

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	ALL (Except B.Arch)	Pass Marks	32
Year / Part	I / I	Time	3 hrs.

**Subject:** - Engineering Mathematics I (SH401)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt All questions.
- ✓ All questions carry equal marks.
- ✓ Assume suitable data if necessary.

- State Leibnitz's theorem. If  $y = (\sin^{-1} x)^2$ , show that  

$$(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$$
- Verify Rolle's Theorem for  $f(x) = \log \frac{x^2 + ab}{(a+b)x}$ ;  $x \in [a, b]$ . How does Rolle's Theorem differ from Lagrange's mean value theorem.
- Evaluate  $\lim_{x \rightarrow 0^+} \left( \frac{\sin x}{x} \right)^{\frac{1}{x}}$
- Find the asymptotes to the curve  $y^3 + 2xy^2 + x^2y - y + 1 = 0$
- Find the radius of curvature at origin for the curve  $x^3 + y^3 = 3axy$ .
- Show that  $\int_0^{\pi} x \log(\sin x) dx = \frac{\pi^2}{2} \log \frac{1}{2}$
- Apply the rule of differentiation under integral sign to evaluate  $\int_0^{\infty} \frac{e^{-ax} \sin x}{x} dx$  and hence deduce that  $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$
- Define Beta function. Apply Beta and Gamma function to evaluate  $\int_0^{2a} x^5 \sqrt{2ax - x^2} dx$
- Find the volume generated by revolution of astroid  $x^{2/3} + y^{2/3} = a^{2/3}$  about x-axis.
- What does the equation  $3x^2 + 3y^2 + 2xy = 2$  becomes when the axes are turned through an angle of  $45^\circ$  to the original axes?
- Find center, length of axes, eccentricity and directrices of the conic  

$$3x^2 + 8xy - 3y^2 - 40x - 20y + 50 = 0$$

**OR**

Describe and sketch the conic  $r = \frac{12}{2 - 6\cos\theta}$

- Deduce standard equation of ellipse.
- Solve the differential equation:  $(1 + y^2) + (x - e^{\tan^{-1}y}) \frac{dy}{dx} = 0$
- Solve:  $xp^2 - 2yp + ax = 0$  where  $p = \frac{dy}{dx}$
- Solve:  $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{2x} \cdot \sin x$
- Resistance of 100 ohms, an inductance of 0.5 Henry are connected in series with battery