01 TRIBHUVAN UNIVERSITY
INSTITUTE OF ENGINEERING
Examination Control Division

2070 Ashad

Exam.	New Back (2066 & Later Batch)		
Level	BE	Full Marks	80
Programme	All (Except B.Arch)	Pass Marks	32
Year / Part	I / I	Time	3 hrs.

Subject: - Mathematics I *(SH401)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ ***All** questions carry equal marks.*
- ✓ *Assume suitable data if necessary.*

1. State Leibnitz's Theorem on higher derivatives. If $y = \sin(m \sin^{-1}x)$ then show that $(1-x^2)\,y_{n+2} - (2n+1)\,xy_{n+1} + (m^2-n^2)y_n = 0$

2. State Rolle's Theorem and verify it for the function $f(x) = \dfrac{x(x+3)}{e^{\frac{x}{2}}}$, $x \varepsilon [-3,0]$

3. Evaluate: $\lim_{x \to 0} \left(\dfrac{\tan x}{x}\right)^{\frac{1}{x^2}}$

4. Find the asymptotes of the curve $(x^2 - y^2)^2 - 2(x^2 + y^2) + x - 1 = 0$

5. Show that the radius of curvature at any point (r,θ) of the curve $r^m = a^m \cos m\theta$ is $\dfrac{a^m}{(m+1)r^{m+1}}$

6. Show that $\int_0^1 \dfrac{\log(1+x)}{1+x^2}\,dx = \dfrac{\pi}{8}\log 2$

7. Evaluate by using the rule of differentiation under the sign of integration $\int_0^\infty \dfrac{e^{-x}\sin bx}{x}dx$

8. Use Gamma function to prove $\int_0^{\frac{\pi}{6}} \cos^4 3\theta.\ \sin^2 6\theta = \dfrac{5\pi}{192}$

9. Find the area bounded by the curve $x^2y = a^2(a-y)$ and $X-$axis

OR

Show that the volume of the solid formed by revolving the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ about the line $x = 2a$ is $4\pi^2a^2b$ cubic units.

10. Solve the differential equation $(1+y^2)\,dy = (\tan^{-1}y - x)\,dx$

11. Solve the differential equation $y = yp^2 + 2px$ where $p = \dfrac{dy}{dx}$

12. Solve the differential equation $(D^2 - 2D + 5)y = e^{2x}.\sin x$

13. Solve the differential equation $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x\log x$

OR

Newton's law of cooling states that the temperature of an object changes at the rate proportional to the difference of temperature between the object and its surroundings. Supposing water at 100°C cools to 80°C in 10 minutes in a room temperature of 30°C find the time when the temperature of water will become 40°C?

14. If the axes be turned through an angle $\tan\theta = 2$ what does the equation $4xy - 3x^2 - a^2 = 0$ becomes.

15. Find the condition that the straight line $x\cos\alpha + y\sin\alpha = p$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

16. Find the centre, length of axes and eccentricity of the conic $9x^2 + 4xy + 6y^2 - 22x - 16y + 9 = 0$

OR

Describe and sketch the graph of the equation $r = \frac{12}{3 + 2\cos\theta}$

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