

TRIBHUVAN UNIVERSITY
 INSTITUTE OF ENGINEERING
Examination Control Division
 2078 Chaitra

Exam.	Regular		
Level	BE	Full Marks	80
Programme	BEL,BEX,BCT BEI,BGE	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics (SH 551)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.



1. Define an analytic function $f(z)$ of complex variable z at a point. If $f(z) = u(x,y) + i v(x,y)$ is analytic, show that $u_x = v_y$ and $u_y = -v_x$. [1+4]
2. Define conformal mapping. Find the linear transformation which maps the points $z=0, 1, \infty$ in to the points $w = -3, -1, 1$ respectively. [5]
3. State and proof Cauchy's Integral theorem. [5]
4. Obtain the Taylor's series expansion of the complex function $f(z) = \frac{z+1}{(z-3)(z-4)}$ about the center $z = 2$ up to four term. [5]
5. State Cauchy residue theorem. Apply it to evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$ where C is the circle $|z| = \frac{3}{2}$. [1+4]
6. Evaluate integrals $\int_0^\pi \frac{1}{3+2\cos\theta} d\theta$ by contour integration. [5]
7. If $x(t) = 0$ for $t < 0$, $Z[x(t)] = X(z)$ for $t \geq 0$, then prove that $Z[e^{-at} x(t)] = X(ze^{aT})$. [5]
8. Obtain the Z- transform of (i) te^{-at} (ii) $\sin at$ [2.5+2.5]
9. Obtain the inverse Z- transform of $X(z) = \frac{(1-e^{-aT})z}{(z-1)(z-e^{-aT})}$ where T is the sampling period. [5]
10. Solve the difference equation $y_{n+2} - 4y_{n+1} + 4y_n = 0$ with given condition $y_0 = 0, y_1 = 1$. [5]
11. A tightly stretched string with fixed ends $x = 0$ and $x = \ell$ is initially in position given by $u(x,0) = u_0 \sin^3 \frac{\pi x}{\ell}$. If it is released from rest in this position, find the displacement at any time t at any distance x from one end. [10]
12. Derive one dimensional heat equation and solve it completely. [10]
13. Obtain the fourier sine and cosine integral of $f(x) = x$ for $0 < x < a$,
 $= 0$ for $x > a$. [5]
14. Find the fourier cosine transform of $f(x) = e^{-x}, x > 0$ and hence parseval's identity, show that $\int_0^\infty \frac{1}{(1+x^2)^2} dx = \frac{\pi}{4}$. [5]

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1. a) State Cauchy-Riemann equation in polar form. Prove that $f(z) = |z|$ is not an analytic function. [1+4]
- b) Find the linear transformation which maps the points $z = 0, 1, \infty$ into the points $w = -3, -1, 1$ respectively. Find also fixed point of the transformation. [4+1]
2. a) State and prove Cauchy's Integral Formula. [1+4]
- b) Find the Laurent's series of $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$ in the region $2 < |z| < 3$. [5]
3. a) State Cauchy Residue theorem and hence evaluate the integral $\int_C \frac{z-1}{(z+1)^2(z-2)} dz$ where $C : |z-i| = 2$. [1+4]
- b) Using counter Integration, evaluate $\int_0^{2\pi} \frac{1}{2 + \cos \theta} d\theta$ in the complex plane. [5]
4. a) State and prove initial value theorem of z-transform. [5]
- b) Find the z-transform of the following sequence for $t \geq 0$. [2.5+2.5]
 - (i) te^{-at}
 - (ii) $\sin at$
5. a) Find the inverse z-transform of the function $\frac{2z^3 + z}{(1-z)^2(z-1)}$. [5]
- b) Solve the difference equation $x(k+2) - 4x(k+1) + 4x(k) = 0$ with conditions $x(0) = 1, x(1) = 0$. [5]
6. Derive one dimensional heat equation and solve it completely. [5+5]
7. A string is stretched and fastened to two points apart. Motion is started by displacing the string in the form $u(n, 0) = u_0 \sin \frac{\pi x}{l}$ from which it is released at time $t=0$. Show that the displacement at any point at a distance x from one end at a time t is given by $u(x, t) = u_0 \sin \frac{\pi x}{l} \cos \frac{\pi c t}{l}$. [10]
8. a) Define the complex form of Fourier integral of a given function with usual notation. Find the Fourier integral representation of the function $f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin w}{w} dw$. [1+3+1]
- b) Find the Fourier sine transformation of $f(x) = e^{-|x|}$ and hence evaluate the integral $\int_0^\infty \frac{s \sin sx}{s^2 + 1} ds$. [5]

TRIBHUVAN UNIVERSITY
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 2076 Baisakh

Exam.	Back		
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Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics (SH 551)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
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1. Obtain polar form Cauchy-Riemann equations for function of complex variable. [5]
2. Find the linear fractional transformation which maps the points $z=0, 1, \infty$ into the points $w= -3, -1, 1$ respectively. [5]
3. Define Complex integration. How does it differ from real integration? Derive Cauchy integral formula for function $f(z)$. [1+1+3]
4. Define Laurent's Series for the function of complex variable. Obtain Taylor's series for function [1+4]

$$f(z) = \frac{z}{z^2 + 4} \text{ about } z=i$$

5. State Cauchy residue theorem. Apply it to evaluate $\oint_c \tan z dz$, where c is the region $|z|=2$. [1+4]
 6. Evaluate integral $\int_0^{2\pi} \frac{d\theta}{a + \sin \theta}$; $a > 1$ by contour integration in complex plane. [5]
 7. Obtain z-transform of $\sin \omega t$ and hence obtain z-transform of $e^{at} \sin \omega t$. [3+2]
 8. Obtain the inverse z-transform of $X(z) = \frac{z^2}{(z-1)^2(z-e^{-aT})}$ [5]
 9. State and prove shifting to the right theorem for z-transform. [5]
 10. Solve the difference equation: [5]
- $x(k+2) - x(k+1) + 0.25x(k) = u(k)$ where $x(0)=1$ and $x(1)=2$ and $u(k)$ is a unit step function; by z-transform method.

11. Find Fourier integral of the function [5]

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ e^{-x} & \text{if } x > 0 \end{cases}$$

12. Find the Fourier Sine transform of e^{-x} , $x \geq 0$ and hence show that $\int_0^{\infty} \frac{x^2}{(1+x^2)^2} dx = \frac{\pi}{4}$ [5]
13. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ with the given boundary conditions; $u(0,t)=0$, $u(\pi,t)=0$, $u(x,0)=0$ and $\left(\frac{\partial u}{\partial t} \right)_{t=0} = 3(Lx - x^2)$. [10]
14. Derive one dimensional heat equation and solve it completely. [10]

TRIBHUVAN UNIVERSITY
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2076 Bhadra

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Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics (SH 551)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt **All** questions.
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1. a) If $u = (x-1)^3 - 3xy^2 + 3y^2$, then find v and construct an analytic function $f(z) = u + iv$. [5]
- b) Define conformal mapping. Find mobius transformation which maps the points $z = 0, 1, \infty$ into the points $w = -3, -1, 1$ respectively. [1+4]
2. a) Define Complex integral. Using Cauchy's integral formula evaluate,

$$\int_C \frac{\sin z}{z^6} dz \text{ where } C : |z|=1. \quad [1+4]$$
- b) Obtain the Laurrent's Series expansion of $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$ in the region $2 < |z| < 3$ [5]
3. a) Evaluate $\int_0^{2\pi} \frac{d\theta}{1 + \frac{1}{2}\cos\theta}$, applying contour integration in the complex plane. [5]

 b) Define the pole of order m of a function of a complex variable. Find the residue of $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ at its poles. [1+4]
- a) Obtain the Fourier integral of the function $f(x) = \begin{cases} 1, & \text{for } |x| < 1, \\ 0, & \text{for } |x| > 1 \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$ [3+2]

 b) State and prove the convolution theorem for Fourier transform. [5]
- a) Find the z-transform of $x(t) = \text{Sin} \omega t$, for $t \geq 0$. [5]

 b) Find the inverse z-transform of:
 - $\frac{z^2}{z^2 - 3z + 2}$ by partial fraction method [2]
 - $\frac{z}{(z+2)^2(z-1)}$ by inversion integral method [3]
- a) State and prove shifting to the right theorem for z-transform. [5]

 b) Solve the difference equation $x(k+2) + 3x(k+1) + 2x(k) = 0$, with the conditions $x(0) = 0, x(1) = 1$ [5]
- Solve the wave equation for a tightly stretched string of length L fixed at both ends if the initial deflection is $u(x, 0) = Lx - x^2$ and the initial velocity is zero. [10]
- Change the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ into polar form. [10]

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- ✓ Attempt All questions.
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1. Define harmonic function of complex variable. Show that $u(x, y) = y^3 - 3x^2y$ is harmonic and find corresponding analytic function. [1+4]
2. Define conformal mapping for function of complex variable. Show that function of complex variable $w = iz$ is transformed through an angle $\frac{\pi}{2}$ in w-plane. [1+4]
3. State and prove Cauchy's integral theorem. [5]
4. Define Laurent's Series for the function of complex variable. Find Laurent's series of the function $f(z) = \frac{z}{(z+2)(z+3)}$ in the region $2 < |z| < 3$. [1+4]
5. Define pole of order m for function of complex variable. Find residues of $f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2 + 1)}$ at its poles. [1+4]
6. Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + 1)(x^2 + 4)} dx$ by contour integration in the complex plane. [5]
7. Find the Z-transform of:
 - i) $t^2 e^{at}$
 - ii) $e^{-at} \cos wt$
8. Find the inverse Z-transform of:
 - i) $X(z) = \frac{2z^2 - 5z}{(z-2)(z-3)}$ (By partial fraction method)
 - ii) $X(z) = \frac{z^{-2}}{(1-z^{-1})^3}$ (By inversion integral method)
[2.5+2.5]
9. State final value theorem for Z-transform. Obtain Z-transform of $(1-e^{-at})$; $a>0$ and hence evaluate $x(\infty)$ by using final value theorem. [1+4]
10. Solve the difference equation: [5]

$$x(k+2) - 3x(k+1) + 2x(k) = 0; \text{ given that } x(0) = 0 \text{ and } x(1) = 1 \text{ by using z-transform method.}$$
11. Find the Fourier integral of the function: [5]

$$f(x) = \begin{cases} 1, & \text{for } 0 < x < \pi \\ 0, & \text{for } x > \pi \end{cases}$$

-
12. Find the Fourier transform of e^{-x^2} . Also verify the convolution theorem for $f(x) = e^{-x^2}$
and $g(x) = e^{-x^2}$ [5]
13. Derive one dimensional wave equation and solve it completely. [10]

14. Solve completely the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ under the conditions: [10]

$$u(0, y) = u(l, y) = u(x, 0) = 0, u(x, \infty) = \sin\left(\frac{n\pi x}{l}\right)$$

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Examination Control Division
 2075 Baisakh

Exam.	Back		
Level	BE	Full Marks	80
Programme	BGE, BEL, BEX, BCT	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics (SH551)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. a) Define harmonic function. Is $V = \arg(z)$ is harmonic? If yes, find a corresponding harmonic conjugate. [1+1+3]
- b) Define conformal mapping. Find the bilinear transformation which maps the points $z = 0, 1, \infty$ into the points $w = -3, -1, 1$ respectively. [1+4]
2. a) Distinguish between Cauchy integral Theorem and Cauchy integral formula. Using Cauchy integral formula evaluate $\int_C \frac{e^z}{(z+1)(z-2)} dz$ where C is the circle $|z-1|=3$. [1+4]
- b) State and Prove Taylor's series for function of complex variable. [5]
3. a) Define an isolated pole. Using Cauchy's residue theorem evaluate $\int_C \frac{z-1}{(z+1)^2(z-2)} dz$ where C is the circle $|z-i|=2$. [5]
- b) Evaluate the integral by contour integration: [5]
- $$\int_{-\infty}^{\infty} \frac{x^2}{(1+x^2)(x^2+4)} dx$$
4. a) Obtain the z-transform of $(1-e^{-at})$, $a > 0$ and hence evaluate $x(\infty)$ by using final value theorem. [2+3]
- b) Obtain the inverse z-transform of:
- $$X(z) = \frac{2z^3+z}{(z-2)^2(z-1)}$$
 by using partial fraction method. [5]
5. a) Define z-transform of function $f(t)$. Find the z-transform of following sequences: [1+2+2]
- (i) $f(k) = \left\{ \begin{matrix} 15, 10, 7, 4, 1, -1, 3, 6 \\ \uparrow \quad \quad \quad \end{matrix} \right\}$
- (ii) $f(k) = \begin{cases} 5^k & ; \quad k < 0 \\ 2^k & ; \quad k \geq 0 \end{cases}$
- b) Solve the difference equation by the application of z-transform:

$$x(k+2) + 3x(k+1) + 2x(k) = 0$$
 with conditions $x(0) = 0, x(1) = 1$. [5]

6. a) A tightly stretched string with fixed ends at $x = 0$ and $x = 1$ is initially at rest in its equilibrium position. Find the deflection $u(x, t)$ if it is set vibrating by giving to each of its points a velocity $3(lx-x^2)$. [10]
- b) Derive two dimensional heat equation. [10]
7. a) Obtain the Fourier sine integral representation of $e^{-x}\cos x$ and hence show that $\int_0^\infty \frac{\omega^3 \sin \omega x}{\omega^4 + 4} d\omega = \frac{\pi}{2} e^{-x} \cos x, \quad x > 0$. [5]
- b) Find the Fourier Cosine transform of $f(x) = e^{-x}, x > 0$ and hence by Parseval's identity, show that $\int_0^\infty \frac{1}{(1+x^2)^2} dx = \frac{\pi}{4}$.

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Year / Part	II / II	Time	3 hrs.

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- a) Define an analytic function for a function of complex variable. Derive Cauchy Riemann equations in Cartesian form. [1+4]
 - b) Define linear fractional mapping. Find bilinear mapping which maps the points $z = 0, 1, -1$ to $w = i, 2, 4$. [5]
 - a) State and Prove Cauchy integral theorem. [1+4]
 - b) Point out difference between Taylor's series and Laurent's series. Find Laurent's series of function $f(z) = \frac{\sin z}{z^6}$, $0 < |z| < TR$ [1+4]
 - a) Define pole of order m. Using Cauchy's residue theorem evaluate $\int_C \cot z dz$; where C is $|z| = 1$. [1+4]
 - b) Using Counter integration evaluate, [5]
$$\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^2}.$$
 - a) Find the z-transform of: [2+3]
 - $\cos at$
 - te^{-at}
 - b) State final value theorem. If $x(t) = 0$ for $t < 0$ and $Z[x(t)] = X(z)$ for $t \geq 0$ then prove that: [1+4]
$$Z[x(t+nT)] = z^n \left[X(z) - \sum_{k=0}^{n-1} x(kT)z^k \right].$$
 - a) Obtain inverse Z-transform of $\frac{z(3z^2 - 6z + 4)}{(z-1)^2(z-2)}$. [5]
 - b) Solve the difference equation by the application of z-transform: [5]
$$x(k+2) - 4x(k+1) + 4x(k) = 0; \text{ with conditions } x(0) = 1; x(1) = 0.$$
 - a) Derive one dimensional wave equation and solve it completely. [5+5]
 - b) A uniform rod of length ℓ has its end maintained at a temperature 0°C and the initial temperature of the rod is:
$$u(x,0) = 3 \sin \frac{\pi x}{\ell} \quad \text{for } 0 < x < \ell.$$

Find the temperature $u(x, t)$. [10]

 - a) Find Fourier integral of the function [5]
$$f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$
 - b) Verify the convolution theorem for Fourier transform for the functions [5]
$$f(x) = g(x) = e^{-x^2}.$$

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Year / Part	II / II	Time	3 hrs.

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- ✓ Attempt All questions.
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1. a) Define harmonic function of complex variable. Determine the analytical function

$$f(z) = u + iv \text{ if } u = y^3 - 3x^2y \quad [1+4]$$

- b) Derive Cauchy-Riemann equations if function of complex variable $f(z) = u + iv$ is analytic in cartesian form. [5]

2. a) What do you mean by conformal mapping? Find the linear transformation which maps points $z_1 = 1, z_2 = i, z_3 = -1$ into the points $w_1 = 0, w_2 = 1, w_3 = \infty$. [1+4]

- b) State and prove Cauchy's integral formula. [5]

3. a) State Taylor's theorem. Find the Laurent's series representation of the function

$$f(z) = \frac{z}{(z+1)(z+2)} \text{ in the annular region between } |z|=1 \text{ and } |z|=2. \quad [1+4]$$

- b) Define zero of order m of function of complex variable .Determine the poles and residue at poles of the functions $f(z) = \frac{1+z}{(z+2)(1-z)^2}$. [1+4]

OR

Evaluate the real integral $\int_{-\infty}^{\infty} \frac{x^2}{(1+x^2)^3} dx$ by contour integration in the complex plane. [5]

4. a) Define z-transform. How does it differ from Fourier transform? Obtain z-transform of

$$(i) t^2 a^t \quad (ii) \cos at \quad [1+1+1.5+1.5]$$

- b) State initial value theorem for z transform. Find the initial value $x(0)$ and $x(1)$ for the function. [1+4]

$$X(z) = \frac{(1-e^{-T})z^{-1}}{(1-z^{-1})(1-e^{-T}z^{-1})}$$

5. a) Obtain the inverse z-transform of $X(z) = \frac{3z^3 + 2z}{(z-3)^2(z-2)}$ by using inversion integral method. [5]

- b) Apply method of z-transform to solve the difference equation

$$x(k+2) - 4x(k+1) + 4x(k) = 0; x(0) = 0, x(1) = 1 \quad [5]$$

6. Solve completely one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the conditions: [10]

$$u(0, t) = 0, u(l, t) = 0, u(x, 0) = 0 \text{ and } \left(\frac{\partial u}{\partial t} \right)_{\text{at } t=0} = 3(lx - x^2)$$

7. Derive one dimensional heat equation and solve it completely. [10]

8. a) State convolution theorem for Fourier transform. Give its importance with suitable example. [2+3]

b) Find the Fourier cosine integral of the function $f(x) = e^{-kx}$ ($x > 0, k > 0$) and hence show that $\int_0^\infty \frac{\cos \omega x d\omega}{k^2 + \omega^2} = \frac{\pi}{2k} e^{-kx}; x > 0, k > 0$ [5]

Exam. Level	New Back (2066 & Later Batch)
Programme	BE BEC, BEX, BCT
Year / Part	Pass Marks II / II
	Time 3 hrs.

Subject: - Applied Mathematics (SH551)

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1. a) Define analytical function of complex-variable. Determine the analytic function $f(z) = u + iv$ if $u = \log \sqrt{x^2 + y^2}$. [1+4]

b) Express Cauchy Riemann equations $U_x = V_y$ and $U_y = -V_x$ into polar form. [5]

2. a) Define bilinear transformation. Obtain the linear transformation which maps points $z_1 = -i, z_2 = 0, z_3 = i$ into $w_1 = -1, w_2 = i, w_3 = 1$ [1+4]

b) Evaluate $\int_C \frac{e^{2z}}{z^2 - 3z + 2} dz$ in the circle $|z|=3$ by using Cauchy integral formula. [5]

3. a) State Laurent's Theorem. Obtain the Taylor's series expansion of $f(z) = \frac{1}{z^2 + 4}$ about the point $z = i$. [1+4]

b) Define residue at poles. Evaluate $\oint_C \frac{\sin z}{z^6} dz$, $C: |z|=1$ by residue method. [1+4]

OR

Evaluate real integral $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta} d\theta$ by contour integration in the complex plane. [5]

4. a) Define Z-transform and its region of convergence. Find the Z-transform of [1+1+1.5+1.5]
i) $t^2 e^{-at}$ ii) $\sin at$

b) State and prove final value theorem for Z-transform. [1+4]

5. a) Find the inverse Z-transform of $f(z) = \frac{z-4}{(z-1)(z-2)^2}$ by partial fraction method. [5]

b) Use the method of Z-transform to solve the difference equation. [5]
 $x(k+2) + 2x(k+1) + 3x(k) = 0; x(0) = 0, x(1) = 2$

6. Derive one dimensional wave equation and solve it completely. [10]

7. Solve completely the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ under the conditions: [10]

$$u(0, y) = u(l, y) = u(x, 0) = 0, u(x, \infty) = \sin\left(\frac{n\pi x}{l}\right)$$

8. a) Define Fourier transform of a function. How does it differ from Fourier series? Support your answer with suitable example. [1.5+1.5+2]

b) Find the Fourier Sine transform of e^{-x} , $x \geq 0$ and hence show that

$$\int_0^{\infty} \frac{x^2}{(1+x^2)^2} dx = \frac{\pi}{4} [5]$$

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Year / Part	BGE	Pass Marks
II / II	II / II	32

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- ✓ Attempt All questions.
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1. a) If $u = (x-1)^3 - 3xy^2 + 3y^2$, determine V so that $u + iv$ is an analytic function of $x+iy$. [5]
- b) Define an analytic function. Express Cauchy Riemann equations $u_x = v_y$ and $u_y = -v_x$ in polar form. [5]
2. a) Find the bilinear transformation which maps points $z_1 = 1, z_2 = i, z_3 = -1$ into the points $w_1 = i, w_2 = -1, w_3 = -i$ respectively. [5]
- b) Evaluate $\int_0^{1+i} (x^2 + iy) dz$ along the path $y = x^2$ [5]
3. a) Express $f(z) = \frac{1}{(z^2 - 3z + 2)}$ as Laurent's series in the region $1 < |z| < 2$. [5]
- b) Evaluate $\int_0^{2\pi} \frac{1}{5 - 4\sin\theta} d\theta$ by contour integration method in complex plane. [5]
4. a) Find z-transform of:
 - te^{-at}
 - $\sin at$
 b) State and prove final value theorem for z- transform. [5]
5. a) Find the inverse z-transform of $\frac{2z^2 - 5z}{(z-2)(z-3)}$ by using partial fraction method. [5]
- b) Solve difference equation $x(k+2) - 3x(k+1) + 2x(k) = 4^k$ for $x(0) = 0$ and $x(1) = 1$. [5]
6. Derive one dimensional wave equation and obtain its solution. [10]
7. Solve one dimensional heat equation: [10]

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \text{ under the conditions:}$$
 - u is not infinite as $t \rightarrow \infty$
 - $\frac{\partial u}{\partial x} = 0$ for $x = 0$ and $x = l$
 - $u(x, 0) = lx - x^2$ for $t = 0$; between $x = 0$ and $x = l$
8. a) Find Fourier integral representation of $f(x) = e^{-x}, x > 0$ and hence evaluate $\int_0^\infty \frac{\cos(sx)}{s^2 + 1} ds$ [5]
- b) Find the Fourier cosine transform of $f(x) = e^{-|x|}$ and hence, by Parseval's identity, shown that $\int_0^\infty \frac{1}{(1+x^2)^2} dx = \frac{\pi}{4}$ [5]

Exam. Level	New Batch (2066 & Later Batch) BE	Full Marks 80
Programme	All (Except B.Arch)	Pass Marks 32
Year / Part	I / II	Time 3 hrs

Subject: - Engineering Mathematics II (SH451)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ All questions carry equal marks.
- ✓ Assume suitable data if necessary.

1. If $u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$ prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$
2. Find the extreme value of $x^2 + y^2 + z^2$ connected by the relation $x + y + z = 3a$
3. Evaluate: $\iint r \sin \theta dr d\theta$ over the area of the cardioid $r = a(1 + \cos \theta)$ above the initial line.
4. Evaluate: $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2 \cdot \sqrt{x^2 + y^2} dy dx$ by changing polar coordinates.

OR

Evaluate: $\iiint x^{l-1} \cdot y^{m-1} \cdot z^{n-1} dx dy dz$

Evaluate: x, y, z are all positive but $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \leq 1$

5. Find the length of perpendicular from the point $(3, -1, 11)$ to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.
Also obtain the equation of the perpendicular.
6. Find the magnitude of the line of the shortest distance between the lines

$$\frac{X}{4} = \frac{Y+1}{3} = \frac{Z-2}{2}, 5x - 2y - 3z + 6 = 0, x - 3y + 2z - 3 = 0$$
7. Find the centre and radius of the circle in which the sphere $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$ is cut by the plane $x - 2y + 2z = 3$
8. The plane through OX and OY include an angle α , show that their line of intersection lies on the cone $z^2(x^2 + y^2 + z^2) = x^2 y^2 \tan^2 \alpha$
9. Solve by power series method of the differential equation $y'' - y = 0$
10. Express $f(x) = x^3 - 5x^2 + x + 2$ in terms of Legendre's polynomials.
11. Prove the Bessel's function: $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{\sin x}{x} - \cos x \right)$

12. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are the reciprocal system of vectors then prove that

$$\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \frac{\vec{a} + \vec{b} + \vec{c}}{[\vec{a} \vec{b} \vec{c}]}, \begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}' & \vec{b}' & \vec{c}' \end{bmatrix} \neq 0$$

13. If $\vec{r} = a \cos t \vec{i} + a \sin t \vec{j} + a \tan \alpha \vec{k}$, find $\left[\vec{r} \frac{d\vec{r}}{dt} \frac{d^2 \vec{r}}{dt^2} \right]$

14. Find the directional derivative of $\phi(x, y, z) = xy^2 + yz^3$ at the point (2, -1, 1) in the direction of vector $\vec{i} + 2\vec{j} + 2\vec{k}$

OR

If \vec{r} be the position vector and \vec{a} is constant vector then prove that $\nabla \cdot \left(\frac{\vec{a} \times \vec{r}}{r} \right) = 0$

15. Determine whether the series $\sum \frac{n}{1+n\sqrt{n+1}}$ is convergent or divergent.

16. Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n \cdot 2^n}$

Subject: - Applied Mathematics (SH531)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ All questions carry equal marks.
- ✓ Assume suitable data if necessary.

1. Determine the analytic function $f(z) = u + iv$ if $u = \log \sqrt{x^2 + y^2}$.
2. State and prove Cauchy's integral formula.
3. Find the Taylor's series of $f(z) = \frac{1}{1-z}$ about $z = 3i$.
4. Evaluate the integral $\oint_C \frac{e^{2z}}{(z+1)(z+3)} dz$ where $C: |z| = 4$, using residue theorem.
5. Define conformal mapping, show that $w = \frac{az+b}{cz+d}$ is invariant to

$$\left(\frac{w-w_1}{w-w_3} \right) \times \left(\frac{w_2-w_3}{w_2-w_1} \right) = \left(\frac{z-z_1}{z-z_3} \right) \times \left(\frac{z_2-z_3}{z_2-z_1} \right)$$

6. Using contour integration, evaluate real integral: $\int_{-\infty}^{+\infty} \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)}$

7. Find the z-transform of $x(z) = \cosh t \sinh t$.
8. State and prove "final value theorem" for the z-transform.

9. Find the inverse z-transform of $x(z) = \frac{z}{z^2 + 7z + 10}$.

10. Using z-transform solve the difference equation:

$$x(K+2) + 6x(K+1) + 9x(K) = 2^K; \quad x_0 = x_1 = 0.$$

11. Derive one-dimensional heat equation.

12. Solve the wave equation for a tightly stretched string of length 'l' fixed at both ends if the initial deflection in $y(x, 0) = lx - x^2$ and the initial velocity is zero.

13. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ under the conditions $u(0, y) = u(l, y) = u(x, 0) = 0, u(x, a) = \sin\left(\frac{n\pi x}{l}\right)$

14. Derive the wave equation (vibrating of a string).

15. Find the Fourier cosine transform of $f(x) = e^{-m|x|}$ and hence show that $\int_0^{\infty} \frac{\cos py}{y^2 + \beta^2} dy = \frac{\pi}{2\beta} e^{-p\beta}$.

16. Find the Fourier integral representation of the function $f(x) = e^{-x}, x \geq 0$ with $f(-x) = f(x)$.

Hence evaluate $\int_0^{\infty} \frac{\cos(sx)}{s^2 + 1} ds$.

Exam. Level	III	New Basic Course	Full Marks
Programme	B.T. B.I.E.	Pass Marks	33
Year / Part	B-B	Time	3 hrs

Subject: - Applied Mathematics (311331)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. a) Determine the analytic function $f(z) = u + iv$ if $u = 3x^3y - y^3$. [5]
- b) Find the linear transformation which maps the points $z = 0, 1, \infty$ into the points $w = -3, -1, 1$ respectively. Find also fixed points of the transformation. [5]

2. a) State and prove Cauchy's integral formula. [5]

b) Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$ where C is the circle $|z| = 3$. [5]

3. a) Find the first four terms of the Taylor's series expansion of the complex function

$$f(z) = \frac{z+1}{(z-3)(z-4)} \text{ about the centre } z = 2. \quad [5]$$

b) Evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$ where C is the circle $|z| = \frac{3}{2}$. [5]

OR

Evaluate $\int_0^{2\pi} \frac{1}{\cos\theta + 2} d\theta$ by contour integration in the complex plane.

4. Derive one dimensional heat equation $u_t = c^2 u_{xx}$ and solve it completely. [10]

- 5. Find all possible solution of Laplace equation $u_{xx} + u_{yy} = 0$. Using this, hence solve $u_{xx} + u_{yy} = 0$, under the conditions $u(0, y) = 0$, $u(x, y) = 0$ when $y \rightarrow \infty$ and $u(x, 0) = \sin x$. [10]

6. a) Find the z-transform of $\sin K\theta$. Use it to find the $z[a^K \sin K\theta]$. [5]

b) If $z[x(K)] = \frac{2z^2 + 3z + 12}{(z-1)^4}$, find the value of $x(2)$ and $x(3)$. [5]

7. a) Find the inverse z-transform of $x(z) = \frac{3z^3 + 2z}{(z-3)^2(z-2)}$ by using inversion integral method. [5]

- b) Using z-transform solve the difference equation $x(K+2) - 4x(K+1) + 4x(K) = 2^K$ given that $x(0) = 0$, $x(1) = 1$. [5]

8. a) Find the Fourier sine integral of the function $f(x) = e^{-Kx}$ and hence show that [5]

$$\int_0^{\infty} \frac{\lambda \sin \lambda x}{\lambda^2 + \beta^2} d\lambda = \frac{\pi}{2} e^{-Kx}, \quad x > 0, K > 0$$

- b) Find the Fourier sine transform of e^{-x} , $x \geq 0$ and hence show that [5]

$$\int_0^{\infty} \frac{x \sin mx}{x^2 + 1} dx = \frac{\pi}{2} e^{-m}, \quad m > 0$$

Exam Level	BE	Regular Full Marks	80
Programme	BEL, BEX, BCT	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics (SH551)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. Show that $u(x, y) = x^2 + 2xy - y^2$ is a harmonic function and determine $v(x, y)$ in such a way that $f(z) = u(x, y) + iv(x, y)$ is analytic. [5]
2. Define complex integral. State and prove Cauchy integral formula. [5]

OR

Obtain bilinear transformation which maps $-i, 0, i$ to $-1, i, 1$. [5]

3. Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$ where C is $|z| = 3$ using Cauchy's integral formula. [5]
4. Obtain the Laurent series which represents the function $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$ $2 < |z| < 3$. [5]
5. Find the Laurent series of $f(z) = \frac{1}{4+z^2}$ about the point $z = i$. [5]
6. State and prove Taylor series of a function $f(z)$. [5]
7. Derive one dimensional wave equation $u_{tt} = c^2 u_{xx}$ and solve it completely. [10]
8. Solve one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the boundary condition $\frac{\partial u}{\partial x} = 0$ when $x = 0$ and $x = L$ and initial condition $u(x, 0) = x$ for $0 < x < L$. [10]
9. Find Z transform of (a) te^{-at} and (b) $\sin at$. [5]
10. Find the inverse z-transform (a) $\frac{z-4}{(z-1)(z-2)^2}$ (b) $\frac{z}{z^2 - 3z + 2}$. [5]
11. Obtain the Z transform of $x(t) = (1 - e^{-at})$, $a > 0$ and hence evaluate $x(\infty)$ by using final value theorem. [5]
12. Solve using z-transform the difference equation $x(K+2) + 2x(K+1) + 3x(K) = 0$. [5]
13. Find the Fourier sine transform of $f(x) = e^{-x}$, $x \geq 0$ and hence evaluate $\int_0^\infty \frac{x \sin x}{(1+x^2)} dx$. [5]
14. State and prove convolution theorem of Fourier transform. [5]

Exam. Level	BE B.E., B.E.N.	Full Marks	80
Programme	BCT	Pass Marks	32
Year / Part	H / II	Time	3 hrs.

Subject: - Applied Mathematics (8H331)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. Define analytic function. Show that the function $f(z) = \frac{1}{z^4}$ is analytic except $z = 0$ [5]

2. Define complex integral. Evaluate $\int_C \log z dz; C : |z| = 1$ [5]

OR

Obtain a bilinear transformation which maps $-i, 0, i$ to $-1, i, 1$.

3. Evaluate $\int_0^{1+i} (x^2 + iy) dz$ along the path $y = x$. [5]

4. Find the Taylor series of $f(z) = \frac{1}{4+z^2}$ about the point $z = i$. [5]

5. Evaluate the integrals by residue theorem $\int_C \frac{1-\cos z}{z^3} dz$ [5]

6. State Cauchy's Residue theorem and use it to evaluate $\int_C \frac{z^2}{3+4z+z^2} dz$ where C is $|z| = 2$ [5]

OR

Evaluate $\int_0^{2\pi} \frac{d\theta}{\cos \theta + 2}$ by contour integration in complex plane.

7. Derive the one dimensional wave equation. [10]

8. A rod of length L has its ends A and B maintained at 0° and 100° respectively until steady state prevails. If the changes are made by reducing the temperature of end B to 85° and increasing that of end A to 15° , then find the temperature distribution in the rod at a time t . [10]

9. Find the z-transform of (i) $e^{-at} \sin wt$ (ii) $\cos at$ [5]

10. Obtain inverse Z-transform of (i) $\frac{z+2}{(z-2)(z-3)}$, (ii) $\frac{z}{(z-2)(z-1)}$ [5]

11. If $x(k) = 0$ for $k < 0$ and $Z\{x(k)\} = X(z)$ for $k > 0$ then prove that $Z\{x(k+n)\} = z^n X(z) - z^n$

$\sum_{k=0}^{n-1} \chi(k) z^{-k}$ where $n = 0, 1, 2, \dots$ [5]

12. Solve the difference equation $x(k+2) - 4x(k+1) + 4x(k) = 0$ with conditions, $x(0) = 0, x(1) = 1$ [5]

13. Find the cosine transform of $f(x) = e^{-mx}$ $m > 0$ show that $\int_0^{\infty} \frac{\cos pr}{r^2 + B^2} = \frac{\Pi}{2B} e^{-PB}$ [5]

14. Find the Fourier transform of $g(x) = \begin{cases} 1-x^2 & \text{if } -1 < x < 1; \\ 0, & \text{otherwise.} \end{cases}$ [5]

and hence use it to evaluate $\int_0^{\infty} \left(\frac{x \cos x - \sin x}{x^3} \right) \cos(x/2) dx$

Exam. Level Programme Year / Part	Regular (2069 & Early Batch)
BE	Full Marks 80
BEL, BEX, BCT	Pass Marks 32
H / H	Time 3 hrs.

Regular (2069 & Early Batch)
Full Marks 80
Pass Marks 32

Subject: - Applied Mathematics (SH551)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ All questions carry equal marks.
- ✓ Assume suitable data if necessary.

1. Determine the analytic function $f(z) = u(x,y) + iv(x,y)$ if $u(x,y) = x^2 - y^2$.
2. Define complex integral. Evaluate: $\oint_C (z+1) dz$ where C is the square with vertices at $z = 0, z = 1, z = 1+i$ and $z = i$.

OR

Find linear fractional transformation mapping of: $-2 \mapsto \infty, 0 \mapsto \frac{1}{2}, 2 \mapsto \frac{3}{4}$.

3. a) State Cauchy's integral formula and evaluate the integral $\oint_C \frac{4-3z}{z(z-1)(z-2)} dz$, where C is circle $|z| = \frac{3}{2}$.
- b) Obtain the Laurent series which represents the function $f(z) = \frac{1}{(1+z^2)(z+2)}$ when $|z| < 2$.
4. a) Find the Taylor's series expansion of $f(z) = \frac{1}{z^2 + 4}$ about the point $z = i$.
- b) Evaluate $\oint_C \tan z dz$ where C is a circle $|z| = 2$ by Cauchy's residue theorem.

OR

Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta} d\theta$ by contour integration in the complex plane.

5. Find the z-transforms of: (i) $\cos h$ (at) $\sin(bt)$ (ii) $n(n-1); n = k$
6. Find the inverse z-transforms of: (i) $\frac{Z}{Z^2 - 3Z + 2}$ (ii) $\frac{Z}{(Z+1)^2(Z-1)}$.
7. a) State and prove convolution theorem for z-transform.
- b) Solve by using z-transform the difference equation $x(k+2) + 2x(k+1) + 3x(k) = 0$ given that $x(0) = 0$ and $x(1) = 2$

8. Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ given that $u = 0$ as $t \rightarrow \infty$ as well as $u = 0$ at $x = 0$ and $x = L$

9. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, which satisfies the condition $u(0,y) = u(L,y) = u(x,0) = 0$ and $u(x,a) = \sin\left(\frac{n\pi x}{L}\right)$.

OR

The diameter of a semi-circular plate of radius a is kept at 0°C and the temperature at the semi-circular boundary is u_0 . Find the steady state temperature in the plate.

10. Find the Fourier integral representation of the function $f(x) = e^{-x}$, $x \geq 0$ with $f(-x) = f(x)$.

Hence evaluate $\int_0^\infty \frac{\cos(sx)}{s^2 + 1} ds$.

11. Find the Fourier transform of:

$$f(x) = 1 - x^2, |x| < 1$$

= 0, $|x| > 1$ and hence evaluate

$$\int_0^\infty \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx.$$

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	BEL, BEK, BCT	Pass Marks	32
Year / Part	H / H	Time	3 hrs

Subject: - Applied Mathematics (SH551)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ All questions carry equal marks.
- ✓ Assume suitable data if necessary.

1. If $f(z) = u + iv$ is any analytic function of complex variable z and $u-v = e^x(\cos y - \sin y)$, find $f(z)$ in terms of z .
2. State and prove Cauchy's integral theorem.
3. Using Cauchy integral formal to evaluate: $\oint_C \frac{z dz}{(z-1)(z-3)}$ where $C: |z| = 3/2$.
4. Find the Laurent's series expansion of $f(z) = \frac{1-\cos z}{z^3}$, $0 < |z| < R$.
5. Define singular points and poles. compute the residue of $f(z) = \frac{z^2}{(z-2)(z^2+1)}$ at its pole(s).

OR

Using contour integration to evaluate $\int_0^{2\pi} \frac{d\theta}{2 + \sin \theta}$.

6. Find the z-transform of the following: (any two)
 - i) $K^2 a^k$ for $K \geq 0$ (ii) $a^k \cos k\pi$ for $k \geq 0$ (iii) $e^{at} \cos wt$ for $t \geq 0$.
7. State and prove final value theorem for the z-transform.
8. Solve the difference equation: $y_{n+2} + 2y_{n+1} + y_n = n$ where $y_0 = y_1 = 0$, and $n = k$
9. A tightly stretched string with fixed end points $x = 0$ and $x = 1$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{1}\right)$. If it is released from rest from this position, find the displacement $y(x, t)$.
10. Change the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in to polar form.
11. Define Fourier series in complex form. Verify the convolution theorem for $f(x) = g(x) = e^{-x^2}$.
12. Find the Fourier cosine transform of $f(x) = e^{-x}$, $x > 0$ and hence by Parseval's identity, show that $\int_0^\infty \frac{1}{(1+x^2)^2} dx = \frac{\pi}{4}$.

Exam. Level	BE	Regular Full Marks	80
Programme	BEL, BEX, BCT	Pass Marks	32
Year / Part	II / II	Time	3 hrs.

Subject: - Applied Mathematics

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ All questions carry equal marks.
- ✓ Assume suitable data if necessary.

1. a) State necessary conditions for a function $f(z)$ to be analytic. Show that the function $f(z) = \log z$ is analytic everywhere except at the origin.
b) Find the linear fractional transformation that maps the points $z_1 = -i$, $z_2 = 0$ and $z_3 = i$ into points $w_1 = -1$, $w_2 = i$, $w_3 = 1$ respectively.
 2. a) State and prove Cauchy's integral formula.
b) Write the statement of Cauchy's integral formula. Use it to evaluate the integral $\oint_C \frac{e^z}{(z-1)(z-3)} dz$ where C is the circle $|z| = 2$.
 3. a) Write the statement of Taylor's theorem. Find the Laurent series for the function $f(z) = \frac{1}{z^2 - 3z + 2}$ in the region $1 < |z| < 2$.
b) State Cauchy-residue theorem. Using it evaluate $\oint_C \frac{\sin z}{z^6} dz$ where $C: |z| = 1$.
- OR**
- Evaluate $\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta}$ by contour integration in the complex plane.
 - a) Show that the Z-transform of $\cos k\theta$ is $\frac{z(z - \cos\theta)}{z^2 - 2z\cos\theta + 1}$. Use this result to find Z-transform of $a^k \cos k\theta$.
b) Obtain the inverse Z-transform of $\frac{2z^3 + z}{(z - 2)^2(z - 1)}$, using partial fraction method.
 - a) Solve the difference equation $x(k+2) - x(k+1) + 0.25x(k) = u(k)$ where $x(0) = 1$ and $x(1) = 2$ and $u(k)$ is unit step function.
b) State and prove shifting theorem of z-transform.
 - Derive one-dimensional wave equation governing transverse vibration of string and solve it completely.

7. Solve the one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the conditions:

- a) u is not infinite as $t \rightarrow \infty$
- b) $\frac{\partial u}{\partial x} = 0$ for $x = 0$ and $x = l$ and
- c) $u(x, 0) = lx - x^2$ for $t = 0$ between $x = 0$ and $x = l$

OR

The diameter of a semi circular plate of radius a is kept at 0°C and temperature at the semi circular boundary is $T^\circ\text{C}$. Show that the steady temperature in the plate is given

$$\text{by } u(r, \theta) = \frac{4T}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \left(\frac{r}{a}\right)^{2n-1} \sin(2n-1)\theta$$

8. a) Find the Fourier cosine integral representation of the function $f(x) = e^{-kx}$ ($x > 0, k > 0$) and hence show that

$$\int_{-\infty}^{\infty} \frac{\cos \omega x}{k^2 + \omega^2} d\omega = \frac{\pi}{2k} e^{-kx} \quad (x > 0, K > 0)$$

b) Obtain Fourier sine transform of e^{-x} , ($x > 0$) and hence evaluate $\int_0^{\infty} \frac{x^2}{(1+x^2)^2} dx$.

Exam.	New Back (2066 & Later Batch)		
Level	BB	Full Marks	80
Programme	BEL, BEA,	Pass Marks	32
Year / Part	II / II	Time	3 hrs

Subject : Applied Mathematics

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ The figures in the margin indicate Full Marks.
- ✓ Assume suitable data if necessary.

1. a) Define harmonic function and show that the function $u = \frac{1}{2} \log(x^2 + y^2)$ is harmonic and hence determine its harmonic conjugate function. [5]
- b) Define conformal mapping. Find the linear fractional transformation that maps three points $2i, -2, -2i$ onto three given points $-2, -2i, 2$. [5]
2. a) State and prove Cauchy integral theorem. [5]
- b) Evaluate the following integral using Cauchy integral formula $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$ where C is the circle $|z| = 3/2$. [5]
3. a) Find the Taylor's series expansion of $f(z) = \frac{2z^3 + 1}{z^2 + z}$ about the point $z = 1$. [5]
- b) Determine the poles and the residue at each pole of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$. [5]

OR

Evaluate $\int_0^{2\pi} \frac{d\theta}{\sqrt{2 - \cos \theta}}$ by contour integration in complex plane.

4. a) Find the z-transform of: (any two) [5]
 - a^k
 - $\sin \omega k$
 - $\cosh \omega t \sin \omega t$
- b) Find inverse z-transform of $\frac{z-4}{(z-1)(z-2)^2}$ by using inversion integral method. [5]
5. a) State and prove final value theorem for z transform. [5]
- b) Solve by using z transform $y_{k+2} - 4y_{k+1} + 4y_k = 0$ where $y_0 = 1$ and $y_1 = 0$. [5]
6. A tightly stretched string with fixed end points $x = 0$ and $x = 1$ is initially in the position given by $y = y_0 \sin^3 \frac{\pi x}{L}$. If $y(0, t) = y(L, t) = 0$, then find the displacement $y(x, t)$ where initial velocity is zero. [10]
7. Deduce the two dimensional Laplace equation into polar form. [10]

OR

Derive one dimensional heat equation for the flow of heat along a metallic rod by conduction and solve it completely.

8. a) State and prove convolution theorem for Fourier transform. [5]
- b) Find the Fourier cosine transform of $f(x) = e^{-mx}$ ($m > 0$) and hence show that $\int_0^{\infty} \frac{\cos py}{y^2 + \beta^2} dy = \frac{\pi}{2\beta} e^{-\beta p}$ ($\beta > 0, p > 0$). [5]