


```

    """## Enter your name below and run the cell:\n",
    "\n",
    "Individual cells can be run with `Ctrl` + `Enter`"
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    "    # ganymede.update(p,True)\n",
    "# check(0)"
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        "\u001b[0;32mIn[3], line 1\u001b[0m\n\u001b[0;32m----> 1\u001b[0m\n\u001b[0;43mYouTubeVideo\u001b[0m(\u001b[49m\u001b[38;5;124m'\u001b[39m\u001b[38;5;124m9vKqVkmQHKk\u001b[39m\u001b[38;5;\nwidth\u001b[38;5;241m=\u001b[39m\u001b[38;5;241m560\u001b[39m,\nheight\u001b[38;5;241m=\u001b[39m\u001b[38;5;241m315\u001b[39m) \u001b[38;5;66;03m# Video by\nhttp://www.3bluelbrown.com/\u001b[39;00m\u001b[0m",
        "\u001b[0;31mNameError\u001b[0m: name 'YouTubeVideo' is not defined"
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```



```

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            "—(x )\n",
            "dx   "
          ]
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```

```

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    "\n",
    "sym.plot(expr, xlim=(-2, 2), ylim=(-10, 10));"
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          "— (- x  + 2)\n",
          "dx"
        ]
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          " 2\n",
          "2\n",

```



```

"ys = np.array([0, 1, 0, 1, 2, 0, 2, 3, 2, 1, 2, 102, 108, 95, 100, 98, 99, 104, 110,
103, 100, 96, 102, 101])\n",
"\n",
"fig,ax = plt.subplots()\n",
"ax.plot([i for i in range(len(ys))], ys);\n",
"# check(1)"
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"Next, let's look at small chunks of our fake signal:"
]
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"print(chunks)\n",
"check(2)"
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"***Question:** Which one of these chunks would you say is the most \"interesting\"?\n",
"Between 2 and 102 is the most interesting because this is where it spikes up.\n",
"\n",
"***Question** If we always divide up the signal as we did above, will we always find
something \"interesting\"?\n",
"Not necessarily, it could just be a straight line."
]
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"## Convolutions\n",
"\n",
"Derivatives and convolutions are one technique to help us tackle the above problem. \n",
"\n",
"First, you'll need to generate windows into the signal. Write a function that can
generate windows with a user-supplied window size, and print them out.\n",
"\n",
"An example signal with 3 window sizes is shown below. Your output does not need to
replicate the formatting shown, but they should produce the same windows. E.g., given an input
signal of `[10,20,30]` and a `window size=2`, your function should return `[[10,20],
[20,30]]`."
]
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"### A window size of 1:\n",
"\n",
"```\n",

```



```

"signal:\n",
"      0  1  0  2  1  0  1 101 100 98 102 101\n",
" 0:      0\n",
" 1:      1\n",
" 2:      0\n",
" 3:      2\n",
" 4:      1\n",
" 5:      0\n",
" 6:      1\n",
" 7:      101\n",
" 8:      100\n",
" 9:      98\n",
"10:      102\n",
"11:      101\n",
"\n",
"\t : ..... \n",
"\n",
"i:  0 | i + window size: 1 | window: [ 0]\n",
"i:  1 | i + window size: 2 | window: [ 1]\n",
"i:  2 | i + window size: 3 | window: [ 0]\n",
"i:  3 | i + window size: 4 | window: [ 2]\n",
"i:  4 | i + window size: 5 | window: [ 1]\n",
"i:  5 | i + window size: 6 | window: [ 0]\n",
"i:  6 | i + window size: 7 | window: [ 1]\n",
"i:  7 | i + window size: 8 | window: [101]\n",
"i:  8 | i + window size: 9 | window: [100]\n",
"i:  9 | i + window size:10 | window: [ 98]\n",
"i: 10 | i + window size:11 | window: [102]\n",
"i: 11 | i + window size:12 | window: [101]\n",
"```\n",
"\n",
"### A window size of 2:\n",
"\n",
"```\n",
"signal:\n",
"      0  1  0  2  1  0  1 101 100 98 102 101\n",
" 0:      0  1\n",
" 1:      1  0\n",
" 2:      0  2\n",
" 3:      2  1\n",
" 4:      1  0\n",
" 5:      0  1\n",
" 6:      1 101\n",
" 7:      101 100\n",
" 8:      100 98\n",
" 9:      98 102\n",
"10:      102 101\n",
"\n",
"\t : ..... \n",
"\n",
"i:  0 | i + window size: 2 | window: [ 0, 1]\n",
"i:  1 | i + window size: 3 | window: [ 1, 0]\n",
"i:  2 | i + window size: 4 | window: [ 0, 2]\n",
"i:  3 | i + window size: 5 | window: [ 2, 1]\n",
"i:  4 | i + window size: 6 | window: [ 1, 0]\n",
"i:  5 | i + window size: 7 | window: [ 0, 1]\n",
"i:  6 | i + window size: 8 | window: [ 1, 101]\n",
"i:  7 | i + window size: 9 | window: [101, 100]\n",
"i:  8 | i + window size:10 | window: [100, 98]\n",
"i:  9 | i + window size:11 | window: [ 98, 102]\n",
"i: 10 | i + window size:12 | window: [102, 101]\n",
"```\n",

```

```

"\n",
"\n",
"### A window size of 3\n",
"\n",
"\\\n",
"signal:\n",
"      0  1  0  2  1  0  1 101 100 98 102 101\n",
"0:      0  1  0\n",
"1:      1  0  2\n",
"2:      0  2  1\n",
"3:      2  1  0\n",
"4:      1  0  1\n",
"5:      0  1 101\n",
"6:      1 101 100\n",
"7:      101 100 98\n",
"8:      100 98 102\n",
"9:      98 102 101\n",
"\n",
"\t :..... \n",
"\n",
"i:  0 | i + window size: 3 | window: [ 0, 1, 0]\n",
"i:  1 | i + window size: 4 | window: [ 1, 0, 2]\n",
"i:  2 | i + window size: 5 | window: [ 0, 2, 1]\n",
"i:  3 | i + window size: 6 | window: [ 2, 1, 0]\n",
"i:  4 | i + window size: 7 | window: [ 1, 0, 1]\n",
"i:  5 | i + window size: 8 | window: [ 0, 1, 101]\n",
"i:  6 | i + window size: 9 | window: [ 1, 101, 100]\n",
"i:  7 | i + window size: 10 | window: [ 101, 100, 98]\n",
"i:  8 | i + window size: 11 | window: [ 100, 98, 102]\n",
"i:  9 | i + window size: 12 | window: [ 98, 102, 101]\n",
"\\\n",
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"The below resources may be helpful:\n",
"\n",
"### List Comprehensions\n",
"\n",
https://www.pythonlikeyoumeanit.com/Module2\_EssentialsOfPython/Generators\_and\_Comprehensions.html#List-&-Tuple-Comprehensions\n",
"\n",
"### Numpy indexing with slices\n",
"\n",
http://www.pythonlikeyoumeanit.com/Module3\_IntroducingNumpy/AccessingDataAlongMultipleDimensions.html#Slice-Indexing
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"### Formatting numbers in python\n",
"\n",
https://pyformat.info/#number\n",
"\n",


```

input: '{:4d}'.format(42)
output: _ _ `4` `2`

```


]

```

```

    """input:** `'{:06.2f}'.format(3.141592653589793)`\n",
    "\n",
    """output:** `003.14`"
]
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    "\n",
    "```python\n",
    ">>> print('a' + 'b' + 'c')\n",
    "abc\n",
    ">>> print(''.join(['a', 'b', 'c']))\n",
    "abc\n",
    ">>> print(''.join(['a', 'b', 'c']))      \n",
    "a,b,c\n",
    "```"
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    "def make_windows(sequence, windowsize):\n",
    "    window = []\n",
    "    for i in range(len(sequence) - windowsize + 1):\n",
    "        new_list = sequence[i:i + windowsize]\n",
    "        window.append(new_list)\n",
    "    print(window)\n",
    "\n",
    "make_windows([10,20,30], 2)"
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    "series = [0, 1, 0, 2, 1, 0, 1, 101, 100, 98, 102, 101]\n",
    "\n",
    "\n",
    "make_windows(sequence=series, windowsize=1)\n",
    "make_windows(sequence=series, windowsize=2)\n",
    "make_windows(sequence=series, windowsize=3)\n",
    "\n",
    "check(3)"
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    "## When you are done:\n",
    "\n",
    "Generate some example outputs in this notebook.\n",
    "\n",
    "1. Double-check that you filled in your name at the top of the notebook!\n",

```

```
    "2. Click `File` -> `Export Notebook As` -> `PDF`\n",  
    "3. Email the PDF to `YOURTEAMNAME@beaver.works` "  
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