Exam.		Regular	
Level	BE	Full Marks	80
Programme	ALL	Pass Marks	32
Year / Part	I / I	Time	3 hrs.

Subject: - Engineering Mathmatics I *(SH 401)*

✓ Candidates are required to give their answers in their own words as far as practicable.
✓ Attempt **All** questions.
✓ **All** questions carry equal marks.
✓ Assume suitable data if necessary.

1. If $y^{1/m} + y^{-1/m} = 2x$ Show that:
   a) $(x^2-1)y_2 + xy_1 - m^2 y = 0$
   b) $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)\, y_n = 0$.

2. State the Rolle's theorem and use it to prove Lagrange's mean value theorem.

3. Evaluate: $\displaystyle\lim_{x \to 0}\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$

4. Find the asymptotes of the curve $a^2 b^2 + 2ab^2 x + b^2 x^2 + a^2 x^2 + 2ax^3 + x^4 - x^2 y^2 = 0$.

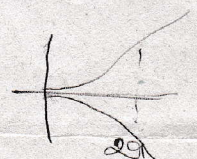5. Find the pedal equation of the curve $r^m = a^m \cos m\theta$.

6. Show that $\displaystyle\int_0^{\frac{\pi}{2}} \frac{x}{(\sin x + \cos x)}\,dx = \frac{\pi}{2\sqrt{2}}\log(\sqrt{2}+1)$

7. Apply differentiation under integral sign to evaluate $\displaystyle\int_0^\infty \frac{e^{-x}\sin bx}{x}\,dx$

8. Use Gamma function to evaluate $\displaystyle\int_0^1 x^6 \sqrt{1-x^2}\,dx$

9. Find the area of curve $y^2(2a - x) = x^3$ and its asymptotes.

OR

   Find the volume of solid formed by the revolution of the cardiode $r = a(1 + \cos\theta)$ about the initial line.

10. Solve the differential equation $\dfrac{dy}{dx} - 2y\tan x = y^2 \tan x$

11. Solve the differential equation $xp^2 - 2yp + ax = 0$ where $p = dy/dx$.

12. Solve $(D^2 - 2D + 5)y = 10\sin x$

13. Solve the differential equation $x^2 \dfrac{d^2 y}{dx^2} + 4x\dfrac{dy}{dx} + 2y = e^x$

14. Derive the equation of an ellipse in standard form.

15. Prove that the normal at a point t of the rectangular Hyperbola $xy = c^2$ meets the curve again at a point t_1 such that $t^3 t_1 = -1$.

16. Find the equation of axes and length of axes of conic $x^2 - 4xy - 2y^2 + 10x + 4y = 0$

OR

Describe and sketch the polar conic $r = \dfrac{12}{}$