CHAPTER 2
TRANSFORMERS

2.1 INTRODUCTION TO TRANSFORMERS

A transformer is a device which uses the phenomenon of mutual induction to change the values of alternating voltages and currents. In fact, one of the main advantages of A.C transmission and distribution is the ease will which an alternating voltage can be increased or decreased by transformers.

Losses in transformers are generally low and thus efficiency is high. Being static they have a long-life and are very reliable.

A transformer consists essentially of two separate windings on an iron-core, one receives energy and is called the primary and the other delivers energy and is called the secondary. Since high voltage links carry low currents and hence incur low losses. Consider a variety of specialist transformer such as current Transformer, the Autotransformer and the air-cored transformer.

The two basic types of transformer construction used for power and distribution applications. Note that the high-voltage coils are smaller cross-section conductor than the low voltage coils.

Shown in Fig.(2.1)(a) has primary and secondary coils wound on different legs, and shown in Fig.(b) has both coils wound on the same leg.

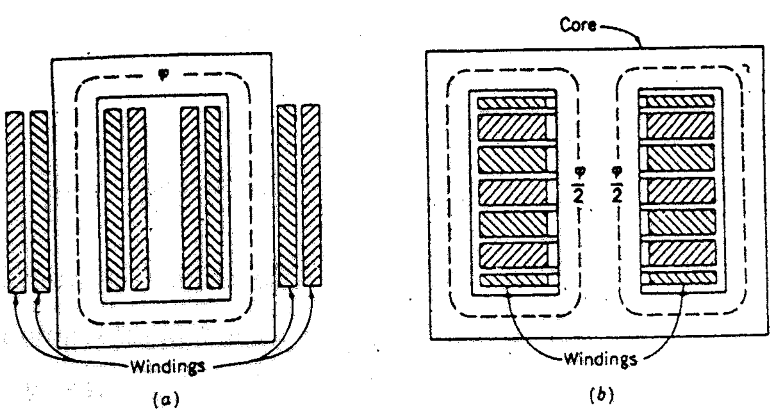


Fig.(2.1) (a). Core-type and (b). Shell-type transformers

2.2. NO-LOAD CONDITIONS

Fig. (2.2) shows a transformer with its secondary circuit open and an alternating voltage v_1 applied to its primary terminals.

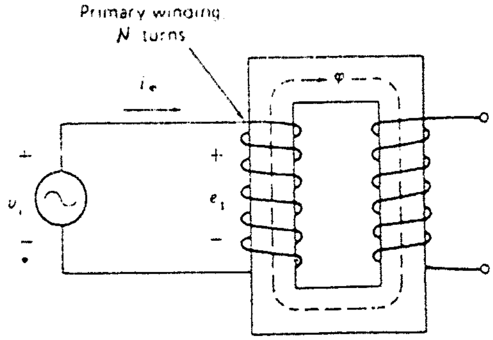


Fig.(2.2) Transformer with open secondary

where i_ϕ = exciting current

$$e_1 = \frac{d\lambda_1}{dt} = N_1\frac{d\phi}{dt} = \text{induce e.m.f in the primary.}$$

λ_1 = flux linkage with primary.

N_1 = number of turns in primary winding.

$\therefore$ The applied voltage v_1 is;

$$v_1 = R_1\, i_\phi + e_1 \tag{2.1}$$

where R_1 = the primary resistance.

If the instantaneous flux is

$$\phi = \phi_{max} \sin \omega t \tag{2.2}$$

$\therefore$ the induced voltage is

$$e_1 = N_1 \frac{d\phi}{dt} = \omega N_1\, \phi_{max} \cos \omega t \text{ volts} \tag{2.3}$$

where, $\omega = 2\pi f$

$$\therefore\ E_1 = \frac{2\pi}{\sqrt{2}}\, f N_1\, \phi_{max}$$

$$E_1 = \sqrt{2}\,\pi f N_1\, \phi_{max} \tag{2.4}$$

If the resistive voltage drop is negligible;

$$v_1 = E_1$$

$$\therefore \phi_{max} = \frac{v_1}{\sqrt{2}\,\pi f N_1} \tag{2.5}$$

$\therefore$ The flux is determined solely by the applied voltage, its frequency, and the number of turns in the winding.

The exciting current can be resolved into two components, are inphase with the counter e.m.f and the other lagging the counter e.m.f by 90. The fundamental in phase component accounts for the power absorbed by hysteresis and eddy-current losses in the core. It is called the core-loss component of the exciting current. When the core-losses component is subtracted from the total exciting current, the remainder is called the magnetizing current.

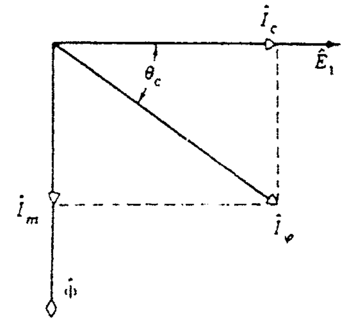


Fig.2.3 No-load phasor diagram

where I_c = core-loss component current

I_m = magnetizing current.

$\therefore P_c = E_1 I_\phi \cos \theta_c$ (2.6)

where P_c is the core loss.

Example 2.1

The core loss and exciting voltamperes for the core of Figure at B_{max} = 1.5 T and 60Hz were found to be P_c = 16W, $(VI)_{rms}$ = 20 VA. And the induced voltage was $275/\sqrt{2}$ = 194 V rms when the winding had 200 turns. Find the power factor, the core loss current I_c, and the magnetizing current I_m.

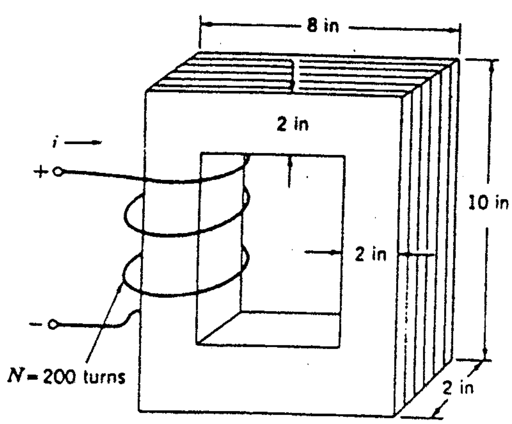


Fig. Reactor with laminated steel core.

Solution

Power factor $\cos\theta_c = \dfrac{16}{20} = 0.80, \quad \theta_c = 36.9°, \quad \sin\theta_c = 0.60$

$$\text{Exciting current } I_c = \frac{20}{194} = 0.10 \text{ A rms}$$

$$\text{Core-loss component } I_c = \frac{16}{194} = 0.082 \text{ A rms}$$

$$\text{Magnetizing component } I_m = I_c \sin\theta_c = 0.060 \text{ A rms}$$

2.3 EFFECT OF SECONDARY CURRENT, I DEAL TRANSORMER

For an ideal transformer, it is assumed that there is no leakage flux, so that all the flux links with both windings. This flux as it grows and dies generator the voltage e_1 and e_2 in the primary and secondary windings, respectively.

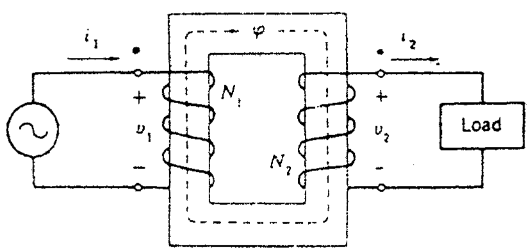


Fig. (2.4) Ideal transformer and load

Since $v_1 = e_1 = N_1 \dfrac{d\phi}{dt}$ (2.7)

and $v_2 = e_2 = N_2 \dfrac{d\phi}{dt}$ (2.8)

From the ratio of eqs.2.7 and 2.8

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} \quad (2.9)$$

Thus an ideal transformer voltage in the direct ratio of the turns in its windings.

Since the full-load efficiency of an ideal transformer is nearly 100 percent, i.e, $P_{in} = P_{out}$.

$\therefore i_1 v_1 = i_2 v_2$ (2.10)

$$\therefore \frac{i_1}{i_2} = \frac{v_2}{v_1} = \frac{N_2}{N_1} \tag{2.11}$$

In phasor form, eq.(2.11) can be expressed as;

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_1}{N_2} \tag{2.12}$$

From eq. 2.12

$$\frac{V_1}{I_1} = \left(\frac{N_1}{N_2}.V_2\right) \Big/ \left(\frac{V_2}{V_1}\right) I_2$$

$$= \frac{N_1}{N_2} \frac{V_1}{V_2} \frac{V_2}{I_2}$$

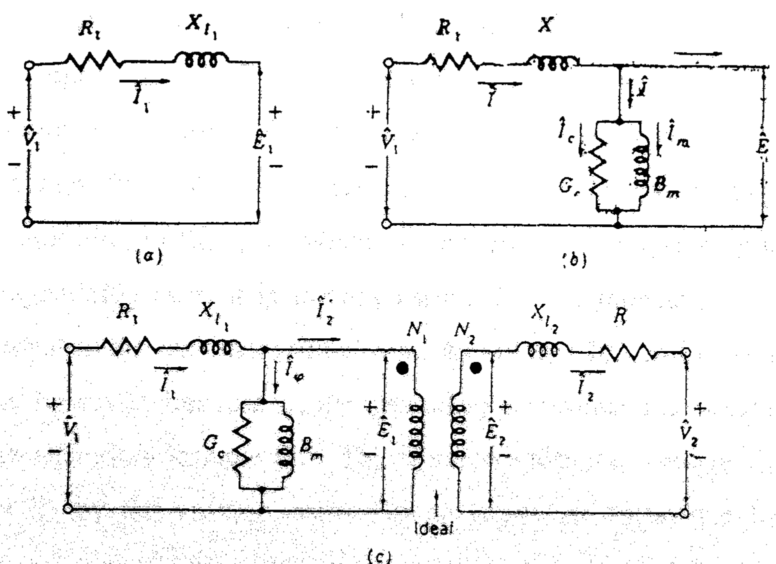
$$\frac{V_1}{I_1} = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

$$Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

$$\therefore \frac{Z_1}{Z_2} = \left(\frac{N_1}{N_2}\right)^2 \tag{2.13}$$

Thus the impedances in the direct ratio squared of the turns in its windings.

2.4 EQUIVALENT CIRCUIT OF A TRANSFORMER



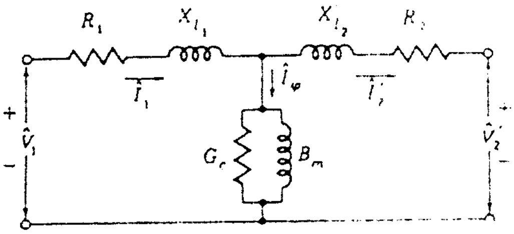


Fig. (2.5) Steps in the development of the transformer equivalent circuit.

Where, I_l = Primary current

R_l = Primary resistance

X_{l1} = Primary leakage reactance

R'_2 = Secondary resistance referred to primary

X'_{L2} = Secondary leakage reactance referred to primary

I_ϕ = Exciting current

I'_2 = Secondary load current referred to primary

G_c = Conductance of the iron core

B_m = Susceptance of the iron core

In Fig. 2.5(a) the impressed terminal voltage V_l is then opposed by three phasor voltages; the I_lR_l drop in the primary resistance, the I_lX_{l1} drop arising from primary leakage flux, and the counter emf E_l induced in the primary by the resultant mutual flux.

In the equivalent circuit (Fig.2.5 (b)) the equivalent sinusoidal exciting current is accounted for by means of a shunt branch connected across E_l, comprising a noninductive resistance whose conductance is G in parallel with a lossless inductance whose susceptance is B_m. In the parallel combination (Fig.2.5b) the power E_l^2 G_c accounts for the core loss due to the resultant mutual flux. When G_c is assumed constant. The magnetizing susceptance B_m varies with the saturation of the iron. When the inductance corresponding to B_m is assumed constant, the magnetizing current is thereby assumed to be independent of frequency and directly proportional to the resultant mutual flux. As in Fig.2.5c e.m.f E_2 is not the secondary terminal voltage, however, because of the secondary resistance and because the secondary current I_2 creates secondary leakage flux. The secondary terminal voltage V_2 differs from the induced voltage E_2 by the voltage drops due to secondary resistance R_2 and secondary leakage reactance X_{l2}, as in the portion of the equivalent circuit (Fig.2.5c) to the right of E_2. In Fig.2.5d the referred values are indicated with primes, for example, X'_{l2} and R'_2, to distinguish them from the actual values of Fig.2.5c. The circuit of Fig.2.5d is called the equivalent T circuit for a transfer.

Example 2.2

A 50kVA 2400: 240V 60Hz distribution transformer has a leakage impedance of 0.72 + j0.92Ω in the high-voltage winding and 0.007 + j 0.009Ω in the low-voltage winding. At rated voltage and frequency, the admittance Y_ϕ of the shunt branch accounting for the exciting current is (0.324 – j2.24) x 10^{-2} mho when viewed from the low-voltage side. Draw the equivalent circuit referred to (a) the high-voltage and (b) the low-voltage side and labeled the impedance numerically.

Solution

Used as step-down transformer

(a) $$\frac{N_1}{N_2} = \frac{2400}{240} = 10$$

Since $\dfrac{Z'_{12}}{Z_{12}} = \left(\dfrac{N_1}{N_2}\right)^2$

$$Z'_{12} = (0.007 + j0.009)(10)^2$$

$$= (0.7 + j0.9)\Omega$$

$$\frac{Y'_\phi}{Y_\phi} = \left(\frac{N_2}{N_1}\right)^2$$

$$Y'_\phi = (0.324 - j2.24) \times 10^{-2} \times (1/10)^2$$

$$= (0.324 - j\,2.24) \times 10^{-4} \text{ mho.}$$

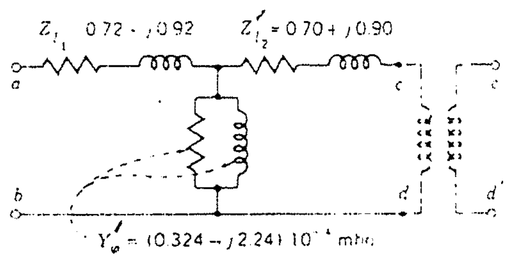


Fig.(a) The equivalent circuit referred to high-voltage side

(b) $$\frac{Z'_{11}}{Z_{11}} = \left(\frac{N_2}{N_1}\right)^2 = \left(\frac{1}{10}\right)^2$$

$$Z'_{11} = (0.0072 + j0.0092)\Omega$$

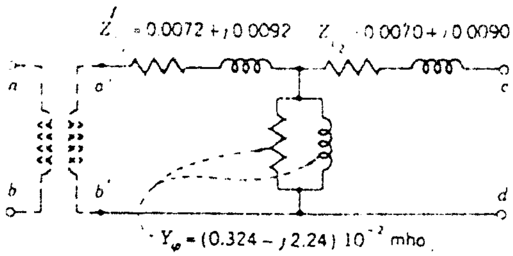


Fig. (b) The equivalent circuit referred to low-voltage side

2.5 APPROXIMATE EQUIVALENT CIRCUITS, POWER TRANSFORMERS

The approximate equivalent circuits commonly used for constant-frequency power transformer analyses are summarized for comparison in Fig.2.6. All quantities in these circuits are referred to either the primary or secondary, and the ideal transformer is not shown.

The computational labor involved often can be appreciably reduced by moving the shunt branch representing the exciting current out from the middle of the T circuit to either the primary or the secondary terminal, as in Fig.2.6 a and b.

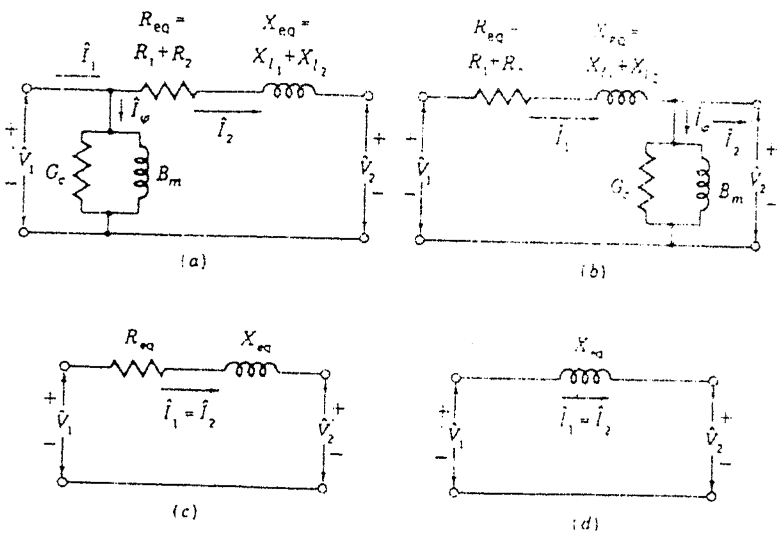


Fig.2.6. Approximate transformer equivalent circuits

Neglecting the exciting current entirely, as in Fig.2.6.c, in which the transformer is represented as an equivalent series impedance. If the transformer is large, the equivalent resistance R_{eq} is small compared, with the equivalent resistance X_{cq} and can frequently be neglected, giving Fig.2.6.d. The circuit of Fig.2.6.c and d are sufficiently accurate for most ordinary power system problems.

Example 2.3

The 50 kVA 2400:240V transformer whose constants are given in Example 2.2 is used to step down the voltage at the load end of a feeder whose impedance is 0.3 + j1.6Ω. The voltage V_s at the sending end of the feeder is 2400 V.

Find the voltage at the secondary terminals of the transformer when the load connected to is secondary draws rated current from the transformer and the power factor of the load is 0.8 lagging. Neglect the voltage drops in the transformer and feeder caused by the exciting current.

Solution

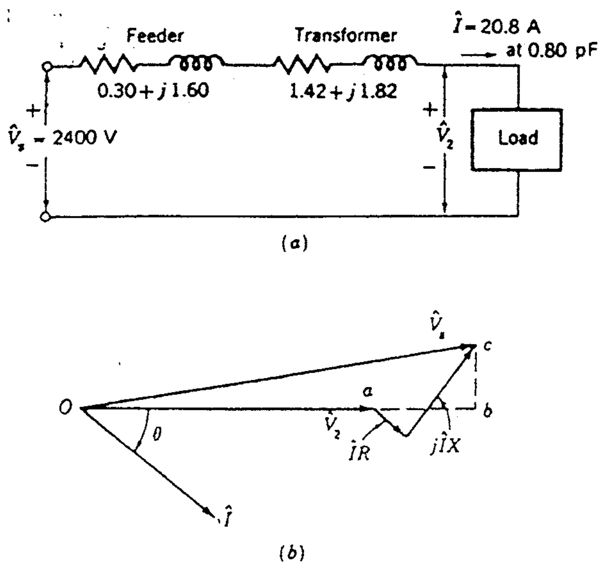


Fig. 2.7 equivalent circuit and phasor diagram

From example 2.2, the equivalent circuit referred to high voltage side is

$Z_{eqt\,1} = Z_{1\,1} + Z'_{1\,2} = 1.42 + j1.82\,\Omega$

$I_{H.V}$ = rated KVA/rated voltage

$$= \frac{50 \times 10^3}{2400} = 20.8\,A$$

$$\therefore\ V_s = I\,(Z_F + Z_{eqt\,1}) + V_2$$

$$= I(Z) + V_2$$

$$= I\,(R + jX) + V_2$$

from phasor diagram,

$$Ob = \sqrt{V_s^2 - (bc)^2} \quad \text{and } V_2 = Ob - ab$$

Note that

$$bc = IX\cos\theta - IR\sin\theta \qquad\qquad ab = IR\cos\theta + IX\sin\theta$$

where R and X are the combined resistance and reactance, respectively.

Thus

$$bc = 20.8\ (3.42)\ (0.80) - 20.8\ (1.72)\ (0.60) = 35.5\ \text{V}$$

$$ab = 20.8\ (1.72)\ (0.80) + 20.8\ (3.42)\ (0.60) = 71.4\ \text{V}$$

Substitution of numerical values shows that Ob very nearly equals V_s, or 2400 V. Then V_2 = 2329 v is referred to high voltage side. The actual voltage at the secondary terminals is 2329/10, or

$$V_2 = 233\ \text{V}$$

2.6 SHORT-CIRCUIT AND OPEN-CIRCUIT TESTS

Two very simple tests serve to determine the power losses in a transformer. These consist of measuring the input voltage, current, and power to the primary, first with the secondary short-circuit and then with the secondary open-circuit.

2.6.1 Short-Circuit Test

From short-circuit test we can calculate R_{eq}t, X_{eqt} and copper loss connect as shown in Fig. And increase the power supply voltage to the rated current of the high voltage side of the transformer. The High Voltage-side is usually taken as the primary in this test. (i.e Low-Voltage side is short-circuit).

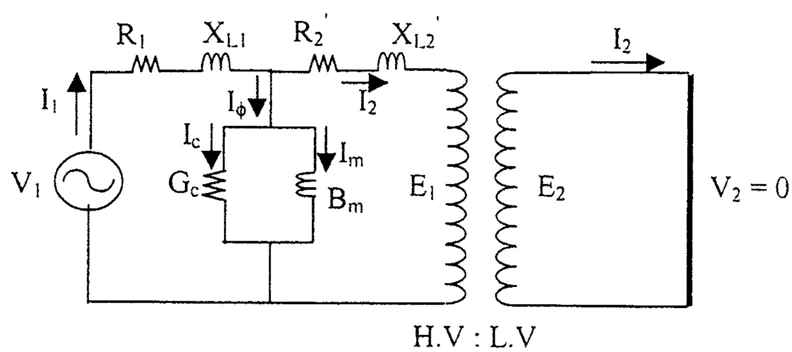


Fig (2.8) Equivalent circuit with short-circuit secondary

Since $V_2 = E_2 = 0$

$$\therefore E_1 = 0 \left(\frac{E_1}{E_2} = \frac{N_1}{N_2} \right)$$

$\therefore$ The equivalent circuit of Fig.2.8 becomes.