FRICTION

5.1 Friction

When a body slides over another body, a force is exerted at a surface of contact by the stationary body on a moving body. This resisting force is called the *force of friction* and acts in the direction opposite to the direction of motion.

Characteristic of friction:

i. It always acts in a direction opposite to that in which motion is intended.

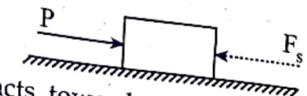
ii. It exists as along as the tractive force acts. So, it is passive force.

iii. It is self-adjusting force i.e., only that much comes to play as is just sufficient to prevent motion.

5.1.1 Static and Dynamic Friction

Static Friction

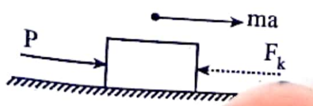
The frictional force that develops between mating surfaces when subjected to external force but there is no relative motion between them is called *static friction.*



If force P acts towards right, then F_s equal to P acts in opposite direction to prevent motion. This force F_s is static friction.

Dynamic Friction

The frictional force that develops between mating surfaces when subjected to external forces and there is relative motion between them is called *dynamic friction* (also known as *kinetic friction*).



If force P acts towards right, then F_k less than P acts in opposite direction and the body moves with acceleration a.

5.1.2 Laws of Solid Friction (Static or Dynamic)

Based on the experiment, following laws of friction have been stated:

i. Friction acts tangential to the surface in contact and is in a direction opposite to that in which motion is to take place.

ii. Frictional force is maximum at the instant of impending motion.

iii. The magnitude of limiting friction bears a constant ratio to the normal reaction between the mating surfaces. This ratio drops to a slightly lower value when motion starts.

iv. Limiting friction is independent of the area and shape of contact surface.

v. Limiting friction depends upon the nature of the surface in contact.

vi. At low velocity, friction is independent of the velocity. But at higher speed, there will be slight reduction in friction.

5.1.3 Advantages and Disadvantages of Friction

Advantages:

i. We can easily walk on road due to good friction which is too difficult to do on ice due to poor friction.

ii. Friction helps to transmit power from the motors and engine to other machines by making use of belts and clutches.

iii. Brakes make use of friction to stop vehicles.

iv. A nail can be easily fixed on the wall due to friction.

Disadvantages:

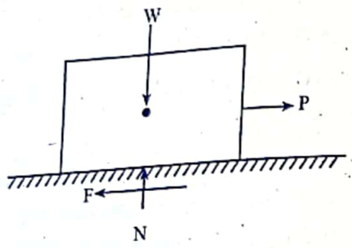
i. Friction causes unnecessary wear and tear of the machinery parts.

ii. Friction causes heat to develop within the machinery which in fact reduces the life of machine.

iii. Efficiency of a machine is decreased due to friction. A part of useful energy is dissipated in overcoming the friction.

5.2 Some Terminologies

Coefficient of Friction: Static and Dynamic



The frictional force is proportional to normal reaction i.e., $F \propto N$. The ratio $\frac{F}{N}$ i.e., the ratio of frictional force to normal reaction is called *coefficient of friction*.

When the system is in state of impending motion (i.e., when the body just comes to motion), the frictional force has maximum value F_s which is known as *static friction*. The ratio $\frac{F_s}{N}$ is called *coefficient of static friction* and is denoted by μ_s.

$$\mu_s = \frac{F_s}{N}$$

When motion starts, the maximum frictional force falls to some lower value F_k which is known as *kinetic friction*. The ratio $\frac{F_k}{N}$ is called *coefficient of kinetic friction* and is denoted by μ_k.

$$\mu_k = \frac{F_k}{N}$$

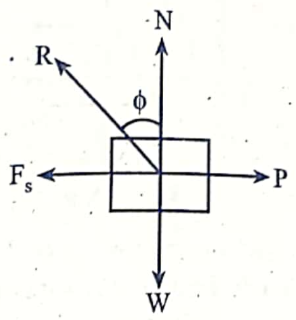
Limiting Friction

The maximum value of frictional force which acts on the body when it just starts to slide over another body is called *limiting friction*.

Mathematically, $F_{max} = F_{lim} = \mu_s N$

Angle of Friction

Angle of friction (ϕ) is defined as the angle which the resultant of normal reaction and limiting frictional force makes with the normal reaction.



where

N = Normal reaction

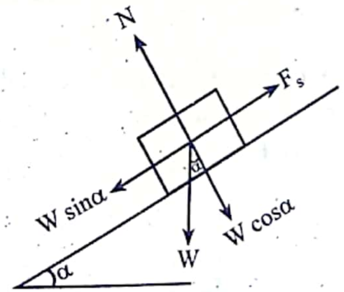
F_s = Limiting friction

R = Resultant of N and $F_s = \sqrt{N^2 + F_s^2}$

$$\tan\phi = \frac{F_s}{N} = \frac{\mu_s N}{N} = \mu_s$$

Angle of Repose

Angle of repose is defined as the angle α of the inclined plane at which the block resting on it is about to slide down the plane.



At equilibrium,

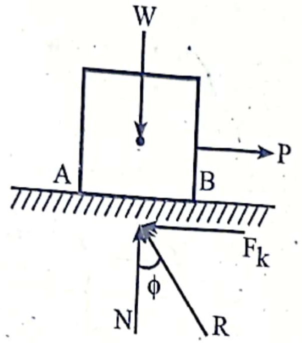
$F_s = W \sin\alpha$

or, $\mu_s N = W \sin\alpha$

or, $\mu_s W \cos\alpha = W \sin\alpha$

or, $\mu_s = \frac{W \sin\alpha}{W \cos\alpha} = \tan\alpha$

5.3 Condition of Tipping (Overturning) and Sliding

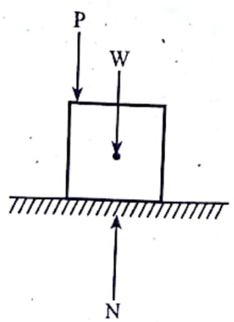


When the applied horizontal force P Increases, frictional force also increases slowly form 0 to F_s (maximum frictional force). Since the applied force is directed towards right end of the block so that couples formed by P and F as well as W and N remain balanced. If R reaches end B of the block before F reaches its maximum value F_s, then the couples will be imbalanced causing overturning or tipping about B. If F reaches to its maximum frictional force F_s earlier than the R reaches end B of the block, the block will slide as P and F will cause more clockwise moment than anticlockwise moment due to N and W.

5.4 No Friction, No Motion, Impending Motion, and Motion

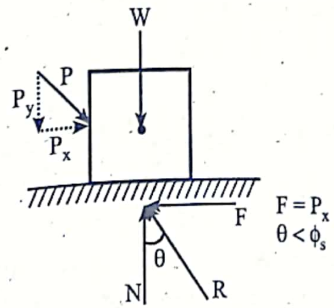
No Friction

When the applied force P is perpendicular to the surface of contact, no friction exists as only vertical forces are present and no horizontal component of force exist.



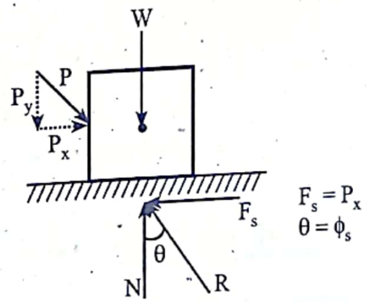
No Motion

In this case P is in inclined position.



This force trends to move the body along the surface but are not large enough to set in motion since angle between N (normal reaction) and resultant (R) i.e., Q is less than angle of static friction (ϕ_s).

Impending Motion



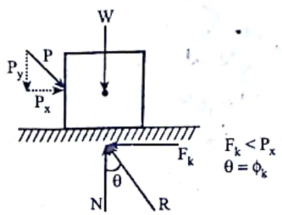
As P_x increases gradually, the resultant R also increases and the angle between N and R becomes ϕ_s. The body is now in the verge of motion.

Here, frictional force = Limiting frictional force

= Maximum frictional force

$$(F_s)_{max} = \mu_s N$$

Motion

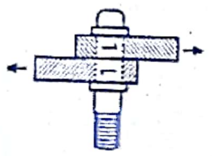


As P_x increases a condition occurs in which the block is in motion. The corresponding frictional force is known as kinetic frictional force F_k and angle is ϕ_k.

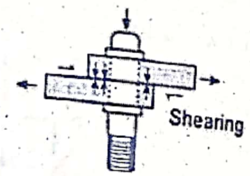
$$F_k = \mu_k N$$

5.5 High Tension Friction Grip Bolts

High tension friction grip bolts are high strength structural bolts which have been tightened such as to induce tension in the bolt shank. Due to the tension in the bolt, the interface between the piles (steel member in joint) cannot move relative to each other because of frictional resistance. The bolt act differently than normal bolts or rivets. Friction along interface takes load in case of high tension friction grip bolt subject to shear.



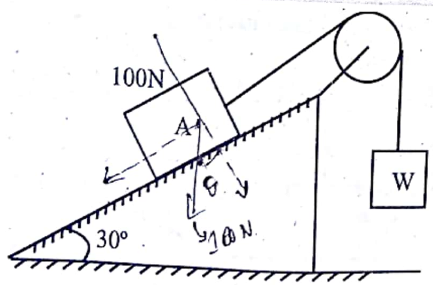
(a) Ordinary bolt



(b) High tension friction grip bolt

SOLVED NUMERICALS

1. A block 'A' of weight 100 N rests on an inclined plane and another weight W is attached to the first weight through a string as shown in figure. If the coefficient of friction between the block and plane is 0.3. determine the maximum value of W so that equilibrium can exist.

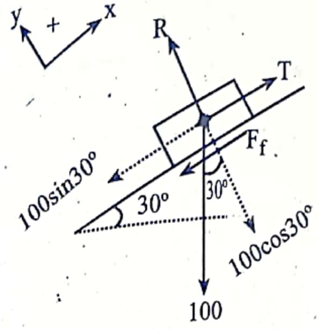


[2070 Bhadra]

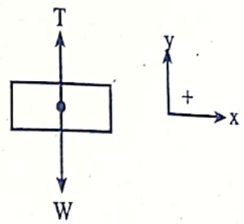
Solution:

Weight of block A, W_A = 100N

Coefficient of friction, μ_s = 0.3



FBD of block A



FBD of W

For block A, applying condition of equilibrium in the direction perpendicular to the inclined plane, we have

$R - 100 \cos 30° = 0$

$\therefore R = 86.6N$

Applying condition of equilibrium in the direction parallel to the inclined plane, we have

$T - 100 \sin 30° - F_f = 0$

Where $F_f = \mu_s \times R = 0.3 \times 86.6 = 25.98$ N

or, $T - 50 - 25.98 = 0$

or, $T = 75.98$ N

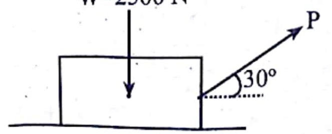
For weight W,

$(\uparrow +)\sum F_y = 0$

or, $T - W = 0$

$\therefore W = T = 75.98$ N

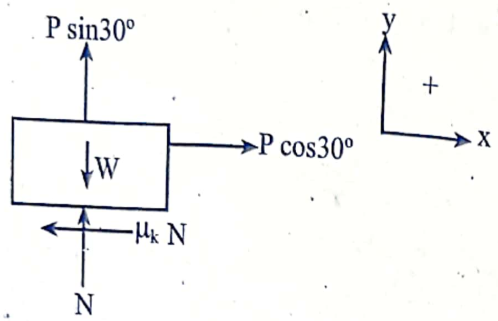
2. **Determine the magnitude of force P required to give, the block acceleration of 10 m/s². Coefficient of friction between the block and the floor is 0.25 N.**



[2070 Ashadh]

Solution:

The FBD of given figure is



$(\uparrow+)\ \sum F_y = ma_y$

or, $N + P\sin\theta - W = 0$ [since the block has no vertical acceleration]

or, $N = W - P\sin\theta$ .....(i)

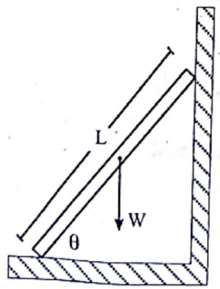
$(\overset{+}{\rightarrow})\ \sum F_x = ma_x$

or, $P\cos\theta - \mu_K N = ma$ $(\because a_x = a)$

or, $P\cos\theta - \mu_K(W - P\sin\theta) = ma$ [from (i)]

$$\therefore P = \frac{ma + \mu_K W}{\cos\theta + \mu_K \sin\theta} = \frac{\frac{2500}{9.81} \times 10 + 0.25 \times 2500}{\cos 30^\circ + 0.25 \sin 30^\circ} = 3202.24\ N$$

3. **Determine the minimum angle θ (made by the ladder AB of length 'L' with the floor) at which a uniform ladder can be placed against a wall without slipping under its own weight (W). The coefficient of friction for all surfaces is 0.2.**



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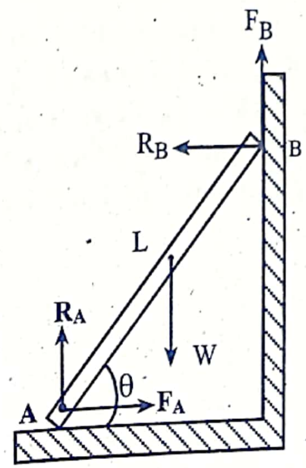
Solution:

When ladder slides downwards top (B) on its own weight the bottom part A slides to the left thereby producing frictional force upwards and towards right respectively at B and A.



$(\overset{+}{\rightarrow})\sum F_x = 0$

or, $F_A - R_B = 0$

or, $R_B = F_A = \mu R_A = 0.2\ R_A$ ......(i)

$(+\uparrow)\sum F_y = 0$

or, $F_B + R_A - W = 0$

or, $F_B + R_A = W$

or, $0.2\ R_B + R_A = W$ $(\because F_B = \mu R_B)$

or, $0.2\ (0.2R_A) + R_A = W$ (using equation (i))

or, $R_A = \frac{W}{1.04} = 0.962W$

From equation (i), $R_B = 0.2\ R_A = 0.192W$

$(\circlearrowleft+)\ \Sigma M_B = 0$

or, $-W \times \frac{L}{2} \cos\theta + R_A \times L\cos\theta - F_A \times L\sin\theta = 0$

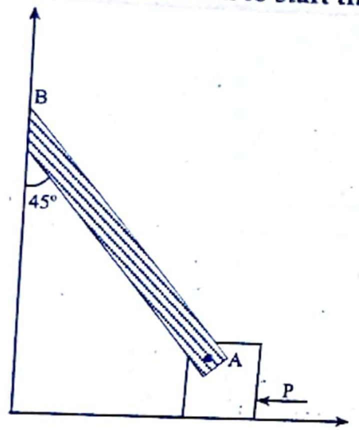
or, $-0.5\ WL\cos\theta + 0.962\ WL\cos\theta - 0.192WL\sin\theta = 0$

or, $0.462 \cos\theta = 0.192 \sin\theta$

or, $\tan\theta = \dfrac{0.462}{0.192}$

$\theta = \tan^{-1}(2.40625) = 67.43°$

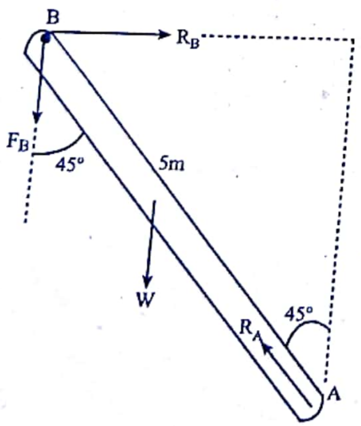
4. **A uniform bar AB having length 5m and weighing 500N is fastened by a frictionless pin to a block, weighing 200N as shown in figure below. At the vertical wall, coefficient of friction is0.3 while under the block is 0.20. Determine the forces P needed to start the motion P to the left.**



[2072 Magh]

Solution:

Considering the free body diagram of bar AB



When the motion just starts to begin, the forces are in equilibrium and so are the moments.

$(\circlearrowright)\Sigma M_A = 0$

or, $R_B \times 5\cos45° - F_B \times 5\sin45° - W_1 \times 2.5\sin45° = 0$

or, $(R_B - F_B)\dfrac{5}{\sqrt{2}} = W_1\dfrac{2.5}{\sqrt{2}}$

or, $(R_B - \mu_1 R_B)2 = W_1$

or, $R_B(1 - \mu_1)\,2 = W_1$

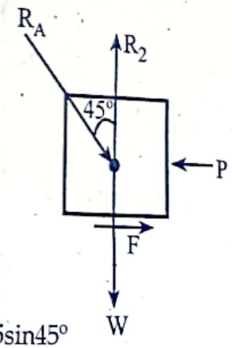
or, $R_B = \dfrac{W_1}{2(1-\mu_1)} = \dfrac{500}{2(1-0.3)} = 357.14$ N

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $R_B - R_A \cos45° = 0$

or, $R_A = \dfrac{R_B}{\cos45°} = 505$ N

On considering the free-body diagram of block,



$(+\uparrow)\Sigma F_y = 0$

or, $-R_A \cos45° - N + R_2 = 0$

or, $R_2 = R_A\cos45° - N = 557.1$ N

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $-P + F + R_A \sin45° = 0$

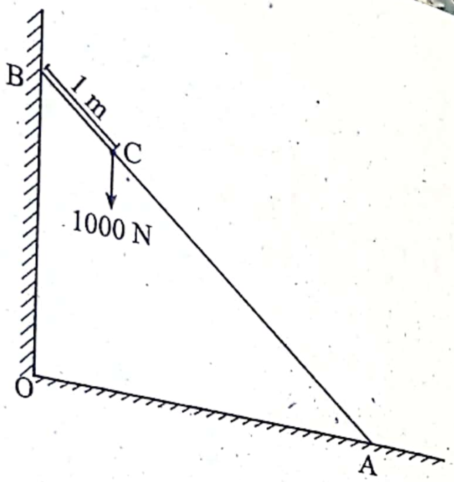
or, $P = F + R_A\sin45° = \mu R_2 + R_A\sin45°$

$= 0.2 \times 557.10 + 505\sin45°$

or, $P = 468.5$ N $(\leftarrow)$

Thus, 468.5N is to be applied to block in order to start the motion to the left.

6. **A ladder shown in figure is 4m long and is supported by a horizontal floor and a vertical wall. The coefficient of friction at the wall is 0.3 and at the floor is 0.45. The weight of the ladder is 300N. The ladder supports a vertical load of 1000N at C. Determine the reactions at A and B and compute the least value of α at which ladder may be placed without slipping to right.**

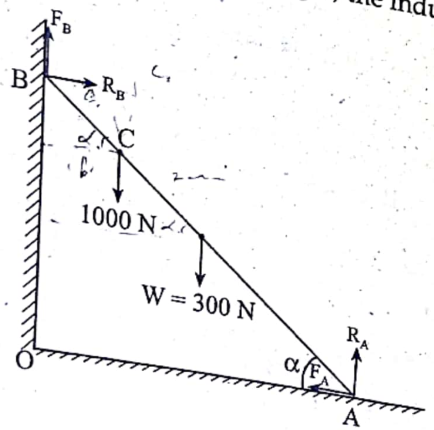


[2075 Ashwin]

Solution:

$\mu_w = 0.3$, $\mu_f = 0.45$

While ladder tends to slide to the right, the induced reaction are:



$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $F_A = R_B$

or, $R_B = F_A = \mu_f R_A = 0.45R_A$ .... (i)

$(+\uparrow)\ \Sigma F_y = 0$

or, $F_B - 1000 - 300 + R_A = 0$

or, $\mu_w R_B - 1300 + R_A = 0$

or, $0.3\times0.45\times R_A - 1300 + R_A = 0$

or, $R_A = \dfrac{1300}{1.135} = 1145.374$ KN

$R_B = 0.45R_A = 0.45\times1145.374 = 515.42$ KN

Taking moment about B, we have

$(\overset{+}{\curvearrowright})\Sigma M_B = 0$

or, $1000\times1\times\cos\alpha + 300\times2\times\cos\alpha + F_A\times4\times\sin\alpha - R_A\times4\times\cos\alpha = 0$

or, $2061.68\sin\alpha = 2981.496\cos\alpha$

or, $\tan\alpha = 1.446$

$\therefore\ \alpha = 55.36°$

CHAPTER - 6

ANALYSIS OF BEAMS AND FRAMES

6.1 Beams and Frames

A structural member designed to support loads applied at various points along the member is known as a *beam*. In most cases, the loads are perpendicular to the axis of the beam and will cause only shear and bending in the beam. When the loads are not at a right angle to the beam, they will also produce axial forces in the beam.

Classification of beam:

i. Cantilever beam
ii. Simply supported beam
iii. Overhanging beam
iv. Fixed beam
v. Continuous beam

A *frame* is a network of beams and columns joined together to carry loads and transfer it to the support.

Classification of frame:

i. Perfect frame
ii. Imperfect frame
  - Deficient
  - Redundant

6.2 Different Types of Load and Support

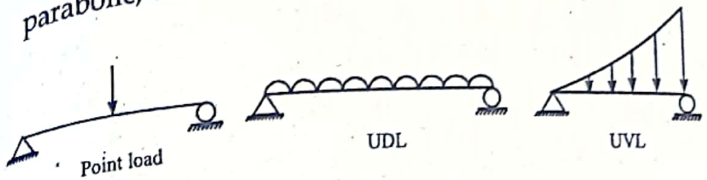
6.2.1 Types of Load

Loads are classified as:

i. **Point load /concentrated load**
Loads which act on smaller areas are called *concentrated* or *point loads*.

ii. **Uniformly distributed load (UDL)**
Load which is distributed uniformly along the length of member is called *uniformly distributed load*.

iii. ***Uniformly varying load (UVL)***
Load that varies along the length of the structure is called *uniformly varying load*. Load variation may be in linear, parabolic, cubic fashion, etc.



iv. **Couple**
It is the combination of two equal and opposite forces separated by a certain distance.

v. **Moment**
It is the product of force and distance.

vi. **Static and dynamic load**
The loads which do not vary with time are called *static loads* whereas the loads which change with time are *dynamic loads*.

vii. **Imposed load**
Imposed load may be defined as the load that is applied to the structure which is not permanent. Examples: Snow load, wind load, earthquake load.

viii. **Dead and live load**
Dead loads are the self weight of the structures i.e., self weight of slab, beam, column, finishing loads. Such loads do not change their position.
Live load is the load imposed on the structure when it fulfills its design purpose. The position and magnitude of live load may change.

6.2.2 Types of Support

Supports are classified as:

i. **Roller support**
It gives rise to one force reaction which is perpendicular to the plane on which it rests.

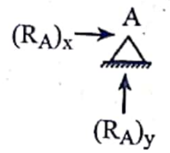
Roller support

- Free to move in x-direction
- Free to rotate about z-direction (axis)
- Degree of freedom = 2

Since it is restrained in y-direction, reaction is developed in this direction only.

ii. Hinge support

It gives rise to one force reaction whose direction is unknown and can be resolved into two forces along x and y axes.

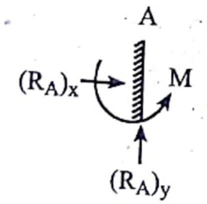


Hinge support

- Free to rotate about z-direction.
  So, its degree of freedom = 1
- Restrained in x and y-direction.
  So, no. of reactions developed = 2

iii. Fixed support

It gives rise to one force reaction having two components and one moment reaction.



Fixed support

- No translation in x and y-direction
- No rotation about z-direction
- Restrained in all three possible directions.
  So, degree of freedom = 0
  No. of reactions = 3

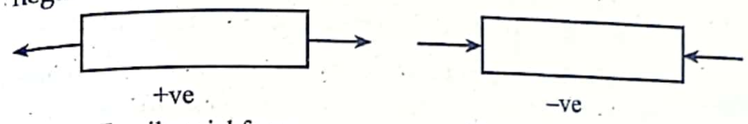
6.3 Axial Force, Shear Force, and Bending Moment

On application of external loads on a structure, following forces are developed:

1. **Axial force**

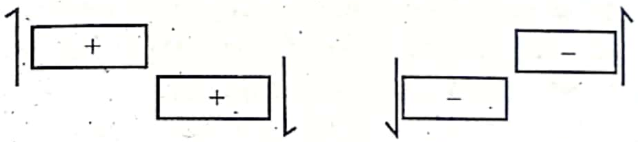
   A force lying along the longitudinal axis of the member which produces either compression or tension is the *axial force*.

   Tensile force is positive axial force and compression is negative axial force.



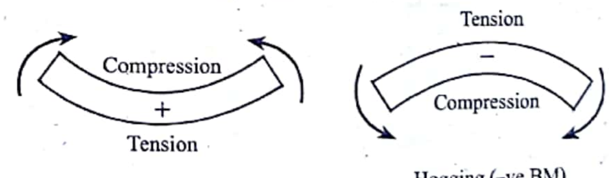
2. **Shear force**

   Unbalanced force along the transverse direction is known as *shear force*. Shearing force having an upward direction to the left side of the section is positive or downward force to the right side of the section is positive. The reverse is negative.



3. **Bending moment**

   Unbalanced moment of any section is known as *bending moment* for a particular section. Sagging bending moment is taken as +ve while hogging bending moment is taken as –ve.



6.4 Static Determinacy and Indeterminacy

A structure is said to be *statically determinate* if all the internal member forces and reactions can be determined using the equation of static equilibrium.

For beam,

External indeterminacy = $r - (3 + c)$

Internal indeterminacy = 0

Total degree of static indeterminacy = $r - (3+c) + 0 = r - (3+c)$

For frame,

External indeterminacy = $r - (3 + c)$

Internal indeterminacy = $3 \times$ total no. of cuts required to have open configuration

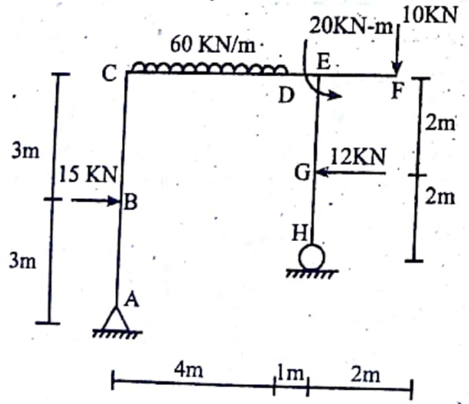
Total degree of static indeterminacy = $(3m + r) - (3j + c)$

where r = no. of support reactions, c = no. of equations due to special condition (internal hinge), j = no. of joints.

Note: Internal indeterminacy = Total indeterminacy – External indeterminacy

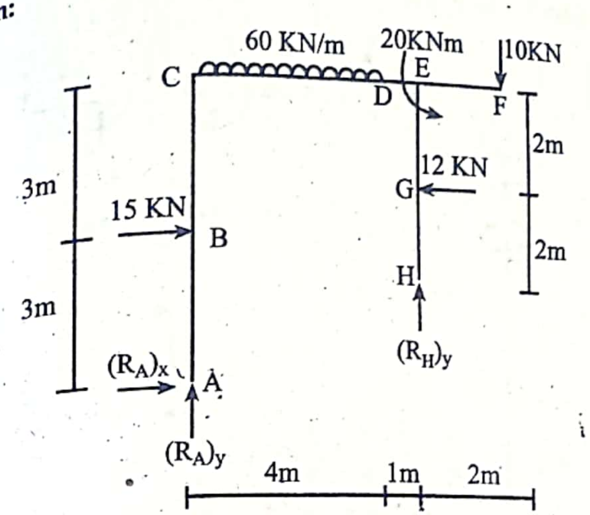
SOLVED NUMERICALS

1. Draw axial force, shear force, and bending moment diagram for the given frame. Also, indicate salient features, if any.



[2069 Chaitra]

Solution:



Calculation of unknown reactions

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $(R_A)_x + 15 - 12 = 0$

or, $(R_A)_x = -3KN = 3KN\ (\leftarrow)$

$(\circlearrowleft +)\ \Sigma M_A = 0$

or, $15\times3 + 60\times4\times2 - 20 + 10\times7 - 12\times4 - (R_H)_y\times5 = 0$

$\therefore (R_H)_y = 105.4\ KN(\uparrow)$

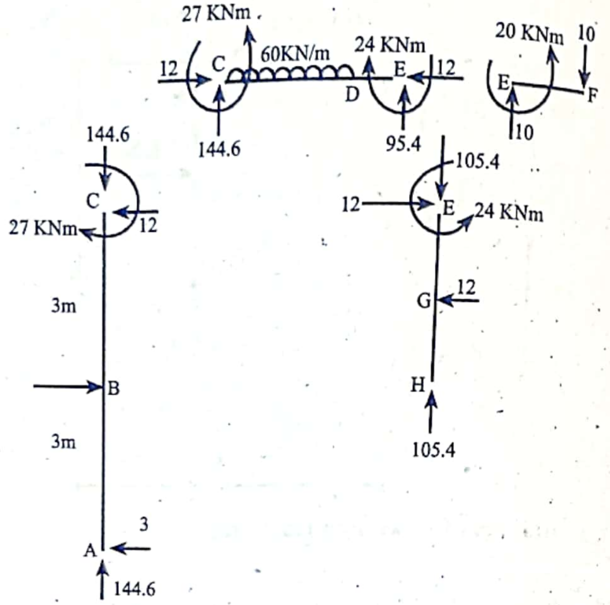
$(+\uparrow)\ \Sigma F_y = 0$

or, $(R_A)_y - 60\times4 + (R_H)_y - 10 = 0$

or, $(R_A)_y - 240 + 105.4 - 10 = 0$

$\therefore (R_A)_y = 144.6\ KN\ (\uparrow)$

Balancing of Beam



Calculation of axial force

Element AC

$(N_A)_L = 0$

$(N_A)_R = -144.6\ KN = 144.6\ KN\ (C) = (N_B) = (N_C)_L$

$(N_C)_R = -144.6 + 144.6 = 0$

Element CE

$(N_C)_L = 0$

$(N_C)_R = -12\ KN = 12\ KN\ (C) = (N_E)_L$

$(N_E)_R = -12 + 12 = 0$

Element EF

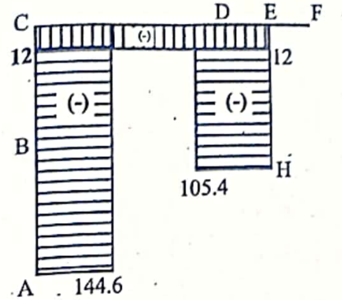
No axial force

Element EH

$(N_E)_L = 0$

$(N_E)_R = -105.4\ KN = 105.4\ (C) = (N_H)_L$

$(N_H)_R = -105.4 + 105.4 = 0$



AFD (←+→, KN)

Calculation of shear force

Element AC

$(V_A)_L = 0$

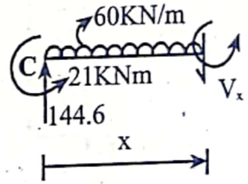
$(V_A)_R = 3KN$

$(V_B)_L = 3KN$

$(V_B)_R = 3 - 15 = -12\ KN$

$(V_C)_L = -12\ KN$

$(V_C)_R = -12 + 12 = 0$



Element CE

Portion CD

$V_x = 144.6 - 60x$

$V_x = 0$ at $x = 2.41m$

$(V_C)_L = 0$

$(V_C)_R = 144.6 - 60\times0 = 144.6\ KN$

$(V_D)_L = 144.6 - 60\times4 = -95.4\ KN$

Portion DE

$(V_D)_R = -95.4\ KN$

$(V_E)_L = -94.4\ KN$

$(V_E)_R = -94.4 + 95.4 = 0$

Element EF

$(V_E)_L = 0$

$(V_E)_R = 10\ KN$

$(V_F)_L = 10$ KN

$(V_F)_R = 10 - 10 = 0$

Element EH

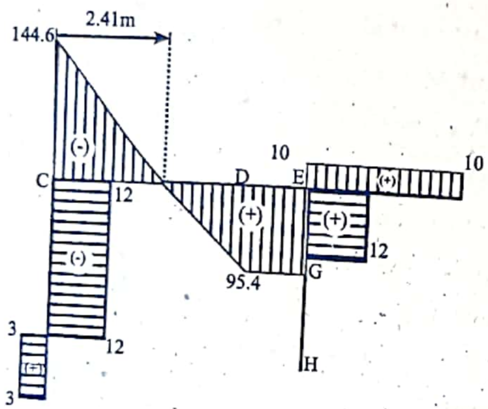
$(V_E)_L = 0$

$(V_E)_R = 12$ KN

$(V_G)_L = 12$ KN

$(V_G)_R = 12 - 12 = 0$

$V_H = 0$



SFD (↑+↓, KN)

Calculation of bending moment

Element AC

$M_A = 0$

$M_B = 3\times3 = 9$ KNm

$(M_C)_L = 3\times6 - 15\times3 = -27$ KNm

$(M_C)_R = -27 + 27 = 0$

Element CE

Portion CD

$$M_x = 144.6x - 60x \times \frac{x}{2} - 27 = 144.6x - 30x^2 - 27$$

$M_x = 0$ at $x = 0.19$ m

$M_{max} = M_{x=2.41} = 144.6\times2.41 - 30\times2.41^2 - 27 = 147.243$ KNm

$M_{mid} = M_{x=2} = 144.6\times2 - 30\times2^2 - 27 = 142.2$ KNm

$(M_C)_L = 0$

$(M_C)_R = -27$ KNm

$(M_D)_L = 144.6\times4 - 30\times4^2 - 27 = 71.4$ KNm

Portion DE

$(M_D)_R = 144.6\times4 - 60\times4\times2 - 27 = 71.4$ KNm

$(M_E)_L = 144.6\times5 - 60\times4\times(2 + 1) - 27 = -24$kNm

$(M_E)_R = -24 + 24 = 0$

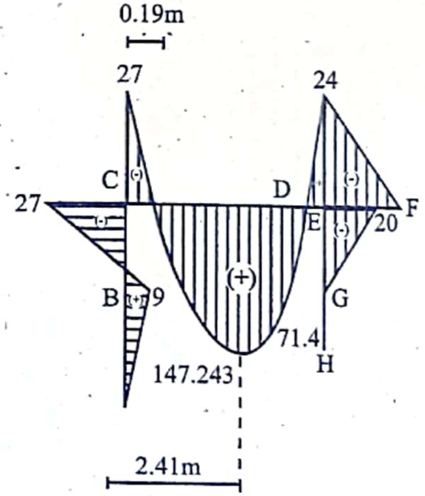
Element EF

$(M_E)_L = 0$

$(M_E)_R = -20$ KNm

$M_F = 10\times2 - 20 = 0$

$M_H = 12\times4 - 12 - 24 = 0$



BMD (↻+↺, KNm)

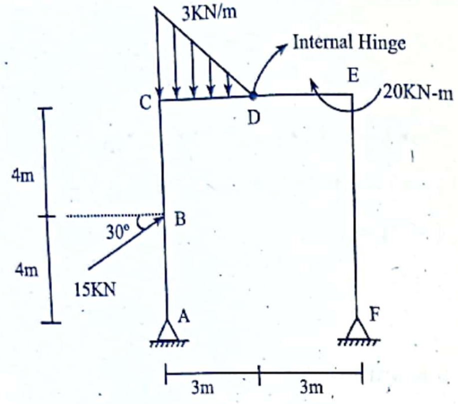
Salient features

$V_x = 0$ at $x = 2.41$ m from C

$M_x = 0$ at $x = 0.19$ m from C

$M_{max} = 147.243$ KNm

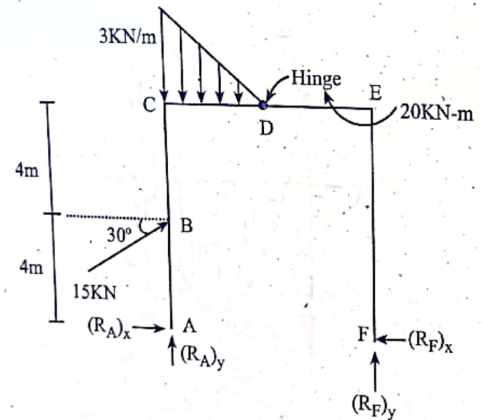
2. **Draw axial force, shear force, bending moment diagram for the loaded frame shown in figure. Indicate also the salient features if any.**



[2070 Bhadra]

Solution:

Calculation of unknown reactions



For equilibrium condition,

$(\overset{+}{\rightarrow})\Sigma F_x = 0$

or, $(R_A)_x - (R_F)_x + 15\cos 30° = 0$

or, $(R_F)_x - (R_A)_x = 12.99$ KN ....(i)

$(\circlearrowright)\sum(M_D)_R = 0$

or, $20 + (R_F)_x \times 8 - (R_F)_y \times 3 = 0$

or, $3(R_F)_y - 8(R_F)_x = 20$ ....(ii)

$(\uparrow +)\sum F_y = 0$

or, $(R_A)_y + 15\sin 30° - \frac{1}{2} \times 3 \times 3 + (R_F)_y = 0$

or, $(R_A)_y + (R_F)_y = -3$ ....(iii)

$(\circlearrowright)\sum M_A = 0$

or, $15\cos 30° \times 4 + \frac{1}{2} \times 3 \times 3 \times \frac{3}{3} + 20 - (R_F)_y \times 6 = 0$

$\therefore (R_F)_y = 12.74$ KN $(\uparrow)$

From equation (ii),

$3 \times 12.74 - 8 R_{FX} = 20$

$\therefore (R_F)_x = 2.277$ KN $(\leftarrow)$

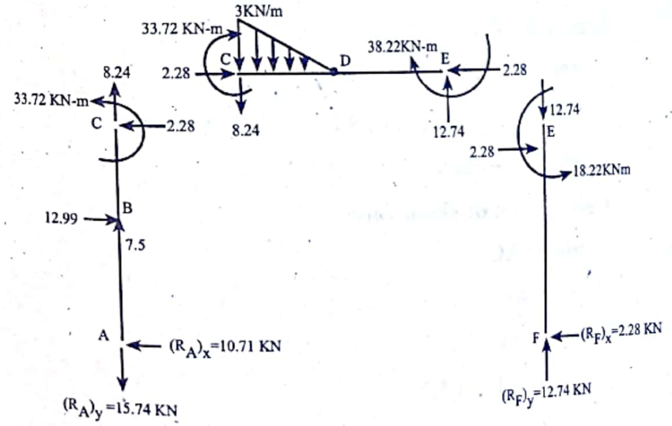
From equation (i),

$R_{(A)x} = (R_F)_x - 12.99 = 2.277 - 12.99 = -10.71$ KN $= 10.71$ KN $(\leftarrow)$

From equation (iii),

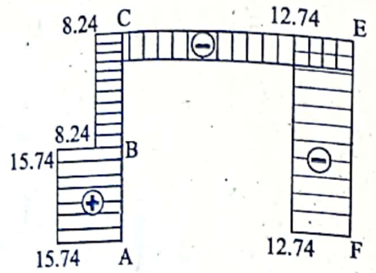
$R_{(A)y} = -3 - (R_F)_y = -3 - 12.74 = -15.74$ KN $= 15.74$ KN $(\downarrow)$

Balancing of frame



Calculation of axial force

Element AC



AFD ($\xleftrightarrow{+}$, KN)

Portion AB

$(N_A)_L = 0$

$(N_A)_R = 15.74$ KN (T) $= (N_B)_L$

Portion BC

$(N_B)_R = 15.74 - 15\sin30° = 8.24$ KN (T) $= (N_C)_L$

$(N_C)_R = 8.24 - 8.24 = 0$

Element CE

$(N_C)_L = 0$

$(N_C)_R = -2.28 = 2.28$ KN (C) $= N_D = (N_E)_L$

$(N_E)_R = 2.28 - 2.28 = 0$

Element EF

$(N_E)_L = 0$

$(N_E)_R = -12.74 = 12.74$ KN (C) $= (N_F)_L$

$(N_F)_R = -12.74 + 12.74 = 0$

Calculation of shear force

Element AC

Portion AB

$(V_A)_L = 0$

$(V_A)_R = 10.71$ KN

$(V_B)_L = 10.71$ KN

$(V_B)_R = 10.71 - 15\cos30° = -2.28$KN

Portion BC

$(V_C)_L = -2.28$ KN

$(V_C)_R = -2.28 + 2.28 = 0$ KN

Element FE(calculation from right)

$(V_F)_R = 0$

$(V_F)_L = 2.28$ KN $= (V_E)_R$

$(V_E)_L = 2.28 - 2.28 = 0$ KN

Element EC(calculation from right)

Portion ED

$(V_E)_R = 0$

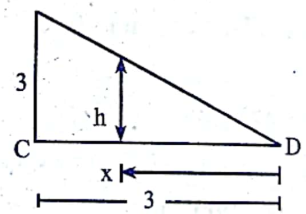
$(V_E)_L = -12.74$ KN

$V_D = -12.74$ KN

Portion DC

$$V_x = V_D + \frac{1}{2} \times x \times h = V_D + \frac{1}{2} \times x \times x$$

$$V_X = -12.74 + \frac{x^2}{2}$$



From Similar triangles,

$$\frac{h}{3} = \frac{x}{3}$$

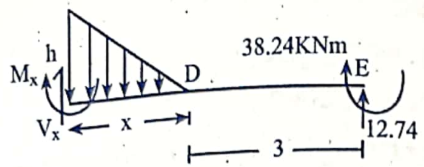
$$\therefore h = x$$

When $x = 0$, $V_D = -12.74$ KN

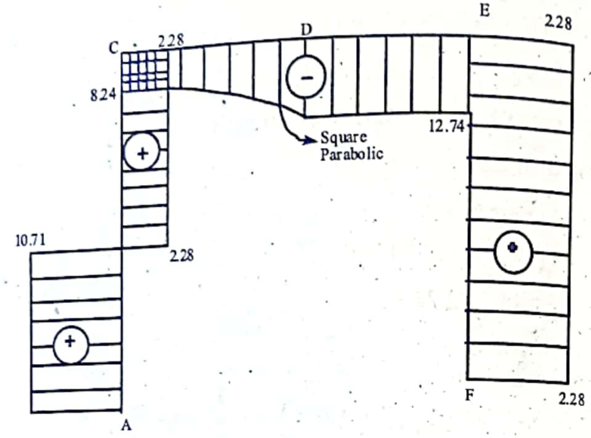
$x = 1.5$, $V_{mid} = -12.74 + \frac{1.5^2}{2} = -11.615$ KN

$x = 3$, $(V_C)_R = -12.74 + \frac{3^2}{2} = -8.24$ KN

$(V_C)_L = -8.24 + 8.24 = 0$ KN



For DC



SFD (↑+↓, KN)

Calculation of bending moment

Element AC (calculation from left)

$M_A = 0$

$M_B = 10.71 \times 4 = 42.84$ KN-m

$M_C = 10.71 \times 8 - 15\cos 30° \times 4 = 33.72$ KN-m

Element FE (calculation from F)

$M_F = 0$

$M_E = -2.28 \times 8 = -18.24$ KN-m

Element EC (calculation from E)

$M_E = -38.22$ KN-m

$M_D = -38.22 + 12.74 \times 3 = 0$

For DC, D as origin $(0 < x < 3)$

$$M_x = -38.22 + 12.74 \times (3 + x) - \frac{1}{2} \times x \times h \times \frac{x}{3}$$

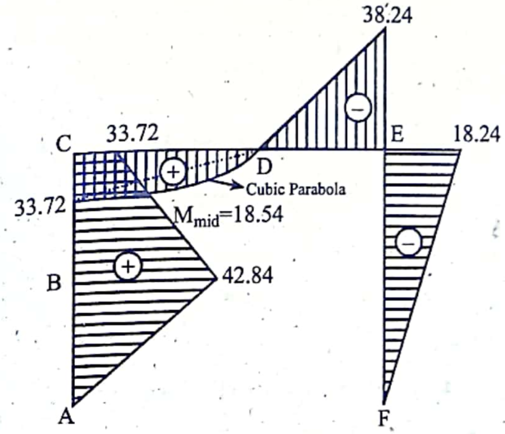
$$\text{or, } M_x = -38.22 + 38.22 + 12.74x - \frac{x^3}{6}$$

$$\text{or, } M_x = 12.74x - \frac{x^3}{6}$$

When $x = 0$, $M_D = 0$

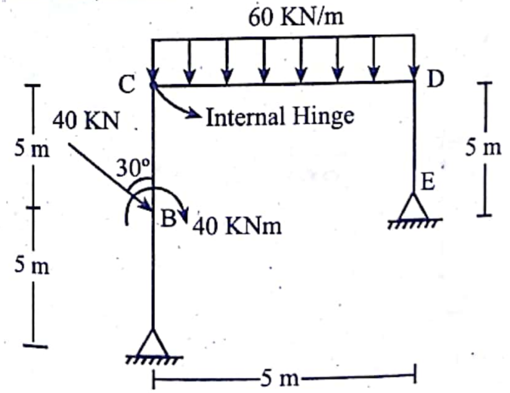
When $x = 1.5$, $M_{mid} = 12.74 \times 1.5 - \frac{1.5^3}{6} = 18.54$ KN-m

When $x = 3$, $M_C = 12.74 \times 3 - \frac{3^3}{6} = 33.72$ KN-m



BMD (↻+↺, KNm)

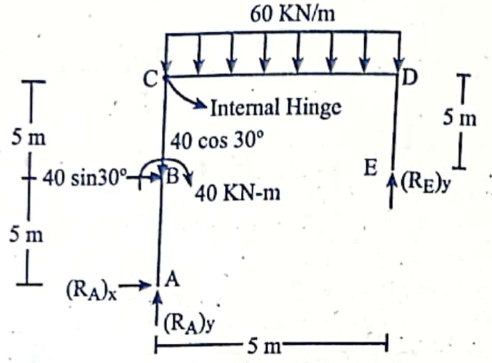
3. **Draw the axial force, shear force and bending moment diagram of the given frame.**



[2070 Ashadh]

Solution:

Calculation of unknown reactions



$(\overset{+}{\rightarrow})\ \sum F_x = 0$

or, $40\sin30° + (R_A)_x = 0$

$\therefore (R_A)_x = -40\sin30° = -20KN = 20\ KN\ (\leftarrow)$

$(\uparrow +)\ \sum F_y = 0$

or, $(R_A)_y - 40\cos30° \times 60 \times 5 + (R_E)_y = 0$

$\therefore (R_A)_y + (R_E)_y = 334.64\ KN$ ....(i)

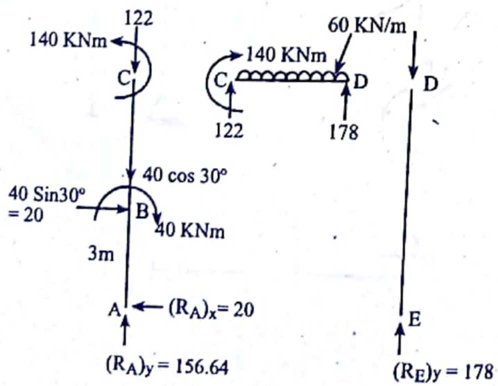
$\sum M_A = 0$

or, $5 \times 40\sin30° + 40 + 60 \times 5 \times 2.5 - (R_E)_y \times 5 = 0$

$\therefore (R_E)_y = 178\ KN$

From equation (i),

$(R_A)_y = 156.64\ KN$



Calculation of axial force

Element AC

$(N_A)_L = 0$

$(N_A)_R = (N_B)_L = -156.64\ KN$

$(N_B)_R = -156.64 + 40\cos30° = -122\ KN = (N_C)_L$

$(N_C)_R = -122 + 122 = 0$

Element CD

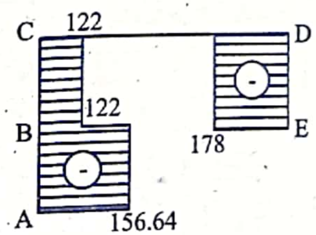
$N_C = N_D = 0$

Element DE

$(N_D)_L = 0$

$(N_D)_R = -178 = (N_E)_L$

$(N_E)_R = -178 + 178 = 0$



AFD ($\overset{+}{\longleftrightarrow}$, KN)

Calculation of shear force

Element AC

$(V_A)_L = 0$

$(V_A)_R = 20 = 20\ KN$

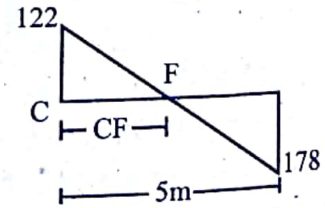
$(V_B)_L = 20\ KN$

$(V_B)_R = 20 - 40\sin30° = 0$

$(V_C)_L = (V_B)_R = 0$

$(V_C)_R = (V_C)_L = 0$

Element CD



Let F be the point of zero S.F.

$$\therefore \frac{CF}{122} = \frac{5-CF}{178}$$

or, $178CF = 122 \times 5 - 122CF$

$\therefore CF = 2.033$ m

$(V_C)_L = 0$

$(V_C)_R = 122$ KN

$(V_D)_L = 122 - 60 \times 5 = 122 - 60 \times 5 = -178$ KN

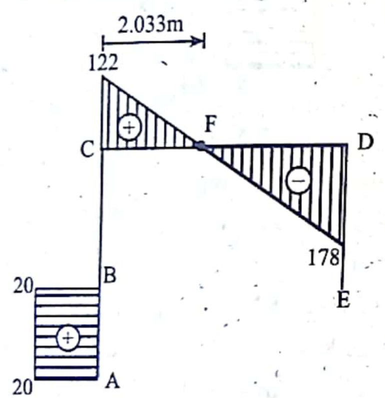
$(V_D)_R = -178 + 178 = 0$

Element DE

$(V_D)_L = 0$

$(V_D)_R = 0$

$(V_E)_L = (V_E)_R = 0$



SFD (↑+↓, KN)

Calculation of bending moment

Element AC

$M_A = 0$

$(M_B)_L = 20 \times 5 = 100$ KN-m

$(M_B)_R = 100 + 40 = 140$ KN-m

$(M_C)_L = 20 \times 10 + 40 - 20 \times 5 = 140$ KN-m

$(M_C)_R = 140 - 140 = 0$

Element CD

$M_C = 140$ KN-m

$$M_x = M_c + 122x - 60 \times x \times \frac{x}{2}$$

When $x = 0$, $M_C = 140$ KN-m

When $x = 2.033$m (at zero SF point),

$$M_{Max/Min} = 140 + 122 \times 2.033 - 60 \times \frac{2.033^2}{2} = 264.033 \text{ KN-m}$$

$\therefore M_{Max} = 264.033$ KN-m

When $x = 2.5$m,

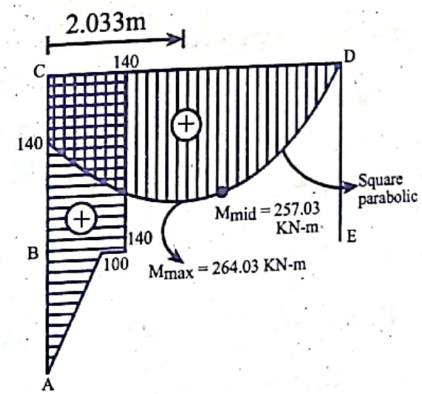
$$M_{mid} = 140 + 122 \times 2.5 - 60 \times \frac{2.5^2}{2} = 257.5 \text{ KN-m}$$

When $x = 5$m,

$$M_D = 140 + 122 \times 5 - \frac{60 \times 5^2}{2} = 0$$

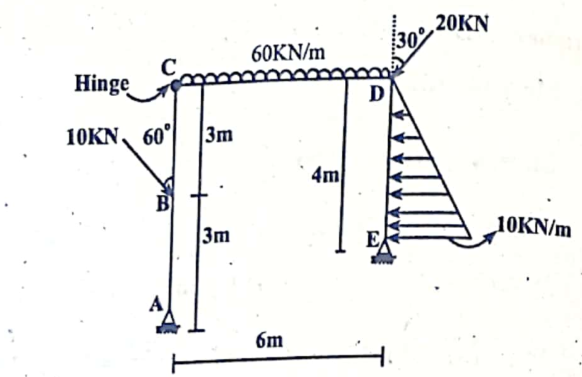
Element DE

$M_D = 0$, $M_E = 0$



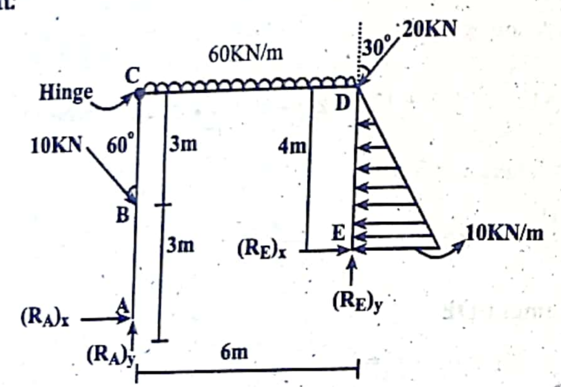
BMD (↻+↺, KNm)

4. **Draw axial force, shear force and bending moment diagram for the given frame. Also, indicate salient features, if any.**



[2071 Shrawan]

Solution:



Calculation of unknown reactions

$(\Sigma M_C)_{left} = 0$

or, $(R_A)_x \times 6 - 10\sin60° \times 3 = 0$

or, $(R_A)_x = 4.33$ KN (←)

$\left(\overset{+}{\rightarrow}\right) \Sigma F_x = 0$

or, $-(R_A)_x + 10\sin60° - \frac{1}{2} \times 10 \times 4 - 20\sin30° + (R_E)_x = 0$

or, $(R_E)_x = 25.67$ KN (→)

$(\Sigma M_A) = 0$

or, $+ 10 \sin60° \times 3 + 5 \times 6 \times 3 + 20 \cos30° \times 6 - 20 \sin30° \times 6 - (R_F)_y \times 6 + (R_E)_x \times 2 - \left(\frac{1}{2} \times 10 \times 4\right) \times \left(2 + \frac{1}{3} \times 4\right) = 0$

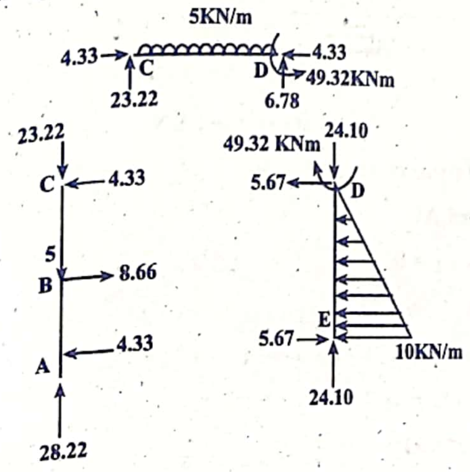
or, $(R_E)_y = 24.10$ KN (↑)

$(+\uparrow)\Sigma F_y = 0$

or, $(R_A)_y - 10 \cos60° - 5 \times 6 - 20 \cos30° + (R_E)_y = 0$

or, $(R_A)_y = 28.22$ KN (↑)

Balancing of frame



Calculation of axial force

Element AC

$(N_A)_L = 0$

$(N_A)_R = -28.22 = 28.22$ KN (C) $= (N_B)_L$

$(N_B)_R = -28.22 + 5 = -23.22 = 23.22$ KN (C) $= (N_C)_L$

$(N_C)_R = -23.22 + 23.22 = 0$

Element CD

$(N_C)_L = 0$

$(N_C)_R = (N_D)_L = -4.33$

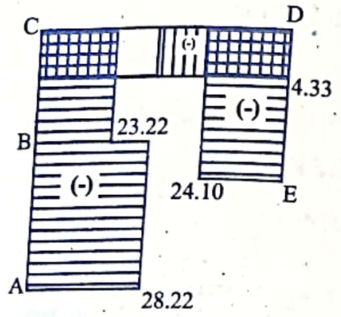
$(N_D)_R = -4.33 + 4.33 = 0$

Element DE

$(N_D)_L = 0$

$(N_D)_R = -24.10 = (N_E)_L$

$(N_E)_R = -24 + 24.10 \approx 0$



AFD ($\xleftrightarrow{+}$, KN)

Calculation of shear force

Element AC

$(V_A)_L = 0$

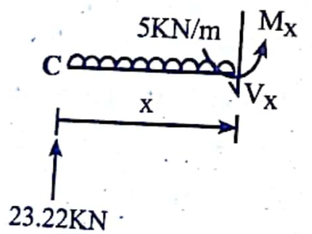
$(V_A)_R = 4.33 = (V_B)_L$

$(V_B)_R = 4.33 - 8.66 = -4.33$ KN $= (V_C)_L$

$(V_C)_R = -4.33 + 4.33 = 0$ KN

Element CD

$V_x = 23.22 - 5x$ $(0 \le x \le 6)$ (origin at C)



$(V_C)_L = 0$

At $x = 0$, $(V_C)_R = 23.22$

At $x = 6$, $(V_D)_L = 23.22 - 5 \times 6 = -6.78$KN

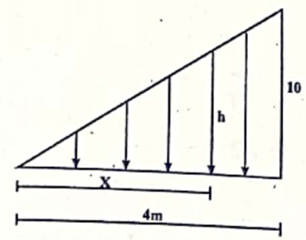
$(V_D)_R = -6.78 + 6.78 = 0$ KN

To know where $V_x = 0$, we write

$V_x = 23.22 - 5x$

or, $0 = 23.22 - 5x$

$\therefore x = \dfrac{23.22}{5} = 4.644$ m



$\dfrac{h}{x} = \dfrac{10}{4}$ or, $h = 2.5x$

Element DE

$V_x = -5.67 - \frac{1}{2} \times x \times h = -5.67 - \frac{1}{2} \times 2.5x \times x = -5.67 - 1.25\,x^2$

$(0 \le x \le 4)$ (origin at D)

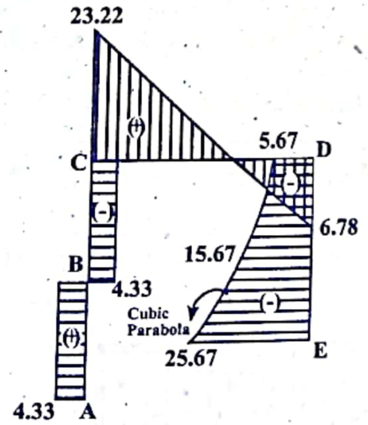
$V_x = 0$ is not possible as x comes out to be imaginary

$(V_D)_L = 0$

When $x = 0$, $(V_D)_R = -5.67$ KN

When $x = 4$, $(V_E)_L = -5.67 - 1.25 \times 4^2 = -25.67$ KN

$(V_E)_R = -25.67 + 25.67 = 0$ KN



SFD (↑+↓, KN)

Calculation of bending moment

Element AC

$M_A = 0$

$M_B = 4.33 \times 3 = 12.99$

$M_C = 4.33 \times 6 - 8.66 \times 3 = 0$

Element CD

$M_x = 23.22x - 5 \times x \times \frac{x}{2} = 23.22x - 2.5x^2 (0 \le x < 6)$ (origin at C)

$M_x = 0$ gives $23.22x - 2.5x^2 = 0$

$\Rightarrow x = 9.28$m and 0m

Since x = 6 is beam span, x = 9.28 m is not possible.

M_{max} occurs at position where $V_x = 0$ i.e., at x = 4.644m

$\therefore M_{max} = M_{x=4.644}$

$= 23.22 \times 4.644 - 2.5 \times 4.644^2 = 53.92$ KNm

At x = 0, $M_C = 0$

At x = 3, M_{mid}

$= 23.22 \times 3 - 2.5 \times 3^2 = 47.16$ KNm

At x = 6, $(M_D)_L = 23.22 \times 6 - 2.5 \times 6^2 = 49.32$ KNm

$(M_D)_R = 49.32 - 49.32 = 0$

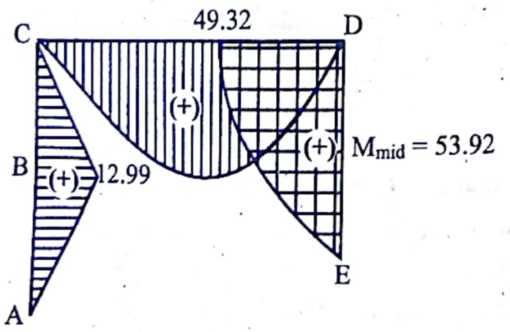
Element DE

$M_x = 49.32 - 5.67x - \left(\frac{1}{2} \times 2.5x \times x\right) \times \frac{1}{3} \times x = 49.32 - 5.67x - \frac{2.5x^3}{6}$

When x = 0, $M_D = 49.32$ KNm

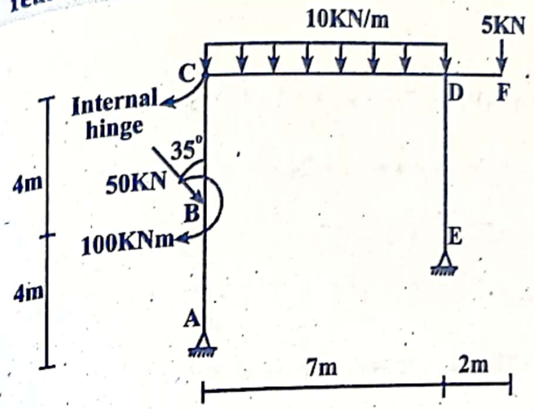
When x = 2, $M_{mid} = 49.32 - 5.67 \times 2 - \frac{2.5 \times 2^3}{6} = 34.65$ KNm

When x = 4, $M_E = 49.32 - 5.67 \times 4 - \frac{2.5 \times 4^3}{6} \approx 0$



BMD (↻+, KNm)

5. **Draw the axial force shear force and bending moment diagram of the given frame. Also, show the salient features.**



[2071 Magh]

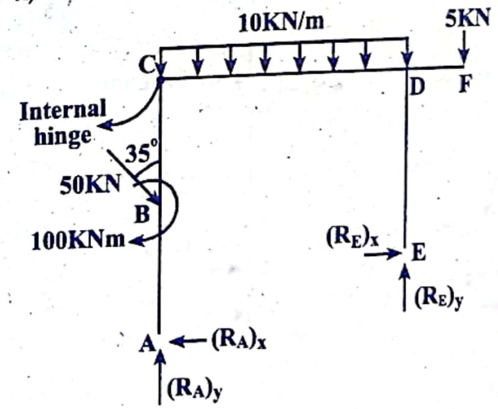
Solution:

Calculation of unknown reactions

Since C is an internal hinge, (↻+) $(\Sigma M_C)_{left} = 0$

or, $(R_A)_x \times 8 + 100 - 50 \sin 35° \times 4 = 0$

or, $(R_A)_x = 1.84$KN (←)



$(\overset{+}{\rightarrow}) \ \Sigma F_x = 0$

or, $-(R_A)_x + 50\sin 35° + (R_E)_x = 0$

or, $(R_E)_x = -50 \sin 35° + (R_A)_x$

or, $(R_E)_x = -50 \sin 35° + 1.84$

$\therefore (R_E)_x = -26.84 \text{ KN} = 26.84 \text{ KN} (\leftarrow)$

Also, $(\circlearrowleft +)\ (\Sigma M_C)_{right} = 0$

or, $(R_E)_x \times 4 - (R_E)_y \times 7 + 5 \times 9 + 10 \times 7 \times 3.5 = 0$

or, $(R_E)_y \times 7 = 26.84 \times 4 + 45 + 245$

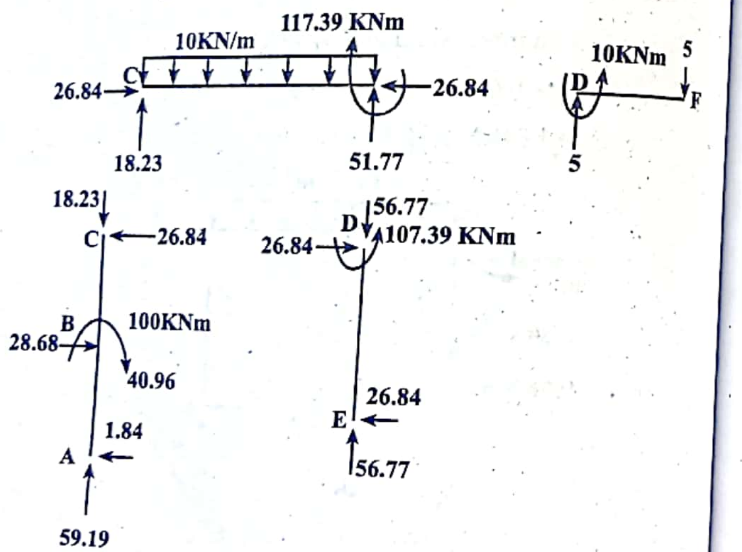
$\therefore (R_E)_y = 56.77 \text{ KN} (\uparrow)$

$(+\uparrow) \Sigma F_y = 0$

or, $(R_A)_y + (R_E)_y - 50 \cos 35° - 10 \times 7 - 5 = 0$

or, $(R_A)_y = 59.19 \text{KN} (\uparrow)$

Balancing of frame



Calculation of axial force

Element AC

$(N_A)_L = 0$, $(N_A)_R = -59.19 \text{ KN} = 59.19 \text{KN(C)} = (N_B)_L$

$(N_A)_R = 40.96 - 59.19 = -18.23 = 18.23 \text{ (C)} = (N_C)_L$

$(N_C)_R = -18.23 + 18.23 = 0 \text{KN}.$

Element CD

$(N_C)_L = 0 \text{KN}$, $(N_C)_R = -26.84 \text{ KN} = 26.84 \text{KN(C)} = (N_D)_L$

$(N_D)_R = -26.84 + 26.84 = 0 \text{KN}$

Element DF

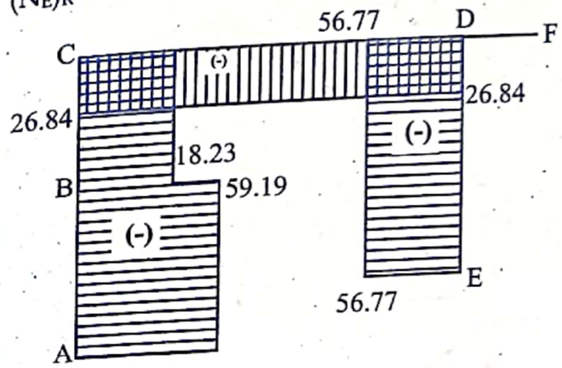
No axial force

Element DE

$(N_D)_L = 0$

$(N_D)_R = -56.77 = 56.77 \text{ KN (C)} = (N_E)_L$

$(N_E)_R = -56.77 + 56.77 = 0$



AFD ($\overset{+}{\longleftrightarrow}$, KN)

Calculation of shear force

Element AC

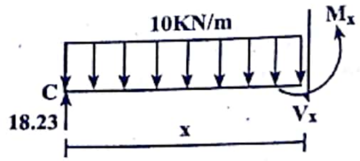
$(V_A)_L = 0 \text{ KN}$

$(V_A)_R = 1.84 \text{ KN} = (V_B)_L$

$(V_B)_R = 1.84 - 28.68 = -26.84 \text{ KN} = (V_C)_L$

$(V_C)_R = -26.84 + 26.84 = 0 \text{ KN}$

Element CD



$V_x = 18.23 - 10x$

To find x where $V_x = 0$

$18.23 - 10x = 0$ or, $x = 1.823$ m

Thus, $V_x = 0$ at $x = 1.823$ m

$(V_C)_L = 0$

At $x = 0$, $(V_C)_R = 18.23$

At $x = 7$, $(V_D)_L = 18.23 - 7\times10 = -51.67$ KN

$(V_D)_R = -51.67 + 51.67 = 0$ KN

Element DF

$(V_D)_L = 0$

$(V_D)_R = 5$ KN $= (V_F)_L$

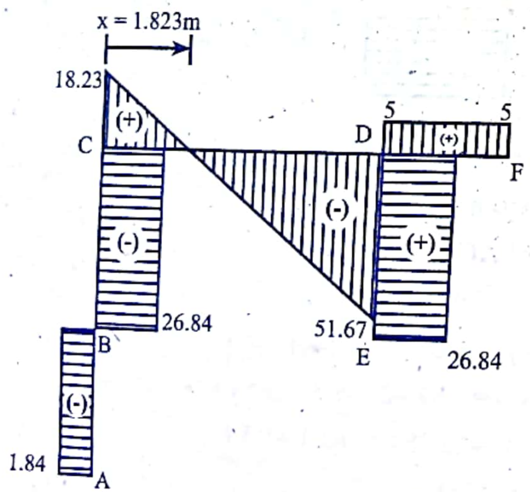
$(V_F)_R = 5 - 5 = 0$ KN

Element DE

$(V_D)_L = 0$

$(V_D)_R = 26.84$ KN$= (V_E)_L$

$(V_E)_R = 26.84 - 26.84 = 0$



SFD (↑+↓, KN)

Calculation of bending moment

Element AC

$M_A = 0$

$(M_B)_L = 1.84 \times 4 = 7.36$ KNm

$(M_B)_R = 1.84 \times 4 + 100 = 107.36$ KNm

$M_C = 1.84 \times 8 + 100 - 28.68\times4 = 0$ KNm

Element CD

$M_x = 18.23x - 10x \times \frac{x}{2} = 18.23x - 5x^2$

For value of x where M_x is zero, we have

$18.23x - 5x^2 - 0$

$\therefore x = 0, 3.645$ m

BM is maximum where SF is zero (i.e., at x = 1.823)

$M_{max} = M_{x=1.823}$

$= 18.23 \times 1.823 - 5 \times 1.823^2 = 16.62$ KNm

$M_{mid} = M_x$

$= 3.5 = 18.23 \times 3.5 - 5\times3.5^2 = 2.56$KNm

At $x = 7$m, $(M_D)_L = 18.23 \times 7 - 5\times7^2 = -117.39$ KNm

$(M_D)_R = -117.39 + 117.39 = 0$ KNm

Element DF

$(M_D)_L = 0$, $(M_D)_R = -10$KNm

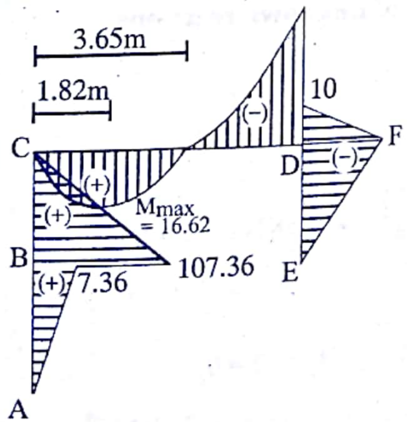
$M_F = -10 + 5 \times 2 = 0$

Element DE

$(M_D)_L = 0$ KNm

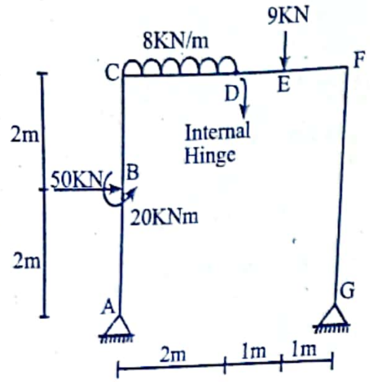
$(M_D)_R = -107.39$ KNm.

$(M_E) = -107.39 + 26.84 \times 4 = 0$



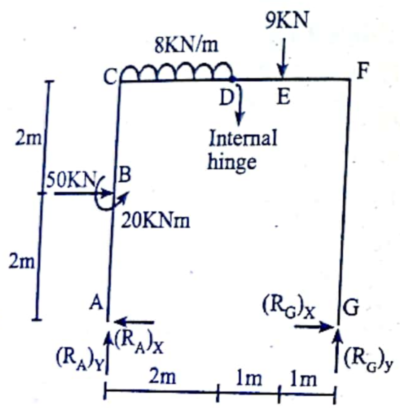
BMD (↶+↷, KNm)

6. **A frame is loaded as shown in the figure. Draw AFD, SFD, and BMD. Also show the salient features of each diagram.**



[2071 Chaitra]

Solution:



Calculation of unknown reactions

$(\circlearrowleft+)\Sigma M_G = 0$

or, $(R_A)_y \times 4 + 50 \times 2 - 20 - \times 2 \times 3 - 9 \times 1 = 0$

or, $(R_A)_y = \frac{-23}{4} = -5.75KN = 5.75\ KN\ (\downarrow)$

$(+\uparrow)\Sigma F_y = 0$

or, $(R_A)_y + (R_G)_y - 9 - 8 \times 2 = 0$

or, $(R_G)_y = 9 + 16 + 5.75 = 30.75\ KN\ (\uparrow)$

Since D is an internal hinge,

$(\circlearrowleft+)\Sigma(M_D)_{right} = 0$

or, $-(R_G)_y \times 2 + 9 \times 1 - (R_G)_x \times 4 = 0$

or, $(R_G)_x = \left(\frac{-30.75 \times 2 + 9 \times 1}{4}\right) = -13.125 = 13.125\ KN\ (\leftarrow)$

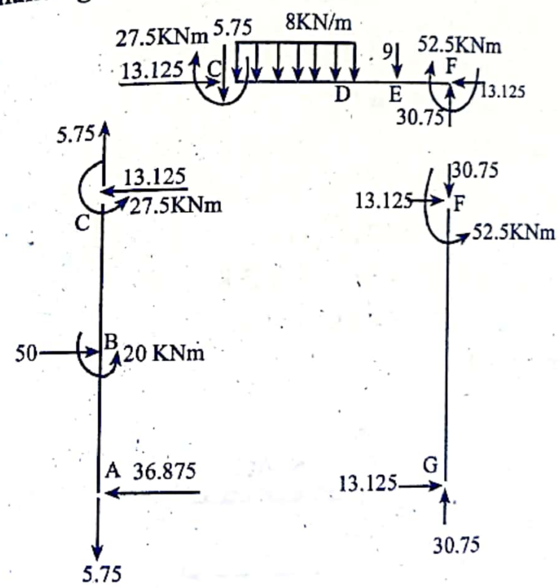
$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $-(R_A)_x + (R_G)_x + 50 = 0$

or, $-(R_A)_x - 13.125 + 50 = 0$

or, $(R_A)_x = 36.375\ KN\ (\leftarrow)$

Balancing of frame



Calculation of axial force

Element AC

$(N_A)_L = 0$, $(N_A)_R = 5.75 = 5.75\ KN\ (T) = N_B = (N_C)_L$

$(N_C)_R = 5.75 - 5.75 = 0$

Element CF

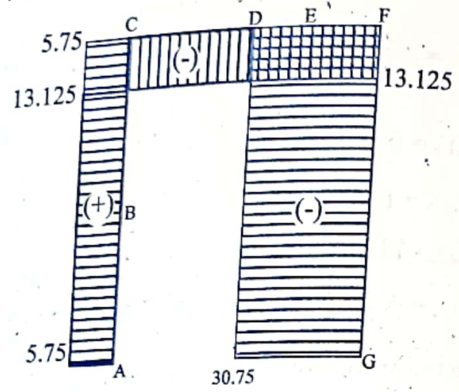
$(N_C)_L = 0$, $(N_C)_R = -13.125 = 13.125KN\ (C) = N_D = N_E = (N_F)_L$

$(N_F)_R = -13.125 + 13.125 = 0\ KN$

Element FG

$(N_F)_L = 0$, $(N_F)_R = -30.75 = 30.75$ KN (C) $= (N_G)_L$

$(N_G)_R = -30.75 + 30.75 = 0$



AFD ($\xleftrightarrow{+}$, KN)

Calculation of shear force

Element AC

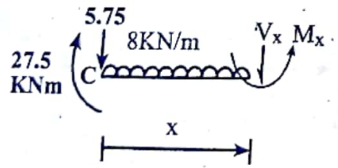
$(V_A)_L = 0$

$(V_A)_R = 36.875$ KN $= (V_B)_L$

$(V_B)_R = 36.875 - 50 = -13.125$ KN $= (V_C)_L$

$(V_C)_R = -13.125 + 13.25 = 0$KN

Element CF



Portion CD

$V_x = -5.75 - 8x$

For location of a point where S.F. is zero

$V_x = 0$

or, $-5.75 - 8x = 0$

$\therefore x = -0.72$m (which is not within the span CF,so it is neglected)

$(V_C)_L = 0$

When $x = 0$, $(V_C)_R = -5.75$KN

When $x = 2$, $V_D = -5.75 - 8 \times 2 = -21.75$ KN

Portion DF

$V_D = (V_E)_L = -21.75$ KN

$(V_E)_R = -21.75 - 9 = -30.75$KN $= (V_F)_L$

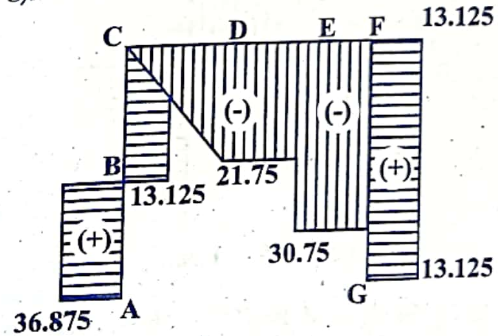
$(V_F)_R = -30.75 + 30.75 = 0$ KN

Element FG

$(V_F)_L = 0$ KN

$(V_F)_R = 13.125$ KN $= (V_G)_L$

$(V_G)_R = 13.125 - 13.125 = 0$



SFD (1+↓, KN)

Calculation of bending moment

Element AC

$M_A = 0$

$(M_B)_L = 36.875 \times 2 = 73.75$ KNm

$(M_B)_R = 73.75 - 20 = 53.75$ KNm

$(M_C)_L = 36.375 \times 4 - 20 - 50 \times 2 = 27.5$ KNm

$(M_C)_R = 27.5 - 27.5 = 0$ KNm

Element CF

Portion CD

$(M_C) = 0$

$(M_C)_R = 27.5$ KNm

$M_D = -5.75 \times 2 - 8 \times 2 \times 1 + 27.5 = 0$

(OK since D is an internal hinge)

$M_{mid} = -5.75 \times 1 + 27.5 - 8 \times 1 \times 0.5 = 17.75$ KNm

Portion DF

$M_D = 0$

$M_E = -5.75 \times 3 - 8 \times 2 \times 2 + 27.5 = 21.75$ KNm

$(M_F)_L = -5.75 \times 4 - 8 \times 2 \times 3 - 9 \times 1 + 27.5 = -52.5$ KNm

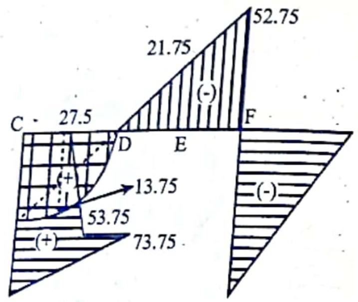
$(M_F)_R = -52.5 + 52.5 = 0$

Element FG

$(M_F)_L = 0$

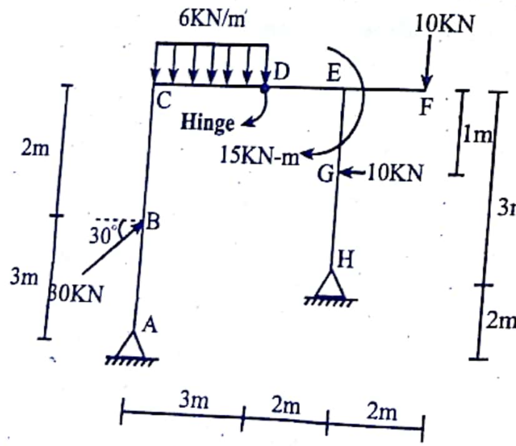
$(M_F)_R = -52.5$ KNm

$M_G = -52.5 + 13.125 \times 4 = 0$



BMD (↻+, KNm)

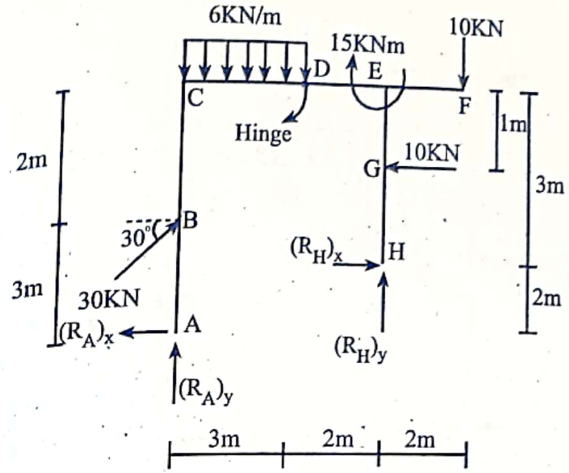
7. **Draw AFD, SFD, and BMD of the given frame. Indicate salient features, if any.**



[2072 Ashwin]

Solution:

Calculation of unknown reactions



(↻+) $\Sigma(M_D)_{right} = 0$

or, $(10\times4) + 15 + (10\times1) - (R_H)_x\times 3 - (R_H)_y\times 2 = 0$

or, $3(R_H)_x + 2(R_H)_y = 65$ ....(i)

(↻+) $\Sigma(M_A) = 0$

or, $(R_H)_x\times2-(R_H)_y\times5+30\cos30°\times3+6\times3\times1.5+15+10\times 7 - 10 \times 4 = 0$

or, $2(R_H)_x - 5(R_H)_y = -149.94$ ....(ii)

On solving equations (i) and (ii), we get

$(R_H)_x = 1.32$ KN (→)

$(R_H)_y = 30.52$ KN (↑)

$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $(R_H)_x + 30\cos30° - 10 - (R_A)_x = 0$

or, $1.32 + 30\cos30° - 10 = (R_A)_x$

or, $(R_A)_x = 17.3$ KN (←)

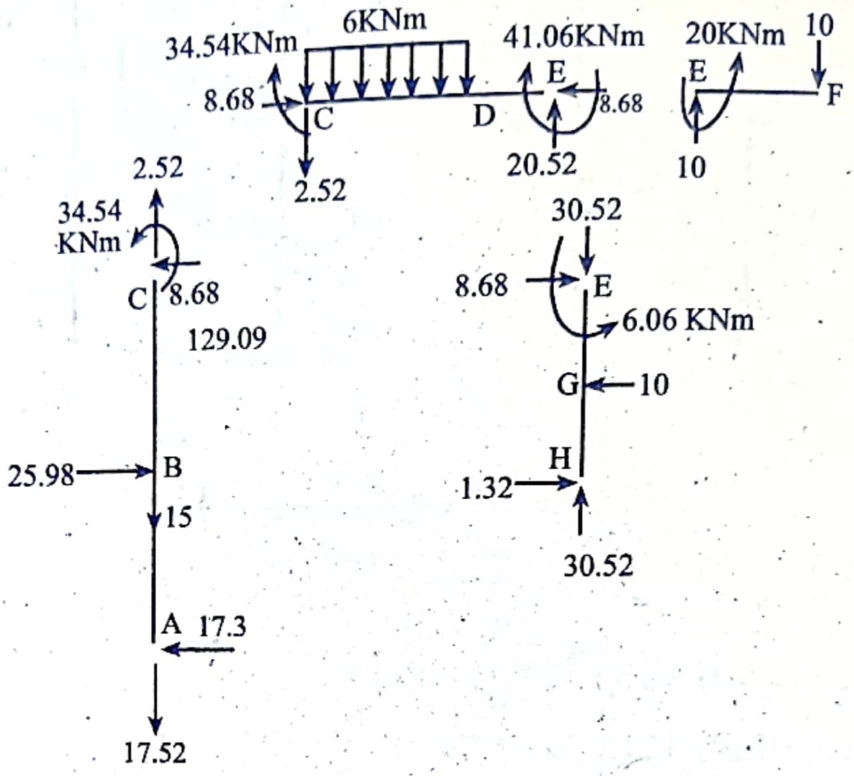
$(+\uparrow)\Sigma F_y = 0$

or, $(R_A)_y + (R_H)_y + 30\sin30° - 6\times 3 - 10 = 0$

or, $(R_A)_y + 30.52 + 30\sin30° - 18 - 10 = 0$

or, $(R_A)_y = 17.52$ KN $(\downarrow)$

Balancing of frame



Calculation of axial force

Element AC

$(N_A)_L = 0$

$(N_A)_R = 17.52\text{ KN} = (N_B)_L$

$(N_B)_R = 17.52 - 15 = 2.52 = (N_C)_L$

$(N_C)_R = 2.52 - 2.52 = 0$

Element CE

$(N_C)_L = 0$

$(N_C)_R = 8.68\text{ KN (C)} = N_D = (N_E)_L$

$(N_E)_R = 8.68 - 8.68 = 0$

Element EF

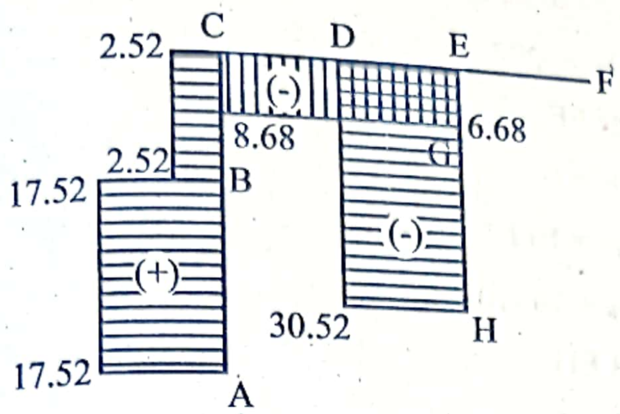
No axial force

Element EH

$(N_E)_L = 0$

$(N_E)_R = -30.52 \text{ KN} = N_G = (N_H)_L$

$(N_H)_R = -30.52 + 30.52 = 0$



AFD ($\overset{+}{\longleftrightarrow}$, KN)

Calculation of shear force

Element AC

$(V_A)_L = 0$

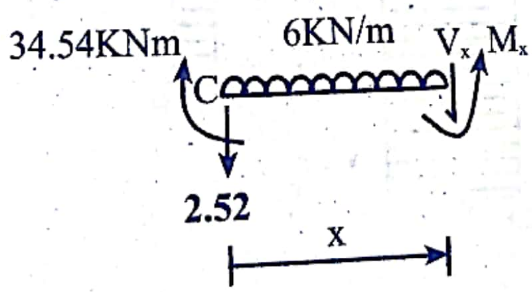
$(V_A)_R = 17.3 \text{ KN} = (V_B)_L$

$(V_B)_R = 17.3 - 25.98 = -8.68 \text{ KN} = (V_C)_L$

$(V_C)_R = -8.68 + 8.68 = 0$

Element CE

Portion CD



$V_x = -2.52 - 6x$

For x where $V_x = 0$

$-2.52 - 6x = 0$

$\therefore x = -0.42$ m which is not possible.

$(V_C)_L = 0$

$(V_C)_R = -2.52 - 6\times3 = -20.52$ KN

Portion DE

$V_D = -20.52\ KN = (V_E)_L$

$(V_E)_R = -20.52 + 20.52 = 0$

Element EF

$(V_E)_L = 0$

$(V_E)_R = 10\ KN = (V_F)_L$

$(V_F)_R = 10 - 10 = 0$

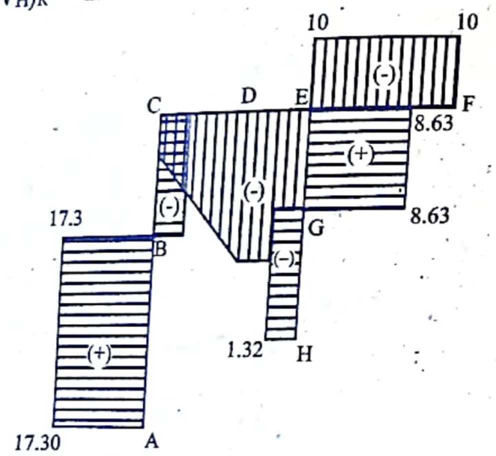
Element EH

$(V_E)_L = 0$

$(V_E)_R = 8.68 = (V_G)_L$

$(V_G)_R = 8.68 - 10 = -1.32\ KN = (V_H)_L$

$(V_H)_R = -1.32 + 1.32 = 0$



SFD (↑+↓, KN)

Calculation of bending moment

Element AC

$M_A = 0$

$M_B = 17.3 \times 3 = 51.9$ KNm

$(M_C)_L = 17.3\times5 - 25.98 \times 2 = 34.54$ KNm

$(M_C)_R = 34.54 - 34.54 = 0$

Element CE

Portion CD

$M_x = +34.54 - 2.52x - 3x^2$

For x where $M_x = 0$

i.e., $34.54 - 2.52x - 3x^2 = 0$

$x = 3m$

$(M_A)_L = 0$

When $x = 0$, $(M_A)_R = 34.54$ KNm

When $x = 1.5$, $M_{mid} = 34.54 - 2.52 \times 1.5 - 3\times1.5^2 = 24.01$ KNm

When $x = 3$, $M_D = 0$

Portion DE

$M_D = 0$

$(M_E)_L = 34.54 - 2.52\times 5 - 6\times 3\times3.5 = 41.06$ KNm

$(M_E)_R = 41.06 - 41.06 = 0$

Element EF (calculation from right)

$M_F = 0$

$(M_E)_R = -10\times2 = -20$ KNm

$(M_E)_L = -20 + 20 = 0$

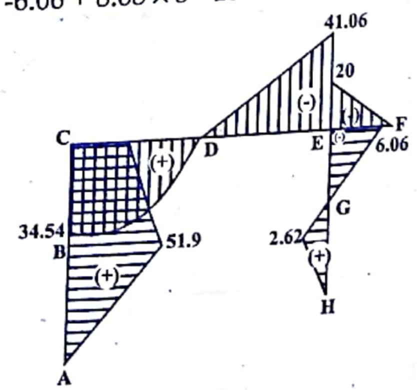
Element EH

$(M_E)_L = 0$

$(M_E)_R = -41.06 + 20 + 15 = 6.06$ KNm

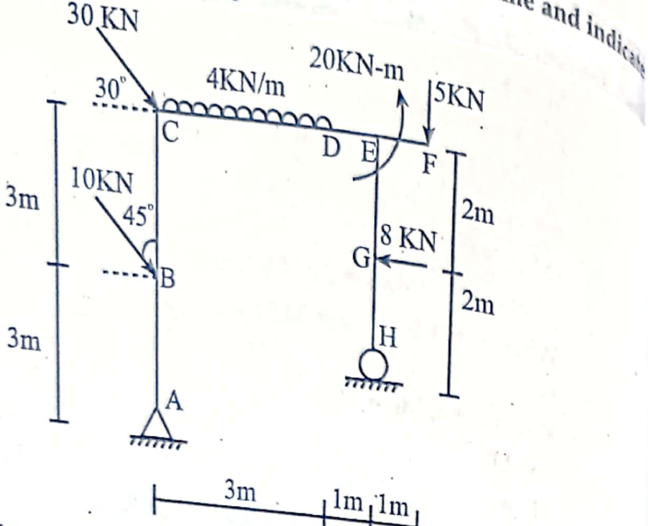
$M_G = -6.06 + 8.68 \times 1 = 2.62$ KNm

$M_H = -6.06 + 8.68 \times 3 - 10\times2 \approx 0$



BMD (↻+↺, KNm)

8. **Draw AFD, SFD and BMD of the given frame and indicate salient feature, if any.**



[2072 Kartik]

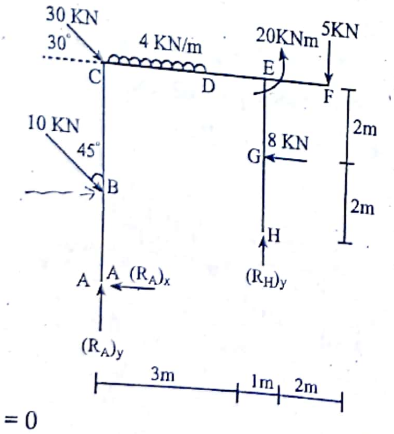
Solution:

Calculation of unknown reactions

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $10 \sin 45° + 30 \cos 30° - 8 - (R_A)_x = 0$

$\therefore (R_A)_x = 25.05$ KN $(\leftarrow)$



$(+\uparrow)\ \Sigma F_y = 0$

or, $(R_A)_y - 10\cos 45° - 30\sin 30° - 4\times 3 - 5 + (R_H)_y = 0$

or, $(R_A)_y + (R_H)_y = 39.07$ KN ……(i)

$(\circlearrowleft +)\ \Sigma M_A = 0$

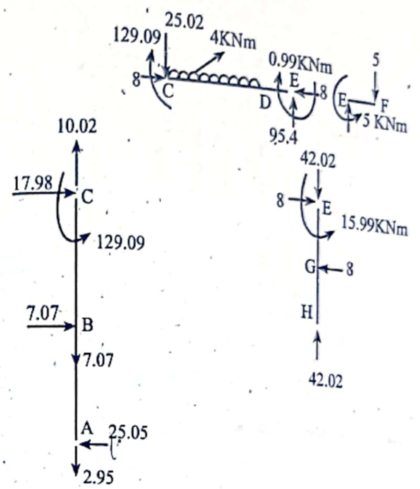
or, $10 \sin 45° \times 3 + 30\cos 30° \times 6 + 4\times 3\times 1.5 - 20 + 5\times 5 - 8\times 4 - (R_H)_y \times 4 = 0$

or, $(R_H)_y = 42.02$ KN $(\uparrow)$

Using equation (i),

$(R_A)_y = 39.07 - 42.02 = -2.95 = 2.95$ KN $(\downarrow)$

Balancing of frame



Calculation of axial force

Element AC

Portion AB

$(N_A)_L = 0$

$(N_A)_R = 2.95$ KN $= (N_B)_L$

Portion BC

$(N_B)_R = 2.95 + 7.07 = 10.02$ KN $= (N_C)_L$

$(N_C)_R = 10.02 - 10.02 = 0$

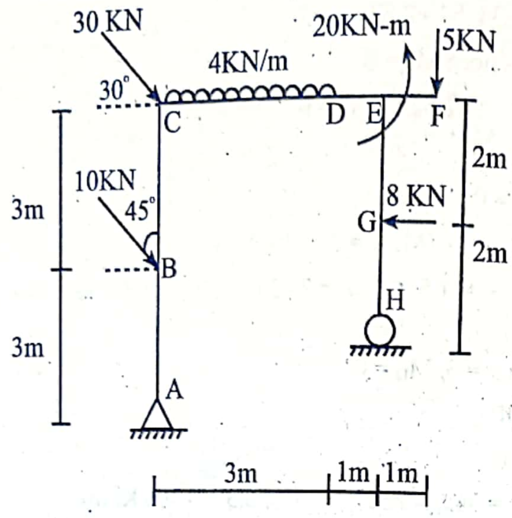
Element CE

$(N_C)_L = 0$

$(N_C)_R = -8$ KN $= (N_D) = (N_E)_L$

$(N_E)_R = -8 + 8 = 0$

8. **Draw AFD, SFD and BMD of the given frame and indicate salient feature, if any.**



[2072 Kartik]

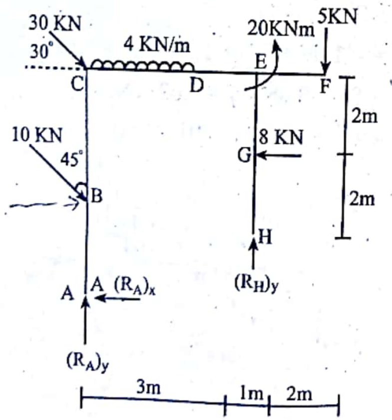
Solution:

Calculation of unknown reactions

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $10\sin 45° + 30\cos 30° - 8 - (R_A)_x = 0$

$\therefore (R_A)_x = 25.05$ KN $(\leftarrow)$



$(+\uparrow)\ \Sigma F_y = 0$

or, $(R_A)_y - 10\cos 45° - 30\sin 30° - 4\times 3 - 5 + (R_H)_y = 0$

or, $(R_A)_y + (R_H)_y = 39.07$ KN ....(i)

$(\circlearrowleft +)\ \Sigma M_A = 0$

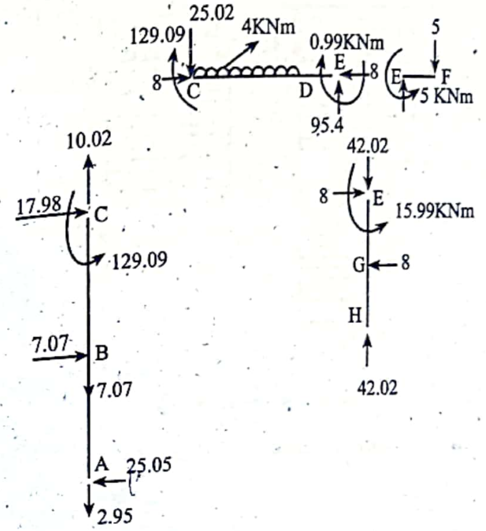
or, $10\sin 45°\times 3 + 30\cos 30°\times 6 + 4\times 3\times 1.5 - 20 + 5\times 5 - 8\times 4 - (R_H)_y\times 4 = 0$

or, $(R_H)_y = 42.02$ KN $(\uparrow)$

Using equation (i),

$(R_A)_y = 39.07 - 42.02 = -2.95 = 2.95$ KN $(\downarrow)$

Balancing of frame



Calculation of axial force

Element AC

Portion AB

$(N_A)_L = 0$

$(N_A)_R = 2.95$ KN $= (N_B)_L$

Portion BC

$(N_B)_R = 2.95 + 7.07 = 10.02$ KN $= (N_C)_L$

$(N_C)_R = 10.02 - 10.02 = 0$

Element CE

$(N_C)_L = 0$

$(N_C)_R = -8$ KN $= (N_D) = (N_E)_L$

$(N_E)_R = -8 + 8 = 0$

Element EF

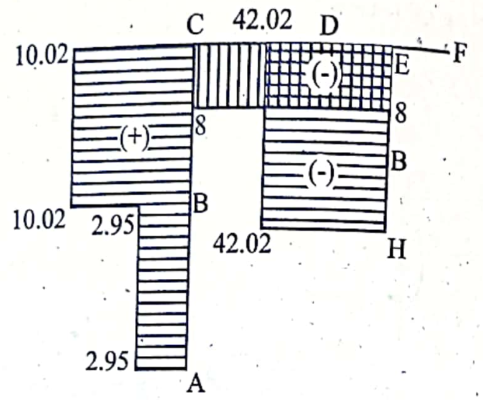
No axial force

Element EH

$(N_E)_L = 0$

$(N_E)_R = -42.02$ KN $= N_G = (N_H)_L$

$(N_H)_R = -42.02 + 42.02 = 0$



AFD ($\overset{+}{\longleftrightarrow}$, KN)

Calculation of shear force

Element AC

$(V_A)_L = 0$ KN

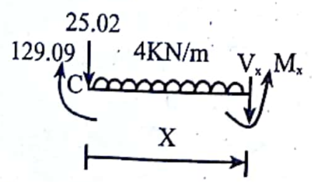
$(V_A)_R = 25.05$ KN $= (V_B)_L$

$(V_B)_R = 25.05 - 7.07 = 17.98$ KN $= (V_C)_L$

$(V_C)_R = 17.98 - 17.98 = 0$

Element CE

Portion CD



$V_x = -25.02 - 4x$

$(V_C)_L = 0$

When $x = 0$, $(V_C)_R = -25.02$ KN

For x where $V_x = 0$, we write

$-25.02 - 4x = 0$

or, $x = -6.255$ m which is not possible.

$\therefore$ V_x is not zero within span CD.

When $x = 3$, $V_D = -25.02. 4\times3 = -37.02$ KN

Portion DE

$V_D = -37.02$ KN $= (V_E)_L$

$(V_E)_R = -37.02 + 37.02 = 0$

Element EF

$(V_E)_L = 0$

$(V_E)_R = 5$ KN $= (V_F)_L$

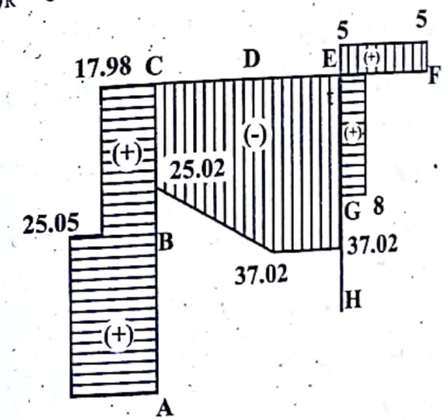
$5 - 5 = 0$

Element EH

$(V_E)_L = 0$

$(V_E)_R = 8$ KN $= (V_G)_L$

$(V_G)_R = 8 - 8 = 0$



SFD (↑+↓, KN)

Calculation of bending moment

Element AC

$M_A = 0$

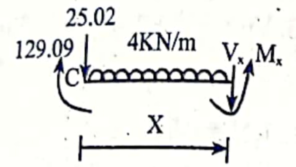
$M_B = 25.05 \times 3 = 75.15$ KNm

$(M_C)_L = 25.05 \times 6 - 7.07 \times 3 = 129.09$ KNm

$(M_C)_R = 129.09 - 129.09 = 0$

Element CE

Portion CD



$(M_C)_L = 0$

$M_x = 129.09 - 25.02x - 4 \times x \times \frac{x}{2} = 129.09 - 25.02x - 2x^2$

For x where $M_x = 0$

$129.09 - 25.02x - 2x^2 = 0$

or, $x = 3.92$ m , -16.44m

Of these two values, negative value is neglected.

When $x = 3$, $M_D = 129.09 - 25.02 \times 3 - 2 \times 3^2 = 36.03$ KNm

Portion DE

$M_D = 36.03$ KNm

$(M_E)_L = 129.09 - 25.02 \times 4 - 4 \times 3 \times 2.5 = -0.99$ KNm

$(M_E)_R = -0.99 + 0.99 = 0$

Element EF

$(M_E)_L = 0$

$(M_E)_R = -5$KNm

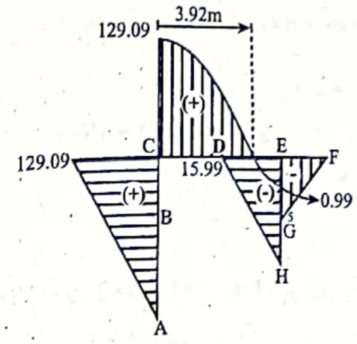
$(M_F) = -5 + 5\times1 = 0$

Element EH

$(M_E)_L = 0$

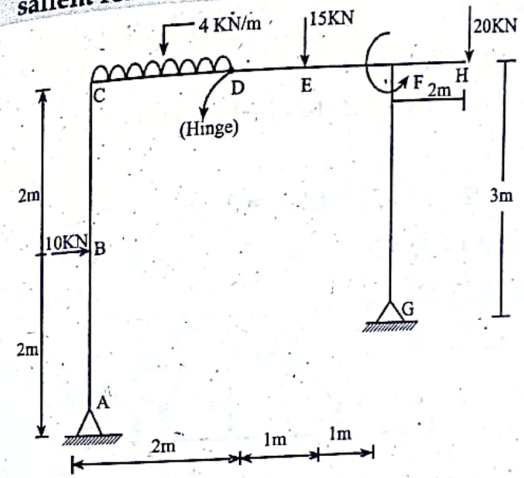
$(M_E)_R = 5 - 0.99 - 20 = -15.99$ KNm

$M_G = -15.99 + 8\times 2 \approx 0$



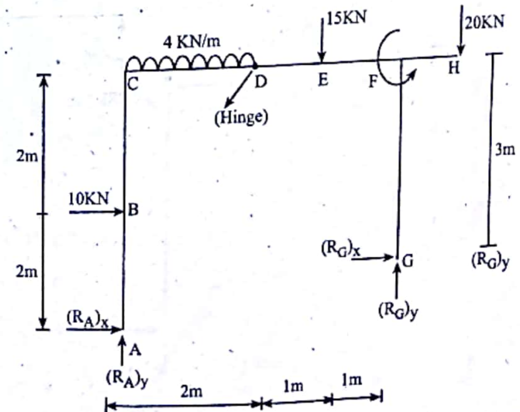
BMD (↺+, KNm)

9. Draw the axial force, shear force, and bending moment diagram for the given frame shown below. Also, show the salient features.



[2072 Chaitra]

Solution:



Calculation of unknown reactions

$(\circlearrowleft+)\Sigma(M_D)_{left} = 0$

or, $-4 \times 2 \times 1 - 10 \times 2 - (R_A)_x \times 4 + (R_A)_y \times 2 = 0$

or, $2(R_A)_x - (R_A)_y = -14$ .... (i)

$(\circlearrowleft+)\Sigma M_G = 0$

or, $(R_A)_y \times 4 - (R_A)_x \times 1 + 10\times1 - 4\times2\times3 - 15\times1 - 20 + 20\times2 = 0$

or, $(R_A)_x - 4(R_A)_y = -9$ ....: (ii)

On solving equations (i) and (ii)

$(R_A)_x = -6.71$ KN = 6.71 KN $(\leftarrow)$

$(R_A)_y = 0.57$ KN$(\uparrow)$

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $-6.71 + 10 + (R_G)_x = 0$

or, $(R_G)_x = -3.29$ KN = 3.29 KN $(\leftarrow)$

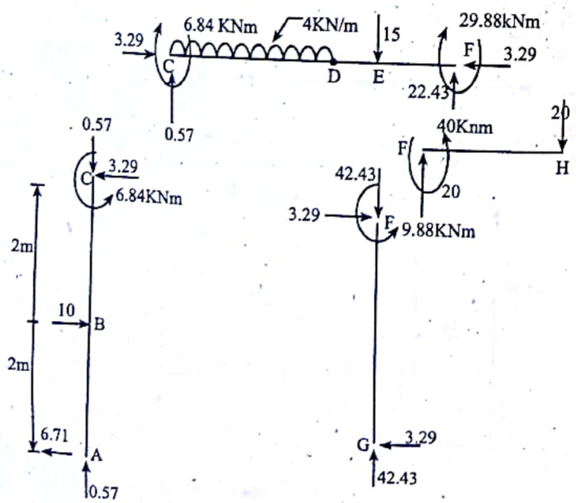
$(+\uparrow)\ \Sigma F_y = 0$

or, $(R_A)_y - 4 \times 2 - 15 - 20 + (R_G)_y = 0$

or, $0.57 - 43 + (R_G)_y = 0$

or, $(R_G)_y = 42.43$ KN $(\uparrow)$

Balancing of frame



Calculation of axial force

Element AC

$(N_A)_L = 0$

$(N_A)_R = -0.57$ KN

$(N_B) = -0.57$KN

$(N_C)_L = -0.57$KN

$(N_C)_R = -0.57 + 0.57 = 0$

Element CF

$(N_C)_L = 0$

$(N_C)_R = -3.29$KN

$(N_E) = (N_F)_L = -3.29$KN

$(N_F)_R = -3.29 + 3.29 = 0$

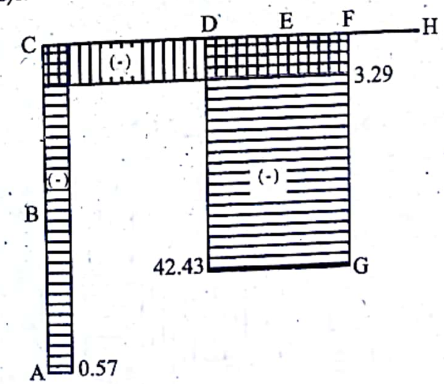
Element FH

No axial force

Element FG

$(N_F)_L = 0$, $(N_F)_R = -42.43$KN $= (N_G)_L$

$(N_G)_R = -42.43 + 42.43 = 0$



AFD ($\overset{+}{\longleftrightarrow}$, KN)

Calculation of shear force

Element AC

$(V_A)_L = 0$

$(V_A)_R = 6.71$KN

$(V_B)_L = 6.71KN$

$(V_B)_R = 6.71 - 10 = -3.29KN = (V_C)_L$

$(V_C)_R = -3.29 + 3.29 = 0$

Element CF

Portion CD

$(V_C)_L = 0$

$(V_C)_R = 0.57\ KN$

For SFD of UDL, it is better to find shear force at its mid-point.

$(V_{CD})_{midpoint} = 0.57 - 4 \times 1 = -3.43\ KN$

$V_D = 0.57 - 4 \times 2 = -7.43\ KN$

Portion DF

$V_D = -7.43\ KN = (V_E)_L$

$(V_E)_R = -7.43 - 15 = -22.43\ KN = (V_F)_L$

$(V_F)_R = -22.43 + 22.43 = 0$

Element FH

$(V_F)_L = 0$

$(V_F)_R = 20KN = (V_H)_L$

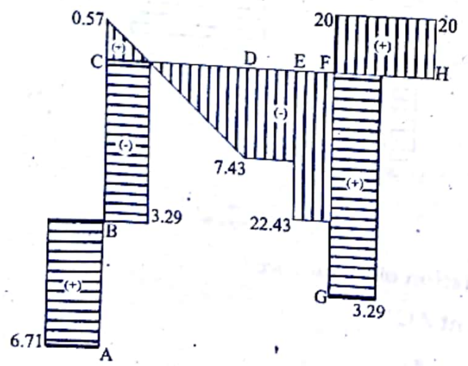
$(V_H)_R = 20 - 20 = 0$

Element FG

$(V_F)_L = 0$

$(V_F)_R = 3.29\ KN = (V_G)_L = 3.29\ KN$

$(V_G)_R = 3.29 - 3.29 = 0$



SFD (↑+↓, KN)

Calculation of bending moment

Element AC

$M_A = 0, M_B = 6.71 \times 2 = 13.42\ KNm$

$(M_C)_L = 6.71 \times 4 - 10 \times 2 = 6.84\ KNm$

$(M_C)_R = 6.84 - 6.84 = 0$

Element CF

Portion CD

$M_x = -4 \times x \times \frac{x}{2} + 0.57x + 6.84 \quad (0 \le x \le 2)$

When $x = 0$, $M_A = 6.84\ KNm$

When $x = 1m$, $M_{mid} = -4 \times 1 \times \frac{1}{2} + 0.57 + 6.84 = 5.41KNm$

When $x = 2m$, $M_D = -4 \times 2 \times \frac{2}{2} + 0.57 \times 2 + 6.84 \approx 0$

Portion DF

$M_D = 0$

$M_E = 0.57 \times 3 + 6.84 - 4 \times 2 \times 2 = -7.45\ KNm$

$(M_F)_L = 0.57 \times 4 + 6.84 - 4 \times 2 \times 3 - 15 \times 1 = -29.88\ KNm$

$(M_F)_R = -29.88 + 29.88 = 0$

Element FH

$(M_F)_L = 0$

$(M_F)_R = -20\ KNm$

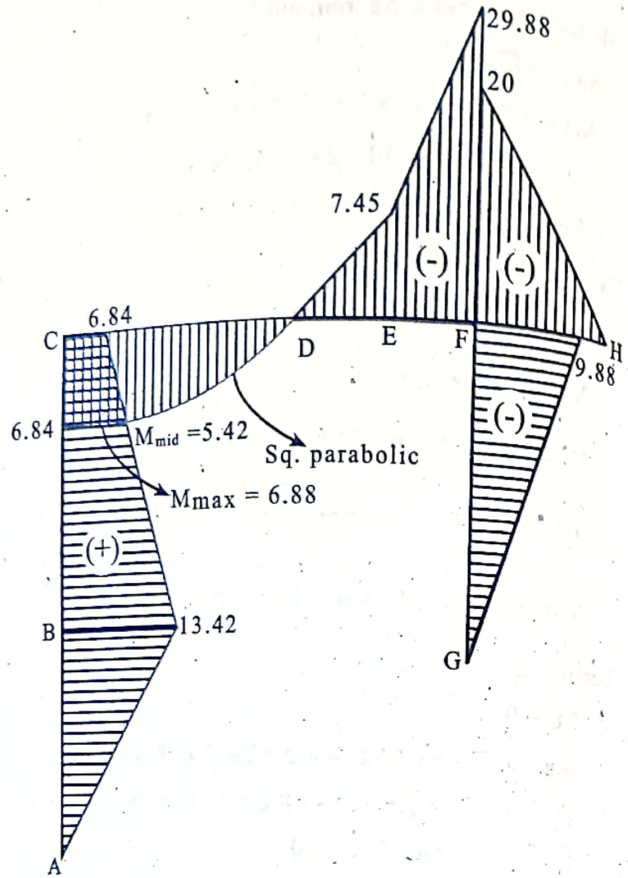
$(M_H) = -20 + 20 = 0$

Element FG

$(M_F)_L = 0$

$(M_F)_R = -9.88\ KNm$

$M_G = -9.88 + 3.29 \times 3 = 0$



BMD (↺+, KNm)

Salient features

For the curve (bending moment diagram) maximum BM occurs at a point where SF = 0.

But $V_x = 0.57 - 4 \times x$

or, $0 = 0.57 - 4x$

$\therefore x = \frac{0.57}{4} = 0.1425m$

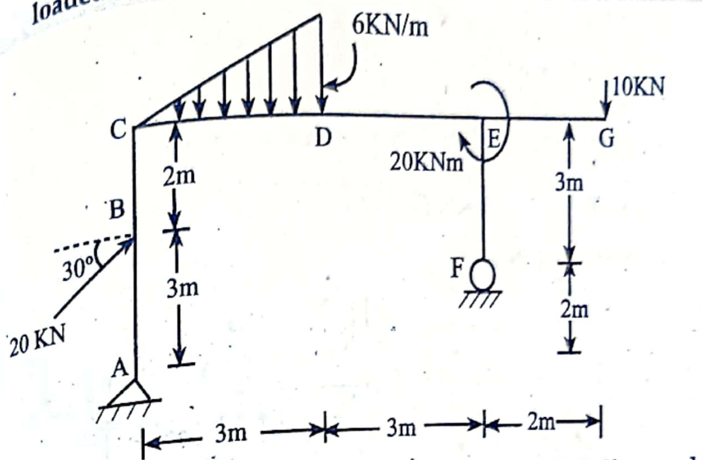
$\therefore M_{max} = -4 \times 0.1425 \times \frac{0.1425}{2} + 0.57 \times 0.1425 + 6.84$

$= 6.88$ KNm

Clearly, from bending moment diagram, point of contraflexure occurs at D.

i.e., x = 2m from C along CD.

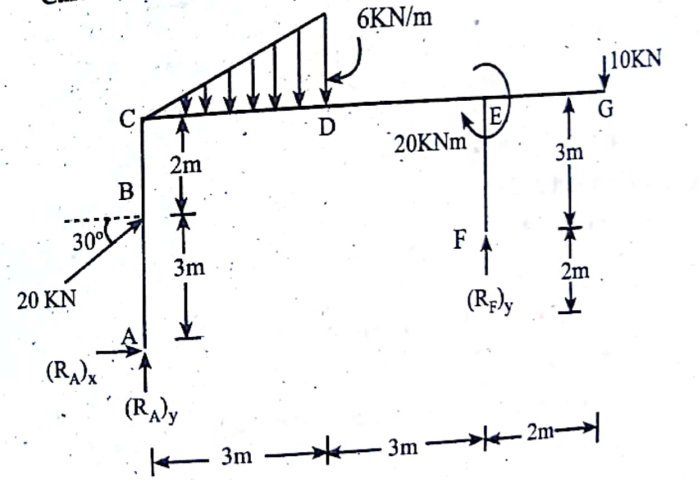
10. **Draw axial force, shear force, and bending moment diagram and obtain salient features for the given frame loaded as shown in figure.**



[2073 Shrawan]

Solution:

Calculation of unknown reactions



(↺+) $\Sigma M_F = 0$

or, $(R_A)_y \times 6 + 20\ Cos30° \times 1 + 20\ Sin30° \times 6 - \frac{1}{2} \times 6 \times 3 \times 4 + 20 + 10 \times 2 = 0$

or, $(R_A)_y = -13.55$ KN $= 13.55$ KN (↓)

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $-(R_F)_x + 20\cos 30° = 0$

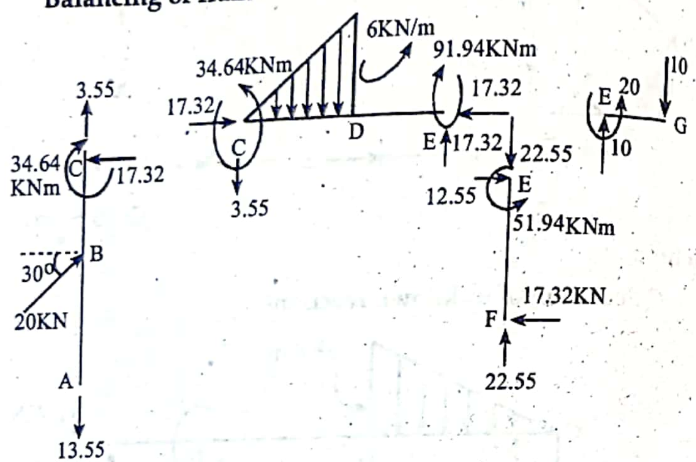
or, $(R_F)_x = 17.32$ KN $(\leftarrow)$

$(+\uparrow)\ \Sigma F_y = 0$

or, $(R_A)_y + (R_F)_y - \frac{1}{2} \times 6 \times 3 + 20\sin 30° - 10 = 0$

or, $(R_F)_y = 22.55$ KN $(\uparrow)$

Balancing of frame



Calculation of axial force

Element AC

Portion AB

$(N_A)_L = 0$

$(N_A)_R = 13.55$ KN $= (N_B)_L$

Portion BC

$(N_B)_R = 13.55 - 10.00 = 3.55$ KN $= (N_C)_L$

$(N_c)_R = 3.55 - 3.55 = 0$

Element CE

$(N_C)_L = 0$

$(N_C)_R = -17.32$ KN $= (N_E)_L$

$(N_E)_R = -17.32 + 17.32 = 0$

Element EG

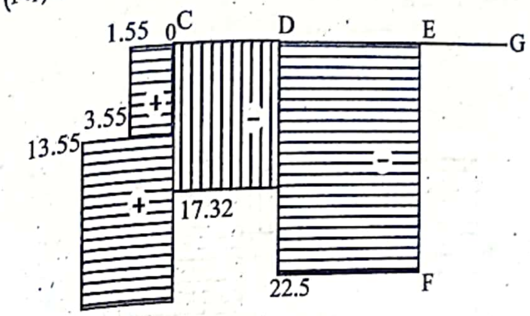
No axial force

Element EF

$(N_E)_L = 0$

$(N_E)_R = -22.55$ KN $= (N_F)_L$

$(N_F)_R = -22.55 + 22.55 = 0$



AFD ($\overset{+}{\longleftrightarrow}$, KN)

Calculation of shear force

Element AC

$(V_A)_L = (V_A)_R = (V_B)_L = 0$

$(V_B)_R = -20\cos 30° = -17.32$ KN $= (V_C)_L$

$(V_C)_R = -17.32 + 17.32 = 0$

Element CE

Portion CD

From figure, $V_x = -3.55 - \frac{1}{2} \times x \times 2x = -3.55 - x^2$

$(V_C)_L = 0$

When $x = 0$, $(V_C)_R = -3.55$ KN

When $x = 1.5$, $V_{mid} = -3.55 - 1.5^2 = -5.8$ KN

When $x = 3$, $V_D = -3.55 - 3^2 = -12.55$ KN

Portion DE

$V_D = -12.55$ KN $= (V_E)_L$

$(V_E)_R = -12.55 + 12.55 = 0$

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $-(R_F)_x + 20\ \text{Cos}30° = 0$

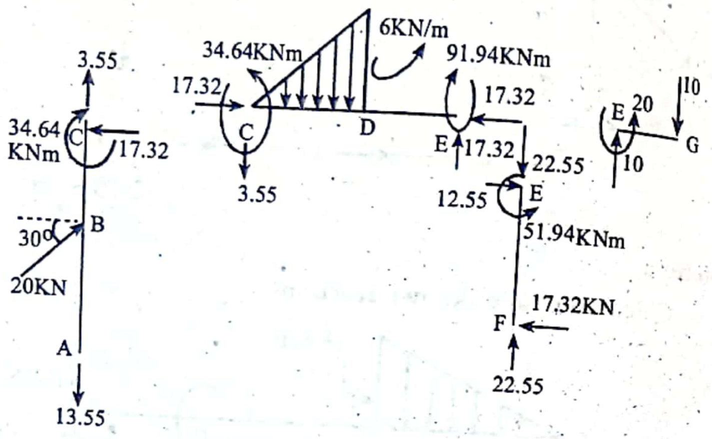
or, $(R_F)_x = 17.32$ KN $(\leftarrow)$

$(+\uparrow)\ \Sigma F_y = 0$

or, $(R_A)_y + (R_F)_y - \frac{1}{2} \times 6\times3 + 20\sin30° - 10 = 0$

or, $(R_F)_y = 22.55$ KN $(\uparrow)$

Balancing of frame



Calculation of axial force

Element AC

Portion AB

$(N_A)_L = 0$

$(N_A)_R = 13.55$ KN $= (N_B)_L$

Portion BC

$(N_B)_R = 13.55 - 10.00 = 3.55$ KN $= (N_C)_L$

$(N_c)_R = 3.55 - 3.55 = 0$

Element CE

$(N_C)_L = 0$

$(N_C)_R = -17.32$ KN $= (N_E)_L$

$(N_E)_R = -17.32 + 17.32 = 0$

Element EG

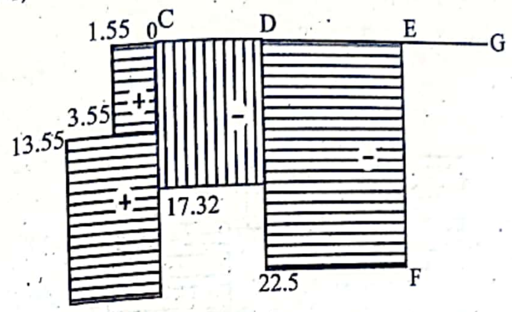
No axial force

Element EF

$(N_E)_L = 0$

$(N_E)_R = -22.55$ KN $= (N_F)_L$

$(N_F)_R = -22.55 + 22.55 = 0$



AFD $(\overset{+}{\leftarrow\!\bullet\!\rightarrow}$, KN)

Calculation of shear force

Element AC

$(V_A)_L = (V_A)_R = (V_B)_L = 0$

$(V_B)_R = -20\ \text{Cos}30° = -17.32$ KN $= (V_C)_L$

$(V_C)_R = -17.32 + 17.32 = 0$

Element CE

Portion CD

From figure, $V_x = -3.55 - \frac{1}{2} x x 2x = -3.55 - x^2$

$(V_C)_L = 0$

When x = 0, $(V_C)_R = -3.55$ KN

When x = 1.5, $V_{mid} = -3.55 - 1.5^2 = -5.8$ KN

When x = 3, $V_D = -3.55 - 3^2 = -12.55$ KN

Portion DE

$V_D = -12.55$ KN $= (V_E)_L$

$(V_E)_R = -12.55 + 12.55 = 0$

Element EG

$(V_E)_L = 0$

$(V_E)_R = 10\ KN = (V_G)_L$

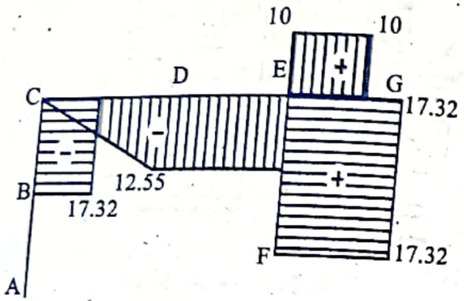
$(V_G)_R = 10 - 10 = 0$

Element EF

$(V_E)_L = 0$

$(V_E)_R = 17.32\ KN = (V_F)_L$

$(V_F)_R = 17.32 - 17.32 = 0$



SFD (↑+↓, KN)

Calculation of bending moment

Element AC

Portion AB

No bending moment

Portion BC

$M_x = -17.32x \quad (0 < x < 2)$

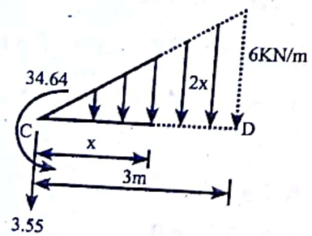
When $x = 0$, $M_B = 0$

When $x = 2$, $(M_C)_L = -17.32 \times 2 = -34.64$ KNm

$(M_C)_R = -34.64 + 34.64 = 0$

Element CE

Portion CD



From figure, $M_x = -34.64 - 3.55x - \left\{\frac{1}{2} \times 2x \times x\right\} \times \frac{1}{3}x (0 \le x \le 3)$

$= -34.64 - 3.55x - \frac{x^3}{3}$

At $x = 0$, $M_C = -34.64$ KNm

At $x = 1.5$, $M_{mid} = -34.64 - 3.55 \times 1.5 - \frac{1.5^3}{3} = -41.09$ KNm

At $x = 3$, $M_D = -34.64 - 3.55 \times 3 - \frac{3^3}{3} = -54.29$ KNm

Portion DE

$M_D = -54.29$ KNm

$(M_E)_L = -34.64 - 3.55 \times 6 - \frac{1}{2} \times 6 \times 3 \times 4 = -91.94$

$(M_E)_R = -91.94 + 91.94 = 0$

Element EG

$M_x = 10x - 20 \quad (0 \le x \le 2)$

$(M_E)_L = 0$

When $x = 0$, $(M_E)_R = 10 \times 0 - 20 = -20$ KNm

When $x = 2$, $M_G = 10 \times 2 - 20 = 0$

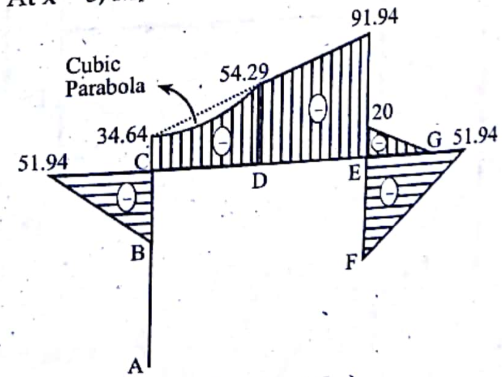
Element EF

$M_x = -51.94 + 17.32x \ (0 \le x \le 3)$

$(M_E)_L = 0$

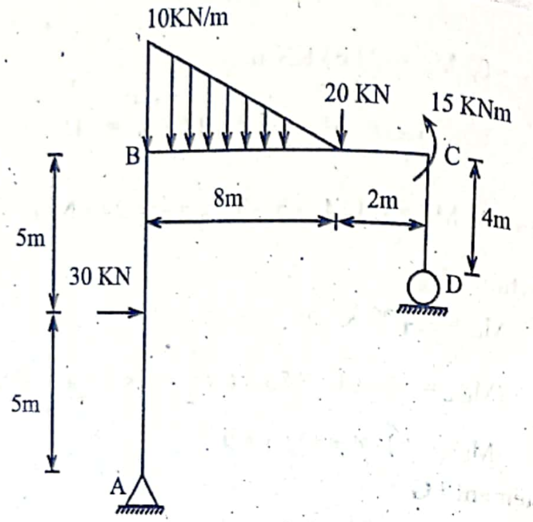
At $x = 0$, $(M_E)_R = -51.94$ KNm

At $x = 3$, $M_F = -51.94 + 17.32 \times 3 = 0.02 \approx 0$



BMD (↻+↺, KNm)

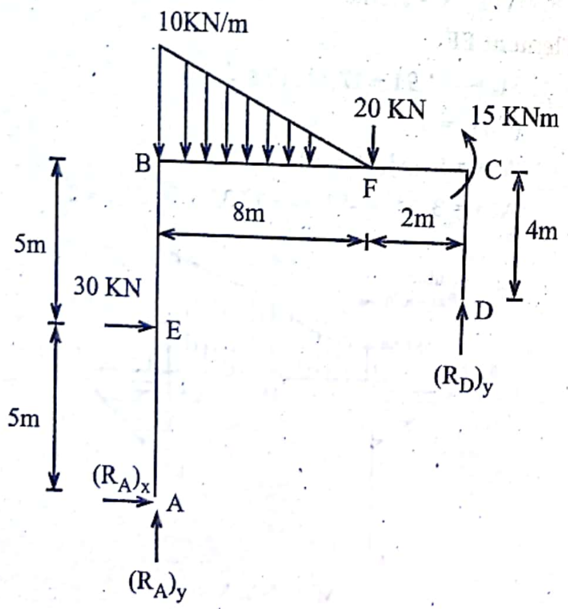
11. **Draw axial force, shear force and bending moment diagram and indicate the salient features if any, for the given frame loaded as shown in figure below.**



[2074 Chaitra]

Solution:

Calculation of unknown reactions



$(\circlearrowleft +)\ \Sigma M_A = 0$

or, $-(R_D)_y \times 10 - 15 + 20 \times 8 + \left(\frac{1}{2} \times 10 \times 8\right) \times \frac{1}{3} \times 8 + 30 \times 5 = 0$

or, $(R_D)_y = 40.17KN\ (\uparrow)$

$(+\uparrow)\ \Sigma F_y = 0$

or, $(R_A)_y + (R_D)_y - \left(\frac{1}{2} \times 10 \times 8\right) - 20 = 0$

or, $(R_A)_y = 60 - 40.17 = 19.83KN\ (\uparrow)$

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $(R_A)_x + 30 = 0$

or, $(R_A)_x = -30KN = 30KN\ (\leftarrow)$

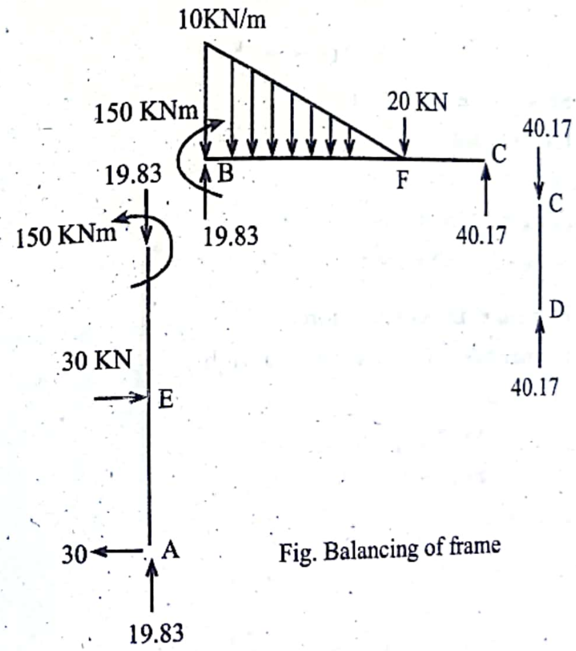


Fig. Balancing of frame

Axial force calculation:

Element AB:

$(N_A)_L = 0$

$(N_A)_R = -19.83KN = 19.83KN(C) = (N_E) = (N_B)_L$

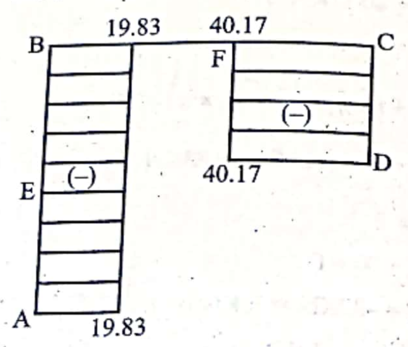
$(N_B)_R = -19.83 + 19.83 = 0$

Element CD:

$(N_C)_L = 0$

$(N_C)_R = -40.17KN = 40.17KN(C) = (N_D)_L$

$(N_D)_R = -40.17 + 40.17 = 0$



AFD ($\overset{+}{\longleftrightarrow}$, KN)

Shear force calculation:

Element AB

$(V_A)_L = 0$

$(V_A)_R = 30KN = (V_E)_L$

$(V_E)_R = 30 - 30 = 0 = V_B$

Element CD: No shear force

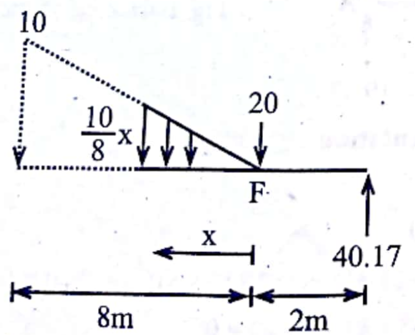
Element BC: (Calculation from right)

$(V_C)_R = 0$

$(V_C)_L = -40.17KN = (V_E)_R$

$(V_F)_L = -40.17 + 20 = -20.17KN$

Portion FB



$$V_x = -20.17 + \frac{1}{2} \times \frac{10x}{8} \times x = -20.17 + \frac{5x^2}{8}$$

When $x = 0$, $(V_F)_L = -20.17KN$

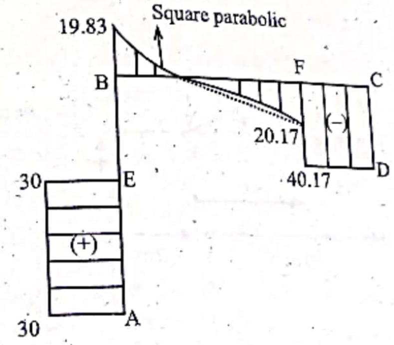
When $x = 8$, $(V_B)_L = 19.83KN$

$(V_B)_L = 19.83 - 19.83 = 0$

When $x = 4$, $(V_{mid})_{BF} = -10.17KN$

$$V_x = 0 \Rightarrow -20.17 + \frac{5x^2}{8} = 0$$

$$\Rightarrow x = 5.68m$$



SFD (↑+↓, KN)

Bending moment calculation

Element AB:

$M_A = 0$

Portion AE

$M_x = 30x$

When $x = 5$, $M_x = 150KNm$

Portion EB

$M_x = 30(5 + x) - 30 = 150KNm$

When $x = 5$, $(M_B)_L = 150KNm$, $(M_B)_R = 150 - 150 = 0$ (O.K.)

Element CD:

No Bending Moment

Element CB: (Calculations from right)

$(M_C)_R = 0$

$(M_C)_L = 15$ KNm

Element CF: $(0 \le x \le 2)$

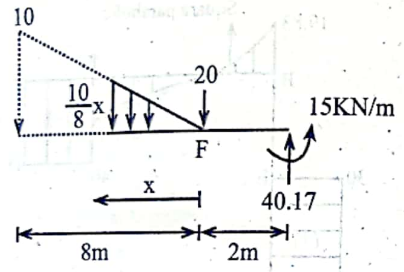
$M_x = 15 + 40.17x$

When x = 2, $M_F = 15 + 40.17 \times 2 = 95.34$ KNm

Portion FB $(0 \le x \le 8)$

$$M_x = 15 + 40.17(2 + x) - 20 \times x - \frac{1}{2} \times \frac{10x}{8} \times x \times \frac{1}{3} \times x$$

$$= 95.34 + 20.17x - \frac{5x^3}{24}$$



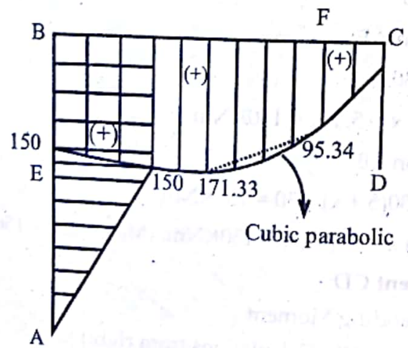
When x = 8

$(M_B)_R = 150$KNm

$(M_B)_L = 150 - 150 = 0$

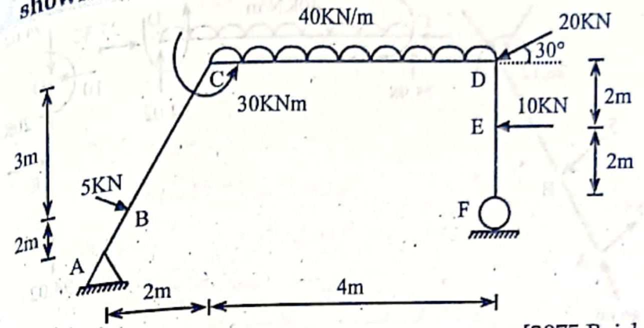
When x = 5.68m,

$M_{max} = 171.73$KNm



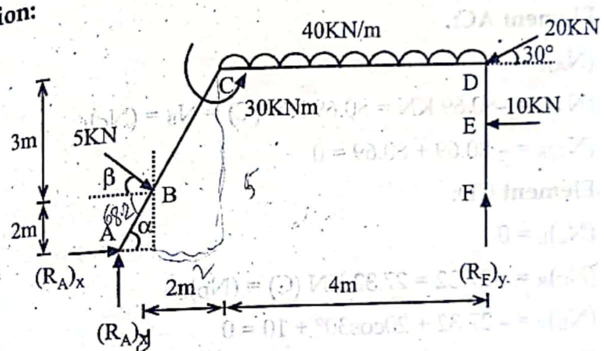
BMD (↻+, KNm)

12. **Draw AFD, SFD and BMD of the given frame loaded as shown in figure below. Indicate the salient features if any.**



[2075 Baishakh].

Solution:



+5-48 ?

$$\alpha = \tan^{-1}\left(\frac{5}{2}\right) = 68.2°$$

$$\beta = 90° - \alpha = 21.8°$$

$(\circlearrowleft +)\Sigma M_A = 0$

or, $-(R_F)_y \times 6 - 10 \times 3 - 20\cos30° \times 5 + 20\sin30° \times 6 + 40 \times 4 \times 4$

$- 30 + 5 \times \frac{2}{\sin 68.2°} = 0$ → or, $5\cos 68.8 \times 2 + 5\sin 21.8 \times \frac{2}{\tan 68.2}$

$= 9.28 + 1.1.48$

$= 10.76$

$\therefore (R_F)_y = 94.03$ KN (↑)

$(+\uparrow)\ \Sigma F_y = 0 \Rightarrow (R_A)_y + (R_F)_y - 5\sin\beta - 40 \times 4 - 20\sin30° = 0$

$\therefore (R_A)_y = 77.83$ KN (↑)

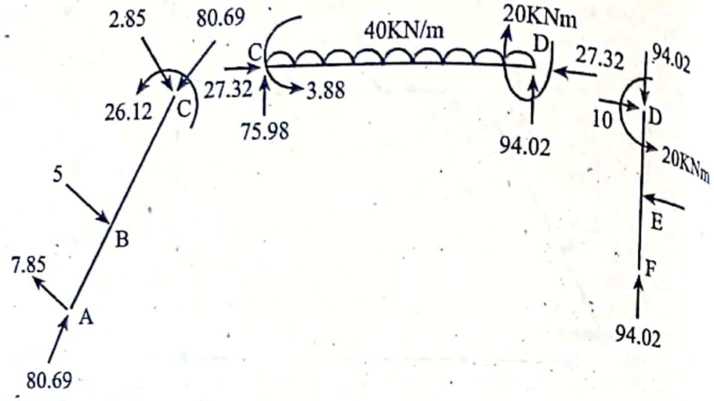
$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $(R_A)_x + 5\cos\beta - 20\cos30° - 10 = 0$

or, $(R_A)_x = 22.68$ KN (→)

Scanned with CamScanner

Balancing of frame:



Axial force calculation:

Element AC:

$(N_A)_L = 0$

$(N_A)_L = -80.69$ KN $= 80.69$ KN (C) $= N_B = (N_C)_L$

$(N_C)_R = -80.69 + 80.69 = 0$

Element CD:

$(N_C)_L = 0$

$(N_C)_R = -27.32 = 27.32$ KN (C) $= (N_D)_L$

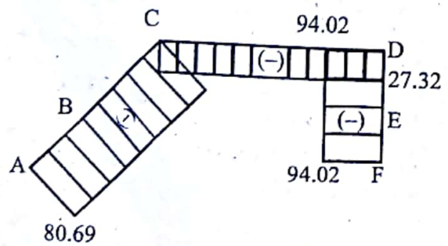
$(N_D)_R = -27.32 + 20\cos 30° + 10 = 0$

Element DF:

$(N_D)_L = 0$

$(N_D)_R = -94.02 = 94.02$ KN (C) $= N_E = (N_F)_L$

$(N_F)_R = -94.02 + 94.02 = 0$ KN



AFD (←+→, KN)

Shear force calculation:

Element AC:

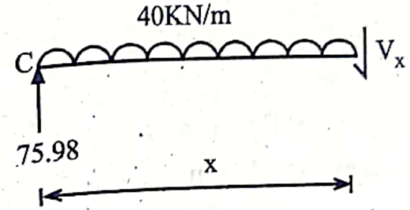
$(V_A)_L = 0$

$(V_A)_R = 7.85$ KN $= (V_B)_L$

$(V_B)_R = 7.85 - 5 = 2.85$ KN $= (V_C)_L$

$(V_C)_R = 2.85 - 2.85 = 0$ KN (O.K.)

Element CD:



$(V_C)_L = 0$

$(V_C)_R = 75.98$ KN

$V_x = 75.98 - 40x$

Location where $V_x = 0$

i.e. $V_x = 0$

or, $75.98 - 40x = 0$

$\therefore x = 1.899$m

(At this location, bending moment is maximum)

When $x = 4$,

$(V_D)_L = 75.98 - 40 \times 4 - 20\sin 30°$

$= -94.02$

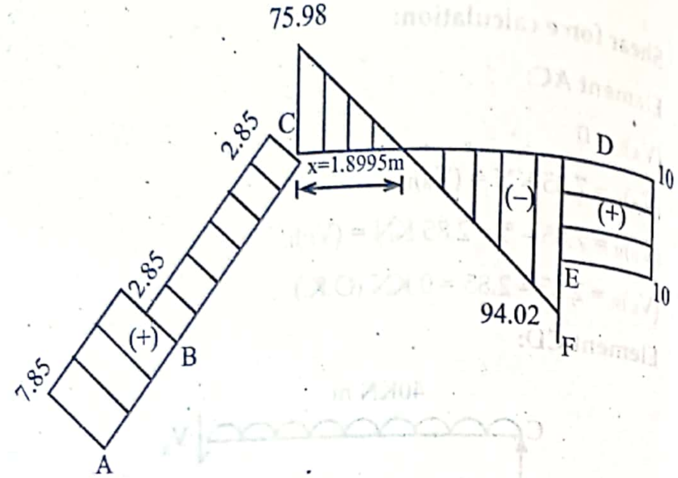
$(V_D)_R = -94.02 + 94.02 = 0$

Element DF:

$(V_D)_L = 0$

$(V_D)_R = 10$ KN $= (V_E)_L$

$(V_E)_R = 10.10 = 0 = V_F$



SFD (↑+↓, KN)

Bending Moment Calculation

Element AC:

Portion AB

$M_A = 0, M_x = 7.85x$ (x is taken along AB)

When $x = 2, M_B = 7.85 \times \frac{2}{\sin 68.2°} = 16.91$ KNm

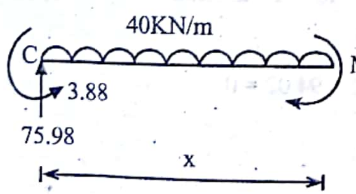
Portion BC

$(M_C)_L = \frac{7.85 \times 5}{\sin(68.2°)} - 5 \times \frac{3}{\sin 68.2°} = 26.12$ KN

$(M_C)_L = 26.12 - 26.12 = 0$

Element CD:

$(M_C)_L = 0$



$M_x = 75.98x - 40 \times x \times \frac{x}{2} - 3.88 = 75.98x - 20x^2 - 3.88$

When $x = 0$

$(M_C)_R = -3.88$ KNm

When $x = 2, M_{mid} = 75.98 \times 2 - 20 \times 2^2 \times 3.88 = 68.08$ KNm

When $x = 4, (M_D)_L = 79.98 \times 4 - 20 \times 4^2 - 3.88 = 19.96$ KNm

$(M_D)_R = -19.96 + 20 = 0.04 \approx 0$

When $x = 1.8995$m, $V_x = 0$ and M_x is maximum

$\therefore M_{max} = M_x = 1.8995\text{m} = 75.98 \times 1.8995 \times 20 \times 1.8995^2 - 3.88$

$= 68.28$ KNm

Location where $M_x = 0$

i.e. $75.96x - 20 \times x^2 - 3.88 = 0$

i.e. $x = 3.75$m, 0.052m

$\therefore$ Points of contra-flexure occur at $x = 0.052$m and $x = 3.75$m

Members DF:

Element DE:

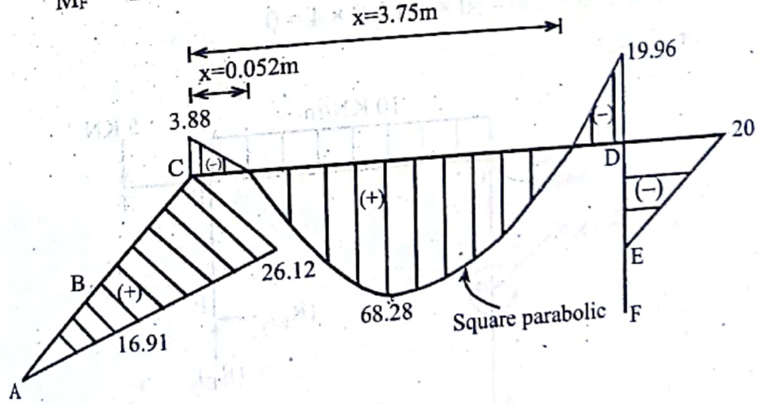
$(M_D)_L = 0$

$(M_D)_R = -20$ KNm

$M_x = -20 + 10x$

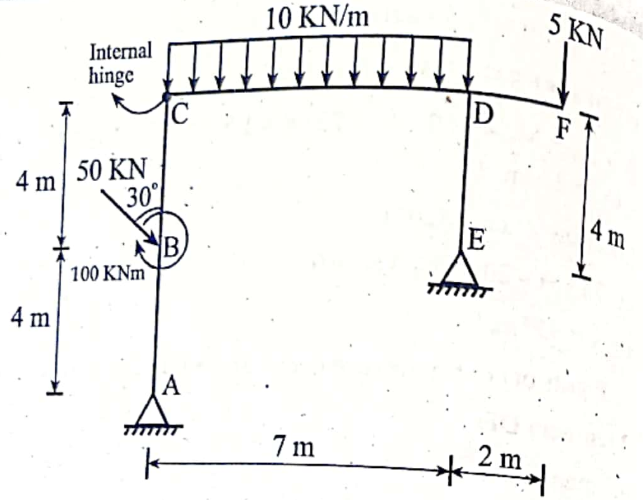
When $x = 2, M_E = -20 + 10 \times 2 = 0$

$M_F = -20 + 10 \times 4 - 10 \times 2 = 0$



BMD (↶+↷, KNm)

13. **Draw AFD, SFD and BMD for the beam loaded as shown in figure. Also show the salient point (if any).**



[2075 Chaitra]

Solution:

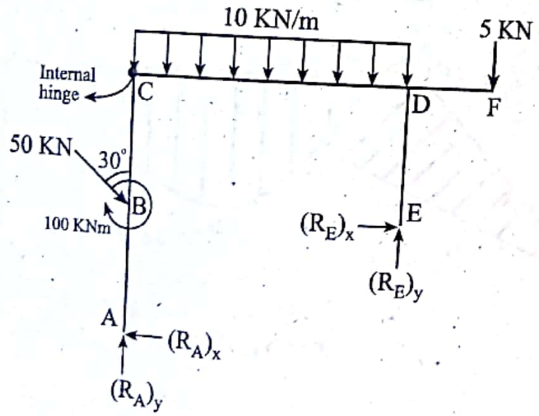
Calculation of unknown reactions

Since C is an internal hinge,

$(\circlearrowright +)\ (\Sigma M_C)_{left} = 0$

or, $(R_A)_x \times 8 + 100 - 50 \times \sin 30° \times 4 = 0$

or, $(R_A)_x = 0$



$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $-(R_A)_x + 50 \sin 30° + (R_E)_x = 0$

or, $(R_E)_x = -50 \sin 30° + 0 = -25KN = 25\ KN\ (\leftarrow)$

Also, $(\circlearrowright +)\ (\Sigma M_C)_{right} = 0$

or, $(R_E)_x \times 4 - (R_E)_y \times 7 + 5\times9 + 10\times7\times3.5 = 0$

or, $(R_E)_y \times 7 = 25\times4 + 45 + 245$

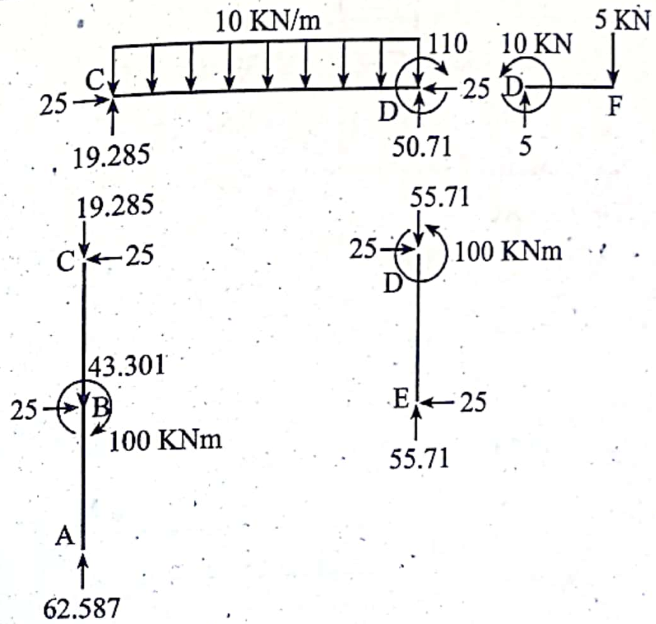
or, $(R_E)_y = 55.71\ KN\ (\uparrow)$

$(+\uparrow)\ \Sigma F_y = 0$

or, $(R_A)_y + (R_E)_y - 50 \cos 30° - 10\times7 - 5 = 0$

or, $(R_A)_y = 62.587\ KN\ (\uparrow)$

Balancing of frame



Calculation of axial force

Element AC

$(N_A)_L = 0,\ (N_A)_R = -62.587\ KN = 62.587\ KN\ (C) = (N_B)_L$

$(N_A)_R = 43.301 - 62.587 = -19.285 = 19.285\ (C) = (N_C)_L$

$(N_C)_R = -19.285 + 19.285 = 0$

Element CD

$(N_C)_L = 0,\ (N_C)_R = -25\ KN = 25\ KN(C) = (N_D)_L$

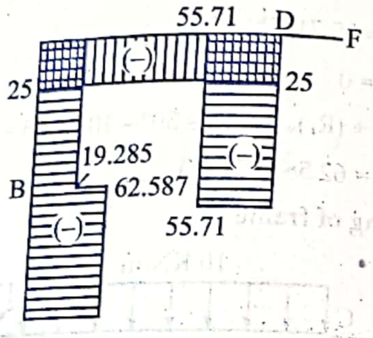
$(N_D)_R = -25+25 = 0$

Element DE

$(N_D)_L = 0$

$(N_D)_R = -55.71 = 55.71\ (C) = (N_E)_L$

$(N_E)_R = -55.71 + 55.71 = 0$



AFD ($\overset{+}{\leftarrow\bullet\rightarrow}$, KN)

Calculation of shear force

Element AC

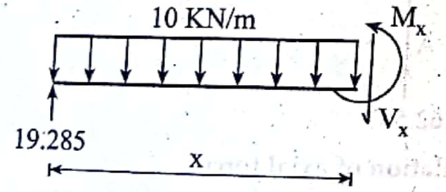
$(V_A)_L = 0$

$(V_A)_R = 0$

$(V_B)_R = -25KN = (V_C)_L$

$(V_C)_R = -25 + 25 = 0$

Element CD



$V_x = 19.285 - 10x$

To find where $V_x = 0$, we write

$19.285 - 10x = 0$

or, $x = 1.928$ m

Thus $V_x = 0$ at $x = 1.928$ m

$(V_C)_L = 0$

At $x = 0$, $(V_C)_R = 19.285$

At $x = 7$, $(V_D)_L = 19.285 - 7\times10 = -50.71$ KN

$(V_D)_R = -50.71 + 50.71 = 0$ KN

Element DF

$(V_D)_L = 0$

$(V_D)_R = 5\ KN = (V_F)_L$

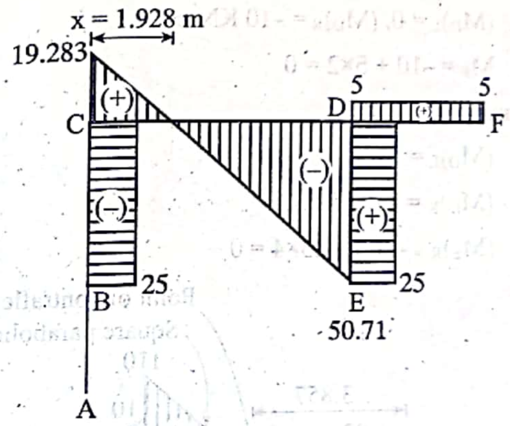
$(V_F)_R = 5 - 5 = 0$ KN

Element DE

$(V_D)_L = 0$

$(V_D)_R = 25\ KN = (V_E)_L$

$(V_E)_R = 25 - 25 = 0$



SFD (↑+↓, KN)

Calculation of bending moment

Element AC

$M_A = 0$

$(M_B)_L = 0$

$(M_B)_R = 100$ KNm

$M_C = 100 - 25\times4 = 0$ KNm

Element CD

$M_x = 19.285x - 10x\frac{x}{2} = 19.285x - 5x^2$

For value of x where M_x is zero, we have

$19.285x - 5x^2 = 0$

$x = 0, 3.857$ m

BM is maximum where SF is zero i.e., at $x = 1.928$.

$M_{max} = M_{x=1.928}$

$= 19.285 \times 1.928 - 5 \times (1.928)^2$

$= 18.595$ KNm

$M_{mid} = M_{x=3.5}$

$= 19.285 \times 3.5 - 5 \times 3.5^2 = 6.2475$ KNm

At $x = 7$, $(M_D)_L = 19.285 \times 7 - 5 \times 7^2 = -110$ KNm

$(M_D)_R = -110 + 110 = 0$ KNm

Element DF

$(M_D)_L = 0$, $(M_D)_R = -10$ KNm

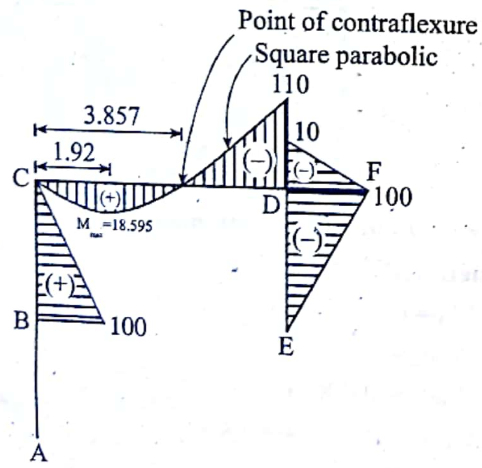
$M_F = -10 + 5 \times 2 = 0$

Element DE

$(M_D)_L = 0$

$(M_D)_R = -100$ KNm

$(M_E)_R = -100 + 25 \times 4 = 0$



BMD (↶+↷, KNm)

CHAPTER - 7

ANALYSIS OF PLANE TRUSSES

7.1 Introduction

A *truss* is a structure composed of slender members that are fastened together at their end joints. The truss is one of the major types of engineering structures which provides both a practical and an economic solution to many engineering situations, especially in the design of bridges and buildings.

The ideal assumptions made while calculating the forces in the members of truss are as follows:

i. Self-weight of the member of a truss is negligible.

ii. The members are connected by smooth frictionless pins.

iii. Loads are applied only at the joints and not on the member.

iv. Truss is statically determinate.

Classification of truss:

i. According to the analysis
   - Plane truss or 2-D truss
   - Space truss or 3-D truss

ii. According to their construction
   - Simple truss
   - Compound truss
   - Complex truss

iii. According to support condition
   - Simply supported truss
   - Cantilever truss

iv. According to purpose of utilization
   - Roof truss
   - Bridge truss
   - Tower truss, etc.

7.2 Determinacy and stability of truss

If m be the number of members, r be the number of support reactions, and j be the number of joints in the truss, then

Total no. of unknowns = m + r

Total no. of equations = 2j

Degree of static determinacy

= total no. of unknowns – total no. of equations available

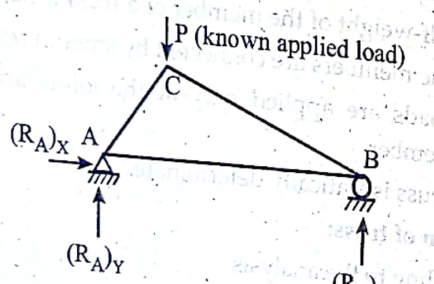
= m + r – 2j

If m + r = 2j, truss is *statically determinate and stable*

If m + r > 2j, truss is *statically indeterminate*

If m + r < 2j, truss is *unstable*

Example:



Members (AC, CB, AB)

Reactions $\begin{Bmatrix} 2 \text{ at A} \\ 1 \text{ at B} \end{Bmatrix}$

Total no. of members (m) = 3

Total no. of unknowns = m + r = 3 + 3 = 6

Total no. of equations= 2j = 2 × 3

Degree of static indeterminacy (DSI) = 6 - 6 = 0

Thus, the truss is stable and determinate.

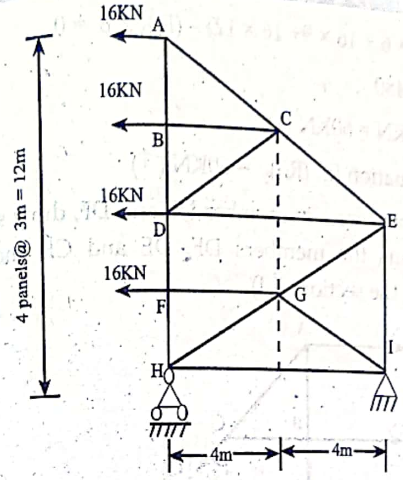
Analysis of truss:

There are two methods to analyze a truss. These are:

- Method of joints
- Method of sections

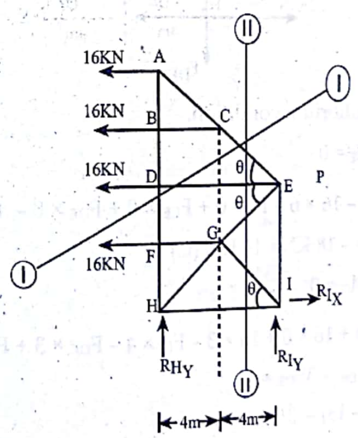
SOLVED NUMERICALS

1. **Use method of section to determine member forces DE, DF and GI for the given pin joined truss and also indicate the nature of forces.**



[2069 Bhadra]

Solution:



Calculation of unknown reactions

$(\overset{\rightarrow}{+})\ \sum F_x = 0$

or, $-16 - 16 - 16 + (R_I)x = 0$

$\therefore (R_I)_x = 64$ KN ($\rightarrow$)

$(\uparrow +) \sum F_y = 0$

or, $(R_H)_y + (R_I)_y = 0$ ....(i)

$\circlearrowleft(+) \sum M_H = 0$

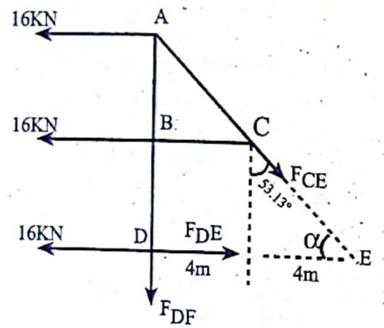
$-(16 \times 3 + 16 \times 6 + 16 \times 9 + 16 \times 12) - (R_I)_y \times 8 = 0$

or, $8(R_I)_y = -480$

$\therefore (R_I)_y = -60$ KN = 60KN ($\downarrow$)

Also, from equation (i), $(R_H)_y = 60$KN ($\uparrow$)

To find member forces in member DE and DF, draw section (I-I) which cuts the members DF, DE and CE and take portion above the section (I-I).



For equilibrium condition,

$\circlearrowleft(+) \sum M_E = 0$

$-16 \times 3 - 16 \times 6 - 16 \times 0 + F_{CE} \times 0 + F_{DE} \times 0 - F_{DF} \times 8 = 0$

$\therefore F_{DF} = -18$ KN= 18 KN (C)

$\circlearrowleft(+) \sum M_C = 0$

$-16 \times 3 + 16 \times 0 + 16 \times 3 - F_{DF} \times 4 - F_{DE} \times 3 + F_{CE} \times 0 = 0$

or, $-4F_{DF} - 3F_{DE} = 0$

or, $-4(-18) - 3F_{DE} = 0$

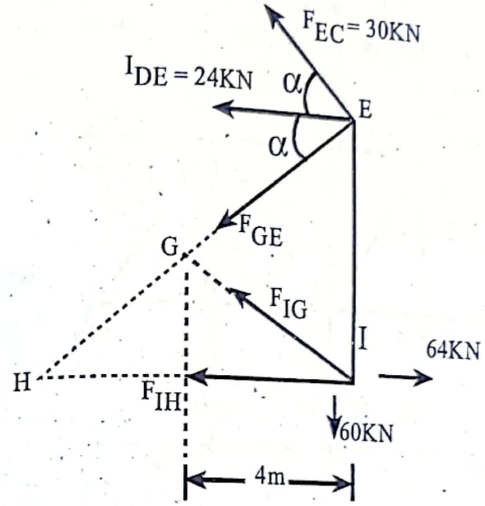
or, $F_{DE} = 24$ KN (T)

$\circlearrowleft(+) \sum M_D = 0$

$-16 \times 3 - 16 \times 6 + F_{CE} \cos 53.13° \times 4 + F_{CE} \sin 53.13° \times 3 = 0$

or, $F_{CE} = \frac{144}{4.8} = 30$KN (T)

To find member force on member GI, draw section (ii) - (ii) vertically which cuts the member CE,DE,GE, GI and HI and take right portion



For equilibrium condition,

$\circlearrowleft(+) \sum M_H = 0$

$-F_{DE} \times 6 - F_{CE} \cos 36.87° \times 6 - F_{GE} \cos 36.87° \times 6 - F_{GE} \cos 36.87° \times 6 = 0$

or, $-48 - F_{GE} \cos 36.87 = 0$

$\therefore F_{GE} = \frac{-48}{\cos 36.87} = -60$ KN = 60 KN (C)

$(\uparrow +) \sum F_y = 0$

$F_{CE} \sin 36.87° - F_{GE} \sin 36.87° - 60 - F_{IG} \sin 36.87° = 0$

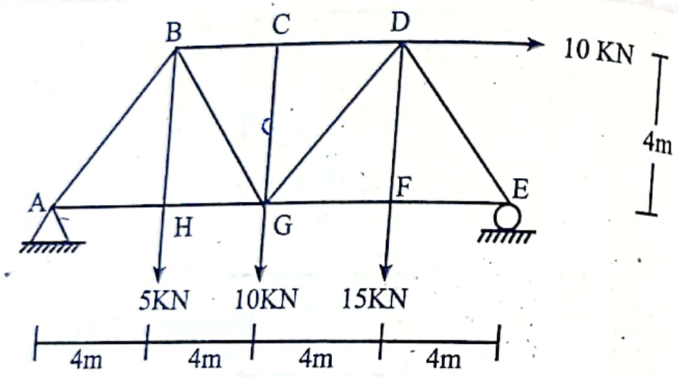
or, $30 \sin 36.87° - (-60) \sin 36.87° - 60 - F_{IG} \sin 36.87° = 0$

or, $54 - 60 - F_{IG} \sin 36.87° = 0$

or, $F_{IG} = \frac{-6}{\sin 36.87} = -10$ KN= 10 KN (C)

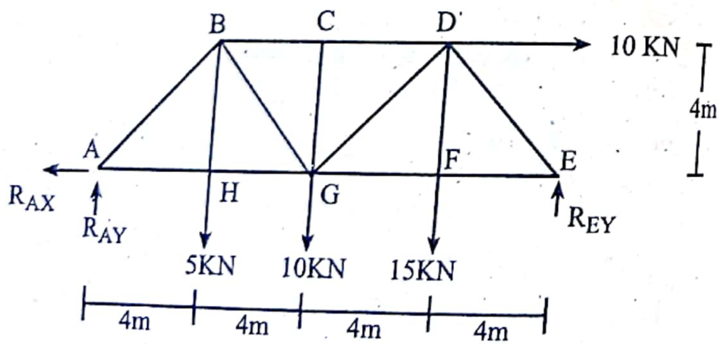
Member	Force (KN)	Type
DE	24	T
DF	18	C
GI	10	C

2. **Determine the total degree of internal, external indeterminacy of given truss. Also determine the forces in members BC, BG, HG and GD.**



[2070 Chaitra]

Solution:



Total no. of members (m) = 13

No. of reactions (r) = 3

No. of joints (j) = 8

Now,

$m + r = 13 + 3 = 16,\ 2j = 2 \times 8 = 16$

For truss, total degree of static indeterminacy $= (m+r) - 2j$

$= 16 - 16 = 0$

$\therefore$ Total degree of static indeterminacy= 0

Since no. of equations available are 3 and no. of unknown reactions (r) is 3, degree of external indeterminacy= r-3=3-3=0.

Now, degree of internal indeterminacy = total degree of static indeterminacy–degree of external indeterminacy= 0 - 0 = 0.

Calculation of unknown reactions

For static equilibrium,

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

$-(R_A)_x + 10 = 0$

$\therefore\ (R_A)_x = 10\ KN\ (\leftarrow)$

$(\uparrow +)\ \Sigma F_y = 0$

$(R_A)_y - 5 - 10 - 15 + (R_E)_y = 0$

$\therefore (R_A)_y + (R_E)_y = 30$ .... (i)

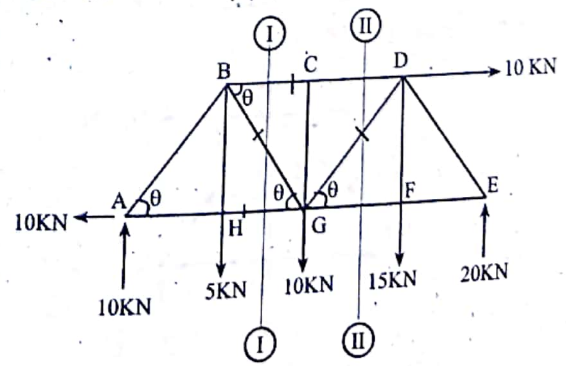
$(\circlearrowleft +)\ \Sigma M_A = 0$

or, $5\times4 + 10\times8 + 15\times12 + 10\times4 - (R_E)_y\times16 + (R_A)_x\times0 + (R_A)_y\times0 = 0$

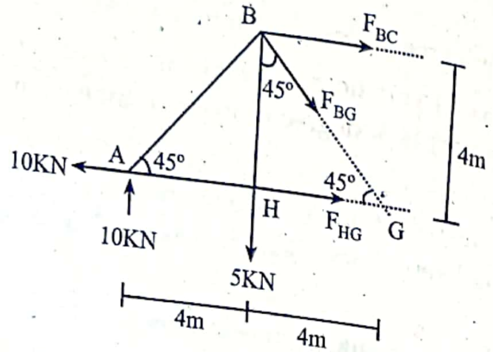
$\therefore\ (R_E)_y = 20\ KN\ (\uparrow)$

From equation (i),

$(R_A)_y = 30 - 20 = 10\ KN\ (\uparrow)$



To calculate member forces F_{BC}, F_{BG}, F_{HG}, draw section (I) - (I) and take left portion



$(\circlearrowright +)\Sigma M_B = 0$

$F_{BC} \times 0 + F_{BG} \times 0 - F_{HG} \times 4 + 5 \times 0 + 10 \times 4 + 10 \times 4 = 0$

$\therefore \quad F_{HG} = \dfrac{80}{4} = 20 \text{ KN (T)}$

Again, $(\circlearrowright +)\Sigma M_G = 0$

$F_{BC} \times 4 + F_{BG} \times 0 + F_{HG} \times 0 - 5 \times 4 + 10 \times 8 + 10 \times 0 = 0$

or, $4F_{BC} - 20 + 80 = 0$

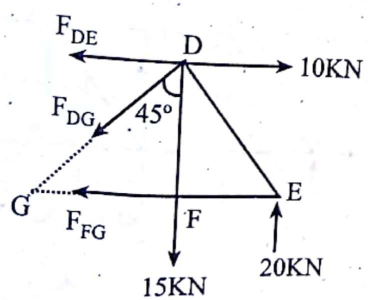
or, $F_{BC} = -\dfrac{60}{4} = -15 \text{ KN} = 15 \text{ KN (C)}$

$(\uparrow +)\Sigma F_y = 0$

or, $10 - 5 - F_{BG} \cos 45° = 0$

or, $F_{BG} = \dfrac{5}{\cos 45°} = 7.07 \text{ KN (T)}$

To calculate member force F_{GD}, draw section (II) - (II) and take right side portion.



For equilibrium condition,

$(\uparrow +)\Sigma F_y = 0$

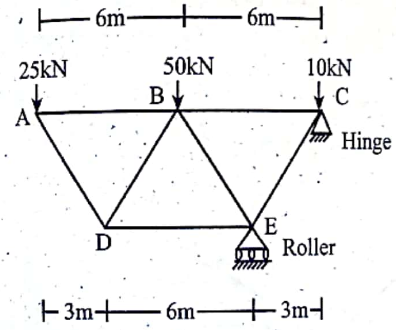
or, $-15 + 20 - F_{DG} \cos 45° = 0$

or, $-F_{DG} \cos 45° = -5$

or, $F_{DG} = \dfrac{5}{\cos 45°} = 7.07 \text{ KN (T)}$

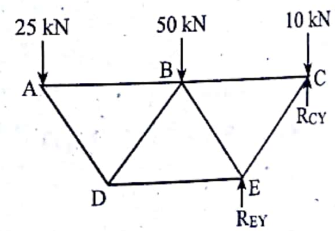
Member	Force	Magnitude
B C	F_{BC}	15 KN (C)
B G	F_{BG}	7.07 KN (T)
H G	F_{HG}	20 KN (T)
G D	F_{GD}	7.07 KN (T)

3. Determine the member forces for given truss loaded as shown in figure below.



[2070 Ashadh]

Solution:



Calculation of unknown reactions

$(\circlearrowright +)\Sigma M_C = 0$

or, $(R_E)_y \times 3 - 25 \times 12 - 50 \times 6 = 0$

$\therefore (R_E)_y = 200 \text{ KN } (\uparrow)$

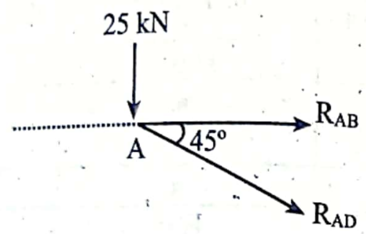
$(\uparrow+)\sum F_y = 0$

or, $(R_C)_y + (R_E)_y - 25 - 50 - 10 = 0$

$\therefore (R_C)_y = -115$ KN $= 115$ KN $(\downarrow)$

Joint Method

Joint A



$(\uparrow+)\sum F_y = 0$

or, $R_{AD} \sin 45° + 25 = 0$

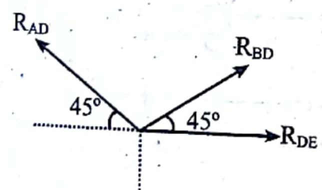
$\therefore R_{AD} = -35.35$ KN $= 35.35$ KN (C)

$(\overset{+}{\rightarrow})\sum F_x = 0$

or, $R_{AD} \cos 45° + R_{AB} = 0$

$\therefore R_{AB} = -R_{AD} \cos 45° = 25$ KN (T)

Joint D



$(\uparrow+)\sum F_y = 0$

or, $R_{AD} \sin 45° + R_{BD} \sin 45° = 0$

$\therefore R_{BD} = -R_{AD} = -(-35.35) = 35.35$ KN (T)

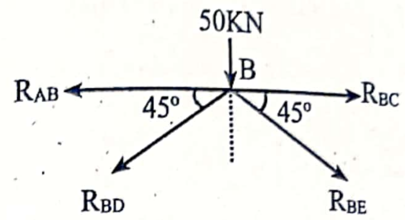
$(\overset{+}{\rightarrow})\sum F_x = 0$

or, $R_{DE} + R_{BD} \cos 45° - R_{AD} \cos 45° = 0$

$\therefore R_{DE} = -50$ KN

$= 50$ KN (C)

Joint B



$(\uparrow+)\sum F_y = 0$

or, $-R_{BD} \sin 45° - R_{BE} \sin 45° - 50 = 0$

or, $R_{BE} \sin 45° = -75$

$\therefore R_{BE} = -106.07$ KN $= 106.07$ KN (C)

$(\overset{+}{\rightarrow})\sum F_x = 0$

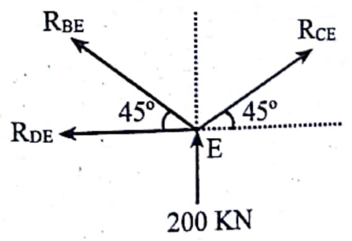
or, $-R_{AB} + R_{BC} + R_{BE} \cos 45° - R_{BD} \cos 45° = 0$

$R_{BC} = R_{AB} - R_{BE} \cos 45° + R_{BD} \cos 45°$

$= 25 + 106.07 \cos 45° + 35.35 \cos 45°$

$\therefore R_{BC} = 125$ KN (T)

Joint E



$(\uparrow+)\sum F_y = 0$

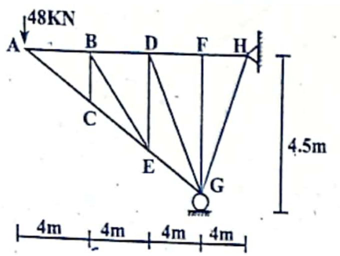
or, $200 + R_{BE} \sin 45° + R_{CE} \sin 45° = 0$

or, $R_{CE} \sin 45° = -200 - R_{BE} \sin 45°$

or, $R_{CE} = -176.78$ KN $= 176.78$ KN (C)

Member	AB	AD	BD	BE	BC	DE	CE
Magnitude of force (KN)	25	35.35	35.35	106.07	125	50	176.78
Nature (T or C)	T	C	T	C	T	C	C

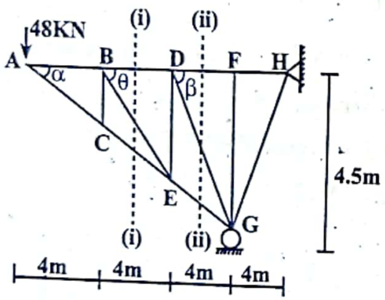
4. **Compute the forces developed in the members (BC, BD, BE, DE, DG and EG of the given truss loaded as shown in figure:**



[2069 Chaitra]

Solution:

Considering left part of section (i - i)



Taking moment about E,

$(\circlearrowleft +)\Sigma M_E = 0$

or, $R_{BD} \times 3 - 48 \times 8 = 0$

$\therefore R_{BD} = 128$ KN (T)

From similar triangles ADE and AFG

$$\frac{DE}{AD} = \frac{FG}{AF}$$

$$\Rightarrow DE = \frac{4.5}{12} AD = \frac{4.5}{12} \times 8 = 3$$

In ΔBDE, $\theta = \tan^{-1}\left(\frac{DE}{BD}\right) = \tan^{-1}\left(\frac{3}{4}\right) = 36.87°$

In ΔAFG, $\alpha = \tan^{-1}\left(\frac{FG}{AF}\right) = \tan^{-1}\frac{4.5}{12} = 20.56°$

$(\circlearrowleft +)\Sigma M_A = 0$

or, $R_{BE} \sin 36.87° \times 4 = 0$

or, $R_{BE} = 0$

$(\overset{+}{\rightarrow})\Sigma F_x = 0$

or, $R_{BD} + R_{CE} \cos 20.56° = 0$

or, $R_{CE} = -136.71$ KN $= 136.71$ KN (C)

Taking left-part of section (ii),

$(\circlearrowleft +)\Sigma M_A = 0$

or, $R_{DG} \sin 48.7° \times 8 = 0$

or, $R_{DG} = 0$

$(+\uparrow)\Sigma F_y = 0$

or, $R_{EG} \sin 20.56° - 48 = 0$

or, $R_{EG} = -136.68$ KN

$= +136.68$ KN (C)

In ΔDFG, $\beta = \tan^{-1}\left(\frac{FG}{DF}\right) = \tan^{-1}\left(\frac{4.5}{4}\right) = 48.37°$

Joint D

$(+\uparrow)\Sigma F_y = 0$

or, $R_{DG} \sin\beta + R_{DE} = 0$

or, $R_{DE} = 0$ (Since $R_{DG} = 0$)

Joint B

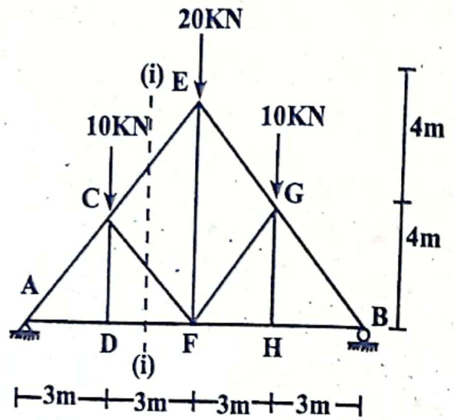
$R_{BE} \sin\theta + R_{BC} = 0$

$\therefore R_{BC} = 0$

Thus, we can tabulate the final result as

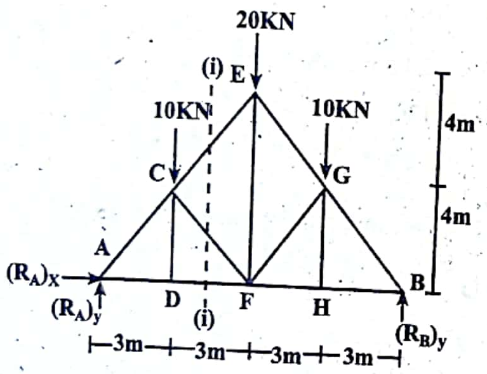
Member	BC	BD	BE	DE	DG	EG
Magnitude of force	0	128	0	0	0	136.68
Nature	-	(T)	-	-	-	(C)

5. **Determine the force developed in members CE, DF, EF, GH of the given truss loaded as shown in the figure.**



[2071 Bhadra]

Solution:



Calculation of unknown reactions

$\circlearrowright^{+} \Sigma M_E = 0$

or, $(R_A)_y \times 12 - 10\times9 - 20\times6 - 10\times3 = 0$

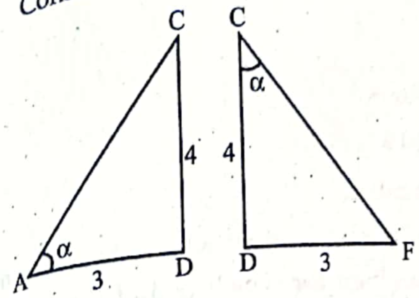
$\therefore (R_A)_y = 20$ KN $(\uparrow)$

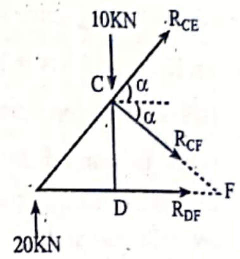
$(+\uparrow)\Sigma F_y = 0$

or, $(R_A)_y + (R_B)_y = 20 - 10 - 10 = 0$

or, $(R_B)_y = 20$ KN $(\uparrow)$

Considering left part of section (I-I),





$\alpha = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$

$\circlearrowright^{+} \Sigma M_C = 0$

or, $20\times3 - (R_{DF}) \times 4 = 0$

or, $R_{DF} = 15$ KN (T)

$\circlearrowright^{+} \Sigma M_F = 0$

or, $20 \times 6 + R_{CE} \sin \alpha\times3 + R_{CE} \cos\alpha\times 4 - 10\times3 = 0$

or, $R_{CE} = \dfrac{-120+30}{3\sin\alpha + 4\cos\alpha} = -18.75$ KN $= 18.75$ KN (C)

$\left(\overset{+}{\rightarrow}\right)\Sigma F_x = 0$

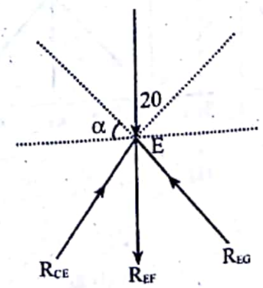
or, $R_{DF} + R_{CF} \cos\alpha + R_{CE} \cos\alpha = 0$

or, $R_{CF} = \dfrac{-R_{DF} - R_{CE} \cos\alpha}{\cos\alpha} = \dfrac{-15-(-18.75)\cos\alpha}{\cos 53.13}$

$= -6.25$ KN

$= 6.25$ KN (C)

At joint E,



$(\xrightarrow{+})\ \Sigma F_x = 0$ gives $R_{CE} = R_{EG}$

$(+\uparrow)\Sigma F_y = 0$

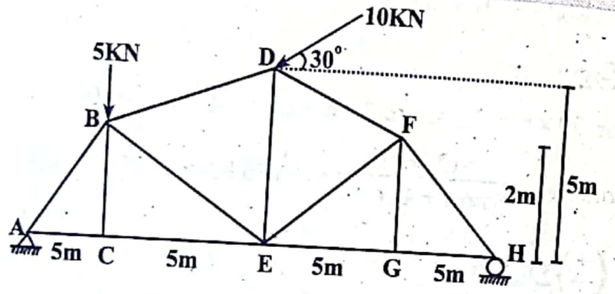
or, $-20 + 2R_{CE}\sin\alpha - R_{EF} = 0$

or, $R_{EF} = +10\text{ KN} = 10\text{ KN (T)}$

GH is a zero force member.

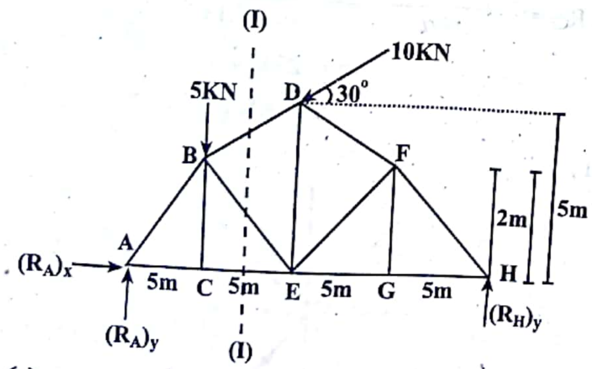
(This is guided by the idea that when three or more members form a truss joint for which two of the members are collinear and the third forms an angle with the first two, then the non-collinear member is zero force member.)

6. **Find the member forces in CE, BE, BD, and DE for the given truss.**



[2071 Shrawan]

Solution:



$(\circlearrowleft +)\Sigma M_A = 0$

or, $-(R_H)_y \times 20 - 10\cos30° \times 5 + 10\sin30° \times 10 + 5\times5 = 0$

or, $(R_H)_y = 1.58\text{ KN} (\uparrow)$

$(+\uparrow)\ \Sigma F_y = 0$

or, $-5 - 10\sin30° + (R_H)_y + (R_A)_y = 0$

or, $(R_A)_y = 5 + 10\sin30° - 1.48 = 8.42\text{KN} = 8.42\text{KN}(\uparrow)$

$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $(R_A)_x - 10\cos30° = 0$

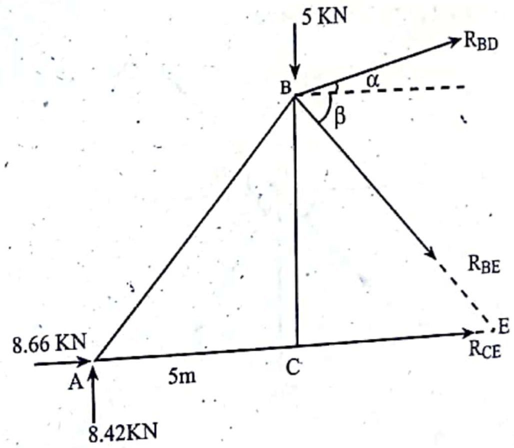
or, $(R_A)_x = 8.66\text{ KN} (\rightarrow)$

Considering left portion of section (I) – (I),

$(\circlearrowleft +)\Sigma M_B = 0$

or, $8.42 \times 5 - 8.66 \times 3 - R_{CE} \times 3 = 0$

or, $R_{CE} = 5.37\text{KN (T)}$

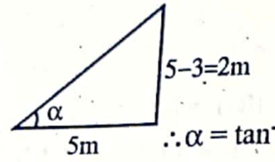


$(\circlearrowleft +)\Sigma M_E = 0$

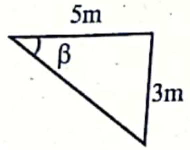
or, $8.42 \times 10 - 5 \times 5 + R_{BD}\cos\alpha \times 3 + R_{BD}\sin\alpha \times 5 = 0$

or, $R_{BD} = \dfrac{5\times5 - 8.42\times10}{3\cos21.8° + 5\sin21.8°}$

$= -12.75\text{KN} = 12.75\text{KN (C)}$



$\therefore \alpha = \tan^{-1}\left(\frac{2}{5}\right) = 21.80°$



$\therefore \beta = \tan^{-1}\left(\frac{3}{5}\right) = 30.96°$

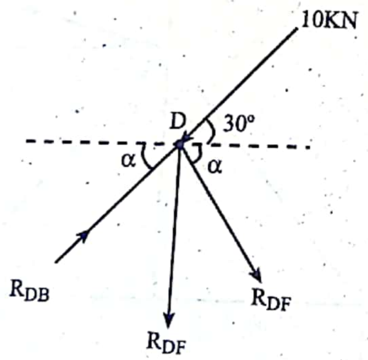
$(+\uparrow)\ \Sigma F_y = 0$

or, $8.42 - 5 + R_{BD}\sin\alpha - R_{BE}\sin\beta = 0$

or, $R_{BE}\sin 30.96° = 8.42 - 5 - 12.75\sin\alpha$

or, $R_{BE} = -2.56$ KN $= 2.56$ KN (C)

Considering joint D, we have



$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $R_{DB}\cos\alpha + R_{DF}\cos\alpha - 10\cos 30° = 0$

or, $R_{DF} = \dfrac{-12.75\cos 21.8 + 10\cos 30°}{\cos(21.80°)} = -3.42$ KN $= 3.42$KN (C)

$(+\uparrow)\ \Sigma F_y = 0$

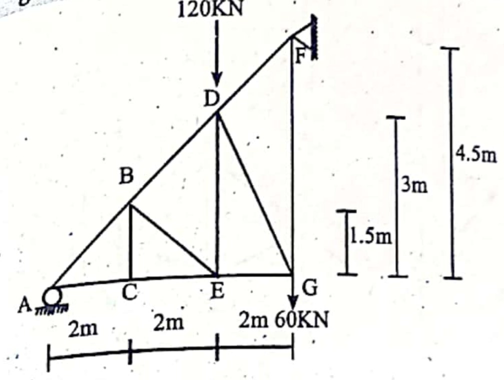
or, $-R_{DE} + R_{DB}\sin 21.8° - R_{DF}\sin 21.8° - 10\sin 30° = 0$

or, $R_{DE} = 12.75\sin 21.8° - (-3.42)\sin 21.8° - 10\sin 30°$

$= 1$ KN (T)

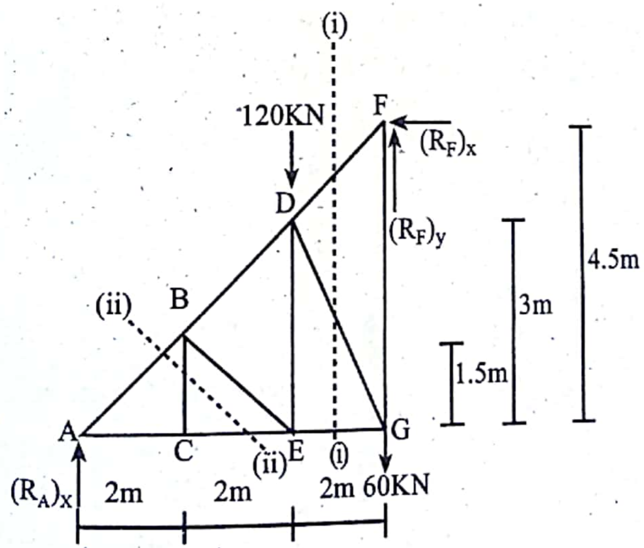
Member	CE	BE	BD	DE
Magnitude of force	5.37	2.56	12.75	1
Nature	0	C	C	T

7. **Determine the member forces BC, DG and FG for the given truss.**



[2071 Magh]

Solution:



$(\circlearrowleft +)\ \Sigma M_F = 0$

or, $(R_A)_y \times 6 - 120 \times 2 = 0$

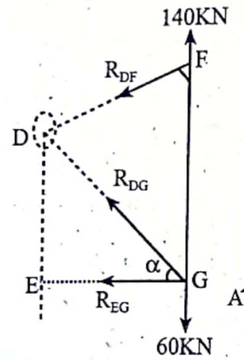
$\therefore (R_A)_y = 40\ KN\ (\uparrow)$

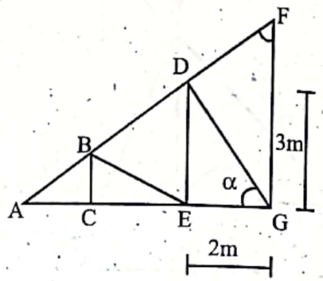
$(+\uparrow)\ \Sigma F_y = 0$

or, $(R_A)_y - 60 - 120 + (R_F)_y = 0$

or, $(R_F)_y = 180 - 40 = 140\ KN\ (\uparrow)$

Taking section (i) - (i) and considering right part, we get





$(\circlearrowleft +)\ \Sigma M_D = 0$

or, $R_{EG} \times 3 + 60 \times 2 - 140 \times 2 = 0$

or, $R_{EG} = 53.33KN = 53.33KN\ (T)$

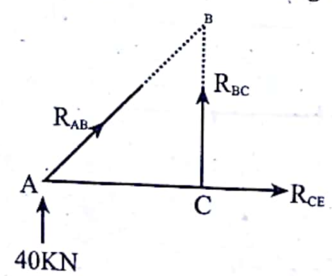
$\alpha = \tan^{-1}\left(\frac{DE}{EG}\right) = 56.31°$

$(+\uparrow)\ \Sigma F_y = 0$

or, $140 - 60 + R_{DG} \sin\alpha = 0$

or, $R_{DG} = \frac{-80}{\sin\beta} = -96.15\ KN = 96.15\ KN\ (C)$

Taking section (ii) - (ii) and considering left part, we have



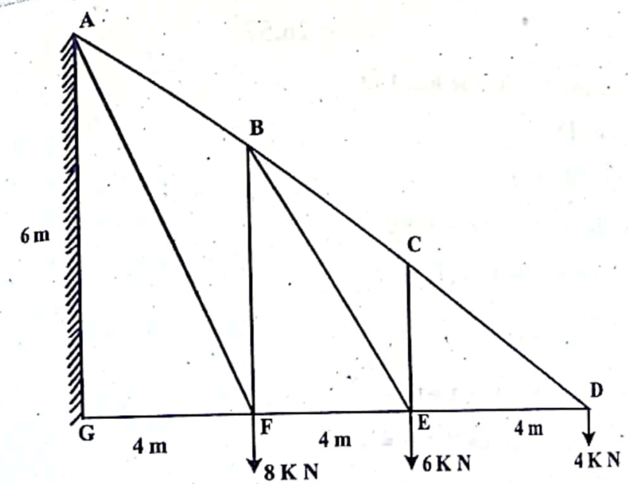
$(\circlearrowleft +)\ \Sigma M_A = 0$

or, $-R_{BC} \times 2 = 0$

$\therefore R_{BC} = 0$

Member	BC	DG	EG
Magnitude of force	0	96.15KN	53.33KN
Nature	-	C	T

8. **Describe the uses of trusses in engineering. Determine the force developed in BC, BE, EF, AB, AF and BF members of cantilever truss loaded as shown in the figure.**



[2071 Chaitra]

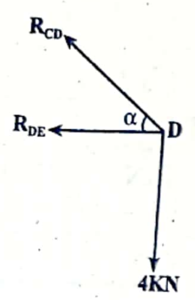
Solution:

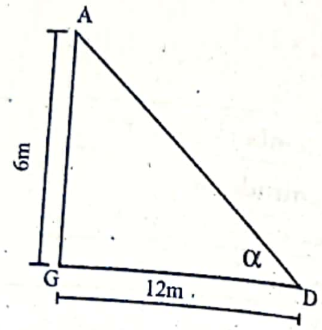
Trusses find their use in various civil engineering structures such as bridge, sports stadium, transmission towers, railway platforms, roofs, water reservoirs etc.

It is light, cheaper and is economical as compared to the R.C.C. structures (frame structures) for heavy loads.

Trusses are also used in construction of fuselage of aeroplane.

Using joint method:





$$\tan\alpha = \left(\frac{6}{12}\right)$$

$$\therefore \alpha = \tan^{-1}\left(\frac{1}{2}\right)$$

$$= \mathbf{26.57^{\circ}}$$

Stating form free joint D,

Joint D

$(+\uparrow)\ \Sigma F_y = 0$

or, $R_{CD} \sin 26.57° - 4 = 0$

$\therefore R_{CD} = 8.94$ KN (T)

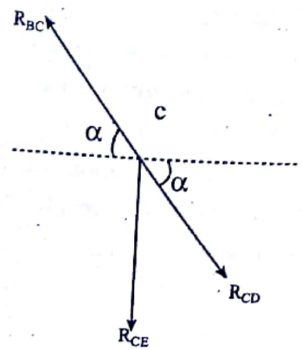
$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $-R_{DE} - R_{CD} \cos\alpha = 0$

or, $-R_{DE} - R_{CD} \cos 26.57° = 0$

or, $R_{DE} = -8.94 \cos 26.57° = -8KN = 8KN$ (C)

Joint C



$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

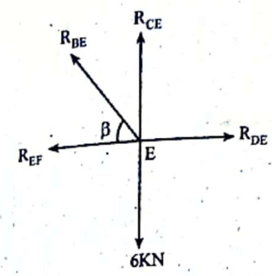
or, $-R_{BC} \cos\alpha + R_{CD} \cos\alpha = 0$

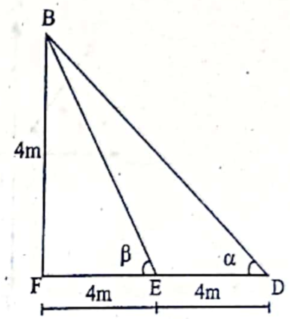
$\therefore R_{BC} = R_{CD} = 8.94$ KN (T)

$(+\uparrow)\ \Sigma F_y = 0$

or, $-R_{CE} + R_{BC} \sin\alpha - R_{CD} \sin\alpha = 0$

or, $R_{CE} = 0$ ($\because R_{BC} = R_{CD}$)





Joint E

$(+\uparrow)\ \Sigma F_y = 0$

or, $R_{BE} \sin 45° - 6 = 0$

$\therefore R_{BE} = 8.485$ KN (T)

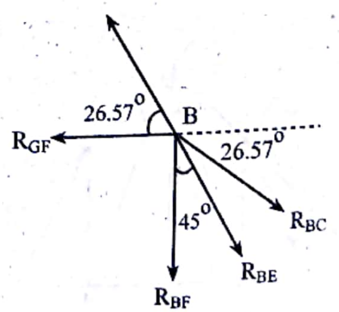
$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $-R_{EF} - R_{BE} \cos 45° + R_{DE} = 0$

or, $-R_{EF} - 6 - 3 = 0$

$\therefore R_{EF} = -14KN = 14KN$ (C)

Joint B



$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $R_{BC} \cos 26.57° + R_{BE} \cos 45° - R_{AB} \cos 26.57° = 0$

or, $8+6-R_{AB}\cos 26.17° = 0$

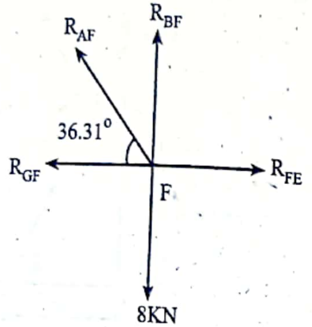
$\therefore R_{AB} = 15.65$ KN (T)

$(+\uparrow)\ \Sigma F_y = 0$

or, $R_{AB}\sin 26.57° - R_{BE}\sin 45° - R_{BC}\sin 26.57° - R_{BF} = 0$

$\therefore R_{BF} = -3$ KN = 3 KN (C)

Joint F



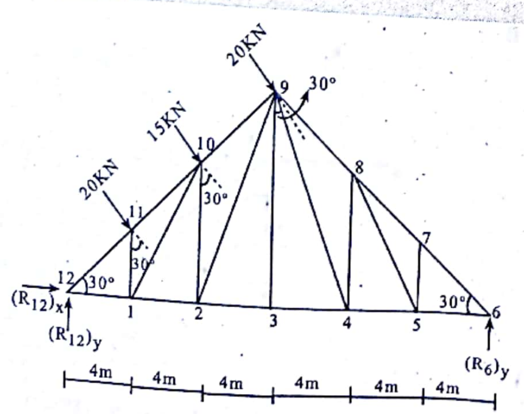
$(+\uparrow)\ \Sigma F_y = 0$

or, $R_{BF} + R_{AF}\sin 56.31° - 8 = 0$

$\therefore R_{AF} = 13.22$ KN (T)

Member	BC	BE	EF	AB	AF	BF
Magnitude of force	8.94	8.485	14	15.65	13.22	3
Nature	(T)	T	C	T	T	C

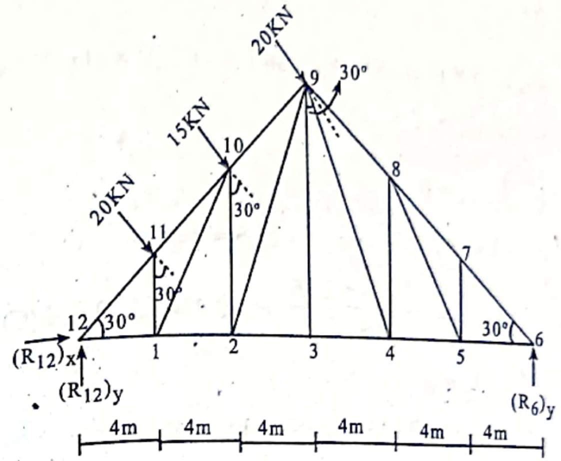
9. **Find the member - force of member 1 - 11, 1 - 10, 1 - 2, 2 - 10 and 10-11 of the simply supported roof truss loaded as shown in figure below:**



[2072 Chaitra]

Solution:

Calculation of unknown reactions:



$(\circlearrowleft +)\ \Sigma M_{12} = 0$

or, $20\cos30° \times 4 + 20\sin30° \times (4\tan30°) + 15\cos30° \times 8 + 15\sin30° \times (8\tan30°) + 20\cos30° \times 12 + 20\sin30° \times (12\tan30°) - 24 \times (R_6)_y = 0$

or, $(R_6)_y = 21.17\ (\uparrow)$

$(+\uparrow)\ \Sigma F_y = 0$

or, $(R_{12})_y + (R_6)_y - 20\cos30° - 15\cos30° - 20\cos30° = 0$

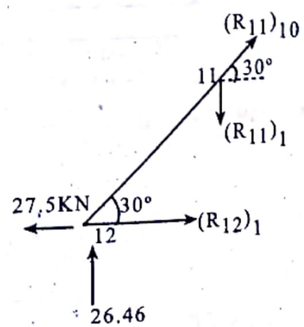
or, $(R_{12})_y = 26.46$ KN

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $(R_{12})_x + (20\sin30°) + 15\sin30° + 20\sin30° = 0$

or, $(R_{12})_x = -27.5$KN= 27.5 KN $(\leftarrow)$

On drawing the section A - A and taking left portion:



This section should be in equilibrium. So,

$(\circlearrowleft +)\ \Sigma M_{11} = 0$

or, $27.5 \times (4\tan 30^\circ) + 26.46 \times 4 - (R_{12})_1 \times 4\tan 30^\circ = 0$

or, $(R_{12})_1 = 73.33KN\ (T)$

$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $73.33 + (R_{11})_{10} \cos 30^\circ - 27.5 = 0$

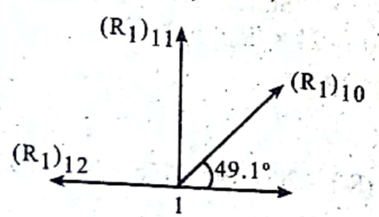
or, $(R_{11})_{10} = \dfrac{-73.33 + 27.5}{\cos 30^\circ} = -52.89 = 52.89\ KN\ (C)$

$(+\uparrow)\ \Sigma F_y = 0$

or, $-(R_{11})_1 + 26.46 - 52.89\sin 30^\circ = 0$

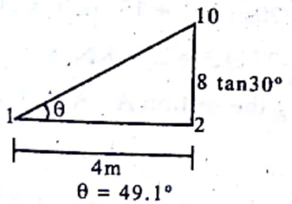
or, $(R_{11})_1 \approx 0KN$ (As compared to 70 KN, 15N can be neglected)

Applying joint - method at joint-1,



$(+\uparrow)\ \Sigma F_y = 0$

or, $(R_1)_{10} \sin(49.1^\circ) + (R_1)_{11} = 0$



or, $(R_1)_{10} \times \sin 49.1^\circ = 0 - 0$

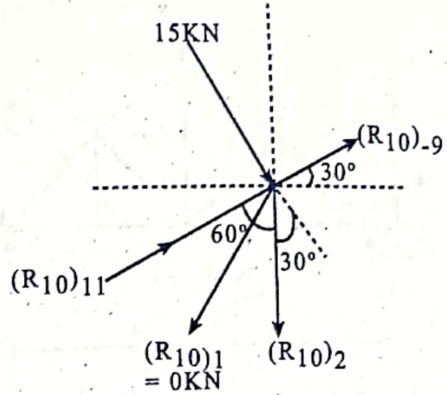
or, $(R_1)_{10} = 0$

$(\xrightarrow{+})\ \Sigma F_x = 0 \Rightarrow (R_1)_2 - (R_1)_{12} = 0 + (R_1)_{10} \cos 49.1^\circ$

or, $(R_1)_2 = (R_1)_{12}$

or, $(R_1)_2 = 73.33KN\ (T)$

Applying method of joint at joint 10,



$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $(R_{10})_9 \cos 30^\circ + 15\sin 30^\circ + (R_{10})_{11} \sin 60^\circ - (R_{10})_1 \sin(40.9) = 0$

or, $(R_{10})_9 \cos 30^\circ + 7.5 + 45.80 + 0 = 0$

or, $(R_{10})_9 \cos 30^\circ = -53.3$

or, $(R_{10})_9 = -61.55\ KN = 61.55KN\ (C)$

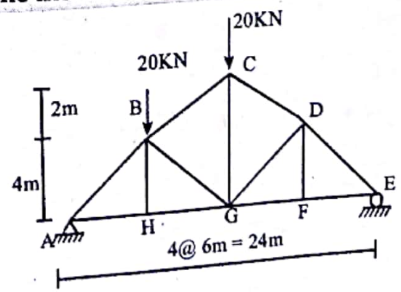
$(+\uparrow)\ \Sigma F_y = 0$

or, $-15\cos 30^\circ - 61.55\sin 30^\circ + 52.89\sin 60^\circ - (R_{10})_2 = 0$

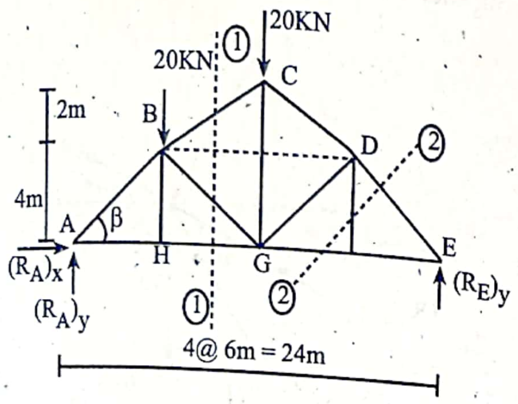
or, $(R_{10})_2 = 2.04KN\ (T)$

Nature	Forces
T	$R_{1\text{-}11} = R_{1\text{-}10} = 0$, $R_{2\text{-}10} = 2.04KN$
	$R_{1\text{-}2} = 73.33\ KN$
C	$(R_{10})_{11} = 52.89\ KN$

10. Determine the member force in member BC, BG and DF.



Solution:



Clearly, $(R_A)_x = 0$ as there is no external force $(\circlearrowleft +)\Sigma M_A = 0$

$-(R_E)_y \times 24 + 20 \times 12 + 20 \times 6 = 0$

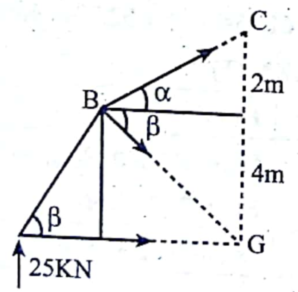
$\therefore R_E = 15$ KN $(\uparrow)$

$(+\uparrow)\ \Sigma F_y = 0$

$\Rightarrow 15 - 90 + (R_A)_y = 0$

$\therefore (R_A)_y = 25$ KN

In order to find BC and BG, let's take left section of (1) - (1)



$\alpha = \tan^{-1}\left(\frac{2}{6}\right) = 18.43°$, $\beta = \tan^{-1}\left(\frac{4}{6}\right) = 33.69°$

$(\circlearrowleft +)\Sigma M_G = 0$

$R_{BC} \cos\alpha \times 4 + R_{BC} \sin\alpha \times 6 - 20 \times 6 + 25 \times 12 = 0$

$\therefore R_{BC} = \dfrac{20 \times 6 - 25 \times 12}{4 \cos\alpha + 6 \sin\alpha}$

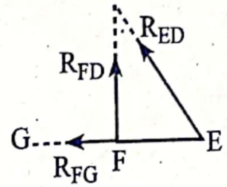
$= -25.27$ KN $= 25.27$ KN (C)

$(+\uparrow)\ \Sigma F_y = 0$

or, $25 - 20 - R_{BC} \sin\alpha - R_{BC} \sin\beta = 0$

or, $R_{BG} = \dfrac{5 - 8}{\sin\beta} = -5.41$ KN$=5.41$ KN (C)

Taking section (2) - (2) [Right of (2) - (2)]



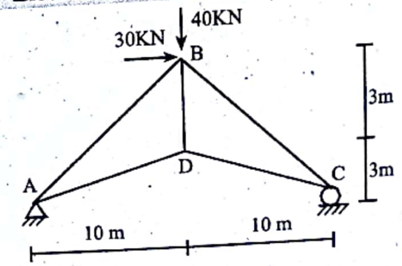
$(\circlearrowleft +)\Sigma M_F = 0$

or, $R_{DF} \times 6 = 0$

$\therefore R_{DF} = 0$

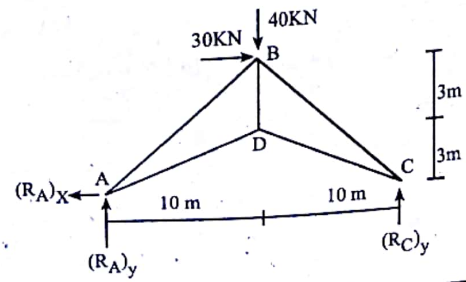
Member	Magnitude/ Nature
BG	5.41 KN (C)
BC	25.27 KN (C)
DF	0 KN

11. **Calculate the member forces in all members of the truss loaded as shown in figure below by using suitable method.**



[2073 Shrawan]

Solution:



$(\circlearrowleft +)\Sigma M_A = 0$

or, $-(R_c)_y \times 20 + 40 \times 10 + 30 \times 6 = 0$

or, $(R_c)_y = 29$ KN $(\uparrow)$

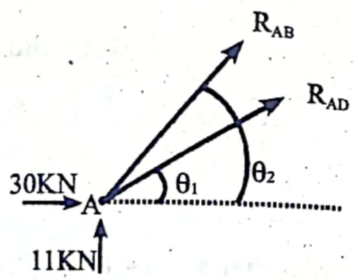
$(+\uparrow)\Sigma F_y = 0$

or, $(R_c)_y + (R_A)_y = 40$

or, $(R_A)_y = 40 - 29 = 11$ KN$(\uparrow)$

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $-(R_A)_x + 30$ KN $= 0$



or, $(R_A)_x = 30$ KN $(\leftarrow)$

$\theta_1 = \tan^{-1}\left(\frac{6}{10}\right) = 30.96°$, $\theta_2 = \tan^{-1}\left(\frac{3}{10}\right) = 16.7°$

Joint A

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $R_{AD}\cos\theta_1 + R_{AB}\cos\theta_2 = 30$

or, $R_{AD}\cos(30.96°) + R_{AB}\,(16.7°) = 30$

$(+\uparrow)\ \Sigma F_y = 0$

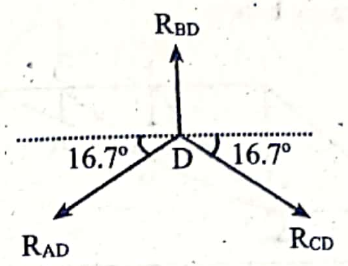
or, $R_{AD}\sin(30.96°) + R_{AB}\sin(16.7°) = -11$

On solving,

$R_{AD} = 100.95$ (T)

$R_{AB} = -77.77$ KN $= 77.77$ KN (C)

Joint D



$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $-R_{AD}\cos 16.7° + R_{DC}\cos 16.7° = 0$

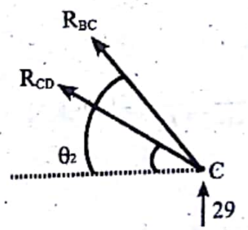
$\therefore R_{AD} = R_{DC}$ (T)

$(+\uparrow)\ \Sigma F_y = 0$

or, $R_{BD} - R_{AD}\sin 16.7° - R_{DC}\sin 16.7° = 0$

or, $R_{BD} = 2R_{DC}\sin 16.7° = 2 \times 100.95 \sin 16.7° = 58$ KN

Joint C



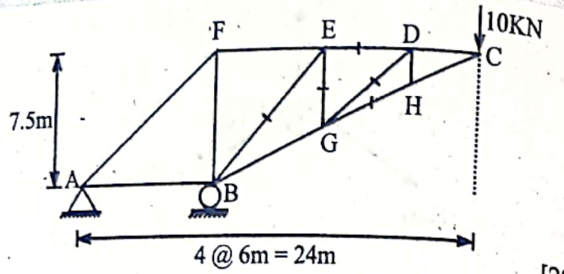
$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $R_{BC}\cos\theta_2 - R_{CD}\cos\theta_1 = 0$

or, $R_{BC} = R_{CD}\dfrac{\cos\theta_1}{\cos\theta_2} = 100.95\dfrac{\cos 16.7°}{\cos 30.96°} = 112.76$ KN (T)

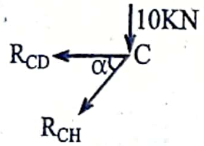
Member	Magnitude/ Nature
AB	77.77KN (C)
AD	100.95KN(T)
BC	112.76 KN (T)
BD	58 KN (T)
CD	100.95 (T)

12. **Find the member forces of indicated members of the truss shown in figure below.**



[2074 Chaitra]

Solution:



Joint C

$\alpha = \tan^{-1}\left(\frac{7.5}{18}\right) = 22.62°$

$(+\uparrow)\ \Sigma F_y = 0$

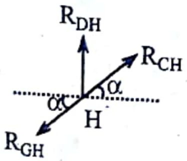
or, $-R_{CH}\sin\alpha - 10 = 0$

or, $R_{CH} = \frac{-10}{\sin\alpha} = \frac{-10}{\sin 22.62°} = -26KN = 26KN(C)$

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $-R_{CD} - R_{CH}\cos\alpha = 0$

or, $R_{CD} = -R_{CH}\cos 22.62° = -(-26)\cos 22.62° = 24\ KN(T)$



Joint H

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

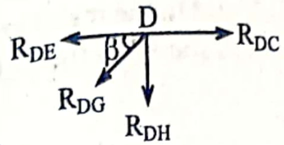
or, $-R_{GH}\cos\alpha + R_{CH}\cos\alpha = 0$

or, $R_{GH} = R_{CH} = -26KN = 26KN(C)$

$(+\uparrow)\ \Sigma F_y = 0$

or, $R_{DH} - R_{GH}\sin\alpha + R_{GH}\sin\alpha = 0$

or, $R_{DH} = 0$



Joint D

$(+\uparrow)\ \Sigma F_y = 0$

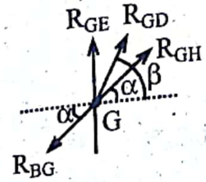
or, $-(R_{DG}\sin\beta + R_{DH}) = 0$

or, $R_{DG} = 0$

$(\overset{+}{\rightarrow})\Sigma F_x = 0$

or, $-R_{DE} + R_{DC} = 0$

or, $R_{DE} = R_{DC} = 24KN(T)$



Joint G

$(\overset{+}{\rightarrow})\Sigma F_x = 0$

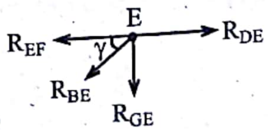
or, $R_{GH}\cos\alpha - R_{BG}\cos\alpha + R_{GD}\cos\beta = 0$

or, $R_{BG} = R_{GH}$ (since $R_{GD} = 0$)

$(+\uparrow)\ \Sigma F_y = 0$

or, $R_{GH}\sin\alpha - R_{BG}\sin\alpha + R_{GD} = 0$

or, $R_{GE} = 0$ (Since $R_{BG} = R_{GH}$)



Joint E

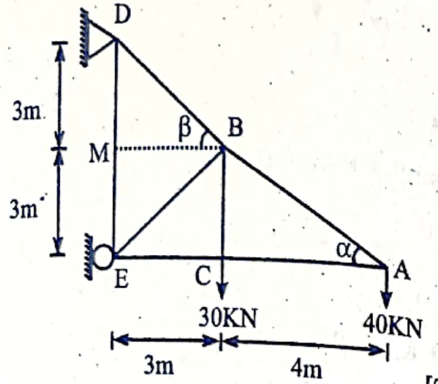
$(+\uparrow)\ \Sigma F_y = 0$

or, $-R_{BE}\sin\gamma - R_{GE} = 0$

or, $R_{BE} = 0$

Member	DH	DE	GH	GE	BE
Nature	None	T	C	None	None
Magnitude of force (KN)	0	24	26	0	0

13. Determine the force developed in the members BC, BE, BD and CE of the truss loaded as shown in figure below.



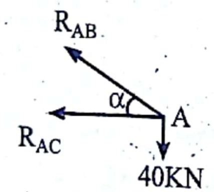
[2075 Baishakh]

Solution:

Joint Method

Joint A

In right angled ΔACB, $\alpha = \tan^{-1}\left(\frac{3}{4}\right) = 36.87°$



$(+\uparrow)\ \Sigma F_y = 0$

or, $-40 + R_{AB}\sin\alpha = 0$

or, $R_{AB} = \dfrac{40}{\sin\alpha} = 66.67$ KN(T)

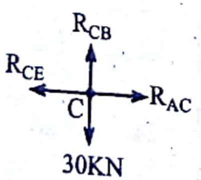
$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $-R_{AC} - R_{AB}\cos\alpha = 0$

or, $R_{AC} = -R_{AB}\cos\alpha$

$= -66.67\cos 36.87 = -53.34\text{KN} = 53.34\text{KN(C)}$

Joint C



$(+\uparrow)\ \Sigma F_y = 0$

or, $R_{CB} - 30 = 0$

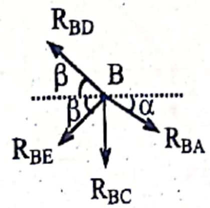
or, $R_{CB} = 30\text{KN(T)}$

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $R_{AC} - R_{CE} = 0$

or, $R_{CE} = R_{AC} = -53.34\text{KN} = 53.34\text{ KN(C)}$

Joint B



In right angled ΔDMB, $\beta = \tan^{-1}\left(\frac{3}{3}\right) = 45°$

$(+\uparrow)\ \Sigma F_y = 0$

or, $R_{BD}\sin\beta - R_{BE}\sin\beta - R_{BC} - R_{BA}\sin\alpha = 0$

or, $(R_{BD}\cos\beta)\sin 45° = 30 + 66.67\sin 36.87°$

or, $R_{BD} - R_{BE} = 70\sqrt{2}$ .... (i)

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $-R_{BD}\cos\beta - R_{BE}\cos\beta + R_{BA}\cos\alpha = 0$

or, $(R_{BD} + R_{BE})\cos\beta = 66.67\cos 36.37°$

or, $R_{BD} + R_{BE} = 75.43$.............. (ii)

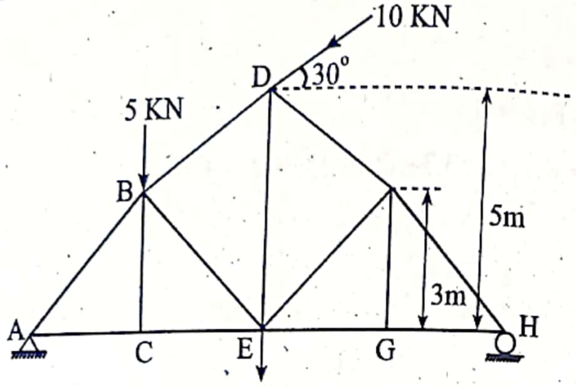
On solving (i) and (ii),

$R_{BD} = 87.21\text{KN} = 87.21\text{KN(T)}$

$R_{BE} = -11.78\text{KN} = 11.78\text{KN(C)}$

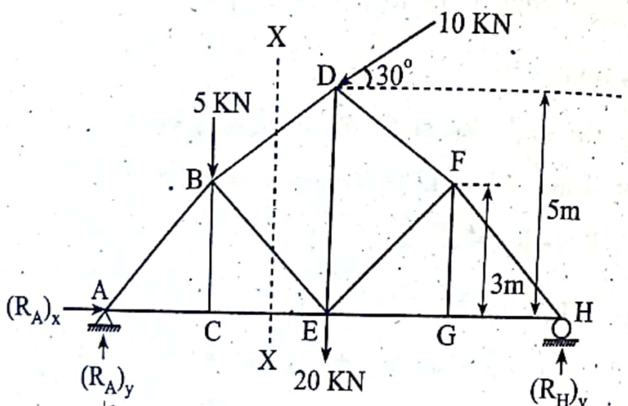
Member	BC	BE	BE	CE
Nature	T	C	T	C
Magnitude of force (KN)	30	11.78	87.21	53.34

14. **Determine the member force in AB, CE, BE, ED and BD for given truss.**



[2075 Chaitra]

Solution:



$(\circlearrowright +)\ \Sigma M_A = 0$

or, $-5\times5 - 20\times10 + (R_H)_y\times20 + 10\cos30°\times5 - 10\sin30°\times10 = 0$

or, $-25 - 200 + 20(R_H)_y + 25\sqrt{3} - 50 = 0$

$\therefore (R_H)_y = 11.5849$ KN (↑)

$(+\uparrow)\Sigma F_y = 0$

or, $(R_A)_y - 5 - 20 - 10\sin30° + 11.5849 = 0$

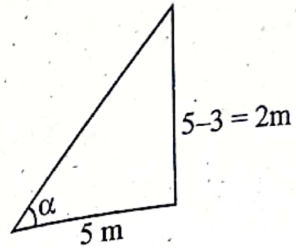
$\therefore (R_A)_y = 18.4151$ KN (↑)

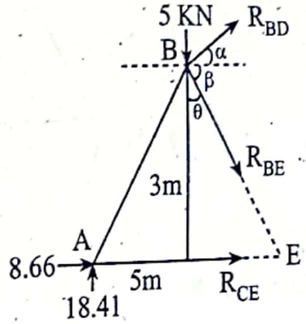
$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $(R_A)_x - 10\cos30° = 0$

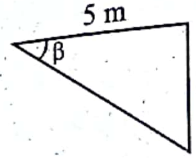
$\therefore (R_A)_x = 8.66$ KN (→)

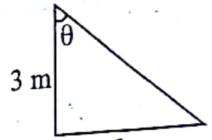
Draw a section X–X as shown and take the left part of section,





$\therefore \alpha = \tan^{-1}\left(\frac{2}{5}\right) = 21.80°$





$\beta = \tan^{-1}\left(\frac{3}{5}\right) = 30.96°$ $\qquad \theta = \tan^{-1}\left(\frac{5}{3}\right) = 59.037°$

$(\circlearrowright +)\ \Sigma M_A = 0$

or, $8.66\times3 - 18.41\times5 + R_{CE}\times3 = 0$

$\therefore R_{CE} = 22.03$ KN

$(\circlearrowright +)\ \Sigma M_E = 0$

or, $-18.41\times10 + 5\times5 - R_{BD}\cos21.8°\times3 - R_{BD}\sin21.8°\times8 = 0$

$\therefore R_{BD} = -34.282$ KN $= 34.282$ KN (C)

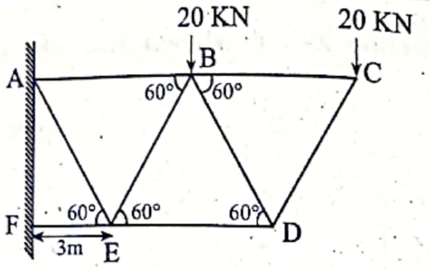
$(+\uparrow)\ \Sigma F_y = 0$

or, $18.415 - 5 - R_{BE}\sin30.96° - 34.28\sin21.81° = 0$

or, $13.4151 - 0.51448\ R_{BE} - 12.7317 = 0$

$\therefore R_{BE} = 1.32816$ KN (T)

15. **Determine the force developed in the members AB, ED, BC and BD of the truss loaded as shown in figure below.**

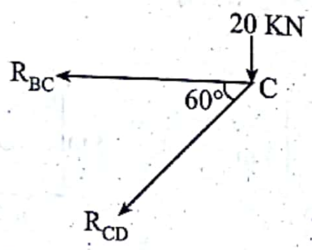


[2076 Bhadra]

Solution:

Joint Method

Joint C



$(+\uparrow)\ \Sigma F_y = 0$

or, $-20 - R_{CD}\sin60° = 0$

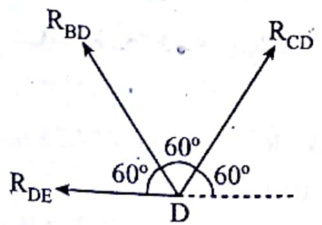
$\therefore R_{CD} = -23.1$ KN = 23.1 KN(C)

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $-R_{BC} - R_{CD}\cos60° = 0$

$\therefore R_{BC} = 11.55$ KN (T)

Joint D



$(+\uparrow)\ \Sigma F_y = 0$

or, $R_{CD}\sin60° + R_{BD}\sin60° = 0$

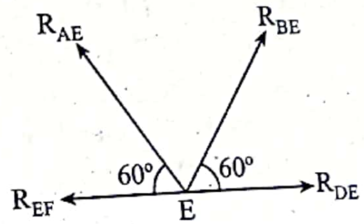
or, $R_{BD} = -R_{CD} = 23.1$ KN (T)

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $R_{CD}\cos60° - R_{BD}\cos60° - R_{DE} = 0$

or, $R_{DE} = -23.1$ KN = 23.1 KN (C)

Joint E



$(+\uparrow)\ \Sigma F_y = 0$

or, $-20 - 23.1\sin60° - R_{BE}\sin60° = 0$

or, $R_{BE} = -46.2$KN = 46.2 KN (C)

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $-R_{AB} + 11.55 + 23.1\cos60° - R_{BE}\cos60° = 0$

or, $R_{AB} = 46.2$ KN (T)

Member	Magnitude and nature
BC	11.55 (T)
BD	23.1 (T)
ED	23.1 (C)
AB	46.2 (T)

CHAPTER - 3

FORCES ACTING ON PARTICLE AND RIGID BODY

3.1 Force

Force is defined as an external agent which changes or tends to change the speed, direction, or shape of system.

The characteristics of force are:

- Force has both magnitude and direction. So, it is a vector quantity.
- Force has point of application.
- Force is a transmissible vector i.e., it can be moved along its line of action.

Forces may be classified as:

1. **Point force**
   - It is a force which is assumed to act through point.
2. **Body force**
   - It is a force which acts on each element of body. Examples: gravitational force, inertia force, electromagnetic force, etc.
3. **Surface force**
   - It is a force which acts on the surface or area elements of the body. When the area considered lies on the actual boundary of the body, the surface force distribution is termed as *surface traction.*

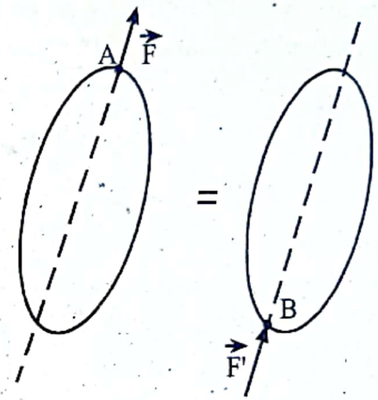
Force may also be classified as:

1. **Translation force**
   - The force which moves or tends to move a body from one point to another point is called *translation force.*
2. **Rotational force**
   - The force which rotates or tends to rotate a body around a central axis is called *rotational force.*

3.2 Principle of Transmissibility and Equivalent Forces

Principle of Transmissibility of Force

It states that the condition of equilibrium or of motion of a rigid body will remain unchanged if a force $\vec{F}$ acting at a given point of the rigid body is replaced by a force $\vec{F'}$, of the same magnitude and direction, but acting at different point, provided that the two forces have the same line of action.



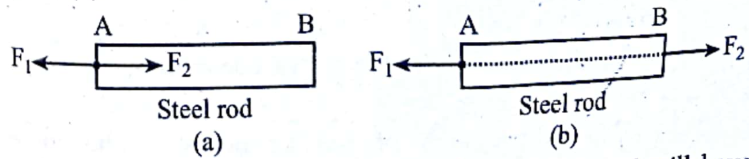
Here, Force $\vec{F}$ acting at point A is replaced by a force $\vec{F'}$ such that $|\vec{F'}| = |\vec{F}|$ at B and having same line of action.

Clearly, $\vec{F'} = \vec{F}$

Limitation:

Consider a steel rod in equilibrium. Apply two forces of same magnitude but in opposite direction at A. The equilibrium condition of steel rod is not disturbed yet.

But as per principle of transmissibility, the force 'F' can be replaced at B without disturbing its line of action as shown in figure (b).

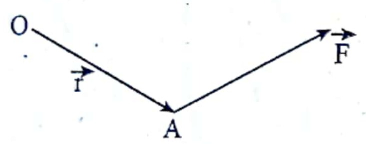


If we observe fig. (b), the rod is in tension and will have some deflections depending upon its area, length, modulus of elasticity magnitude of force. This tensile force will result in the elongation of that steel bar which is contradiction of principle of transmissibility.

Thus, principle of transmissibility is applicable only to the rigid body in which we neglect the internal effects.

3.3 Moments and Couples

Moment of force about a point is defined as the turning tendency of a force about that point. It is given by the product of force and the perpendicular distance of the line of action of force from that point.



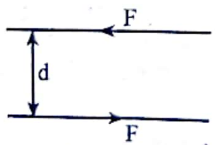
In vector form, moment of $\vec{F}$ about O is expressed as

$\vec{M}_O = \vec{r} \times \vec{F}$

where $\vec{r}$ = position vector of A w.r.t. O

A system formed by two parallel forces equal in magnitude but opposite in direction separated by a finite distance is known as *couple.*

Moment of couple = F × d



Characteristics of couple:

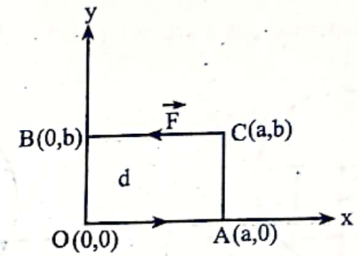
A couple (whether clockwise or anticlockwise) has the following characteristics:

1. The algebraic sum of the forces, constituting the couple is zero.
2. Th algebraic sum of the moments of the forces, constituting the couple, about any point is the same, and equal to the moment of the couple itself.
3. A couple cannot be balanced by a single force, but can be balanced only by a couple, but of opposite sense.
4. Any number of coplanar couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.

Couple is a Free Vector

Proof:

To show couple is a free vector, we have to show moment of couple about any point will be same.



Consider a couple as shown in figure acting on a xy-plane. The couple is acting on a rectangular element of length a and breadth b.

Taking moment about O,

$\vec{M}_O = \vec{r}_{OB} \times (-F\hat{i})$ (Note: $\vec{r}_{AB} = \vec{r}_{OB} - \vec{r}_{OA}$)

$= \{(0,b) - (0,0)\} \times (-F\hat{i})$

$= +b\hat{j} \times -F\hat{i} = bF\hat{k}$

Taking moment about A,

$\vec{M}_A = \vec{r}_{AC} \times \vec{F}$

$= \{(a,b) - (a,0)\} \times (-F\hat{i})$

$= (a\hat{i} - b\hat{j}) \times (-F\hat{i}) = bF\hat{k}$

Taking moment about B,

$\vec{M}_B = \vec{r}_{BO} \times F\hat{i}$

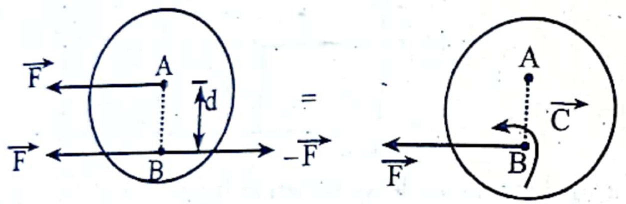
$= \{(0,0) - (0,b)\} \times F\hat{i}$

$= -b\hat{j} \times F\hat{i} = bF\hat{k}$

Since $M_O = M_A = M_B$, it is proved that couple is a free vector.

3.4 Resolution of a Force into Force and a Couple

Any force $\vec{F}$ acting on a rigid body can be moved to an arbitrary point O provided that a couple is added whose moment is equal to the moment of $\vec{F}$ about O. Let a force $\vec{F}$ acts at point A on the body as shown in figure below. If it is desired to transfer it to point B, we replace equal and opposite force $\vec{F}$ at point B. The second equivalent system can be viewed as force $\vec{F}$ acting at B and couple $\vec{C}$ whose magnitude is F×d, where d = distance between A and B.



This couple $\vec{C}$ and $\vec{F}$ are perpendicular to each other

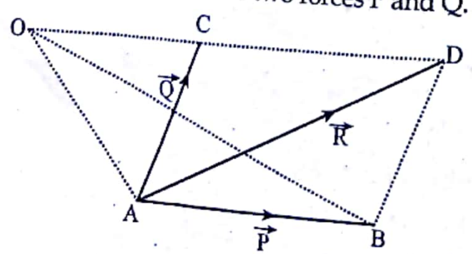
$$|\vec{C}| = F \times d$$

$$\vec{C} = \vec{r}_{BA} \times \vec{F}$$

3.5 Varignon's Theorem

Varignon's theorem states that moment of a resultant of two forces about a point lying in the plane of the forces is equal to the algebraic sum of moments of these forces about the same point.

Consider two concurrent forces P and Q as shown. Let O be the point about which moment is to be calculated. Through O, draw a line parallel to the direction of force P which meets the line of action of force Q at C. Let R be the resultant of two forces P and Q.



Moment of force P about O (M_P) = 2×ΔAOB

Moment of force Q about O (M_Q) = 2×ΔAOC

Moment of force P about O (M_R) = 2×ΔAOD

From figure,

ΔAOD = ΔAOC + ΔACD

or, ΔAOD = ΔAOC + ΔABD

or, ΔAOD = ΔAOC + ΔAOB

or, 2 × ΔAOD = 2 × ΔAOC + 2 × ΔAOB

$\therefore M_R = M_Q + M_P$

This principle can be extended for number of forces i.e., moment of resultant of a number of forces about a point lying in the plane of forces is equal to the algebraic sum of moments of these forces about the same point.

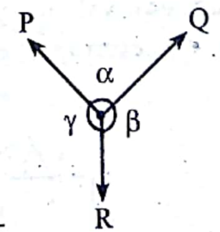
$$\vec{r} \times (\vec{F_1} + \vec{F_2} + \vec{F_3} + \ldots\ldots) = \vec{r} \times \vec{F_1} + \vec{r} \times \vec{F_2} + \vec{r} \times \vec{F_3} + \ldots\ldots$$

$$\therefore \vec{r} \times \vec{F} = \vec{r} \times \vec{F_1} + \vec{r} \times \vec{F_2} + \vec{r} \times \vec{F_3} + \ldots\ldots$$

3.6 Lami's Theorem

Lami's theorem states that if a body is in equilibrium under the action of three forces, then, each force is proportional to the sine of the angle between the other two forces.

Mathematically,

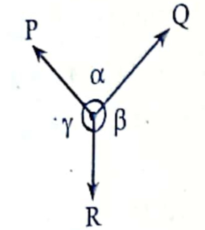


$$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin\alpha}$$

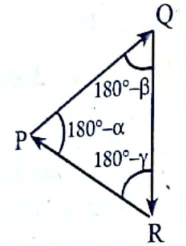
All the forces must be outgoing from a common point.

Proof:

Let P, Q, R be the three concurrent forces in equilibrium as shown in figure.



Forming a triangle with these forces,



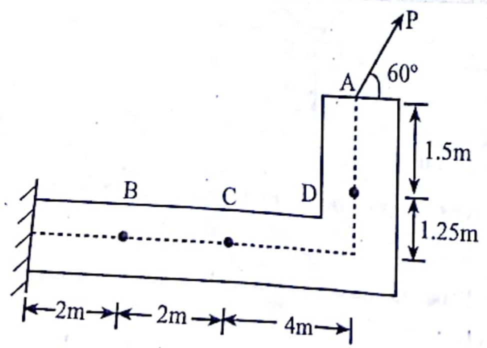
Using sine law,

$$\frac{P}{\sin(180°-\beta)} = \frac{Q}{\sin(180°-\gamma)} = \frac{R}{\sin(180°-\alpha)}$$

$\therefore \frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin\alpha}$ proved.

SOLVED NUMERICALS

1. **A 160 N force P is applied at point A of a structural member. Replace P with (a) an equivalent force couple system at C (b) an equivalent system consisting of a vertical force at B and a second force at D.**



[2069 Bhadra]

Solution:

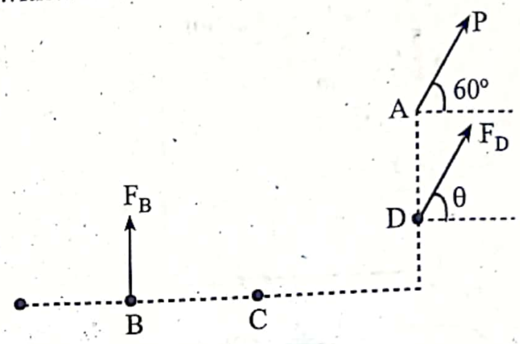
a) Equivalent force at C = $\vec{P}$ = 160 N ∠60°

Equivalent couple is moment of this force P about C and is calculated as

$M_C = -P\cos60° \times (1.5 + 1.25) + P\sin60° \times 4$

$= -160\cos60° \times 2.75 + 160\sin60° \times 4 = 334.25$ Nm

b) Let F_B be the vertical force at B and F_D be the force at D at an angle θ with x-axis.



Using Varignon's theorem,

Moment of P about D = Moment of F_B about D + moment of F_D about D

or, $P\cos60° \times 1.5 = F_B \times 6 + 0$

or, $160\cos60° \times 1.5 = F_B \times 6 + 0$

$\therefore$ $F_B = 20$ N (↑)

Equating vertical component,

$P\sin60° = F_B + F_D\sin\theta$

or, $160\sin60° = 20 + F_D\sin\theta$

or, $F_D\sin\theta = 118.56$ .... (i)

Equating horizontal component,

$P\cos60° = F_D\cos\theta$

or, $160\cos60° = F_D\cos\theta$

or, $F_D\cos\theta = 80$ .... (ii)

From equations (i) and (ii),

$\tan\theta = 1.482$

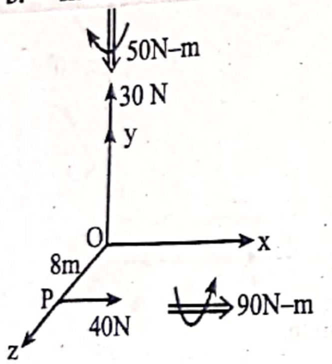
or, $\theta = 56°$

From equation (ii),

$F_D \cos 56° = 80$

$\therefore\ F_D = 143.03$ N

2. **Replace the two wrenches as shown in figure by a single equivalent wrench and determine:**
   a. **The resultant force**
   b. **Indicate its line of action.**



[2069 Chaitra]

Solution:

a. First of all reducing the given system of wrench into force-couple system at O.

This force-couple system consists of a force $\vec{R}$ and couple vector $\vec{M}_O^R$ defined as follows:

$\vec{R} = \sum\vec{F}$ and, $\vec{M}_O^R = \sum(\vec{r} \times \vec{F})$

Computations are arranged in tabular form.

$\vec{r}$ (m)	$\vec{F}$ (N)	$\vec{r} \times \vec{F}$ (N-m)
0	$30\hat{j}$	0
$8\hat{k}$	$40\hat{i}$	$320\hat{j}$

$\vec{C}_1 = 90\hat{i}$, $\vec{C}_2 = -50\hat{j}$

$\vec{R} = 40\hat{i} + 30\hat{j}$

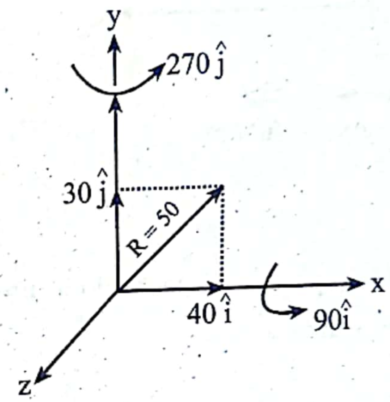
$\vec{M}_O^R = \vec{M}_{\text{due to 40N}} + \vec{C}_1 + \vec{C}_2$

$= 320\hat{j} + 90\hat{i} - 50\hat{j} = 90\hat{i} + 270\hat{j}$

Magnitude of $\vec{R}$ is

$|\vec{R}| = R$

$= \sqrt{40^2 + 30^2} = 50$ N



For calculation of wrench,

$$\text{Pitch of wrench (P)} = \frac{\vec{R}\cdot\vec{M}_o^R}{R^2}$$

$$= \frac{(40\hat{i} + 30\hat{j}).(90\hat{i} + 270\hat{j})}{50^2}$$

$$= \frac{3600 + 8100}{2500}$$

$= 4.68$

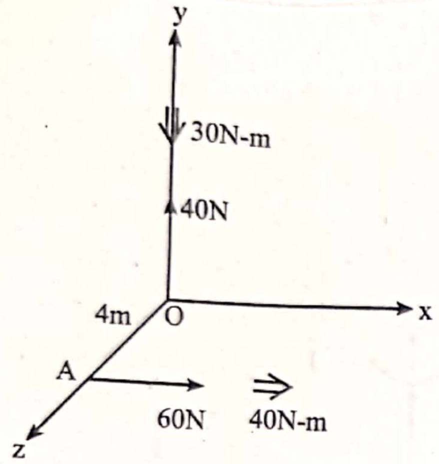
Resolving $\vec{M}_o^R$ in direction of resultant force $\vec{R}$

$\vec{M}_1 = 4.68\,(40\hat{i} + 30\hat{j})$

$= 187.2\hat{i} + 140.4\hat{j}$

and, $\vec{R} = 40\hat{i} + 30\hat{j}$ is new wrench.

3. **Obtain the resultant of the two pairs of wrench shown in the figure. Indicate its line of action.**



[2070 Chaitra]

Solution:

Resultant of two pairs of wrench and its line of action:

Resultant force at O is

$\vec{F_R} = \vec{F_1} + \vec{F_2}$

$\vec{F_1} = 40\hat{j}$ N, $\vec{C_1} = -30\hat{j}$ N–m

$\vec{F_2} = 60\hat{i}$ N, $\vec{C_2} = 40\hat{i}$ N–m

$\therefore \vec{F_R} = 60\hat{i} + 40\hat{j}$ N

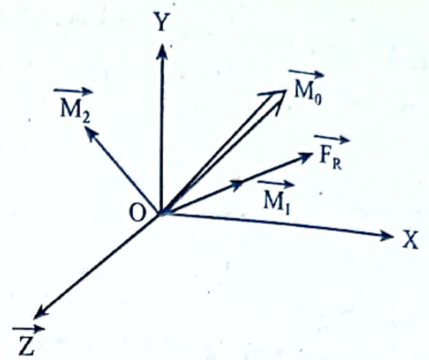
Resultant moment about O is

$\vec{M_O} = \vec{r_1} \times \vec{F_1} + \vec{r_2} \times \vec{F_2} + \vec{C_1} + \vec{C_2}$

$= 0 + 4\hat{k} \times 60\,\hat{i} - 30\hat{j} + 40\hat{i}$

$= 240\hat{j} - 30\hat{j} + 40\hat{i} = 40\hat{i} + 210\hat{j}$ N-m

Now, pair of wrenches can be replaced by a system of forces i.e., $\vec{F_R}$ and $\vec{M_O}$ at point O.



Direction of $\vec{M_O}$ and $\vec{F_R}$ are different and hence, this system is not wrench.

So, resolve $\vec{M_O}$ along $\vec{F_R}$ as $\vec{M_1}$

$\therefore M_1 = \vec{M_O}\,\hat{f}$

$\hat{f} = \dfrac{\vec{F_R}}{|\vec{F_R}|} = \dfrac{60\hat{i} + 40\hat{j}}{\sqrt{60^2 + 40^2}} = 0.832\hat{i} + 0.554\hat{j}$

$M_1 = (40\hat{i} + 210\hat{j}).(0.832\hat{i} + 0.554\hat{j})$

$= 33.28 + 116.34 = 144.62$ N-m

$\vec{M_1} = M_1\hat{f}$

$= 149.62(0.832\hat{i} + 0.554\hat{j}) = 124.48\hat{i} + 82.9\hat{j}$ N-m

Component of moment perpendicular to $\vec{F_R}$ is

$\vec{M_2} = \vec{M_O} - \vec{M_1} = 40\hat{i} + 210\hat{j} - (124.48\hat{i} + 82.9\hat{j})$

$= -84.48\hat{k} + 127.1\hat{j}$ N-m

Now, replacing $\vec{F_R}$ to new point P(x, y, z) from O so that it can cancel $\vec{M_2}$.

$\vec{OP} \times \vec{F_R} = \vec{M_2}$

or, $(x\hat{i} + y\hat{j} + z\hat{k}) \times (60\hat{i} + 40\hat{j}) = 84.48\hat{i} + 127.1\hat{j}$

or, $40x\hat{k} - 60y\hat{k} + 60z\hat{j} - 40z\hat{i} = -84.48\hat{i} + 127.1\hat{j}$

Equating coefficients of like vectors:

For $\hat{i}$,

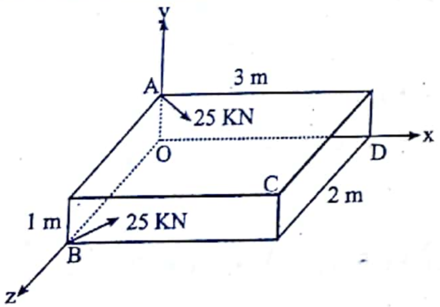
$-40z = -84.48$

$\therefore z = 2.112$ m

For $\hat{k}$, $40x - 60y = 0$

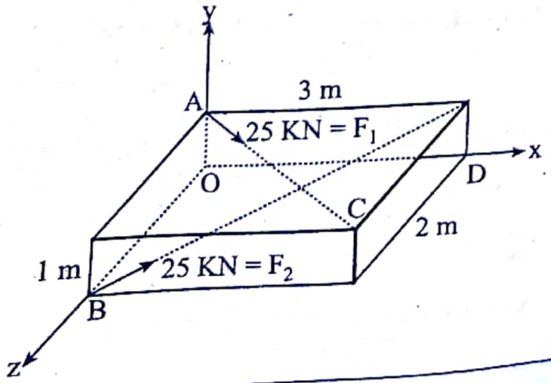
Force also passes through $x = 0$, $y = 0$. So, after cancelling $\vec{M_2}$; $\vec{M_1}$ and $\vec{F_R}$ make wrench at point P along z-axis at P(0, 0, 2.112 m).

4. **If two forces of same magnitude 25 KN act at points A and B as shown in figure and force at A passes through C and force at B passes through D, (a) find equivalent force couple system at 'O' (b) find equivalent wrench, give pitch and axis of wrench.**



[2070 Bhadra]

Solution:



Coordinate of O = O(0, 0, 0); coordinate of A = A(0, 1, 0)
coordinate of B = B(0, 0, 2); coordinate of C = C(3, 1, 2)
coordinate of D = D(3, 0, 0)

Direction of F_1 is along $\hat{f}_{AC}$

$\vec{F_1} = F_1 \hat{f}_{AC}$

$$\hat{f}_{AC} = \frac{\vec{AC}}{|\vec{AC}|} = \frac{(3-0)\hat{i} + (1-1)\hat{j} + (2-0)\hat{k}}{\sqrt{3^2 + 2^2}} = 0.832\hat{i} + 0.554\hat{k}$$

$\therefore \vec{F_1} = 25(0.832\hat{i} + 0.554\hat{k}) = 20.8\hat{i} + 13.87\hat{k}$ KN

Similarly, $\vec{F_2} = F_2 \hat{f}_{BD} = 25\dfrac{\vec{BD}}{|\vec{BD}|}$

$$= 25\frac{\{(3-0)\hat{i} + (0-2)\hat{k}\}}{\sqrt{3^2 + 2^2}}$$

$= 25(0.832\hat{i} - 0.554\hat{k}) = 20.8\hat{i} - 13.87\hat{k}$ KN

Resultant force at O, $\vec{F_R} = \vec{F_1} + \vec{F_2}$

$= 20.8\hat{i} + 13.87\hat{k} + 20.8\hat{i} - 13.87\hat{k}$

$= 41.6\hat{i}$ KN

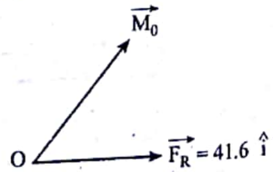
Moment about O, $\vec{M_O} = \vec{r_1} \times \vec{F_1} + \vec{r_2} \times \vec{F_2}$

$\vec{r_1} = \vec{OA} = \hat{j}$, $\vec{r_2} = \vec{OB} = 2\hat{k}$

$\vec{M_O} = \hat{j} \times (20.8\hat{i} + 13.87\hat{k}) + 2\hat{k}(20.8\hat{i} - 13.87\hat{k})$

$= -20.8\hat{k} + 13.87\hat{i} + 41.6\hat{j} + 0$

$= 13.87\hat{i} + 41.6\hat{j} - 20.8\hat{k}$ KN-m



To make wrench, $\vec{M_O}$ is resolved along $\vec{F_R}$ and perpendicular to $\vec{F_R}$.

Resolve $\vec{M_R}$ along $\vec{F_R}$ i.e., $\vec{M_1}$, $M_1 = \vec{M_1}.\hat{f}$

$\hat{f}$ = unit vector along $\vec{F_R} = \hat{i}$

$M_1 = \vec{M_O}.\hat{i} = (13.87\hat{i} + 41.6\hat{j} - 20.8\hat{k}).\hat{i} = 13.87$ KN-m

$\therefore \vec{M_1} = 13.87\hat{i}$ KN-m

$\vec{M_2}$ is moment along perpendicular direction of $\vec{F_R}$ which is given as

$$\vec{M_2} = \vec{M_O} - \vec{M_1}$$
$$= (13.87\hat{i} + 41.6\hat{j} - 20.8\hat{k}) - 13.87\hat{i} = 41.6\hat{j} - 20.8\hat{k} \text{ KN-m}$$

Cancel $\vec{M_2}$ by replacing $\vec{F_R}$ from O to point P(x, y, z). Now,

$\vec{OP} \times \vec{F_R} = \vec{M_2}$

or, $(x\hat{i} + y\hat{j} + z\hat{k}) \times 41.6\hat{i} = 41.6\hat{j} - 20.8\hat{k}$

or, $-41.6y\hat{k} + 41.6z\hat{j} = 41.6\hat{j} - 20.8\hat{k}$

Equating coefficients of like vectors:

For $\hat{j}$,

$41.6z = 41.6 \Rightarrow z = 1$ m

For $\hat{k}$,

$-41.6y = -20.8$

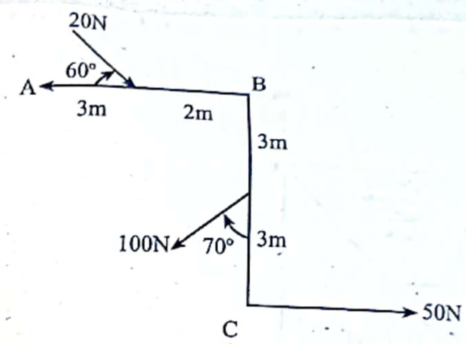
$\therefore y = 0.5$ m

Point P is P(0, 0.5, 1).

$$\text{Pitch of wrench (P)} = \frac{\vec{F_R}.\vec{M_O}}{F_R^2}$$
$$= \frac{(41.6\hat{i}).(13.87\hat{i} + 41.6\hat{j} - 20.8\hat{k})}{41.6^2}$$
$$= \frac{865.28}{41.6^2} = 0.5$$

Thus, $\vec{M_1} = 13.87\hat{i}$ and $\vec{F_R}$ at point P(0, 0.5, 1) = $41.6\hat{i}$ makes the wrench whose axis is along x-axis.

5. **Determine magnitude, direction, and line of action of the resultant of forces acting in the system shown in figure below.**



[2070 Ashadh]

Solution:

Let A be the origin. Resolving all the forces in x and y direction respectively, we get

Taking $(\xrightarrow{+})$,

$$\vec{F_x} = 20\cos 20°\hat{i} - 100\cos 20°\hat{i} + 50\hat{i}$$
$$= -33.9\hat{i}$$

Taking $(+\uparrow)$,

$$\vec{F_y} = -20\sin 60°\hat{j} - 100\cos 70°\hat{j}$$
$$= -51.52\hat{j}$$

Resultant of force in the system is

$$\vec{R} = \vec{F_x} + \vec{F_y}$$
$$= -33.9\hat{i} - 51.52\hat{j}$$

Magnitude of resultant is

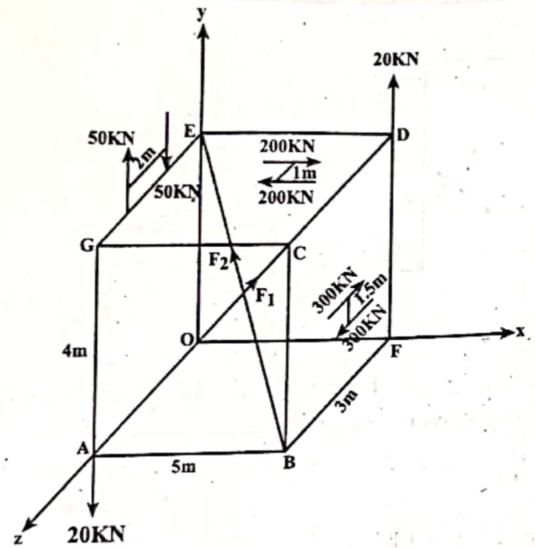
$$|\vec{R}| = \sqrt{(-33.9)^2 + (-51.52)^2} = 61.61 \text{ N}$$

Direction of resultant force is

$$\tan\theta = \frac{F_y}{F_x} = \frac{51.52}{33.97}$$

$$\therefore \theta = \tan^{-1}\left(\frac{51.52}{33.97}\right) = 56.6^\circ$$

6. Find the resultant of force couple system at point 'A' as shown in figure below. Take F_1 = 100 KN, F_2 = 300 KN.



[2071 Shrawan]

Solution:

Coordinates are O(0, 0, 0), A(0, 0, 3), B(5, 0, 3), C(5, 4, 3), D(5, 4, 0), E(0, 4, 0), F(5, 0, 0), G(0, 4, 3).

$\vec{F_1}$ is directed from O to C.

$$\vec{F_1} = 100\left\{\frac{5\hat{i} + 4\hat{j} + 3\hat{k}}{\sqrt{5^2+4^2+3^2}}\right\} = 70.71\hat{i} + 56.56\hat{j} + 42.43\hat{k}$$

$\vec{F_2}$ is directed from B to E.

$$\vec{F_2} = 300\left\{\frac{-5\hat{i} + 4\hat{j} - 3\hat{k}}{\sqrt{5^2+4^2+3^2}}\right\} = -212.13\hat{i} + 169.68\hat{j} - 127.27\hat{k}$$

We have to find resultant of force couple system at point A.

Resultant force at A

$$= \vec{F_1} + \vec{F_2}$$

$$= 70.71\hat{i} + 56.56\hat{j} + 42.43\hat{k} - 212.13\hat{i} + 169.68\hat{j} - 127.27\hat{k}$$

$$= -141.42\hat{i} + 226.27\hat{j} - 84.85\hat{k}$$

Couple due to $\vec{F_1}$ when transferring $\vec{F_1}$ at O to point A is

$$\vec{C_1} = \vec{r_{AO}} \times \vec{F_1}$$

$$= -3\hat{k} \times \{(70.71\hat{i} + 56.56\hat{j} + 42.4\hat{k})\}$$

$$= -212.1\hat{j} + 169.68\hat{i} = 169.68\hat{i} - 212.1\hat{j}$$

Similarly, due to $\vec{F_2}$

$$\vec{C_2} = \vec{r_{AB}} \times \vec{F_2}$$

$$= \{(5,0,3) - (0,0,3)\} \times (-12.12\hat{i} + 169.8\hat{j} - 127.27\hat{k})\}$$

$$= 849\hat{k} + 636\hat{j}$$

$$= 636\hat{j} + 849\hat{k}$$

$$\vec{C_3} = (-\hat{k}) \times 200\hat{i} = -200\hat{j}$$

$$\vec{C_4} = 2\hat{k} \times 50\hat{j} = -100\hat{i}$$

$$\vec{C_5} = 1.5\hat{k} \times 300\hat{j} = -450\hat{i}$$

$\vec{C_6}$ (due to 20 KN at A and D) $= \vec{r_{AD}} \times 20\hat{j}$

$$= \{(5, 4, 0) - (0,0,3)\} \times 20\hat{j}$$

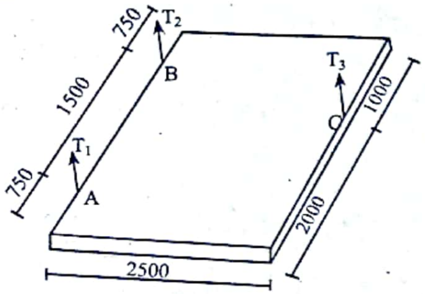
$$= (5\hat{i} + 4\hat{j} - 3\hat{k}) \times 20\hat{j}$$

$$= 100\hat{k} + 60\hat{i}$$

$$\therefore \vec{C} = \Sigma\vec{C_i} = \vec{C_1} + \vec{C_2} + \vec{C_3} + \vec{C_4} + \vec{C_5} + \vec{C_6}$$

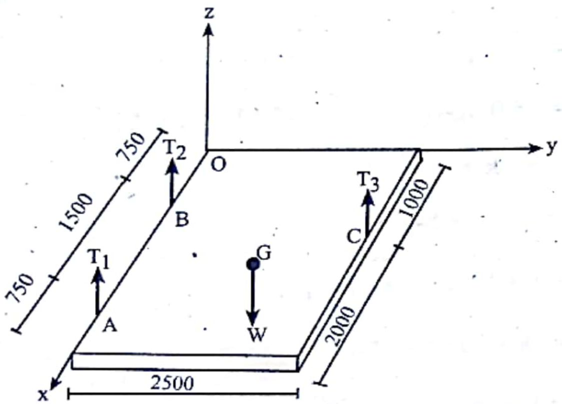
$$= -320.32\hat{i} + 223.9\hat{j} + 949\hat{k}$$

7. **Three vertical wires as shown in figure support a plate of 50 kg. Determine the tension in each wire. All dimensions are in mm.**



[2071 Bhadra]

Solution:



Co-ordinates of A, B, C with respect to origin O are A (2250, 0, 0), B (750, 0, 0) and C(1000, 2500, 0). Co-ordinate of G (1500, 12500).

Since the plate is an equilibrium,

$(+\uparrow)\ \Sigma F_y = 0$

or, $T_1 + T_2 + T_3 = 50$ kg.......(i)

At origin, $(\circlearrowleft +)\Sigma M = 0$

or, $2250\hat{i} \times T_1\hat{k} + 750\hat{i} \times T_2\hat{k} - (1500\hat{i} + 1250\hat{j}) \times N\hat{k} + (1000\hat{i} + 12500\hat{j}) \times T_3\hat{k} = 0$

Once again rewriting,

$2250\hat{i} \times T_1\hat{k} + 750\hat{i} \times T_2\hat{k} + (1500\hat{i} + 1250\hat{j}) \times (-W\hat{k}) + (1000\hat{i} + 2500\hat{j}) \times T_3\hat{k} = 0$

On reducing it,

$(-2250T_1 - 750T_2 + 1500W - 1000T_3)\hat{j} + (-1250W + 2500T_2)\hat{i} = 0\hat{i} + 0\hat{j}$

Equating coefficient of like components of vectors,

$-1250W + 2500\ T_3 = 0$

$\therefore T_3 = \dfrac{1250}{2500} W = \dfrac{50}{2} = 25$ kg

Similarly,

$-2250\ T_1 - 750\ T_2 + 1500\ W - 1000\ T_3 = 0$

or, $2250T_1 + 750T_2 = 1500 \times 50 - 1000 \times 25$

or, $2250T_1 + 750T_2 = 50{,}000$.........(ii)

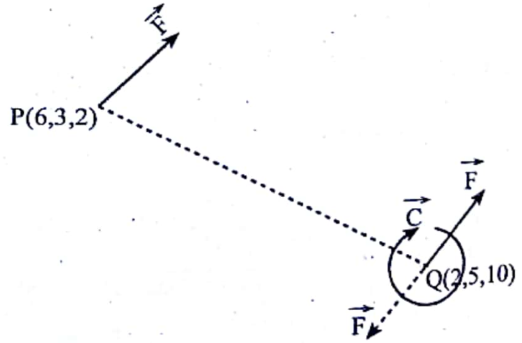
Solving equations (i) and (ii), we get

$T_1 = 20.83$ kg , $T_2 = 4.17$ kg

8. **Force $\vec{F} = (3\hat{i} - 6\hat{j} + 4\hat{k})$ N passes through the point (6, 3, 2) m. Release this force with an equivalent system where the force $\vec{F}$ passes through the point (2, 5, 10) m.** [2071 Bhadra]

Solution:

When the force $\vec{F}$ is replaced to some other point, couple is additionally developed.



This couple thus formed is given by

$\vec{C} = \vec{r}_{QP} \times \vec{F}$

$= (\overrightarrow{OP} - \overrightarrow{OQ}) \times \vec{F}$

$= \{(6,3,2) - (2,5,10)\} \times (3, -6, 4)$

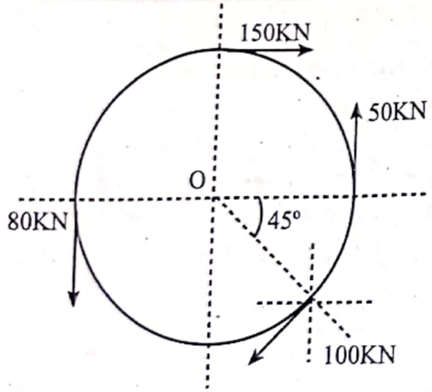
$= (4,-2,-8) \times (3,-6,4)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & -8 \\ 3 & -6 & 4 \end{vmatrix}$$

$= \hat{i}\,(-8-48) - \hat{j}\,(16+24) + \hat{k}\,(-24+6)$

$= -56\hat{i} - 40\hat{j} - 18\hat{k}$

9. **Determine the resultant of the forces acting tangentially to a circle of radius 3m as shown in figure. What will be the location of the resultant with respect to the centre of the circle?**



[2071 Magh]

Solution:

$(\overset{+}{\rightarrow})\sum F_x = 150 - 100\cos 45^\circ$

$= 79.29$ KN $(\rightarrow)$

$(+\uparrow)\Sigma F_y = 50 - 80 - 100\sin 45^\circ$

$= -100.71$ N$= 100.71$ N $(\downarrow)$

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{100.71^2 + 79.29^2} = 128.18$KN

$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \tan^{-1}\left(\frac{100.71}{79.29}\right) = 51.786^\circ$

(clockwise with horizontal direction).

$\therefore R = 128.18$KN

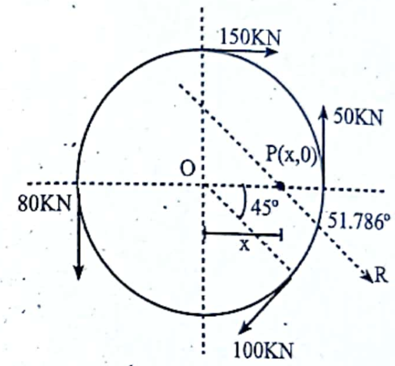
To find position,

$(\circlearrowright +)\ \Sigma M_O = 0$

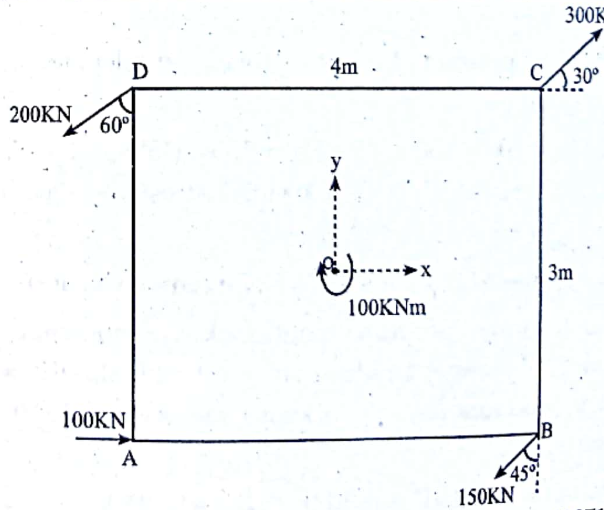
or, $+150\times3 - 50\times3 - 80\times3 + 100\times3 = R\sin 51.786^\circ \times x = 0$

or, $360 = 128.18 \sin 51.786^\circ \times x$

$\therefore x = 3.57$ m (right of O)



10. **Determine the magnitude, direction, and position w.r.t. center 'O' of the resultant of the forces acting on the rectangular plate ABCD as shown in figure below.**



[2071 Chaitra]

Solution:

At first find the sum of the all forces in horizontal and vertical direction.

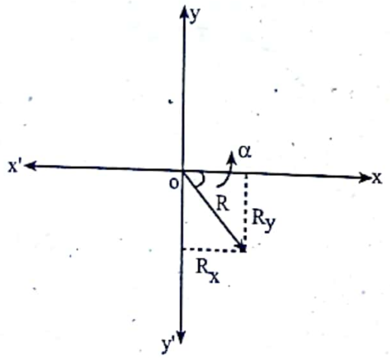
$(\overset{+}{\rightarrow})\sum F_x = 300\cos30^\circ - 200\sin60^\circ + 100 - 150\sin45^\circ$

$= 80.53$ KN

$(+\uparrow)\Sigma F_y = 300\sin30^\circ - 200\cos60^\circ - 150\cos45^\circ$

$= -56.07$ KN

Since ΣF_x is +ve and ΣF_y is –ve, the resultant lies in fourth quadrant.



$\therefore R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{30.53^2 + (-56.07)^2} = 98.12$ KN

$\alpha = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \tan^{-1}\left(\frac{-56.07}{80.53}\right) = 34.84^\circ$

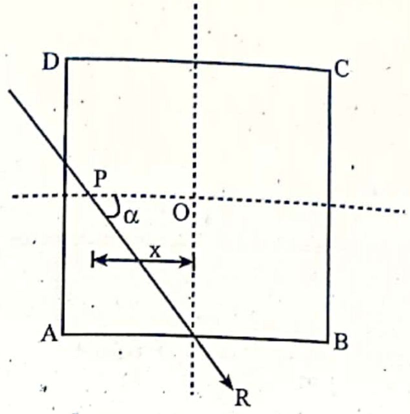
To find the position of resultant force, we calculate moment about point (say O for now).

$(\circlearrowleft +)\Sigma M_O = 100 + 150\cos45^\circ\times1.5 + 150\sin45^\circ\times2 - 300\sin30^\circ\times2 + 300\cos30^\circ\times1.5 - 100\times1.5 - 200\cos60^\circ\times2 - 200\sin60^\circ\times 1.5$

$= -48.86$ KNm $= 48.86$ KNm (anticlockwise)

So, all the forces produce an anticlockwise moment of 48.86 KNm. Thus, we get an idea that resultant R should lie such that it produces an anticlockwise moment of 48.86 KNm about O.

For this to happen, R should lie to the left of O.



Moment of R about O = R sin34.84° × x

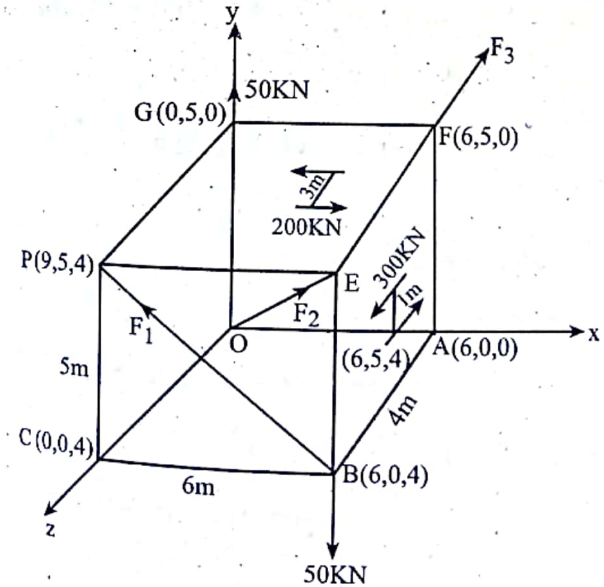
or, $\frac{48.86}{98.12\sin34.84^\circ} = x$

or, x = 0.87 m = OP

OM = OP tanα = 0.87 tan 34.84° = 0.606 m

Thus, the resultant cuts EF 0.87 m left and GH 0.606 m below O at an angle of α = 34.84° with horizontal.

11. **Determine the force couple system at origin of given system. Take $F_1 = 100$ KN, $F_2 = 300$ KN and $F_3 = 200$ KN.**



[2072 Ashwin]

Solution:

$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3}$

Now, $\vec{F_1} = F_1 \hat{f}_{BD}$

$$= 100\left\{\frac{(0,5,4) - (6,0,4)}{\sqrt{(0-6)^2 + (5-0)^2 + (4+4)^2}}\right\}$$

$$= 100\left\{\frac{-6\hat{i} + 5\hat{j}}{\sqrt{36+25}}\right\} = -76.82\hat{i} + 64.02\hat{j}$$

Similarly, $\vec{F_2} = F_2 \hat{f}_{OE}$

$$= 300\left\{\frac{(6,5,4) - (0,0,0)}{\sqrt{(6-0)^2 + (5-0)^2 + (4-0)^2}}\right\}$$

$$= 34.19\,(6\hat{i} + 5\hat{j} + 4\hat{k})$$

$$= 205.14\hat{i} + 170.95\hat{j} + 136.76\hat{k}$$

$\vec{F_3} = -200\hat{k}$

$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3}$

$$= (-76.82\hat{i} + 64.02\hat{j}) + (205.14\hat{i} + 170.95\hat{j} + 136.76\hat{k}) - 200\hat{k}$$

$$= (128.32\hat{i} + 234.97\hat{j} - 63.24\hat{k})$$

Couple due to transferring of $\vec{F_1}$, $\vec{F_2}$ and $\vec{F_3}$ at origin is calculated as:

Due to transfer of $\vec{F_1}$, $\vec{C_1} = \vec{r_{OB}} \times \vec{F_1}$

$$= (6\hat{i} + 4\hat{k}) \times (-76.82\hat{i} + 64.02\hat{j})$$

$$= -256.08\hat{i} - 307.28\hat{j} + 384.12\hat{k}$$

Due to transfer of $\vec{F_3}$, $\vec{C_3} = \vec{r_{OF}} \times \vec{F_3}$

$$= (6\hat{i} + 5\hat{j}) \times (-200\hat{k})$$

$$= +1200\hat{j} - 1000\hat{i}$$

$$= -1000\hat{i} + 1200\hat{j}$$

Two couples are lying on the planes DEFG and ABEF respectively.

Couple on plane DEFG, $\vec{C_4} = 3\hat{k} \times 200\hat{i} = 600\hat{j}$

Couple on plane ABEF, $\vec{C_5} = \hat{j} \times 300\hat{k} = 300\hat{i}$

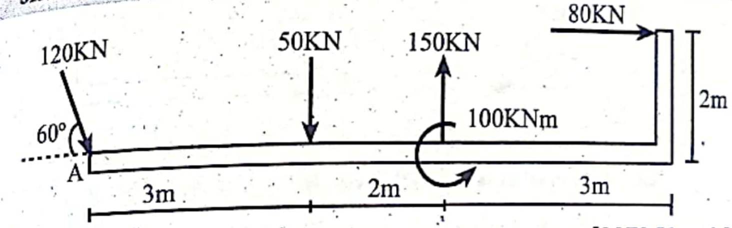
Total couple system at origin ($\vec{C}$)

$$= \vec{C_1} + \vec{C_2} + \vec{C_3} + \vec{C_4} + \vec{C_5}$$

$$= -956.08\hat{i} + 1492.72\hat{j} + 384.12\hat{k}$$

Total force at O, $\vec{R} = 128.32\hat{i} + 234.97\hat{j} - 63.24\hat{k}$

12. **Determine the magnitude, direction, and position of the resultant of the system of forces with respect to point A as shown in figure below.**



[2072 Kartik]

Solution:

On finding the sum of all forces in horizontal and vertical direction, we get

$(\overset{+}{\rightarrow})\ \Sigma F_x = 80 + 120\cos 60°$

$= 80 + 60 = 140\text{KN} (\rightarrow)$

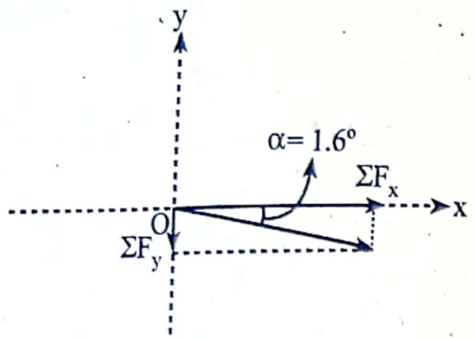
$(+\uparrow)\ \Sigma F_y = 150 - 50 - 120\sin 60°$

$= -3.923 \text{ KN} = 3.923\text{KN}(\downarrow)$

Since ΣF_x is +ve, ΣF_y is –ve, the resultant lies in 4th quadrant.

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{140^2 + (-3.923)^2} = 140.05$ KN

$\alpha = \tan^{-1}\left(\frac{3.923}{140}\right) = 1.6°$

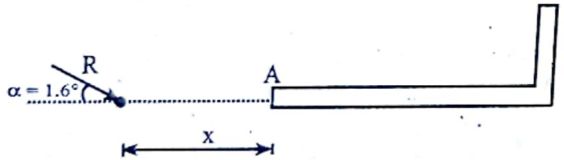


To find position, we have to calculate moment about A.

$(\circlearrowleft +)\Sigma M_A = 50 \times 3 - 150 \times 5 - 100 + 80 \times 2$

$= -540\ KNm = 540\ KNm$ (anticlockwise)

This means the resultant should lie such that it produces anticlockwise moment of 540 KNm about A. For this, R should lie to the left of A.



Let R acts at a distance x from A. Then,

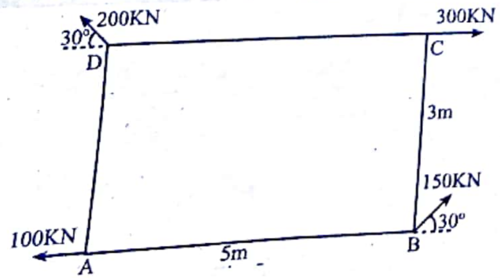
Moment of R about A = $(R\sin\alpha) \times x$

or, $540 = R\sin 1.6° \times x$

or, $x = \dfrac{540}{140.05\sin 1.6°} = 138.09m$

Thus, the resultant force should act far away i.e., 138.09m at an angle $\alpha = 1.6°$ with horizontal, left of A.

13. Find the magnitude, direction, and position of resultant force of the following system as shown in figure.



[2072 Chaitra]

Solution:

On finding the sum of all forces in horizontal and vertical direction, we get

$(\overset{+}{\rightarrow})\ \Sigma F_x = 300 + 150\cos 30° - 100 - 200\cos 30°$

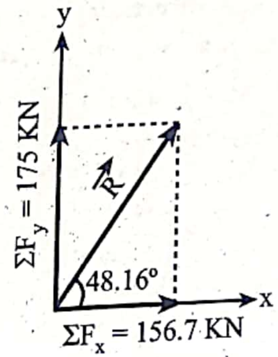
$= 156.7\ KN$

$(\uparrow +)\Sigma F_y = 150\sin 30° + 200\sin 30°$

$= 175\ KN$

Since ΣF_x and ΣF_y both are positive, the resultant lies in the first quadrant.

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(156.7)^2 + (175)^2} = 234.90\ KN$



If α is the angle made by resultant with x-direction, then

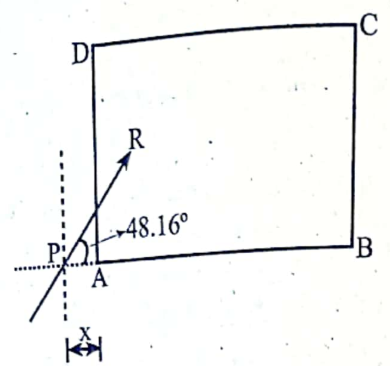
$\tan\alpha = \dfrac{\Sigma F_y}{\Sigma F_x} \Rightarrow \alpha = \tan^{-1}\left(\dfrac{175}{156.7}\right) = 48.16°$

To find the position of resultant force, we have to calculate moment about any point (say A for now).

$(\circlearrowright +)\Sigma M_A = -200\cos 30° \times 3 + 300 \times 3 - 150\sin 30° \times 5$

$= 5.385\ KNm$

So, all the forces produce a moment of 5.385 KNm clockwise. Thus, we get an idea that the resultant R should lie such that it produces clockwise moment of 5.385KNm about point A. For this to happen, R should lie to the left of A.

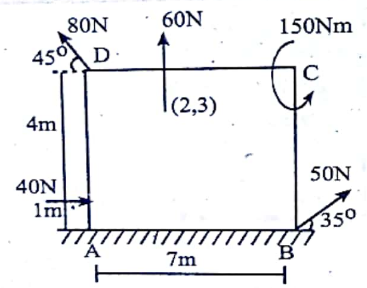


Moment of R about A = R sin(48.16°) × x

or, 5.385 = 234.90 sin(48.16°) × x

or, x = 0.030m = 3 cm (left of A)

14. Find the magnitude, direction of resultant force and locate two points on the edge of plate where the resultant meets.



[2073 Bhadra]

Solution:

Let's collect all forces at A.

$\therefore R_x = -80\cos45° + 50\cos35° + 40 = 24.4$ N

$R_y = 80\sin45° + 60 + 50\sin35° = 145.25$ N

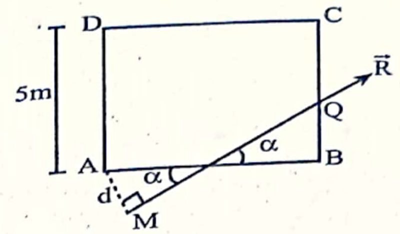
where R_x = resultant of all forces in x–direction and R_y = resultant of all forces in y–direction.

$\therefore \vec{R} = R_x\hat{i} + R_y\hat{j} = 24.4\hat{i} + 145.25\hat{j}$; R = 147.3 N

Since R_x and R_y both are positive, resultant lies in first quadrant.

Total moment at A,

$$(+)\Sigma M_A = -60\times2 - 150 - 50\sin35°\times7 + 40\times1 - 80\cos45°\times5$$
$$= -713.6 = 713.6 \text{ KNm (anticlockwise)}$$



$$\alpha = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{145.24}{24.4} = 80.5°$$

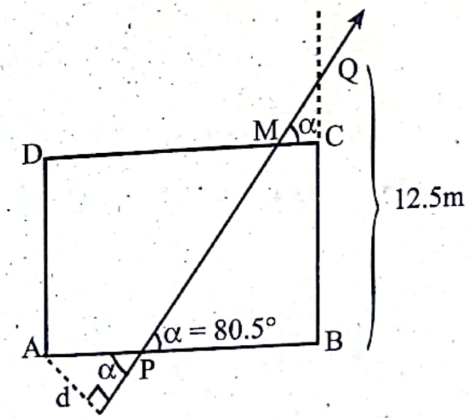
The resultant R should lie such that it produces an anticlockwise moment of 713.6 KNm about A. For this, it should lie to the right of A.

$R \times d = 713.6$

or, $d = \dfrac{713.6}{147.3} = 4.84$ m

$AP = \dfrac{d}{\sin\alpha} = \dfrac{4.84}{\sin80.5°} = 4.91$ m and

$BQ = PB\tan\alpha = (7 - 4.91)\tan80.5° = 12.5$ m

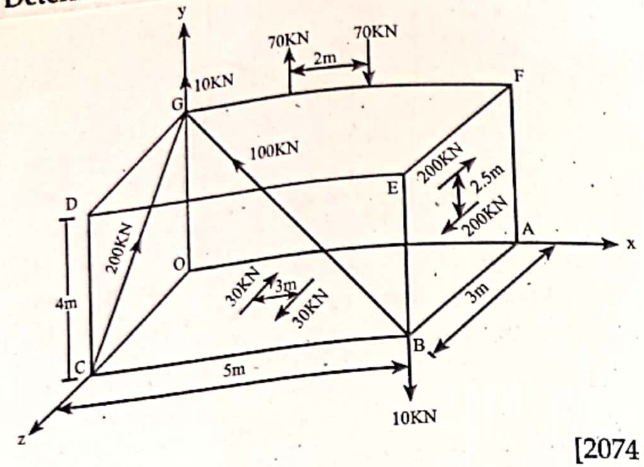


From ΔQMC and ΔQPB (using similarity property),

$$MC = \frac{QC}{QB}\times PB = \frac{(12.5-5)}{12.5}\times(7-4.91) = 1.254 \text{ m}$$

$\therefore$ Co-ordinate of P is (4.91, 0) and M is (7–1.254, 5) i.e., (5.746, 5).

15. **Determine the resultant force and moment about point O.**



[2074 Bhadra]

Solution:

Co-ordinates are 0 (0,0,0), A (5,0,0), B (5,0,3), C(0,0,3), D(0,4,3), E(5,4,3), F(5,4,0), G(0,4,0).

Let's collect all the forces and the couples at O.

$\vec{F_1} = 100\,\hat{f}_{BG}$ ($\hat{f}_{BG}$ is unit vector along BG)

$$= 100 \times \left\{ \frac{(0,4,0) - (5,0,3)}{\sqrt{(0\text{-}5)^2 + (4\text{-}0)^2 + (0\text{-}3)^2}} \right\}$$

$= (-70.71, 56.57, -42.42) = -70.71\hat{i} + 56.57\hat{j} - 42.42\hat{k}$ KN

$\vec{F_2} = 200\ \hat{f}_{CG}$

$$= 200 \left\{ \frac{(0,0,3) - (0,4,0)}{\sqrt{(0,0)^2 + (0\text{-}4)^2 + (3,0)^2}} \right\}$$

$= (0, -160, 120) = -160\hat{j} + 120\hat{k}$ KN

Resultant force, $\vec{F_R} = \vec{F_1} + \vec{F_2}$

$= -70.71\hat{i} - 103.43\hat{j} + 77.58\hat{k}$ KN

On transferring $\vec{F_1}$ and $\vec{F_2}$ to O, couples are developed.

$\vec{C}_{\text{due to F1}} = \vec{r}_{OB} \times \vec{F_1}$

$= (5,0,3) \times (-70.71, 56.57, -42.42)$

$= (-169.71, -0.03, 282.85) \approx -169.71\hat{i} + 282.85\hat{k}$ KNm

$\vec{C}_{\text{due to F2}} = \vec{r}_{OC} \times \vec{F_2}$

$= (0,0,3) \times (0,-160,120)$

$= (480,0,0)$

$= 480\hat{i}$ KNm

Couple due to 70KN, $\vec{C_1} = 2\hat{i} \times (-70\hat{j}) = -140\hat{k}$ KNm

Couple due to 10KN, $\vec{C_2} = \vec{r}_{BG} \times 10\hat{j}$

$= \{(0,4,0) - (5,0,3)\} \times 10\hat{j}$

$= 30\hat{i} - 50\hat{k}$ KNm

Couple due to 200KN, $\vec{C_3} = 2.5\hat{j} \times (-200\hat{k})$

$= -500\hat{i}$ KNm

Couple due to 30 KN, $\vec{C_4} = 3\hat{i} \times 30\hat{k}$

$= -90\hat{j}$ KNm

Total moment about point O,

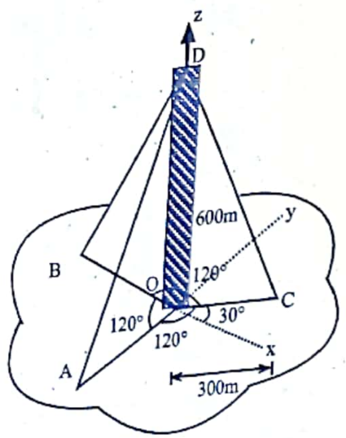
$= \vec{C}_{\text{due to F1}} + \vec{C}_{\text{due to F2}} + \vec{C_1} + \vec{C_2} + \vec{C_3} + \vec{C_4}$

$= (-169.71, 0, 282.85) + (480, 0, 0) + (0, 0, -140) + (30, 0, -50) + (-500, 0, 0) + (0, -90, 0)$

$= (-159.71, -90, 92.85)$

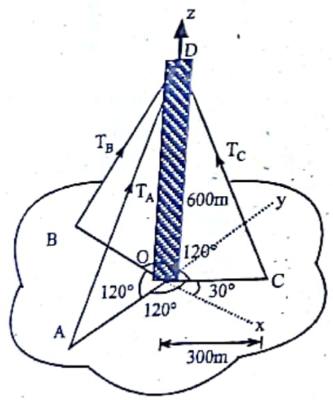
$= -159.71\hat{i} - 90\hat{j} + 92.85\hat{k}$ KNm

16. **Three guy wires are used in the support system for a television transmission tower that is 600 m tall. Wires A and B are tightened to a tension of 60 KN, whereas wire C has only 30 KN of tension. What is the moment of wire forces about the base of the tower? The y-axis is collinear with AO.**



[2074 Chaitra]

Solution:



Assume OA = OC = OB = 300m

Co-ordinates:

O (0, 0, 0)

A (0, –300, 0)

B (–300 sin30°, 300 cos30°, 0) = (–150, 259.81, 0)

C (300 cos30°, 300 sin30°, 0) = (259.81, 150, 0)

D (0, 0, 600)

Unit vectors:

$$\text{Along AD, } \hat{f}_{AD} = \frac{(0, 0, 600) - (0, -300, 0)}{\sqrt{(0-0)^2 + (0-(-300)^2 + (600-0)^2)}}$$

$$= \frac{300\hat{j} + 600\hat{k}}{\sqrt{300^2 + 600^2}} = 0.45\,\hat{j} + 0.89\hat{k}$$

$$\text{Along BD, } \hat{f}_{BD} = \frac{(0, 0, 600) - (-150, 259.81, 0)}{\sqrt{(0-(-150))^2 + (0-259.81)^2 + (600-0)^2)}}$$

$$= \frac{150\hat{i} - 259.81\hat{j} + 600\hat{k}}{670.82} = 0.22\hat{i} - 0.39\hat{j} + 0.89\hat{k}$$

$$\text{Along CD, } \hat{f}_{CD} = \frac{(0, 0, 600) - (259.81, 150, 0)}{\sqrt{(0-259.81)^2 + (0-150)^2 + (600-0)^2}}$$

$$= \frac{(-259.81, -150, 600)}{670.82} = -0.39\hat{i} - 0.22\hat{j} + 0.89\hat{k}$$

$$\text{Tension along AD, } \vec{T}_{AD} = |\vec{T}_{AD}|\,\hat{f}_{AD} = 60(0.45\hat{j} + 0.89\hat{k})$$

$$= 27\hat{j} + 53.4\hat{k}$$

$$\text{Tension along BD, } \vec{T}_{BD} = |\vec{T}_{BD}|\,\hat{f}_{BD} = 60\,(0.22\hat{i} - 0.39\hat{j} + 0.89\hat{k})$$

$$= 13.2\hat{i} - 23.4\hat{j} + 53.4\hat{k}$$

$$\text{Along CD, } \vec{T}_{CD} = |\vec{T}_{CD}|\,\hat{f}_{CD} = 30\,(-0.39\hat{i} - 0.22\hat{j} + 0.89\hat{k})$$

$$= -11.7\hat{i} - 6.6\hat{j} + 26.7\hat{k}$$

Position vectors of A, B and C w.r.t. O,

$$\vec{r}_{OA} = -300\hat{j},\ \vec{r}_{OB} = -150\hat{i} + 259.81\hat{j},\ \vec{r}_{OC} = 259.81\hat{i} + 150\hat{j}$$

$$\Sigma\vec{M}_O = (\vec{r}_{OA} \times \vec{T}_{AD}) + (\vec{r}_{OB} \times \vec{T}_{BD}) + (\vec{r}_{OC} \times \vec{T}_{CD})$$

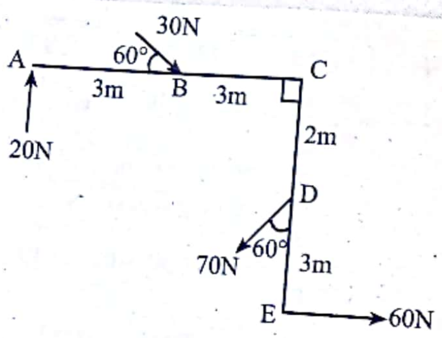
$$= (-300\hat{j}) \times (27\hat{j} + 53.4\hat{k}) + (-150\hat{i} + 259.81\hat{j}) \times (13.2\hat{i} - 23.4\hat{j} + 53.4\hat{k}) + (259.81\hat{i} + 150\hat{j}) \times (-11.7\hat{i} - 6.6\hat{j} + 26.7\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -300 & 0 \\ 0 & 27 & 53.4 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -150 & 259.81 & 0 \\ 13.2 & -23.4 & 53.4 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 259.81 & 150 & 0 \\ -11.7 & -66 & 26.7 \end{vmatrix}$$

$$= (-16020\hat{i}) + (13873.85\hat{i} + 8010\hat{j} + 80.51\hat{k}) + (4005\hat{i} - 6936.93\hat{j} + 40.254\hat{k})$$

$$= (1858.85\hat{i} + 1073.07\hat{j} + 120.7645\hat{k})$$

17. **Determine magnitude, directions and line of action of the resultant of force acting in the system as shown in figure below.**



[2075 Baishakh]

Solution:

Collecting all forces along x- and y-direction at A,

$\vec{R_x} = (30\cos60° - 70\sin60° + 60)\hat{i}$

$= 14.38\hat{i}$

$\vec{R_y} = (20 - 30\sin60° - 70\cos60°)\hat{j}$

$= -40.98\hat{j}$

Resultant $(\vec{R}) = \vec{R_x} + \vec{R_y} = 14.38\hat{i} - 40.98\hat{j}$

$|\vec{R}| = \sqrt{(R_x)^2 + (R_y)^2} = \sqrt{(14.38)^2 + (-40.98)^2} = 43.43N$

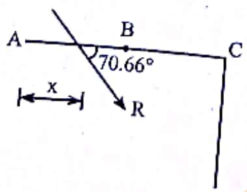
$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = 70.66°$ (4^{th} quadrant)

Calculating moment of all forces about A,

$\Sigma M_A = 30\sin60° \times 3 + 70\sin60° \times 2 + 70\cos60° \times 6 - 60 \times 5$

$= 109.19$ Nm(↻)

Thus, resultant R should lie such that it produces clockwise moment of 109.19 Nm about A. For this to happen, it should lie to the right of A.



Moment of R about A

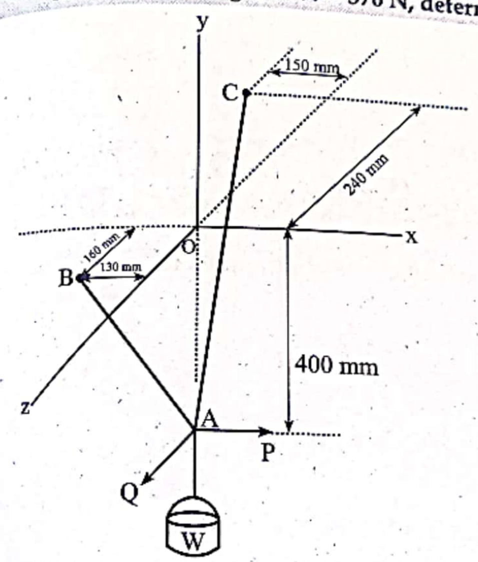
$= 43.43 \sin(70.66°) \times x$

or, $109.19 = 43.43 \sin(70.66°) \times x$

$\therefore x = 2.66$ m

Thus resultant R cuts AC at a distance of 2.66 m right of A along AC.

18. **A container of weight W is suspended from ring A. Cable BAC passes through the ring and is attached to fixed supports at B and C. Two forces $\vec{P} = P\hat{i}$ and $\vec{Q} = Q\hat{k}$ are applied to the ring to maintain the container in position shown. Knowing that W = 376 N, determine P and Q.**



[2075 Chaitra]

Solution:

Coordinates are: A(0, –400, 0), B(–130, 0, 160), C(–150, 0, –240).

$T_{AB} = T_{AC} = T$ [Same cable BAC]

$$\vec{T}_{AB} = T\frac{\vec{AB}}{|\vec{AB}|}$$

$$= T\frac{(-130\hat{i} + 400\hat{j} + 160\hat{k})}{450}$$

$$= T\left(\frac{-13}{45}\hat{i} + \frac{40}{45}\hat{j} + \frac{16}{45}\hat{k}\right)$$

$$\vec{T}_{AC} = T\frac{\vec{AC}}{|\vec{AC}|}$$

$$= T\frac{(-150\hat{i} + 400\hat{j} - 240\hat{k})}{490}$$

$$= T\left(\frac{-15}{49}\hat{i} + \frac{40}{49}\hat{j} - \frac{24}{49}\hat{k}\right)$$

We have,

$$\Sigma\vec{F} = 0$$

or, $\vec{T}_{AB} + \vec{T}_{AC} + P\hat{i} + Q\hat{k} - W\hat{j} = 0$

or, $\left(\frac{-13}{45}T - \frac{15}{49}T + P\right)\hat{i} + \left(\frac{40}{45}T + \frac{40}{49}T - W\right)\hat{j} + \left(\frac{16}{45}T - \frac{24}{49}T + Q\right)\hat{k} = 0$

On equating the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$ to zero, we get

$0.595\,T = P$

$1.705\,T = W$

$0.134\,T = Q$

Substituting W = 376N, we get

$T = 220.5$ N

$\therefore \quad P = 0.595 \times 220.5 = 131.2$N

$\quad Q = 0.134 \times 220.5 = 29.6$ N

19. The direction cosines of the line of action of a force with magnitude 200 N passing through point A(2, –2, 2) is (0.5, 0.707, 0.5). Find moment of force about point P(–2, 2, –2).

[2076 Baishakh]

Solution: Given F = 200 N

Unit vector in the direction of $\vec{F}$ is

$\hat{f} = 0.5\hat{i} + 0.707\hat{j} + 0.5\hat{k}$

Force $(\vec{F}) = F\hat{f} = 200\,(0.5\hat{i} + 0.707\hat{j} + 0.5\hat{k})$

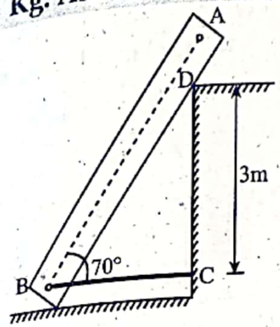
$= 100\hat{i} + 141.4\hat{j} + 100\hat{k}$

$\vec{r}_{PA} = (2 + 2)\,\hat{i} + (-2 - 2)\hat{j} + (2 + 2)\,\hat{k} = 4\hat{i} - 4\hat{j} + 4\hat{k}$

$\vec{M} = \vec{r}_{PA} \times \vec{F} = (4\hat{i} - 4\hat{j} + 4\hat{k}) \times (100\hat{i} + 141.4\hat{j} + 100\hat{k})$

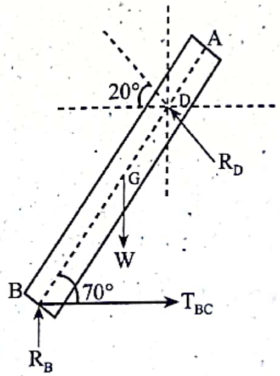
$= -965.6\hat{i} + 965.6\hat{k}$

20. Determine the tension in the cable BC which holds a part AB of length 4 m from sliding. The part has a mass of 10 Kg. Assume all the contact surfaces are smooth.



[2074 Ashwin]

Solution:



Free body diagram

Length BD = $\frac{3}{\sin 70°} = 3.1925$

BG = 2m as G is the point C.G.

$(+\circlearrowright)\ \Sigma M_B = 0$

or, $W \times 2\cos 70° - R_D \times 3.1925 = 0$

or, $mg \times 2\cos 70° - R_D \times 3.1925 = 0$

or, $R_D = 21.02$ N

$$\vec{T}_{AC} = T\frac{\overrightarrow{AC}}{|\overrightarrow{AC}|}$$

$$= T\frac{(-150\hat{i} + 400\hat{j} - 240\hat{k})}{490}$$

$$= T\left(\frac{-15}{49}\hat{i} + \frac{40}{49}\hat{j} - \frac{24}{49}\hat{k}\right)$$

We have,

$\Sigma\vec{F} = 0$

or, $\vec{T}_{AB} + \vec{T}_{AC} + P\hat{i} + Q\hat{k} - W\hat{j} = 0$

or, $\left(\frac{-13}{45}T - \frac{15}{49}T + P\right)\hat{i} + \left(\frac{40}{45}T + \frac{40}{49}T - W\right)\hat{j} + \left(\frac{16}{45}T - \frac{24}{49}T + Q\right)\hat{k} = 0$

On equating the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$ to zero, we get

$0.595\ T = P$

$1.705\ T = W$

$0.134\ T = Q$

Substituting W = 376N, we get

T = 220.5 N

$\therefore$ P = 0.595×220.5 = 131.2N

Q = 0.134×220.5 = 29.6 N

19. **The direction cosines of the line of action of a force with magnitude 200 N passing through point A(2, −2, 2) is (0.5, 0.707, 0.5). Find moment of force about point P(−2, 2, −2).**

[2076 Baishakh]

Solution: Given F = 200 N

Unit vector in the direction of $\vec{F}$ is

$\hat{f} = 0.5\hat{i} + 0.707\hat{j} + 0.5\hat{k}$

Force $(\vec{F}) = F\hat{f} = 200\ (0.5\hat{i} + 0.707\hat{j} + 0.5\hat{k})$

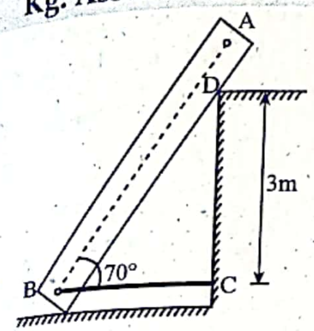
$= 100\hat{i} + 141.4\hat{j} + 100\hat{k}$

$\vec{r}_{PA} = (2+2)\ \hat{i} + (-2-2)\hat{j} + (2+2)\ \hat{k} = 4\hat{i} - 4\hat{j} + 4\hat{k}$

$\vec{M} = \vec{r}_{PA} \times \vec{F} = (4\hat{i} - 4\hat{j} + 4\hat{k}) \times (100\hat{i} + 141.4\hat{j} + 100\hat{k})$

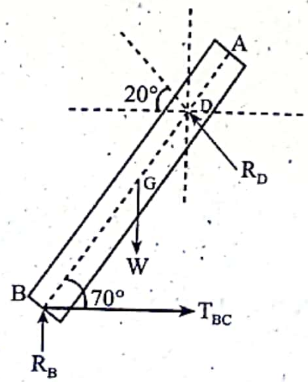
$= -965.6\hat{i} + 965.6\hat{k}$

20. **Determine the tension in the cable BC which holds a part AB of length 4 m from sliding. The part has a mass of 10 Kg. Assume all the contact surfaces are smooth.**



[2074 Ashwin]

Solution:



Free body diagram

Length BD = $\frac{3}{\sin 70°}$ = 3.1925

BG = 2m as G is the point C.G.

(+↺) $\Sigma M_B = 0$

or, W × 2cos70° − R_D × 3.1925 = 0

or, mg × 2cos70° − R_D × 3.1925 = 0

or, R_D = 21.02 N

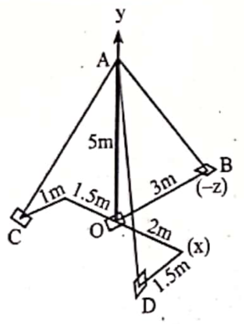
$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $T_{BC} - R_D \cos 20° = 0$

or, $T_{BC} = 21.02 \times \cos 20° = 19.752$ N

$\therefore$ The tension in BC is 19.752 N

21. In the system shown in figure, a 5 m long pole is held in vertical position by three gay wires AB, AC and AD. If the tension of 600 N is induced in AD and the resultant forces at A is to be vertical, determine the tension in cables AB and AC.



[2076 Bhadra]

Solution:

The forces applied at A are $\vec{T}_{AC}$, $\vec{T}_{AD}$, $\vec{T}_{AB}$, and $\vec{P}$ where $\vec{P} = P\hat{j}$ as it is vertical.

The coordinate of different points are A(0, 5, 0), B(0, 0, –3), C(–1.5, 0, 1), D(2, 0, 1.5).

$$\overrightarrow{AB} = 0\hat{i} - 5\hat{j} - 3\hat{k},\ AB = 5.831$$

$$\overrightarrow{AC} = (-1.5\hat{i} - 5\hat{j} + 1\hat{k}),\ AC = 5.315$$

$$\overrightarrow{AD} = (2\hat{i} - 5\hat{j} + 1.5\hat{k}),\ AD = 5.590$$

$$\vec{T}_{AB} = T_{AB}\hat{f}_{AB} = T_{AB}\left(\frac{0\hat{i} - 5\hat{j} - 3\hat{k}}{5.831}\right)$$

$$\vec{T}_{AC} = T_{AC}\hat{f}_{AC} = T_{AC}\left(\frac{-1.5\hat{i} - 5\hat{j} + 1\hat{k}}{5.315}\right)$$

$$\vec{T}_{AD} = T_{AD}\hat{f}_{AD} = T_{AD}\left(\frac{2\hat{i} - 5\hat{j} + 1.5\hat{k}}{5.590}\right)$$

Applying equilibrium condition,

$\Sigma\vec{F} = 0$

or, $\vec{T}_{AB} + \vec{T}_{AC} + \vec{T}_{AD} + \vec{P} = 0$

or, $T_{AB}\left(\frac{0\hat{i} - 5\hat{j} - 3\hat{k}}{5.831}\right) + T_{AC}\left(\frac{-1.5\hat{i} - 5\hat{j} + 1\hat{k}}{5.315}\right) + T_{AD}\left(\frac{2\hat{i} - 5\hat{j} + 1.5\hat{k}}{5.590}\right) + P\hat{j} = 0$

or, $(-0.2822T_{AC} + 0.3578T_{AD})\hat{i} + (-0.8575T_{AB} - 0.941T_{AC} - 0.8944T_{AD} + P)\hat{j} + (-0.5145T_{AB} + 0.1881T_{AC} + 0.2683T_{AD})\hat{k} = 0\hat{i} + 0\hat{j} + 0\hat{k}$

Equating like coefficients, we get

$-0.2822T_{AC} + 0.3578T_{AD} = 0$ .... (i)

$-0.8575T_{AB} - 0.941T_{AC} - 0.8944T_{AD} + P = 0$ .... (ii)

$-0.5145T_{AB} + 0.1881T_{AC} + 0.2683T_{AD} = 0$ .... (iii)

Given $T_{AD} = 600$ N.

From equation (i), $T_{AC} = 760.74$ N

From equation (iii), $T_{AB} = 591.01$ N

From equation (ii), $P = 1759.29$ N

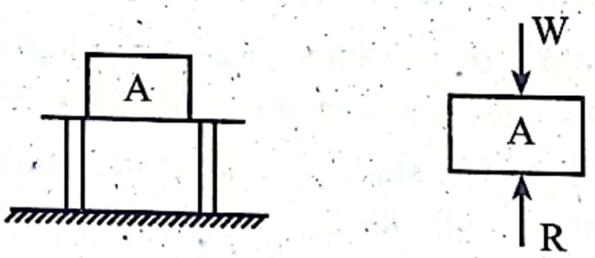
CHAPTER - 2

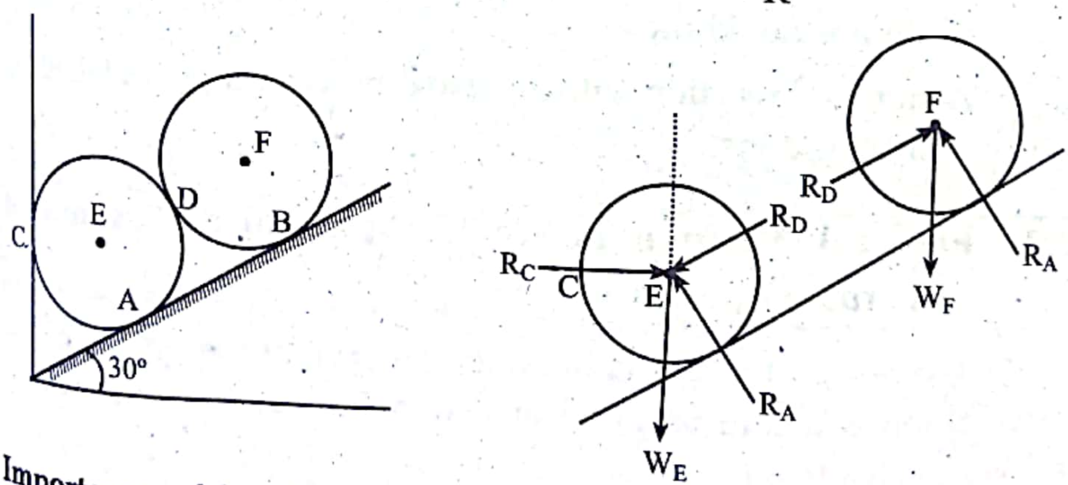
BASIC CONCEPT IN STATICS AND STATIC EQUILIBRIUM

2.1 Concept of Particle and Free Body Diagram

Particle means a very small amount of matter which may be assumed to occupy a single point in space. A *rigid body* is a combination of a large number of particles occupying fixed positions with respect to each other.

Free body diagram is a sketch of the body (space diagram) drawn in such a way that it shows all the reaction forces, applied forces, and moment on the body.





Importance of free body diagram:

i. We can analyze the complex body by isolating it from the system of bodies.

ii. We can easily apply equilibrium equation on free body diagram.

iii. Free body diagram is sketch of the isolated body showing all the forces acting on it by vector.

iv. We can adopt any co-ordinate system whose axes are not only in horizontal and vertical direction.

v. Free body diagram indicates each applied load including the weight of isolated body.

vi. Free body diagram indicates all the dimensions including slope.

Guidelines for drawing good free body diagram:

i. The body to be freed for consideration may be the entire system or any portion of the system. So, it is important to make a clear decision as to which portion of the system is to be freed.

ii. The free body diagram should have no supports or connections.

iii. Any adopted coordinate system whose axes are not in the horizontal or vertical directions should be shown.

iv. Appropriate dimensions are needed for defining appropriate configuration of force system.

v. Each applied load should be indicated with an arrow and labeled either with its known magnitude or with a letter when it is not known.

vi. Action and reaction with an arrow head along with labelling should be done.

2.2 Physical Meaning of Equilibrium and its Essence in Structural Application

The physical meaning of *equilibrium* is that the system of the external forces will impart no translational or rotational motion to the body considered.

Principle of Equilibrium

Principle of equilibrium states that a stationary body which is subjected to coplanar forces (concurrent or parallel) will be in equilibrium if the algebraic sum of all the external forces is zero and also algebraic sum of moments of all the external forces about any point in their plane is zero.

Mathematically,

$\Sigma F = 0, \Sigma M = 0$

Importance of equilibrium in structure analysis:

Condition of equilibrium is the central theme for structural analysis. Civil engineers analyze and design the structures (bridges, buildings, transmission tower) standing on the concept of equilibrium.

2.3 Equation of Equilibrium in Two and Three Dimension

Two-dimensional (2-D) analysis for equilibrium condition:

i. **For a particle**

A particle will be in equilibrium condition if algebraic sum of all the coplanar forces acting on it is zero.

$\Sigma \vec{F}_p = 0$; where p denotes particle

Generally, the forces are resolved into horizontal and vertical components.

$\therefore \Sigma(F_x)_p = 0$ and $\Sigma(F_y)_p = 0$

ii. **For a rigid body**

A rigid body will be in equilibrium condition if the algebraic sum of all coplanar external forces is zero and the algebraic sum of moments of all the forces about any point in their plane is zero.

Mathematically,

$\Sigma \vec{F} = 0, \Sigma \vec{M} = 0$

which can be resolved as:

$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma M = 0$

Three-dimensional (3-D) analysis for equilibrium condition:

i. **For a particle**

A particle will be in equilibrium condition if algebraic sum of all external forces acting on a point is zero. The external forces considered now may not be necessarily coplanar as 3-D analysis is being performed.

Thus, we can write

$\Sigma\vec{F}_p = 0$

On resolving into three reference directions, we have

$\Sigma(F_x)_p = 0, \Sigma(F_y)_p = 0, \Sigma(F_z)_p = 0$

ii. **For a rigid body**

For a rigid body to be in equilibrium condition under the action of external space forces, the algebraic sum of all these forces should be equal to zero.

Thus, the equilibrium equations for 3-D are

$\Sigma\vec{F} = 0, \Sigma\vec{M} = 0$

which can be further resolved as:

$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0$

$\Sigma M_x = 0, \Sigma M_y = 0, \Sigma M_z = 0$

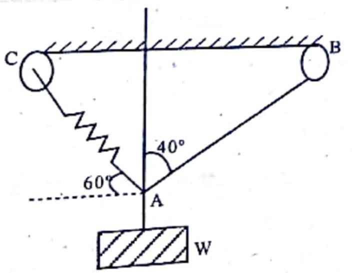
where ΣM_x = algebraic sum of moment about yz plane.

ΣM_y = algebraic sum of moment about xz plane.

ΣM_z = algebraic sum of moment about xy plane.

SOLVED NUMERICALS

1. **A container of weight W is subjected from ring A to which cable AB and spring AC are attached. The constant of spring is 100 N/m and its unstretched length is 3 m. Determine the tension in the cable, when (a) W = 120 N (b) W = 160 N.**

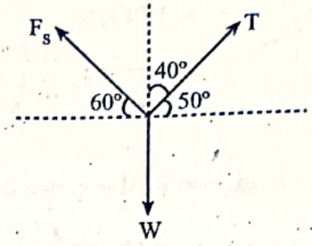


[2069 Bhadra]

Solution:

Force exerted by spring $(F_s) = Kx$

where K = spring constant = 100N/m and x = elongation of spring



FBD of the system

Let T be the tension in the cable AB.

$(\overset{+}{\rightarrow})\ \sum F_x = 0$

or, $T\cos 50° - F_s \cos 60° = 0$

or, $F_s = \dfrac{T\cos 50°}{\cos 60°}$

$(\uparrow +)\sum F_y = 0$

or, $T\sin 50° + F_s \sin 60° - W = 0$

or, $T\sin 50° + \dfrac{T\cos 50°}{\cos 60°}\sin 60° = W$

or, $T = \dfrac{W}{\sin 50° + \dfrac{\cos 50°}{\cos 60°}\sin 60°} = \dfrac{W}{1.88}$

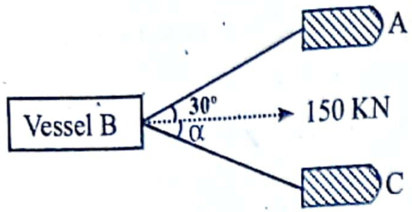
a) When W = 120 N,

$T = \dfrac{120}{1.88} = 63.829\ N$

b) When W = 160 N,

$T = \dfrac{160}{1.88} = 85.106\ N$

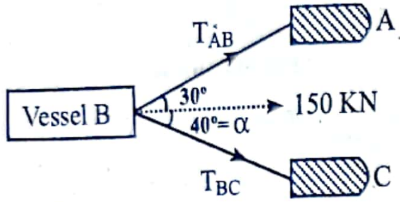
2. **A commercial vessel is being pulled into harbour for unloading by two tugboats as shown in figure. Knowing the vessel requires 150 KN along its axis to move it steadily, compute the tensions in rope AB and BC when α = 40°.**



[2070 Ashadh]

Solution:

Let T_{AB} and T_{BC} be tension in the ropes as shown in figure.



$(\overset{+}{\rightarrow})\sum F_x = 0$

or, $T_{AB}\cos 30° + T_{BC}\cos 40° = 150$ ....(i)

$(+\uparrow)\sum F_y = 0$

or, $T_{AB}\sin 30° = T_{BC}\sin 40°$

or, $T_{AB} = 2T_{BC}\sin 40°$ ....(ii)

From (i) & (ii),

$2T_{BC}\cos 30° + T_{BC}\cos 40° = 150°$

Solving, we get

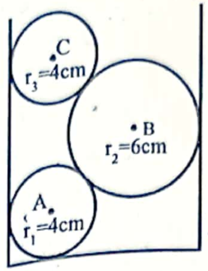
$T_{BC} = 79.81$ KN

Substituting this value of T_{BC} in (ii), we get

$T_{AB} = 2\times 79.81\sin 40° = 102.61$ KN

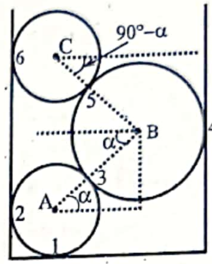
Hence, the tensions in the rope AB & BC are $T_{AB} = 102.61$ KN and $T_{BC} = 79.81$ KN respectively.

3. **Determine the reactions at the contact points, if three cylinders are piled in a rectangular ditch as shown in figure. Given that the weight of the cylinders are: W_A= 2KN, W_B = 5 KN, W_C = 3 KN.**



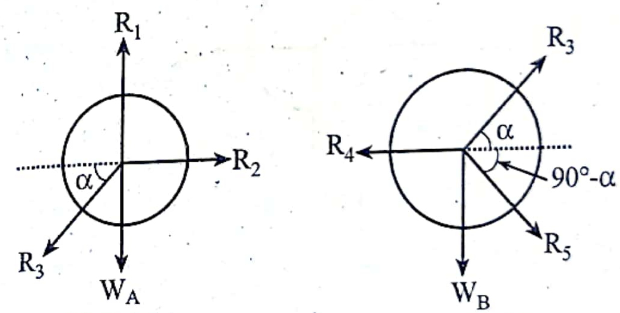
[2070 Chaitra]

Solution:



From the figure above, $\alpha = \cos^{-1}\left(\frac{18-4-6}{4+6}\right) = 36.87°$

The FBD of individual spheres are shown below.



FBD of A **FBD of B**

From the condition of equilibrium for sphere A, we get

$(\overset{+}{\rightarrow})\sum F_x = 0$

or, $R_2 - R_3\cos\alpha = 0$ .....(i)

$(\downarrow +)\sum F_y = 0$

$W_A - R_1 + R_3\sin\alpha = 0$ ....(ii)

Using equilibrium condition for sphere B, we get

$(\overset{+}{\rightarrow})\sum F_x = 0$

or, $R_3\cos\alpha + R_5 \cos(90^\circ - \alpha) - R_4 = 0$ ......(iii)

$(\downarrow +)\sum F_y = 0$

or, $W_B + R_5 \sin(90^\circ - \alpha) - R_3\sin\alpha = 0$ ....(iv)

Using equilibrium condition for sphere C, we get

$(\overset{+}{\rightarrow})\sum F_x = 0$

or, $R_6 - R_5 \cos(90^\circ - \alpha) = 0$ ......(v)

$(\downarrow +)\sum F_y = 0$

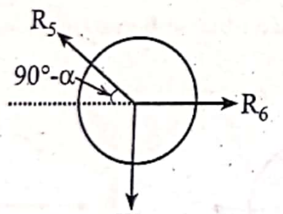
or, $W_C - R_5 \sin(90^\circ - \alpha) = 0$

or, $R_5 = \dfrac{W_C}{\sin(90^\circ - \alpha)} = \dfrac{3}{\sin(53.13^\circ)} = 3.75$ KN

Substituting this value of R_5 in equation (v) and (iv) respectively, we get

$R_6 - 3.75 \cos(53.13^\circ)$ KN $= 0$

or, $R_6 = 2.25$ KN



FBD of C

$R_3 = \dfrac{W_B + R_5 \sin(53.13^\circ)}{\sin(36.87^\circ)}$

$= \dfrac{5 + 3.75 \sin(53.13^\circ)}{\sin(36.87^\circ)} = 13.33$ KN

From equation (iii),

$R_4 = R_3 \cos\alpha + R_5 \cos(90^\circ - \alpha)$

$= 13.33 \cos(36.87^\circ) + 3.75 \cos(53.13^\circ) = 12.91$ KN

From equation (ii),

$R_1 = W_A + R_3 \sin\alpha$

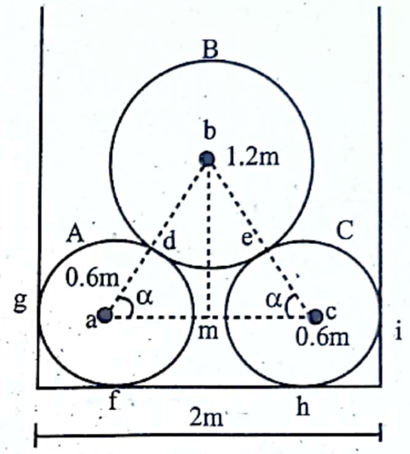
$= 2 + 13.33 \sin(36.87^\circ) = 10$ KN

From equation (i),

$R_2 = R_3 \cos\alpha$

$= 13.33 \cos(36.87^\circ) = 10.66$ KN

4. **The cylinders A and C weigh 1000 N each and the weight of the cylinder is 2000 N. Determine the forces exerted at the contact points.**



(The given values 1.2m and 0.6m are diameter)

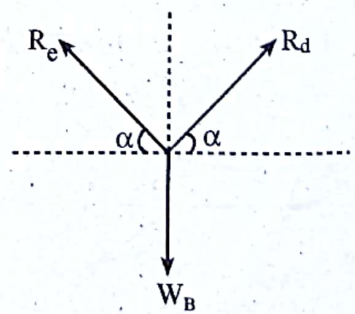
[2071 Magh]

Solution:

Cylinder B has only two unknown (reactions at d and reactions at e)

$ac = 2 - 0.3 - 0.3 = 1.4$ m

$am = \dfrac{1.4}{2} = 0.7$m



FBD of cylinder B

Since ab = bc,

$\angle bac = \angle bca = \alpha$.

$\cos\alpha = \frac{am}{ab} = \frac{0.7}{0.9} \Rightarrow \alpha = 38.94^\circ$

For cylinder B,

$(\overset{+}{\rightarrow})\Sigma F_x = 0$

or, $R_d \cos\alpha - R_e \cos\alpha = 0$

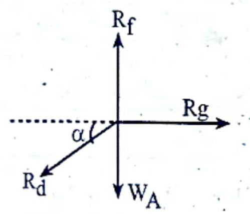
or, $R_d = R_e$

$(+\uparrow)\ \Sigma F_y = 0$

or, $R_d \sin\alpha + R_e \sin\alpha = W_B$

or, $R_d = \frac{2000}{2\sin 38.94^\circ} = 1591.07$ N

$\therefore R_e = R_d = 1591.07$ N



FBD of cylinder A

For cylinder A,

$(\overset{+}{\rightarrow})\Sigma F_x = 0$

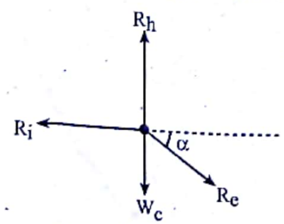
or, $-R_d \cos\alpha + R_g = 0$

or, $R_g = R_d \cos\alpha = 1597.07 \cos 38.94^\circ = 1237.54$ N

$(+\uparrow)\ \Sigma F_y = 0$

or, $R_f - R_d \sin\alpha - W_A = 0$

or, $R_F = 1237.54 \sin 38.94^\circ + 1000 = 1777.80$ N



FBD of cylinder C

For cylinder C,

$(\overset{+}{\rightarrow})\Sigma F_x = 0$

or, $(R_e) \cos\alpha - R_i = 0$

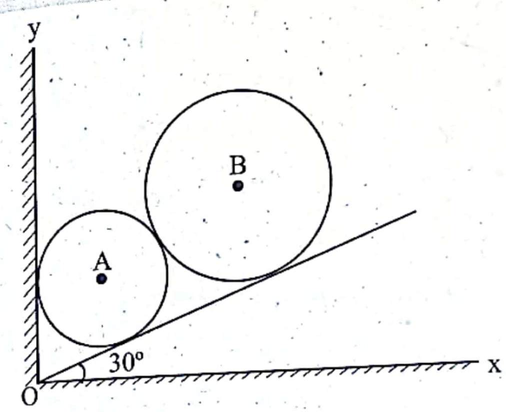
or, $R_i = (R_e) \cos\alpha = 1591.07 \cos 38.99^\circ = 1237.54$ N

$(+\uparrow)\ \Sigma F_y = 0$

or, $R_h - W_c - R_e \sin\alpha = 0$

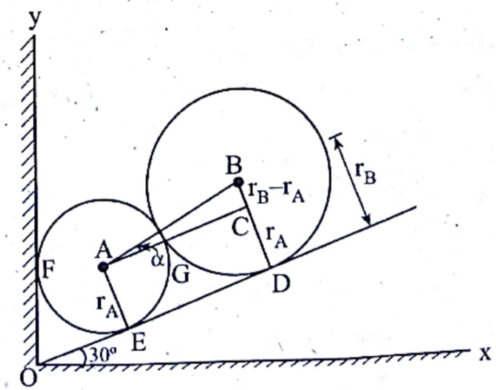
or, $R_h = 1000 + 1237.54 \sin 38.94^\circ = 1777.30$ N

5. **Determine the reactions at all contact points. Assume all contact surfaces are smooth. Take weight of sphere A = 200 KN, weight of sphere B = 400 KN, radius of sphere A = 120 mm, radius of sphere B = 250 mm.**



[2072 Ashwin]

Solution:



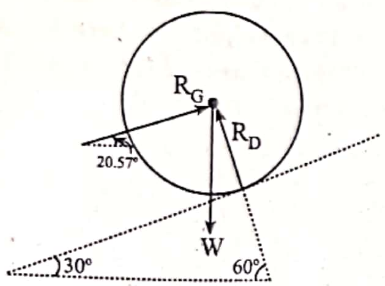
The contact points are named D, E, F and G. We notice AC is parallel to DE.

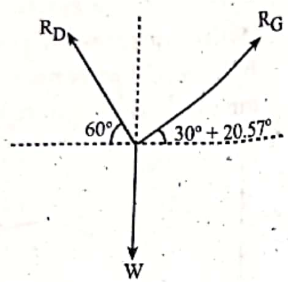
From ΔACB,

$$\sin\alpha = \frac{BC}{AB} = \frac{r_B - r_A}{r_B + r_A}$$

$$\text{or, } \alpha = \sin^{-1}\left(\frac{250 - 120}{250 + 120}\right) = 20.57^\circ$$

Let's consider sphere B first as there are only 2 unknown reactions.





FBD of sphere B

$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $-R_D \cos 60^\circ + R_G \cos 50.57^\circ = 0$

or, $\frac{-R_D}{2} = -R_G \times 0.6351$

or, $R_D = 1.2702\, R_G$ ....(a)

$(+\uparrow)\ \Sigma F_y = 0$

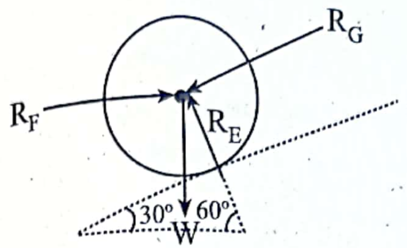
or, $R_D \sin 60^\circ + R_G \sin 50.57^\circ - 400 = 0$

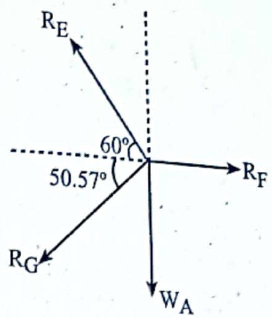
or, $1.2702 R_G \sin 60^\circ + R_G \sin 50.57^\circ = 400$ [using equation (a)]

$\therefore R_G = 213.6$ KN

Putting the value of R_G in (a), we get

$R_D = 271.35$ KN





FBD of sphere A

$(+\uparrow)\ \Sigma F_y = 0$

or, $-W_A - R_G \sin 50.57^\circ + R_E \sin 60^\circ = 0$

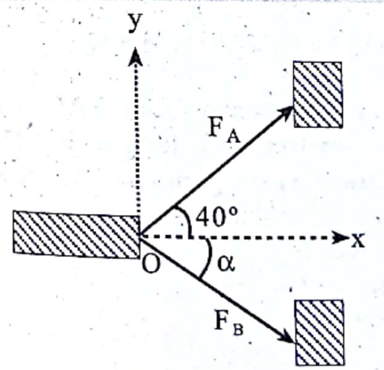
or, $R_E = \frac{W_A + R_G \sin 50.57^\circ}{\sin 60^\circ} = 421.45$ KN $(\uparrow)$

$(\xrightarrow{+})\ \Sigma F_x = 0$

or, $R_F - R_E \cos 60^\circ - R_G \sin 50.57^\circ = 0$

or, $R_F = 421.45 \cos 60^\circ + 213.6 \sin 50.57^\circ = 357.71$ KN $(\rightarrow)$

6. **A vehicle needs to be moved forward by two pullers A and B. Puller 'A' is at 40° to the axis of movement. Compute the value of angle 'α' for which puller 'B' has to exert minimum force. Also compute the respective values of pull to be exerted.**



[2072 Magh]

Solution:

Let F_A be the applied force along OA and F_B be along OB. We have to move the vehicle in the forward direction. For moving vehicle,

$(\overset{+}{\rightarrow}) \ \Sigma F_x = 50KN$

or, $F_A \cos40° + F_B \cos\alpha = 50$ KN

or, $0.76\cos40° + F_B\cos\alpha = 0$ ....(i)

In vertical direction, the vehicle can't move. So,

$(+\uparrow) \ \Sigma F_y = 0$

or, $F_A \sin40° - F_B \sin\alpha = 0$

or, $F_A = 1.56 \ F_B \sin\alpha$ ....(ii)

Using the value of F_A in (i),

$1.192 \ F_B \sin\alpha + F_B \cos\alpha = 50$

or, $F_B = \dfrac{50}{1.192\sin\alpha + \cos\alpha}$ ......(iii)

For force F_B to be minimum, $\dfrac{dF_B}{d\alpha} = 0$

or, $\dfrac{d}{d\alpha}\left(\dfrac{50}{1.192 \sin\alpha + \cos\alpha}\right) = 0$

Solving, we get

$1.192 \cos\alpha = \sin\alpha$

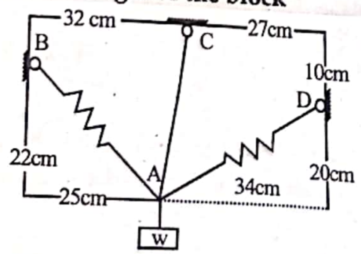
or, $\tan\alpha = 1.192$

$\therefore \alpha = 50°$

From eq. (iii), $F_B = \dfrac{50}{1.192\sin50°+\cos50°} = 32.14$ KN

From eq. (ii), $F_A = 1.56 \times 32.14 \sin50° = 38.31$ KN

7. **A block of weight W is suspended by a cord AC and two springs of which the unstretched length of 25 cm. Knowing that the constant of spring are AB = 10N/cm and AD = 3.5 N/cm, determine:**
   a. Tension in the cord AC
   b. Weight of the block



[2074 Bhadra]

Solution:

Unstretched length of spring AB and AD = 25 cm

Stretched length of spring AB = $\sqrt{25^2 + 22^2} = 33.30$ cm

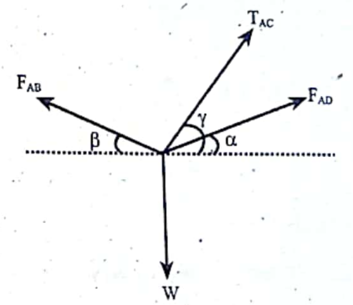
Total extension of spring AB (x) = 33.30 – 25 = 8.30 cm

$F_{AB} = K_{AB} \times x = 10 \times 8.30 = 83$ N

Stretched length of spring AD = $\sqrt{34^2 + 20^2} = 39.45$ cm

Total extension of spring AD (x) = 39.45 – 25 = 14.45 cm

$F_{AD} = K_{AD} \times x = 3.5 \times 14.45 = 50.58$ N



FBD of A

From the fig. (see question),

$\alpha = \tan^{-1}\left(\dfrac{20}{34}\right) = 30.47°$, $\beta = \tan^{-1}\left(\dfrac{22}{25}\right) = 41.35°$,

$\gamma = \tan^{-1}\left(\dfrac{30}{34-27}\right) = 76.87°$

$(\overset{+}{\rightarrow}) \ \Sigma F_x = 0$

$F_{AD} \cos\alpha - F_{AB} \cos\beta + T_{AC} \cdot \cos\gamma = 0$

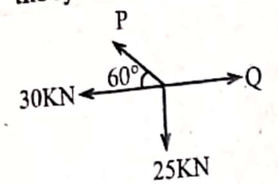
or, $T_{AC} = \dfrac{F_{AD} \cos\alpha - F_{AB} \cos\beta}{\cos\gamma} = 82.38$ N

$(+\uparrow) \ \Sigma F_y = 0$

or, $T_{AC} \sin\gamma + F_{AB} \sin\beta + F_{AB} \sin\alpha - W = 0$

$\therefore W = 160.71N$

8. Determine the values of the unknown forces P and Q for the system of forces to be in equilibrium.



[2074 Chaitra]

Solution:

$(+\uparrow)\ \Sigma F_y = 0$

or, $P\sin 60° - 25 = 0$

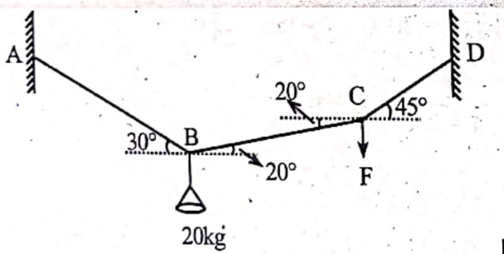
or, $P = \dfrac{25}{\sin 60°} = 28.87$ KN $(\uparrow)$

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $Q - 30 - P\cos 60° = 0$

or, $Q = 30 + 28.87\cos 60° = 44.43$ KN $(\rightarrow)$

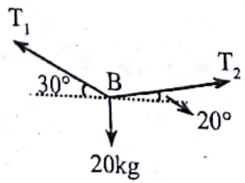
9. Determine the force in each cable and the force 'F' needed to hold the 20 kg lamp in the position shown in figure as below.



[2075 Baisakh]

Solution:

Considering FBD of lamp,



$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $-T_1\cos 30° + T_2\cos 20° = 0$

or, $T_1\cos 30° = T_2\cos 20°$

or, $T_1 = 1.085T_2$................ (i)

$(+\uparrow)\ \Sigma F_y = 0$

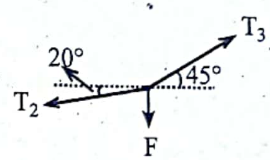
or, $T_1\sin 30° + T_2\sin 20° - 20 = 0$

or, $1.085\sin 30° T_2 + T_2\sin 20° = 20$

or, $T_2 = \dfrac{20}{1.085\sin 30° + \sin 20°} = 22.61$ N

$\therefore T_1 = 22.61 \times 1.085 = 24.53$ N

Considering FBD at point C,

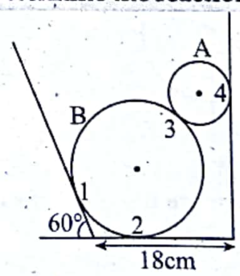


$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $-T_2\cos 20° + T_3\cos 45° = 0$

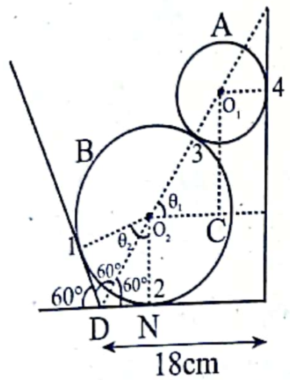
or, $T_3 = \dfrac{T_2\cos 20°}{\cos 45°} = \dfrac{22.61 \times \cos 20°}{\cos 45°} = 30.05$ N

10. Two cylinders A and B rest in a channel as shown in figure. The cylinder A has diameter of 10 cm and weighs 200 N whereas the cylinder B has diameter of 18 cm and weighs 500 N. Determine the reactions at all contact points.



[2076 Baishakh]

Solution:



In ΔO_2DN, $\angle DO_2N = 30°$

$$\tan 30° = \frac{DN}{NO_2}$$

or, $DN = \tan 30° \times 9 = 5.2$ cm

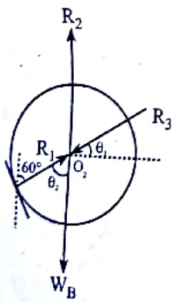
$O_2C = 18 - 5.2 - 5 = 7.8$ cm

In ΔO_2O_1C,

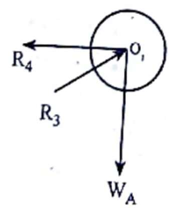
$$\cos\theta_1 = \frac{O_2C}{O_1O_2} = \frac{7.8}{14}$$

or, $\theta_1 = 56.14°$

$\therefore \theta_2 = 30° + 30° = 60°$



FBD of sphere B



FBD of sphere A

$\angle O_2O_1C = 90° - 56.14° = 33.86°$

For sphere A,

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $R_3 \sin 33.86° - R_4 = 0$.......... (i)

$(+\uparrow)\ \Sigma F_y = 0$

or, $R_3 \cos 33.86° - W_A = 0$

or, $R_3 = \dfrac{W_A}{\cos 33.86°} = \dfrac{200}{0.83} = 240.964$ N

From equation (i),

$R_4 = R_3 \sin 33.86° = 240.964 \times \sin 33.86° = 134.257$ N

For sphere B,

$(\overset{+}{\rightarrow})\ \Sigma F_x = 0$

or, $R_1 \sin 60° - R_3 \cos 56.14° = 0$

or, $R_1 = \dfrac{240.964 \times \cos 56.14°}{\sin 60°} = 155.0264$ N

$(+\uparrow)\ \Sigma F_y = 0$

or, $R_2 - W_B - R_3 \sin 56.14° + R_1 \cos 60° = 0$

or, $R_2 = 500 + 240.964 \sin 56.14° - 155.0264 \cos 60° = 623.028$N

Therefore, $R_1 = 155.0264$ N, $R_2 = 623.0280$ N, $R_3 = 240.964$ N, $R_4 = 134.257$ N.