1 Vector

◆ Definition

A vector is an ordered list of numbers that represents magnitude and direction in space.

◆ Mathematical Form

$$v = (x_1, x_2, x_3, \ldots, x_n)$$

◆ In Your Project

Each student is represented as a 6-dimensional vector:

$$v = (Math, Physics, Chemistry, English, Computer, Statistics)$$

Vectors allow us to perform mathematical operations on student performance.

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2 Norms (Vector Length)

Norm measures the magnitude of a vector.

◆ L1 Norm (Manhattan Norm)

$$|| v ||_1 = | x_1 | + | x_2 | + \ldots + | x_n |$$

Used to measure total absolute performance.

◆ L2 Norm (Euclidean Norm)

$$|| v ||_2 = \sqrt{x_1^2 + x_2^2 + \ldots + x_n^2}$$

Represents actual geometric length of vector.

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3 Dot Product

◆ Definition

Dot product measures similarity between two vectors.

◆ Formula

$$v \cdot w = x_1 y_1 + x_2 y_2 + \ldots + x_n y_n$$

◆ Interpretation

- Large value → Similar performance
- Zero → Independent (orthogonal)

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4 Cross Product (3D Only)

◆ **Definition**

Produces a vector perpendicular to two 3D vectors.

◆ **Formula**

$$v \times w = \begin{vmatrix} i & j & k \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

Used in geometric interpretation.

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5 Projection of Vector

◆ **Definition**

Projection measures component of one vector in direction of another.

◆ **Formula**

$$proj_w(v) = \frac{v \cdot w}{w \cdot w} w$$

Used in regression and dimensionality reduction.

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6 Matrix

◆ **Definition**

A matrix is a rectangular arrangement of numbers in rows and columns.

◆ **In Your Project**

Your dataset is a 250 × 6 matrix:

- Rows → Students
- Columns → Subjects

Matrix representation allows advanced operations.

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7 Determinant

◆ **Definition**

Determinant is a scalar value that indicates whether matrix is invertible.

If:

$$det(A) \neq 0$$

→ Inverse exists.

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8 Inverse Matrix

◆ **Definition**

Inverse matrix reverses transformation of original matrix.

$$AA^{-1} = I$$

Used in solving linear systems.

9️⃣ Covariance Matrix

🔹 **Definition**

Covariance measures how two variables vary together.

$$Cov(X,Y) = E[(X - \bar{X})(Y - \bar{Y})]$$

Covariance matrix shows relationship between all subjects.

🔟 Eigenvalues & Eigenvectors

🔹 **Definition**

Eigenvectors represent directions of maximum variance.
Eigenvalues represent magnitude of variance.

🔹 **Formula**

$$Av = \lambda v$$

Very important in PCA.

1️⃣1️⃣ LU Decomposition

🔹 **Definition**

Factorizes matrix into lower and upper triangular matrices.

$$A = LU$$

Used to simplify solving linear equations.

1️⃣2️⃣ Singular Value Decomposition (SVD)

🔹 **Definition**

Decomposes matrix into three matrices.

$$A = USV^T$$

Used in dimensionality reduction and pattern extraction.

1️⃣3️⃣ Principal Component Analysis (PCA)

🔹 **Definition**

PCA reduces dimensionality by projecting data onto new orthogonal axes that maximize variance.

🔹 **Steps:**

1. Compute covariance matrix
2. Find eigenvalues and eigenvectors
3. Select top components

1️⃣4️⃣ Linear Discriminant Analysis (LDA)

🔹 Definition

LDA finds a hyperplane that maximizes separation between classes.

🔹 Formula

$$w^T x + b = 0$$

Used for classification.

1️⃣5️⃣ Line, Plane, Hyperplane

🔹 Line (2D)

$$y = mx + c$$

🔹 Plane (3D)

$$z = ax + by + d$$

🔹 Hyperplane (n-Dimension)

$$w_1 x_1 + w_2 x_2 + \ldots + w_n x_n + b = 0$$

Used as decision boundary in ML models.

1️⃣6️⃣ Dimensionality Concept

As number of features increases:

- 2 features → 2D space
- 3 features → 3D space
- n features → n-dimensional space

Higher dimensions require reduction techniques like PCA.