Acoustic Streaming in a Microchannel Cross Section

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Abstract

Acoustic streaming is a nonlinear phenomenon that arises due to the viscous dissipation of acoustic waves in a fluid medium, leading to a steady flow. This project focuses on modeling and analyzing acoustic streaming in a two-dimensional microchannel cross section. Using analytical theory and simulation results from COMSOL Multiphysics, we investigate the generation of acoustic streaming rolls, temperature gradients, and particle transport under the influence of the acoustic radiation and viscous drag forces. The results demonstrate how microscale streaming effects can be harnessed for particle manipulation in lab-on-a-chip and biomedical applications.

1. Introduction

Acoustic streaming refers to the steady flow induced in a fluid medium due to the nonlinear interaction of oscillatory acoustic waves and viscous effects. It plays a critical role in microfluidic applications where precise control over fluid and particle motion is required, such as in cell sorting, mixing, and targeted delivery.

In confined geometries like microchannels, boundary-layer driven acoustic streaming (Rayleigh streaming) emerges due to the dissipation of acoustic energy in the thin viscous and thermal boundary layers adjacent to rigid walls. This results in characteristic flow patterns that can be analytically predicted and numerically simulated.

The focus of this study is to understand and analyze acoustic streaming within a rectangular microchannel cross section. Both analytical formulations and numerical simulation (performed in COMSOL Multiphysics) are used to investigate the acoustic pressure field, the resulting steady streaming velocity field, and the transport of microparticles due to the combined effects of acoustic radiation forces and viscous drag.

This report aims to build a comprehensive understanding of the mechanisms underlying acoustic streaming and demonstrate the alignment of analytical models with numerical results.

2. Fundamental Concepts

2.1 Acoustic Fields in Fluids

Acoustic waves are small perturbations in pressure, density, and velocity propagating through a compressible medium. These perturbations are typically modeled using the linearized Navier–Stokes, continuity, and energy equations. Assuming harmonic time dependence, the pressure field can be written as:

$$p(\mathbf{r}, t) = p_0(\mathbf{r})e^{i\omega t}$$

where p_0 is the complex amplitude, ω is the angular frequency, and $\mathbf{r}$ is the position vector.

2.2 Viscous and Thermal Boundary Layers

In microscale acoustics, boundary layers form near solid-fluid interfaces where viscous and thermal effects become significant. These layers have characteristic thicknesses:

$$\delta_\nu = \sqrt{\frac{2\mu}{\rho\omega}}, \quad \delta_T = \sqrt{\frac{2k}{\rho c_p \omega}}$$

where:

- μ is the dynamic viscosity,
- ρ is the fluid density,
- ω is the angular frequency,
- k is the thermal conductivity,
- c_p is the specific heat capacity at constant pressure.

These thin layers play a key role in the generation of acoustic streaming.

2.3 Nonlinear Effects and Time Averaging

Though the acoustic field is described by first-order (linear) equations, the second-order nonlinear effects give rise to phenomena like acoustic streaming. Time-averaging the nonlinear terms in the Navier–Stokes equations reveals steady forces and flows.

For instance, the time-averaged momentum equation for the second-order flow $\langle \mathbf{v}_2 \rangle$ includes a body force derived from the time-averaged Reynolds stress tensor:

$$\langle \rho \mathbf{v}_1 \cdot \nabla \mathbf{v}_1 \rangle$$

This term acts as a driving force for the steady streaming flow.

2.4 Acoustic Streaming

Acoustic streaming is the steady, time-averaged flow resulting from the interaction of acoustic fields with fluid viscosity. Two primary types exist:

- **Bulk-driven streaming** (e.g., Eckart streaming): occurs in the fluid bulk due to attenuation of the acoustic wave.Eckart streaming induced from viscous damping in the domain of the fluid. Eckart streaming is often important for traveling acoustic fields.

- **Boundary-driven streaming** (e.g., Rayleigh streaming): arises due to shear in the viscous boundary layers near rigid walls.Rayleigh streaming induced by stresses in the viscous boundary layers. Rayleigh streaming is well known and is often dominant in microfluidic systems with a standing pressure field.

In microchannels, Rayleigh streaming dominates and forms symmetric vortices or rolls due to the reflection of acoustic waves off hard boundaries.

2.5 Acoustic Radiation Force

Microparticles suspended in an acoustic field experience a net force due to the acoustic radiation pressure. For small spherical particles, the primary radiation force is given by:

$$\mathbf{F}_{\mathrm{rad}} = -\nabla U_{\mathrm{ac}}$$

$$U_{\mathrm{ac}} = \frac{4\pi a^3}{3}\left(\frac{1}{2} f_1 \kappa_0 |p|^2 - \frac{3}{4} f_2 \rho_0 |v|^2\right)$$

where:

- a is particle radius,
- p is the acoustic pressure,
- v is the acoustic velocity,
- f_1, f_2 are contrast factors depending on the properties of the particle and fluid,
- κ_0 is the compressibility of the fluid,
- ρ_0 is the fluid density.

This force interacts with the drag force due to the streaming flow, determining the trajectory of the particle.

3. Acoustic Streaming: Theory

3.1 Acoustic Streaming from Thermoviscous Acoustics

The multiphysics interface Acoustic Streaming from Thermoviscous Acoustics couples the Thermoviscous Acoustics, Frequency Domain interface with a Laminar Flow interface. In the thermoviscous acoustic interface, the viscous and thermal boundary layers are resolved numerically and therefore their streaming contribution is part of the Acoustic Streaming Domain Coupling. Therefore, it is important that the mesh has a good resolution of the boundary layers. The Acoustic Streaming Domain Coupling includes all the effects from the viscous stresses both in the boundary layers and in the domain of the fluid and is therefore often the dominant source of acoustic streaming when coupling from thermoviscous acoustics. The Acoustic Streaming Boundary Coupling only contains the Stokes-slip contribution and is only important if the boundary of the fluid-solid interfaces is vibrating.

3.2 Acoustic Streaming Domain Coupling

The Acoustic Streaming Domain Coupling () is a multiphysics coupling from an acoustic interface to a fluid flow (CFD) model, used to add the domain source contributions necessary to model an acoustic streaming flow. The multiphysics coupling should be used in combination with the Acoustic Streaming Boundary Coupling to ensure that all sources are modeled.The forces applied on the fluid flow depends on the derivatives of the acoustic fields. Therefore, it is recommended to use quadratic element order for the acoustic pressure, and when coupling to Thermoviscous Acoustics, Frequency Domain use cubic element order for the acoustic velocity and temperature.

3.3 Acoustic Streaming Boundary Coupling

The Acoustic Streaming Boundary Coupling () is a multiphysics coupling from an acoustic interface to a fluid flow (CFD) model, used to add the boundary source contributions necessary to model an acoustic streaming flow. The multiphysics coupling should be used in combination with the Acoustic Streaming Domain Coupling to ensure that all sources are modeled. The coupling is between either the Thermoviscous Acoustics, Frequency Domain or the Pressure Acoustics, Frequency Domain and a fluid flow interface. When Pressure Acoustics, Frequency Domain is used as the acoustic source interface, the Fluid model should be set to the Thermally conducting and viscous option (see Thermally Conducting and/or Viscous Fluid Model) and the Thermoviscous Boundary Layer Impedance condition is best used to include boundary layer losses. In situations with highly curved or sharp edges, when the boundary layer impedance formulation is not valid, the acoustic interface should be changed to thermoviscous acoustics instead of pressure acoustics.

4. Acoustic Streaming: Physics

4.1 Perturbation Approach

Acoustic streaming is a second-order effect derived from the nonlinear nature of the Navier–Stokes equations. The fluid variables are expanded as perturbation series:

$$\begin{aligned}
\mathbf{v} &= \varepsilon \mathbf{v}_1 + \varepsilon^2 \mathbf{v}_2 + \dots \\
p &= p_0 + \varepsilon p_1 + \varepsilon^2 p_2 + \dots \\
\rho &= \rho_0 + \varepsilon \rho_1 + \varepsilon^2 \rho_2 + \dots
\end{aligned}$$

where $\varepsilon \ll 1$ is a small parameter representing the amplitude of the acoustic field. The first-order terms represent the primary acoustic field, while the second-order terms give rise to a steady flow—acoustic streaming.

4.2 First-Order Acoustic Field

The first-order velocity $\mathbf{v}_1$ and pressure p_1 satisfy the linearized momentum and continuity equations:

$$\rho_0 \frac{\partial \mathbf{v}_1}{\partial t} = -\nabla p_1 + \mu \nabla^2 \mathbf{v}_1$$

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \nabla \cdot \mathbf{v}_1 = 0$$

These describe a time-harmonic wave solution, often solved using the Helmholtz equation for pressure.

4.3 Second-Order Streaming Flow

Time-averaging the second-order momentum equation leads to:

$$\rho_0\langle \mathbf{v}_1 \cdot \nabla \mathbf{v}_1 \rangle = -\nabla p_2 + \mu \nabla^2 \langle \mathbf{v}_2 \rangle$$

This equation describes the steady, incompressible streaming flow $\langle \mathbf{v}_2 \rangle$, driven by non-linear Reynolds stresses from the oscillatory field $\mathbf{v}_1$.

4.4 Rayleigh Streaming Near Walls

For a plane standing wave between two parallel plates, the boundary-layer driven streaming velocity (Rayleigh streaming) is approximated by:

$$\mathbf{v}_{\text{stream}} = \frac{3}{4}\frac{v_0^2}{c}\left(\frac{\delta_\nu}{L}\right)\sin(2kx)\,\hat{y}$$

where:

- v_0 is the oscillatory velocity amplitude,
- c is the speed of sound,
- δ_ν is the viscous boundary layer thickness,
- L is the distance between plates,
- $k = \frac{\pi}{W}$ is the wave number for width W.

This expression shows the quadrupole-like roll structures known as Rayleigh streaming rolls.

4.5 Thermoviscous Boundary Effects

In microchannels, the viscous and thermal boundary layers are thin compared to the wavelength. Their effects are incorporated analytically through the **Thermoviscous Boundary Layer Impedance (BLI)**. It modifies:

- The damping of the acoustic field,
- The slip velocity at boundaries,
- The boundary heat source term for temperature rise.

4.6 Acoustic Heating

Energy dissipated in boundary layers leads to heating. The total acoustic power dissipation per unit area at a boundary is given by:

$$Q_{\text{tot}} = \mathrm{Re}\left[\mathbf{v}_0 \cdot \mathbf{n} \cdot (\sigma_1 \cdot \mathbf{n})^*\right]$$

This heat source contributes to a small temperature increase, especially in regions with poor thermal conductivity (e.g., glass lids in microfluidic chips).

4.7 Forces on Particles

Microparticles in the channel experience two primary forces:

1. **Acoustic Radiation Force**:

$$\mathbf{F}_{\text{rad}} = -\nabla U_{\text{ac}}, \quad U_{\text{ac}} = \frac{4\pi a^3}{3}\left(\frac{1}{2}f_1\kappa_0|p|^2 - \frac{3}{4}f_2\rho_0|v|^2\right)$$

2. **Viscous Drag Force** from streaming:

$$\mathbf{F}_{\text{drag}} = 6\pi\mu a\left(\mathbf{v}_{\text{stream}} - \mathbf{v}_{\text{particle}}\right)$$

The interplay of these forces governs the trajectory of particles, enabling particle focusing or separation depending on their size and material properties.

5. Analytical Formulation

5.1 First-Order Acoustic Field: Helmholtz Equation

The acoustic pressure p_1 in a homogeneous, isotropic fluid medium satisfies the linearized wave equation. For harmonic time dependence $e^{i\omega t}$, this reduces to the Helmholtz equation:

$$\nabla^2 p_1 + k^2 p_1 = 0$$

where $k = \frac{\omega}{c}$ is the acoustic wave number, ω is the angular frequency, and c is the speed of sound.

For a standing wave in a rectangular microchannel of width W, we can assume the pressure field as:

$$p_1(x) = p_0 \cos(kx), \quad k = \frac{n\pi}{W}$$

where n is the mode number (usually $n = 1$ for the fundamental half-wave mode).

5.2 Viscous and Thermal Boundary Layer Thickness

The characteristic thicknesses of the viscous and thermal boundary layers are given by:

$$\delta_\nu = \sqrt{\frac{2\mu}{\rho\omega}}, \quad \delta_T = \sqrt{\frac{2k}{\rho c_p \omega}}$$

These thin boundary layers influence both the streaming flow and the acoustic energy dissipation, especially near solid walls.

5.3 Rayleigh Streaming Velocity

In the region just outside the viscous boundary layer, the second-order streaming velocity due to Rayleigh streaming for a 1D standing wave is:

$$v_{\text{stream}}(y) = \frac{3}{4}\frac{v_0^2}{c}\left(\frac{\delta_\nu}{H}\right)\sin(2kx)$$

where:

- v_0 is the oscillatory velocity amplitude at the boundary,
- c is the speed of sound,
- δ_ν is the viscous boundary layer thickness,
- H is the microchannel height,
- x is the position along the channel.

5.4 Acoustic Radiation Force on Particles

Particles in the acoustic field experience a net force due to radiation pressure. For small spherical particles ($a \ll \lambda$), the acoustic potential energy is:

$$U_{\mathrm{ac}} = \frac{4\pi a^3}{3}\left(\frac{1}{2}f_1\kappa_0|p_1|^2 - \frac{3}{4}f_2\rho_0|v_1|^2\right)$$

The acoustic radiation force is then:

$$\mathbf{F}_{\mathrm{rad}} = -\nabla U_{\mathrm{ac}}$$

where:

- a: particle radius,
- κ_0: fluid compressibility,
- ρ_0: fluid density,
- p_1, v_1: first-order pressure and velocity amplitudes,
- f_1, f_2: contrast factors dependent on material properties of the particle and fluid.

5.5 Drag Force from Streaming Flow

The particle is also subject to a viscous drag force due to the relative motion between the particle and the streaming flow:

$$\mathbf{F}_{\mathrm{drag}} = 6\pi\mu a(\mathbf{v}_{\mathrm{stream}} - \mathbf{v}_{\mathrm{particle}})$$

This force is linear in the velocity difference and opposes motion.

5.6 Equations of Motion for a Particle

Neglecting particle inertia (low Reynolds number), the motion is governed by a force balance:

$$\mathbf{F}_{\mathrm{rad}} + \mathbf{F}_{\mathrm{drag}} = 0$$

Solving this gives the instantaneous velocity of a particle:

$$\mathbf{v}_{\mathrm{particle}} = \mathbf{v}_{\mathrm{stream}} + \frac{1}{6\pi\mu a}\mathbf{F}_{\mathrm{rad}}$$

This is the key result for computing particle trajectories under acoustic streaming.

6. Problem Description and Geometry

6.1 Microchannel Geometry

The system consists of a two-dimensional cross section of a microchannel, modeled as a rectangular fluid domain. The geometry is defined as:

- Width $W = 380\,\mu\text{m}$
- Height $H = 160\,\mu\text{m}$

The walls of the channel are assumed to be acoustically hard, except for one vibrating wall that introduces the acoustic excitation.

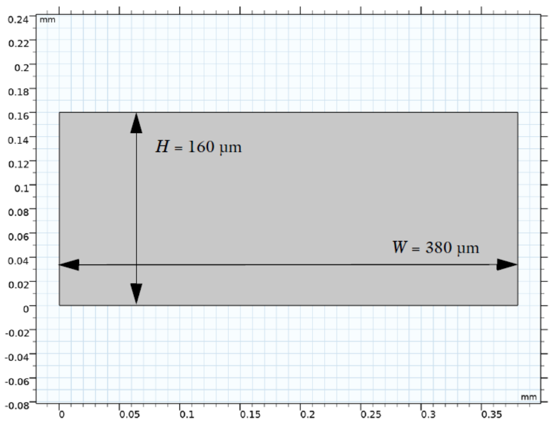

Figure 1: 2D cross section of the microchannel.

6.2 Material Properties

The fluid medium is assumed to be water at room temperature. The relevant physical parameters are:

- Density $\rho_0 = 997\,\text{kg/m}^3$
- Speed of sound $c = 1500\,\text{m/s}$
- Dynamic viscosity $\mu = 8.9 \times 10^{-4}\,\text{Pa}\cdot\text{s}$
- Thermal conductivity $k = 0.6\,\text{W/(m}\cdot\text{K)}$
- Specific heat $c_p = 4181\,\text{J/(kg}\cdot\text{K)}$

6.3 Excitation and Operating Conditions

The channel is excited by an oscillating boundary velocity at the resonance frequency:

$$f_0 = 1.9652\,\text{MHz}$$

This corresponds to a horizontal half-wave mode in the microchannel width.

6.4 Boundary Conditions

- **Acoustics:** One wall is excited with a harmonic velocity. Remaining walls are rigid (zero normal velocity).
- **Flow:** No-slip conditions are applied to all solid boundaries.
- **Thermal:** Bottom and side walls are held at a constant temperature T_0. The top boundary is thermally insulating, modeling a glass lid.

6.5 Assumptions

To simplify the analytical and numerical modeling, the following assumptions are made:

- The system is two-dimensional.
- The fluid is Newtonian and incompressible for the streaming flow.
- The acoustic wavelength is much larger than the boundary layer thickness.
- Viscous and thermal effects are confined to boundary layers.
- Particles are small enough for Stokes drag to be valid.

These assumptions are consistent with microfluidic regimes and enable the use of analytical approximations for streaming and particle motion.

7. COMSOL Implementation Overview

7.1 Physics Interfaces and Coupling

The simulation of acoustic streaming in the microchannel cross section was performed using COMSOL Multiphysics 6.1. The following physics interfaces were used:

- **Pressure Acoustics, Frequency Domain:** To compute the harmonic acoustic field in the channel.
- **Laminar Flow (spf):** To solve the time-averaged second-order streaming velocity field.
- **Heat Transfer in Fluids (ht):** To model acoustic heating due to viscous dissipation in boundary layers.
- **Particle Tracing for Fluid Flow (fpt):** To evaluate particle trajectories based on acoustic radiation and drag forces.

These interfaces are coupled using:

- **Acoustic Streaming Domain Coupling (ASDC):** Applies volume forces from the acoustic field to the fluid flow.
- **Acoustic Streaming Boundary Coupling (ASBC):** Applies slip velocities on walls due to boundary-layer-driven effects.

7.2 Study Sequence

The problem was solved in three sequential studies:

1. **Study 1 – Frequency Domain:** Computes the harmonic acoustic pressure field p_1 and velocity field $\mathbf{v}_1$ at the resonance frequency $f_0 = 1.9652\,\text{MHz}$.
2. **Study 2 – Stationary:** Computes the time-averaged streaming flow $\langle \mathbf{v}_2 \rangle$ and temperature distribution resulting from acoustic heating. Inputs come from Study 1.
3. **Study 3 – Time-Dependent:** Solves for particle trajectories using forces derived from the acoustic and streaming fields.

7.3 Thermoviscous Boundary Layer Impedance

To avoid numerically resolving extremely thin viscous and thermal boundary layers, the model uses the **Thermoviscous Boundary Layer Impedance** (TVBLI) boundary condition. This condition analytically incorporates:

- Damping of the acoustic wave
- Streaming slip velocity on the wall
- Heat generation due to viscous dissipation

This significantly reduces computational complexity while retaining the accuracy of second-order effects. The Thermoviscous Boundary Layer Impedance condition adds the losses due to thermal and viscous dissipation in the acoustic boundary layers at a wall. The condition is sometimes known simply as the BLI model. The losses are included in a locally homogenized manner, where the losses are integrated through the boundary layers analytically. The condition is applicable in cases where boundary layers are not overlapping. That is, it is not applicable in a very narrow waveguide (with dimensions comparable to the boundary layer thickness) or on very curved boundaries. Other than that, there are no restrictions on the shape of the geometry. This is in contrast to the Narrow Region Acoustics feature which is applicable only in waveguides of constant cross section, but also applicable for all frequencies, that is, also the very narrow case where boundary layers are overlapping. The thickness of the viscous and thermal boundary layers is given by

$$\delta_\nu = \sqrt{\frac{2\mu}{\rho\omega}}, \quad \delta_T = \sqrt{\frac{2k}{\rho c_p \omega}}$$

The Thermoviscous Boundary Layer Impedance condition adds an impedance-like boundary condition by defining the inward normal velocity -n·v at the boundary in terms of the pressure and its tangential derivatives:

$$-\mathbf{n}\cdot\mathbf{v} = -i\omega\left(T_{\text{bnd}} - \frac{\alpha_p T}{\rho C_p} p_t\right)\frac{\delta_{\text{th}}\alpha_p}{1+i} - v_n + \frac{\delta_v}{1+i}\left(\nabla_{||}\cdot\mathbf{v}_{||}^0 + \frac{1}{i\omega\rho}\Delta_{||} p_t\right)$$

Figure 2: Thermoviscous Boundary Layer Impedance

where Tbnd is a possible boundary temperature variation source, vn is a possible normal velocity source, and is a possible tangential velocity source (normal and tangential components are computed from a velocity vector).

7.4 Particle Tracking Configuration

Particles were released in a grid distribution and subjected to:

- **Acoustic Radiation Force:** Calculated from pressure and velocity fields.
- **Drag Force:** Based on the streaming flow.

Two particle sizes were simulated:

- Large particles ($a = 3\,\mu$m): Focused at pressure nodes.
- Small particles ($a = 0.4\,\mu$m): Dragged by the streaming rolls.

7.5 Mesh and Solver Notes

A single thin boundary layer mesh element was applied to all boundaries to ensure accurate evaluation of normal gradients needed for the TVBLI condition. Default solvers were used with minor adjustments to time-stepping tolerances for stability.

8. Analytical vs Simulation Approach

8.1 Comparison Overview

The acoustic streaming phenomena in the microchannel were explored using both analytical formulations and numerical simulation. While analytical models provide insight and predictive capability, simulations offer high-resolution visualizations and incorporate boundary conditions that are difficult to treat analytically.

8.2 Pressure Field

Analytically, the acoustic pressure field was assumed as a 1D standing wave:

$$p(x) = p_0 \cos(kx), \quad \text{with} \quad k = \frac{\pi}{W}$$

This closely matches the COMSOL output in the frequency domain at resonance frequency $f_0 = 1.9652\,\text{MHz}$, where the standing wave exhibits nodes and antinodes aligned with theoretical expectations.

8.3 Streaming Flow Field

The analytical Rayleigh streaming velocity near boundaries:

$$v_{\text{stream}} \propto \frac{v_0^2}{c}\left(\frac{\delta_\nu}{H}\right)\sin(2kx)$$

was consistent with the quadrupole streaming rolls observed in COMSOL. However, COMSOL's simulation captures 2D effects, curved streamlines, and variation in both directions, whereas the analytical model assumes a simplified 1D driving field.

8.4 Temperature Rise

Analytical boundary layer models estimate heating through:

$$Q_{\text{tot}} = \text{Re}\left[\mathbf{v}_0 \cdot (\sigma_1 \cdot \mathbf{n})^*\right]$$

COMSOL directly uses this predefined variable to simulate the temperature gradient across the microchannel. The simulation showed a small temperature rise (on the order of millikelvin), aligned with analytical expectations.

8.5 Particle Trajectories

The motion of particles under the influence of acoustic radiation and drag forces was predicted using:

$$\mathbf{v}_{\text{particle}} = \mathbf{v}_{\text{stream}} + \frac{1}{6\pi\mu a}\mathbf{F}_{\text{rad}}$$

In COMSOL, large particles ($a = 3\,\mu$m) focused at pressure nodes due to dominant radiation forces, while small particles ($a = 0.4\,\mu$m) followed the streaming rolls due to dominant viscous drag — exactly as predicted by theory.

8.6 Discussion of Discrepancies

While the analytical model captures the essential physics and trends, some differences arise:

- Analytical solutions assume simplified boundary conditions and ignore 2D curvature effects.
- COMSOL includes temperature dependence, curvature, and full viscous damping through impedance conditions.
- Particle interactions and 3D flow effects are not captured in 2D analytical models.

Despite these limitations, the close match between simulation and theory validates the use of analytical models for understanding and predicting acoustic streaming behavior in microscale systems.

9. Results and Interpretation

9.1 1. Acoustic Pressure Field

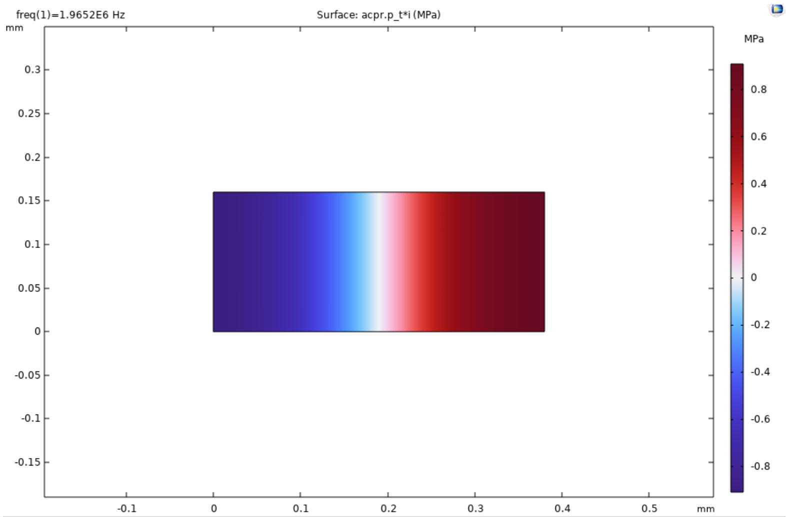

Figure 3: Acoustic pressure field in the rectangular channel at the horizontal half-wave resonance.

This plot shows the spatial distribution of the acoustic pressure at the resonance frequency $f_0 = 1.9652\,\text{MHz}$. A clear standing wave pattern is observed, with pressure nodes and antinodes along the channel width. The simulation confirms the analytical pressure field assumption $p(x) = p_0 \cos(kx)$, validating the fundamental mode shape predicted by the Helmholtz solution.

9.2 2. Acoustic Streaming Flow

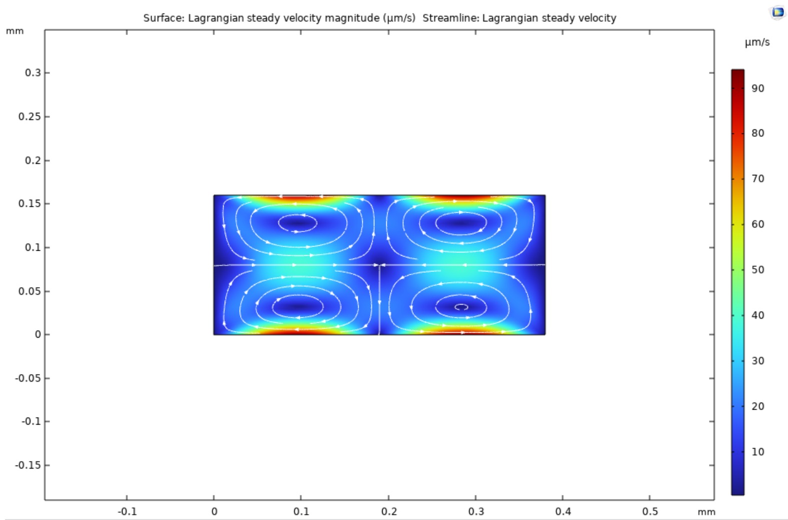

Figure 4: Acoustic streaming flow in the microchannel cross section. The color scale shows velocity magnitude; black lines indicate flow direction.

The second-order steady streaming velocity field exhibits four Rayleigh streaming rolls, typical of boundary-driven streaming in rectangular geometries. The rolls originate due to shear stresses in the viscous boundary layers, as predicted by Rayleigh streaming theory. The simulation captures the quadrupole symmetry and the sinusoidal $\sin(2kx)$ dependence of streaming velocity along the channel.

9.3 3. Temperature Field from Acoustic Heating

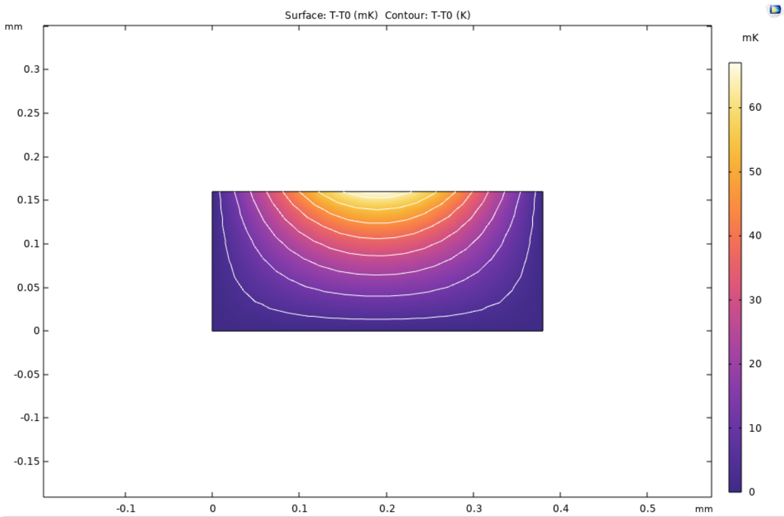

Figure 5: Temperature field due to acoustic heat generation in viscous boundary layers. White lines represent temperature contours.

Acoustic energy dissipated in the viscous boundary layers results in local heating, modeled as a boundary heat source. The temperature field shows a gradient across the channel, with maximum values near the top boundary (assumed thermally insulating). The increase is on the order of millikelvins, consistent with analytical expectations from Q_{tot}. These gradients can influence fluid properties and flow in long-term operation.

9.4 4. Particle Trajectories – Large Particles

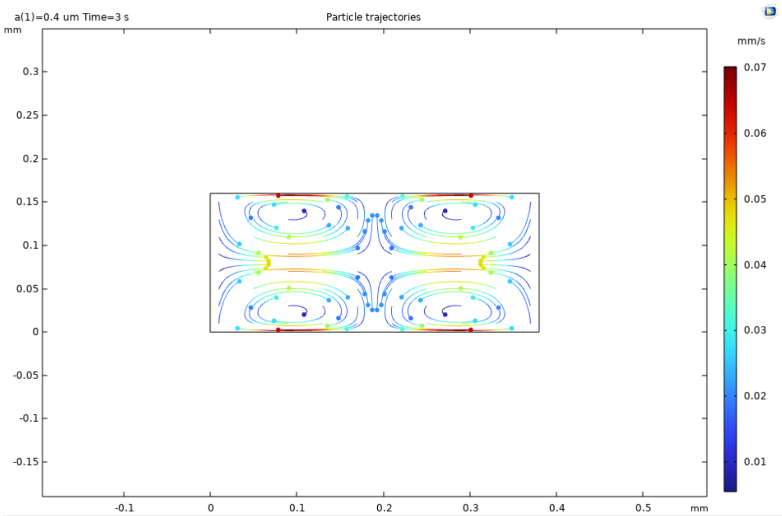

Figure 6: Trajectory and velocity amplitude of particles with radius $a = 3\,\mu$m at $t = 0.1\,$s.

Larger particles experience strong acoustic radiation forces and are focused toward pressure nodes. Their motion is primarily governed by the gradient of acoustic pressure. This focusing effect demonstrates the utility of acoustic streaming in particle separation or concentration applications. The colored traces also show higher particle speeds compared to smaller ones.

9.5 5. Particle Trajectories – Small Particles

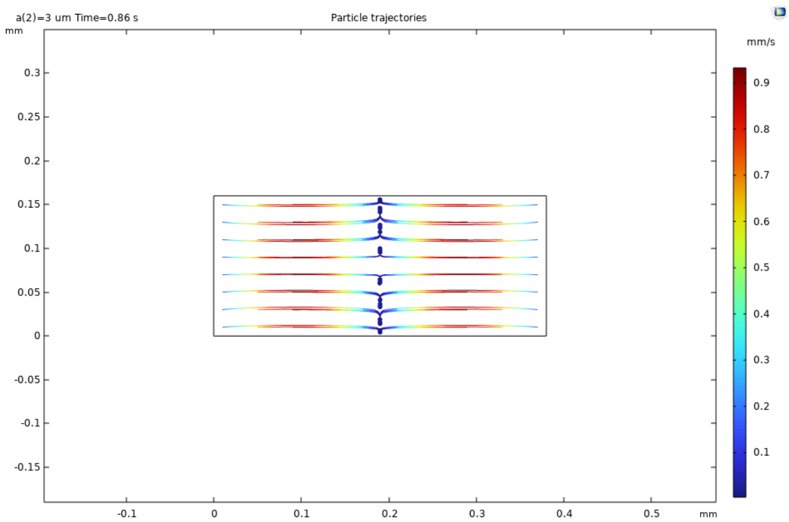

Figure 7: Trajectory and velocity amplitude of particles with radius $a = 0.4\,\mu$m at $t = 3\,$s.

Smaller particles are dominated by the viscous drag force from the streaming flow. They do not respond to the acoustic radiation force due to their small size. As a result, they follow the fluid streamlines and circulate within the Rayleigh rolls. Their velocity magnitudes are significantly lower than those of the large particles, consistent with theoretical predictions.

10. Conclusion

This project explored the phenomenon of acoustic streaming in a two-dimensional microchannel cross section, with emphasis on both analytical understanding and computational validation using COMSOL Multiphysics.

Through the application of perturbation theory and boundary layer analysis, we derived key expressions governing the acoustic pressure field, streaming velocity, and forces acting on particles. These theoretical insights were validated through simulation, which reproduced classical Rayleigh streaming rolls, temperature gradients due to viscous dissipation, and size-dependent particle motion.

Simulation results showed that:

- The acoustic pressure field matches the predicted standing wave pattern at the fundamental resonance frequency.
- Streaming rolls emerge due to boundary-layer-induced slip velocities, aligning with Rayleigh streaming theory.
- Larger particles are primarily influenced by the acoustic radiation force and focus at pressure nodes.
- Smaller particles are entrained in the steady streaming flow due to dominant viscous drag forces.
- Acoustic heating is small but measurable, potentially affecting long-term operation in microfluidic systems.

Overall, the combination of analytical modeling and simulation provides a powerful toolkit for designing and optimizing acoustofluidic systems. This study demonstrates how acoustic streaming can be effectively used for particle manipulation, with applications in biomedical diagnostics, cell sorting, and lab-on-a-chip technologies.

Future work could extend this analysis to three-dimensional channels, nonlinear acoustics at higher amplitudes, and interactions between multiple particles or external fields (e.g., magnetic or electric).

11. References

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Appendix A: Parameters Used

Parameter	**Value**	**Description**
W	$380\,\mu$m	Channel width
H	$160\,\mu$m	Channel height
f_0	1.9652 MHz	Resonance frequency
ρ_0	$997\,\mathrm{kg/m^3}$	Fluid density (water)
μ	8.9×10^{-4} Pa·s	Dynamic viscosity
c	1500 m/s	Speed of sound in water
c_p	4181 J/(kg·K)	Specific heat capacity
k	0.6 W/(m·K)	Thermal conductivity
a	$0.4 - 3\,\mu$m	Particle radius (varied)

Table 1: Physical and geometric parameters used in the simulation and analysis.

Appendix B: Key Derived Equations

- Helmholtz equation for pressure:

$$\nabla^2 p_1 + k^2 p_1 = 0$$

- Boundary layer thicknesses:

$$\delta_\nu = \sqrt{\frac{2\mu}{\rho\omega}}, \quad \delta_T = \sqrt{\frac{2k}{\rho c_p \omega}}$$

- Rayleigh streaming velocity:

$$v_{\text{stream}} = \frac{3}{4}\frac{v_0^2}{c}\left(\frac{\delta_\nu}{H}\right)\sin(2kx)$$

- Acoustic radiation potential:

$$U_{\text{ac}} = \frac{4\pi a^3}{3}\left(\frac{1}{2}f_1\kappa_0|p|^2 - \frac{3}{4}f_2\rho_0|v|^2\right)$$

- Radiation force:

$$\mathbf{F}_{\text{rad}} = -\nabla U_{\text{ac}}$$

- Drag force:

$$\mathbf{F}_{\text{drag}} = 6\pi\mu a(\mathbf{v}_{\text{stream}} - \mathbf{v}_{\text{particle}})$$

- Particle velocity:

$$\mathbf{v}_{\text{particle}} = \mathbf{v}_{\text{stream}} + \frac{1}{6\pi\mu a}\mathbf{F}_{\text{rad}}$$