1. $a, b, c \in \mathbb{R}$, $S = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$

1. $P_S(x) = (x-a)(x-c) - b^2$

2. $P_S(x) = x^2 - ax - cx + ac - b^2 = 0$

$P_S(x) = x^2 - (a+c)x + ac - b^2 = 0$

$$x = \frac{a+c \pm \sqrt{(a+c)^2 - 4(ac-b^2)}}{2}$$

$$x = \frac{a+c \pm \sqrt{(a-c+2b)(a-c-2b)}}{2}$$

↗ eigenvalues of S

3. For each eigenvalue λ of S:

Solve for v in $Sv = \lambda v$

$\begin{pmatrix} a & b \\ b & c \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \lambda x \\ \lambda y \end{pmatrix}$

$\begin{pmatrix} ax+by \\ bx+cy \end{pmatrix} = \begin{pmatrix} \lambda x \\ \lambda y \end{pmatrix}$

plug in eigenvalue for λ and solve for x & y

(way too ugly so im not doing it by hand)

2. I don't know ∴

Differential Equations

$\dot{x} = f(x)$, $f: \mathbb{R}^n \to \mathbb{R}^n$, $A = \begin{pmatrix} 3 & -1 \\ 3 & 2 \end{pmatrix}$

I don't know what a Jacobian is ∴.

1. $P_A(x) = (x-3)(x+2) + 3$

$P_A(x) = x^2 - x - 3$

eigenvalue	eigenvector
$\frac{1+\sqrt{13}}{2}$	$(\frac{3+\sqrt{13}}{6}, 1)$
$\frac{1-\sqrt{13}}{2}$	$(\frac{3-\sqrt{13}}{6}, 1)$

2. I don't know

3. ?