Practicing Advanced Feedback Control

In this unit we are going to explore basic feedback control on a simplified, 1-dimensional model of the quadrotor-camera. This system is more complicated than the Simple Slide Camera model because it incorporates a dependency between the velocity and a pitch angle as well as a dependency of the output angle, γ, on the pitch angle, θ.

Pitch Slide Camera

Imagine a camera attached to a cart that is allowed to slide along a 1-dimensional track. The track is elevated some height h above the ground and the camera is pitched at an angle θ relative to vertical. There is some target on the ground that the camera can observe; more specifically the camera can measure the angle $-\pi/2 < \gamma < \pi/2$ from the centerline of the camera to the target. Since the view angle depends on the pitch of the camera, γ is a function of pitch in the form:

$$\gamma = -\left(\tan^{-1}\left(\frac{x_{des}-x}{h}\right) + \theta\right) = \tan^{-1}\left(\frac{x-x_{des}}{h}\right) - \theta$$

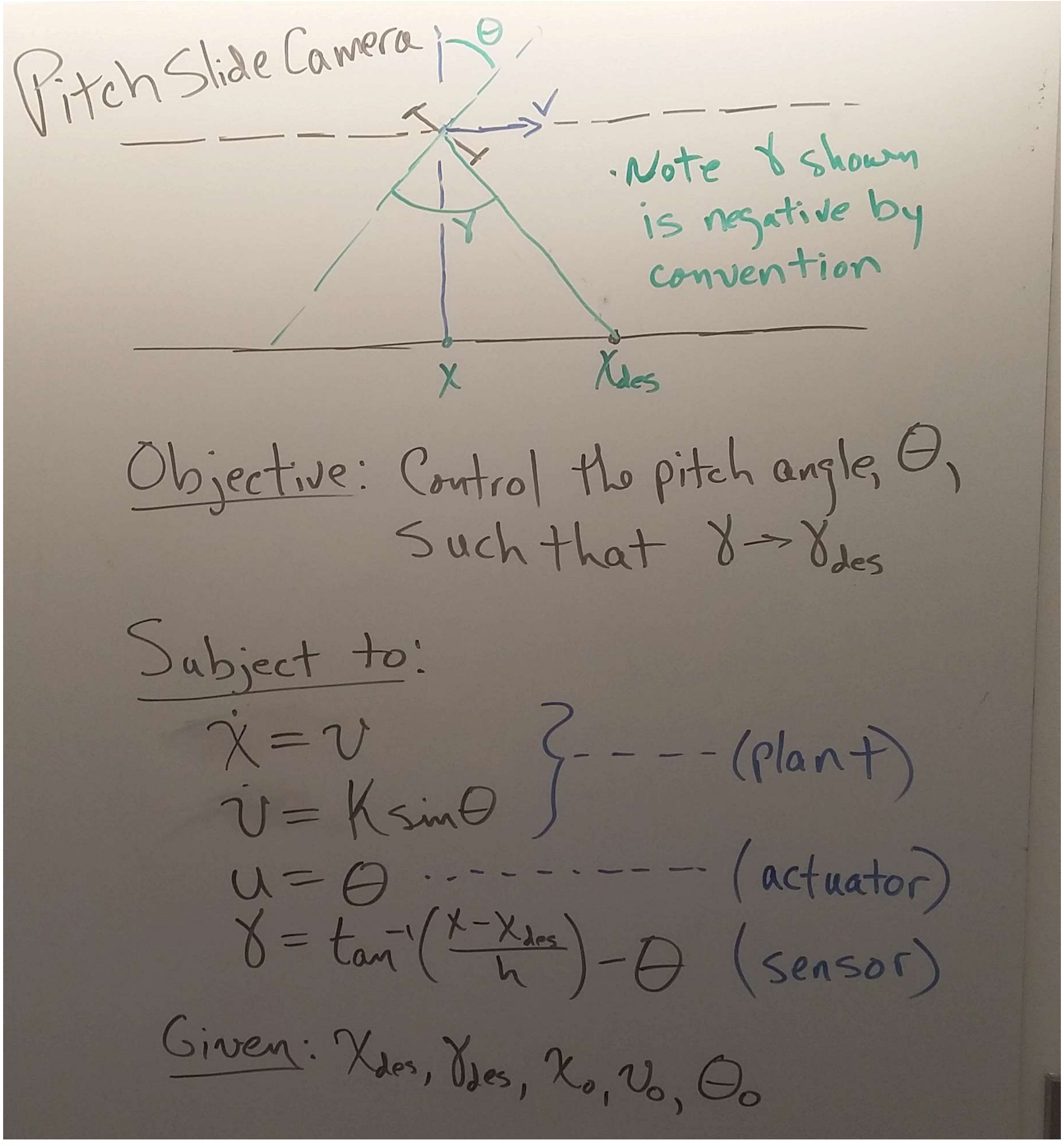
Note that orientation of γ (i.e. the minus sign at the beginning of the expression) is by convention.

The velocity of camera-cart system is also dependent upon the pitch angle; i.e. the larger the pitch, the greater the change in velocity (you could think of this somewhat similar to a segway scooter). The velocity dependence is defined by:

$$\dot{v} = K \sin(\theta)$$

The objective is to issue pitch commands commands, θ_{cmd}, in order to move the cart to a position such that $\gamma \to \gamma_{des}$ (e.g. if $\gamma_{des} = 0$, then the objective is simply to move the cart directly over the target.)

This system is depicted in the below diagram:



This somewhat contrived system can be thought of as a simplified model of the quadrotor and it's downward-facing camera. The velocity of a quadrotor is in fact a function of the pitch of the quadrotor, similar to this system. The major simplification here is that the system is constrained to move in 1-dimension, which is not the case for a quadrotor.

Note that the position and velocity of the cart, x and v, and position of the target x_{des} are not directly measured, only γ is measured.

Now we will provide the code necessary to simulate this system

In [ ]:
```
from __future__ import division, print_function
import numpy as np
import matplotlib.pyplot as plt

_HEIGHT = 1.0
_VEL_CONST = 1.0
```

```
_TIME_STEP = 0.1
_THETA_LIMIT = np.pi/4.0
```

Plant Dynamics, Sensors, and Actuators

the following object contains functions for the plant dynamics, sensing of the target angle γ, and actuator for v_{cmd}

In [ ]:
```
class PitchSlideCamera():
    '''Object that defines the dynamics of the simple slide-camera'''

    def __init__(self, x_0, v_0, theta_0, x_d, gamma_d=0.0, h=_HEIGHT, k=_VEL_CONST, t

        # state variables (hidden)
        self.__x = x_0
        self.__v = v_0

        # reference position (hidden)
        self.__x_d = x_d

        # reference angle (observed)
        self.gamma_d = gamma_d

        # parameters
        self.__h = h
        self.__k = k
        self.__theta_limit = theta_limit

        # control variables (observed, commanded)
        self.__theta = theta_0

    def get_theta(self):
        return self.__theta

    def sense_gamma(self):
        # calculate angle from camera center line to target
        return  np.arctan2(self.__x - self.__x_d, self.__h) - self.__theta

    def _get_hidden_position(self):

        return self.__x

    def _get_hidden_position_desired(self):
        return self.__x_d

    def _get_hidden_velocity(self):
        return self.__v

    def actuate_theta_command(self, theta_cmd, dt=_TIME_STEP):
        self.__theta = min(self.__theta_limit, max(theta_cmd, -self.__theta_limit))
        self.__v += self.__k*np.sin(self.__theta)*dt
        self.__x += self.__v*dt
```

Controllers

Functions for control algorithms such as proportional control, proportional-derivative control, etc, as well as any custom controllers you may wish to try

In [ ]:
```
def p_control(y_err, kp):
    ''' compute the actuator command based on proportional error between output and de
    Args:
    y_err: y_des - y where y is the output variable of the plant
    '''

    # TODO: write a proportional control law (hint: it is a single line, very simple e
    # YOUR CODE HERE
    cmd = y_err*kp

    return cmd
```

In [ ]:
```
def pd_control(y_err, y_err_prev, dt, kp, kd):
    '''compute the actuator command based on proportional and derivative error between
    Args:
    y_err: y_des - y where y is the output variable of the plant
    y_err_prev: previous step y_des - y
    '''

    # TODO: write a proportional+derivative control law
    # YOUR CODE HERE
    cmd = (y_err * kp) + (((y_err - y_err_prev)/dt)*kd)

    return cmd
```

In [ ]:
```
assert np.isclose(pd_control(0.0, 1.0, 0.1, 1.0, 1.0), -10.0)
```

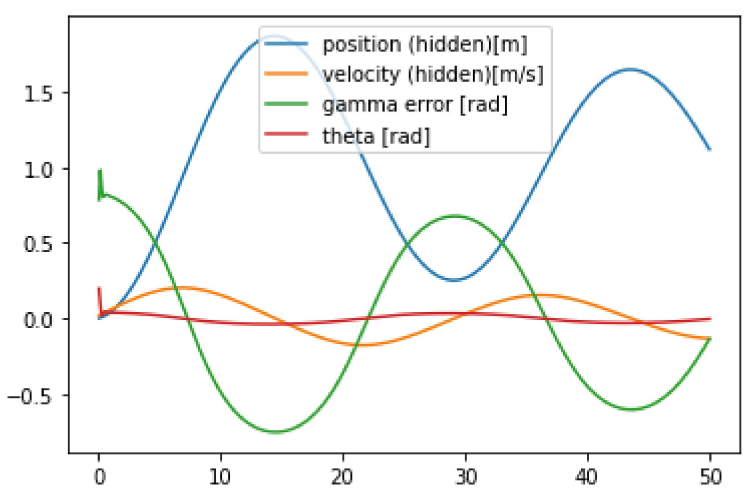
In [ ]:
```
def custom_control():
    '''custom-made controller, if you want to develop one
    Args:
    '''
    pass
```

Simulation Script

below is a script for testing various controllers for the `SimpleSlideCamera` plant as well as plotting the results.

In [ ]:
```
# Control gains
# YOUR CODE HERE
kp = 0.05
kd = 0.02

# Control inputs
dt = _TIME_STEP
t_final = 50.0

# intial conditions (position, velocity and targe position)
x_0 = 0.0
v_0 = 0.0
theta_0 = 0.0
x_des = 1.0
```

```
# create SimpleSlideCamera with initial conditions
pscam = PitchSlideCamera(x_0, v_0, theta_0, x_des)

# initialize data storage
data = dict()
data['t'] = []
data['theta_cmd'] = []
data['theta'] = []
data['err_gamma'] = []
data['x_hidden'] = []
data['v_hidden'] = []
t = 0.0
err_gamma_prev = 0.0
while t < t_final:
    t += dt

    # SENSOR: sense output variable gamma (angle from camera centerline to target) and
    err_gamma = pscam.gamma_d - pscam.sense_gamma()

    # CONTROLLER: call theta control algoritm
    # theta_cmd = p_control(err_gamma, kp)
    theta_cmd = pd_control(err_gamma, err_gamma_prev, dt, kp, kd)

    # ACTUATOR: send velocity command to plant
    pscam.actuate_theta_command(theta_cmd)

    # store data
    err_gamma_prev = err_gamma
    data['t'].append(t)
    data['theta_cmd'].append(theta_cmd)
    data['theta'].append(pscam.get_theta())
    data['err_gamma'].append(err_gamma)
    data['x_hidden'].append(pscam._get_hidden_position())
    data['v_hidden'].append(pscam._get_hidden_velocity())

# Plot Data
handle_position, = plt.plot(data['t'], data['x_hidden'], label='position (hidden)[m]')
handle_velocity, = plt.plot(data['t'], data['v_hidden'], label='velocity (hidden)[m/s]
handle_err_gamma, = plt.plot(data['t'], data['err_gamma'], label='gamma error [rad]')
handle_theta, = plt.plot(data['t'], data['theta'], label='theta [rad]')
plt.legend(handles=[handle_position, handle_velocity, handle_err_gamma, handle_theta])
plt.show()
```



Questions

Q1. Can you design a controller that is capable of converging the gamma error to zero?

Not really, but I can get it to oscillate around zero within pretty small bounds.

Q2. The time scale to make this control converge is on the order of 10s of seconds. Our real drones, which are much more complex control problems, are capable of converging on a target much faster with far less oscillation. Can you give explanations why our drone controllers perform so much better than this controller?

The drone controllers perform much better because their control algorithms have been fine-tuned and optimized for much longer than we have had working on this practical. They also likely incorporate an integral term as well, which further increases the accuracy and speed at which they converge.