

Exam.	Regular		
Level	BE	Full Marks	80
Programme	All except BAR	Pass Marks	32
Year / Part	1 / 1	Time	3 hrs.

Subject: - Engineering Mathematics I (SH 401)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. If $y=(x^2-1)^n$, then prove that: $(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$ [5]
2. Assuming the validity of expansion, expand $\log(1+x)$ by using Maclaurin's theorem. [5]

3. Give an example of indeterminate form. Evaluate: $\lim_{x \rightarrow 0} (\cot x)^{\frac{1}{\log x}}$ [5]

4. Find the asymptote of the curve: $(x^2 - y^2)^2 - 2(x^2 + y^2) + x - 1 = 0$ [5]

5. Find the radius of curvature for the curve $r^m = a^m \cos m\theta$. [5]

OR

Find the pedal equation of the following curves $y^2 = 4a(x+a)$. [5]

6. Evaluate: $\int_0^1 \frac{\log(1+x)}{(1+x^2)} dx$ [5]

7. Evaluate by using the rule of differentiation under the sign of integration:

$$\int_0^\infty \frac{\log(1+a^2x^2)}{1+b^2x^2} dx \quad [5]$$

8. Define Gamma function. Use it to prove: $\int_0^{\pi/8} \cos^3 4x dx = \frac{1}{6}$ [5]

9. Find the area of a loop of the curve: $a^2y^2 = a^2x^2 - x^4$ [5]

OR

Prove that the volume and surface area of a sphere of radius 'a' is $\frac{4}{3}\pi a^3$ and $4\pi a^2$ respectively. [5]

10. Solve: $\frac{dy}{dx} + \frac{y}{x} \log y = \frac{y}{x^2} (\log y)^2$ [5]

11. Find the general solution of the differential equation $y = (1+p)x + ap^2$. [5]

12. Solve: $(D^2+3D+2)y = e^{2x} \sin x$ [5]

13. Solve: $(x^2D^2 - 2)y = x^2 + \frac{1}{x}$

OR

A certain culture of bacteria grows at rate proportional to its size. If the size doubles in 4 days, find the time required for the culture to increase to 10 times to its original size. [5]

14. Through what angle must the axes be rotated to remove the term containing xy in $11x^2 + 4xy + 14y^2 = 5$. [5]

15. Prove that: $2x^2 + 3y^2 - 4x - 12y + 13 = 0$ represents equation of ellipse. Find its center, length of axes, eccentricity, and directrices of ellipse. [5]

16. Show that the line $x \cos \alpha + y \sin \alpha = p$ will be a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if

$$a^2 \cos^2 \alpha - b^2 \sin^2 \alpha = p^2. \quad [5]$$