Subject: - Engineering mathematics I *(SH 401)*

✓ Candidates are required to give their answers in their own words as far as practicable.
✓ *Attempt **All** questions.*
✓ ***All** questions carry equal marks.*
✓ *Assume suitable data if necessary.*

1. If $y = a\cos(\log x) + b \sin(\log x)$ prove that:

   (i) $x^2 y_2 + xy_1 + y = 0$
   (ii) $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$

2. State and prove Lagrange's mean value theorem.

3. State L' Hospital's Rule and hence evaluate $\lim_{x \to 0}(\cot x)^{\sin 2x}$

4. Find the asymptote of $(x+y)^2(x+2y+2) = x+9y-2$

5. Find the radius of curvature of the curve $r = a(1 - \cos\theta)$.

   Or,

   Find the pedal equation of $y^2 = 4a(x+a)$

6. Evaluate $\int_0^{\pi/2} \dfrac{x \sin x \cos x}{\cos^4 x + \sin^4 x} dx$

7. Using the rule of differentiation under the integral sign, evaluate $\int_0^{\infty} \dfrac{\log(1 + a^2 x^2)}{1 + b^2 x^2} dx$

8. Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x\, dx$ and hence evaluate $\int_0^{\pi/2} \cos^{10} x\, dx$.

9. Obtain the area of a loop of the curve $y^2(a^2 + x^2) = x^2(a^2 - x^2)$

   Or,

   Find the volume of the solid formed by the revolution of the cycloid $x = a(\theta + \sin\theta)$

10. Solve the differential equation: $\dfrac{dy}{dx} = \dfrac{y}{x} + \tan\dfrac{y}{x}$

11. Find the general solution of $y = Px + x^4 p^2$

12. Solve $(D^2 - 2D + 5)y = e^{2x}\sin x$

13. Solve $x^2 \dfrac{d^2 y}{dx^2} - 2x\dfrac{dy}{dx} - 4y = x^4$

    Or,

    A radio active material has an initial mass 100mg. After two years, it is left to 75mg. Find the amount of the material at any time t.

14. What does the equation $3x^2 + 3y^2 + 2xy = 2$ become when the axes are turned through an angle 45° with the original axes.

15. Obtain the equation of hyperbola in standard form.

16. Find the center for the conic $3x^2 + 8xy - 3y^2 - 40x - 20y + 50 = 0$.

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