BWSI HW 1

Linear Algebra

1.

1. $S = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$ $P_S(x) = \det(S - xI) = \det\begin{pmatrix} a-x & b \\ b & c-x \end{pmatrix}$

   $P_S(x) = (a-x)(c-x) - b^2$

   $P_S(x) = x^2 - x(a+c) + (ac-b^2)$

2. $x = \dfrac{-B \pm \sqrt{B^2 - 4C}}{2}$

   $x = \dfrac{a+c \pm \sqrt{(a+c)^2 - 4(ac-b^2)}}{2}$

   $x_{1,2} = \dfrac{a+c \pm \sqrt{(a-c)^2 + 4b^2}}{2}$

   (Using λ to not get mixed up with x and y of eigenvector)

3. For both λ_1 & λ_2 solve $(S - xI)v = 0$

   $\begin{pmatrix} a-\lambda & b \\ b & c-\lambda \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0$

   $\vec{V_1} = \begin{pmatrix} b \\ \dfrac{c-a + \sqrt{(a-c)^2 + 4b^2}}{2} \end{pmatrix}$ $\vec{V_2} = \begin{pmatrix} b \\ \dfrac{c-a - \sqrt{(a-c)^2 + 4b^2}}{2} \end{pmatrix}$

Differential Equations

2.

1. $\det \begin{pmatrix} 3-\lambda & -1 \\ 3 & -2-\lambda \end{pmatrix} \to \lambda^2 - \lambda - 3 = 0$

$$\lambda = \frac{1 \pm \sqrt{(-1)^2 - 4 \cdot -3}}{2}$$

$$\lambda_1 = \frac{1 + \sqrt{13}}{2}$$

$$\lambda_2 = \frac{1 - \sqrt{13}}{2}$$

$$\vec{V}_1 = \begin{pmatrix} -1 \\ \frac{-5 - \sqrt{13}}{2} \end{pmatrix} \qquad \vec{V}_2 = \begin{pmatrix} -1 \\ \frac{\sqrt{13} - 5}{2} \end{pmatrix}$$

2. Saddle point

3. Unsure about this