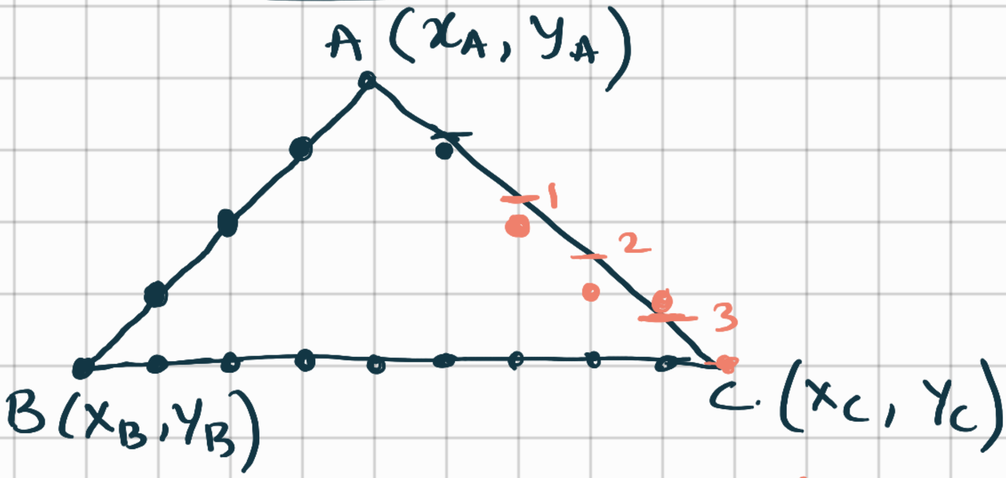
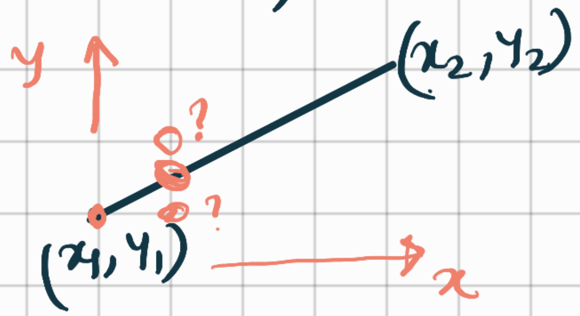
Scan Conversion.





$$y = mx + b$$

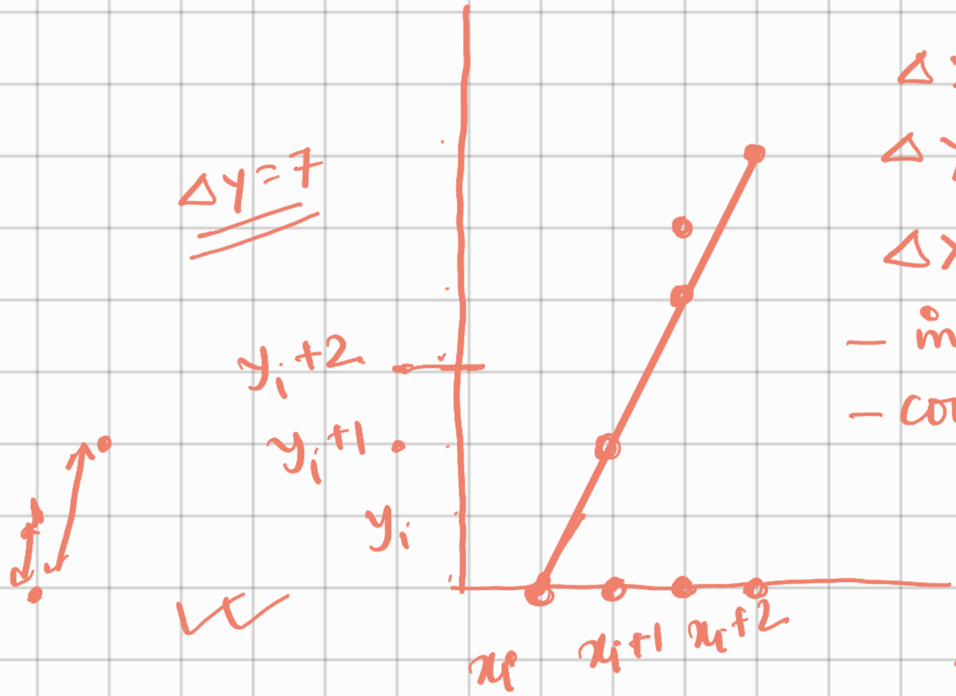
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$b = \frac{y_1 x_2 - y_2 x_1}{x_2 - x_1}$$

$$\boxed{\begin{aligned} x_{i+1} &= x_i + 1 \\ y_{i+1} &= \frac{y_2 - y_1}{x_2 - x_1} x_{i+1} + \frac{y_1 x_2 - y_2 x_1}{x_2 - x_1} \end{aligned}}$$

$(y_i - y_{i+1})) \geq 0.5 \quad y_{i+1} = y_i + 1$

else $\quad y_{i+1} = y_i$



Δx

Δy

$\Delta x > \Delta y \Rightarrow$

- increment x by 1
- compute y

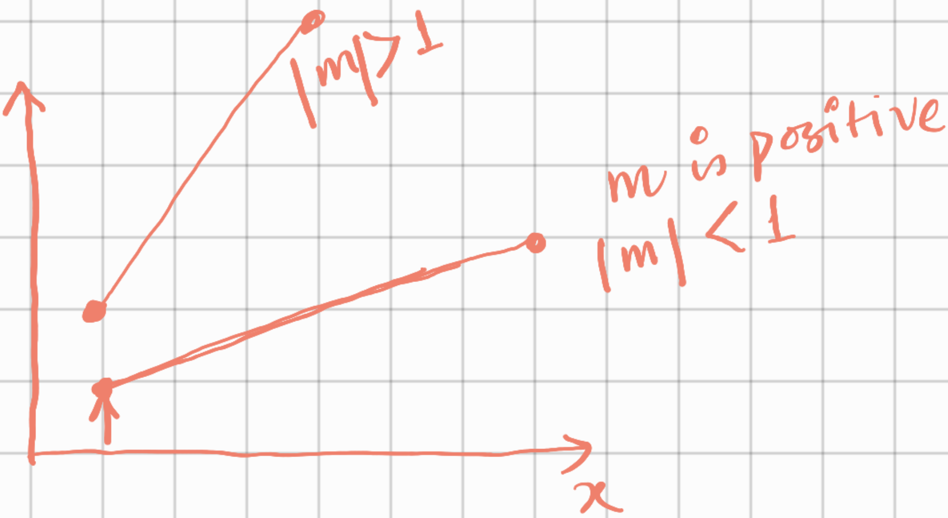
$\Delta y > \Delta x \Rightarrow$

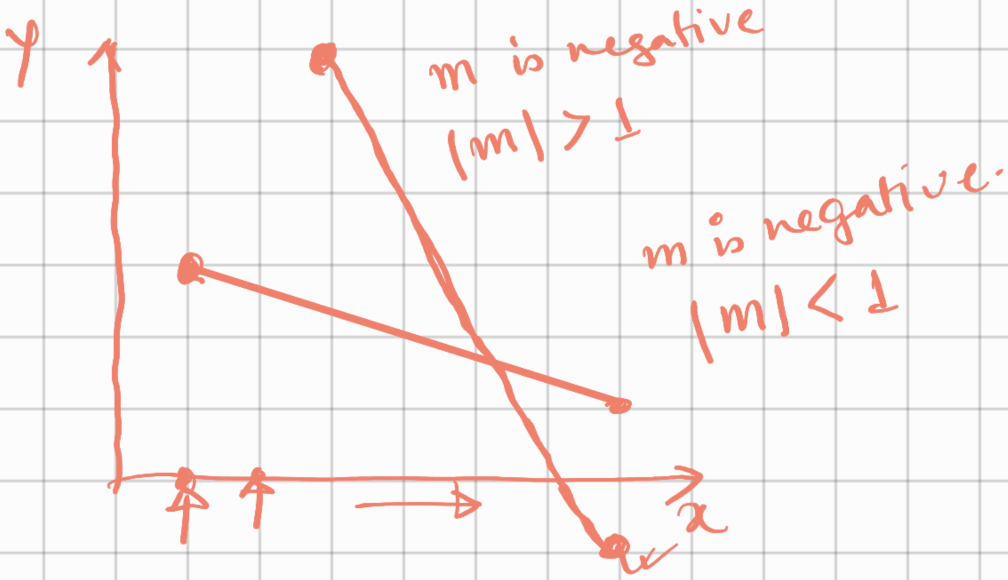
→ increment y by 1
→ compute x

→ Line.
→ Circle.
→ Ellipse.

- Polynomial Algo
- DDA Algo
- Bresenham's Algo
- Mid-point Algo

Line.





$|m| > 1$ $\quad |\Delta y| > |\Delta x| \Rightarrow$ positive slop.

decrement / increment y by 1

negative slope

compute x

$|m| < 1$ $\quad |\Delta x| > |\Delta y| \Rightarrow$

increment x by 1.

compute y.

Polynomial Method for Line drawing

$$y = mx + b$$

$|m| < 1$ $\Rightarrow$

$$y_{i+1} = \frac{y_2 - y_1}{x_2 - x_1} x_{i+1} + \frac{y_1 x_2 - y_2 x_1}{x_2 - x_1}$$

+ve slope $\Rightarrow$

$$x_{i+1} = x_i + 1$$

-ve slope $\Rightarrow$ $x_{i+1} = x_i + 1$

$|m| > 1$

$$x_{i+1} = \frac{\left[y_{i+1} - \left(\frac{y_1 x_2 - y_2 x_1}{x_2 - x_1}\right)\right](x_2 - x_1)}{(y_2 - y_1)}$$

+ve slope ⇒

$$y_{i+1} = y_i + 1$$

-ve slope

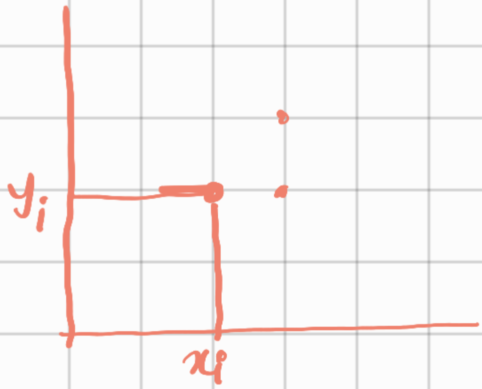
$$y_{i+1} = y_i - 1$$

Digital Difference Analyzes (DDA).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

$$\Delta y = m \, \Delta x$$

$$\Delta x = \frac{1}{m} \Delta y$$



$$y_1 = m x_1 + b$$

$$\boxed{y_i = m x_i + b}$$

$$\boxed{x_{i+1} = x_i + \Delta x}$$

$$y_{i+1} = m \, \underline{x_{i+1}} + b$$

$$= m(x_i + dx) + b$$

$$= \underline{m x_i} + m \, dx \; \underline{+ b}$$

$$= y_i + m \, dx$$

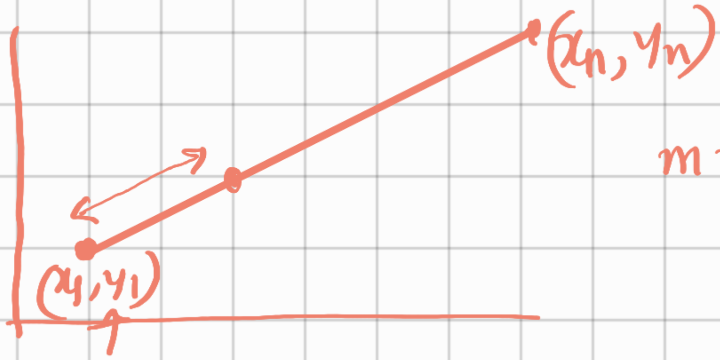
$$x_i = \frac{y_i}{m} - \left(\frac{b}{m}\right)$$

$-d$

$$x_{i+1} = \frac{(y_i + dy)}{m} - d$$

$$= x_i + \frac{1}{m}(\underline{dy})$$

$|m| < 1$



$m = \frac{\Delta y}{\Delta x}$

$y_2 = y_1 + m\,dx$

$y_2 = y_1 + m.$

$y_{i+1} = y_i + m \Rightarrow$ Roundoff( )

$|m| > 1$

$$\boxed{x_{i+1} = x_i + \left(\frac{1}{m}\right)}$$

→ Input of two end points of a line.

① → Draw the line using Polynomial & DDA algorithm.

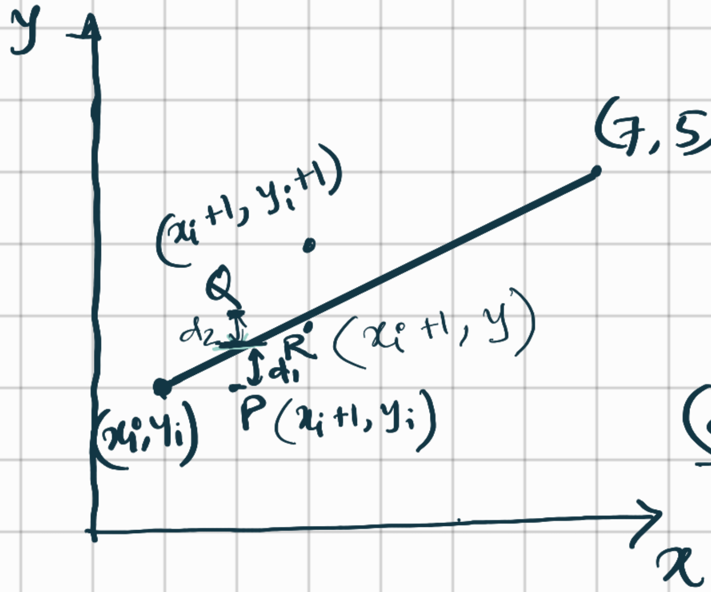
② $|m| > 1 \Rightarrow$ increment X by 1
compute y

$|m| < 1 \rightarrow$ increment Y by 1
compute X

Deadline : 23 st Jan, 2022 / 5 pm.

Bresenham's Algorithm

$\boxed{|m| < 1}$



$(d_1 - d_2) > 0 \rightarrow Q$

$(d_1 - d_2) < 0 \rightarrow P$

(x_i, y_i)

$$y_i = m x_i + b$$

$$y_{i+1} = m x_{i+1} + b$$

$$= m(x_i + 1) + b = y$$

$$d_1 = y - y_i = m(x_i+1) + b - y_i \qquad \left[m = \frac{\Delta y}{\Delta x} \right.$$

$$d_2 = y_i + 1 - y = y_i + 1 - m(x_i + 1) - b$$

$$d_1 - d_2 = 2m(x_i + 1) - 2y_i + 2b - 1 \rightarrow (1)$$

(x_i, y_i) $\quad P_i = \Delta x (d_1 - d_2) = 2\Delta y x_i - 2\Delta x y_i + c \rightarrow (2)$

$$\boxed{c = 2\Delta y + \Delta x(2b - 1)}$$

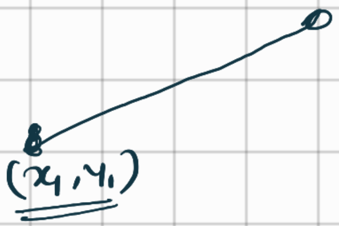
(x_{i+1}, y_{i+1})

$$P_{i+1} = 2\Delta y x_{i+1} - 2\Delta x y_{i+1} + c \rightarrow (3)$$

$$p_{i+1} - p_i = 2\Delta y (x_{i+1} - x_i) - 2\Delta x (y_{i+1} - y_i)$$

$$p_{i+1} = p_i + 2\Delta y - 2\Delta x (y_{i+1} - y_i) \rightarrow (4)$$

(x_1, y_1)



$$\boxed{p_1 = 2\Delta y - \Delta x}$$

$$p_1 = 2\Delta y\, x_1 - 2\Delta x\, y_1 + 2\Delta y + \Delta x(2b-1)$$

$$= 2\Delta y\, x_1 - 2\Delta x\, y_1 + 2\Delta y + \Delta x\left(2\cdot\frac{y_1x_2 - y_2x_1}{x_2-x_1} - 1\right)$$

$$+ 2y_1x_2 - 2y_2x_1 - \Delta x$$

$$= 2(y_2-y_1)x_1 - 2(x_2-x_1)y_1 + 2\Delta y$$

$$+ 2y_1x_2 - 2y_2x_1 - \Delta x$$

$$= 2\Delta y - \Delta x$$

$p_1 > 0$ $\quad x_{i+1} = x_i + 1, \quad y_{i+1} = y_i + 1$

$p < 0$ $\quad x_{i+1} = x_i + 1 \quad y_{i+1} = y_i$

$$p_{i+1} = p_i + 2\Delta y - 2\Delta x$$

$$p_{i+1} = p_i + 2\Delta y$$

Mid Point Line Drawing Algorithm

(x, y)

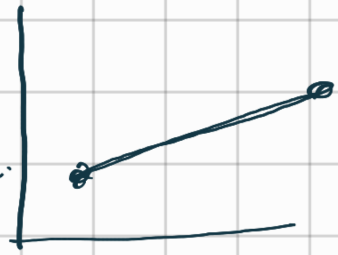
$$f(x,y) = 0$$

$$\Rightarrow \frac{\Delta y}{\Delta x} x - y + d = 0$$

$$\Rightarrow \Delta y\, x - \Delta x\, y + \Delta x\, d = 0$$

$$\Rightarrow ax + by + c = 0$$

where $a = \Delta y$, $b = -\Delta x$
$c = \Delta x\, d$



$f(x,y) < 0 \Rightarrow$ above the line.

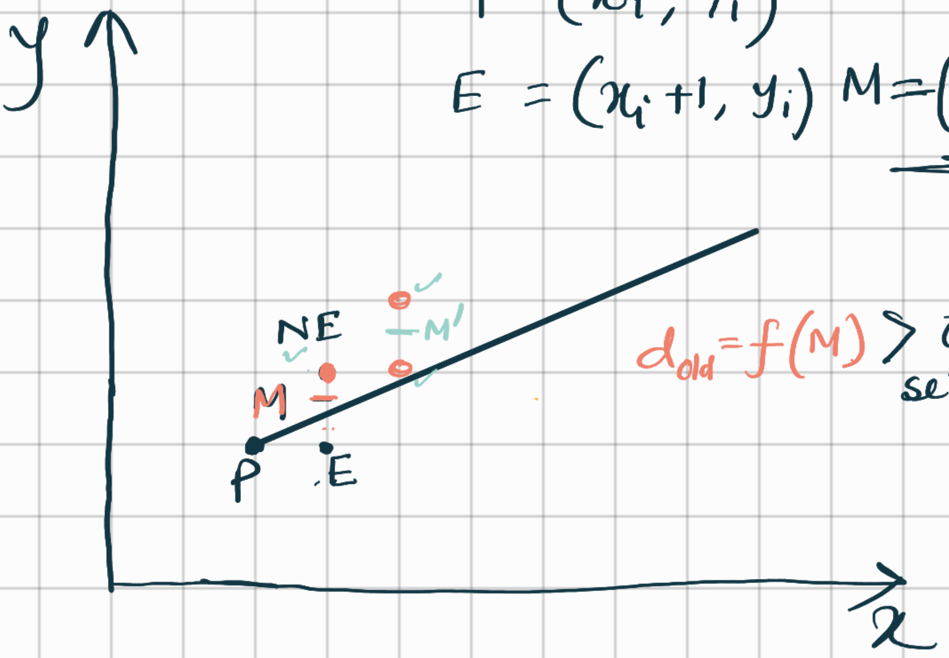
$= 0$ on the line

> 0 below the line.

$P = (x_i, y_i)$ $NE = (x_i + 1, y_i + 1)$

$E = (x_i + 1, y_i)$ $M = (x_i + 1, y_i + \frac{1}{2})$

$|m| < 1$



$f(M) < 0 \Rightarrow$ M is above the line.

Line is closer to E

$f(M) > 0 \Rightarrow$ M is below the line.

Line is closer to NE

$$d_{old} = f\left(x_i + 1, y_i + \frac{1}{2}\right)$$

$$= a(x_i + 1) + b\left(y_i + \frac{1}{2}\right) + C$$

$d_{old} < 0 \Rightarrow$ select E

next mid point will be

$x_i + 2, \ y_i + \frac{1}{2}$

$$d_{new} = f\left(x_i + 2,\ y_i + \tfrac{1}{2}\right)$$

$$= a(x_i + 1) + a + b\left(y_1 + \tfrac{1}{2}\right) + c$$

$$= d_{old} + a$$

$$= d_{old} + \Delta y$$

$d_{old} > 0$ select NE

Next midpoint will be.

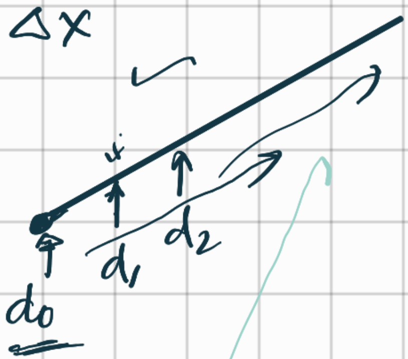
$$\left(x_i + 2,\ y_i + 3/2\right)$$

$$f\left(x_i + 2, y_i + 3/2\right) = a(x_i + 2) + b(y_i + 3/2) + c$$

$$= d_{old} + a + b.$$

$f(x, y)$

$$= d_{old} + \Delta y - \Delta x$$



$$d_0 = f\left(x_1 + 1,\ y_1 + \tfrac{1}{2}\right)$$

$$= ax_1 + a + by_1 + b/2 + c$$

$$= a + b/2$$

$$\boxed{2f(x, y)}$$

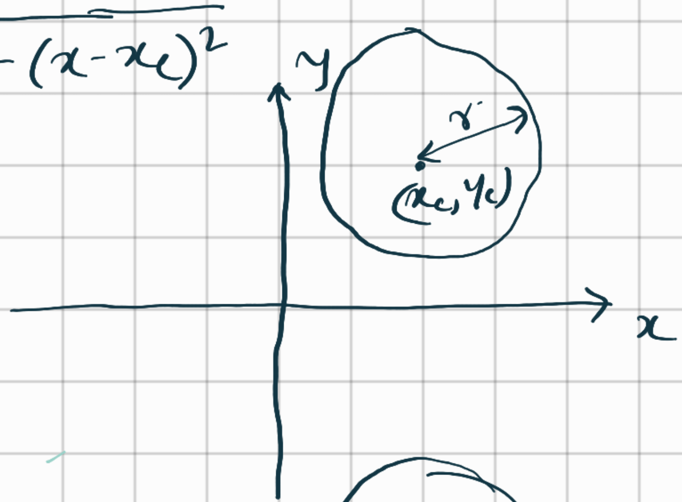
Circle Drawing Algorithm

→ Polynomial Algorithm

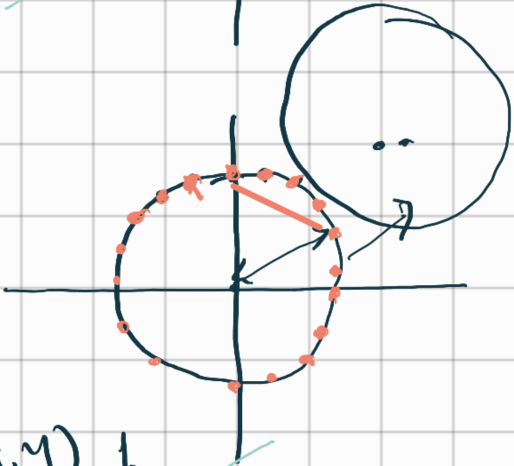
x_c, y_c with r

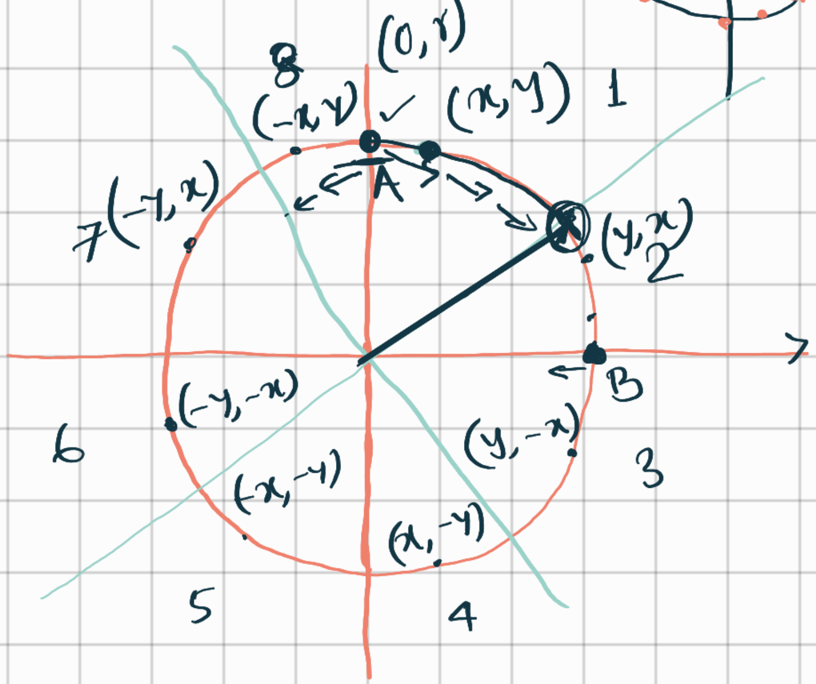
$$(x-x_c)^2+(y-y_c)^2=r^2$$

$$y = y_c \pm \sqrt{r^2-(x-x_c)^2}$$

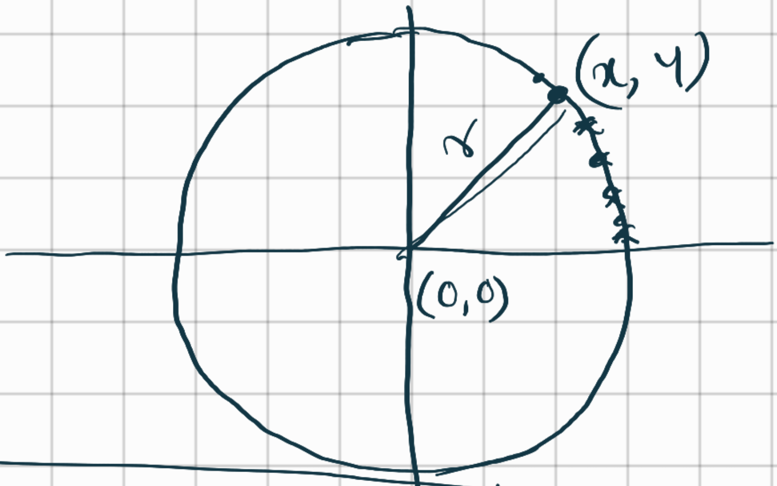


$$y=\sqrt{r^2-x^2}$$





Polar Co-ordinate System



$$\boxed{\cos\delta\theta \approx 1 \quad \sin\delta\theta \approx 0}$$

$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$\boxed{\delta\theta = \frac{1}{\text{radius}(r)}}$$

$\theta = 0 \to 45$

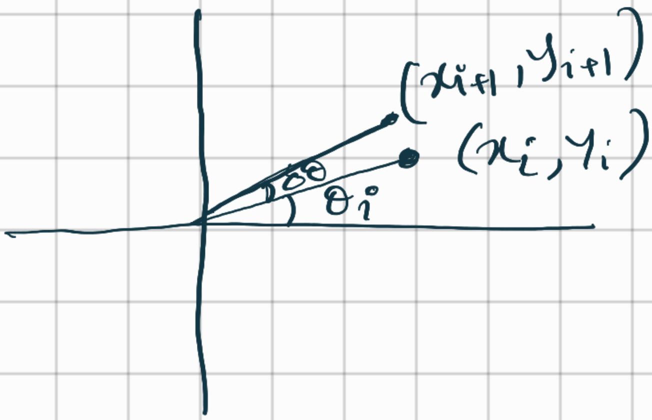
$\delta\theta$

DDA Algorithm

$x = r\cos\theta$ for $0 \leq 2\theta \leq \pi$

$y = r\sin\theta$ for $0 \leq 2\theta \leq \pi$

$(x_i, y_i) \rightarrow$ curve angle is θ_i



$$x_i = r\cos\theta_i$$

$$y_i = r\sin\theta_i$$

$$x_{i+1} = r\cos(\theta_i + \delta\theta)$$

$$y_{i+1} = r\sin(\theta_i + \delta\theta)$$

$\theta \geqslant 45^\circ$

$$x_{i+1} = r\cos\theta_i \cos\delta\theta - r\sin\theta_i \sin\delta\theta$$

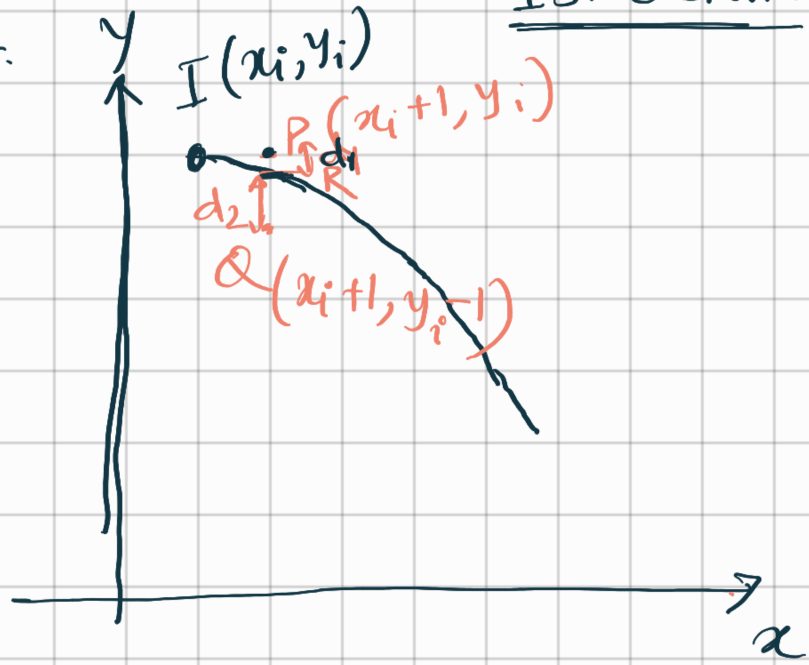
$$= x_i \cos\delta\theta - y_i \sin\delta\theta$$

$$y_{i+1} = y_i \cos\delta\theta + x_i \sin\delta\theta$$

Bresenham's Circle Algorithm

1st octant

$$x^2+y^2=r^2$$



$$d_1-d_2>0$$

$$y=r^2-x^2$$

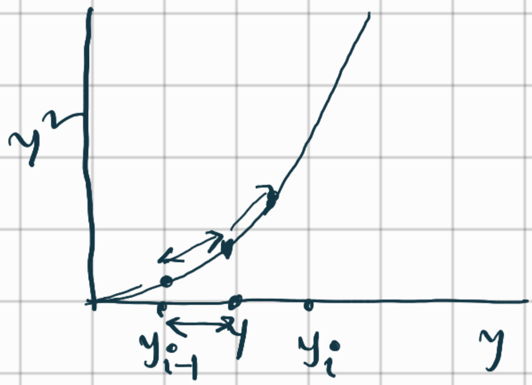
At point $x_{i+1}=x_i+1$ $\quad y_{i+1}=y$.

$$y^2=r^2-(x_i+1)^2$$

$$d_1=y_i^2-y^2=y_i^2-r^2+(x_i+1)^2$$

$$d_2=y^2-(y_i-1)^2$$

$$=r^2-(x_i+1)^2-(y_i-1)^2$$



$\boxed{p_0=?}$

$$\boxed{p_i=d_1-d_2=2(x_i+1)^2+y_i^2+(y_i-1)^2-2r^2}$$

$$p_{i+1}=2(x_{i+1}+1)^2+y_{i+1}^2+(y_{i+1}-1)^2-2r^2$$

$$=2(x_i+1+1)^2+y_{i+1}^2+(y_{i+1}-1)^2-2r^2$$

$[x_{i+1}=x_i+1]$

$$p_{i+1} = p_i + 4x_i + 6 + 2(y_{i+1}^2 - y_i^2) - 2(y_{i+1} - y_i)$$

$p_i < 0$ $\quad x_{i+1} = x_i + 1$

$\qquad\quad y_{i+1} = y_i$

$$\boxed{p_{i=0} = ?}$$

$$\boxed{p_{i+1} = p_i + 4x_i + 6} \quad (1)$$

$p_i > 0$ $\quad x_{i+1} = x_i + 1$

$\qquad\quad y_{i+1} = y_i - 1$

$$\boxed{p_{i+1} = p_i + 4x_i + 10 - 4y_i}$$

$x_0 = 0$ $\quad y_0 = r$

$$P_0 = 2 + r^2 + (r-1)^2 - 2r^2$$

$$\boxed{p_0 = 3 - 2r}$$

Find out the coordinates in 1st octant

→ $\boxed{(5, 3) \text{ with } 8.}$

$p_0 = 3 - 2\times 8$

$=$

$p_0 < 0$ $\quad p_{i+1}$

x	y	x'	y'
0	8	0+5	8+3
1	8	1+5	8+3