STAT 410

Examples for Week 01 (1)

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Functions of One Random Variable

Let X be a continuous random variable.

Let $Y = g(X)$. What is the probability distribution of Y ?

Cumulative Distribution Function approach:

$$F_Y(y) = P(Y \le y) = P(g(X) \le y) = \int_{\{x:\, g(x) \le y\}} f_X(x)\, dx = \ldots$$

Moment-Generating Function approach:

$$M_Y(t) = E(e^{Y \cdot t}) = E(e^{g(X) \cdot t}) = \int_{-\infty}^{\infty} e^{g(x) \cdot t} f_X(x)\, dx = \ldots$$

1. Let U be a Uniform$(0, 1)$ random variable:

$$f_U(u) = \begin{cases} 1 & 0 < u < 1 \\ 0 & \text{o.w.} \end{cases} \qquad F_U(u) = \begin{cases} 0 & u < 0 \\ u & 0 \le u < 1 \\ 1 & u \ge 1 \end{cases}$$

Consider $Y = U^2$. What is the probability distribution of Y ?

Change-of-Variable Technique:

Theorem 1.7.1 X – continuous r.v. with p.d.f. $f_X(x)$

$Y = g(X)$ $g(x)$ – one-to-one, differentiable

$$\frac{dx}{dy} = \frac{d}{dy}\, g^{-1}(y)$$

$$\Rightarrow \qquad f_Y(y) = f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right|$$

1. Let U be a Uniform$(0, 1)$ random variable:

Consider $Y = U^2$. What is the probability distribution of Y?

2. Consider a continuous random variable X with p.d.f.

$$f_X(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{o.w.} \end{cases}$$

a) Find the probability distribution of $Y = \sqrt{X}$.

b) Find the probability distribution of $W = \dfrac{1}{X+1}$.

3. Consider a continuous random variable X with the p.d.f. $f_X(x) = \dfrac{24}{x^4}$, $x > 2$.

Find the probability distribution of $Y = 1/X^2$.