Subject :- Engineering Mathematics I
2068 - Shrawan

1. If $y = \log(x + \sqrt{a^2 + x^2})$, show that $(a^2 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + n^2 y_n = 0$.

2. State and prove Lagrange's mean value theorem.

3. Evaluate: $\lim_{x \to 0}\left(\dfrac{\tan x}{x}\right)^{1/x}$

4. Find the asymptotes of the curve $(x^2 - y^2)(x + 2y + 1) + x + y + 1 = 0$.

5. Show that for the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, the radius of curvature at the extremity of the major axis is equal to half of the latus rectum.

6. Evaluate: $\displaystyle\int_0^{\pi/2} \dfrac{dx}{1 + \sqrt{\tan x}}$.

7. Use Gamma function to prove that $\displaystyle\int_0^1 \dfrac{dx}{(1 - x^6)^{1/6}} = \dfrac{\pi}{2}$.

8. Using method of differentiation under integral sign, evaluate: $\displaystyle\int_0^\infty \dfrac{e^{-x} \sin bx}{x} dx$.

9. Find the angle through which the axes must be turned so that the equation $ax^2 + 2hxy + by^2 = 0$ may become an equation having no term involving xy.

11. Obtain the equation of an ellipse in the standard form.

12. Find the Center of Conic $3x^2 + 8xy - 3y^2 - 40x - 20y + 50 = 0$

13. Solve the differential equation $(x + y + 1)\dfrac{dy}{dx} = 1$.

14. Find the general Solution of the differential equation: $P^3 - 4xyp + 8y^2 = 0$

15. Find the general solution of the differential equation: $(D^2 + 2D + 1)y = e^x\cos x$.

16. Newton's Law of cooling states that " The temperature of an object changes at a rate proportional to the differences of tempretures between the object and it's surrounding". Supposing water at a tempreture 100°C. cools to 80°c in 10minutes., in a room maintained at 30°c. Find when the tempreture of water will becomes 40°c.

OR

Solve: $x2\dfrac{d^2y}{dx^2} - 4x\dfrac{dy}{dx} + 6y = x$