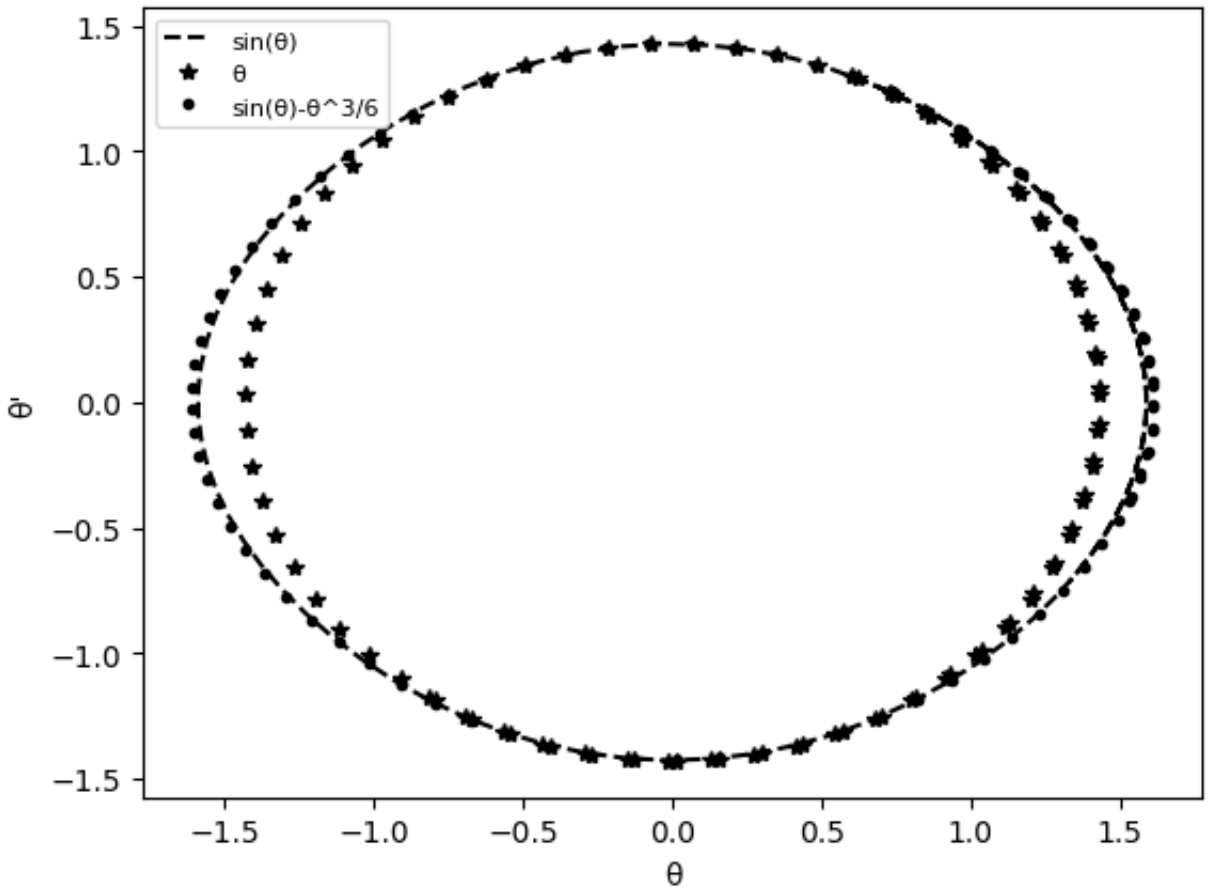
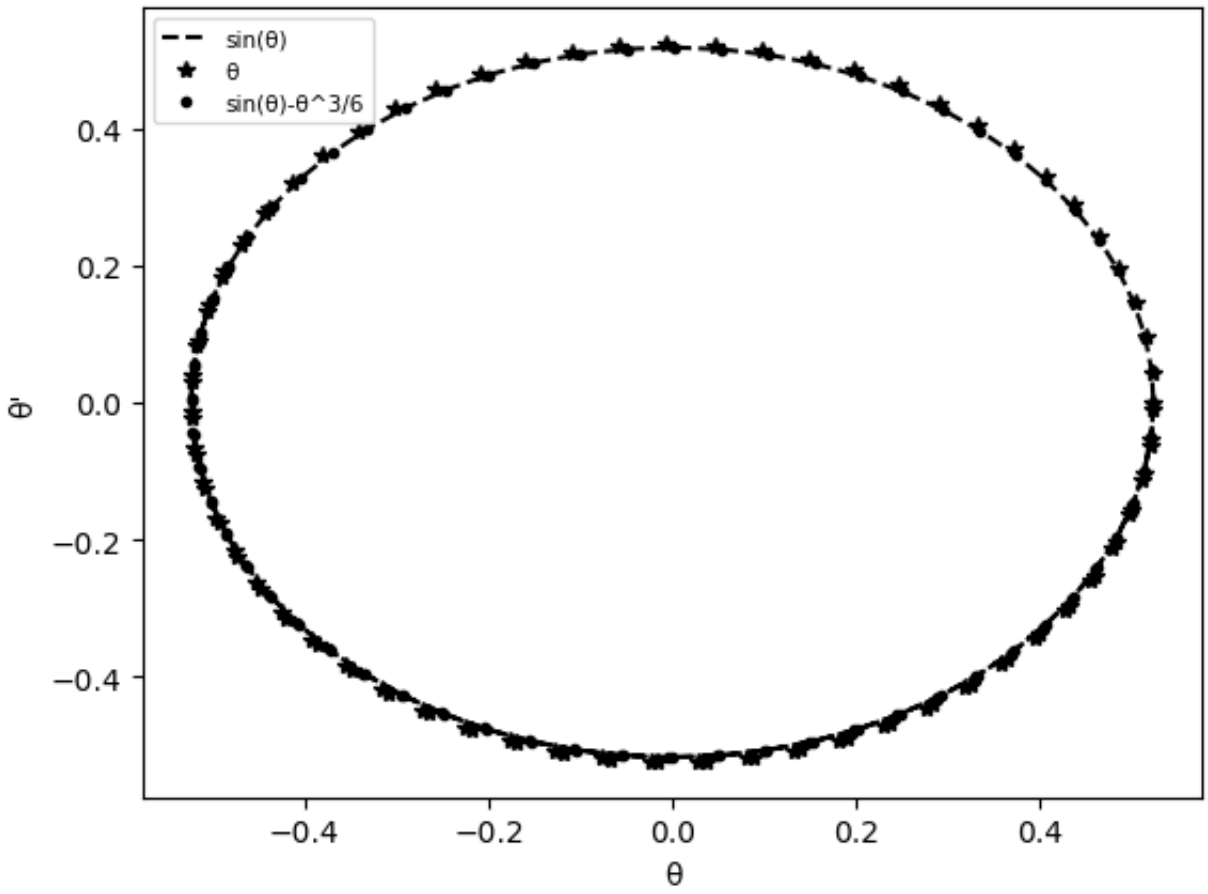
In [50]:
```
import numpy as np
import matplotlib.pyplot as plt
def RK4(f1,f2,a,b,alp1,alp2,h):
    n=int((b-a)/h)
    w=np.zeros([n+1,2])
    t=np.arange(a,b+h,h)
    w[0,0]=alp1
    w[0,1]=alp2
    for i in range(n):
        k11=h*f1(t[i],w[i,0],w[i,1])
        k12=h*f2(t[i],w[i,0],w[i,1])
        k21=h*f1(t[i]+h/2,w[i,0]+k11/2,w[i,1]+k12/2)
        k22=h*f2(t[i]+h/2,w[i,0]+k11/2,w[i,1]+k12/2)
        k31=h*f1(t[i]+h/2,w[i,0]+k21/2,w[i,1]+k22/2)
        k32=h*f2(t[i]+h/2,w[i,0]+k21/2,w[i,1]+k22/2)
        k41=h*f1(t[i]+h,w[i,0]+k31,w[i,1]+k32)
        k42=h*f2(t[i]+h,w[i,0]+k31,w[i,1]+k32)
        w[i+1,0]=w[i,0]+(k11+2*k21+2*k31+k41)/6
        w[i+1,1]=w[i,1]+(k12+2*k22+2*k32+k42)/6
    return w

f1= lambda t,u1,u2: u2
f2= lambda t,u1,u2: -np.sin(u1)
f3= lambda t,u1,u2: -u1
f4= lambda t,u1,u2:-(u1-u1**3/6)
a=1/3
b=10
alp1=.6
alp2=1.3
h=0.1
s1=RK4(f1,f2,a,b,alp1,alp2,h)
s2=RK4(f1,f3,a,b,alp1,alp2,h)
s3=RK4(f1,f4,a,b,alp1,alp2,h)
plt.plot(s1[:,0],s1[:,1],'--',color='k',label='sin(θ)')
plt.plot(s2[:,0],s2[:,1],'*',color='k',label='θ')
plt.plot(s3[:,0],s3[:,1],'.',color='k',label='sin(θ)-θ^3/6')
plt.legend(loc='upper left',fontsize=8)
plt.xlabel("θ")
plt.ylabel("θ'")
plt.savefig("Q1")
plt.show()
```


sin(θ)
θ
sin(θ)-θ^3/6
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
θ'
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
θ

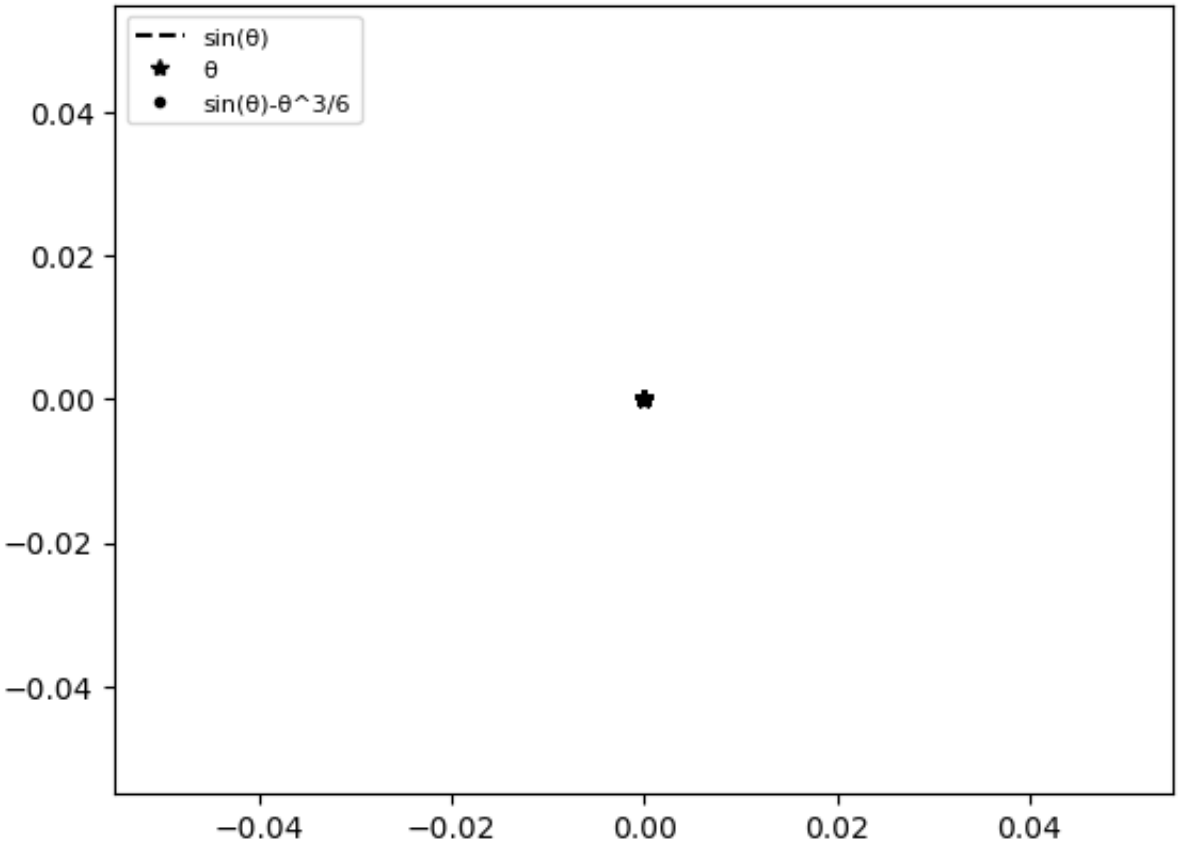
In [49]:
```
import numpy as np
import matplotlib.pyplot as plt
def RK4(f1,f2,a,b,alp1,alp2,h):
    n=int((b-a)/h)
    w=np.zeros([n+1,2])
    t=np.arange(a,b+h,h)
    w[0,0]=alp1
    w[0,1]=alp2
    for i in range(n):
        k11=h*f1(t[i],w[i,0],w[i,1])
        k12=h*f2(t[i],w[i,0],w[i,1])
        k21=h*f1(t[i]+h/2,w[i,0]+k11/2,w[i,1]+k12/2)
        k22=h*f2(t[i]+h/2,w[i,0]+k11/2,w[i,1]+k12/2)
        k31=h*f1(t[i]+h/2,w[i,0]+k21/2,w[i,1]+k22/2)
        k32=h*f2(t[i]+h/2,w[i,0]+k21/2,w[i,1]+k22/2)
        k41=h*f1(t[i]+h,w[i,0]+k31,w[i,1]+k32)
        k42=h*f2(t[i]+h,w[i,0]+k31,w[i,1]+k32)
        w[i+1,0]=w[i,0]+(k11+2*k21+2*k31+k41)/6
        w[i+1,1]=w[i,1]+(k12+2*k22+2*k32+k42)/6
    return w

f1= lambda t,u1,u2: u2
f2= lambda t,u1,u2: -np.sin(u1)
f3= lambda t,u1,u2: -u1
f4= lambda t,u1,u2:-(u1-u1**3/6)
a=0
b=10
alp1=np.pi/6
alp2=0
h=.1
s1=RK4(f1,f2,a,b,alp1,alp2,h)
s2=RK4(f1,f3,a,b,alp1,alp2,h)
s3=RK4(f1,f4,a,b,alp1,alp2,h)
plt.plot(s1[:,0],s1[:,1],'--',color='k',label='sin(θ)')
plt.plot(s2[:,0],s2[:,1],'*',color='k',label='θ')
plt.plot(s3[:,0],s3[:,1],'.',color='k',label='sin(θ)-θ^3/6')
plt.legend(loc='upper left',fontsize=7.5)
plt.xlabel("θ")
plt.ylabel("θ'")
plt.savefig("Q2")
plt.show()
```


sin(θ)
θ
sin(θ)-θ^3/6
0.4
0.2
0.0
−0.2
−0.4
θ'
−0.4
−0.2
0.0
0.2
0.4
θ

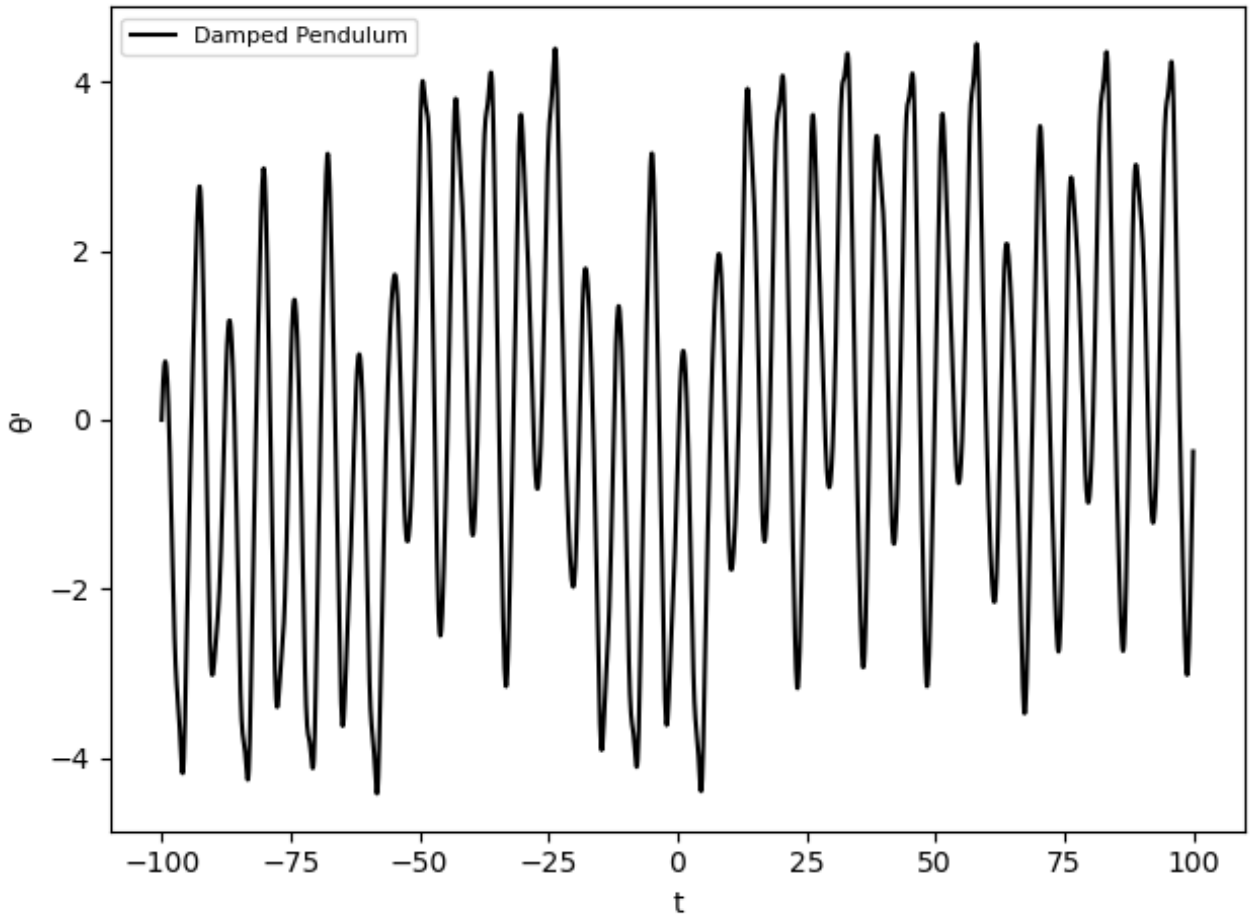
In [51]:
```
import numpy as np
import matplotlib.pyplot as plt
def RK4(f1,f2,a,b,alp1,alp2,h):
    n=int((b-a)/h)
    w=np.zeros([n+1,2])
    t=np.arange(a,b+h,h)
    w[0,0]=alp1
    w[0,1]=alp2
    for i in range(n):
        k11=h*f1(t[i],w[i,0],w[i,1])
        k12=h*f2(t[i],w[i,0],w[i,1])
        k21=h*f1(t[i]+h/2,w[i,0]+k11/2,w[i,1]+k12/2)
        k22=h*f2(t[i]+h/2,w[i,0]+k11/2,w[i,1]+k12/2)
        k31=h*f1(t[i]+h/2,w[i,0]+k21/2,w[i,1]+k22/2)
        k32=h*f2(t[i]+h/2,w[i,0]+k21/2,w[i,1]+k22/2)
        k41=h*f1(t[i]+h,w[i,0]+k31,w[i,1]+k32)
        k42=h*f2(t[i]+h,w[i,0]+k31,w[i,1]+k32)
        w[i+1,0]=w[i,0]+(k11+2*k21+2*k31+k41)/6
        w[i+1,1]=w[i,1]+(k12+2*k22+2*k32+k42)/6
    return w

f1= lambda t,u1,u2: u2
f2= lambda t,u1,u2: -np.sin(u1)
f3= lambda t,u1,u2: -u1
f4= lambda t,u1,u2:-(u1-u1**3/6)
a=.3
b=10
alp1=0
alp2=0
h=.1
s1=RK4(f1,f2,a,b,alp1,alp2,h)
s2=RK4(f1,f3,a,b,alp1,alp2,h)
s3=RK4(f1,f4,a,b,alp1,alp2,h)
plt.plot(s1[:,0],s1[:,1],'--',color='k',label='sin(θ)')
plt.plot(s2[:,0],s2[:,1],'*',color='k',label='θ')
plt.plot(s3[:,0],s3[:,1],'.',color='k',label='sin(θ)-θ^3/6')
plt.legend(loc='upper left',fontsize=8)
plt.show()
```


sin(θ)
θ
sin(θ)-θ^3/6
0.04
0.02
0.00
−0.02
−0.04
−0.04
−0.02
0.00
0.02
0.04

In [33]:
```
import numpy as np
import matplotlib.pyplot as plt
def RK4(f1,f2,a,b,alp1,alp2,h):
    n=int((b-a)/h)
    w=np.zeros([n+1,2])
    t=np.arange(a,b+h,h)
    w[0,0]=alp1
    w[0,1]=alp2
    for i in range(n):
        k11=h*f1(t[i],w[i,0],w[i,1])
        k12=h*f2(t[i],w[i,0],w[i,1])
        k21=h*f1(t[i]+h/2,w[i,0]+k11/2,w[i,1]+k12/2)
        k22=h*f2(t[i]+h/2,w[i,0]+k11/2,w[i,1]+k12/2)
        k31=h*f1(t[i]+h/2,w[i,0]+k21/2,w[i,1]+k22/2)
        k32=h*f2(t[i]+h/2,w[i,0]+k21/2,w[i,1]+k22/2)
        k41=h*f1(t[i]+h,w[i,0]+k31,w[i,1]+k32)
        k42=h*f2(t[i]+h,w[i,0]+k31,w[i,1]+k32)
        w[i+1,0]=w[i,0]+(k11+2*k21+2*k31+k41)/6
        w[i+1,1]=w[i,1]+(k12+2*k22+2*k32+k42)/6
    return w

f1= lambda t,u1,u2: u2
f5= lambda t,u1,u2:-0.22*u2-np.sin(u1)+2.7*np.cos(t)
a=-100
b=100
alp1=np.pi/6
alp2=0
h=.1
s4=RK4(f1,f5,a,b,alp1,alp2,h)
t=np.arange(a,b+h,h)
plt.plot(t,s4[:,1],'-',color='k',label='Damped Pendulum')
plt.legend(loc='upper left',fontsize=8)
plt.xlabel("t")
plt.ylabel("θ'")
plt.tight_layout()
plt.savefig("Q3")
plt.show()
```

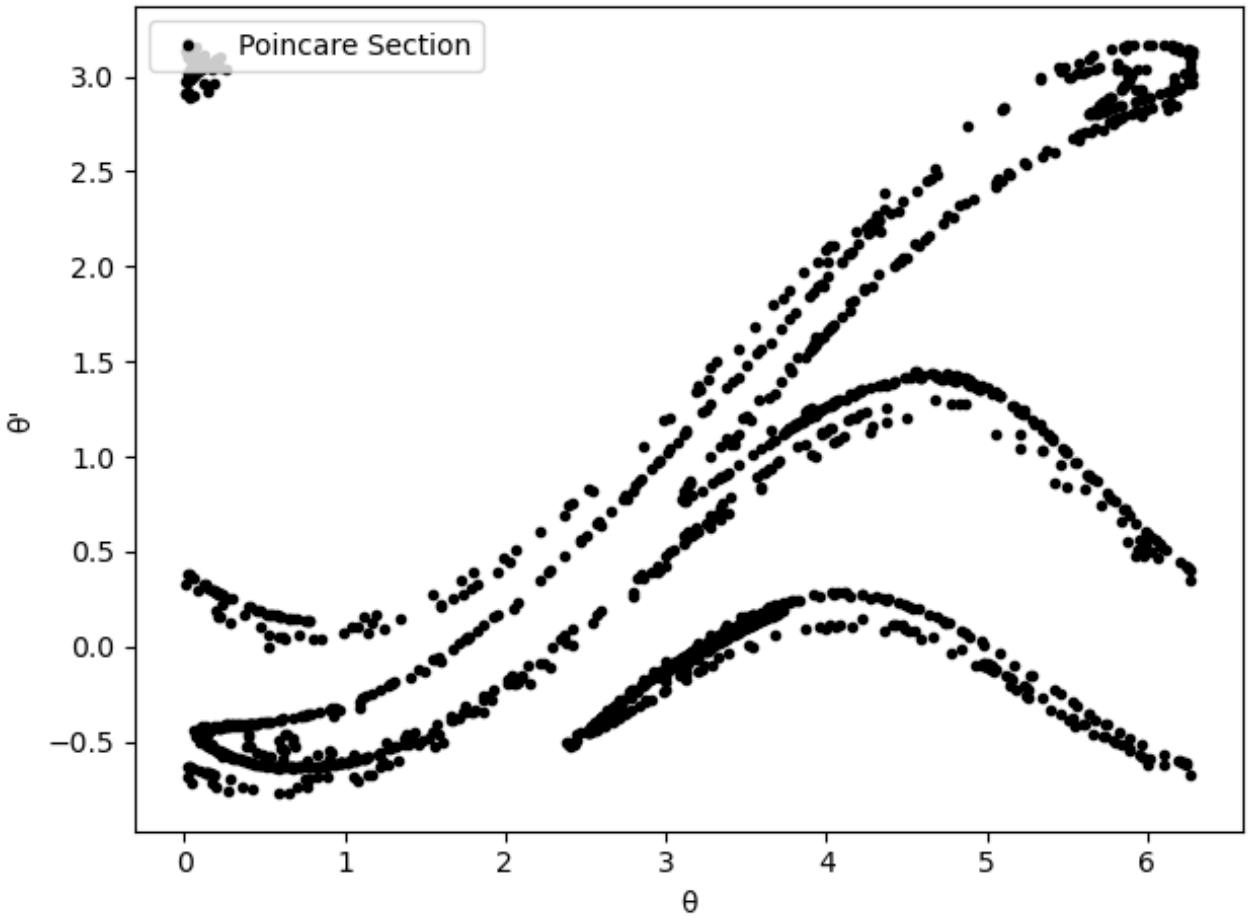

Damped Pendulum
θ'
t
4
2
0
−2
−4
−100
−75
−50
−25
0
25
50
75
100

In [34]:
```
import numpy as np
import matplotlib.pyplot as plt
def RK4(f1,f2,a,b,alp1,alp2,h):
    n=int((b-a)/h)
    w=np.zeros([n+1,2])
    t=np.arange(a,b+h,h)
    w[0,0]=alp1
    w[0,1]=alp2
    k=1
    for i in range(n):
        k11=h*f1(t[i],w[i,0],w[i,1])
        k12=h*f2(t[i],w[i,0],w[i,1])
        k21=h*f1(t[i]+h/2,w[i,0]+k11/2,w[i,1]+k12/2)
        k22=h*f2(t[i]+h/2,w[i,0]+k11/2,w[i,1]+k12/2)
        k31=h*f1(t[i]+h/2,w[i,0]+k21/2,w[i,1]+k22/2)
        k32=h*f2(t[i]+h/2,w[i,0]+k21/2,w[i,1]+k22/2)
        k41=h*f1(t[i]+h,w[i,0]+k31,w[i,1]+k32)
        k42=h*f2(t[i]+h,w[i,0]+k31,w[i,1]+k32)
        w[i+1,0]=w[i,0]+(k11+2*k21+2*k31+k41)/6
        w[i+1,1]=w[i,1]+(k12+2*k22+2*k32+k42)/6
    return w

f1= lambda t,u1,u2: u2
f5= lambda t,u1,u2:-0.22*u2-np.sin(u1)+2.7*np.cos(t)
a=0
b=10000
alp1=np.pi/6
alp2=0
h=np.pi/1000


s4=RK4(f1,f5,a,b,alp1,alp2,h)

plt.plot(s4[::2000,0]%(2*np.pi),s4[::2000,1],'.',color="k",label="Poincare S
plt.xlabel("θ")
plt.ylabel("θ'")
plt.legend(loc="upper left")
plt.tight_layout()
plt.savefig("Q4")
plt.show()
```


Poincare Section
θ'
θ
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
0
1
2
3
4
5
6

In [36]:
```
import numpy as np
import matplotlib.pyplot as plt
def runge_kutta(f,f1, a, b, alp1, alp2, h):
    n = int((b-a)/h)
    t=np.arange(a,b+h,h)
    y = np.zeros([n+1,2])
    y[0,0] = alp1
    y[0,1]=alp2
    w=np.zeros([n+1,2])
    k=1
    for i in range(n):
        k1 = h * y[i,1]
        l1 = h * f(t[i], y[i,0], y[i,1])
        k2 = h * (y[i,1]+ 0.5*l1)
        l2 = h * f(t[i] + 0.5*h, y[i,0] + 0.5*k1, y[i,1] + 0.5*l1)
        k3 = h * (y[i,1] + 0.5*l2)
        l3 = h * f(t[i] + 0.5*h, y[i,0] + 0.5*k2, y[i,1] + 0.5*l2)
        k4 = h * (y[i,1] + l3)
        l4 = h * f(t[i] + h, y[i,0] + k3, y[i,1] + l3)

        y[i+1,1] = y[i,1] + (1/6) * (l1 + 2*l2 + 2*l3 + l4)
        y[i+1,0] = y[i,0] + (1/6) * (k1 + 2*k2 + 2*k3 + k4)

    for i in range(len(y)):
        if np.isclose(t[i],2*np.pi*k):
            w[i+1,1]=y[i,1]
            w[i+1,0]=y[i+1,0]
    return w

f1= lambda t,u1,u2: u2
f5= lambda t,u1,u2:-0.22*u2-np.sin(u1)+2.7*np.cos(t)
a=-30
b=30
alp1=np.pi/5
alp2=0
h=0.1
print(runge_kutta(f5,f1,a,b,alp1,alp2,h))
```

```
[[0. 0.]
 [0. 0.]
 [0. 0.]
 ...
 [0. 0.]
 [0. 0.]
 [0. 0.]]
```

In [ ]: