Sigma M_H = 0$

$R_{Ay} \times 8 - 2 \times 8 - 3 \times 6 - 4 \times 4 - 3 \times 2 = 0$

$\therefore \quad R_{Ay} = 7\ kN\ (\uparrow)$

From equation (1); we get,

$R_{Hy} = 7\ kN\ (\uparrow)$,

Now, using joint method to calculate the forces in the members DE, CD, AB and AC

Joint A

$\Sigma F_y = 0$

$-R_{AC} \sin 26.57° + 7 - 2$

$\therefore \quad R_{AC} = 11.18\ kN\ (T)$

$\Sigma F_x = 0$

$R_{AB} + R_{AC} \cos 26.57° = 0$

$\therefore \quad R_{AB} = -10\ kN = 10\ kN\ (C)$

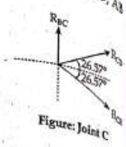


Figure: Joint C

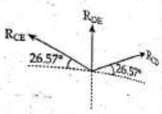


Figure: Joint E

Joint B

$\Sigma F_x = 0$

$R_{BD} - R_{AB} = 0$

$\therefore \quad R_{BD} = R_{AB} = -10\ kN = 10 KN\ (C)$

$\Sigma F_y = 0$

$-R_{BC} - 3 = 0$

$\therefore \quad R_{BC} = -3 KN = 3\ kN\ (C)$

Joint C

$\Sigma F_y = 0$

$R_{CD} \sin 26.57 - R_{CE} \sin 26.57 = -2$ (2)

$\Sigma F_x = 0$

$R_{CD} \cos 26.57° + R_{CE} \cos 26.57° = 10$ (3)

Solving equation (2) and (3); we get,

$R_{CD} = 3.35\ kN\ (T)$

$R_{CE} = 7.83\ kN\ (T)$

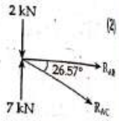


Figure: Joint A

Joint E

$\Sigma F_y = 0$

$R_{DE} + R_{GE} \sin 26.57° = -3.5$ (4)

$\Sigma F_x = 0$

$R_{GE}\ w\cos 26.57° - R_{CE} \cos 26.57 = 0$

$\therefore \quad R_{GE} = 7.83\ kN\ (T)$

$\therefore$ From equation (4); we get,

$R_{DE} = -7\ kN = 7\ kN\ (C)$

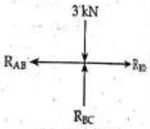
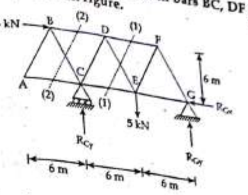


Figure: Joint B

Hence,

Member	DE	CD	AB	AC
Member force (kN)	7	3.35	7.83	11.18
Nature (T or C)	C	T	T	T

12. Use method of sections to compute the force in bars BC, DF and CE of the warren truss loaded as shown in figure. [2068 Chaitra]



Solution:

First of all calculating the reactions at hinge and roller;

$+\rightarrow \Sigma F_x = 0$

$-R_{GX} + 3 = 0$

$\therefore \quad R_{GX} = 3\ kN\ (\leftarrow)$

$+\uparrow \Sigma F_y = 0$

$R_{Cy} + R_{Gy} - 4 - 5 = 0$

$\therefore \quad R_{cy} + R_{Gy} = 9$ (1)

$+\circlearrowleft \Sigma M_G = 0$

$R_{Cy} \times 12 - 4 \times 9 - 5 \times 6 + 3 \times 6 = 0$

$\therefore \quad R_{Cy} = 4\ kN\ (\uparrow)$

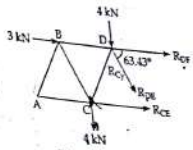


Figure: Section (1)–(1)

From equation (1); we get,

$R_{Gy} = 5\ kN\ (\uparrow)$

Taking section (1)–(1) and considering left part;

$\Sigma F_y = 0$

$-R_{DE} \sin 63.43° - 4 + 4 = 0$

$\therefore \quad R_{DE} = 0$

$\Sigma F_x = 0$

$R_{CE} + R_{DE} + 3 = 0$ (2)

$\Sigma M_C = 0$

$-R_{DF} \times 6 + 3 \times 6 + 4 \times 3 = 0$

$\therefore \quad R_{DF} = 5\ kN\ (T)$

From equation (2); we get,

$R_{CE} = -8\ kN = 8\ kN\ (C)$

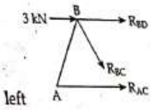


Figure: Section (2)-(2)

Again taking section (2)–(2) and considering left portion from the condition it can be said that,

$R_{AC} = 0$

i.e., AC is zero force members.

$\Sigma F_x = 0$

$R_{BC} \cos 63.43° + R_{BD} = 3$ (3)

$\Sigma M_A = 0$

$R_{BD} \times 6 + R_{BC} \cos 63.43° \times 6 + R_{BC} \sin 63.43° \times 3 + 3 \times 6 = 0$

$(6 \cos 63.43 + 3 \sin 63.43)\ R_{BC} + 6\ R_{BD} = -18$ (4)

Solving equation (3) and (4); we get,

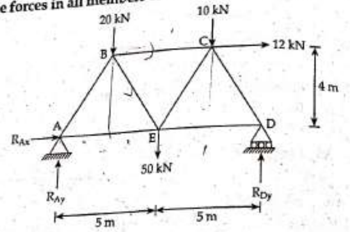
$R_{BC} = 0$

$R_{BD} = -3\text{ kN} = 3\text{ kN (C)}$

Hence,

Member	BC	DF	CE
Member force (kN)	0	5	8
Nature (T or c)	–	T	C

13. Determine forces in all members of truss in figure below. [2069 Ashadh Back]



Solution:

Let us first calculate reactions;

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$R_{Ax} + 12 = 0$

$\therefore \quad R_{Ax} = 12\text{ kN } (\leftarrow)$

$+\uparrow \Sigma F_y = 0$

$R_{Ay} + R_{Dy} - 20 - 10 - 50 = 0$

$\therefore \quad R_{Ay} + R_{Dy} = 80 \qquad (1)$

$+\circlearrowleft \Sigma M_D = 0$

$R_{Ay} \times 10 - 20 \times 7.5 - 50 \times 5 - 10 \times 2.5 + 12 \times 4 = 0$

$\therefore \quad R_{Ay} = 37.7\text{KN } (\uparrow)$

From equation (1); we get,

$R_{Dy} = 42.3\text{ kN } (\uparrow)$

Joint A

$\Sigma F_y = 0$

$R_{AB} \sin 58° + 37.7 = 0$

$\therefore \quad R_{AB} = -44.45\text{ kN} = 44.45\text{ kN (C)}$

and, $\Sigma F_x = 0$

$R_{AE} + R_{AB} \cos 558° - 12 = 0$

$\therefore \quad R_{AE} = 35.55\text{ kN (T)}$

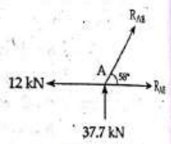


Figure: Joint A

Joint B

$\Sigma F_y = 0$

$-R_{BE} \sin 58° - R_{AB} \sin 58° - 20 = 0$

$\therefore \quad R_{BE} = 20.87\text{ kN (T)}$

$\Sigma F_x = 0$

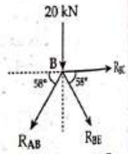


Figure: Joint B

$R_{BC} + R_{BE} \cos 58° - R_{AB} \cos 58° = 0$

$R_{BC} = -34.61\text{ kN}$

$\therefore \quad R_{BC} = 34.61\text{ kN (C)}$

Joint E

$\Sigma F_y = 0$

$R_{CE} \sin 58° + R_{BE} \sin 58° - 50 = 0$

$\therefore \quad R_{CE} = 38.09\text{ kN (T)}$

$\Sigma F_x = 0$

$R_{DE} - R_{AE} + R_{CE} \cos 58° - R_{BE} \cos 58° = 0$

$\therefore \quad R_{DE} = 26.42\text{ kN (T)}$

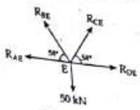


Figure: Joint E

Joint D

$\Sigma F_x = 0$

$-R_{DE} - R_{CD} \cos 58° = 0$

or, $R_{CD} = -49.86\text{ kN}$

$\therefore \quad R_{CD} = 49.86\text{ kN (C)}$

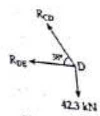


Figure: Joint D

Check

$\Sigma F_x = 0$

$R_{CD} \sin 58° + 42.3 = 0$

$\therefore \quad R_{CD} = 49.86\text{ kN (C)}$

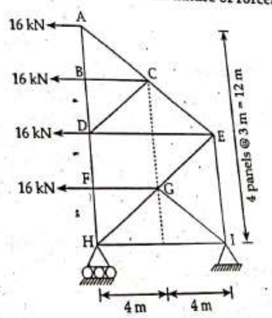
Hence,

Member	AB	AE	BC	BE	CD	CE	DE
Member force (kN)	44.45	35.55	34.61	20.87	49.86	38.09	26.42
Nature (T or C)	C	T	C	T	C	T	T

Note

While using joint method for particular joint keeps all the forces in tension and use the actual positive or negative values of force calculated during the process to calculate the forces in other members.

14. Use method of section to determine members forces DE and DF for the given pin jointed truss and also indicate the nature of forces. [2069 Bhadra]



Solution:

The truss is supported by a hinge at and roller at 'H'. So, calculating the reaction at these points;

$$+\uparrow \Sigma F_y = 0$$

or, $R_{Hy} + R_{Iy} = 0$

and, $+\circlearrowleft \Sigma M_I = 0$

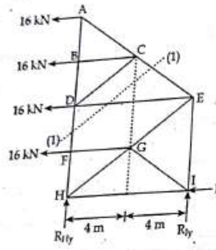
or, $R_{Hy} \times 8 - 16 \times 3 - 16 \times 6 - 16 \times 9 - 16 \times 12 = 0$

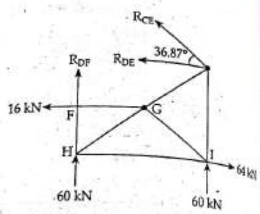
$\therefore$ $R_{Hy} = 60 \text{ kN } (\uparrow)$

and, $R_{Iy} = -60 \text{ kN} = 60 \text{ kN } (\downarrow)$

$$\xrightarrow{+} \Sigma F_x = 0$$

or, $R_{Ix} = 16 + 16 + 16 + 16 = 64 \text{ kN } (\leftarrow)$





Now, taking section (1)–(1) and considering lower section as shown in figure.

$$\Sigma F_x = 0$$

or, $-R_{DE} - 16 + 64 - R_{CE} \cos 36.87° = 0$

or, $-R_{DE} - R_{CE} \cos 36.87° = -48$

or, $R_{DE} + R_{CE} \cos 36.87° = 48$ (1)

and, $\Sigma F_y = 0$

or, $R_{DF} + R_{CE} \sin 36.87° = 0$ (2)

and, $+\circlearrowleft M_E = 0$

or, $R_{DF} \times 8 + 60 \times 8 + 16 \times 3 - 64 \times 6 = 0$

$\therefore$ $R_{DF} = -18 \text{ kN} = 18 \text{ kN (C)}$

From equation (2); we get,

$$-18 + R_{CE} \sin 36.87° = 0$$

$\therefore$ $R_{CE} = 30 \text{ kN (T)}$

From equation (1); we get,

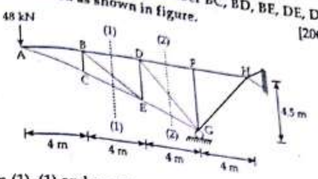
$$R_{DE} = 24 \text{ kN (T)}$$

Hence,

Member	DE	DF
Member force (kN)	24	18
Natural (T or C)	T	C

15. **Compute the force developed in the member BC, BD, BE, DE, DG and EG of the given truss loaded as shown in figure.**

[2069 Chaitra]



Solution:

Taking section (1)–(1) and considering left part,

$$\sum F_x = 0$$

or, $R_{BD} + R_{BE} \cos 36.87° + R_{CE} \cos 20.55° = 0$ (1)

$$\sum M_A = 0$$

or, $R_{BE} \sin 36.87° \times 4 + R_{CE} \sin 20.55° \times 4 - R_{CE} \cos 20.55° \times 1.5 = 0$

$\therefore$ $R_{BE} = 0$

$$\sum F_y = 0$$

or, $-R_{CE} \sin 20.55° - 48 = 0$

$\therefore$ $R_{CE} = -136.74 \text{ kN(C)}$

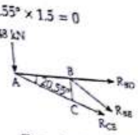


Figure: Section (1)–(1)

From equation (1); we get,

$$R_{BD} = 128.04 \text{ kN(T)}$$

Now, taking section (2)–(2) and considering left part,

$$\sum F_x = 0$$

or, $R_{DF} + R_{DG} \cos 48.37° + R_{EG} \cos 20.55° = 0$ (2)

$$\sum M_A = 0$$

or, $R_{DG} \sin 48.37° \times 8 + R_{EG} \sin 20.55° \times 8 - R_{EG} \cos 20.55° \times 3 = 0$

$\therefore$ $R_{DG} = 0$

$$\sum F_y = 0$$

or, $-48 - R_{EG} \sin 20.55° = 0$

$\therefore$ $R_{EG} = -136.74 \text{ kN} = 136.74 \text{ kN(C)}$

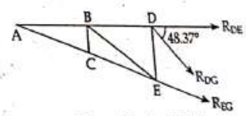


Figure: Section (2)–(2)

It can easily be analyzed at joint 'B' and 'D' that member BC and DE carry no force and hence are zero force members.

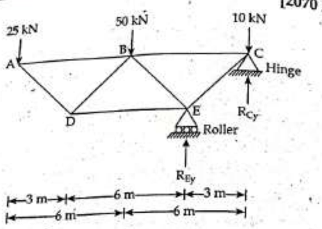
$\therefore$ $R_{BC} = 0$

and, $R_{DE} = 0$

Hence,

Member	BC	BD	BE	DE	DG	EG
Member force (KN)	0	128.04	0	0	0	136.74
Nature (T or C)	-	T	-	-	-	C

16. **Determine the member forces for given truss loaded as shown in figure below.** [2070 Ashadh Back]



Solution:

Let us calculate the reaction at support first.

$+\circlearrowleft \sum M_C = 0$

or, $R_{Ey} \times 3 - 25 \times 12 - 50 \times 6 = 0$

$\therefore$ $R_{Ey} = 200\ kN(\uparrow)$

$+\uparrow \sum F_y = 0$

or, $R_{Cy} + R_{Ey} - 25 - 50 - 10 = 0$

$\therefore$ $R_{Cy} = -115\ kN = 115\ kN(\downarrow)$

Using joint method to calculate member forces for given truss;

Joint A

$\sum F_y = 0$

or, $R_{AD} \sin 45° - 25 = 0$

$\therefore$ $R_{AD} = -35.35\ kN(C)$

$\sum F_x = 0$

or, $R_{AD} \cos 45° + R_{AB} = 0$

or, $R_{AB} = -R_{AD} \cos 45°$

$\therefore$ $R_{AD} = -(-35.35) \cos 45° = 25\ kN(T)$

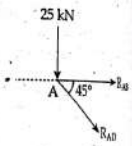


Figure: Joint A

Joint D

$\sum F_y = 0$

or, $R_{AD} \sin 45° + R_{BD} \sin 45° = 0$

$\therefore$ $R_{BD} = -R_{AD} = 35.35\ kN(T)$

$\sum F_x = 0$

or, $R_{DE} + R_{BD} \cos 45° - R_{AD} \cos 45° = 0$

$\therefore$ $R_{DE} = -50\ kN = 50\ kN(C)$

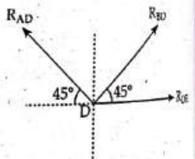


Figure: Joint D

Joint B

$\sum F_y = 0$

or, $-R_{BD} \sin 45° - R_{BE} \sin 45° - 50 = 0$

or, $R_{BE} \sin 45° = -75$

$\therefore$ $R_{BE} = -106.07\ kN = 106.07\ kN(C)$

$\sum F_x = 0$

or, $-R_{AB} + R_{BC} + R_{BE} \cos 45° - R_{BD} \cos 45° = 0$

$\therefore$ $R_{BC} = 125\ kN(T)$

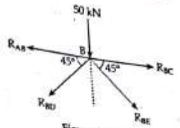


Figure: Joint B

Joint E

$\sum F_y = 0$

$200 + R_{BE} \sin 45° + R_{CE} \sin 45° = 0$

or, $R_{CE} \sin 45° = -200 - R_{BE} \sin 45°$

or, $R_{CE} = -176.78\ kN$

$\therefore$ $R_{BE} = 176.78\ kN(C)$

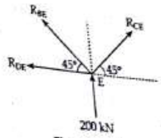


Figure: Joint E

Check

$\sum F_x = 0$

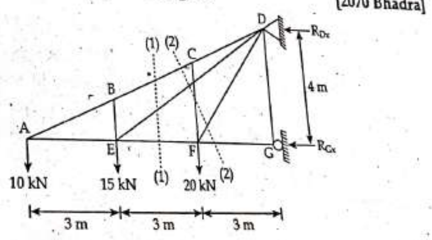
or, $-R_{OE} - R_{BE} \cos 45° + R_{CE} \cos 45° = 0$

$\therefore$ $R_{BC} = -176.78\ kN = 176.78\ kN(C)$

Hence;

Member	AB	AD	BD	BE	BC	DE	CE
Member force (KN)	25	35.35	35.35	106.07	125	50	176.78
Nature (T or C)	T	C	T	C	T	C	C

17. **Calculate the force developed in members BC, EC, FC, FD and FG of the cantilever truss loaded as shown in figure.** [2070 Bhadra]



Solution:

Taking section (1) –(1) and considering left part,

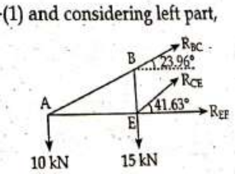


Figure: Section at (1)–(1)

$\sum F_x = 0$

or, $R_{EF} + R_{CE}\cos 41.63° + R_{BC}\cos 23.96° = 0$ (1)

$\sum F_y = 0$

or, $R_{BC}\sin 23.96° + R_{CE}\sin 41.63° = 25$ (2)

$M_E = 0$

or, $R_{BC}\cos 23.96° \times 1.33 - 10 \times 3 = 0$

∴ $R_{BC} = 24.68$ kN(T)

From equation (1) and (2);

$R_{CE} = 22.58$ kN(T) and

$R_{EF} = 39.43$ kN(C)

Now, taking section (2) –(2) and considering left part,

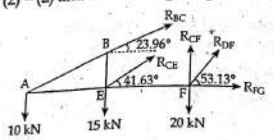


Figure: Section at (2)–(2)

$\sum F_x = 0$

or, $R_{BC}\cos 23.96° + R_{CE}\cos 41.63° + R_{DF}\cos 53.13° + R_{FG} = 0$

∴ $0.6R_{DF} + R_{FG} = -39.43$ (3)

$\sum F_y = 0$

or, $R_{BC}\sin 23.96° + R_{CE}\sin 41.63° + R_{CF} + R_{DF}\sin 53.13° - 10 - 15 - 20 = 0$

or, $R_{CF} + R_{DF}\sin 53.13° = 20$

∴ $R_{CF} = 20 - R_{DF}\sin 53.13°$ (4)

$^{+}\circlearrowleft \sum M_B = 0$

or, $-R_{FG} \times 1.33 - R_{DF}\cos 53.13° \times 1.33 - R_{DF}\sin 53.13° \times 3 - R_{CF} \times 3$
$- R_{CE}\cos 41.63° \times 1.33 - 10 \times 3 + 20 \times 3 = 0$

or, $-R_{FG} \times 1.33 - 3.2 \times R_{DF} - R_{CF} \times 3 - 22.45 - 30 + 60 = 0$

or, $-R_{FG} \times 1.33 - 3.2 \times R_{DF} - 3(20 - R_{DF}\sin 53.13°) + 7.55 = 0$

or, $-R_{FG} \times 1.33 - 3.2 \times R_{DF} - 60 + 3R_{DF}\sin 53.13° + 7.55 = 0$

∴ $R_{FG} \times 1.33 + 0.8R_{DF} = 52.45$ (5)

Solving equation (3) and (5), we get;

$R_{FG} = -37$ kN $= 37$ kN(C)

$R_{DF} = -4.05$ kN $= 4.05$ kN(C)

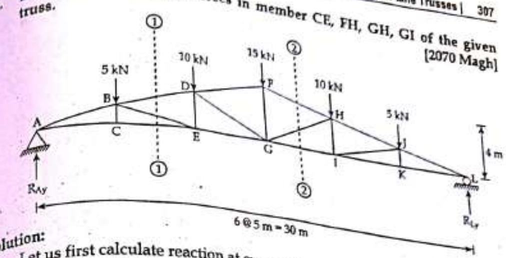
From equation (4);

$R_{CF} = 23.24$ kN(T)

Hence;

Member	BC	EC	EF	FC	FD	FG
Member force (kN)	24.68	22.58	39.43	23.24	4.05	37
Nature (T or C)	T	T	C	T	C	C

18. Determine the member forces in member CE, FH, GH, GI of the given truss. [2070 Magh]



Solution:

Let us first calculate reaction at supports;

$\sum M_L = 0$

or, $R_{Ay} \times 30 = 5 \times 25 + 10 \times 20 + 15 \times 15 + 10 \times 10 + 5 \times 5$

∴ $R_{Ay} = 22.5$ kN (↑)

and, $\sum F_y = 0$

or, $R_{Ay} + R_{Ly} = 5 + 10 + 15 + 10 + 5$

∴ $R_{Ly} = 22.5$ kN (↑)

Taking section (1)–(1) and considering left part; we have,

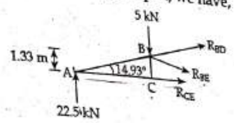


Figure: Section (1) – (1)

$\circlearrowleft \sum M_B = 0$

or, $22.5 \times 5 - R_{CE} \times 1.33 = 0$

∴ $R_{CE} = 84.375$ kN (T)

Taking section (2)–(2) and considering right part; we have,

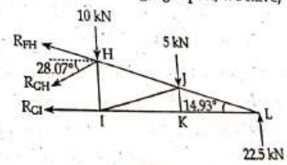


Figure: Section (2) – (2)

$\sum F_y = 0$

or, $R_{FH}\sin 14.93° - R_{GH}\sin 28.07° - 10 - 5 + 22.5 = 0$

∴ $0.258R_{FH} - 0.471R_{GH} + 7.5 = 0$ (1)

and, $\sum M_L = 0$

or, $-R_{FH}\sin 14.93° \times 10 + R_{FH}\cos 14.93° \times 2.67 + R_{GH}\sin 28.07° \times 10 + R_{GH}\cos 28.07° \times 2.67 + 10 \times 10 + 5 \times 5 = 0$

or, $-2.58R_{FH} + 2.58R_{FH} + 4.71R_{GH} + 2.36R_{GH} + 125 = 0$

$\therefore$ $7.07R_{GH} + 125 = 0$

$\therefore$ $R_{GH} = -17.68\ kN = 17.68\ kN\ (C)$

From equation (1); on putting the value of R_{GH}; we get,

$0.258R_{FH} - 0.471 \times (-17.68) + 7.5 = 0$

$\therefore$ $R_{FH} = -61.34\ kN = 61.34\ kN\ (C)$

and, $\sum F_x = 0$

or, $-R_{GI} - R_{FH}\cos 14.93° - R_{GH}\cos 28.07° = 0$

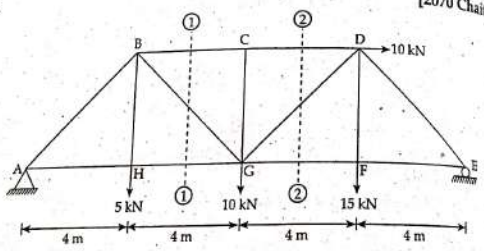
or, $-R_{GI} + 59.275 + 15.6 = 0$

$\therefore$ $R_{GI} = 74.88\ kN\ (T)$

Hence,

Member	CE	FH	GH	GI
Member force	84.375	61.34	17.68	74.88
Nature (T or C)	T	C	C	T

19. **Determine the total degree of internal, external indeterminacy of the given truss. Also determine the member forces in members BC, BG, HG and GD.** [2070 Chaitra]



Solution:

We have,

Total external indeterminacy (EI) $= r - 3 = 3 - 3 = 0$

Total internal indeterminacy (II) $= m - (2j - 3) = 13 - (2 \times 8 - 3) = 0$

Total degree of static indeterminacy $= EI + II = 0 + 0 = 0$

Let us first calculate reaction at supports;

$\sum M_E = 0$

or, $R_{Ay} \times 16 + 10 \times 4 = 5 \times 12 + 10 \times 8 + 15 \times 4$

$\therefore$ $R_{Ay} = 10\ kN\ (\uparrow)$

$\sum F_y = 0$

or, $10 - 5 - 10 - 15 + R_{Ey} = 0$

$\therefore$ $R_{Ey} = 20\ kN\ (\uparrow)$

$\sum F_x = 0$

or, $-R_{Ax} + 10 = 0$

$\therefore$ $R_{Ax} = 10\ kN\ (\leftarrow)$

Taking section (1)-(1) and considering left part; we have,

$\sum F_y = 0$

or, $-5 + 10 - R_{BG}\sin 45° = 0$

$\therefore$ $R_{BG} = 7.071\ kN\ (T)$

$\sum M_B = 0$

or, $10 \times 4 + 10 \times 4 - R_{GH} \times 4 = 0$

$\therefore$ $R_{GH} = 20\ kN\ (T)$

$\sum F_x = 0$

or, $-10 + R_{BC} + R_{GH} + R_{BG}\cos 45° = 0$

or, $-10 + R_{BC} + 20 + 7.071\cos 45° = 0$

$\therefore$ $R_{BC} = -15\ kN = 15\ kN\ (C)$

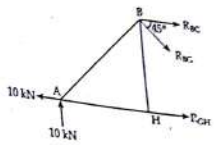


Figure: Section (1) - (1)

Taking section (2)-(2) and considering right part; we have,

$\sum F_y = 0$

or, $20 - 15 - R_{GD}\sin 45° = 0$

$\therefore$ $R_{GD} = 7.071\ kN\ (T)$

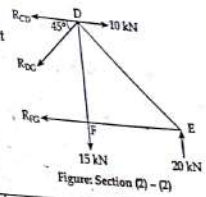
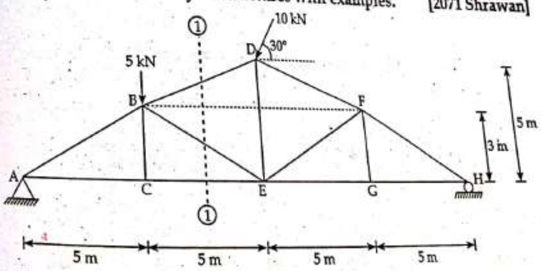


Figure: Section (2) - (2)

Hence,

Member	BC	BG	HG	GD
Member force	15	7.071	20	7.071
Nature (T or C)	C	T	T	T

20. **Find the member forces in CE, BE, BD and DE for the given truss. Define stability and determinacy of structures with examples.** [2071 Shrawan]



Solution:

Let us first calculate the reaction at supports;

$\sum M_H = 0$

or, $R_{Ay} \times 20 - 5 \times 15 - 10 \sin 30° \times 10 - 10 \cos 30° \times 5 = 0$

$\therefore$ $R_{Ay} = 8.42$ kN (↑)

$\sum F_y = 0$

or, $8.42 - 5 - 10 \sin 30° + R_{Hy} = 0$

$\therefore$ $R_{Hy} = 1.58$ kN (↑)

$\sum F_x = 0$

or, $R_{Ax} - 10 \cos 30° = 0$

$\therefore$ $R_{Ax} = 8.66$ kN (→)

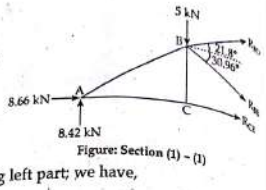


Figure: Section (1) - (1)

Taking section (1)-(1) and considering left part; we have,

$\sum M_B = 0$

or, $8.42 \times 5 - 8.66 \times 3 - R_{CE} \times 3 = 0$

$\therefore$ $R_{CE} = 5.37$ kN (T)

$\sum F_x = 0$

or, $5.37 + 8.66 + R_{BD} \cos 21.8° + R_{BE} \cos 30.96° = 0$

$\therefore$ $R_{BD} \cos 21.8° + R_{BE} \cos 30.96° = -14.03$ (1)

$\sum F_y = 0$

or, $8.42 + R_{BD} \sin 21.8° - R_{BE} \sin 30.96° - 5 = 0$

$\therefore$ $R_{BD} \sin 21.8° - R_{BE} \sin 30.96° = -3.42$ (2)

Solving equation (1) and (2); we get,

$R_{BD} = -12.75$ kN (C)

and, $R_{BE} = -2.56$ kN (C)

Now, considering joint D; we have,

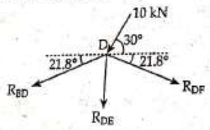


Figure: Joint D

$\sum F_x = 0$

or, $-R_{BD} \cos 21.8° + R_{DF} \cos 21.8° - 10 \cos 30° = 0$

$\therefore$ $R_{DF} = 3.42$ kN (C)

$\sum F_y = 0$

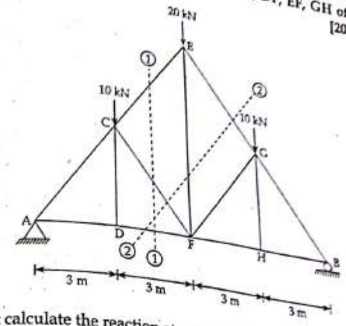
or, $-R_{DE} - R_{BD} \sin 21.8° - R_{DF} \sin 21.8° - 10 \sin 30° = 0$

$\therefore$ $R_{DE} = 1$ kN (T)

Hence,

Member	CE	BE	BD	DE
Member force	5.37	2.56	12.75	1
Nature (T or C)	T	C	C	T

21. **Determine the force developed in members CE, DF, EF, GH of the given truss loaded as shown in the figure.** **[2071 Bhadra]**



Solution:

Let us first calculate the reaction at supports;

$\sum M_E = 0$

or, $R_{Ay} \times 12 = 10 \times 9 + 20 \times 6 + 10 \times 3 = 0$

$\therefore$ $R_{Ay} = 20$ kN (↑)

$\sum F_y = 0$

or, $20 - 10 - 20 - 10 + R_{By} = 0$

$\therefore$ $R_{By} = 20$ kN (T)

Taking section (1)-(1) and considering left part; we have,

$\sum M_C = 0$

or, $20 \times 3 - R_{DF} \times 4 = 0$

$\therefore$ $R_{DF} = 15$ kN (T)

$\sum F_x = 0$

or, $R_{DF} + R_{CF} \cos 53.13° + R_{CE} \cos 53.13° = 0$

$\therefore$ $R_{CF} \cos 53.13° + R_{CE} \cos 53.13° = -15$

$R_{CE} + R_{CF} = -25$ (1)

$\sum F_y = 0$

or, $-10 + 20 + R_{CE} \sin 53.13° - R_{CF} \sin 53.13° = 0$

$\therefore$ $R_{CE} - R_{CF} = -12.5$ (2)

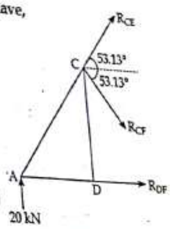


Figure: Section (1) - (1)

Solving equation (1) and (2); we get,

$R_{CE} = -18.75$ kN (C)

and, $R_{CF} = -6.25$ kN (C)

Taking section (2)-(2) and considering left part; we have,

$\sum F_x = 0$

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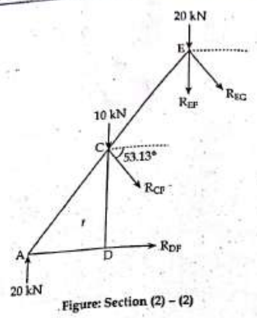


Figure: Section (2) – (2)

or, $R_{DF} + R_{CF}\cos 53.13° + R_{EG}\cos 53.13° = 0$

or, $15 - 6.25\cos 53.13° + R_{EG}\cos 53.13° = 0$

∴ $R_{EG} = -18.75 = 18.75$ kN (C)

$\sum F_y = 0$

or, $20 - R_{CF}\sin 53.13° - R_{EG}\sin 53.13° - R_{EF} = 0$

or, $20 + 5 + 15 - R_{EF} = 0$

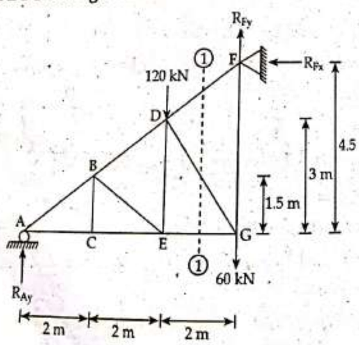
∴ $R_{EF} = 40$ kN (T)

R_{GH} is a zero force member.

Hence,

Member	CE	DF	EF	GH
Member force	18.75	15	40	0
Nature (T or C)	C	T	T	

22. **Write down the ideal assumptions of truss. Determine the member forces BC, DG and EG for the given truss.** [2071 Magh]



Solution:

The followings are the ideal assumption of truss:

i) The ends of the member are frictionless and hinged.
ii) Self weight of the member is negligible.
iii) Loads are applied only at the joints not on the member.
iv) Cross-section of the member is uniform.
v) The centroidal axis of the member should meet at a point.

Let us first calculate the reaction at supports;

$\sum M_F = 0$

or, $R_{Ay} \times 6 - 120 \times 1.5 = 0$

∴ $R_{Ay} = 30$ kN (↑)

$\sum F_y = 0$

or, $R_{Ay} - 60 - 120 + R_{Fy} = 0$

or, $30 - 60 - 120 + R_{Fy} = 0$

∴ $R_{Fy} = 150$ kN (↑)

BC is a zero force member.

∴ $R_{BC} = 0$

Taking section (1)-(1) and considering right part; we have,

$\sum M_G = 0$

or, $R_{DF}\sin 53.13° = 0$

∴ $R_{DF} = 0$

$\sum F_y = 0$

or, $R_{DG}\sin 56.31° - 60 + 150 = 0$

∴ $R_{DG} = -108.17$ kN (C)

$\sum M_F = 0$

or, $R_{EG} \times 4.5 + R_{DG}\cos 56.31° \times 4.5 = 0$

or, $4.5R_{EG} - 20 \times 108.17 = 0$

∴ $R_{EG} = 60$ kN (T)

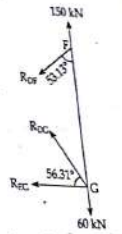
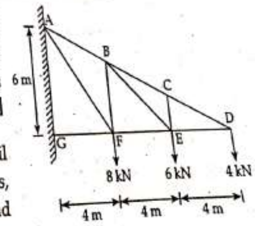


Figure: Section (1) – (1)

Hence,

Member	BC	DG	EG
Member force	0	108.17	60
Nature (T or C)		C	T

23. **Describe the use of trusses in engineering. Determine the force developed in BC, EF, AB, AF and BF members of cantilever truss loaded as shown in the figure.** [2071 Chaitra]



Solution:

Trusses are used in the various civil engineering structures such as bridges, roofs, transmission towers, over load water tanks, etc. It is light and cheaper as compared to the R.C.C. structures. In a large span roof, light truss roofs are used instead of heavy R.C.C. slabs. Trusses are used for conveyor frames in the material handling systems. They are used in the various bridge structures.

Joint D

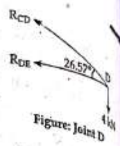


Figure: Joint D

$$\sum F_y = 0$$

or, $R_{CD}\sin 26.57° - 4 = 0$

∴ $R_{CD} = 8.94$ kN (T)

$\Sigma F_x = 0$

or, $-R_{DE} - R_{CD}\cos 26.57° = 0$

∴ $R_{DE} = -8$ kN $= 8$ kN (C)

Joint C

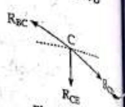


Figure: Joint C

($\searrow$) $\Sigma F_x = 0$

or, $-R_{BC} + R_{CD} = 0$

∴ $R_{BC} = 8.94$ kN (T)

$R_{CE} = 0$ (being a zero force member by external appearance)

Joint E

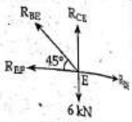


Figure: Joint E

$$\sum F_y = 0$$

or, $R_{BE}\sin 45° - 6 = 0$

∴ $R_{BE} = 8.485$ kN (T)

$\Sigma F_x = 0$

or, $-R_{EF} - R_{BE}\cos 45° + R_{DE} = 0$

or, $-R_{EF} - 6 - 8 = 0$

∴ $R_{EF} = 14$ kN (C)

Joint B

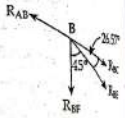


Figure: Joint B

$\Sigma F_x = 0$

or, $R_{BC}\cos 26.57° + R_{BE}\cos 45° - R_{AB}\cos 26.57° = 0$

or, $8 + 6 - R_{AB}\cos 26.57° = 0$

∴ $R_{AB} = 15.65$ kN (T)

$$\sum F_y = 0$$

or, $R_{AB}\sin 26.57° - R_{BE}\sin 45° - R_{BC}\sin 26.57° - R_{BF} = 0$

or, $R_{BF} = 7 - 6 - 4$

∴ $R_{BF} = -3$ kN $= 3$ kN (C)

Joint F

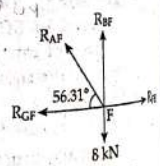


Figure: Joint F

$$\sum F_y = 0$$

or, $R_{BF} + R_{AF}\sin 56.31° - 8 = 0$

∴ $R_{AF} = 13.22$ kN (T)

$\Sigma F_x = 0$

or, $R_{EF} + R_{AF}\cos 56.31° - R_{GF} = 0$

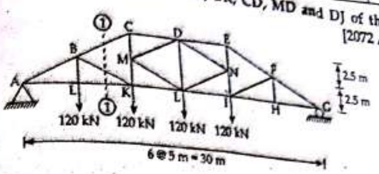
or, $-14 + 7.33 - R_{GF} = 0$

∴ $R_{GF} = 6.66$ kN (C)

Hence,

Member	BC	BE	EF	AB	AF	BF
Member force	8.94	8.485	14	15.65	13.22	3
Nature (T or C)	T	T	C	T	T	C

24. Determine the member forces of BC, BK, CD, MD and DJ of the given truss. [2072 Ashwin]



Solution:

Let us first calculate the reaction at supports;

$\Sigma M_G = 0$

or, $R_{Ay} \times 30 - 120 \times 25 - 120 \times 20 - 120 \times 15 - 120 \times 10 = 0$

∴ $R_{Ay} = 280$ kN (↑)

$$\sum F_y = 0$$

or, $280 - 120 - 120 - 120 - 120 + R_{Gy} = 0$

∴ $R_{Gy} = 200$ kN

Taking section (1)-(1) and considering left part; we have,

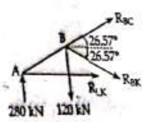


Figure: Section (1) - (1)

$\Sigma M_B = 0$

or, $280 \times 5 - R_{LK} \times 2.5 = 0$

∴ $R_{LK} = 560$ kN

$\Sigma F_x = 0$

or, $R_{BK}\cos 26.57° + R_{BC}\cos 26.57° = -560$ (1)

$$\sum F_y = 0$$

or, $280 + 120 + R_{BC}\sin 26.57° - R_{BK}\sin 26.57° = 0$

∴ $R_{BC}\sin 26.57° - R_{BK}\sin 26.57° = -400$ (2)

Solving equation (1) and (2); we get,

$R_{BC} = -760.2$ kN (C)

and, $R_{BK} = 134$ kN (T)

Now, considering the equilibrium of joint C; we have,

$\Sigma F_x = 0$

or, $-R_{BC}\cos 26.57° + R_{CD} = 0$

$\therefore \quad R_{CD} = -679.91$ kN (C)

$\sum F_y = 0$

or, $\quad -R_{CM} - R_{BC}\sin 26.57° = 0$

$\therefore \quad R_{CM} = 340$ kN (T)

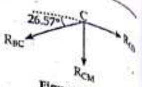


Figure: Joint C

Joint K

$\sum F_y = 0$

or, $\quad -120 + R_{BK}\sin 26.57° + R_{MK} = 0$

$\therefore \quad R_{MK} = 60$ kN (T)

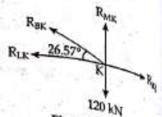


Figure: Joint K

Joint M

$\sum F_x = 0$

or, $\quad R_{MD}\cos 26.57° + R_{MJ}\cos 26.57° = 0$

$\therefore \quad R_{MD} + R_{MJ} = 0$ (3)

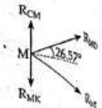


Figure: Joint M

$\sum F_y = 0$

or, $\quad 340 - 60 + R_{MD}\sin 26.57° - R_{MJ}\sin 26.57° = 0$

$\therefore \quad R_{MD} - R_{MJ} = -280$ kN (4)

Solving equation (3) and (4); we get,

$R_{MD} = -140$ kN (C)

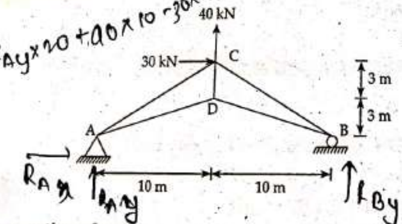
and, $R_{MJ} = 140$ kN (T)

Hence,

Member	BC	BK	CD	MD	DJ
Member force (kN)	760.2	134	679.91	−140	0
Nature (T or C)	C	T	C	C	-

25. **Write down the ideal assumption of truss. Calculate the force developed in all members of the truss loaded as shown in the figure by using suitable methods.** [2072 Kartik]

(+) $R_{Ay} \times 20 + 40 \times 10 - 30 \times 6$



Solution:

Ideal assumption of truss

See the solution of Q. no. 22 on page no. 312

Numerical

Let us first calculate the reaction at supports;

$\sum M_B = 0$

or, $\quad R_{Ay} \times 20 - 40 \times 10 + 30 \times 6 = 0$

$\therefore \quad R_{Ay} = 11$ kN (↑)

$\sum F_x = 0$

$\therefore \quad R_{Ax} = 30$ kN (←)

$\sum F_y = 0$

or, $\quad 11 - 40 + R_{By} = 0$

$\therefore \quad R_{By} = 29$ kN (↑)

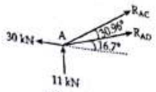


Figure: Joint A

Joint A

$\sum F_x = 0$

or, $\quad R_{AD}\cos 16.7° + R_{AC}\cos 30.96° = 30$ (1)

$\sum F_y = 0$

or, $\quad R_{AD}\sin 16.7° + R_{AC}\sin 30.96° = 11$ (2)

Solving equation (1) and (2); we have,

$R_{AD} = 100.95$ kN (T)

$R_{AC} = -77.77$ kN (C)

Joint D

$\sum F_x = 0$

or, $\quad -R_{AD}\cos 16.7° + R_{BD}\cos 16.7° = 0$

$\therefore \quad R_{BD} = 100.95$ kN (T)

$\sum F_y = 0$

or, $\quad R_{CD} - R_{BD}\sin 16.7° - R_{AD}\sin 16.7° = 0$

$\therefore \quad R_{CD} = 58$ kN (T)

Joint B

$\sum F_x = 0$

or, $\quad -R_{BD}\cos 16.7° - R_{BC}\cos 30.96° = 0$

$\therefore \quad R_{BC} = -112.76$ kN (C)

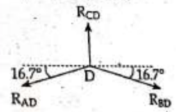


Figure: Joint D

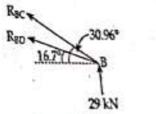


Figure: Joint B

Check

$\sum F_y = 0$

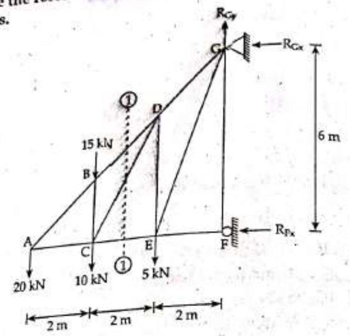
or, $\quad R_{BD}\sin 16.7° + R_{BC}\sin 30.96° + 29 = 0$

$\therefore \quad R_{BC} = -112.76$ kN (C)

Hence,

Member	AC	AD	CD	BC	BD
Member force (kN)	77.77	100.95	58	−112.76	100.95
Nature (T or C)	C	T	T	C	T

26. Determine the force developed in the members BD, CD, EG and DE of the given truss. [2072 Magh]



Solution:

Let us calculate the reaction at supports.

$\sum M_G = 0$

or, $[(-20) \times 6] - (10 \times 4) - (5 \times 2) - (15 \times 4) - (R_{Fx} \times 6) = 0$

$\therefore$ $R_{Fx} = 38.33$ kN (←)

$\sum F_x = 0$

or, $R_{Fx} + R_{Gx} = 0$

$\therefore$ $R_{Gx} = 38.33$ kN (→)

$\sum F_y = 0$

or, $R_{Gy} - 20 - 10 - 5 - 15 = 0$

$\therefore$ $R_{Gy} = 50$ kN (↑)

Taking section (1)-(1) and considering left part; we have,

$\sum M_C = 0$

or, $[(-20) \times 2] - (R_{BD} \cos 45°) = 0$

$\therefore$ $R_{BD} = 56.57$ kN (T)

$\sum F_y = 0$

or, $R_{BD} \sin 45° - 20 - 15 - 10 + R_{CD} \sin 63.43° = 0$

$\therefore$ $R_{CD} = 5.59$ kN (T)

$\sum F_x = 0$

or, $R_{CE} + R_{CD} \cos 63.43° + R_{BD} \cos 45° = 0$

$\therefore$ $R_{CE} = 42.5$ kN (C)

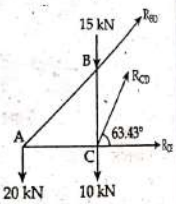


Figure: Section (1) - (1)

Now, considering the equilibrium of joint D; we have,

$\sum F_x = 0$

or, $R_{DG} \cos 45° - R_{BD} \cos 45° - R_{CD} \cos 63.43° = 0$

$\therefore$ $R_{DG} = 60.11$ kN (T)

$\sum F_y = 0$

or, $R_{DG} \sin 45° - R_{BD} \sin 45° - R_{CD} \sin 63.43° - R_{DE} = 0$

$\therefore$ $R_{DE} = 2.5$ kN (C)

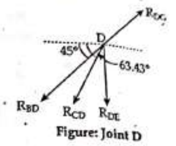


Figure: Joint D

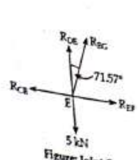


Figure: Joint E

Considering the equilibrium of joint E; we have,

$\sum F_y = 0$

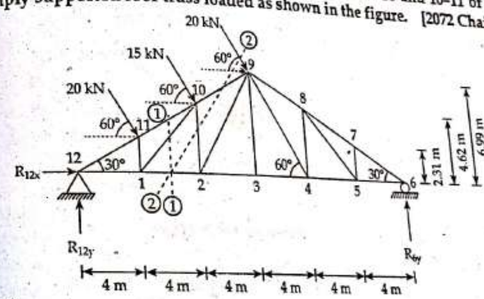
or, $R_{DE} + R_{EG} \sin 71.57° - 5 = 0$

$\therefore$ $R_{EG} = 7.91$ kN (T)

Hence,

Member	BD	CD	EG	DE
Member force (kN)	56.57	5.59	7.91	2.5
Nature (T or C)	T	T	T	C

27. **Find the member force of members 1-11, 1-10, 1-2, 2-10 and 10-11 of the simply supported roof truss loaded as shown in the figure.** [2072 Chaitra]



Solution:

Let us calculate the unknown reaction forces.

$\sum M_6 = 0$

or, $(R_{12y} \times 24) + 20 \cos 60° \times 2.31 - 20 \sin 60° \times 20 + 15 \cos 60° \times 4.62$
$- 15 \sin 60° \times 16 + 20 \cos 60° \times 6.93 - 20 \sin 60° \times 12 = 0$

$\therefore$ $R_{12y} = 26.46$ kN (↑)

$\sum F_y = 0$

or, $R_{12y} + R_{6y} = 20 \sin 60° + 15 \sin 60° + 20 \sin 60°$

$\therefore$ $R_{6y} = 21.17$ kN (↑)

$$\Sigma F_x = 0$$

or, $R_{12x} = 20\cos 60° + 15\cos 60° + 20\cos 60°$

∴ $R_{12x} = 27.5\ \text{kN}\ (\leftarrow)$

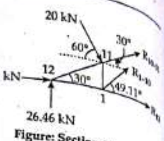


Figure: Section (1) – (1)

Considering section (1)-(1) and taking left part for the analysis; we have,

$$\Sigma M_1 = 0$$

or, $26.46 \times 4 + 20\cos 60° \times 2.31 + R_{10\text{-}11}\cos 30° \times 2.31 = 0$

∴ $R_{10\text{-}11} = 64.45\ \text{kN (C)}$

$$\sum F_y = 0$$

or, $26.46 - 20\sin 60° + R_{10\text{-}11}\sin 30° + R_{1\text{-}10}\sin 49.11° = 0$

∴ $R_{1\text{-}10} = 30.54\ \text{kN (T)}$

$$\Sigma F_x = 0$$

or, $-27.5 + 20\cos 60° + R_{10\text{-}11}\cos 30° + R_{1\text{-}10}\cos 49.11° + R_{12} = 0$

∴ $R_{12} = 53.32\ \text{kN (T)}$

Considering the equilibrium of joint 1; we have,

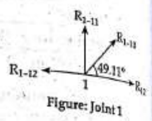


Figure: Joint 1

$$\sum F_y = 0$$

or, $R_{1\text{-}10}\sin 49.11° + R_{1\text{-}11} = 0$

∴ $R_{1\text{-}11} = 23.09\ \text{kN (C)}$

Taking section (2)–(2) and considering the left portion; we have,

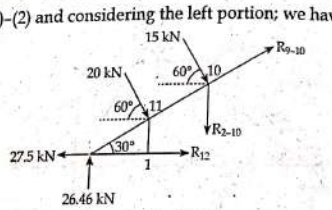


Figure: Section (2) – (2)

$$\Sigma F_x = 0$$

or, $-27.5 + 20\cos 60° + 15\cos 60° + R_{9\text{-}10}\cos 30° + R_{12} = 0$

∴ $R_{9\text{-}10} = -50.02\ \text{kN} = 50.02\ \text{kN (C)}$

$$\sum F_y = 0$$

or, $26.46 - 20\sin 60° - 15\sin 60° + R_{9\text{-}10}\sin 30° - R_{2\text{-}10} = 0$

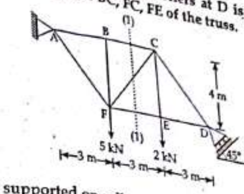
∴ $R_{2\text{-}10} = -28.86\ \text{kN} = 28.86\ \text{kN (C)}$

Hence,

Member	1-11	1-10	1-2	2-10	10-11
Member force (kN)	23.09	30.54	53.32	28.86	64.45
Nature (T or C)	C	T	T	C	C

ADDITIONAL PROBLEMS

1. **A truss hinged at A, and supported on rollers at D is shown in figures. Find the forces in the members BC, FC, FE of the truss.**



Solution:

Since the truss is supported on rollers at the end 'D', therefore the reaction at this support will be normal to the support. *i.e.*, inclined at 45° with the horizontal, so, the reaction at D will be the resultant of horizontal and vertical forces.

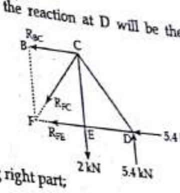
and, $R_{Dx} = R_{Dy}$

$$+\circlearrowright \Sigma M_A = 0$$

$$-R_{Dy} \times 9 + R_{Dx} \times 4 + 5 \times 3 + 2 \times 6 = 0$$

∴ $R_{Dx} = 5.4\ \text{kN}\ (\leftarrow)$

and, $R_{Dy} = 5.4\ \text{kN}\ (\uparrow)$



Now, taking section (1)–(1) and considering right part;

$$+\circlearrowright \Sigma M_F = 0$$

$$-R_{BC} \times 4 - 5.4 \times 6 + 2 \times 3 = 0$$

$$+\circlearrowright \Sigma M_C = 0$$

$$R_{FE} \times 4 - 5.4 \times 3 + 5.4 \times 4 = 0$$

∴ $R_{FE} = 1.35\ \text{kN (C)}$

and, $+\circlearrowright \Sigma M_B = 0$

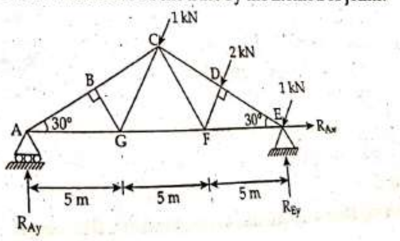
$$-R_{FCy} \times 2.4 + R_{FE} \times 4 - 2 \times 3 + 5.4 \times 6 - 5.4 \times 4 = 0$$

∴ $R_{FC} = 4.25\ \text{kN (T)}$

Hence,

Member	BC	FC	FE
Member force	6.6	4.25	1.35
Nature (T or C)	C	T	C

2. **The figure given below of truss has a span of 15 and loaded as shown. Find the forces in the members of the truss by the method of joints.**



Solution:

Firstly calculating the length of inclined members;

$$AC = CE = \frac{7.5}{\cos 30^\circ} = 8.66 \text{ m}$$

$$CD = DE = \frac{8.66}{2} = 4.33 \text{ m}$$

Height of the truss $= \sqrt{8.66^2 - 7.5^2} = 4.33$ m

$+\circlearrowleft \Sigma M_E = 0$

or, $R_{Ay} \times 15 - 1 \times 8.66 - 2 \times 4.33 = 0$

$\therefore$ $R_{Ay} = \frac{17.32}{15} = 1.15 \text{ kN } (\uparrow)$

But, $\Sigma F_y = 0$

$R_{Ay} + R_{Ey} = 1 \times \sin 60^\circ + 2 \sin 60^\circ + 1 \sin 60^\circ$

or, $R_{Ay} + R_{Ey} = 3.46$ kN

$\therefore$ $R_{Ey} = 2.31 \text{ kN } (\uparrow)$

and, $\Sigma F_x = 0$

or, $R_{Ex} = 1 \cos 60^\circ + 2 \cos 60^\circ + 1 \cos 60^\circ$

$\therefore$ $R_{Ex} = 2$ kN

Joint A

$\Sigma F_y = 0$

or, $R_{AB} \sin 30^\circ + 1.15 = 0$

$\therefore$ $R_{AB} = -2.3 \text{ kN} = 2.3 \text{ kN (C)}$

$\Sigma F_x = 0$

or, $R_{AB} \cos 30^\circ + R_{AG} = 0$

$\therefore$ $R_{AG} = 2 \text{ kN (T)}$

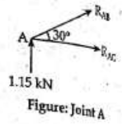


Figure: Joint A

Joint B

Resolving the forces parallel to, ABC; we get,

$R_{BC} = R_{AB} = -2.3 \text{ kN} = 2.3 \text{ k N(C)}$

Resolving the forces perpendicular to ABC;

$R_{BG} = 0$

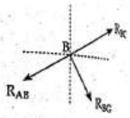


Figure: Joint B

Joint G

$\Sigma F_y = 0$

$R_{CG} = 0$

$\Sigma F_x = 0$

$R_{GF} = R_{AG} = 2$ kN

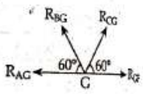


Figure: Joint G

Joint E

$\Sigma F_y = 0$

or, $R_{DE} \sin 30^\circ + 2.31 = \sin 60^\circ$

$\therefore$ $R_{DE} = -2.89 \text{ kN} = 2.89 \text{ kN (C)}$

$\Sigma F_x = 0$

or, $R_{EF} + R_{DE} \cos 30^\circ + \cos 60^\circ = 2$

$\therefore$ $R_{EF} = 4 \text{ kN (T)}$

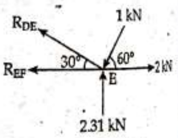


Figure: Joint E

Joint D

Resolving the forces parallel to the member CDE; we get,

$R_{CD} = R_{DE} = -2.89 \text{ kN} = 2.89 \text{ kN (C)}$

Resolving forces perpendicularly to CDE; we get,

$R_{DF} + 2 = 0$

$\therefore$ $R_{DF} = -2 \text{ kN} = 2\text{KN (C)}$

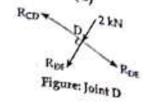


Figure: Joint D

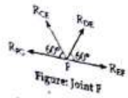


Figure: Joint F

Joint E

$\Sigma F_y = 0$

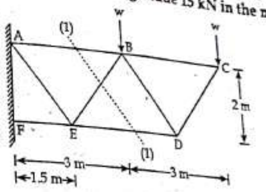
or, $R_{CF} \sin 60^\circ + R_{DF} \sin 60^\circ = 0$

$\therefore$ $R_{CF} = 2$ kN

Hence,

Member	AB	BC	CD	DE	EF	FG	GA	BG	CG	CF	DF
Member force (kN)	2.31	2.31	2.89	2.89	4	2	2	0	0	2	2
Nature (T or C)	C	C	C	C	T	T	T	–	–	T	C

3. **A cantilever truss is loaded as shown in figure. Find out the value 'w' which would produce the force of magnitude 15 kN in the member AB.**



Solution:

First of all let us find out force in the member AB of the truss in terms of 'w'. Now, taking section (1)-(1) and considering the equilibrium of the right part of the truss.

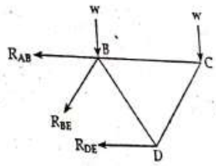


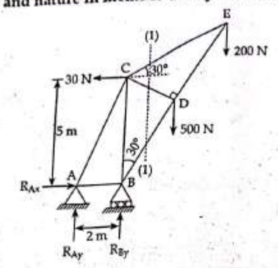
Figure: Section (1)-(1)

$R_{AB} \times 2 = w \times 1.5 + w \times 4.5$

or, $R_{AB} = \frac{6w}{2} = 3w$

Hence, the value of 'w' which would produce the force of 15 kN in the member AB $= \frac{w}{3w} \times 15 = 5$ kN

4. A jib crane truss is shown in figure calculate the magnitude and nature of forces in all members of the crane truss by using method of joints. Also check the force and nature in member CD by method of section.



Solution:

In ΔABC; we have,

$$\measuredangle CAB = \theta = \tan^{-1}\left(\frac{5}{2}\right) = 68.2°$$

Joint E

$\Sigma F_x = 0$

$-R_{CE}\cos 30° - R_{DE}\sin 30° = 0$

$\therefore\ R_{CE} + R_{DE}\tan 30° = 0$ **(1)**

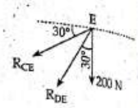


Figure: Joint E

$\Sigma F_y = 0$

$-R_{CE}\sin 30° - R_{DE}\cos 30° - 200 = 0$

$\therefore\ R_{CE} + R_{DE}\cot 30° = -400$ **(2)**

Subtracting equation (1) from (2); we get,

$R_{DE} = -346.41\text{ N} = 346.41\text{ N (C)}$

From equation (1); we get,

$R_{CE} = 200\text{ N (T)}$

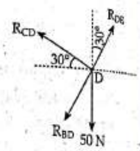


Figure: Joint D

Joint D

$\Sigma F_x = 0$

$R_{DE}\sin 30° - R_{CD}\cos 30° - R_{BD}\sin 30° = 0$

$\therefore\ R_{CD} + 0.577\,R_{BD} = -200$ (3)

$\Sigma F_y = 0$

$R_{DE}\cos 30° + R_{CD}\sin 30° - R_{BD}\cos 30° - 5° = 0$

$\therefore\ R_{CD} - 1.732\,R_{BD} = 700$ (4)

Subtracting equation (3) from (4); we get,

$2.309\,R_{BD} = -900$

$\therefore\ R_{BD} = -389.7\text{ kN} = 389.7\text{ kN (C)}$

From equation (3); we get,

$R_{CD} = 25\text{ N (T)}$

Joint C

$\Sigma F_x = 0$

$R_{CE}\cos 30° + R_{CD}\cos 30° - R_{AC}\cos 68.2° - 30 = 0$

$\therefore\ R_{AC} = 443.9\text{N (T)}$

$\Sigma F_y = 0$

$R_{CE}\sin 30° - R_{CD}\sin 30° - R_{BC} - R_{AC}\sin 68.2° = 0$

$\therefore\ R_{BC} = -324.6\text{ N} = 324.6\text{ N (C)}$

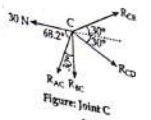


Figure: Joint C

Joint B

$\Sigma F_x = 0$

$-R_{AB} + R_{BD}\sin 30° = 0$

$\therefore\ R_{AB} = -194.85\text{ N} = 194.85\text{ N (C)}$

$\Sigma F_y = 0$

$R_{By} + R_{BC} + R_{BD}\cos 30° = 0$

$\therefore\ R_{By} = 662\text{ N} (\uparrow)$

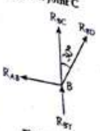


Figure: Joint B

Now, checking the magnitude and nature of force in member CD by using method of sections and taking section (1)-(1) while considering upper part.

$+\circlearrowright \Sigma M_E = 0$

$R_{CD} \times DE - 50 \times R_{DE}\sin 30° = 0$

$\therefore\ R_{CD} = 25\text{ N (T)}$

Hence, magnitude and nature of force in the member 'CD' by using method of section satisfies the answer by method of joint.

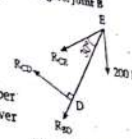
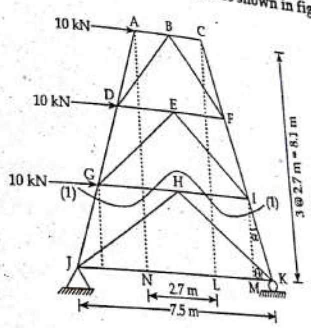


Figure: Section (1)-(1)

5. **Determine the force in the member Ik of the truss shown in figure below.**

Solution:



Here,

$Lk + JN = 7.5 - 2.7$

$2Lk = 4.8$

$\therefore\ Lk = 2.4\text{ m}$

and, $\tan\theta = \dfrac{3 \times 2.7}{2.4}$

$\therefore\ \theta = 73.49°$

$\alpha = 90° - 73.49° = 16.5°$

Now,

$Mk = 2.7 \times \tan 16.5° = 0.8\text{ m}$

Taking section (1)-(1) and considering lower part;

$\circlearrowleft^{+} M_G = 0$

$10 \times 2.7 + 10 \times (2 \times 2.7) + R_{Ik} \cos\alpha \times OF = 0$

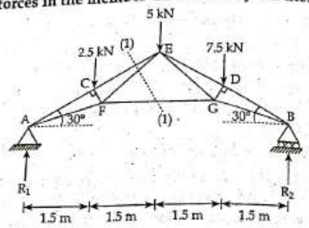
or, $27 + 54 + R_{Ik} \cos 16.51° \times 5.9 = 0$

or, $81 + R_{Ik} \times 5.656 = 0$

or, $R_{Ik} = -14.31$ kN

$\therefore$ $R_{Ik} = 14.31$ kN (C)

6. **The given figure shows a roof truss supported and loaded as shown. Determine the forces in the member CE and FG by the method of sections.**



Solution:

The truss is supported by a hinge at 'A' and on rollers at 'B'. Since, all the external loads are vertical the reaction at 'A' (R_1) and (R_2) will be vertical. There will be no horizontal reaction.

$\Sigma M_B = 0$

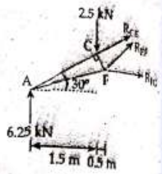
$R_1 \times 6 = 7.5 \times 1.5 + 5 \times 3 + 2.5 \times 4.5$

$\therefore$ $R_1 = 6.25$ kN (↑)

$\Sigma F_y = 0$

$R_1 + R_2 = 2.5 + 5 + 7.5$

$\therefore$ $R_2 = 15 - R_1 = 8.75$ kN (↑)



From the geometry of figure;

Vertical distance of 'E' from $F_G = 3 \tan 30° - 0.5 = 1.23$ m

Length of member $AF = \sqrt{2^2 + 0.5^2} = 2.06$ m

$\tan \measuredangle FAB = \frac{0.5}{2} = 0.25$

$\measuredangle FAB = \tan^{-1}(0.25) = 14.04°$

$\measuredangle CAF = 30 - 14.04 = 15.96°$

Length of member $CF = AF \sin 15.96° = 0.57$ m

Length of member $AC = \frac{AE}{2} = \frac{3}{2 \times \cos 30°} = 1.732$ m

Distance between line of action of load at 'C' and 'F' = 2 − 1.5 = 0.5 m

Now, taking section (1)-(1) and considering the equilibrium of left part;

$\Sigma M_E = 0$

$R_{FG} \times 1.23 + 2.5 \times 1.5 = 6.25 \times 3$

$\therefore$ $R_{FG} = 12.19$ kN

$\Sigma M_F = 0$

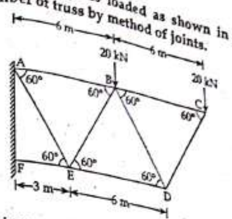
$R_{CE} \times 0.57 + 6.25 \times 2 = 2.5 \times 0.5$

$\therefore$ $R_{CE} = -19.73$ kN = 19.73 kN (C)

Hence, force in member CE = 19.73 kN (C)

Force in member FG = 12.19 kN (T)

7. **A cantilever truss warren type is loaded as shown in figure. Find the forces in all the member of truss by method of joints.**



Solution:

In case of cantilever trusses, the reactions need not to be calculated.

Joint C

$\Sigma F_y = 0$

$-20 - R_{CD} \sin 60° = 0$

$\therefore$ $R_{CD} = -23.1$ kN = 23.1 kN (C)

$\Sigma F_x = 0$

$-R_{BC} - R_{CD} \cos 60° = 0$

$\therefore$ $R_{BC} = 11.55$ kN (T)

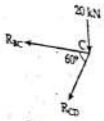


Figure: Joint C

Joint D

$\Sigma F_y = 0$

$R_{CD} \sin 60° + R_{BD} \sin 60° = 0$

$R_{CD} + R_{BD} = 0$

$\therefore$ $R_{BD} = -R_{CD} = 23.1$ kN (T)

$\Sigma F_x = 0$

$R_{CD} \cos 60° - R_{BD} \cos 60° - R_{DE} = 0$

$R_{DE} = -23.1$ kN = 23.1 kN (C)

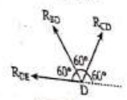


Figure: Joint D

Joint B

$\Sigma F_y = 0$

$-20 - 23.1 \sin 60° - R_{BE} \sin 60° = 0$

or, $R_{BE} = -46.2$ kN

$\therefore$ $R_{BE} = 46.2$ kN (C)

$\Sigma F_x = 0$

$-R_{AB} + 11.55 + 23.1 \cos 60° - R_{BE} \cos 60° = 0$

$\therefore$ $R_{AB} = 46.2$ kN (T)

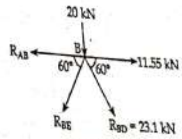


Figure: Joint B

Joint E

$\Sigma F_y = 0$

$R_{AE} \sin 60° + (-46.2) \sin 60° = 0$

$\therefore$ $R_{AE} = 46.2$ kN (T)

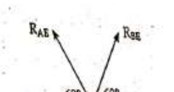


Figure: Joint E

$\Sigma F_x = 0$

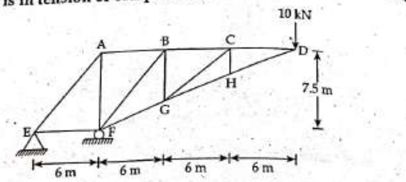
$-R_{EF} - R_{AE}\cos 60° + (-46.2)\cos 60° + (-23.1) = 0$

$\therefore \quad R_{EF} = -69.3\ \text{kN} = 69.3\ \text{kN (C)}$

Hence,

Member	AB	BC	CD	DE	EF	AE	BE	BD
Member force (kN)	46.2	11.55	23.1	23.1	69.3	46.2	46.2	23.1
Nature (T or c)	T	T	C	C	C	T	C	T

8. **Determine the force in each member of the truss shown state whether each member is in tension or compression.**



Solution:

By inspection of Joint H;

$R_{CH} = 0$ and $R_{DH} = R_{GH}$

($\because$ Note: see conditions for zero force members in theory part)

By inspection of Joint C,

$R_{CG} = 0$ and $R_{BC} = R_{CD}$

By inspection of Joint G,

$R_{BG} = 0$ and $R_{FG} = R_{GH}$

By inspection of Joint B,

$R_{BF} = 0$ and $R_{AB} = R_{BC}$

Joint D

$\sum F_y = 0$

or, $\quad -10 - R_{DH}\sin 22.62° = 0$

$\therefore \quad R_{DH} = -26\ \text{kN} = 26\ \text{kN(C)}$

$\sum F_x = 0$

or, $\quad -R_{CD} - R_{DH}\cos 22.62° = 0$

$\therefore \quad R_{CD} = 24\ \text{kN(T)}$

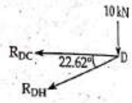


Figure: Joint D

Joint A

$\sum F_x = 0$

or, $\quad -R_{AE}\cos 51.34° + R_{AB} = 0$

$\therefore \quad R_{AE} = 38.42\ \text{kN(T)}$

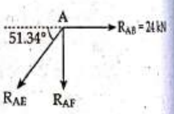


Figure: Joint A

$\sum F_y = 0$

or, $\quad -R_{AE}\sin 51.34° - R_{AF} = 0$

$\therefore \quad R_{AF} = -30\ \text{kN} = 30\ \text{kN(C)}$

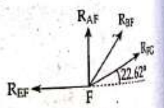


Figure: Joint F

Joint F

$\Sigma F_x = 0$

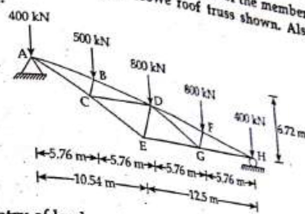
or, $\quad -R_{EF} + R_{FG}\cos 22.62° = 0$

$\therefore \quad R_{EF} = -24\ \text{kN(C)}$

Hence;

Member	AB	BC	CD	DH	GH	FG	EF	AF	BF	BG	CG	CH	AE
Member force (kN)	24	24	24	26	26	26	24	30	0	0	0	0	38.42
Nature (T or C)	T	T	T	C	C	C	C	C	-	-	-	-	T

9. **Determine the force in member DE and in each of the member located to the left of DE for the inverted Howe roof truss shown. Also state the nature of force.**



Solution:

Due to symmetry of load condition,

$R_{AY} = R_{HY} = 1600\ \text{kN(T)}$

Joint A

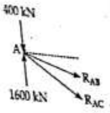


Figure: Joint A

$(^{+}\nearrow)\sum F_y = 0$

or, $\quad (1600 - 400)\cos 16.26° - R_{AC}\sin 16.26° = 0$

$\therefore \quad R_{AC} = 4114.3\ \text{kN(T)}$

$(^{+}\rightarrow)\sum F_x = 0$

or, $\quad R_{AC}\cos 32.52° - R_{AB}\cos 16.26° = 0$

$\therefore \quad R_{AB} = 3613.5\ \text{kN(C)}$

Joint B

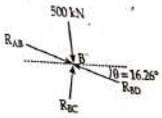


Figure: Joint B

$(^{+}\searrow)\sum F_x = 0$

or, $\quad 3613.5 - R_{BD} + 800\sin 16.26° = 0$

$\therefore \quad R_{BD} = 3837.5\ \text{kN(C)}$

$(^{+}\nearrow)\sum F_y = 0$

or, $\quad R_{BC} - 800\cos 16.26° = 0$

$\therefore \quad R_{BC} = -768\ \text{kN} = 768\ \text{kN(C)}$

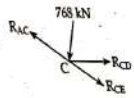


Figure: Joint C

Joint C

$(^{+}\nearrow)\sum F_y = 0$

or, $\quad R_{CD}\sin 32.52° - 768\cos 16.26° = 0$

$\therefore \quad R_{CD} = 1371.4\ \text{kN(T)}$

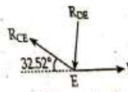


Figure: Joint E

$(^+\searrow)\sum F_x = 0$

or, $R_{CE} + 1371.4\cos 32.52° - 768\sin 16.26° - 4114.3 = 0$

$\therefore$ $R_{CE} = 2742.9$ kN(T)

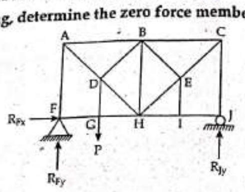
Joint E

$+\uparrow \sum F_y = 0$

or, $2742.9\sin 32.52° - R_{DE}\cos 16.26° = 0$

$\therefore$ $R_{DE} = -1536.01$ kN $= 1536.01$ kN(C)

10. For the given loading, determine the zero force members in truss shown.



Solution:

$\sum F_x = 0$

$\therefore$ $R_{CF} = 0$

By inspection of joint F; $R_{FG} = 0$

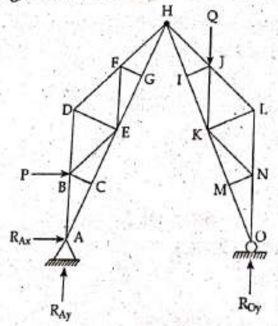
By inspection of joint G; $R_{GH} = 0$

By inspection of joint J; $R_{IJ} = 0$

By inspection of joint I; $R_{HI} = 0$, $R_{EI} = 0$

By inspection of joint E; $R_{BE} = 0$

11. For the given loading, determine the zero force members in the truss shown.



Solution:

By inspection of joint C; $R_{BC} = 0$

By inspection of joint G; $R_{FG} = 0$

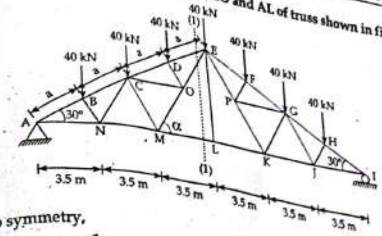
By inspection of joint F; $R_{FE} = 0$

By inspection of joint I; $R_{IJ} = 0$

By inspection of joint M; $R_{MN} = 0$

By inspection of joint N; $R_{KN} = 0$

12. Determine the force in member CD, DE, EO and AL of truss shown in figure.



Solution:

Due to symmetry,

$$R_{AY} = R_{IY} = \frac{1}{2}\times \text{total load} = 140 \text{ kN}(\uparrow)$$

Taking the section (1)-(1) as shown in figure;

Since, AB = BC = CD = DE, the horizontal distance of B, C, D from E are $\frac{1}{2}(10.5)$, $\frac{1}{2}(10.5)$ and $0.25(10.5)$

Now,

$\sum M_E = 0$, gives;

$$140\times(3.5\times 3) - 40\times\frac{3}{4}(10.5) - 40\times\frac{1}{2}(10.5) - 40\times 0.25(10.5) - R_{ML}\times 10.5\tan 30° = 0$$

$$\therefore\ R_{ML} = \frac{140-30-2010}{\tan 30°} = 138.56 \text{ kN}$$

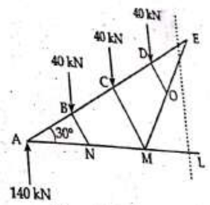


Figure: Section at (1)-(1)

Now,

$EL = 10.5\tan 30°$

$ML = 3.5$

$\measuredangle EML = \alpha = \tan^{-1}\left(\frac{EL}{ML}\right) = 60°$

$\sum F_y = 0$, gives;

$140 - 40 - 40 - 40 + R_{DE}\sin 30° - R_{EO}\sin 60° = 0$

$0.5R_{DE} - 0.866R_{EO} = 20$ (1)

$\sum F_x = 0$, gives;

$R_{ML} + R_{DE}\cos 30° - R_{EO}\cos 60° = 0$

$$138.56 + 0.866R_{DE} - 0.5R_{EO} = 0$$
$$R_{EO} = 277.12 + 1.732R_{DE} \qquad (2)$$

Substituting it in equation (1), we get;

$$0.5R_{DE} - 0.866(277.12 + 1.732R_{DE}) = 20$$
$$\therefore \quad R_{DE} = 219.88 \text{ kN(C)}$$

From equation (2); we get,

$$R_{EO} = 103.88 \text{ kN(T)}$$

Now, considering the equilibrium for the joint D;
The force acting at the joint is shown in figure.

$$(^{+}\searrow)\ \sum F_x = 0 \text{ (Force parallel to AE)}$$
$$R_{CD} - 219.98 - 40\cos 60^\circ = 0$$
$$\therefore \quad R_{CD} = 239.98 \text{ kN(C)}$$
$$(^{+}\nearrow)\ \sum F_y = 0 \text{ (Force normal to AE)}$$
$$R_{DO} - 40\sin 60^\circ = 0$$
$$\therefore \quad R_{DO} = 34.64 \text{ kN}$$

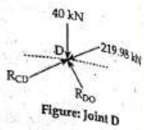
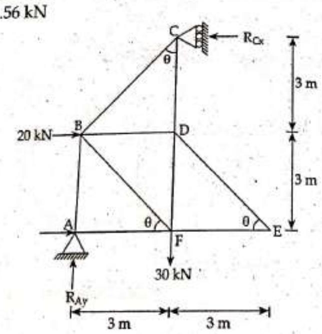


Figure: Joint D

Thus,

$$R_{DE} = 129.98 \text{ kN}$$
$$R_{EO} = 103.98 \text{ kN}$$
$$R_{CD} = 239.98 \text{ kN}$$
$$R_{ML} = 138.56 \text{ kN}$$



13. Determine the forces developed in all member of the truss shown in figure.

Solution:

$\sum M_A = 0$ gives;

$$R_{Cx} \times 6 - 20 \times 3 - 30 \times 3 - 30 \times 6 = 0$$
$$\therefore \quad R_{Cx} = 55 \text{ kN}(\leftarrow)$$
$$\sum F_y = 0$$
$$R_{Ay} - 30 - 30 = 0$$
$$\therefore \quad R_{Ay} = 60 \text{ kN}(\uparrow)$$

$$\sum F_x = 0$$
$$\text{or,} \quad R_{Ax} + 20 - R_{Cx} = 0$$
$$\therefore \quad R_{Ax} = 35 \text{ KN}(\rightarrow)$$

Joint C

$$\theta = \tan^{-1}\frac{3}{3} = 45^\circ$$
$$\sum F_x = 0$$
$$\text{or,} \quad -R_{BC}\sin 45^\circ - 55 = 0$$
$$\therefore \quad R_{BC} = -77.78 \text{ kN} = 77.78 \text{ kN(C)}$$
$$\sum F_y = 0$$
$$\text{or,} \quad -R_{CD} - R_{BC}\cos 45^\circ = 0$$
$$\therefore \quad R_{CD} = 55 \text{ kN(T)}$$

Joint E

$$\sum F_y = 0$$
$$\text{or,} \quad R_{DE}\sin 45^\circ - 30 = 0$$
$$\therefore \quad R_{DE} = 42.43 \text{ KN(T)}$$
$$\sum F_x = 0$$
$$\text{or,} \quad -R_{EF} - R_{DE}\cos 45^\circ = 0$$
$$\therefore \quad R_{EF} = -30 \text{ kN} = 30 \text{ kN(C)}$$

Joint D

$$\sum F_x = 0$$
$$\therefore \quad R_{BD} = R_{DE}\sin 45^\circ = 30 \text{ kN(T)}$$
$$\sum F_y = 0$$
$$\text{or,} \quad R_{DF} - 55 + 42.43\cos 45^\circ = 0$$
$$\therefore \quad R_{DF} = 25 \text{ kN(T)}$$

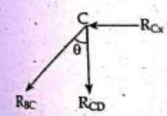


Figure: Joint C

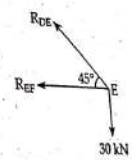


Figure: Joint E

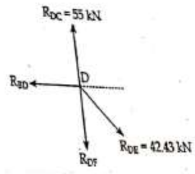


Figure: Joint D

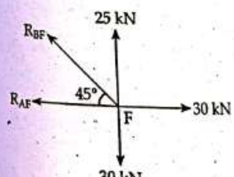


Figure: Joint F

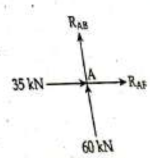


Figure: Joint A

Joint F

$$\sum F_y = 0$$

or, $R_{BF} \sin 45° + 25 - 30 = 0$

$\therefore$ $R_{BF} = 7.07\ kN(T)$

$\Sigma F_x = 0$

or, $-R_{AF} - 30 - R_{BF} \cos 45° = 0$

$\therefore$ $R_{AF} = -35\ KN = 35\ kN(C)$

Joint A

$\Sigma F_x = 0$

$\therefore$ $R_{AF} = -35\ KN = 35\ kN(C)$

$$\sum F_y = 0$$

$\therefore$ $R_{AB} = -60\ KN = 60\ kN(C)$

Hence;

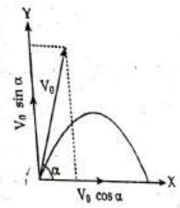
Member	AF	EF	DE	BF	AB	BD	BC	CD	DF
Member force (kN)	35	30	42.43	7.07	60	30	77.78	55	25
Nature (T or C)	C	C	T	T	C	T	C	T	T

CHAPTER 8 Kinematics of Particles

DEFINITIONS

- The study of the bodies under motion is known as *dynamics*.
- The study of motion of bodies without considering the forces causing the motion is termed as *kinematics*.
- The study of motion of bodies together with the forces causing the motion is known as *kinetics*.
- When the motion of a particle moves along a straight line, the motion is said to be *one-dimensional or rectilinear motion*.
- When a particle moves along a curve path then the motion is said to be *curvilinear*.
- *Position* of a particle is defined with respect to a fixed reference frame.
- *Displacement* of a particle is defined as the change in position of the particles with respect to time.
- *Velocity* of a particle is defined as the rate of change of displacement.
- *Acceleration* of a particle is defined as the rate of change of velocity with respect to time.
- *Uniform rectilinear motion* is defined as that type of motion in which the acceleration is zero or in other words velocity is uniform or constant.
- *Uniformly acceleration rectilinear motion* is that type of motion in which the acceleration is constant or the velocity is uniformly varying
- When bodies thrown upward acceleration is taken as negative (*i.e.*, $a = -g$)
- When bodies thrown downward acceleration is taken as positive (*i.e.*, $a = +g$)
- *Projectile motion* is a special type of curvilinear motion with constant acceleration occurring in vertical plane.

In projectile motion



Motion along the x – direction	Motion along the y-direction
$a_x = 0$	$a_y = -g$
$V_x = V_0 \cos \alpha$	$V_y = V_0 \sin \alpha - gt$
$X = (V_0 \cos \alpha)\, t$	$Vy^2 = (V_0 \sin \alpha)^2 - 2gy$
	$Y = (V_0 \sin \alpha)\, t - \frac{1}{2} gt^2$

- $y = x \tan\alpha - \frac{1}{2} g \frac{x^2}{(V_0 \cos\alpha)^2}$
- Horizontal range (R) $= \frac{V_0 \sin 2\alpha}{g}$
- Maximum height
- $(H_{max}) = \frac{V_0^2 \sin^2\alpha}{2g}$
- Time of flight (T) $= \frac{2 V_0 \sin\alpha}{g}$
- When acceleration is the function of time *i.e.,* $a = f(t)$. Then we can write,

$$a = \frac{dV}{dt} = f(t)$$

- When acceleration is the function of velocity *i.e.,* $a = f(V)$. Then, we can write,

$$a = \frac{dV}{dt} = f(V)$$

or, $$\frac{dV}{f(V)} = dt \qquad (1)$$

Also,

$$V\frac{dV}{dx} = f(V)$$

or, $$\frac{Vdv}{f(V)} = dx \qquad (2)$$

- When acceleration is the function of position *i.e.,* $a = f(x)$. Then we can write,

$$a = f(x)$$

or, $$\frac{V\,dV}{dx} = f(x)$$

or, $$V\,dV = f(x)\,dx$$

- Tangential and normal components of acceleration:
- $\vec{a} = a_t \hat{e}_t + \frac{V^2}{\rho}\hat{e}_n$
- Radial and transverse component of acceleration.

$$\vec{r} = r\hat{e}_r$$

$$\vec{V} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$

$$\vec{a} = [\ddot{r} - r(\dot{\theta})^2]\,\hat{e}_r + r\ddot{\theta} + 2\dot{r}\,\dot{\theta}]\hat{e}_\theta$$

- First law of motion

It states that "everybody continuous in state of rest or of uniform motion in a straight line unless it is compelled to change that state by force acting on it.'

EXAM SOLUTION

1. **The position of particle is defined by $\vec{r} = [4(t - \sin t)\,\vec{i} - (2t^2 - 3)\,\vec{j}\,]$ m where, 't' is in second and argument for sine is in radian. Determine speed for a particle and its normal and tangential components of acceleration when t = 1 sec.** [2059 Chaitra]

Solution:

Given that;

$$\vec{r} = [4(t - \sin t)\vec{i} - (2t^2 - 3)\vec{j}\,]$$

$$\therefore \vec{V} = \frac{d\vec{r}}{dt} = [4(1 - \cos t)\vec{i} - (4t)\,\vec{j}\,]$$

$$\vec{a} = \frac{d\vec{v}}{dt} = (4 \sin t)\,\vec{i} - 4\vec{j}$$

Again,

$$V = |\vec{V}| = \sqrt{V_x^2 + V_y^2} = [\{4(1 - \cos t)\}^2 + (-4t)^2]^{1/2}$$
$$= [16(1 - \cos t)^2 + 16t^2]^{1/2}$$

At t = 1 sec.

$$V = [\{4(1 - \cos 1)\}^2 + (-4 \times 1)^2]^{1/2}$$

[∵ Since, in cosine component is in radian so, 1 has to change in degree]

$$\therefore V = \left[\left\{4\left(1 - \cos\frac{180^0}{\pi}\right)\right\}^2 + 16\right]^{1/2}$$
$$= [\{4(1 - 0.54)\}^2 + 16]^{1/2} = 4.403 \text{ m/s} \qquad \left[\because 1^c = \frac{180^0}{\pi}\right]$$

Also,

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2}$$
$$= [(4 \sin t)^2 + 4^2]^{1/2}$$
$$= [16 \sin^2 t + 16]^{1/2}$$

At t = 1 sec.

$$a = [16 \sin^2(1) + 16]^{1/2}$$

where, 1 is in radian.

$$= \left[16 \times \left(\sin\frac{180^0}{\pi}\right)^2 + 16\right]^{1/2} = 5.23 \text{ m/s}^2$$

Tangential component of equation $(a_t) = \frac{dV}{dt}$

$$= \frac{d}{dt}[16(1 - \cos t)^2 + 16t^2]^{1/2}$$

$$= \frac{[32(1 - \cos t) \times \sin t + 32t]}{2[16(1 - \cos t)^2 + 16t^2]^{1/2}}$$

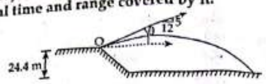
At t = 1 sec $a_t = 5.04 \text{ m/s}^2$

Now,

$$a = \sqrt{a_t^2 + a_n^2}$$

$$\therefore a_n = (a^2 - a_t^2)^{1/2} = (5.23^2 - 5.04^2)^{1/2} = 1.40 \text{ m/s}^2$$

2. **A projectile is projected with a velocity 30.5 m/s as shown in figure. Determine the total time and range covered by it.** [2061 Baishakh]



Solution:

Given that;

Initial velocity $(V_0) = 30.5$ m/s

$\tan\theta = \frac{5}{12}$

$\therefore \quad \sin\theta = \frac{5}{13}$

and, $\cos\theta = \frac{12}{13}$

Here,

Total time taken $(T) = \frac{2 v_0 \sin\theta}{g} = \frac{2 \times 11.73}{9.81} = 2.39$ sec.

Now, motion in x-direction

$x = V_0 \cos\theta \times t$ (1)

Motion in y-direction

$y = (v_0 \sin\theta) \times t - \frac{1}{2} g t^2$ (2)

Combining equation (1) and (2); we get,

$y = x\tan\theta - \frac{1}{2} g \frac{x^2}{v_0^2 \cos^2\theta}$ (3)

or, $-24.4 = x \times \frac{5}{12} - 4.905 \times \frac{x^2}{30.5^2 \times \cos^2\theta}$

[$\because$ – ve sign is taken. since 'O' is taken as origin]

or, $-24.4 = 0.42x - 6.19 \times 10^{-3} x^2$

or, $6.19 \times 10^{-3} x^2 - 0.42x - 24.4 = 0$

Solving; we get,

$x = -37.44$ or, 105.29 m

Thus,

$R = 105.29$ m

Note

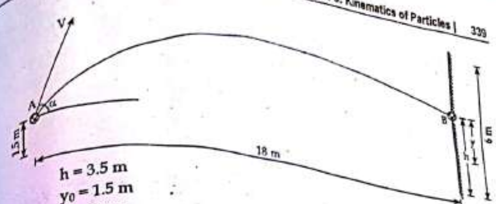
While calculating range, we must bear in mind that it is always the horizontal distance between the point of projection and point of fall at the same level as that projection, otherwise use equation (3).

3. **A player thrown a ball with an initial velocity 'V' of 10 m/s from a point 'A' located 1.5 m from the floor. Knowing that h = 3.5 m determine the angle α for which the ball will strike the wall at point B.** [2063 Baishakh]

Solution:

Given that;

Initial velocity $(V_0) = 10$ m/s



$h = 3.5$ m

$y_0 = 1.5$ m

$x = 18$ m

$\therefore \quad y = (3.5 - 1.5)$ m $= 2$ m

Angle of projection $(\alpha) = ?$

We have,

$y = x\tan\alpha - \frac{1}{2} g \frac{x^2}{(V_0 \cos\alpha)^2}$

or, $2 = 18\tan\alpha - 4.905 \times \frac{18^2}{16^2 \times \cos^2\alpha}$

or, $2 = 18\tan\alpha - 6.21 \sec^2\alpha$

or, $2 = 18\tan\alpha - 6.21(1 + \tan^2\alpha)$

or, $6.21\tan^2\alpha + 2 + 6.21 - 18\tan\alpha = 0$

or, $\tan^2\alpha - 2.90\tan\alpha + 1.39 = 0$

Solving; we get,

$\tan\alpha = 2.33$ or 0.567

$\therefore \quad \alpha = 66.77°$ or $29.55°$

4. **The acceleration of a particle is defined by the relation $a = 25 - 3x^2$, where 'a' is in mm/sec² and 'x' in mm. The particle starts with no initial velocity at the position x = 0. Determine; (a) the velocity when x = 2 mm (b) the position where the velocity is again zero.** [2063 Baishakh]

Solution:

We have given that;

$a = 25 - 3x^2$

or, $V\frac{dv}{dx} = 25 - 3x^2$

or, $V\,dv = (25 - 3x^2)\,dx$

or, $\int V\,dv = \int (25 - 3x^2)\,dx$

or, $\frac{V^2}{2} = 25x - x^3 + c_1$ (1)

To find c_1, at $x = 0$ and $v = 0$ then,

$0 = 0 + c_1 \Rightarrow c_1 = 0$

Then, equation (1) becomes,

$\frac{V^2}{2} = 25x - x^3$

or, $V^2 = 50x - 2x^3$

a) At $x = 2$ mm

$V^2 = 50 \times 2 - 2 \times (2)^3 = 100 - 16 = 84$

$\therefore$ $V = 9.17$ mm/sec

b) When $V = 0$ then,

or, $0 = 50x - 2x^3$

or, $x^3 - 25x = 0$

or, $x(x^2 - 25) = 0$

$\therefore$ $x = 0, \pm 5$

$\therefore$ Position = 5 mm

5. **The acceleration of a particle is defined by the relation, $a = kt^2$. Knowing that the velocity is -32 m/sec when time is zero second and again velocity is $+32$ m/s when time is 4 sec.**
   i) **Determine the value of constant k**
   ii) **Also develop equation of motion of particle, knowing that position that the position of particle is zero at the instant of 4 sec. [2064 Jestha]**

Solution:

Given that;

$a = kt^2$

or, Initial velocity $(V_0) = -32$ m/s

Velocity at 4 sec $(V_4) = +32$ m/s

Now,

$a = kt^2$ (1)

or, $\frac{dv}{dt} = kt^2$

or, $dv = kt^2 dt$

Integrating both sides; we get,

$V = \frac{kt^3}{3} + c_1$ (2)

To find c_1,

At $t = 0$, $V = -32$ m/s

$-32 = 0 + c_1$

or, $c_1 = -32$

$\therefore$ $V = \frac{kt^3}{3} - 32$ (3)

At $t = 4$ sec,

$V = +32$ m/sec

$32 = \frac{k \times 4^3}{3} - 32$

or, $k = 3$ m/sec^4

Then equation (3) becomes,

$V = \frac{3t^3}{3} - 32 = -32 = t^3 - 32$ (4)

or, $\frac{dx}{dt} = t^3 - 32$

or, $dx = (t^3 - 32)\,dt$

Integrating both sides; we get,

$x = \frac{t^4}{4} - 32t + c_2$ (5)

To find c_2,

At $t = 4$ sec and $x = 0$, then,

or, $0 = \frac{4^4}{4} - 32 \times 4 + c_2$

or, $c_2 = 64$

Then equation (5) becomes,

$\therefore$ $x = \frac{t^4}{4} - 32t + 64$

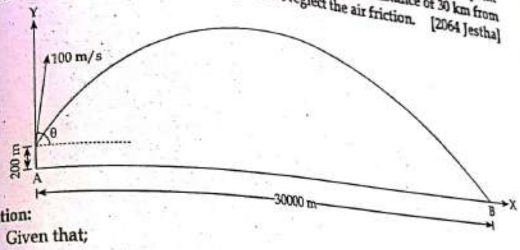
Thus, required expressions are;

$a = kt^2 = 3t^2$

$V = t^3 - 32$ (6)

$x = \frac{t^4}{4} - 32t + 64$

6. **A gun is placed on a cliff of 200 m above a plain. The muzzle velocity of the gun is 100 m/sec. At what angle (θ) to the horizontal axis must the gun point in order to hit a target, which is located at a horizontal distance of 30 km from the vertical axis of the gun on a plane? Neglect the air friction. [2064 Jestha]**



Solution:

Given that;

Height of cliff (y) = 200 m

Initial velocity $(V_0) = 100$ m/s

Horizontal distance (x) = 30 km = 30000 m

$\therefore$ Angle (θ) = ?

Motion in x-direction

$x = (V_0)_x t = V_0 \cos\theta \times t$ (1)

Motion in y-direction

$y = (V_0)_y t - \frac{1}{2} g t^2 = V_0 \sin\theta \times t - \frac{1}{2} g t^2$

or, $-200 = V_0 \sin\theta \times \frac{x}{V_0 \cos\theta} - 4.905 \times \frac{\cancel{x^2}\, x^2}{(V_0 \cos\theta)^2}$ [By equation (1)]

or, $-200 = x \tan\theta - 4.905 \times \frac{(30{,}000)^2}{(1000)^2 \times \cos^2\theta}$

or, $-200 = 30{,}000 \tan\theta - 4414.5 \sec^2\theta$

or, $-200 = 30,000 \tan\theta - 4114.5(1 + \tan^2\theta)$

or, $4414.5 \tan^2\theta - 30,000 \tan\theta - 200 + 4414.5$

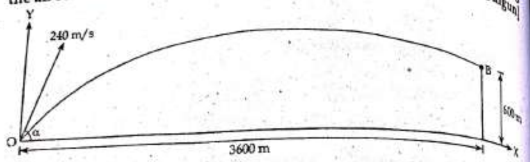
or, $\tan^2\theta - 6.80 \tan\theta + 0.955 = 0$

Solving; we get,

$\tan\theta = 6.66$ or 0.14

$\therefore \theta = 81.46°$ or, $7.97°$

7. **A projectile is fixed with an initial velocity of 240 m/s at a target located 600 m above and horizontal distance of 3600 m from the gun. Neglecting the air resistance determine the value of firing angle.** [2064 Falgun]



Solution:

Given that;

Initial velocity $(V_0) = 240$ m/s

Vertical distance (y) = 600 m

Horizontal distance (x) = 3600 m

Firing angle $(\alpha) = ?$

Now, motion in x-direction

$$x = v_0 \cos\alpha \times t \quad (1)$$

Motion in y-direction

$$y = V_0 \sin\alpha \times t - \frac{1}{2} g t^2 \quad (2)$$

Combining equation (1) and (2); we get,

$$y = x \tan\alpha - \frac{1}{2} g \frac{x^2}{v_0^2 \cos^2\alpha}$$

or, $600 = 3600 \tan\alpha - 4.905 \times \dfrac{3600^2}{240^2 \times \cos^2\alpha}$

or, $600 = 3600 \tan\alpha - 1103.63 \sec^2\alpha$

or, $\tan^2\alpha - 3.26 \tan\alpha + 1.54 = 0$

Solving; we get,

$\tan\alpha = 2.69$ or 0.573

$\therefore \alpha = 69.61°$ or, $29.81°$

8. ***The position of the particle which moves along a straight line is defined by the relation $x = t^3 - 6t^2 - 15t + 40$ where x expressed in meters and t is in second. Determine; (i) the time at which velocity is zero (ii) the position and distance travelled by particle at that time (iii) the acceleration of the particle at that time (iv) the distance travelled by the particle during the interval of t = 4 sec. to 6 sec.*** [2064 Falgun]

Solution:

We have given that;

$$x = t^3 - 6t^2 - 15t + 40 \quad (1)$$

or, $V = \dfrac{dx}{dt} = 3t^2 - 12t - 15 \quad (2)$

a) $t = ?$ when $V = 0$

$V = 3t^2 - 12t - 15$

$0 = 3t^2 - 12t - 15$

or, $t^2 - 4t - 5 = 0$

or, $(t - 5)(t + 1) = 0$

or, $t = 5$ or -1 sec

$\therefore t = +5$ sec (Neglecting – ve value)

$\therefore$ **Position when t = 5 sec**

From equation (1); we have,

$x = t^3 - 6t^2 - 15t + 40 = 5^3 - 6 \times 5^2 - 15 \times 5 + 40 = -60$ m

Distance travelled when t = 5 sec

From equation (1); we have,

$x = t^3 - 6t^2 - 15t + 40$

At $t = 0$ then $x = x_0$

$\therefore x_0 = 0 - 0 - 0 + 40 = 40$ m

$\therefore$ Distance travelled $= |x_5 - 2_0| = |-60 - 40|$ m $= 100$ m

Acceleration at t = 5 sec

From equation (2); we have,

$V = 3t^2 - 12t - 15$

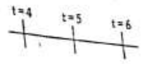
$\therefore a = \dfrac{dV}{dt} = 6t - 12$

$\therefore a_{(t=5\,sec)} = 6 \times 5 - 12 = 18$ m/s²

For t = 4 sec to t = 6 sec

$\therefore d = |x_5 - x_4| + |x_6 - x_5$

Now, from equation (1); we have,



$x_5 = 5^3 - 6 \times 5^2 - 15 \times 5 + 40 = -60$ m

$x_4 = 4^3 - 6 \times 4^2 - 15 \times 4 + 40 = -52$ m

$x_6 = 6^3 - 6 \times 6^2 - 15 \times 6 + 40 = -50$ m

$\therefore d = |-60 + 52| + |-50 + 60| = |-8| + |10| = 18$ m

9. **The rectangular component of acceleration for particle are $a_x = 3t$ and $a_y = 30 - 10t$ where, 'a' is in m/s². If the particle starts from rest at the origin find the radius of curvature of path at instant of 2 sec.** [2065 Shrawan]

Solution:

Given that;

$a_x = 3t$

and, $a_y = 30 - 10t$

Now,

$a_x = 3t$

$\dfrac{dv_x}{dt} = 3t$

or, $\int dv_x = \int 3t\, dt$

or, $v_x = \dfrac{3t^2}{2} + c_1$

To find c_1, At $t = 0$, $v_x = 0$

Then, $c_1 = 0$

$\therefore \quad v_x = \frac{3t^2}{2}$ (1)

Also,

$a_y = 30 - 10t$

or, $\frac{dv_y}{dt} = (30 - 10t)$

or, $\int dv_y = \int (30 - 10t)\, dt$

or, $v_y = 30t - \frac{10t^2}{2} + c_2$

At, $t = 0$, $v_y = 0$

$c_2 = 0$

$v_y = 30t - 5t^2$ (2)

We know,

$$\text{Radius of curvature } (\rho) = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \quad (3)$$

Here,

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

From equation (1); we have,

$$V_x = \frac{dx}{dt} = \frac{3t^2}{2}$$

From equation (2); we have,

$$V_y = \frac{dy}{dt} = 30t - 5t^2$$

$$\therefore \quad \frac{dy}{dx} = (30t - 5t^2) \times \frac{2}{3t^2} = \frac{20}{t} - \frac{10}{3}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(\frac{20}{t} - \frac{10}{3}\right)$$

$$= \frac{d\left(\frac{20}{t}\right)}{dt} \times \frac{dt}{dx} - 0$$

$$= -20t^{-2} \times \frac{2}{3t^2} = \frac{-40t^{-2}}{3t^2}$$

$$= -\frac{40}{3t^4}$$

At $t = 2$ sec.

$$\frac{dy}{dx} = \frac{20}{2} - \frac{10}{3} = \frac{20}{3}$$

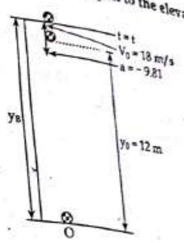
$$\frac{d^2y}{dx^2} = -\frac{40}{3t^4} = -\frac{40}{3 \times 24} = 0.833$$

From equation (3); we get,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1 + \left(\frac{20}{3}\right)^2\right]^{3/2}}{0.833} = 368 \text{ m}$$

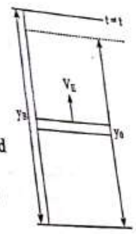
10. **A ball is thrown vertically upward from the 12m level in as elevator shaft with an initial velocity of 18 m/s. At the same instant an open platform elevator passes the 5 m level moving upward with a constant velocity of 2 m/s. Determine;**
   i) **when and where ball hits the elevator.**
   ii) **the relative velocity of ball with respect to the elevator when the ball hits the elevator.**

[2065 Shrawan]



Solution:

Since the ball has constant acceleration, its motion is uniformly accelerated. Consider 'O' be the origin and choosing +ve direction upward.



Given that;

$y_0 = 12$ m, $V_0 = 18$ m/s

a − 9.81 m/s²

Now, from equation of uniformly accelerated motion; we have,

$$V_B = V_0 + at = 18 - 9.81\, t \quad (1)$$

Also,

$$y_B = y_0 + v_0 t + \frac{1}{2} a t^2$$

$$= 12 + 18t - 4.905\, t^2 \quad (2)$$

Motion of elevator:

Since the elevator has constant velocity its motion is uniform.

$y_0 = 5$m

$V_E = 2$ m/s

$$\therefore \quad y_E = y_0 + V_E t = 5 + 2t \quad (3)$$

Ball hits the elevator only when

$y_E = y_B$

From equation (2) and (3); we have,

$$5 + 2t = 12 + 18t - 4.905t^2$$

or, $4.905t^2 - 16t - 7 = 0$

or, $t = -0.39$ or 3.65 sec.

Taking + ve time; we have,

$$y_E = 5 + 2 \times 3.65 = 12.30 \text{ m}$$

∴ Elevation from ground = 12.30 m

Also, the relative velocity of the ball with respect to the elevator is;

$$v_{b/E} = v_B - v_E$$
$$= (18 - 9.81\,t) - 2$$
$$= 16 - 9.81\,t$$
$$= 16 - 9.81 \times 3.65$$
$$= -19.81 \text{ m/s}$$

– ve sign indicates that the ball observed from the elevator to be downward.

11. *The motion of a particle is defined by the position vector, $\vec{r} = 3t^2\vec{i} + 4t^3\vec{j} + 5t^4\vec{k}$, where, 'r' is in meter and t is in sec. At the instant when t = 4 sec. Find the normal and tangential component of acceleration and the prime is radius of curvature.* [2067 Ashadh]

Solution:

We have given that;

$$\vec{r} = 3t^2\vec{i} + 4t^3\vec{j} + 5t^4\vec{k}$$

$$\therefore \quad \vec{v} = \frac{d\vec{r}}{dt} = 6t\vec{i} + 12t^2\vec{j} + 20t^3\vec{k}$$

$$\therefore \quad \vec{a} = \frac{d\vec{v}}{dt} = 6\vec{i} + 24t\vec{j} + 60t^2\vec{k}$$

Again,

$$V = |\vec{v}| = [(6t)^2 + (12\,t^2)^2 + (20\,t^3)^2]^{1/2}$$
$$\therefore \quad = [36t^2 + 144t^4 + 400t^6]^{1/2}$$

At, t = 4 sec.

$$V = [36 \times 4^2 + 144 \times 4^4 + 400 \times 4^6]^{1/2} = 1294.54 \text{ m/s}$$

Also,

$$a = |\vec{a}| = [6^2 + (24t)^2 + (60\,t^2)^2]^{1/2} = [36 + 576\,t^2 + 3600t^4]^{1/2}$$

At, time t = 4 sec.

$$a = [36 \times 576 \times 4^2 + 3600 \times 4^4]^{1/2} = 964.81 \text{ m/s}^2$$

Tangential component of acceleration is;

$$a_t = \frac{dv}{dt} = \frac{d}{dt}(36t^2 + 144t^4 + 400t^6)^{1/2}$$
$$= \frac{1}{2(36t^2 + 144t^4 + 400t^6)^{1/2}} \times (72t + 576t^3 + 2400t^5)$$
$$= \frac{(72t + 576t^3 + 2400t^5)}{2\,(36t^2 + 144t^4 + 400t^6)^{1/2}}$$

At, t = 4 sec.

$$a_t = 963.56 \text{ m/s}^2$$

Hence,

$$a_n = (a^2 - a_t^2)^{1/2} = [(964.81)^2 - (963.56)^2]^{1/2} = 49.1 \text{ m/s}^2$$

Again,

$$\rho = \frac{V^2}{a_n} = \frac{(1294.54)^2}{49.1} = 34131.03 \text{ m}$$

12. Define angular momentum and also prove that rate of change of angular momentum is equal to the moment of the force acting on that particle about the same point. [2067 Ashadh]

Solution:

Let the particle of mass m moving in the x–y plane as shown in figure 1(a). The momentum (or linear momentum) of the particle is equal to the vector $m\vec{v}$. The moment about O of the vector $m\vec{v}$ is called angular momentum of the particle about O at that instant and denoted by $\vec{H}_0$.

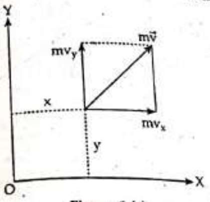


Figure: 1 (a)

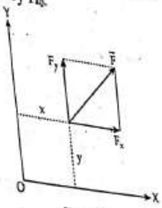


Figure: 1 (b)

mv_x and mv_y are components of $m\vec{v}$ in x and y direction.

Then, taking moment about O;

$$(+\uparrow)\ H_0 = x(mv_y) - y(mv_x)$$

$$\therefore \quad H_0 = m(xv_y - yv_x) \qquad (1)$$

Differentiating equation (1) with respect to time; we get,

$$\dot{H}_0 = m\frac{d}{dt}(xv_y - yv_x) = m(\dot{x}v_y + x\dot{v}_y - \dot{y}v_x - v_x\dot{y})$$

Here,

$$\dot{x} = v_x,\ \dot{y} = v_y,\ \dot{v}_x = a_x \text{ and } \dot{v}_y = a_y$$

$$\therefore \quad \dot{H}_0 = m(xa_y - ya_x) = xma_y - yma_x = xF_y - yF_x$$
$$= \text{moment of the force about O.}$$

Thus,

$$\dot{H}_0 = M_0 \qquad (2)$$

Hence the rate of change of angular momentum of the particle about any point at any instant is equal to the moment of the force ($\vec{F}$) acting on that particle about the same point.

13. **A particle is projected at an angle of 30° to horizontal axis with an initial velocity of 61 m/s hit the target located at 'h' meter below the horizontal axis and having the inclined slope of $\frac{3}{4}$ downward from the axis of target. Find the sloping distance covered by the projectile and the maximum height achieved by particle from the target.** [2067 Ashadh]

Solution:

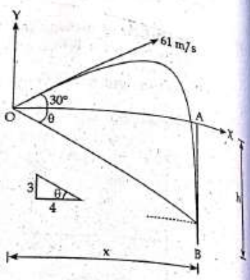
Given that;

Initial velocity $(V_0) = 61\ m/s$

Components of initial velocity $(V_0)_y = V_0 \sin\alpha = 61 \sin 30°$

Motion in x-direction;

$$x = (V_0)_x t - \frac{1}{2}gt^2$$



Motion in y-direction;

$$y = (V_0)_y t - \frac{1}{2}gt^2$$

or, $-h = 61 \sin 30° \times t - \frac{1}{2}gt^2$

$\therefore$ $h = -61 \sin 30° \times t + 4.905\ t^2$

From figure; we have,

$$\tan\theta = \frac{h}{x}$$

or, $\frac{3}{4} = \frac{-61 \sin 30° \times t + 4.05\ t^2}{61 \cos 30° \times t}$

or, $39.62\ t = 4.905\ t^2 - 30.5\ t$

or, $4.905\ t^2 - 70.12\ t = 0$

or, $t\,(4.905\ t - 70.12) = 0$

$\therefore$ $t = 0$ or 14.29 sec.

Now,

At $t = 14.29$ sec.

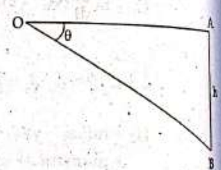
$\therefore$ $h = 4.905 \times (14.29)^2 = 61 \times \sin 30° \times 14.29 = 566.39\ m$

Also,

Sloping distance (OB) = ?

From Δ OAB; we have,

$$OB = \frac{AB}{\sin\theta} = \frac{h}{\sin\theta}$$



or, $OB = \frac{566.39}{\frac{3}{5}} = 944\ m$

Now,

$$\text{Maximum height } (h_{max}) = \frac{(V_0 \sin 30°)^2}{2g} = \frac{(30.5)^2}{2 \times 9.81}$$

$$= 47.41\ m$$

$\therefore$ Maximum weight achieved by particle from target $= (566.39 + 47.41)\ m$

$= 613.8\ m$

14. **The acceleration of a particle is directly proportional to the time. At time (t) = 0, the velocity of the particle is v = 16 m/s. Knowing that velocity (v) = 15 m/s position (x) = 20 m and time (t) = 1 sec determine the velocity, the position and total distance and travelled when time (t) = 7 sec.** [2067 Ashadh]

Solution:

The given equation is;

$a \propto t$

i.e., $a = kt$

where, k is constant.

or, $\frac{dv}{dt} = k\,t.$

or, $dv = kt\,dt$

Integrating; we have,

$$V = \frac{kt^2}{2} + c_1 \quad (1)$$

To find c_1, at $t = 0$ sec. and $V = 16\ m/s$. Then,

$16 = 0 + c_1$

or, $c_1 = 16\ m/s$

At, $t = 1$ sec. and $V = 15\ m/s$

$$V = \frac{kt^2}{2} + 16 \quad (2)$$

or, $15 = \frac{k \times 1}{2} + 16$

$\therefore$ $k = -2\ m/sec^3$

Then, equation (2) becomes;

$$V = \frac{-2 \times t^2}{2} + 16$$

$\therefore$ $V = 16 - t^2$

or, $\frac{dx}{dt} = 16 - t^2 \quad (3)$

or, $dx = (16 - t^2)\,dt$

Integrating; we have,

$$x = 16t - \frac{t^3}{3} + c_2 \quad (4)$$

To find c_2

At, $t = 1$ sec. and $x = 20$ m

Then,

$$20 = 16 \times 1 - \frac{1}{3} + c_2$$

$\therefore$ $c_2 = 4 + \frac{1}{3} = \frac{13}{3}$

By using c_2 in equation (4); then,

$$x = 16t - \frac{t^3}{3} + \frac{13}{3} \quad (5)$$

Now,

At $t = 7$ sec.

Then,

$$V = 16 - t^2 = 16 - 7^2 = -33 \text{ m/s}$$

$$x = 16 \times 7 - \frac{7^3}{3} + \frac{13}{3} = 2 \text{ m}$$

From equation (3) making $V = 0$ then,

$$0 = 16 - t^2$$

or, $t = \pm 4$ sec $= 4$ sec (+ ve taken)

To find distance;

At $t = 7$ sec.

Position when $t = 7$ sec *i.e.*, $x_7 = 2$m

Position when $t = 0$ *i.e.*, $x_0 = \frac{13}{3}$ m

Position when $t = 4$ sec *i.e.*, $x_4 = 16 \times 4 - \frac{4^3}{3} + \frac{13}{3} = 47$ m

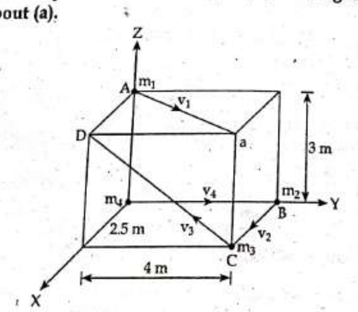
$\therefore$ Total distance travelled $= |x_4 - x_0| + |x_7 - x_4| = \left|47 - \frac{13}{3}\right| + |2 - 47|$

$$= 42.67 + 45 = 87.67 \text{ m}$$

15. **Shown in figure below is a system of particles at time 't'. The following data apply at this instant.**

$V_1 = 7$ m/s **$m_1 = 0.5$ kg**
$V_2 = 6$ m/s **$m_2 = 1,5$ kg**
$V_3 = 5$ m/s **$m_3 = 1$ kg**
$V_4 = 1.5$ m/s **$m_4 = 0.5$ kg**

Determine (a) The total linear momentum of the system (b) The angular momentum of the system about the origin (C) The angular momentum of the system about (a). [2067 Ashadh]



Solution:

We have given that,

$V_1 = 7$ m/s $m_1 = 0.5$ kg
$V_2 = 6$ m/s $m_3 = 1.5$ kg
$V_4 = 1.5$ m/s $m_4 = 0.5$ kg

From the figure co-ordinates of point are;

O (0, 0, 0), A (0, 0, 3), B (0, 4, 0), C (2.5, 4, 0), D (2.5, 0, 3) and a (2.5, 4, 3)

Now,

$$\vec{V}_1 = \frac{7[(2.5-0)\vec{i} + (4-0)\vec{j} + (3-3)\vec{k}]}{\sqrt{(2.5)^2 + (4)^2}} = 3.7\vec{i} + 5.92\vec{j}$$

$$\vec{V}_2 = \frac{6[(2.5-0)\vec{i} + (4-4)\vec{j}]}{\sqrt{(2.5)^2}} = 6\vec{i}$$

$$\vec{V}_3 = \frac{5[(2.5-2.5)\vec{i} + (0-4)\vec{j} + (3-0)\vec{k}]}{\sqrt{(-4)^2 + (3)^2}} = -4\vec{j} + 3\vec{k}$$

and, $\vec{V}_4 = 1.5\vec{j}$

a) Total linear momentum of particle $= \sum_{i=1}^{n} m_i\vec{v}_i$

$$= m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + m_4\vec{v}_4$$

$$= 0.5(3.7\vec{i} + 5.92\vec{j}) + 1.5(6\vec{i}) + (-4\vec{j} + 3\vec{k}) + 0.5(1.5\vec{j})$$

$$= 10.85\vec{i} + 0.260\vec{j} + 3\vec{k}$$

Angular momentum about O

From the figure; we have,

$$\vec{r}_1 = 3\vec{k}, \vec{r}_2 = 4\vec{j}$$

$$\vec{r}_3 = 2.5\vec{i} + 4\vec{j}$$

and, $\vec{r}_4 = 0$

$\therefore$ Angular momentum $= \sum_{T=1}^{n} \vec{r}_i \times m_i\vec{v}_i$

$$= \vec{r}_1 \times m_1\vec{V}_1 + \vec{r}_2 \times m_2\vec{V}_2 + \vec{r}_3 \times m_3\vec{V}_3 + \vec{r}_4 \times m_4\vec{V}_4$$

$$3\vec{k} \times (1.85\vec{i} + 2.99\vec{j}) + 4\vec{j} \times (9\vec{i}) + (2.5\vec{i} + 4\vec{j}) \times (-4\vec{j} + 3\vec{k})$$

$$= 5.55\vec{j} - 8.97\vec{i} - 30\vec{k} - 10\vec{k} - 7.5\vec{j} + 12\vec{i}$$

$$= 3.03\vec{i} - 1.95\vec{j} - 46\vec{k}$$

c) Angular moment about point 'a'

Here,

$$\vec{r}_1 = 3\vec{k} - (2.5\vec{i} + 4\vec{j} + 3\vec{k}) = -2.5\vec{i} - 4\vec{j}$$

$$\vec{r}_2 = 4\vec{j} - (2.5\vec{i} + 4\vec{j} + 3\vec{k}) = -2.5\vec{i} - 3\vec{k}$$

$$\vec{r}_3 = (2.5\vec{i} + 4\vec{j}) - (2.5\vec{i} + 4\vec{j} + 3\vec{k}) = -3\vec{k}$$

$$\vec{r}_4 = -(2.5\vec{i} + 4\vec{j} + 3\vec{k})$$

$$\therefore \quad \vec{H}_0 = \sum_{T=1}^{n} \vec{r}_i \times m_i\vec{v}_i$$

$$= \vec{r}_1 \times m_1\vec{V}_1 + \vec{r}_2 \times m_2\vec{V}_2 + \vec{r}_3 \times m_3\vec{V}_3 + \vec{r}_4 \times m_4\vec{V}_4$$

$$= (-2.5\,\vec{i} - 4\,\vec{j}) \times (1.85\vec{i} + 2.99\vec{j}) + (-2.5\,\vec{i} - 3\vec{k})$$

$$\times (9\,\vec{i}) + (-3\vec{k}) \times (-4\vec{j} + 3\vec{k}) - (2.5\,\vec{i} + 4\,\vec{j} + 3\vec{k}) \times (0.75\vec{j})$$

$$= -7.475\,\vec{k} + 7.4\,\vec{k} - 2.7\vec{j} - 12\vec{j} - 1.875\,\vec{k} + 2.25\vec{i}$$

$$= 2.25\,\vec{i} - 39\,\vec{j} - 1.95\vec{k}$$

16. **State Newton's second law of motion and derive the relation between linear momentum and force.** [2067 Mangshir]

Solution:

It states that, "If the resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of the resultant and direction of this resultant fore."

From the experiment;

i) When a force of constant magnitude and direction is applied on a block on a frictionless surface, it can be found that acceleration have constant magnitude and same direction as that of applied force.

i.e., $\vec{a} \propto \vec{F}$ (1)

ii) The acceleration is inversely proportional to the mass of the body.

$$\vec{a} \propto \frac{1}{m} \quad (2)$$

Combining, both equations; we have,

$$\vec{a} \propto \frac{\vec{F}}{m}$$

or, $\vec{a} = k\dfrac{\vec{F}}{m}$

where, k is proportionality constant.

At, m = 1 kg, $\vec{a} = 1\ m/s^2$ and $\vec{F} = 1$ N.
Then, k = 1.

$$\vec{a} = \frac{\vec{F}}{m}$$

or, $\vec{F} = m\vec{a}$;

which is mathematical expression of Newton's second law of motion.

Now,

$$m\vec{a} = m\frac{d\vec{v}}{dt}$$

Since, mass 'm' of the particle is constant so,

The product of the mass and velocity vector is defined as linear momentum.

Thus,

$$\vec{F} = \frac{d(m\vec{v})}{dt}; \text{ which is required relation.}$$

17. **A ball is thrown vertically upward with a velocity of 25 m/s. After 2 seconds another ball is thrown with the same velocity. Find the height at which the two balls pass each other.** [2067 Mangshir]

Solution:

Let, initial velocity of both balls

$$V_0 = V_1 = v_2 = 25\ m/s$$

Let, 'h' be the height at which two balls pass each other.

t_1 = time elapsed by the first ball before passing second ball.

t_2 = time elapsed by second ball.

From the question; we have,

$$t_1 - t_2 = 2$$

For first ball

$$h = V_0t_1 - \frac{1}{2}gt_1^2 \quad (1)$$

For second ball

$$h = V_0t_2 - \frac{1}{2}g\,t_2^2 \quad (2)$$

Subtracting equation (2) from (3) gives; (3)

$$\frac{1}{2}g\,(t_1^2 - t_2^2) = V_0(t_1 - t_2)$$

or, $t_1 + t_2 = \dfrac{25}{4.905} = 5.97$

$\therefore$ $(t_1 + t_2) = 5.97$

Adding equation (1) and (4); we get, (4)

$$2t_1 = 7.97$$

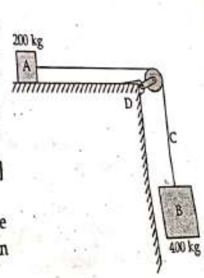
$\therefore$ $t_1 = 3.985$ sec.

and, $t_2 = 3.985 - 2 = 1.985$ sec.

$$h = V_0t_2 - \frac{1}{2}g\,t_1^2 = 25 \times 3.985 - 4.905 - (3.985)^2 = 21.72\ m$$

Thus, both balls meet at the height = 21.72 m.

18. **Two blocks shown in figure below start from rest. The horizontal plane and pulley are frictionless and pulley is assumed to be negligible mass. Determine the acceleration of each block and the tension in each chord.**



[2068 Baishakh]

Solution:

The free body diagrams of two blocks are shown below. Let T_1 and T_2 be the tension in block A and B respectively.

From the figure, As Block 'A' moves through distance x_A then, block 'B' moves distance $2x_B$. *i.e.,* $x_A = 2x_B$.

or, $x_B = \frac{x_A}{2}$

Difference,

$$V_B = \frac{V_A}{2}$$

and, $a_B = \frac{a_A}{2}$ (1)

Motion of block A

$$\xrightarrow{+}\Sigma F_x = m_A a_A$$

or, $T_1 = 200\ a_A$ (2)

Motion of block B

$$(+\downarrow)\Sigma F_y = m_B a_B$$

or, $w_B - T_2 = 400a_B = 400 \times \frac{a_A}{2} = 200\ a_A$

$\therefore$ $T_2 = w_B - 200a_A$ (3)

Motion of pulley

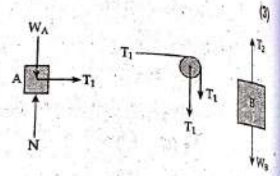
$$(\downarrow +)\Sigma F_y = m_c a_c = 0$$

i.e., $2T_1 = T_2$

or, $2 \times 200a_A = w_B - 200a_A$.

or, $600a_A = w_B = 400 \times 9.81$

$\therefore$ $a_A = 6.54\ m/s^2$



From equation (2); we get,

$$T_1 = 200 \times 6.54 = 1308\ N$$

$$T_2 = 2 \times 1308 = 2616\ N$$

and, $a_B = \frac{a_A}{2} = \frac{6.54}{2} = 3.27\ m/s^2$

19. What is linear momentum? Explain about rate of change of it? [2068 Baishakh]

Solution:

From Newton's second law of motion; we have,

$$\vec{F} = m\vec{a}$$

or, $\vec{F} = m\frac{d\vec{v}}{dt}$ $\left(\because \vec{a} = \frac{d\vec{v}}{dt}\right)$

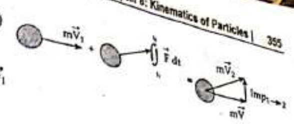
$\therefore$ $\vec{F} = \frac{d}{dt}(m\vec{v})$ (1)

Here, the terms $m\vec{v}$ is called linear momentum *i.e.,* product of mass and velocity of particle is called linear momentum of particle.

or, $\vec{F}dt = d\,(m\vec{v})$

Integrating above equation and taking as time variable from t_1 to t_2 and velocity varies from v_1 to v_2.

$$\int_{t_1}^{t_2} \vec{F}\,dt = \int_{v_1}^{v_2} d\,(m\vec{v})$$



or, $\int_{t_1}^{t_2} \vec{F}\,dt = m\vec{v}_2 - m\vec{v}_1$

or, $m\vec{v}_1 + \int_{t_1}^{t_2} \vec{F}\,dt = m\vec{v}_2$

The term $\int_{t_1}^{t_2} \vec{F}\,dt$ is called impulse.

$\therefore$ Initial momentum + $\vec{I}_{mp1\to 2}$ = final momentum

or, $\vec{L}_1 + \vec{L}_{mp1\to 2} = \vec{L}_2$

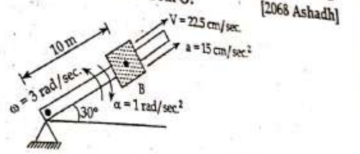
where, $L = m\vec{v}$ = linear momentum.

In diagram; we have,

When several forces act on a particle the impulse of each of forces must be considered. Then,

$$m\vec{v}_1 + \sum \vec{L}_{mp1-2} = m\vec{v}_2;$$ which is required expression.

20. Discuss dynamic equation with an example. In the figure below, the motion of block 'B' on a smooth horizontal table is controlled by the rod which rotates about a vertical axis at 'O'. At the position shown, 'B' has the given values of velocity and acceleration relative to the rod which is rotating with the given values of angles velocity ω and angular acceleration α. If 'B' weighs 60 N, What moment does it exert about O? [2068 Ashadh]



Solution:

From Newton's second law of motion; we have,

$$\sum \vec{F} - m\,\vec{a} = 0$$

The above expression represents the condition of equation. Thus, by adding a fictitious force $-\,m\,\vec{a}$ to the system of forces, we can bring the particle into equation. Since, the particle is not at rest in this condition, but actually moving we term it *dynamic equilibrium.*

Some examples;

$$\left.\begin{array}{l} \Sigma F_x = ma_x = 0 \\ \Sigma F_y = ma_y = 0 \end{array}\right\} \quad (1)$$

$$\left.\begin{aligned}\Sigma F_t &= ma_t = 0\\ \Sigma F_\theta &= ma_n = 0\end{aligned}\right\}$$

$$\left.\begin{aligned}\Sigma F_\theta &= ma_\theta = ma_\theta = 0\\ \Sigma F_r &= ma_r = 0\end{aligned}\right\} \qquad (2)$$

(3)

Numerical part

From the figure; we have,

$$\vec{v} = \dot{r} = 22.5 \text{ cm/sec}$$

$$\vec{a} = \ddot{r} = 15 \text{ cm/sec}$$

$$\omega = \dot{\theta} = 3 \text{ rad/ sec}$$

$$\alpha = \ddot{\theta} = 1 \text{ rad/sec}^2$$

$$r = 10 \text{ cm}$$

and, $w_B = 60$ N

We know that,

$$\boxed{\vec{a} = a_r\hat{e}_r + a_\theta\hat{e}_\theta}$$

$$\therefore \quad \boxed{\vec{a} = \ddot{r} - r\dot{\theta}^2 + r\ddot{\theta} + 2\dot{r}\dot{\theta}}$$

$$= 15 - 10 \times 3^2 + 10 \times 1 + 2 \times 22.5 \times 3$$

$$= 70 \text{ cm/sec}^2$$

$$= 0.70 \text{ m/sec}^2$$

Now,

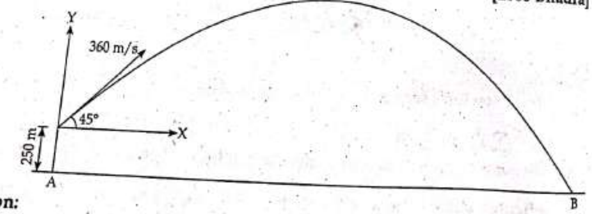
$$\vec{F} = m\vec{a} = \frac{W_B}{9.81} \times 0.70 = \frac{60}{9.81} \times 0.70 = 4.28 \text{ N}$$

$$\therefore \quad \text{Moment} = \vec{r} \times \vec{F} = 0.1 \times 4.28 = 0.428 \text{ Nm}$$

21. A projectile is fired from the edge of a 250 m cliff with an initial velocity of 360 m/s at an angle of 45° with the horizontal. Neglecting air resistance

i) The greatest elevation above the ground reached by the projectile.

ii) The horizontal distance from the gun to the point where the projectile strikes the ground. **[2068 Bhadra]**



Solution:

Given that;

Initial velocity $(V_0) = 360$ m/s

Initial angle $(\alpha) = 45°$

Components of initial velocities;

$(V_0)_x = V_0\cos\alpha = 254.56$ m/s

$(V_0)_y = (V_0 \sin\alpha) = 254.56$ m/s

$y = 250$ m = height of cliff

a) Maximum height of the cliff $(h_{max}) = \dfrac{V_0^2 \sin^2\alpha}{2g}$

$$= \frac{(360 \sin 45^0)^2}{2g} = 3302.75 \text{ m}$$

$\therefore$ Total elevation above ground $= (3302.75 + 250)$ m $= 3552.75$ m

b) Motion in x-direction $(x) = (V_0)_x \times t = V_0 \cos\alpha \times t$ (1)

Motion in y-direction

$$y = (V_0)_y t - \frac{1}{2}gt^2 = (V_0 \sin\alpha) \times t - \frac{1}{2}gt^2 \qquad (2)$$

Combining equation (1) and (2); we get,

$$y = x\tan\alpha - \frac{1}{2}g\frac{x^2}{(V_0\cos\alpha)^2}$$

or, $$-250 = x\tan(45^0) - \frac{4.905\,x^2}{(254.56)^2}$$

[y is – ve, since below form point of projection 'O']

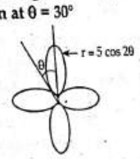
or, $-250 = x - 7.57 \times 10^{-5} x^2$

On solving; we have,

$x = 13455.48$ or -245.4

$\therefore$ $x = 13455.48$ m (– ve value neglected)

22. The particle having the position vector of $r = 5\cos(2\theta)$ m, is travelled in a curvilinear path as shown in figure below, where $\theta = 3t^2$ (rad /sec). Find the velocity and acceleration at $\theta = 30°$ **[2068 Bhadra]**



Solution:

Given relation are;

$r = 5\cos(2\theta)$

and, $\theta = 3t^2$ rad/sec.

Differentiating radial and angular displacement with respect to time,

$$\dot{r} = -5\sin(2\theta) \times 2 = -10\sin(2\theta)$$

$$\ddot{r} = -20\cos(2\theta)$$

$$\dot{\theta} = 6t,\ \ddot{\theta} = 6$$

When $\theta = 30°$

Now,

$$30° = \frac{\pi^c}{6} = 3t^2$$

or, $$t^2 = \frac{\pi^c}{18}$$

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or, $t = 0.418$ sec.

Now,

$$\dot{r} = -10\sin(2\times30°) = -8.66$$

$$\ddot{r} = -2\cos(60°) = -10$$

$$\dot{\theta} = 6\times0.418 = 2.51$$

$$\ddot{\theta} = 6$$

and, $r = 5\cos(2\times30°) = 2.5$

so, $V_r = \dot{r} = -8.66$

$$V_\theta = r\dot{\theta} = 2.5\times2.51 = 6.28$$

Now,

$$\vec{V} = V_r\hat{e}_r + V_\theta\hat{e}_\theta = -8.66\,\hat{e}_r + 6.28\hat{e}_\theta$$

$\therefore$ $V = |\vec{V}| = \sqrt{(-8.66)^2 + (6.28)^2} = 10.70\ \text{m/s}$

Again,

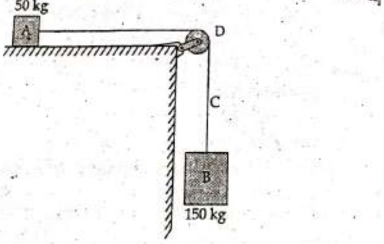
$$a_r = \ddot{r} - r\dot{\theta}^2 = -10 - 2.5\times2.51^2 = -16.28$$

or, $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.5\times6 + 2\times(-8.66)\times2.51 = -28.47$

$\therefore$ $\vec{a} = a_r\hat{e}_r + a_\theta\hat{e}_\theta = -16.28\,\hat{e}_r - 28.47\hat{e}_\theta$

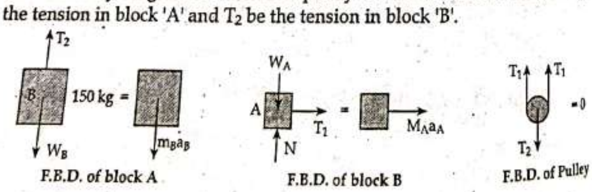
$\therefore$ $\vec{a} = |\vec{a}| = \sqrt{(-16.28)^2 + (-28.47)^2} = 32.80\ \text{m/s}^2$

23. **The two blocks shown in figure below start forms rest. The horizontal plane and pulley are friction less and the mass of the pulley is negligible. Determine the acceleration of each block and the tension in each chord.** [2068 Bhadra]



Solution:

The free body diagram of block and pulley are shown below. Also, let T_1 be the tension in block 'A' and T_2 be the tension in block 'B'.



F.B.D. of block A F.B.D. of block B F.B.D. of Pulley

From the figure as block 'A' moves through distance x_A then block 'B' moves distance $2x_B$.

i.e., $x_A = 2x_B$

Differentiating; we get,

$$V_A = 2V_B$$

and, $a_A = 2a_B$

or, $a_B = \dfrac{a_A}{2}$ (1)

Motion of block A

$$\overset{+}{\rightarrow} \Sigma F_x = m_A a_A$$

or, $T_1 = 50\,a_A$ (2)

Motion of block B

$$(+\downarrow)\ \Sigma F_y = m_B a_B$$

or, $w_B - T_2 = m_B a_B$

$\therefore$ $T_2 = w_B - m_B a_A$ (3)

Motion of pulley

$$(+\downarrow)\ \Sigma F_y = 0$$

or, $2T_1 = T_2$

or, $2\times50a_A = w_B - m_B a_B$

or, $2\times50a_A = 9.8.1\times150 - 150\times\dfrac{a_A}{2}$

or, $a_A = 6.54\ \text{m/s}^2$

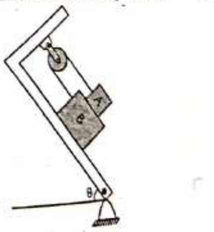
$\therefore$ $a_B = \dfrac{a_A}{2} = \dfrac{6.54}{2} = 2.27\ \text{m/s}^2$

Also, form equation (1); we get,

$$T_1 = m_A a_A = 50\times6.54 = 327\ \text{N}$$

and, $T_2 = 2\times327 = 654$ N

24. **The 100 N block 'A' and the 150 N block 'B' are supported by an incline that is held in the position shown in figure below knowing that coefficient of static friction is 0.15 between all surfaces of contact, determine the value of θ for which motion is impending.** [2068 Magh]



Solution:

The free body diagram is shown below. Let, 'T' be the tension in the blocks 'A' and 'B'. Also,

Given that;

$$W_A = 100\ \text{N}$$

$W_B = 150$ N

and. $\mu_s = 0.15$

Block A

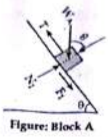


Figure: Block A

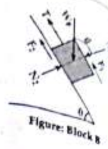


Figure: Block B

$+\downarrow (\Sigma F_y) = 0$

or, $w_A \cos\theta - N_1 = 0$

or, $N_1 = w_A \cos\theta$ (1)

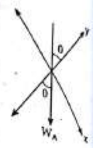
$\xrightarrow{+} (\Sigma F_x) = 0$

or, $w_A \sin\theta - T + F_1 = 0$

or, $T = w_A \sin\theta + F_1$ (2)

Block B

$+\swarrow (\Sigma F_y) = 0$



or, $w_B \cos\theta - N_1 - N_2 = 0$

or, $N_2 = w_B \cos\theta + N_1$ (3)

Also,

$+\searrow \Sigma F_x = 0$

or, $w_B \sin\theta - T - F_1 - F_2 = 0$

or, $w_B \sin\theta - w_A \sin\theta - F_1 - F_1 - F_2 = 0$ [Using equation (2)]

or, $w_B \sin\theta - w_A \sin\theta - 2\mu N_1 - \mu N_2 = 0$

or, $w_B \sin\theta - w_A \sin\theta - \mu(2N_1 + N_2) = 0$

or, $w_B \sin\theta - w_A \sin\theta - \mu(2N_1 + w_B \cos\theta + N_1) = 0$

or, $150 \sin\theta - 100 \sin\theta - 0.15\,(34 \times w_A \cos\theta + w_B \cos\theta) = 0$

or, $50 \sin\theta - 0.15 \times 540 \cos\theta = 0$

or, $50 \sin\theta - 0.15 \times 450 \cos\theta = 0$

or, $50 \sin\theta = 67.5 \cos\theta$

$\therefore \dfrac{\sin\theta}{\cos\theta} = \dfrac{67.5}{50} = 1.35$

$\therefore \theta = \tan^{-1}(1.35) = 53.47°$

25. The acceleration of a particle is defined be the relation; $a = -k\sqrt{v}$; where, 'k' is a constant. Knowing that; $x = 0$ and $v = 81$ m/s at $t = 0$ and that $v = 36$ m/s when $x = 18$ m. Determine;

i) the velocity of the particle $x = 20$ m

ii) time required for the particle to come to rest. [2068 Magh, New back]

Solution:

We have,

or, $a = -k\sqrt{v}$

or, $V\dfrac{dv}{dx} = -k\sqrt{v}$

or, $\dfrac{Vdv}{-k\sqrt{V}} = dx$

Integrating both sides; we get,

or, $-\dfrac{1}{k}\int \sqrt{V}\,dv = \int dx$

or, $-\dfrac{1}{k}\dfrac{V^{3/2}}{\frac{3}{2}} = x + c_1$

or, $-\dfrac{2}{3k} V^{3/2} = x + c_1$ (1)

To find c_1 at $x = 0$ and $V = 81$ m/s

Then,

$-\dfrac{2}{3k} \times (81)^{3/2} = 0 + c_1$

or, $c_1 = -\dfrac{486}{k}$

Therefore, equation (1) becomes;

$-\dfrac{2}{3k} V^{3/2} = x - \dfrac{486}{k}$

To find K, At $V = 36$ m/s and $x = 18$ m

Then,

$-\dfrac{2}{3k} \times (36)^{3/2} = 18 - \dfrac{486}{k}$

or, $-\dfrac{144}{k} + \dfrac{486}{k} = 18$

$\therefore k = 19$

Again,

$a = -k\sqrt{v}$

or, $\dfrac{dV}{dt} = -k\sqrt{V} = -19\sqrt{V}$

or, $V^{-1/2}\,dv = -19\,dt$

Integrating; we get,

$\dfrac{V^{1/2}}{\frac{1}{2}} = -19t + c_2$

or, $2V^{1/2} = -19t + c_2$

To find c_2, at $t = 0$ and $V = 81$ m/s

Then,

$2 \times 9 = -19 \times 0 + c_2$

or, $c_2 = 18$

Then, equation becomes,

$2\sqrt{V} = -19t + 18$

When $V = 0$ then,

$0 = -19t + 18$

or, $t = \dfrac{18}{19} = 0.947$ sec.

26. In the figure below is shown a system of particles at time t moving in xy plane. The following data apply.

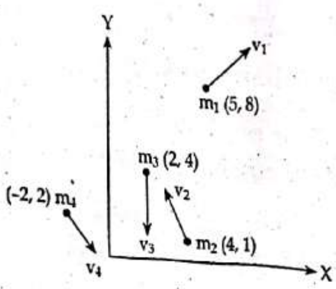
$m_1 = 0.5$ kg $\quad V_1 = 1.5\vec{i} - 1.5\vec{j}$ m/s

$m_2 = 0.35$ kg $\quad V_2 = -1.3\vec{i} + 1\vec{j}$

$m_3 = 1$ kg $\quad V_3 = -1.3\vec{j}$

$m_4 = 0.75$ kg $\quad V_4 = 1\vec{i} - 1.3\vec{j}$

a) What is the linear momentum of the system?
b) What is the linear momentum of centre of mass?
c) What is the total moment of system about the origin and point (2, 6)? [2068 Chaitra]



Solution:

Given that;

$m_1 = 0.5$ kg $\quad V_1 = 1.5\vec{i} + 1.5\vec{j}$

$m_2 = 0.35$ kg $\quad V_2 = -1.3\vec{i} + 1\vec{j}$

$m_3 = 1$ kg $\quad V_3 = -1.3\vec{i}$

$m_4 = 0.75$ kg $\quad V_4 = 1\vec{i} - 1.3\vec{j}$

a) Total linear momentum of the system

$$m\vec{v} = \sum_{i=1}^{n} m_i\vec{v}_i$$

$$= m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + m_4\vec{v}_4$$

$$= 0.5(1.5\vec{i} + 1.5\vec{j}) + 0.35(-1.3\vec{i} + \vec{j}) + 1(1.3\vec{i} + 0.75)(1\vec{i} - 1.3\vec{j})$$

$$= -0.255\vec{i} + 0.125\vec{j}$$

b) Linear momentum with respect to centre of mass

We have,

$$M_0 = m_1 + m_2 + m_3 + m_4 = (0.5 + 0.35 + 1 + 0.75) \text{ kg} = 2.6 \text{ kg}$$

Linear momentum of centre of mass is,

$$m\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + m_4\vec{v}_4$$

or, $2.6\,\vec{v}_0 = -0.255\vec{i} + 0.125\vec{j}$

$\therefore\ \vec{v}_0 = -0.098\vec{i} + 0.048\vec{j}$

Now,

Velocity of m_1 with respect to $\vec{v}_0$ is;

$$\vec{v}_{1/0} = \vec{v}_1 - \vec{v}_0 = (1.5\vec{i} + 1.5\vec{j}) - (-0.098\vec{i} + 0.048\vec{j})$$

$$= 1.598\vec{i} - 1.452\vec{j}$$

Similarly,

$$\vec{v}_{2/0} = \vec{v}_2 - \vec{v}_0 = (-1.3\vec{i} + 1\vec{j}) - (-0.098\vec{i} + 0.048\vec{j})$$

$$= 1.398\vec{i} - 0.952\vec{j}$$

$$\vec{v}_{3/0} = \vec{v}_3 - \vec{v}_0 = -1.202\vec{i} - 0.048\vec{j}$$

$$\vec{v}_{4/0} = \vec{v}_4 - \vec{v}_0 = 1.098\vec{i} - 1.348\vec{j}$$

So, linear momentum with respect to mass centre is;

$$P_0 = \sum_{i=1}^{n} m_i\vec{v}_{i/0} = 0.5(1.598\vec{i} - 1.452\vec{j}) + 0.35(1.398\vec{i} - 0.952\vec{j}) + (-1.202\vec{i} - 0.048\vec{j}) + 0.75(1.098 - 1.348\vec{j})$$

$$= 0.9098\vec{i} - 2.1182\vec{j}$$

c) Moment of moment about origin (0, 0)

Here,

$$\vec{r}_1 = (5, 8) - (0, 0) = 5\vec{i} + 8\vec{j}$$

$$\vec{r}_2 = (4, 1) - (0, 0) = 4\vec{i} + \vec{j}$$

$$\vec{r}_3 = (2, 4) - (0, 0) = 2\vec{i} + 4\vec{j}$$

$$\vec{r}_3 = (-2, 2) - (0, 0) = -2\vec{i} + 2\vec{j}$$

$\therefore$ Moment of moment $= \sum_{i=1}^{n} \vec{r}_i \times \vec{v}_i$

$$= (5\vec{i} + 8\vec{j}) \times (1.5\vec{i} + 1.5\vec{j}) + (4\vec{i} + \vec{j}) \times (-1.3\vec{i} + 1\vec{j}) + (2\vec{i} + 4\vec{j}) \times (-1.3\vec{i}) + (-2\vec{i} + 2\vec{j}) \times (1\vec{i} - 1.3\vec{j})$$

$$= 7.5\vec{k} - 12\vec{k} + 4\vec{k} + 1.3\vec{k} + 5.2\vec{k} + 2.6\vec{k} - 2\vec{k}$$

$$= 6.6\vec{k}$$

About point (2, 6)

Here,

$$\vec{r}_1 = (5, 8) - (2, 6) = 3\vec{i} + 2\vec{j}$$

$$\vec{r}_2 = (4, 1) - (2, 6) = 2\vec{i} - 5\vec{j}$$

$$\vec{r}_3 = (2, 4) - (2, 6) = -2\vec{j}$$

$$\vec{r}_4 = (-2, 2) - (2, 6) = -4\vec{i} - 4\vec{j}$$

$\therefore$ Moment of momentum $= \sum_{i=1}^{n} \vec{r_i} \times \vec{v_i}$

$$= (3\vec{i} + 2\vec{j}) \times (1.5\vec{i} + 1.5\vec{j}) + (2\vec{i} - 3\vec{j}) \times (-1.3\vec{i} + \vec{j}) + (-2\vec{j}) \times (-1.3\vec{i}) + (-4\vec{i} - 4\vec{j}) \times (\vec{i} - 1.3\vec{j})$$

$$= 4.5\vec{k} - 3\vec{k} + 2\vec{k} - 6.5\vec{k} - 2.6\vec{k} + 5.2\vec{k} + 4\vec{k}$$

$$= 3.6\vec{k}$$

27. **Define uniformly rectilinear and acceleration rectilinear motion. Automobile 'A' is travelling east at the constant speed of 20 km/hr. As automobile 'A' crosses intersection shown automobile 'B' starts rest 35 m North of a intersection and moves south with a constant acceleration of 2 m/s². Determine the position, velocity and acceleration relative to 'A' 10 sec after 'A' crosses the intersection.** [2068 Chaitra]

Solution:

Uniformly rectilinear motion is defined as that type of motion in which the acceleration is zero or velocity is uniform or constant. *For example;*

$V = V_0$ and $x = x_0 + V_0 t$

Uniformly accelerated rectilinear motion is that type of motion in which the acceleration is constant or velocity is uniformly varying. *For example;*

$V = V_0 + at$ and $x = x_0 + V_0 t + \frac{1}{2}at^2$

Numerical part

For automobile A

$V_A = 20\text{km/hr} = 5.56 \text{ m/s}$

$\vec{a_A} = 0$ (since velocity is constant)

Now,

$$x_A = (x_A)_0 + V_A t = 0 + 5.56 \times 10 = 55.6 \text{ m}$$

i.e., $\vec{x_A} = \vec{r_A} = 55.6 \text{ m } (\rightarrow)$

For automobile B

$a_B = -2\text{m/s}^2 = 2\text{m/s}^2 (\downarrow)$

$V_B = (V_B)_0 + at = 0 + (-2) \times 10 = -20 \text{ m/s} = 20 \text{ m/s} (\downarrow)$

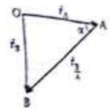
Now,

$$y_B = (y_B)_0 + (V_B)_0 t + \frac{1}{2}a_B t^2 = 50 + 0 \times t + \frac{1}{2} \times (-2) \times 10^2 = 50 - 100 = -50 \text{ m}$$

$\therefore \vec{y_B} = \vec{r_B} = 50 \text{ m}(\downarrow)$

Motion of B relative to A

$\vec{r_{B/A}} = \vec{r_B} - \vec{r_A} = -50\vec{j} - 55.6\vec{i}$



$\therefore \vec{r_{B/A}} = \sqrt{(-50)^2 + (-55.6)^2} = 74.78 \text{ m}$

and, $\alpha = \tan^{-1}\left(\frac{50}{55.6}\right) = 41.96°$

Again,

$\vec{V_{B/A}} = \vec{V_B} - \vec{V_A} = -20\vec{j} - 5.56\vec{i}$

$\therefore |\vec{V_{B/A}}| = \sqrt{(-20)^2 + (-5.56)^2} = 20.76 \text{ m/s}$

and, $\beta = \tan^{-1}\left(\frac{20}{5.56}\right) = 74.46°$

Also,

$\vec{a_{B/A}} = \vec{a_B} - \vec{V_A} = -2\vec{j} - 0$

$\therefore \vec{a_{B/A}} = 2\text{m/sec}^2$

and, $\gamma = \tan^{-1}\left(\frac{2}{0}\right) = 90°$

28. **Define position, velocity a particle. The acceleration on the particle is defined by relation a = – 2 m/s². Initially if velocity of the instant f 5 sec.** [2069 Ashadh]

Solution: See the definition part on page no. 335

We have given that,

$a = -2 \text{ m/sec}^2$

or, $\frac{dV}{dt} = -2$

or, $dV = -2\,dt$

Integrating; we get,

$V = -2t + c_1$ (1)

To find c_1, when $t = 0$ (initially)

and, $V = 10$ m/s

$\therefore 10 = 0 + c_1$

or, $c_1 = 10$

Then, equation (1) becomes,

$V = -2t + 10$

or, $\frac{dx}{dt} = -2t + 10$

or, $dx = (-2t + 10)\,dt$

Integrating; we get,

$x = -t^2 + 10t + c_2$

To find c_2 at $t = 0$ and $x = 0$

$\therefore\ 0 = 0 + c_2$

or, $c_2 = 0$ (2)

Then equation (2) becomes,

$x = -t^2 + 10t$ (3)

Now,

At t = 5 sec.

i) $V = -2t + 10 = -2 \times 5 + 10 = 0\ m/s$

ii) $x = -t^2 + 10t = -(5)^2 + 10 \times 5 = 25\ m$

Total distance travelled

When, V = 0 then,

$0 = -2t + 10$

or, $t = 5$ sec.

$\therefore\ x_5 = -(5)^2 + 10 \times 5 = 25\ m$

$x_0 = 0\ m/s$

$\therefore$ Total distance $x_5 - x_0| + |x_5 - x_5| = |25 - 0| = 25\ m$

29. A particle moving in a straight line has an acceleration $a = \sqrt{v}$. It's displacement and velocity at time t = 2 sec are $\frac{128}{3}$ m and 16 m/s. Find the displacement velocity and acceleration at time t = 3 sec. **[2069 Baishakh]**

Solution:

Given that;

$$a = \sqrt{v}$$

or, $$\frac{Vdv}{dx} = \sqrt{v}$$

or, $$\sqrt{v}\ dv = dx$$

Integrating both sides; we get,

$$\frac{2}{3}v^{3/2} = x + C_1$$

To find C_1 at $V = 16\ m/s$ and $x = \frac{128}{3}$ m; then,

$$\frac{2}{3} \times (16)^{3/2} = c_1 + \frac{128}{3}$$

or, $c_1 = 0$

$$\therefore\ \frac{2}{3}v^{3/2} = x$$

or, $$V^{3/2} = \frac{3x}{2}$$

or, $$V = \left(\frac{3x}{2}\right)^{2/3}$$

or, $$\frac{dx}{dt} = \left(\frac{3x}{2}\right)^{2/3}$$

or, $$\frac{dx}{\left(\frac{3}{2}x\right)^{2/3}} = dt$$

or, $$\left(\frac{3}{2}\right)^{2/3} x^{-2/3}\,dx = dt$$

Integrating; we get,

$$\left(\frac{3}{2}\right)^{-2/3} \times 3 \times x^{1/3}\,dx = t + c_2$$

or, $2.29\,x^{1/3} = t + c_2$

To find c_2 at $x = \frac{128}{3}$ m and t = 2 sec

$$\therefore\ 2.29 \times \left(\frac{128}{3}\right)^{1/3} = 2 + c_2$$

or, $c_2 = 6.002$

Now, at t = 3 sec

i) $2.29\,x^{1/3} = 3 + 6.002$

or, $x^{1/3} = 3.93$

$\therefore\ x = 60.70\ m$

ii) Velocity $(V) = \left(\frac{3x}{2}\right)^{2/3} = \left(\frac{3}{2} \times \frac{128}{3}\right)^{2/3} = (64)^{2/3} = 16\ m/s$

iii) Acceleration $(a) = \sqrt{v} = 4\ m/s^2$

$\therefore$ Displacement $= \left(60.70 - \frac{128}{3}\right) m = 18.03\ m$

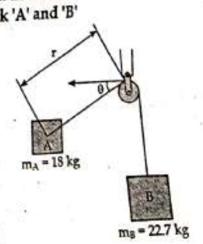
30. A particle projected at an angle of 20° with the horizontal axis with an initial velocity of 50 m/s hit the target located at 'h' meter below the horizontal axis having the axis of the target. Determine the sloping distance covered by the projectile from the target. **[2069 Ashadh]**

Solution: See the solution of Q. no. 13 on page no. 348

31. The two blocks as shown in figure below are released from rest when r = 0.73 m and θ = 30°. Neglecting the mass of pulley and effect of friction in the pulley and between block 'A' and horizontal surface. Determine;

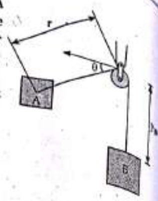
a) the initial tension in the cable

b) acceleration of block 'A' and 'B' **[2069 Bhadra]**



Solution:

Consider r and θ be polar co-ordinate of block A as shown and let y_B be the position co-ordinate (+ve downward, origin at the pulley) for rectilinear motion of block B.



Now, constant of cable;

$$r + y_B = \text{Constant}$$

Difference,

$$\dot{r} + V_B = 0$$

Again, difference,

$$\ddot{r} + a_B = 0$$

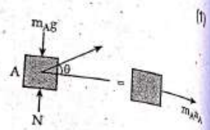
or, $\ddot{r} = -a_B$ (1)

For Block A

$$+\rightarrow \Sigma F_X = m_A a_A$$

or, $T\cos\theta = m_A a_A$

∴ $T = m_A a_A \sec\theta$ (2)

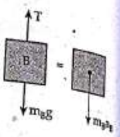


For Block B

$$+\downarrow \Sigma F_y = m_B a_B$$

or, $m_B g - T = m_B a_B$

or, $T = m_B g - m_B a_B$ (3)



From equation (2) and (3); we get,

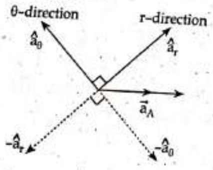
$$m_B g = m_A a_A \sec\theta + m_B a_B \quad (4)$$

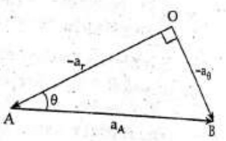
Now, radial and transverse composition a_A;

From ΔOAB; we have,

$$\cos\theta = \frac{OA}{AB} = \frac{-a_r}{a_A}$$

∴ $a_r = -a_A\cos\theta$ (5)





Also,

$$a_r = \ddot{r} - r\dot{\theta}^2$$

Since, initially $\dot{\theta} = 0$

so, $a_r = \ddot{r} - 0 = \ddot{r}$

or, $\ddot{r} = -a_A\cos\theta$ [From equation (5)]

or, $-a_B = -a_A\cos\theta$ [From equation (1)]

∴ $a_B = a_A\cos\theta$ (6)

From equation (4) and (6); we get,

$$m_B g = m_A a_A \sec\theta + m_B a_A \cos\theta$$

$$\therefore\ a_A = \frac{m_B g}{m_A \sec\theta + m_B\cos\theta}$$

$$= \frac{22.7 \times 9.81}{18 \times \sec 30^\circ + 22.7 \times \cos 30^\circ}$$

$$= 5.51\,\text{m/s}^2$$

∴ $a_B = 5.51\cos 30^\circ = 4.77\,\text{m/s}^2$

From equation (2); we get,

$$T = m_A a_A \sec\theta$$

$$= 18 \times 5.51 \times \sec 30^\circ$$

$$= 114.52\,\text{N}$$

32. **Define uniformly rectilinear and uniformly accelerated rectilinear motion. A projectile is fired with an initial velocity of 244 m/s at a target B located 610 m above the level of gun A and horizontal distance of 3658 m. Neglecting air resistance, determine the value of the firing angle.** [2069 Chaitra]

Solution:

The uniformly rectilinear motion is defined as that type of motion in which acceleration is zero or in other words velocity is uniform or constant.

Uniformly accelerated rectilinear motion is that type of motion in which the acceleration is constant or the velocity is uniformly varying.

Numerical part

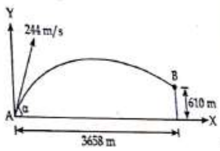
Given that;

Initial velocity $(v_0) = 244\,\text{m/s}$

Vertical distance $(y) = 610\,\text{m}$

Horizontal distance $(x) = 3658\,\text{m}$

Firing Angle = α (let)



Now, motion in x-direction;

$$x = v_0 \sin\alpha \times t \quad (1)$$

and, motion in y-direction;

$$y = v_0 \sin\alpha \times t - \frac{1}{2}gt^2 \quad (2)$$

Combining equation (1) and (2), we get;

$$y = x\tan\alpha - \frac{1}{2}g\frac{x^2}{v_0^2\sin^2\alpha}$$

or, $610 = 3658\tan\alpha - \dfrac{4.905 \times (3658)^2}{244 \times \cos^2\alpha}$

or, $610 = 3658\tan\alpha - 1102.42\sec^2\alpha$

or, $610 = 3658\tan\alpha - 1102.42(1 + \tan^2\alpha)$

or, $0.55 = 3.32\tan\alpha - 1 - \tan^2\alpha$

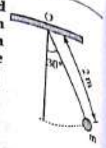
or, $\tan^2\alpha - 3.32\tan\alpha + 0.55 = 0$

Solving; we get,

$\tan\alpha = 2.76$ or 0.562

∴ $\alpha = 70.08^\circ$ or 29.34°

33. Define the linear momentum and angular momentum. Find the velocity and acceleration of the bob in the given position. The bob of a 2 m pendulum describes an arc of a circle in a vertical plane. Tension in the cord is 2.5 times the weight of the bob for the position shown. [2069, Chaitra]



Solution:

Definition

See the Q. no. 12 and 19 on page no. 347 and 354 respectively

Numerical part

Given that;

Length of pendulum $(\rho) = 2\,m$

Angle $(\theta) = 30°$

The free body diagram of the bob is shown below.

Applying Newton's 2nd law in tangential and normal direction we get;

$$(^{+}\swarrow)\sum F_t = ma_t$$

or, $mg\tan 30° = ma_t$

$\therefore$ $a_t = 4.9\,m/s^2(^{+}\swarrow)$

Again,

$$(^{+}\nwarrow)\sum F_n = ma_n$$

or, $2.5\,mg - mg\cos 30° = ma_n$

or, $2.5\,g - g\cos 30° = a_n$

$\therefore$ $a_n = 16.03\,m/s^2$

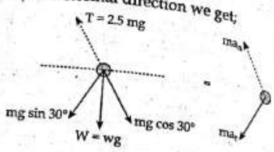


Figure: F.B.D. of bob

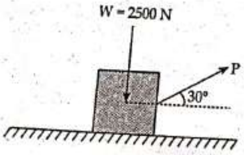
$$\therefore \quad a = \sqrt{a_t^2 + a_n^2} = \sqrt{4.9^2 + 16.03^2} = 16.76\,m/s^2$$

Also we know that,

$$a_n = \frac{v^2}{\rho}$$

$$\therefore \quad v = \sqrt{a_n \times \rho} = \sqrt{2 \times 16.76} = 5.79\,m/s$$

34. Determine the magnitude of force P required to give, the block an acceleration of $10\,m/s^2$. Coefficient of friction between the block and the floor is 0.25 [2070 Ashadh]



Solution:

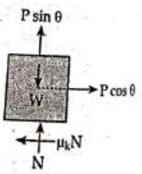
Given that;

Acceleration $(a) = 10\,m/s^2$

Coefficient of friction $(\mu) = 0.25$

Weight of block $(w) = 2500\,N$

The free body diagram of the block is shown in the figure.



Writing equation of motions along X and Y direction, then,

$$(^{+}\uparrow)\sum F_y = m\,a_y$$

or, $N + P\sin\theta - w = 0$

$\therefore$ $N = w - P\sin\theta$ (1)

Note that as the block moves horizontally, its acceleration along Y-direction is zero. Also,

$$\sum F_x = m\,a_x$$

or, $P\cos\theta - \mu_k N = ma$

or, $P\cos\theta - \mu_k(w - P\sin\theta) = ma$

$$\therefore \quad P = \frac{\mu_k \times w + ma}{\mu_k\sin\theta + \cos\theta} = \frac{0.25 \times 2500 + \left(\frac{2500}{9.81}\right) \times 10}{0.25\sin 30° + \cos 30°} = 3202.24\,N$$ [By equation (1)]

35. The motion of a vibrating particle is defined by the equations $x = 100\sin\pi t$ and $y = 25\cos 2\pi t$. where, x and y are expressed in mm and 't' in sec.

a) Determine the velocity and acceleration when $t = 1$ sec.

b) Find the nature of path of the particle. [2070 Ashadh, Back]

Solution:

Given that;

$x = 100\sin\pi t$

and, $y = 25\cos 2\pi t$

Now,

$v_x = \dot{x} = 100\pi\cos\pi t$

$a_x = \ddot{x} = -100\pi^2\sin\pi t$

Also,

$v_y = \dot{y} = -50\pi\sin 2\pi t$

$a_y = \ddot{y} = -100\pi^2\cos 2\pi t$

a) At $t = 1$ sec;

$$V = \sqrt{v_x^2 + v_y^2} = \sqrt{(100\pi \times \cos\pi \times 1)^2 + (-50\pi \times \sin 2\pi \times 1)^2}$$

$$= 100\pi\ mm/sec$$

and, $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\frac{(-50\pi \times \sin 2\pi)}{(100\pi \times \cos\pi)} = 0°$

and, $a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-100\pi^2 \times \sin\pi \times 1)^2 + (-100\pi^2 \times \cos 2\pi \times 1)^2}$

$$= 100\pi^2\ mm/sec^2$$

and, $\alpha = \tan^{-1}\left(\frac{a_y}{a_x}\right) = 270°$

b) We have,

$x = 100\sin\pi t$

or, $\sin\pi t = \frac{x}{100}$

$\therefore$ $\left(\frac{x}{100}\right)^2 = \sin^2\pi t$

Again,

$y = 25\cos 2\pi t$

or, $\frac{y}{25} = 2\cos^2 \pi t - 1$

or, $\left(\frac{y}{25} + 1\right) = 2\cos^2 \pi t$

$\therefore \left(\frac{y+25}{50}\right)^2 = \cos^2 \pi t$ (2)

Adding equation (1) and (2); we get,

$$\left(\frac{x}{100}\right)^2 + \frac{y+25}{50} = \sin^2 \pi t + \cos^2 \pi t$$

or, $\frac{x^2}{10000} + \frac{y+25}{50} = 1$

or, $x^2 + 200y + 5000 = 10000$

$\therefore$ $x^2 + 200y = 5000$; which is the equation of parabola.

36. Deduce the relationship of radial and transverse components of velocity and acceleration for a particle moving along the curve path. The acceleration of a particle is defined by the relation $a = kt^2$, knowing that velocity is –32 m/s when time is zero second and again velocity is +32 m/sec when time is 4 sec. (a) Determine the value of constant k. (b) Write the equations of motion knowing also that position of the particle is zero at the instant of 4 sec.

[2070 Bhadra, Regular]

Solution:

In certain problem of plane motion the position of particle P is defined by its co–ordinate (r, θ) it is then convient to resolve the velocity and acceleration particle into component parallel and perpendicular respectively (to line OP). Consider a particle is moving along a curve path as shown in figure. At any instant the particle reaches to P' as shown in figure.

Suppose,

r = Radius

θ = Angle in radian

$\hat{e}_r$ and $\hat{e}_\theta$ are unit vector along radial and transverse direction.

We have,

$$\vec{r} = r\hat{e}_r$$

$$\vec{\vartheta} = \frac{d\vec{r}}{dt} = \frac{d(r\hat{e}_r)}{dt}$$

or, $\vec{\vartheta} = \frac{d\vec{r}}{dt}\hat{e}_r + r\frac{d\hat{e}_r}{dt}$

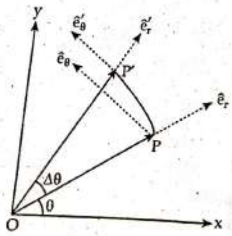


Figure: (1)

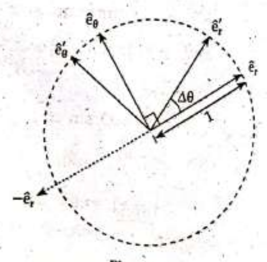


Figure: (2)

or, $\vec{\vartheta} = r\hat{e}_r + r\frac{d\hat{e}_r}{dt}\cdot\frac{d\theta}{dt}$

Now,

$$\frac{d\hat{e}_r}{d\theta} = \lim_{\Delta\theta \to 0}\frac{(1\times\Delta\theta)\hat{e}_\theta}{\Delta\theta}$$

Similarly;

$$\frac{d\hat{e}_r}{d\theta} = \frac{d\hat{e}_\theta}{d\theta}\cdot\frac{d\theta}{dt} = -\hat{e}_r\cdot\theta$$

Since,

$$\frac{d\hat{e}_\theta}{d\theta} = \lim_{\Delta\theta \to 0}\frac{(1\times\Delta\theta)\hat{e}_\theta}{\Delta\theta} = -\hat{e}_r$$

$\therefore$ $\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

and, $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta\right)$

$$= \ddot{r}\hat{e}_r + \dot{r}\frac{d\hat{e}_r}{dt} + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d\hat{e}_\theta}{dt}$$

$$= \ddot{r}\hat{e}_r + \dot{r}\frac{d\hat{e}_\theta}{d\theta}\cdot\frac{d\theta}{dt} + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d\hat{e}_\theta}{d\theta}\cdot\frac{d\theta}{dt}$$

$$= \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}^2(-\hat{e}_r)$$

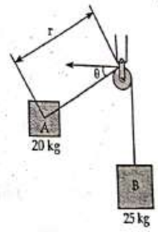
$\therefore$ $\vec{a} = \left(\ddot{r} - \dot{r}\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$

Numerical part

See the solution of Q. no. 5 on page no. 340

37. The velocity of block 'A' is 2 m/s to the right at the instant when r = 0.8 and $\theta = 30°$. Neglecting the mass of pulleys and the effect of friction in the pulley and between block 'A' and the horizontal surfaces. Determine at this instant (a) the tension in the cable, (b) the acceleration of block A. (C) the acceleration block B.

[2070 Bhadra, Regular]

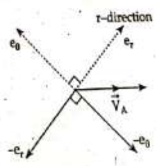


Solution:

Let y_B be the position co–ordinate (+ ve downward, origin at the pulley) for the rectilinear motion of block B.

Now, radial and transverse component of v_A;

From vector triangle; we have,



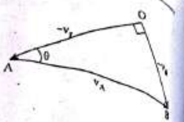
$$\cos\theta = \frac{-v_r}{v_A}$$

or, $\dot{r} = v_r = -v_A\cos\theta = -2\cos 30° = -1.732$ m/s

Also,

$$r\dot{\theta} = v_\theta = -v_A \sin 30^\circ$$

or, $\dot{\theta} = \frac{v_\theta}{r} = \frac{-2\sin 30^\circ}{0.8} = -1.25 \text{ rad/sec}$



Now, constant of cable,

$$r + y_B = \text{Constant}$$

Differentiating; we get,

$$\dot{r} + v_B = 0$$

or, $\ddot{r} + a_B = 0$

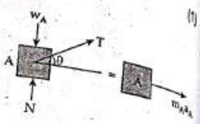
$\therefore$ $\ddot{r} = -a_B$ (1)

For block A

$$\overset{+}{\rightarrow}\Sigma F_x = m_a a_A$$

$\therefore$ $T\cos\theta = m_A a_A$

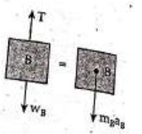
or, $T = m_A a_A \sec\theta$ (2)



For block B

$$+\downarrow(\Sigma F_y) = m_B a_B$$

or, $m_B g - T = m_B a_B$ (3)



Adding equation (2) and (3); we get,

$$m_B g = m_A a_A \sec\theta + m_B a_B \quad (4)$$

Radial and transverse component of a_A.

From vector triangle OAB then,

$$\cos\theta = \frac{OA}{AB}$$

or, $a_r = -a_A \cos\theta$

We know that;

$$a_r = \ddot{r} - r\dot{\theta}^2$$

or, $-a_A\cos\theta = \ddot{r} - r\dot{\theta}^2$ (5)

Using equation (1) to eliminate $\ddot{r}$; we get,

$$a_B = a_A\cos\theta - r\dot{\theta}^2 \quad (6)$$

Substituting equation (6) into equation (4) and solving for a_n; then,

$$a_A = \frac{m_B(g + r\dot{\theta}^2)}{m_A\sec\theta + m_B\cos\theta} = \frac{25[9.81 + 0.81\times(-1.25)^2]}{20\sec 30^\circ + 25\cos 30^\circ} = 6.18 \text{ m/s}^2$$

From equation (6); we get,

$$a_B = 6.18\cos 30^\circ - 0.8\times 1.25^2 = 4.10 \text{ m/s}^2$$

From equation (2); we get,

$$T = 20\times 6.18\sec 30^\circ = 142.7 \text{ N}$$

Thus,

Acceleration of block A $(a_A) = 6.18 \text{ m/s}^2$

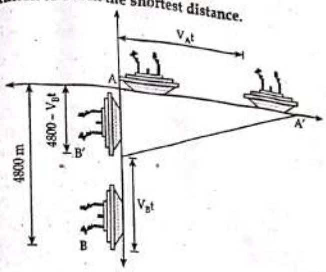
Acceleration of block B $(a_B) = 4.10 \text{ m/s}^2$

and, Tension (T) = 142.7 N

18. Two ships A and B are at a distance of 4800 m apart B being south east of A. Speed of A is 1.6 m/sec due east and B is travelling at the speed of 4.47 m/sec. due north. Determine:

a) the relative velocity of B with respect to A.
b) the shortest distance between them.
c) time taken to reach the shortest distance.

[2070 Magh]



Solution:

Here,

Distance between two ships (AB) = 4800 m

Velocity of ship A $(v_A) = 2.6$ m/sec. (East)

Velocity of ship B $(v_B) = 4.7$ m/sec. (North)

Let, t (sec.) be the time taken to reach the position of shortest approach. At that time, the positions of ships are A' and B' respectively.

Given that;

$$AB = 4800 \text{ m}$$

$$AA' = V_A \times t = 2.6t \text{ m/sec.}$$

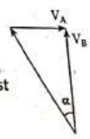
$$BB' = V_B \times t = 4.47t \text{ m/sec.}$$

$\therefore$ $AB' = 4800 - 4.47t$

a) The relative velocity of ship B with respect to ship A is calculates as;

$$v_{\frac{B}{A}} = \sqrt{(v_A)^2 + (v_B)^2} = \sqrt{(2.6)^2 + (4.7)^2} = 5.17 \text{ m/sec.}$$

and, $\tan\alpha = \frac{v_A}{v_B} = \frac{2.6}{4.47} = 0.5817$



$\therefore$ $\alpha = \tan^{-1}(0.5817) = 30.18^\circ$

Distance between ships when at the position of shortest approach is;

$$A'B' = S = \sqrt{(2.6t)^2 + (4800 - 4.47t)^2}$$

or, $S^2 = (2.6t)^2 + (4800 - 4.47t)^2$

or, $S^2 = 6.76t^2 + 23040000 - 42912t + 19.9809t^2$

At shortest approach S is minimum. Therefore, S^2 is also minimum.

$$\therefore \quad \frac{dS^2}{dt} = 0$$

or, $\frac{d}{dt}(6.76t^2 + 23040000 - 42912t + 19.9809t^2) = 0$

or, $2 \times 6.76t + 42912 + 19.9809 \times 2 \times t = 0$

or, $53.4818t = 42912$

$\therefore$ $t = 802.37$ sec.

Hence, the shortest distance between two ships is given by;

$$= \sqrt{(2 \times 802.37)^2 + (4800 - 4.47 \times 802.37)^2} = 2413.38 \text{ m}$$

and, Time taken to reach shortest distance = 2413.38 m

39. What do you mean by principle of impulse and moment? The resultant external force acting on 30 N particle in the space is $\vec{F} = (12t\hat{i} - 24t^2\hat{j} + 30t^3\hat{k})$ N, where, t is the time measured in seconds. Determine y-component of acceleration, velocity and position at the instant of 5 sec. [2070 Magh]

Solution:

For the first part

See the solution of Q. no. 19 on page no. 354

Numerical part

Given that;

Resultant external force on particle in space $(\vec{F}) = (12t\hat{i} - 24t^2\hat{j} + 30t^3\hat{k})$ N.

Weight of particle = 30 N

$\therefore$ Mass of particle $= \frac{30}{10} = 3$ kg

$|a_y|_{t=5} = ?$

$|V_y|_{t=5} = ?$

and, $|S_y|_{t=5} = ?$

We know that;

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

Comparing this equation with the given equation; we obtain,

$F_x = 12t$

$F_y = -24t^2$

and, $F_z = 30t^3$

and, $F_y = ma_y$

or, $-24t^2 = 3a_y$

$\therefore$ $a_y = -8t^2$ (1)

Now, acceleration at t = 5 sec. is;

$$|a_y|_{t=5} = -8t^2 = -8 \times (5)^2 = -200 \text{ m/s}^2$$

Velocity

The velocity of the particle can be related to its acceleration by applying;

$$a_y = \frac{dv_y}{dt}$$

or, $\frac{dv_y}{dt} = -8t^2$

or, $dv_y = -8t^2\,dt$

Integrating both sides; we have,

$$\int dv_y = \int -8t^2\,dt$$

Since, the particle at origin and at rest, so,

Initial velocity $(v_i) = 0$

Limiting the integration up to $v_i = 0$ to $v_f = v$ and $t = 0$ to t; we get,

$$\int_0^V dv_y = \int_0^t -8t^2\,dt$$

or, $v_y = -\frac{8t^3}{3}$

$\therefore$ $|v_y|_{t=5} = -\frac{8 \times (5)^3}{3} = -333.33$ m/sec.

Position

The position of the particle can be obtained by applying;

$$dt = \frac{dS_y}{V_y}$$

or, $dS_y = v_y\,dt$

or, $dS_y = -\frac{8t^3}{3}\,dt$

Integrating both sides; we have,

$$\int_0^S dS_y = \int_0^5 -\frac{8t^3}{3}\,dt$$

$\therefore$ $S_y = \left|-\frac{8t^4}{3 \times 4}\right|_0^5 = -416.67$ m

40. The acceleration of a particle is given by a relation $a = V^3$. It is known that at time t = 0, position is −2 m and velocity is 2 m/sec. Find the displacement, position, velocity and acceleration at instant of $\frac{1}{2}$ sec. What do you mean by projectile and obtain the equation of projectile motion. [2070 Chaitra]

Solution:

The acceleration of a particle is given by a relation is;

$a = V^3$ (1)

Velocity

The velocity of the particle can be related to its position by applying;

$$dS = \frac{v\,dv}{a}$$

or, $\int_{-2m}^{S} dS = \int_2^V \frac{v\,dv}{v^3}$

or, $[S]_{-2}^{S} = \left[\frac{v^{-2+1}}{-2+1}\right]_2^v$

or, $S + 2 = -\left(\frac{1}{v} - \frac{1}{2}\right)$

or, $\frac{1}{v} = \frac{-2S-3}{2}$

$\therefore \quad v = \frac{2}{(-2S-3)}$

Position

The position of the particle can be related to the time by applying; (2)

$$dt = \frac{dS}{v}$$

or, $\int_0^t dt = \int_{-2}^{S} \frac{(-2S-3)}{2}\, dS$

or, $t = -\frac{1}{2} \times \left[\frac{2S^2}{2} + 3S\right]_{-2}^{S}$

or, $2t = -[S^2 + 3S - (4-6)]$

or, $2t = -(S^2 + 3S + 2)$

Now,

At $t = 0.5$ sec.;

$2 \times 0.5 = -(S^2 + 3S + 2)$

or, $S^2 + 3S + 2 = 0$

On solving; we get,

$S = 0.30$

Now, [Choose < -2 m]

$$v = \frac{2}{[\{(-2) \times 0.3\} - 3]} = -0.56 \text{ m/sec.}$$

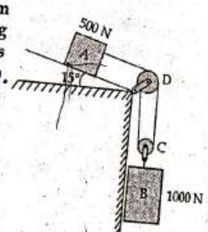
Acceleration

$$\text{Acceleration (a)} = \frac{dv}{dt} = V^3 = (-0.56)^3 = -0.18 \text{ m/s}^2$$

Displacement

$$\text{Displacement } (\Delta S) = |S|_{t=0.5} - |S|_t = 0.30 - (-2) = 2.3 \text{ m}$$

41. What do you mean by impulse momentum principle? Two blocks A and B having respective weights 500 N and 1000 N starts from rest. The pulley is frictionless and also practically mass less. The kinematic coefficient of the friction between block A and the inclined surface is 0.35. Determine the acceleration of each block and tension in the chord. **[2070 Chaitra]**



Solution:

Impulse momentum principle

See the solution of Q. no. 19 on page no. 354

Numerical

Given that;

Weight of block A $(W_A) = 500$ N

Weight of block B $(W_B) = 1000$ N

Kinematic coefficient of friction $(\mu_k) = 0.35$

The free body diagram of the block A and B are shown in the figure. Let, T_1 be the tension in the block A and T_2 be the tension in the block B.

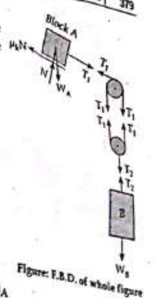


Figure: F.B.D. of whole figure

For block A

Applying equation of motion; we have,

$(+\nearrow) \sum F_y = ma_y$

or, $N - W_A \cos 15° = m_A \times 0$

$\therefore \quad N = 500 \times \cos 15° = 482.96$ N

Also,

$(+\searrow) \sum F_x = m_A a_x$

or, $W_A \sin 15° + T_1 - \mu_k N = m_A a_A$

or, $500 \times \sin 15° + T_1 - 0.35 \times 482.96 = \frac{500}{9.81} \times a_A$

$\therefore \quad T_1 = 50.97a_A + 39.63$ (1)

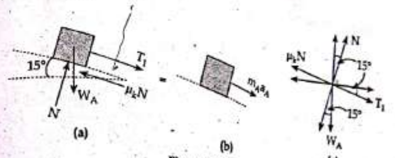


Figure: F.B.D. of A

For block B

$(+\downarrow) \sum F_y = m_B a_y$

or, $W_B - T_2 = \frac{1000}{9.81} \times a_B$

or, $1000 - T_2 = \frac{1000}{9.81} \times a_B$

$\therefore \quad T_2 = 1000 - 101.94a_B$ (2)

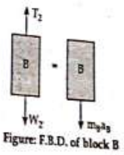


Figure: F.B.D. of block B

Also, from free body diagram of whole figure; we get,

$T_1 + T_1 = T_2$

$\therefore \quad 2T_1 = T_2$ (3)

Again, from free body diagram of the whole figure, as block A moves through distance v_A then block B moves distance $2v_B$.

i.e., $x_A = 2x_B$

Differentiating twice; we get,

$a_A = 2a_B$

$\therefore \quad a_B = \frac{a_A}{2}$ (4)

Now, from equation (3); we get,

$2T_1 = T_2$

or, $2(50.97a_A + 39.63) = 1000 - 101.94a_B$

or, $2(50.97a_A + 39.63) = 1000 - \left(101.94 \times \frac{a_A}{2}\right)$

or, $152.91a_A = 920.74$

$\therefore$ $a_A = 6.02\ m/s^2$

and, $a_B = \frac{a_A}{2} = \frac{6.02}{2} = 3.01\ m/s^2$

Again, from equation **(2)**; we get,

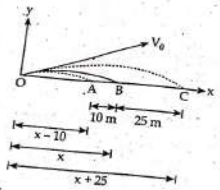
$$T_2 = 1000 - 101.94a_B = 1000 - (101.94 \times 3.01) = 693.16\ N$$

and, $T_1 = \frac{T_2}{2} = \frac{693.16}{2} = 346.58\ N$

42. **A projectile is aimed at a marked on horizontal plan through the point of projection and falls 10 shorts when the angle of projection is 15° while overshoots the mark by 25 m when the inclination is 40°. Calculate the distance of target and the required angle of projection, if the velocity remains constant. Neglecting air resistance, define dependent motion of the particle with example.** [2071 Shrawan]

Solution:

The given figure is;



If x be the distance of the target from the point of projection and V_0 be the velocity of projection as shown in the figure.

Range of projection is given by the expression as;

$$R = \frac{v_0^2 \sin 2\alpha}{g}$$

Applying first case; we get,

$$(x - 10) = \frac{v_0^2 \sin(2 \times 15°)}{g}$$

or, $(x - 10) = \frac{v_0^2}{g} \times \frac{1}{2}$

$\therefore$ $x = 10 + \frac{v_0^2}{2g}$ **(1)**

Also, applying second case; we get,

$$(x + 25) = \frac{v_0^2 \sin(2 \times 40°)}{g}$$

or, $(x + 25) = 0.985 \times \frac{v_0^2}{g}$

$\therefore$ $x = \frac{0.984v_0^2}{g} - 25$ **(2)**

From equation **(1)** and **(2)**; we get,

$$10 + \frac{v_0^2}{2g} = \frac{0.984V_0^2}{g} - 25$$

or, $35 = 0.485 \times \frac{v_0^2}{g}$

or, $v_0^2 = 707.94$

$\therefore$ $v_0 = 26.61\ m/sec.$

Again from equation **(1)**; we get,

$\therefore$ $x = 10 + \frac{707.94}{2 \times 9.81} = 46.09\ m$

Let, the correct angle of projection be α; then,

$$46.09 = \frac{v_0^2}{g} \times \sin 2\alpha$$

or, $\frac{46.09g}{v_0^2} = \sin 2\alpha$

or, $\frac{46.09 \times 9.81}{707.94} = \sin 2\alpha$

or, $\sin 2\alpha = 0.6385$

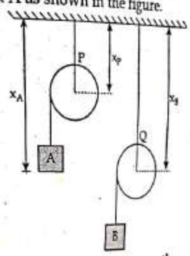
or, $2\alpha = \sin^{-1}(0.6385) = 39.68$

$\therefore$ $\alpha = 19.84°$

For the second part

When the position of the particle will depend upon the position of another or several other particles, then motion are called depend motion.

In the case of pulleys system in which the position of block B depends upon the position of block A as shown in the figure.



Let us measure the distances with respect to the support. As the length of rope, l going around the pulleys, and the distance of the pulley, P from the support is constant; therefore,

$$(x_A - x_P) + (x_B - x_P) + x_B = l$$

or, $x_A + 2x_B - 2x_P = l$

$\therefore$ $x_A + 2x_B = l + 2x_P$

Giving an increment Δx_A to block A (*i.e.*, lower the block A by Δx_A)

i.e., $\Delta x_A + 2\Delta x_B = 0$

$\therefore \quad \Delta x_B = -\frac{\Delta x_A}{2}$

i.e., the displacement of the block B is half the displacement of the block A and is in opposite direction.

43. Define the dynamic equilibrium. Determine the velocity and acceleration of the particle, if it moves along a curved path defined by $r = 5\theta$ and $\theta = \frac{t^2}{3}$, where, r is in metre and t is in seconds. Given that the instant angle is $\theta = \frac{\pi}{3}$ [2071 Shrawan]

Solution:

From Newton's second law of motion; we have,

$$\sum \vec{F} + (-m\vec{a}) = 0$$

The above equation is called dynamic equilibrium where, $(-m\vec{a})$ are the inertia forces added to the system of forces acting on the particle in the direction opposite to the direction of acceleration $\vec{a}$.

For the second part

Given paths are:

$$r = 5\theta \text{ m}$$

$$\theta = \frac{t^2}{3} \text{ rad/sec.}$$

Differentiating radial (r) and angular displacement (θ) with respect to time; we get,

$$\dot{r} = 5$$

and, $\ddot{r} = 5$

Similarly,

$$\dot{\theta} = \frac{2t}{3}$$

and, $\ddot{\theta} = \frac{2}{3}$

When $\theta = \frac{\pi}{2}$, then,

$$\frac{t^2}{3} = \frac{\pi}{2}$$

or, $t = \sqrt{\frac{3\pi}{2}} = 2.17$ sec.

Now, at t = 2.17 sec.;

$$\dot{\theta} = \frac{2t}{3} = \frac{2 \times 2.17}{3} = 1.45$$

and, $r = 5\theta = 5 \times \frac{\pi}{2} = 7.85$

Now,

$$v_r = \dot{r} = 5$$

$$v_\theta = r\dot{\theta} = 7.85 \times 1.45 = 11.38$$

$\therefore \quad \vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta = 5\hat{e}_r + 11.38\hat{e}_\theta$

$\therefore \quad v = |\vec{v}| = \sqrt{(5)^2 + (11.38)^2} = 12.43$ m/sec.

Again,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 7.85 \times (1.45)^2 = -16.50$$

and, $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 7.85 \times \frac{2}{3} + 2 \times 5 \times 1.45 = 19.73$

$\therefore \quad \vec{a} = a_r\hat{e}_r + a_\theta\hat{e}_\theta = -16.50\hat{e}_r + 19.73\hat{e}_\theta$

$\therefore \quad a = |\vec{a}| = \sqrt{(-16.50)^2 + (19.73)^2} = 25.72$ m/s²

44. Define uniformly rectilinear motion and uniformly accelerated rectilinear motion. For the pulleys system as shown in the figure, calculate the velocity and acceleration of block C, if the velocities and acceleration of block A and B are 3 m/sec. (↓), 2 m/s² (↑), 4 m/sec. (↑) and 5 m/s² (↓) respectively. [2071 Bhadra]

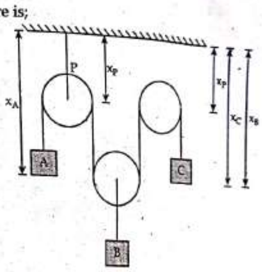
Solution:

For the first part

See the definition part on page no. 335

For the second part

The given figure is;



$$v_A = 3 \text{ m/sec. } (\downarrow)$$

$$a_A = 2 \text{ m/s}^2 (\uparrow)$$

and, $v_B = 4$ m/sec. (↑)

$$a_B = 5 \text{ m/s}^2 (\downarrow)$$

Let us measure the distances with respect to the support. As the length of rope, l going around the pulleys and distance of the pulley, P from the support is constant.

$\therefore \quad (x_A - x_P) + (x_B - x_P) + (x_C - x_P) = l$

or, $x_A + 2x_B + x_C - 3x_P = l$

or, $x_A + 2x_B + x_C = l + 3x_P$

or, $x_A + 2x_B + x_C = \text{Constant}$

Differentiating; we get,

$$v_A + 2v_B + v_C = 0 \qquad (1)$$

Again, differentiating; we get,

$$a_A + 2a_B + a_C = 0 \qquad (2)$$

From equation (1); we get,

$$v_A + 2v_B + v_C = 0$$

or, $3(\downarrow) + (2 \times 4)(\uparrow) + v_C = 0$

Taking ($\uparrow$) positive and ($\downarrow$) negative; we get,

$$-3 + 8 + v_C = 0$$

$\therefore$ $v_C = -5 = 5\ m/sec.\ (\downarrow)$

Also, from equation (2); we get,

$$a_A + 2a_B + a_C = 0$$

or, $2(\uparrow) + (2 \times 5)(\downarrow) + a_C = 0$

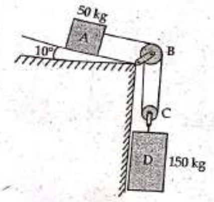
Taking ($\uparrow$) positive and ($\downarrow$) negative; we get,

or, $2 - 10 + a_C = 0$

$\therefore$ $a_C = 8\ m/s^2\ (\uparrow)$

Thus, velocity and acceleration of the block C are $5\ m/s\ (\downarrow)$ and $8\ m/s^2\ (\uparrow)$.

45. Two blocks shown in the figure starts from rest. The pulleys are frictionless and having no mass. The kinematic co-efficient of the friction between block A and inclined plane is 0.37. Determine the acceleration of each block and tension in each chord. What do you mean by dynamic equilibrium? [2071 Bhadra]



Solution:

For the first part

Proceed as the solution of Q. no. 41 on page no. 378

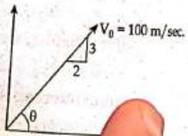
Dynamic equilibrium

See the solution of Q. no. 20 on page no. 355

46. A particle starting from origin is subjected to acceleration such that $a_x = -4\ m/s^2$ and $a_y = -8\ m/s^2$. If the initial velocity is 100 m/sec. directed at a slope of 2 : 3, compute the radius of curvature of path after 5 sec. Also calculate the position at the end of 5 sec. What are the possible equations of motion for a particle in terms of kinematics? [2071 Magh]

Solution:

Here,



$$\tan\theta = \frac{3}{2} = 1.5$$

$$\sin\theta = \frac{3}{\sqrt{(2)^2 + (3)^2}} = \frac{3}{\sqrt{13}} = 0.832$$

$$\cos\theta = \frac{2}{\sqrt{(2)^2 + (3)^2}} = \frac{2}{\sqrt{13}} = 0.555$$

Components of initial velocities;

$$(v_0)_x = 100\cos\theta = 100 \times 0.555 = 55.5\ m/sec.$$

$$(v_0)_y = 100\sin\theta = 100 \times 0.832 = 83.2\ m/sec.$$

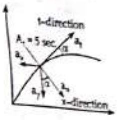
Components of initial velocities at the end of 5 sec.;

$$v_{5x} = (v_0)_x + a_x t = 55.5 + (4 \times 5) = 35.5\ m/sec.$$

$$v_{5y} = (v_0)_y + a_y t = 83.2 - (8 \times 5) = 43.2\ m/sec.$$

Inclination of V_5 with x-axis (α) $= \tan^{-1}\left(\frac{43.2}{35.5}\right) = 50.59°$

Normal components of acceleration;



i.e., $a_n = a_y\cos\alpha + a_x\sin\alpha$

$$= (8 \times \cos 50.59°) - (4 \times \sin 50.59°)$$

$\therefore$ $a_n = 1.988\ m/s^2$

Also,

$$a_n = \frac{v_5^2}{\rho_5} = \frac{v_{5x}^2 + v_{5y}^2}{\rho_5} = \frac{(35.5)^2 + (43.2)^2}{\rho_5} = \frac{3126.49}{\rho_5}$$

$\therefore$ $\rho_5 = \frac{3126.49}{1.988} = 1572.68\ m$

$V_5^1 =$

Hence, radius of curvature at the end of 5 sec. (ρ_5) = 1572.68 m.

$a_n = \frac{v}{\rho}$

To find position

$$x_5 = (v_0)_x t + \frac{1}{2}a_x t^2 = 55.5 \times 5 + \frac{1}{2} \times (-4) \times (5)^2 = 227.5\ m$$

and, $y_5 = (v_0)_y t + \frac{1}{2}a_y t^2 = 83.2 \times 5 + \frac{1}{2} \times (-8) \times (5)^2 = 316\ m$

$\therefore$ Position at the end of 5 sec. $(x_5, y_5) = (227.5\ m, 316\ m)$

47. a) Define the dynamic equilibrium and impulse momentum principle for particle. [2071 Magh]

Solution: See the solution of Q. no. 20 on page no. 335

b) A particle moves along a curved path defined by $r = 5\theta$ and $\theta = \frac{t^2}{3}$ where, r is in metre and t is in seconds. Determine the velocity and acceleration of the particle when $\theta = 90°$. [2071 Magh]

Solution: See the solution of Q. no. 43 on page no. 382

48. a) What is uniformly accelerated motion? Also define the angular momentum and its rate of change. [2071 Chaitra]

Solution:

Uniformly accelerated motion

See the definition part on page no. 335

Angular momentum and its rate of change

See the solution of Q. no. 12 on page no. 347

b) **Motion of particle is defined by a relation $x = \frac{t^3}{3} - 3t^2 + 8t + 15$. Determine the position of particle when velocity is 2.5 m/sec. Also determine the position of the particle when acceleration is 3.6 m/s².** [2071 Chaitra]

Solution:

The given relation is;

$$x = \frac{t^3}{3} - 3t^2 + 8t + 15 \quad (1)$$

Differentiating relation (1); we get,

$$v = \frac{dx}{dt} = \frac{3t^2}{3} - 6t + 8 = t^2 - 6t + 8$$

and, $a = \frac{dv}{dt} = 2t - 6$

When the velocity of the particles is 2.5 m/sec; then,

$$v = t^2 - 6t + 8$$

or, $2.5 = t^2 - 6t + 8$

or, $t^2 - 6t + 5.5 = 0$

On solving; we get,

$t = 4.87$ sec.

and, $t = 1.37$ sec.

Now, position of the particle when $t = 4.87$ sec.;

$$x = \frac{t^3}{3} - 3t^2 + 8t + 15 = \frac{(4.87)^3}{3} - \{3 \times (4.87)^2\} + (8 \times 4.87) + 15$$

$\therefore \quad x = 21.31$ m

Similarly, when $t = 1.37$ sec.; then,

$$x = \frac{t^3}{3} - 3t^2 + 8t + 15 = \frac{(1.37)^3}{3} - \{3 \times (1.37)^2\} + (8 \times 1.37) + 15$$

$\therefore \quad x = 21.06$ m

Again, when acceleration of the particle is 3.6 m/s²; then,

$$a = 2t - 6$$

or, $3.6 = 2t - 6$

$\therefore \quad t = 4.8$ sec.

and, position of the particle when $t = 4.8$ sec.;

$$x = \frac{t^3}{3} - 3t^2 + 8t + 15 = \frac{(4.8)^3}{3} - \{3 \times (4.8)^2\} + (8 \times 4.8) + 15$$

$\therefore \quad x = 21.14$ m

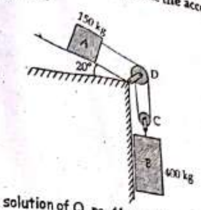
49. a) What is uniformly accelerated rectilinear motion? [2072 Ashwin]

Solution: *See the definition part on page no. 335*

b) **The acceleration of a particle is given by a relation $a = V^3$. It is known that at time $t = 0$, position is -2 m and velocity is 1 m/sec. Find the displacement position, velocity and acceleration at instant of $\frac{1}{4}$ sec.** [2072 Ashwin]

Solution: *Proceed as the solution of Q. no. 40 on page no. 377*

50. a) Two blocks as shown in the figure start from rest. The pulleys are frictionless and have no mass. The kinematic friction co-efficient between block A and inclined plane is 0.4. Determine the acceleration of each block and tension in each chord. [2072 Ashwin]



Solution: Proceed as the solution of Q. no. 41 on page no. 378

b) **Prove that the rate of change of angular momentum about any point is equal to moment of the force about the same point.** [2072 Ashwin]

Solution: See the solution of Q. no. 12 on page no. 347

51. **What do you mean by dependent motion of a particle? Illustrate it with suitable example. A particle starting from origin is subjected to acceleration such that $a_x = -2$ m/s² and $a_y = -5$ m/s². The initial velocity is 60 m/sec. directed at slope of 30° with respect to horizontal. Compute the radius of curvature at the end of 3 sec. Also determine its position at the end of 3 sec.** [2072 Kartik]

Solution:

- **Dependent motion of a particle**

  See the solution of Q. no. 42 on page no. 380

- **Numerical part**

  See the solution of Q. no. 46 on page no. 384

52. **Show that, "rate of change of angular momentum about a point is equal to moment of the force about same point". The resultant external force acting on a 5 kg particle in space is $\vec{F} = (12t\vec{i} - 24t\vec{j} + 40t^3\vec{k})$ N; where, t is in seconds. The particle is initially at rest at origin. Determine the v component of acceleration, velocity and position at the instant of 5 sec.** [2072 Kartik]

Solution:

For the first part

See the solution of Q. no. 12 on page no. 347

53. **A ball is tossed with the velocity 10 m/sec. directed vertically upward from a window located 20 m above the ground. Knowing that the acceleration of the ball is constant and equal to 9.81 m/s² downward, determine:**

i) **the velocity 'v' and elevation 'y' of the ball above the ground at any time 't'.**

ii) **the highest elevation reached by the ball and corresponding value to 't'.**

iii) **the time when the ball hit the ground and corresponding velocity.**

What do you mean by dependent motion? Explain with example. [2072 Kartik]

Solution:

Considering upward direction of the velocity and acceleration as +ve and downward direction as −ve; we have,

$$a = -9.81$$

or, $\frac{dv}{dt} = -9.81$

or, $dv = -9.81\, dt$ ... (1)

Integrating both sides and using condition $v_0 = 10$ to $v = v$ and $t = 0$ to $t = t$ then,

$$\int_{v_0=10}^{v} dv = -\int_{t=0}^{t} 9.81\, dt$$

∴ $v = 10 - 9.81t$

Again,

$$v = \frac{dy}{dt}$$ ... (2)

or, $10 - 9.81t = \frac{dy}{dt}$

or, $dy = (10 - 9.81t)\, dt$

Integrating both sides and using conditions $y_0 = 20$ m to y and $t = 0$ to t; then,

$$\int_{y_0=20}^{y} dy = -\int_0^t (10 - 9.81t)\, dt$$

or, $y - 20 = 10t - \frac{9.81}{2}t^2$

∴ $y = 20 + 10t - 4.91t^2$ ... (3)

When the ball reached the highest position then final velocity of the ball is zero *i.e.*, $v = 0$.

From equation (2); we have,

$$v = 10 - 9.81t$$

or, $0 = 10 - 9.81t$

∴ $t = 1.02$ sec.

Now, from equation (3); we have,

$$y = 20 + (10 \times 1.02) - [4.91 \times (1.02)^2] = 25.09 \text{ m}$$

When the ball hit the ground then; $y = 0$, from equation (3); we get,

$$y = 20 + 10t - 4.91t^2$$

or, $0 = 20 + 10t - 4.91t^2$

∴ $t = 3.28$ sec. or $t = -1.24$ sec.

Hence, from starting of motion is 3.28 sec.

From equation (2); we have,

$$v = 10 - 9.81t = 10 - (9.81 \times 3.28) = -22.18 \text{ m/sec.}$$

∴ $v = 22.18$ m/sec.(↓)

54. The acceleration of a particle is given by the relation $a = 21 - 12x^2$; where, a is expressed in m/s^2 and x in metres. The particle starts with no initial velocity at origin. Determine:

i) the velocity when $x = 1.5$ m.

ii) the position where the velocity is again zero.

iii) the position where the velocity is maximum. [2072 Magh]

Solution:

Here, the given relation is;

$$a = 21 - 12x^2$$

or, $v \times \frac{dv}{dx} = 21 - 12x^2$

or, $v\, dv = (21 - 12x^2)\, dx$

Integrating both sides and using the conditions; we have,

When $x = 0$ m; then,

$$v = 0 \text{ m/sec.}$$

or, $\int_0^v v\, dv = \int_0^x (21 - 12x^2)\, dx$

or, $\frac{v^2}{2} = 21x - \frac{12x^3}{3}$

or, $v^2 = 42x - 8x^3$ ... (1)

i) When $x = 1.5$ m; then,

$$v^2 = (42 \times 1.5) - [8 \times (1.5)^3] = 36$$

∴ $v = \pm 6$ m/sec.

ii) For the velocity (v) = 0; then,

$$(0)^2 = 42x - 8x^3$$

or, $2x(21 - 4x^2) = 0$

∴ $x = 0$ or $x = \pm 2.29$ m

Thus, velocity is again zero at $x = \pm 2.29$ m.

iii) The velocity is maximum when acceleration is zero; then,

$$a = 21 - 12x^2$$

For maximum $a = 0$;

i.e., $\frac{dv}{dt} = 0$

or, $a = 21 - 12x^2$

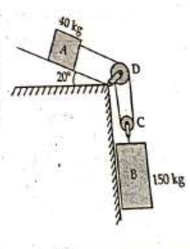
∴ $x = \pm 1.323$ m

Taking ±ve sign;

$$x = 1.323 \text{ m}$$

Thus, velocity is maximum at $x = 1.323$ m.

55. Two block in the figure starts from rest. The pulleys are frictionless and having no mass. The kinetic coefficient of the friction between the block A and the inclined plane is 0.4. Determine the acceleration of each block and tension in each chord. [2072 Magh]



Solution:

Given that;

Mass of block A (m_A) = 40 kg

Mass of block B (m_B) = 150 kg

Kinematic coefficient of friction $(\mu_k) = 0.4$

Let T_1 be the tension in the block A and T_2 be the tension in the block B. From the figure, it is clear that, of block A moves through x_A then block B moves through $2x_B$.

i.e., $x_A = 2x_B$

Differentiating with respect to time; we get,

$a_A = 2a_B$

$\therefore \quad a_B = \frac{a_A}{2}$

Using Newton's second law for block A and B pulley C; we have,

Block A (1)

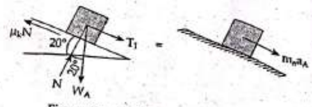


Figure: Free body diagram of block A

$(^+\nearrow) \sum F_y = m_a a_y$

or, $N - W_A \cos 20° = m_A \times 0$ [Since, vertical acceleration $a_y = 0$]

or, $N = m_A g \cos 20°$

$\therefore \quad N = 40 \times 9.81 \times \cos 20° = 368.74\ N$

Also,

$(^+\searrow) \sum F_x = m_a a_x$

or, $T_1 - \mu_k N + W_A \sin 20° = m_a a_A$

or, $T_1 - 0.4 \times 368.74 + 40 \times 9.81 \times \sin 20° = 40a_A$

$\therefore \quad T_1 = 40a_A + 13.28$

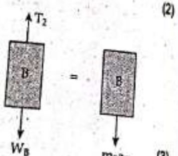
Block B (2)

$(^+\downarrow) \sum F_y = m_B a_B'$

or, $T_2 + W_B = m_B a_B$

or, $-T_2 + 150 \times 9.81 = 150a_B$

$\therefore \quad T_2 = 1471.5 - 150a_B$



(3)

Figure: Free body diagram of block B

Pulley C

Since mass of pulley is assumed to be zero; we have,

$(^+\downarrow) \sum F_y = m_C a_C = 0$

or, $T_2 - 2T_1 = 0$

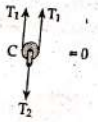


Figure: F.B.D. of pulley C

or, $T_2 = 2T_1$

Putting the values of T_1 and T_2 in the equation (4); we get,

$1471.5 - 150a_B = 2(40a_A + 13.28)$ (4)

$1471.5 - \left(150 \times \frac{a_A}{2}\right) = 2(40a_A + 13.28)$

or, $155a_A = 1458.22$

or, $a_A = 9.41\ m/s^2$

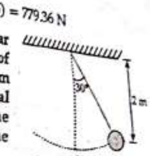
$\therefore$ and, $a_B = \frac{9.41}{2} = 4.705\ m/s^2$

Now,

$T_1 = 40a_A + 13.28 = 40 \times 9.41 + 13.28 = 389.68\ N$

and, $T_2 = 1471.5 - 150a_B = 1471.5 - (150 \times 4.705) = 779.36\ N$

96. **Define linear momentum and angular momentum. Find the velocity and acceleration of the bob in the given position. The bob of a 2 m pendulum describes an arc of a circle in a vertical plane, which is shown in the figure, if the tension in the chord is 2.5 times the weight of the bob for the position shown.**



[2072 Chaitra]

Solution:

The free body diagram of the bob after displacing from mean position is shown below.

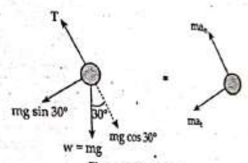


Figure: F.B.D. of bob

Applying Newton's second law in tangential direction; then,

$(^+\swarrow) \sum F_t = ma_t$

or, $mg \sin 30° = ma_t$

$\therefore \quad a_t = g \sin 30° = 9.81 \sin 30° = 4.91\ m/s^2 (\swarrow)$

Also, applying Newton's second law in its normal direction; then,

$(^+\nwarrow) \sum F_x = ma_n$

or, $T - mg \cos 30° = ma_n$

or, $2.5mg - mg \cos 30° = ma_n$

or, $a_n = 2.5g - g \cos 30°$

$\therefore \quad a_n = 16.03\ m/s^2$

$\therefore \quad a = \sqrt{a_t^2 + a_n^2} = \sqrt{(4.91)^2 + (16.03)^2} = 16.76\ m/s^2$

and, Velocity of the bob $(v) = \sqrt{\rho a_n} = \sqrt{2 \times 16.03} = 5.66\ m/sec.$

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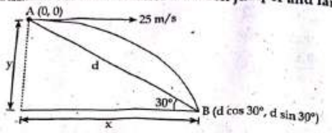
ADDITIONAL PROBLEMS

1. A sky jumper starts with a horizontal take off velocity of 25 m/s and land on a straight landing hill inclined at 30°. Determine;
   a) The time between take-off and landing
   b) The maximum vertical distance between jumper and landing hill.



Solution:

Given that;

$(V_x)_0 = 25$ m/s

$(V_y)_0 = 0$ m/s

Motion in x–direction

$x = (V_0)_x \times t = 25t$ (1)

Motion in y =–direction

$y = y_0 + (VB_y)_0 t - \frac{1}{2} g t^2$

or, $y = 0 + 0 - 4.905\, t^2$

or, $y = -4.905\, t^2$ (2)

For point B;

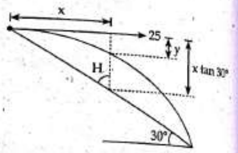
$x = d \cos 30° = 0.866\, d$

and, $y = d \sin 30° = -0.5\, d$

From equation (1); we have,

$0.866\, d = 25\, t$

or, $t = \frac{0.866}{25} d$



From equation (2); we have,

$-0.5\, d = -4.905 \times \left(\frac{0.866}{25}\right)^2$

Solving; we get,

$d = 84.95$ m

$\therefore\ t = \frac{0.866}{25} \times 84.85 = 2.94$ sec

Maximum vertical distance (H) $= x \tan 30° - y$

$= x \tan 30° - y = x \tan 30° - 4.905\, t^2$

For 'H' to be maximum $\frac{dH}{dt} = 0$

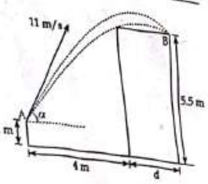
or, $H = 0.5\, t \times \tan 30° - 4.905\, t^2$

$\therefore\ \frac{dH}{dt} = 25 \tan 30° - 2 \times 4.905\, t = 0$

or, $t = 1.471$ sec.

$\therefore\ H = x \tan 30° - 4.905\, t^2 = 25 \times 1.471 \tan 30° - 4.905 \times (1.471)^2 = 10.618$ m

2. A nozzle at 'A' distance water with an initial velocity 11 m/s at an angle α with the horizontal. Determine;
   a) the distance 'd' to the farthest point 'B' on the roof that the water can reach
   b) the correspond angle α. Check that the stream will clear the edge of the roof.



Solution:

Given that;

$V_x = 11 \cos \alpha$

and, $x = (4 + d)$ m

$V_y = 11 \sin \alpha$

$y_0 = 1$ m

and, $y = 5.5$ m

Now, motion in x–direction

$x = x_0 + (V_x)t$

or, $x = 11 \cos \alpha \times t$ (1)

Motion in y–direction

$y = y_0 + (V_y)_0 t - \frac{1}{2} g t^2$ (2)

or, $5.5 = 1 + (11 \sin \alpha) \times t - 4.905\, t^2$

or, $4.5 = (11 \sin \alpha) \times \frac{x}{11 \cos \alpha} - \frac{4.905 \times x^2}{(11 \cos \alpha)^2}$ [By using equation (1)]

or, $4.5 = x \tan \alpha - \frac{4.905 x^2}{1.21 \cos^2 \alpha}$

or, $4.5 = x \tan \alpha - \frac{4.905 x^2}{1.21} (1 + \tan^2 \alpha)$

Let, $x \tan \alpha = u$ then,

$4.5 = u - (x^2 + u^2) \times \frac{4.905}{1.21}$

or, $x^2 = 121 \left(\frac{4 - 4.5}{4.905}\right) - u^2$

For 'x' to be maximum $\frac{d}{du}(x^2) = 0$

or, $\frac{121}{4.905} - 2u = 0$

or, $u = 12.33$

$\therefore\ x^2 = 121 \left(\frac{12.33 - 4.5}{4.905}\right) - (12.33)^2 = 41.413$

$\therefore\ x = 6.41$ m

Now,

$x \tan \alpha = u$

or, $6.41 \times \tan \alpha = 12.33$

or, $\alpha = 62.5°$

Also,

$x = d + 4$

or, $d = 2 - 4 = (6.41 - 4)\ m = 2.41\ m$

At, $x = 4\ m$

$$t = \frac{4}{11 \cos(62.5)} = 0.7875 \text{ sec.}$$

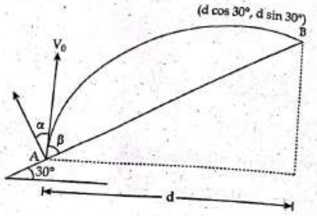
From equation (2); we get,

$$y = 1 + 11 \sin(62.5) \times (0.7875) - 4.905 \times (0.7875)^2 = 5.64\ m$$

$\therefore$ $y > 5.5\ m$

So, the stream clears the edge of the roof.

3. **A projectile is launched from point A with an initial velocity V_0 of 36.6 m/s at an angle α with the vertical. Determine; (a) the distance 'd' to the far west point 'B' on the hill that the projectile can reach (b) the corresponding angle α.**



Solution:

Given,

$V_0 = 36.6\ m/s$

Let, $\theta = \beta$ = Angle made with line AB and projectile.

Then,

$(V_x)_0 = 36.6 \cos\beta$

and, $(V_x)_0 = 36.6 \sin\beta$

Motion in x-direction

$x = x_0 + (V_x)_0 t$

or, $x = (36.6 \cos\beta) \times t$ (1)

Motion in y-direction

$$y = y_0 + (V_y)_0 t - \frac{1}{2} g t^2$$

or, $y = (36.6 \sin\beta) \times t - 4.905\ t^2$ (2)

Now, at point B;

$x = d \cos 30°$ (3)

and $y = d \sin 30°$ (4)

From equation (3) and (1); we get,

$$d \times \frac{\sqrt{3}}{2} = (36.6 \cos\beta) \times 1$$

or, $t = \dfrac{\sqrt{3}\ d}{2(36.6 \cos\beta)}$

From equation (2); we have,

$$\frac{d}{2} = (36.6 \sin\beta) \times \frac{0.866\ d}{(36.6 \sin\beta)} - 4.905 \times \frac{(0.866\ d)^2}{(36.6 \cos\beta)^2}$$

or, $0.5\ d = 0.866\ d \tan\beta - 2.74 \times 10^{-3} (1 + \tan^2\beta)$ (5)

Put $d \tan\beta = u$ then,

$0.5\ d = 0.866\ u - 2.74 \times 10^{-3} (u^2 + d^2)$

or, $2.74 \times 10^{-3} (u^2 + d^2) - 0.866u + 0.5d = 0$

For 'd' to be maximum;

$$\frac{d}{du}(d^2) = 0$$

or, $0 + 0.866 - 5.492 \times 10^{-3}\ u = 0$

or, $u = 157.68$

When $u = 157.67$ then equation (5) becomes,

$2.74 \times 10^{-3} (157.67^2 + d^2) - 0.866 \times 157.68 + 0.5 \times d = 0$

or, $d = 91.2\ m$

Now,

$d \tan\beta = u$

or, $91.2 \tan\beta = 157.67$

$\therefore$ $\beta = 60°$

$\therefore$ $\alpha = (90 - 60)° = 30°$

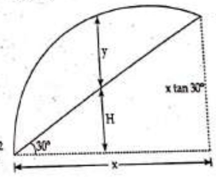
Maximum height (H) = $(x \tan 30° - y)$

or, $H = 36.6 \tan 30° \times t - (36.6 \sin 60° \times - 4.905\ t^2)$

or, $H = -21.13\ t + 4.905\ t^2$

For maximum height;

$$\frac{dH}{dt} = 0$$



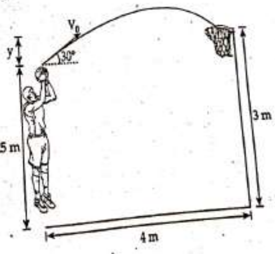
or, $0 = -21.13\ 1 + 9.81\ t$

$\therefore$ $t = 2.154$ sec.

$\therefore$ $H = -21.31 \times 2.154 + 4.905\ (2.154)^2$

$= 22.75\ m$

4. **A basket ball player who is 2 m tall jumps 0.25 m above the ground and shoots the ball into the basket from a point that is 4m from the basket as shown in figure. Determine the velocity with which he must throw the ball so that the ball enters the basket.**



Solution:

Here; we have,

$y = (3 - 2.25)\ m = 0.75\ m$

The equation of motion of the ball can be written as;

$$y = x\tan\alpha - \frac{1}{2}\frac{g}{(v_0\cos\alpha)^2}x^2$$

or, $0.75 = 4\tan 30° - \frac{1}{2}\frac{9.81}{(v_0\cos 30°)^2}\times 4^2$

or, $v_0 = 8.19\ m/s$

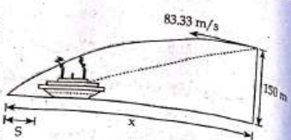
5. **A bomber flying horizontally at a speed of 300 kmph at an altitude of 150 m releases a bomb targeting a ship moving in same direction as the plane at a constant speed of 10 m/s. How far from the ship should it release the bomb to hit it?**

Solution:

Given that;

$$V_0 = 300\ km/hr = \frac{300\times 1000}{3600} = 83.33\ m/s$$

and, Altitude = 150 m



When bomb is released, we know that its speed is the same as that of the plane and the direction is horizontal. Hence, the angle of projection is taken as 0°. Therefore the equation of motion of the bomb can be written as;

$$y = x(\tan\alpha) - \frac{1}{2}\frac{g}{(V_0\cos\alpha)^2}\times x^2$$

Substituting the values; we get,

$$-150 = 0 - \frac{1}{2}\frac{g}{2\times(83.33)^2}\times x^2$$

or, $x = 460.82\ m$

Time taken by the bomb to hit the target is obtained as;

$$t = \frac{x}{v_0\cos\alpha} = \frac{460.82}{83.33} = 5.53\ sec.$$

Hence, the distance travelled by the ship during this time is obtained as;

$$S = V_s\times t = 10\times 5.53 = 55.3\ m$$

Hence, the distance from which the bomb must be released in order to hit the ship $= x - s = (460.82 - 55.3) = 405.52$ m.

$$\text{Angle of sight E } (\tan\phi) = \frac{|x - x_s|}{|y|} = \frac{405.52}{150} = 2.703$$

$\therefore\ \phi = 69.7°$

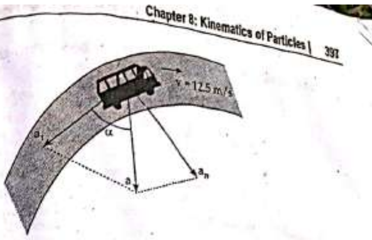
6. **A bus moving along a curved path with a constant speed of 45 km/hr decelerates at a constant rate to a halt in 10 second. Determine; the total acceleration at the instant brake is applied. Radius of curvature is 100 m.**

Solution:

Given that;

Initial speed of bus $V_0 = 45\ km/hr = \frac{45\times 1000}{3600} = 12.5\ m/s$

Final speed of bus $V = 0\ m/s$



We know that;

$$V = v_0 + at$$

Hence tangential component of acceleration is given as;

$$a_t = \frac{0 - 12.5}{10} = -1.25\ m/s^2$$

–ve sign tangential acceleration is opposite to that of velocity.

Again, we have,

$$a_n = \frac{V^2}{\rho} = \frac{12.5^2}{100} = 1.56\ m$$

Hence,

Total acceleration (a) $= \sqrt{a_t^2 + a_n^2} = \sqrt{(-1.25)^2 + (1.56)^2} = 2\ m/s^2$

and, $\alpha = \tan^{-1}\frac{|a_n|}{|a_t|} = \tan^{-1}\left(\frac{1.56}{1.25}\right) = 51.3°$

A man standing at a bus stand sees that a bus just leaves when he is about 20 m from the bus. If the bus accelerates at a constant rate of 1 m/s² then determine the acceleration with which the man must run to catch the bus with in a distance of 30 m. Also, determine the speeds of the bus and the man at that instant.

Solution:

As the distance travelled by the man before catching the bus is 30 m, the distance travelled by the bus during this time is;

$$(30 - 20)\ m = 10\ m$$

Hence, the kinematic equation of motion of the bus can be written as;

$$S_b = V_{b0}\times t + \frac{1}{2}a_B\times t^2$$

or, $10 = 0 + \frac{1}{2}\times 1\times t^2$

$\therefore\ t = 4.47$ sec.

The equation of motion of the man can be written as;

$$S_m = (V_{m0})\,t + \frac{1}{2}a_m t^2$$

or, $30 = 0 + \frac{1}{2}a_m\times(4.47)^2$

or, $a_m = 3\ m/s^2$

Speed of the man when he catches the bus is given as;

$$V_m = V_{m0} + a_m t = 0 + 3\times(4.47) = 13.41\ m/s$$

and, speed of the bus at that instant is given as;

$$V_b = V_{bo} + a_b t = 0 + 1(4.47) = 4.47\ m/s$$

8. The motion of a rocket fixed vertically upwards is tracked by radar as shown in figure. At a particular instant, it is observed that

$r = 15$ km, $\theta = 70°$, $\dot{\theta} = 0.02$ rad/sec and $\ddot{\theta} = 0.0011$ rad/s^2 determine the velocity and acceleration of the rocket at that instant.

Solution:

We know that the velocity vector is given as;

$$\vec{v} = V_r\hat{e}_r + V_\theta\hat{e}_\theta = \dot{r}\,\hat{e}_r + r\,\dot{\theta}\,\hat{e}_\theta$$

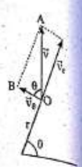
We see that the component of velocity along radial direction is $V_r = \dot{r}$ and component along the transverse direction is $V_\theta = r\dot{\theta}$.

As the rocket moves vertically upwards, the direction of its velocity is also vertically upward. Thus from velocity triangle

$$\frac{V_\theta}{V} = \cos 70°$$

where, $V_\theta = r\dot{\theta} = 15000(0.02) = 300$ m/s.

$$\therefore \quad V = \frac{V_\theta}{\cos 70°} = 877.14\ m/s$$



Also, from ΔOAB; we have,

$$\frac{V_r}{V} = \sin 70°$$

$$\therefore \quad V_r = \dot{r} = V \sin 70° = 824.24\ m/s$$

and, acceleration vector is given as,

$$\vec{a} = [\ddot{r} - r(\dot{\theta})^2]\,\hat{e}_r + [r\ddot{\theta} + 2\,\dot{r}\,\dot{\theta}]\hat{e}_\theta$$

Here,

$$a_\theta = r\,\ddot{\theta} + 2\,\dot{r}\,\dot{\theta} = 15000(0.0001) + 2\,(824.24)\,(0.02) = 47.97\ m/s^2$$

Acceleration triangle is similar to velocity triangle;

$$\frac{a_\theta}{a} = \cos 70°$$

$$\therefore \quad a = \frac{a_\theta}{\cos 70°} = 140.25\ m/s^2$$

9. **A particle moves along a curvilinear path defined by $y = ax^2$ where x and y are meters. The velocity and acceleration of the particle at a point (5 m, 2.5 m) are respectively 5 m/s and 2 m/s^2. Determine the total acceleration of the particle at that instant.**

Solution:

The equation of path of the particle is given as;

$$y = ax^2 \qquad (1)$$

Given that;

When $x = 5$ m, $y = 2.5$ m

$$\therefore \quad a = \frac{y}{x^2} = \frac{2.5}{25} = 0.1$$

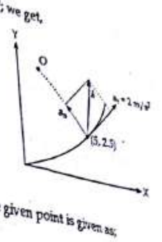
Therefore equation (1) becomes,

$$y = 0.1x^2$$

Differentiating the equation with respect to 'x', we get,

$$\frac{dy}{dx} = (0.1) \times 2x = 0.2x$$

$$\frac{d^2y}{dx^2} = 0.2$$



Hence, at given point (5 m, 2.5 m)

$$\frac{dy}{dx} = 1$$

and, $$\frac{d^2y}{dx^2} = 0.2$$

$\therefore$ Radius of curvature of the path at the given point is given as;

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{[1 + (1)^2]^{3/2}}{0.2} = 14.14\ m$$

Now,

Normal component of acceleration $(a_n) = \frac{V^2}{\rho}$

or, $$\frac{5^2}{14.14} = 1.77\ m/s^2$$

$\therefore$ Total acceleration (a) $= \sqrt{a_t^2 + a_n^2} = \sqrt{2^2 + (1.77)^2} = 2.67\ m/s^2$

10. **Two cars 'A' and 'B' start from rest at the same instant, with car 'A' initially trailing at some distance behind the car 'B'. The uniform acceleration of the cars 'A' and 'B' are respectively 3m/s^2 and 2m/s^2. If car 'A' overtakes the car 'B', when 'B' moved 200 m. Determine; (i) the time taken to overtake (ii) how far was the car 'A' behind 'B' initially (iii) determine the speed of each car at this instant.**

Solution:

Since, two cars start from rest their initial velocities are zero i.e.,

$$V_{A0} = V_{B0} = 0$$

$$a_A = 3m/s^2$$

and, $a_B = 2m/s^2$

i) Time taken to overtake

As the car B moves through a distance of 200 m before the car A overtakes it, we can write the equation of motion of the car B as;

$$S_B = (V_{B0})t + \frac{1}{2}a_B t^2$$

or, $$200 = 0 + \frac{1}{2} \times 2 \times t^2$$

or, $t = 14.14$ sec.

Thus, car A overtakes the car B after 14.14 sec from start.

ii) **Initial distance between two cars**

If 'x' be the initial distance between cars A and B then total distance travelled by the car A before overtaking the car B is;

$$S_B = x + 200$$

Hence, the equation of motion of car A can be written as;

$$S_A = V_{A0}t + \frac{1}{2}a_At^2$$

or, $x + 200 = 0 + \frac{1}{2} \times 3 \times (14.14)^2$

$\therefore$ $x = 100$ m

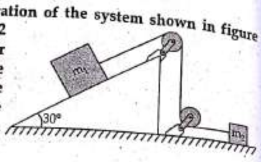
iii) *Speed of the two cars at the instant of overtaking.*

The speeds of two cars at the instant of overtaking can be determined as;

$$v_A = V_{A0} + a_At = 0 + 3 \times 14.14 = 42.43 \text{ m/s}$$

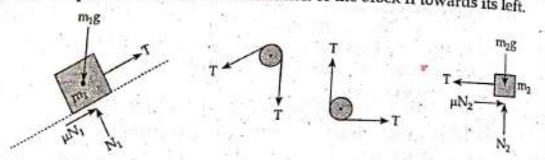
and, $v_B = B_{B0} + a_Bt = 0 + 2 \times 14.14 = 28.28$ m/s

11. **Find the expressions for the acceleration of the system shown in figure and the tension in the string. If $m_1 = 2$ kg, $m_2 = 1$ kg, $\theta = 30°$ and $\mu = 0.2$ for all contact surface, determine the tension in the string and the acceleration of the system. Assume the pulleys are mass less and friction less and the string is inextensible.**



Solution:

The free–body diagrams of the two blocks and pulleys are shown below. As the pulleys are frictionless, the tensions in different portion of the string are equal. In addition, as the string is inextensible, the acceleration of block I down the plane is same as the acceleration of the block II towards its left.



Motion of block I

$$\Sigma F_y = ma_y$$

or, $N_1 - m_1g \cos\theta = 0$

$\therefore$ $N_1 = m_1g \cos\theta$

Also,

$$\Sigma F_x = ma_x$$

or, $m_1g \sin\theta - \mu N_s - T = m_1a$

or, $m_1g \sin\theta - \mu m_1g \cos\theta - T = m_1a$ (1)

Motion of block II

$$\Sigma F_y = ma_y$$

or, $N_2 - m_2g = 0$

or, $N_2 = mg$

and, $\Sigma F_x = ma_x$

or, $T - \mu N_2 = m_2a$

or, $T - \mu m_2g = m_2a$

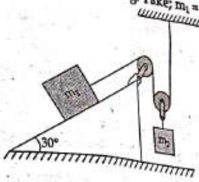
From equation (1) and (2); solving for 'a' and 'T'; then,

$$a = \frac{m_1g(\sin\theta - \mu\cos\theta) - \mu m_2g}{m_1 + m_2} \quad (2)$$

$$= \frac{2 \times 9.81[\sin 30^0 - 0.2 \times \cos 30^0] - 0.2 \times 1 \times 9.81}{2 + 1} = 1.48 \text{ m/s}^2$$

and, $T = \frac{m_1m_2[\sin\theta + \mu(1 - \cos\theta)]}{m_1 + m_2} = 3.45$ N

12. **Determine the acceleration of the system of the blocks shown in figure. The coefficient of kinetic friction between the block 'I' and the inclined plane is 0.45. Also, determine the tension in the string. Take; $m_1 = 75$ kg, $m_2 = 50$ kg**



Solution:

The free body diagram ϕ + blocks are shall below. Also, $m_1 > m_2$ so, blocks I moves downward.

If block I moves distance x_1 and block II moves distance $2x_2$.

i.e., $x_1 = 2x_2$

Differentiating; we get,

$$a_1 = 2a_2$$

or, $a_2 = \frac{a_1}{2}$ (1)

where, T_1 = Tension in block I and T_2 = Tension in block II.

Motion of block I

$$^+\uparrow \Sigma F_y = ma_y$$

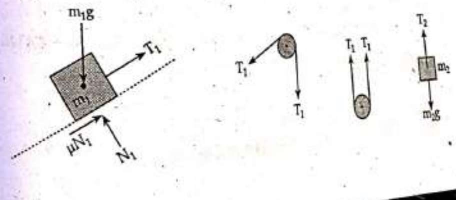
$$N_1 - m_1g\cos\theta = 0$$

or, $N_2 = m_1g\cos\theta$

$$^+\downarrow \Sigma F_x = ma_x$$

or, $m_1g\sin\theta - \mu N_1 - T_1 = m_1a_1$

or, $m_1g\sin\theta - \mu m_1g\sin\theta - T_1 = m_1a_1$ (2)



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Motion of block II

$\Sigma F_y = ma_y$

or, $T_2 - m_2g = m_2a_2 = m_2\frac{a_1}{2}$ (3)

Motion of pulley

$\Sigma F_y = ma_y$

or, $2T_1 - T_2 = 0$ (4)

or, $2 \times m_1 g \sin\theta - \mu m_1 g \sin\theta - \mu m_1 g \cos\theta - m_1a_1 - m_2g + m_2\frac{a_1}{2} = 0$

or, $2 \times m_1 g (\sin\theta - \mu\cos\theta) - m_1a_1 - m_2\left(g - \frac{a_1}{2}\right) = 0$

or, $2 \times 75 \times 9.81 (\sin 30° - 0.15 \times \cos 30°) - 75 \times a_1 - 50\left(9.81 - \frac{a_1}{2}\right) = 0$

$\therefore$ $a_1 = 0.309 \text{ m/s}^2$

$\therefore$ $a_2 = \frac{a_1}{2} = 0.155 \text{m/s}^2$

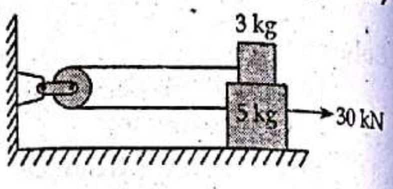
From equation (2); we have,

$T_1 = 249.11$ N

and, from equation (4); we have,

$T_2 = 498.22$ N

13. **In the figure a force of 30 N is applied on the lower block of 5 kg mass, over which another block of 3 kg mass rests. Determine the acceleration of blocks and tension in the string assuming to be inextensible. The coefficient of kinetic friction for all contact surfaces is 0.15.**



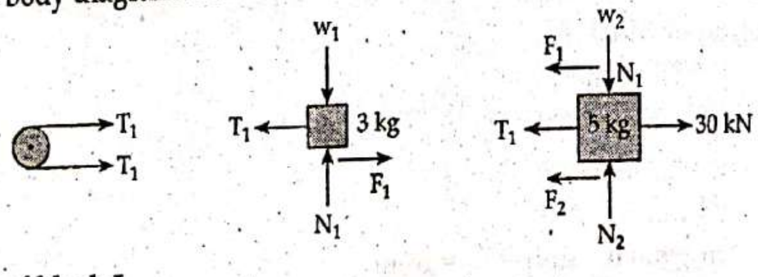
Solution:

Since, pulley is frictionless so, tension in the string is same. Also string is inextensible so, acceleration same.

Let, T_1 = tension in the string

a = acceleration in the string

The free body diagram is shown below as;



Motion of block I

$\overset{+}{\leftarrow} \Sigma F_x = ma_x$

or, $T_1 - F_1 = m_1a$

or, $T_1 = F_1 + m_1a = \mu mg + m_1a = 0.15 \times 3 \times 9.81 + 3a = 3a + 4.4145$

Motion of block II

$\overset{+}{\rightarrow} \Sigma F_x = ma$

or, $-T_1 - F_2 - F_1 + 30 = ma$

or, $-T_1 - (N_1 + w_2)\mu_k - F_1 + 30 = m_1a_1$ $[\because N_2 = N_1 + w_2]$

15. The rectilinear motion of a particle is governed by $a = \frac{-16}{x^3}$, where a is in m/s² and x is in meters. Given that; at time t = 15 x = 2m and v = 2m/s (i) write the equation of motion, (ii) determine the position velocity and acceleration at t = 4 sec.

Solution:

We have given relation that,

$$a = \frac{-16}{x^3} \qquad [\because a = f(x)]$$

or, $$V\frac{dv}{dx} = \frac{-16}{x^3}$$

or, $$V\,dv = -16\,x^{-3}\,dx$$

Integrating; we get,

$$\frac{V^2}{2} = \frac{-16\,x^{-2}}{-2} + c_1$$

or, $$\frac{V^2}{2} = 8x^{-2} + c_1 \qquad (1)$$

To find c_1 at v = 2 m/s and x = 2 m then,

$$\frac{4}{2} = \frac{8}{2^2} + c_1$$

or, $c_1 = 0$

$\therefore$ $$\frac{V^2}{2} = 8x^{-2}$$

or, $v^2 = 16x^{-2}$

or, $$v = \frac{4}{x}$$

Now,

$$\frac{dx}{dt} = \frac{4}{x}$$

or, $x\,dx = 4\,dt$

Again, integrating; we get,

$$\frac{x^2}{2} = 4t + c_2 \qquad (2)$$

To find c_2 at x = 2 m and t = 15 sec. then,

$$\frac{2^2}{2} = 4 \times 1 + c_2$$

or, $c_2 = -2$

Therefore, equation **(2)** becomes,

$$\frac{x^2}{2} = 4t - 2$$

or, $x^2 = 8t - 4$

Thus, required equation of motion is;

$v^2 = 16x^{-2}$

and, $x^2 = 8t - 4$

ii) Position velocity and acceleration at t = 45 sec.; then,

$$x(4) = \sqrt{8 \times 4 - 4} = 5.29\text{ m}$$

$$v(4) = \frac{4}{x} = \frac{4}{5.29} = 0.756\text{ m/s}$$

and, $$a = \frac{-16}{x^3} = \frac{-16}{(5.29)^3} = -0.108\text{ m/s}^2$$

16. The acceleration of a particle is defined by relation a = 0.6 (1 − kv), where, 'k' is a constant knowing that at t = 0 the particles stars from rest at x = 6 m and that v = 6 m/sec when t = 20 sec. Determine; (a) the constant k (b) the position of the particles when v = 7.5 m/s (C) the maximum velocity of the particle.

Solution:

We have given that,

$$a = 0.6\,(1 - kv)$$

or, $$\frac{dv}{dt} = 0.6(1 - kv)$$

or, $$dt = \frac{dv}{0.6(1 - kv)}$$

Integrating both sides and using condition v = 0 m/s when t = 0 sec.; then,

$$\int_0^t dt = \int_0^v \frac{dv}{0.6(1 - kv)}$$

or, $$[t]_0^t = -\frac{1}{0.6k}\left[\ln(1 - kv)\right]_0^v$$

or, $$t = -\frac{1}{0.6k}\ln(1 - kv) \qquad (1)$$

Using t = 20 sec. when v = 6 m/sec.; then,

$$20 = -\frac{1}{0.6k}\ln(1 - k \times 6)$$

a) Solving k = 0.1328 m/s

b) Position when v = 7.5 m/s

$$a = 0.6\,(1 - kv)$$

or, $$\frac{Vdv}{dx} = 0.6(1 - kv)$$

or, $$dx = \frac{Vdv}{0.6(1 - kv)}$$

Integrating both sides and using condition x = 6 m when v = 0 m/s; then,

$$\int_6^x x\,dx = \int_0^v \frac{Vdv}{0.6\,(1 - kv)}dv$$

Now,

$$\frac{V}{1 - kv} = \frac{1}{k}\left(-1 + \frac{1}{1 - kv}\right)$$

so, $$[x]_6^x = \frac{1}{0.6k}\int_0^v \left(-1 + \frac{1}{1 - kv}\right)dv$$

or, $x - 6 = \frac{1}{0.6\,k}\left[-v - \frac{1}{k}\ln(1 - kv)\right]_0^v$

or, $x = 6 - \frac{1}{0.6\,k}\left[V + \frac{1}{k}\ln(1 - kv)\right]$

Using V = 7.5 m/s and k = 0.1328 s/m; then,

$$x = 6 - \frac{1}{0.6 \times 0.1328}\left[7.5 + \frac{1}{0.1328}\ln(1 - 0.1328 \times 7.5)\right] = 434\text{ m}$$

c) Maximum velocity occurs when acceleration is zero.

i.e., $a = 0$

or, $0 = 0.6(1 - kv)$

or, $V = \frac{1}{k}$

$\therefore$ $V_{max} = \frac{1}{0.1328} = 7.53\text{ m/s}^2$

17. The acceleration of a particle is defined by the relation $a = k(1 - e^{-x})$, where, 'k' is a constant. Knowing that; the velocity of the particle is V = 9 m/s when x = − 3 m and that the particle comes to rest at the origin. Determine; (a) the value of 'k' (b) the velocity of the particle when x = − 2 m.

Solution:

We have given that,

$a = k(1 - e^{-x})$

or, $\frac{V\,dv}{dx} = k(1 - e^{-x})$ (1)

or, $V\,dv = k(1 - e^{-x})\,dx$

Integrating and using the condition; we have,

V = 9 m/s when x = − 3 m

and, V = 0 m/s when x = 0 m.

Then,

$$\int_9^0 V\,dv = \int_{-3}^0 k(1 - e^{-x})\,dx$$

or, $\left[\frac{V^2}{2}\right]_9^0 = k[x + e^{-x}]_{-3}^0$

or, $\frac{-9^2}{2} = k[0 + 1 - (-3) - e^3]$

or, $\frac{-81}{2} = -16.0855\,k$

$\therefore$ $K = 2.52$

b) Velocity of particular when x = − 2 m

From equation (1); we get,

$a = k(1 - e^{-x})$

or, $\frac{V\,dv}{dx} = k(1 - e^{-x})$

or, $V\,dv = k(1 - e^{-x})\,dx = 2.52(1 - e^{-x})\,dx$

Integrating with the limit V = 0 when x = 0; then,

$$\int_0^v V\,dv = 2.52\int_0^x (1 - e^{-x})\,dx$$

or, $\frac{V^2}{2} = 2.52[x + e^{-x}]_0^x = 2.52(x + e^{-x} - 1)$

or, $V^2 = 5.04(x + e^{-x} - 1)$

or, $V = \pm 2.24(x + e^{-x} - 1)^{1/2}$

$\therefore$ At x = − 2 m; then,

$V = \pm 2.24(-2 + e^2 - 1)^{1/2}$

or, $V = \pm 4.70$ m/s

Rejecting − ve sign; we have,

$V = 4.70$ m/s

18. The acceleration of point 'A' is defined be the relation $a = -1.8\sin kt$, where, 'a' and 't' are expressed in m/s² and second respectively and k = 3 rad/sec. Knowing that; x = 0 and v = 0.6 m/s when t = 0 determine the velocity and position of point 'A' when t = 0.5 sec.

Solution:

We have given that;

$a = -1.8\sin kt$

$V_0 = 0.6$ m/s (1)

$x_0 = 0$ m

and, k = 3 rad/sec.

Since acceleration is the function of time so,

$a = -1.8\sin kt$

or, $\frac{dv}{dt} = -1.8\sin kt$

or, $dv = (-1.8\sin kt) \times t$

Integrating above equation and taking limit time varies from 0 to t and velocity varies from v_0 to v. Then,

$$\int_{v_0}^{v} dv = \int_0^t -1.8\sin kt\,dt$$

or, $v - v_0 = -1.8\int_0^t \sin kt\,dt$

or, $v - 0.6 = \left[\frac{1.8\cos kt}{k}\right]_0^t$

or, $v - 0.6 = \frac{1.8}{k}(\cos kt - 1)$

or, $v = 0.6 + \frac{1.8}{3}(\cos kt - 1)$

$\therefore$ $v = 0.6\cos kt$ m/s

or, $\frac{dx}{dt} = 0.6 \cos kt$

or, $dx = 0.6 \cos kt \times dt$

Again, integrating and taking limit; we have, $x_0 t_0$ x and 0 to t.

$$\int_{x_0}^{x} dx = 0.6 \int_{0}^{t} \cos kt \, dt$$

or, $x - x_0 = \left| \frac{0.6}{k} \sin kt \right|_0^t$

or, $x - 0 = \frac{0.6}{3} (\sin kt - 0)$

$\therefore$ $x = 0.2 \sin kt$

When t = 0.5 sec.; then,

$kt = 3 \times 0.5 = 1.5$ rad

$V = 0.6 \cos \left(1.5 \times \frac{180°}{\pi}\right) = 0.0424$ m/s

and, $x = 0.2 \sin \left(1.5 \times \frac{180°}{\pi}\right) = 0.1995$ m

19. **The acceleration of point A is defined by the relation a = 200x (1 + kx²) where 'a' and 'x' are expressed in m/s² and meters respectively and 'k' is constant. Knowing that; the velocity of 'A' is 2.5 m/s when x = 0 and 5 m/s when x = 0.15 m determine the value of 'k'.**

Solution:

We have given that,

$a = 200x (1 + kx^2)$

or, $\frac{V\,dv}{dx} = 200x (1 + kx^2)$

or, $V\,dv = 200x (1 + kx^2)\, dx$

Integrating and using limit; we have,

When x = 0;

$V = 2.5$ m/s

When x = 0.15 m, then,

$V = 5$ m/s

$$\int_{2.5}^{5} V dv = \int_{0}^{0.15} (200x + 200kx^3)\, dx$$

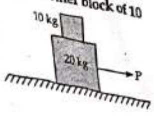
or, $\left| \frac{V^2}{2} \right|_{2.5}^{5} = \left[\frac{200}{2} x^2 + \frac{200}{4} kx^4 \right]_0^{0.15}$

or, $\frac{5^2}{2} - \frac{2.5^2}{2} = 100 \times (0.15)^2 + 50 \times k \times 0.15^4$

or, $9.375 = 2.25 + 0.0253125k$

$\therefore$ $K = 281\ m^{-2}$

A 20 kg block rest on a rough horizontal surface over which another block of 10 kg mass is placed. A horizontal force 'P' is applied to the lower block. Determine the maximum force that can be applied without allowing the upper block to slide backward over the lower block. Assume coefficient of static and dynamic friction to be respectively 0.25 and 0.2 for all contact surface.



Solution:

The F.B.D. of the blocks is shown below. The upper block is also pulled along with the lower block due to friction between the blocks. The friction is sliding friction, while the friction between the lower block and the plane is kinematic in nature due to motion.

Let a be the acceleration of the block,

Upper block

Applying equation of motion; we have,

$\sum F_y = m\, a_y$

$N_1 - W_1 = 0$

or, $N_1 = w_1$

Also,

$\Sigma F_x = m\, a_x$

or, $F_1 = w_1 a$

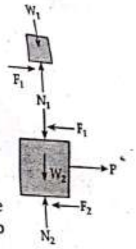


Figure: F.B.D.

We know that as the upper block is about to slide the frictional force reaches maximum value and it is equal to the limiting static friction *i.e.*,

$F_1 = \mu_s N_1 = \mu_s W_1$

Hence, above equation can be written as;

$\mu_s W_1 = \mu_s m_1 g = m_1 a$

$\therefore$ $a = \mu_s g = 0.25 \times 9.81 = 2.45\ m/s^2$

Lower block

Applying equation of motion; we have,

$(^+\uparrow) \sum F_y = m\, a_y$

or, $N_2 - N_1 - W_2 = 0$

$\therefore$ $N_2 = N_1 + W_2$

Also,

$\Sigma F_x = m\, a_x$

or, $P - F_1 - F_2 = m_2 a$

or, $P = F_1 + F_2 + m_2 a$ $\quad [\because F_2 = \mu_k N_2]$

or, $P = \mu_s W_1 + \mu_k (W_1 + W_2) + m_2 a$

$\therefore$ $P = 0.25 \times 10 \times 9.81 + 0.2 \times (10 + 20) \times 9.81 + 20 \times 2.45 = 132.39$ N

21. **The rectangular components of acceleration for a particle are $a_x = 3t$ and $a_y = (30 - 10t)$ where, 'a' in ms^{-2}. If the particle starts from rest at the origin, find the radius of curvature of the path at the instant of 2 sec.**

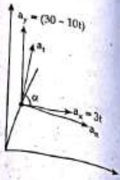
Solution:

Given that;

$a_x = 3t$

$a_y = (30 - 10t)$

$t = 2\,sec$



Now,

$$a_x = \frac{dv_x}{dt}$$

or, $$3t = \frac{dv_x}{dt}$$

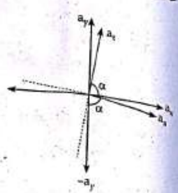
Integrating; we get,

$$v_x = \frac{3t^2}{2} + C_1$$

At $t = 0, v = 0$

$\therefore \quad C_1 = 0$

$$\therefore \quad v_x = \frac{3t^2}{2}$$



Also,

$$a_y = \frac{dv_y}{dt}$$

or, $(30 - 10t)dt = v_y$

Integrating; we get,

$$v_y = 30t - 5t^2 + C_2$$

At initial condition $C_2 = 0$

$\therefore \quad v_y = 30t - 5t^2$

Now, at $t = 2\,sec$

$$v_x = \frac{3 \times 2^2}{2} = 6\,m/s$$

$$v_y = 30 \times 2 - 5 \times 2^2 = 40\,m/s.$$

Also,

$$a_x = 3 \times 2 = 6\,m/s^2$$

$$a_y = 30 - 10 \times 2 = 10\,m/s^2$$

Now,

$$\alpha = \tan^{-1}\frac{v_{2y}}{v_{2x}} = \tan^{-1}\frac{40}{6} = 81.46°$$

Now from figure,

$$a_x = a_x \sin\alpha - a_y \cos\alpha$$

$$= 6 \times \sin 81.46° - 10 \times \cos 81.46° = 4.45\,m/s^2$$

Also,

$$a_x = \frac{v^2}{\rho} = \frac{\left(\sqrt{v_x^2 + v_y^2}\right)^2}{\rho}$$

or, $$\rho = \frac{(6^2 + 40^2)}{4.45} = 367.64\,m$$

$\therefore$ Radius of curvature $(\rho) = 367.64\,m$

For a particle moving rectilinearly, $a = -8x^{-2}$, where, 'a' is the acceleration in m/s^2 and x in metre units. If is known that when $t = 1$ sec., $x = 4$ m and $v = 2$ m/sec. Determine its acceleration when $t = 2$ sec.

Solution:

We have the relation,

$$a = v\frac{dv}{dx} = -8x^{-2}$$

or, $$v\,dv = -8x^{-2}dx$$

Integrating; we have,

$$\frac{v^2}{2} = 8x^{-2} + K_1$$

where, K_1 is a constant.

At $v = 2\,m/sec$, $x = 4$ m, then,

$$K_1 = 0$$

$$\therefore \quad v^2 = 16\,x^{-1}$$

Again,

$$v = \frac{dx}{dt} = \sqrt{16\,x^{-1}} = \frac{dx}{dt}$$

$$\therefore \quad \frac{dx}{dt} = \pm 4\,x^{-\frac{1}{2}} \qquad (1)$$

Case I

Taking positive sign only;

$$\frac{dx}{dt} = 4\,x^{-\frac{1}{2}}$$

$$x^{\frac{1}{2}}dx = 4\,dt$$

Integrating; we have,

$$\frac{2x^{\frac{3}{2}}}{3} = 4t + K_2$$

where, K_2 is a constant.

At $t = 1$ sec, $x = 4$ m, then,

$$K_1 = \frac{4}{3}$$

$$\therefore \quad x^{\frac{3}{2}} = 6t + 2$$

$$x = (6t + 2)^{\frac{2}{3}} \qquad (2)$$

Differentiating equation **(2)**; we get,

$$v = \frac{dx}{dt} = \frac{d}{dt}(6t + 2)^{\frac{2}{3}} = \frac{2}{3}(6t + 2)^{-\frac{1}{3}} \times 6 = 4(6t + 2)^{-\frac{1}{3}}$$

Again differentiating with respect to time; we get,

$$a = \frac{dv}{dt} = \frac{d}{dt}\left\{4(6t + 2)^{-\frac{1}{3}}\right\} = -\frac{4}{3}(6t + 2)^{-\frac{4}{3}} \times 6 = -8(6t + 2)^{-\frac{4}{3}}$$

At $t = 2$ sec.;

$$\text{Acceleration } (a_2) = -8(6t + 2)^{-\frac{4}{3}} = -0.24\,m/s^2$$

Case II

Taking negative sign; we get,

$$x^{\frac{1}{2}}dx = -4\,dt$$

Integrating; we have,

$$\frac{2}{3}x^{\frac{3}{2}} = -4t + K_3$$

where, K_3 is a constant.

At $x = 4$ m and $t = 1$ sec.;

$$\therefore \quad K_3 = \frac{28}{3}$$

Hence,

$$\therefore \quad x^{\frac{3}{2}} = -6t + 14$$

$$x = (-6t + 14)^{\frac{2}{3}}$$

Differentiating above equation with respect to time; we have,

$$v = \frac{dx}{dt} = \frac{d}{dt}\left\{(-6t + 14)^{\frac{2}{3}}\right\} = \frac{2}{3}(-6t + 14)^{-\frac{1}{3}} \times (-6) \qquad (3)$$

$$\therefore \quad v = -4(-6t + 14)^{-\frac{1}{3}}$$

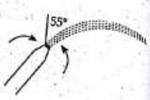
Again differentiating; we get;

$$a = \frac{dv}{dt} = -8(-6t + 14)^{-\frac{4}{3}}$$

At $t = 2$ sec.;

$$a = -8(-6 \times 2 + 14)^{-\frac{4}{3}} = -3.18 \text{ m/s}^2$$

23. **A nozzle discharge a stream of water in the direction shown with an initial velocity of 8 m/sec. Determine the radius of curvature of the stream (a) as it leaves the nozzle (b) at the maximum height of the stream.**



Solution:

Given that;

Initial velocity $(v_0) = 8$ m/sec

We know that,

$$\text{Radius of curvature } (\rho) = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

Now,

$$\frac{dx}{dt} = (v_x)_0 = v_0 \cos(90° - 55°) = 8 \cos 35° = 6 \text{ m/s} \qquad (1)$$

and, $y = (v_y)_0 t - \frac{1}{2}gt^2$

$$y = 4.59\,t - 4.915\,t^2$$

$$\frac{dy}{dt} = 4.59 - 9.81\,t \qquad (2)$$

From equation (1) and (2); we get;

$$\frac{dy}{dt} = \frac{4.59 - 9.81\,t}{6.55} = 0.7 - 1.497\,t$$

and, $$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dt}\right) = \frac{d}{dx}(0.7 - 1.497\,t) = -1.497\frac{dt}{dx} = \frac{-1.497}{\left(\frac{dy}{dx}\right)}$$

$$= \frac{-1.497}{6.55} = -0.228$$

Now,

$$\rho = \left[\frac{\left\{1 + (0.7 - 1.497\,t)^2\right\}^{\frac{3}{2}}}{-0.228}\right]$$

At $t = 0$ sec,

$$\rho = \frac{\left[1 + (0.7)^2\right]}{-0.228} = 7.96 \text{ m}$$

b) At the maximum height;

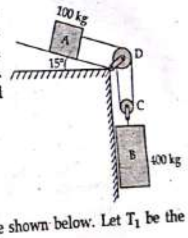
$$v_y = 0$$

i.e., $4.59 - 9.81\,t = 0$

or, $t = 0.4678$ sec.

$$\therefore \quad \rho = \frac{\left[1 + (0.7 - 1.497 \times 0.4678)^2\right]^{\frac{3}{2}}}{0.228} = 4.38 \text{ m}$$

24. **Two blocks shown in figure start from rest. The pulleys are frictionless and having no mass. The kinetic coefficient of friction between the block A and inclined plane is 0.4. Determine the acceleration of each block and tension in each chord.**



Solution:

Given that;

Mass of block A $(M_A) = 100$ kg

Mass of block B $(M_B) = 400$ kg

The free body diagram of block A and B are shown below. Let T_1 be the tension in block A and T_2 be tension in block B.

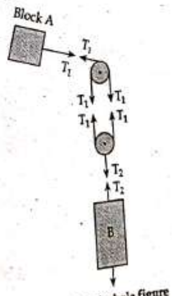


Figure: F.B.D. of whole figure

For Block A

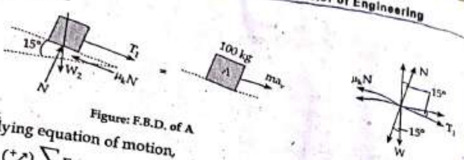


Figure: F.B.D. of A

Applying equation of motion,

$$(^+\nearrow)\sum F_y = m\,a_y$$

or, $N - W\cos 15° = m \times 0$

$\therefore$ $N = 100 \times 9.81 \times \cos 15° = 947.58\ N$

Also,

$$(^+\searrow)\sum F_x = m\,a_x$$

or, $W\sin 15° + T_1 - \mu_k N = ma_A$

or, $9.81 \times 100 \times \sin 15° + T_1 - 0.4 \times 947.58 = 100 \times a_A$

$\therefore$ $T_1 = 100a_A + 125.13$ (1)

For Block B

$$(\uparrow^+)\sum F_y = m\,a_y$$

or, $W - T_2 = ma_B$

or, $400 \times 9.81 - T_2 = 400a_B$

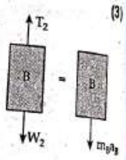
$\therefore$ $T_2 = 3924 - 400a_B$ (2)

Also from F.B.D. of whole figure,

$T_1 + T_1 = T_2$

$\therefore$ $2T_1 = T_2$ (3)

Also, from figure as block 'A' moves through distance x_A then block 'B' moves distance $2x_B$.



i.e., $x_A = 2x_B$

Differentiating twice; we get,

$v_A = 2v_B$

and, $a_A = 2a_B$

$a_B = \frac{a_A}{2}$ (4)

Now, from equation (3); we have,

$2T_1 = T_2$

or, $2(100a_A + 125.13) = 3924 - 400 \times \frac{a_A}{2}$ $\left[\because a_B = \frac{a_A}{2}\right]$

or, $200a_A + 250.26 = 3924 - 200a_A$

$\therefore$ $a_A = 9.18\ m/s^2$

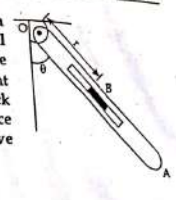
and, $a_B = \frac{a_A}{2} = 4.59\ m/s^2$

Also, from equation (2); we have,

$T_2 = 3924 - 400a_B = 3924 - 400 \times 4.59 = 2088\ N$

and, $T_1 = \frac{T_2}{2} = 1044\ N$

A 1.2 kg block 'B' slides without friction inside a slot cut in arm OA which rotates in vertical plane. The motion of the rod is defined by the relation $\ddot{\theta} = 10\ rad/s^2$ constant. At the instant when $\theta = 45°$, $r = 2.4\ m$ and the velocity of block are zero. Determine at this instant (a) the force exerted on the block by the arm. (b) the relative acceleration of the block with respect to arm.



Solution:

Given that;

$\theta = 45°$

$e = 2.4\ m$

$\ddot{\theta} = 10\ rad/s^2$

Since, at above condition velocity of block is zero. So,

$v_r = \dot{r} = 0$

$\therefore$ $v_\theta = r\dot{\theta} = 0$

Now,

$$(\nearrow_+)\sum F_\theta = m\,a_\theta$$

or, $N - W\cos 45° = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

or, $N = W\cos 45° + \frac{W}{g}(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

$= 1.2 \times \cos 45° + \frac{1.2}{9.81}(2.4 \times 10 + 0)$

$= 3.78$ Newton

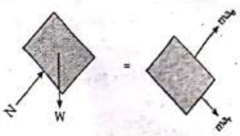


Figure: F.B.D. of Block B

Also,

$$(\searrow^+)\sum F_r = m\,a_r$$

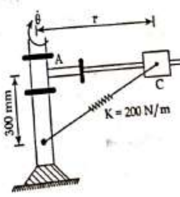
or, $W\sin 45° = m\,a_r = \frac{W}{g}(\ddot{r} + r\dot{\theta}^2)$

or, $\ddot{r} = g\sin 45° + r\dot{\theta}^2$

$= 9.81 \times \sin 45° + 0 = 6.94\ m/s^2$

$\therefore$ $a_B/rad = 6.94\ m/s^2$

26. **If the position of the 3 kg collar C on the smooth rod AB is held r = 720 mm, determine the constant angular velocity $\dot{\theta}$ at which the mechanism is rotating about the vertical axis. The spring has an outstretched length 400 mm. Neglect the mass of rod and size of collar.**



Solution:

Given that;

Spring constant (k) = 200 N/m

Mass of collar (m) = 3 kg

Outstretched length = 40 mm

Now,

Force in the spring $(F_{SP}) = K \times S = 200 \times \left[\sqrt{0.72^2 + 0.3^2} - 0.4\right]$

The free body diagram of the body is shown below.

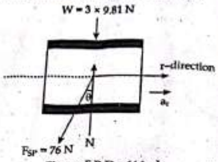


Figure: F.B.D. of block

Here, a_r is assumed to be directed towards the +ve r axis.

Equation of motion

$$\overset{+}{\rightarrow} \sum F_r = m\, a_r$$

or, $-76 \cos\theta = 3\, a_r$

or, $-76 \cos 22.62° = 3\, a_r$

or, $a_r = -23.38\ \text{m/s}^2$

Kinematics

Since $r = 0.72$m is constant

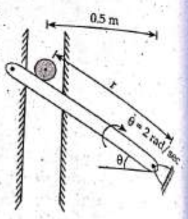
so, $\dot{r} = \ddot{r} = 0$

$\therefore \quad a_r = \ddot{r} - r\dot{\theta}^2$

or, $-23.38 = 0 - 0.72 \times \dot{\theta}^2$

$\therefore \quad \dot{\theta} = 5.70\ \text{rad/sec.}$

27. **The ball has a mass of 1 kg is confined to more along the smooth vertical slot due to the rotation of the smooth arm OA. Determine the force of rod on the ball and the normal force of the slot on the ball when θ = 30°. The rod is rotating with a constant angular velocity $\dot{\theta} = 3$ rad/sec. Assume the ball constants only one side of the slot of any instant.**



Solution:

Given that;

$$\dot{\theta} = 3\ \text{rad/sec and } \theta = 30°$$

$\therefore \quad \ddot{\theta} = 0\ \text{rad/sec}$

Now, from figure,

$$\cos\theta = \frac{0.5}{r}$$

$$r = \frac{0.5}{\cos\theta} \tag{1}$$

Now,

At $\theta = 30°$,

$$r = \frac{0.5}{\cos\theta} = 0.5774\ \text{m}$$

Differentiating equation (1) with respect to time 't'; we get,

$$\frac{dr}{dt} = \frac{d}{d\theta}\left(\frac{0.5}{\cos\theta}\right) \times \frac{d\theta}{dt}$$

$$\dot{r} = \frac{0.5 \sin\theta}{\cos^2\theta} \times \dot{\theta} = 0.5 \tan\theta \cdot \sec\theta \times \dot{\theta}$$

Also,

At $\theta = 30°$ and $\dot{\theta} = 3$ rad/sec.; then,

$$\dot{r} = 0.5 \tan 30° \cdot \sec 30° \times 3 = 1\ \text{m/s}$$

Also,

$$\ddot{r} = \left[\tan\theta \times \sec\theta \times \ddot{\theta} + \left(\sec^3\theta + \tan^2\theta \cdot \sec\theta\right) \times \dot{\theta}^2\right]$$

At, $\theta = 30°$, $\dot{\theta} = 3$ rad/sec and $\ddot{\theta} = 0$ then,

$$\ddot{r} = 8.66\ \text{m/s}^2$$

Applying equations,

$$a_r = \ddot{r} - r\dot{\theta}^2$$
$$= 8.66 - 0.5774 \times 3^2$$
$$= 3.464\ \text{m/s}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$
$$= 0.5774 \times 0 + 2 \times 3 \times 1$$
$$= 6\ \text{m/s}^2$$

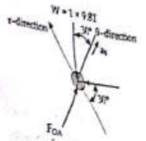


Figure: Free body diagram of body

Equation of motion

$$\overset{+}{\nwarrow} \sum F_r = m a_r$$

or, $N \cos 30° - 1 \times 9.81 \times \sin 30° = 1 \times 3.464$

$\therefore \quad N = 9.66\ \text{N}$

and, $(\overset{+}{\nearrow}) \sum F_\theta = m\, a_\theta$

or, $F_{OA} - 1 \times 9.81 \times \cos 30° - 9.66 \sin 30° = 1 \times 6$

$\therefore \quad F_{OA} = 19.3\ \text{N}$

28. **The motion of a particle is defined by the equations $x = \left[\frac{(t-4)^3}{6}\right] + t^2$ and $y = \frac{t^3}{6} - \frac{(t-1)^3}{4}$; where, x and y are expressed in metres and t is expressed in seconds. Determine: (a) the magnitude of the smallest velocity reached by the particle, (b) the corresponding time, position and direction of the velocity.**

Solution:

a) Given that;

$$x = \frac{1}{6}(t-4)^3 + t^2$$

and, $y = \frac{1}{6}t^3 - \frac{1}{4}(t-1)^2$

By differentiation above relations; we get,

$$v_x = \frac{dx}{dt} = \frac{1}{2}(t-4)^2 + 2t = \frac{1}{2}t^2 - 2t + 8$$

$$v_y = \frac{dy}{dt} = \frac{1}{2}t^2 - \frac{1}{2}(t-1) = \frac{1}{2}t^2 - \frac{1}{2}t + \frac{1}{2}$$

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$$\frac{dv_x}{dt} = t - 2$$

and, $$\frac{dv_y}{dt} = t - \frac{1}{2}$$

Magnitude of velocity $(v) = \sqrt{v_x^2 + v_y^2}$

or, $v^2 = v_x^2 + v_y^2$

Note

V is minimum when v^2 is minimum.

Differentiating and setting equal to zero; we have,

or, $$2V\frac{dv}{dt} = 2v_x\frac{dv_x}{dt} + 2v_y\frac{dv_y}{dt} = 0$$

or, $$v_x\frac{dv_x}{dt} + 2v_y\frac{dv_y}{dt} = 0$$

or, $$\left(\frac{1}{2}t^2 - 2t + 8\right)(t-2) + \left(\frac{1}{2}t^2 - \frac{1}{2}t + \frac{1}{2}\right)\left(t - \frac{1}{2}\right) = 0$$

or, $$t^3 - 3.75t^2 + 12.75t - 16.25 = 0$$

The only real root of the cubic equation is $t = 1.757$ sec.

The corresponding values of v_x and v_y are;

$$v_x = \frac{1}{2}t^2 - 2t + 8 = \frac{1}{2}\times(1.757)^2 - (2\times 1.757) + 8 = 6.03 \text{ m/sec.}$$

$$v_y = \frac{1}{2}t^2 - \frac{1}{2}t + \frac{1}{2} = \frac{1}{2}\times(1.757)^2 - \left(\frac{1}{2}\times 1.757\right) + \frac{1}{2} = 1.165 \text{ m/sec.}$$

$\therefore$ $$v_{min} = \sqrt{(6.03)^2 + (1.165)^2} = 6.14 \text{ m/sec.}$$

b) Here,

Time (t) = 1.757 sec.

$$x = \frac{1}{6}(t-4)^3 + t^2 = \frac{1}{6}(1.757-4)^3 + (1.757)^2 = 1.206 \text{ m}$$

$$y = \frac{1}{6}t^3 - \frac{1}{4}(t-1)^2 = \frac{1}{6}(1.757)^3 - \frac{1}{4}(1.757-1)^2 = 0.761 \text{ m}$$

and, $$\tan\theta = \frac{V_y}{V_x} = \frac{1.165}{6.03} = 0.1932$$

$\therefore$ $$\theta = \tan^{-1}(0.1932) = 10.9°$$

29. The motion of a particle is defined by the relation $x = 2t^3 - 12t^2 - 72t - 80$; where, x and t are expressed in metres and seconds respectively. Determine: (a) when the velocity is zero, (b) the velocity, the acceleration and the total distance travelled when $x = 0$.

Solution:

Given that;

The position is;

$$x = (2t^3 - 12t^2 - 72t - 80) \text{ m}$$

By differentiating; we get,

$$\text{Velocity } (v) = \frac{dx}{dt} = 6t^2 - 24t - 72 \text{ m/sec.}$$

and, $$\text{Acceleration } (a) = \frac{dv}{dt} = 12t - 24 \text{ m/s}^2$$

a) When $v = 0$; then,

$$6t^2 - 24t - 72 = 0$$

On solving; we get,

$$t = -2S$$

and, $6t = 6S$

Neglecting the negative time; then,

For $0 \le t \le 6S$; v is negative x is decreasing.

For $t > 6S$; v is positive x is increasing.

Minimum value of x occurs when t = 6 sec.;

$\therefore$ $$x_{min} = [2\times(6)^3] - [12\times(6)^2] - (72\times 6) - 80 = -512 \text{ m}$$

and, when $t = 0$; then,

$$x_0 = -80 \text{ m}$$

b) Distance travelled over $0 \le t \le 6S$

$\therefore$ $$d_1 = |x_{min} - x_0| = |-512 - (-80)| = 432 \text{ m}$$

When $x = 0$; then,

$$2t^3 - 12t^2 - 72t - 80 = 0$$

On solving; we get,

$$t = 10 \text{ sec.}$$

and, $t = -2$ sec. (twice)

Reject the negative roots; we have,

When $t = 10$ sec.; then,

$$v = 6t^2 - 24t - 72 = [6\times(10)^2] - (24\times 10) - 72$$

$\therefore$ $$v = 288 \text{ m/sec.}$$

and, $$a = (12\times 10) - 24 = 96 \text{ m/s}^2$$

Now, distance travelled over $6S \le t \le 10S$;

$$d_2 = |x_f - x_{min}| = |0 - (-512)| = 512 \text{ m}$$

$\therefore$ Total distance travelled $(d) = d_1 + d_2 = 432 + 512 = 944$ m

30. The motion of a particle is defined by the relation $x = 2t^3 - 18t^2 + 48t - 16$; where, x and t are expressed in millimeters and second respectively. Determine: (a) when the velocity is zero, (b) the position and the total distance travelled when the acceleration is zero.

Solution:

The given relation is;

$$\text{Position } (x) = (2t^3 - 18t^2 - 48t - 16) \text{ mm}$$

By differentiating; we get,

$$\text{Velocity } (v) = \frac{dx}{dt} = (6t^2 - 36t + 48) \text{ mm/sec.}$$

$$\text{Acceleration } (a) = \frac{dv}{dt} = (12t - 36) \text{ mm/s}^2$$

a) When $v = 0$; then,

$$6t^2 - 36t + 48 = 0$$

On solving; we get,

$$t = 2 \text{ sec.}$$

and, $t = 4$ sec.

For $0 \le t \le 2S$; v is positive and x is increasing

For $2S \le t \le 4S$; v is negative and x is decreasing

For $t \ge 4S$; v is positive and x is increasing

b) When $a = 0$; then,

$$12t - 48 = 0$$

or, $t = 3$ sec.

When $t = 3$ sec.; then,

$$x_3 = 2 \times (3)^3 - 18 \times (3)^2 - 48 \times 3 - 16 = 20 \text{ mm}$$

When $t = 0$; then,

$$x_0 = -16 \text{ mm}$$

When $t = 2$ sec.; then,

$$x_2 = 2 \times (2)^3 - 18 \times (2)^2 - 48 \times 2 - 16 = 24 \text{ mm}$$

Distance travelled over $0 \le t \le 2S$ is;

$$d_1 = |x_2 - x_0| = |24 - (-16)| = 40 \text{ mm}$$

Distance travelled over $2S \le t \le 4S$ is;

$$d_2 = |x_3 - x_2| = |20 - 24| = 4 \text{ mm}$$

$\therefore$ Total distance travelled $(d) = d_1 + d_2 = 40 + 4 = 44$ mm

31. The velocity of a particle travelling along a straight line is $v = (3t^2 - 6t)$ m/sec. If $S = 4$ m when $t = 0$, determine the position of the particle $t = 4$ sec. What is the total distance travelled during the time interval $t = 0$ to $t = 4$ sec.? Also, what is the acceleration when $t = 4$ sec.?

Solution:

The position of the particle can be determined by integrating the kinematic equation $dS = v\,dt$ using the initial condition $S = 4$ m when $t = 0$ sec.; then,

$$dS = V\,dt$$

or, $\int_4^S dS = \int_0^t (3t^2 - 6t)\,dt$

or, $|S|_4^S = |t^3 - 3t^2|_0^t$

or, $S - 4 = t^3 - 3t^2$

$\therefore$ $S = (t^3 - 3t^2 + 4)$ m

When $t = 4$ sec.; then,

$$S_4 = (4)^3 - [3 \times (4)^2] + 4 = 20 \text{ m}$$

The velocity of the particle changes direction at the instant when it is momentarily brought to rest. Thus,

$$v = 3t^2 - 6t = 0$$

or, $t(3t - 6) = 0$

$\therefore$ $t = 0$

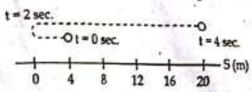
and, $t = 2$ sec.

The position of the particle at $t = 0$ and $t = 2$ sec. is;

$$S_0 = 0 - [3 \times (0)^2] + 4 = 4 \text{ m}$$

$$S_2 = (2)^3 - [3 \times (2)^2] + 4 = 0 \text{ m}$$

Using the above result, the path of the particle shown in the figure is plotted.



From the figure; we have,

$$S_{Tot} = 4 + 20 = 24 \text{ m}$$

Now,

$$\text{Acceleration (a)} = \frac{dv}{dt} = \frac{d}{dt}(3t^2 - 6t)$$

$\therefore$ $a = (6t - 6)$ m/s^2

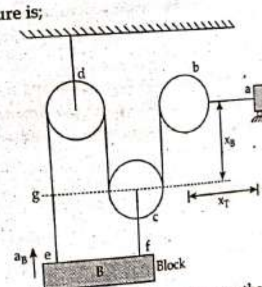
When $t = 2$ sec.; then,

$$|a|_{t=2\text{sec.}} = (6 \times 2) - 6 = 6 \text{ m/s}^2$$

32. A trolley resting on a horizontal plane starts from rest and is moved to the right with a constant acceleration of 0.18 m/s^2. Determine: (a) acceleration of the block B connected to the trolley, (b) velocities of the trolley and block after a time of 4 sec. and the distance moved by each of them.

Solution:

The given figure is;



The length of string abcde connecting to the trolley, T and block B is constant. This is to be expressed in terms of the distance of the connected bodies (e.g.; trolley and block) from some fixed points (e.g.; bd)

i.e., $x_T + 3x_B + cf + cg = \text{Constant}$ (1)

It may be noted that the motions of centre of pulley C and block B are identical, hence expressed in terms of x_B otherwise distance df and eg remains constant.

Giving an increment; we get,

$$\Delta x_T + 3\Delta x_B = 0$$ (2)

Differentiating equation (1) with respect to time; we get,

$$\frac{dx_T}{dt} + 3\frac{dx_B}{dt} = 0$$

or, $V_T + 3V_B = 0$

Differentiating again; we have,

$$\frac{d^2x_T}{dt^2} + 3\frac{d^2x_B}{dt^2} = 0 \quad (3)$$

or, $a_T + 3a_B = 0$

Acceleration of the block is given by the equation (4) gives;

$$a_T = 0.18\ m/s^2 \quad (4)$$

$\therefore \quad a_B = -\frac{a_T}{3} = \frac{0.18}{3} = 0.06\ m/s^2$ [negative sign indicates direction]

Velocity of the trolley 4 seconds after starting from rest is given by;

$$v = u + at$$

$\therefore \quad v_T = 0 + 018 \times 4 = 0.72\ m/sec.$

Velocity of block is given by the equation (3) is;

$$v_B = -\frac{v_T}{3} = \frac{0.72}{3} = 0.24\ m/s\ (\uparrow)$$

Distance moved by trolley in 4 seconds is given by;

$$S = ut + \frac{1}{2}at^2$$

$\therefore \quad S_T = 0 + \frac{1}{2} \times 0.18 \times (4)^2 = 1.44\ m$

Distance moved by the block is given by the equation (2) is;

$$S_B = -\frac{S_T}{3} = \frac{1.44}{3} = 0.48\ m$$

33. The motion of a particle is defined by the equations $x = (2t + t^2)$ m and $y = (t^2)$ m; where, t is in seconds. Determine the normal and tangential components of the particle's velocity and velocity and acceleration when $t = 2$ sec.

Solution:

The given particle equations are;

$$x = (2t + t^2)\ m$$

and, $y = (t^2)\ m$

$\therefore \quad r = x\vec{i} + y\vec{j} = (2t + t^2)\vec{i} + (t^2)\vec{j} \quad (1)$

To determine velocity differentiation equation (1) with respect to t; we get,

$$v = \frac{dr}{dt} = [(2 + 2t)\vec{i} + (2t)\vec{j}]\ m/sec. \quad (2)$$

When $t = 2$ sec.; then,

$$v = [(2 + (2 \times 2))\vec{i} + (2 \times 2)\vec{j}] = 6\vec{i} + 4\vec{j}$$

$\therefore \quad v = \sqrt{(6)^2 + (4)^2} = 7.21\ m/sec.$

Since, the velocity is always directed tangent to the path;

$$v_x = 0$$

and, $v_T = 7.21\ m/sec.$

The velocity, V makes an angle $\theta = \tan^{-1}\left(\frac{4}{6}\right) = 33.69°$ with x-axis.

Acceleration

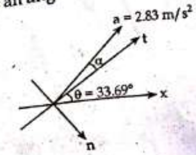
To determine acceleration differentiating equation (2); we get,

$$a = \frac{dv}{dt} = (2\vec{i} + 2\vec{j})\ m/s^2$$

Then,

$$a = \sqrt{(2)^2 + (2)^2} = 2.83\ m/s^2$$

The acceleration, a makes an angle $\phi = \tan^{-1}\left(\frac{2}{2}\right) = 45°$ with the x-axis.



From the figure; we have,

$$\alpha = 45° - 33.69° = 11.31°$$

$\therefore \quad a_n = a \sin\alpha = 2.83 \sin(11.31°) = 0.555\ m/s^2$

and, $a_t = a\cos\alpha = 2.83\cos(11.31°) = 2.78\ m/s^2$

34. The position of a particle is defined by $r = (5\cos 2t\vec{i} + 4\sin 2t\vec{j})$ m; where, t is in seconds and the arguments for the sine and cosine are given in radians. Determine the magnitudes of the velocity and acceleration of the particle when $t = 1$ sec. Also prove that the path of the particle is elliptical.

Solution:

The given position of particle is;

$$r = 5\cos 2t\vec{i} + 4\sin 2t\vec{j} \quad (1)$$

The velocity can be obtained by differentiating equation (1); we get,

$$v = \frac{dr}{dt} = -10\sin 2t\vec{i} + 8\sin 2t\vec{j}$$

When $t = 1$ sec.;

$$v = -10\sin(2 \times 1)\vec{i} + 8\sin(2 \times 1)\vec{j} = -9.093\vec{i} - 3.329\vec{j}$$

$$\text{Magnitude of velocity } (v) = \sqrt{v_x^2 + v_y^2} = \sqrt{(-9.093)^2 + (-3.329)^2} = 9.68\ m/sec.$$

$$\text{Acceleration } (a) = \frac{dv}{dt} = (-20\cos 2t\vec{i} - 16\sin 2t\vec{j})\ m/s^2$$

When $t = 1$ sec.;

$$(a) = -20\cos(2 \times 1)\vec{i} - 16\sin(2 \times 1)\vec{j} = (8.323\vec{i} - 14.549\vec{j})\ m/s^2$$

Thus, the magnitude of the acceleration is;

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(8.323)^2 + (-14.549)^2} = 16.8\ m/s^2$$

Travelling path

Here,

$$x = 5\cos 2t$$

and, $y = 4\sin 2t$

Now,

$$\frac{x^2}{25} = \cos^2 2t \quad (1)$$

and, $\frac{y^2}{16} = \sin^2 2t \quad (2)$

Adding equation (1) and (2); we get,

$$\frac{x^2}{25} + \frac{y^2}{16} = \cos^2 2t + \sin^2 2t$$

$\therefore \quad \frac{x^2}{25} + \frac{y^2}{16} = 1$; which is the equation of ellipse.

35. The velocity of a particle is $v = [3i + (6 - 2t)j]$ m/sec.; where, t is in seconds. If $r = 0$ when $t = 0$ determine the displacement of the particle during the time interval $t = 1$ sec. to $t = 3$ sec..

Solution:

The position, r of the particle can be determined by integrating the kinematic equation $dr = v\,dt$. Using the initial condition $r = 0$ at $t = 0$ as the integration limit. Thus,

$$dr = v\,dt$$

or, $\int_0^r dr = \int_0^t [3i + (6 - 2t)j]\,dt$

$\therefore \quad r = [3ti + (6t - t^2)j]$

When $t = 1$ sec. and $t = 3$ sec.

$$r|_{t=1\,sec.} = (3 \times 1)i + \{(6 \times 1) - (1)^2\}j = (3i + 5j)\ m/sec.$$

$$r|_{t=3\,sec.} = (3 \times 3)i + \{(6 \times 3) - (3)^2\}j = (9i + 9j)\ m/sec.$$

Thus, displacement of the particle is;

$$\Delta r = r|_{t=3\,sec.} - r|_{t=1\,sec.} = (9i + 9j) - (3i + 5j) = (6i + 4j)\ m$$