

min/max-Heap

Pop() =

Push() = $O(1)$

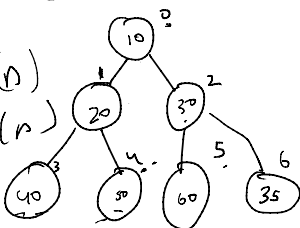
add() = $\log(n)$

remove() = $\log(n)$

Size() = $O(1)$

min/max Priority Queue

[10, 20, 30, 40, 50, 60, 35,]



$$\begin{aligned} l_i &= 2 \times i + 1 \\ r_i &= 2 \times i + 2 \\ p_i &= \frac{(i-1)}{2} \end{aligned}$$

heapify
up down

- Tree should be CBT till $(h-1)$
- Parent should always be smaller than both children

Pop {
AL[INT] data =

upheapify(i) {

$p_i = (i-1)/2$

if $(i \geq 0 \ \& \ data[i] < data[p_i])$ {

swap(i, p_i);

upheapify(p_i);

}

add(d) {

data.add(d);

upheapify(i);

}

downheapify(i) {

$res = i$

$l_i = 2 \times i + 1$

if $(l_i < data.size \ \& \ data[res] > data[l_i])$

$res = l_i$

if $(r_i < data.size \ \& \ data[res] > data[r_i])$

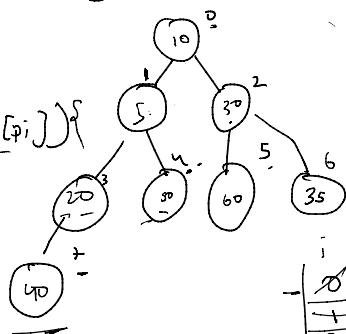
$res = r_i$

if $(res == i)$ return;

Swap(res, i)

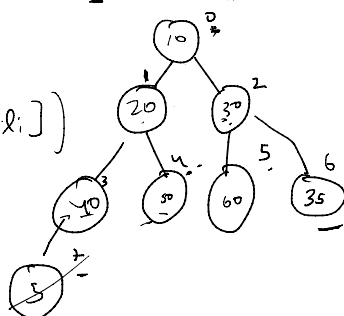
if $(res < data.size)$ downheapify(res);

[10, 20, 30, 40, 50, 60, 35, 5]



| i | p _i |
|---|----------------|
| 0 | 0 |
| 1 | 0 |
| 2 | 1 |
| 3 | 1 |
| 4 | 2 |
| 5 | 2 |
| 6 | 3 |
| 7 | 3 |

[40, 10, 30, 20, 50, 60, 35, 5]



$$\begin{aligned} l_i &= 2 \times i + 1 \\ r_i &= 2 \times i + 2 \\ p_i &= \frac{(i-1)}{2} \end{aligned}$$

| i | res | l _i | r _i |
|---|-----|----------------|----------------|
| 0 | 0 | 1 | 2 |
| 1 | 1 | 3 | 4 |
| 2 | 2 | 5 | 6 |
| 3 | 3 | 7 | 8 |

7

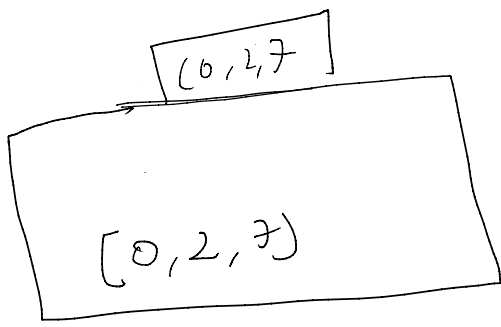
```

remove() {
    res = dat[0];
    swap(0, size-1);
    downHeapify;
    return res;
}

```

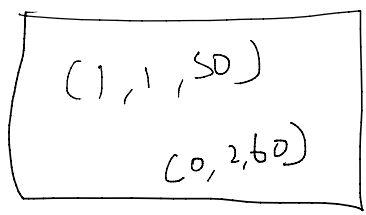
K - sorted lists
 ([1, 2, 3]), [4, 5, 6], [7, 8, 9])
 0 1 2
 [1, 2, 3, 4, 5, 6]

Data {
idx;
lidx;
value;



1 - 4 = -3
 4 - 1 = 3

{ [2, 3], [4, 5], [6, 7] } (0, 0, 1) - (0, 1, 1)
 0 1 2



d = (0, 1, 70)
 on d, val

```

public static ArrayList<Integer> mergeKSortedLists(int[][] arr) {
    ArrayList<Integer> res = new ArrayList<>();
    java.util.PriorityQueue<DataIndex> pq = new java.util.PriorityQueue<>();
    for (int i = 0; i < arr.length; i++)
        pq.add(new DataIndex( idx: 0, i, arr[i][0]));

    while (!pq.isEmpty()) {
        DataIndex d = pq.remove();
        res.add(d.val);
        if (d.idx + 1 < arr[d.lidx].length)
            pq.add(new DataIndex( idx: d.idx + 1, d.lidx, arr[d.lidx][d.idx+1]));
    }
    return res;
}

```

res = [20, 30, 40]