ME647 - Introduction to Turbulence

Assignment 2

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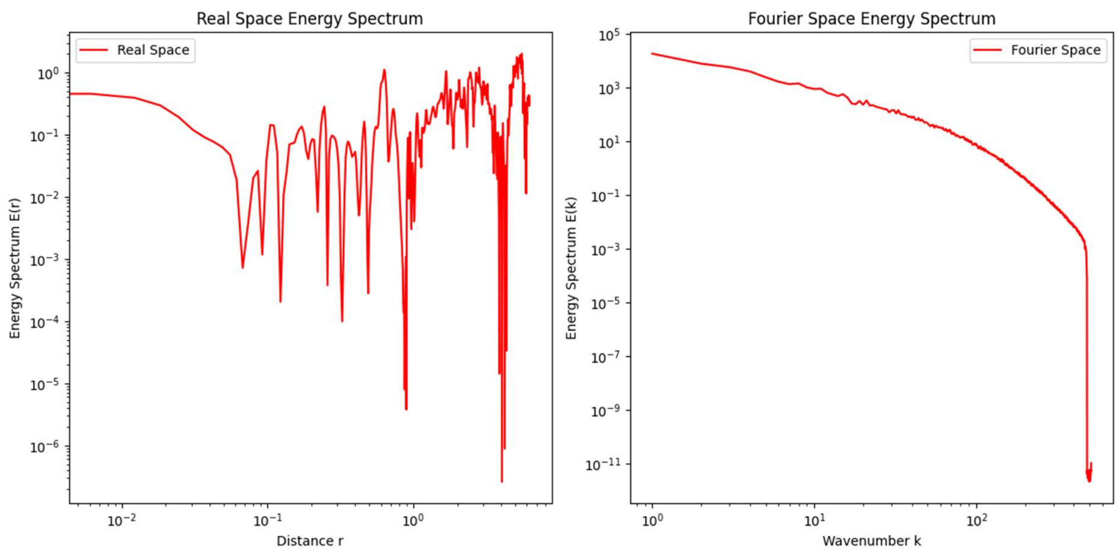
1. Energy in real space: 428.7784573503308

Energy in Fourier space: 428.7784573503309

Parseval's theorem, states that the total energy (or power) of a signal is conserved when transformed between the time and frequency domains



Plot of energy spectra in real and Fourier space

Parseval's theorem relates the total energy in real space to the energy in Fourier space. For a discrete signal , it is stated as:

$$\sum |u(x)|^2 = \frac{1}{N} \sum |\hat{u}(k)|^2$$

Where:

- $u(x)$ is the velocity field in real space.
- $\hat{u}(k)$ is the Fourier transform of the velocity field.
- N is the number of grid points.

Purpose

To ensure that energy is conserved during the Fourier transform, Parseval's theorem is validated by comparing the real-space energy and the Fourier-space energy

2. Inertial range lies in 60*Eta and L/6

And wavenumber, k = $2\pi/L$

- The total Energy Spectrum Along x-direction Averaged Over y:

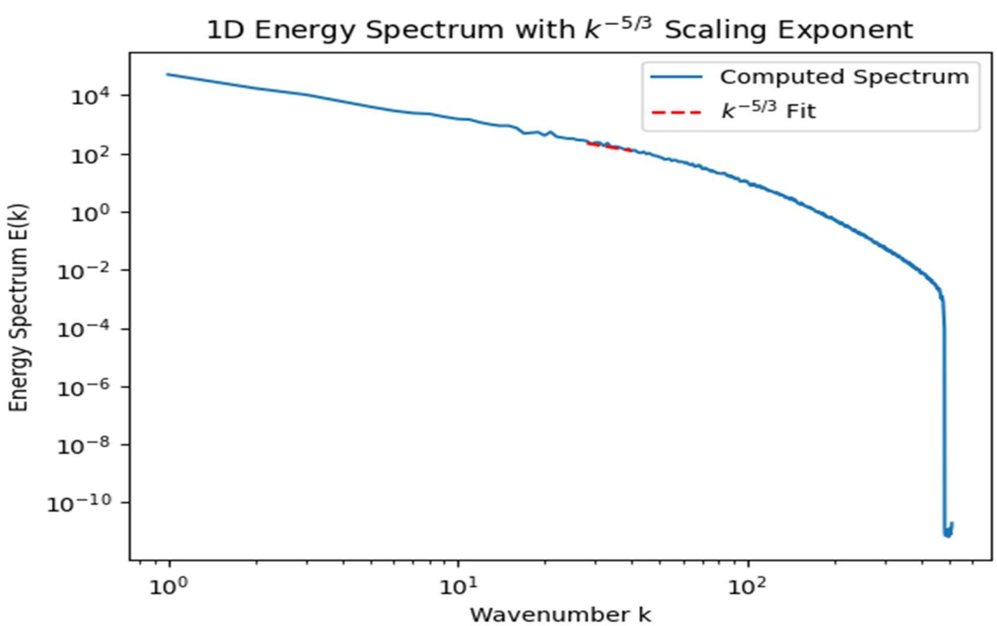
  524811.6514194038 units

- Inertial range: 0.133339 to 0.227333
- Wavenumber range: 27.638645 to 47.121875 ~ 28 to 47

The energy spectrum describes how energy is distributed across different scales (wavenumbers) in turbulence. The discrete energy spectrum can be calculated as:

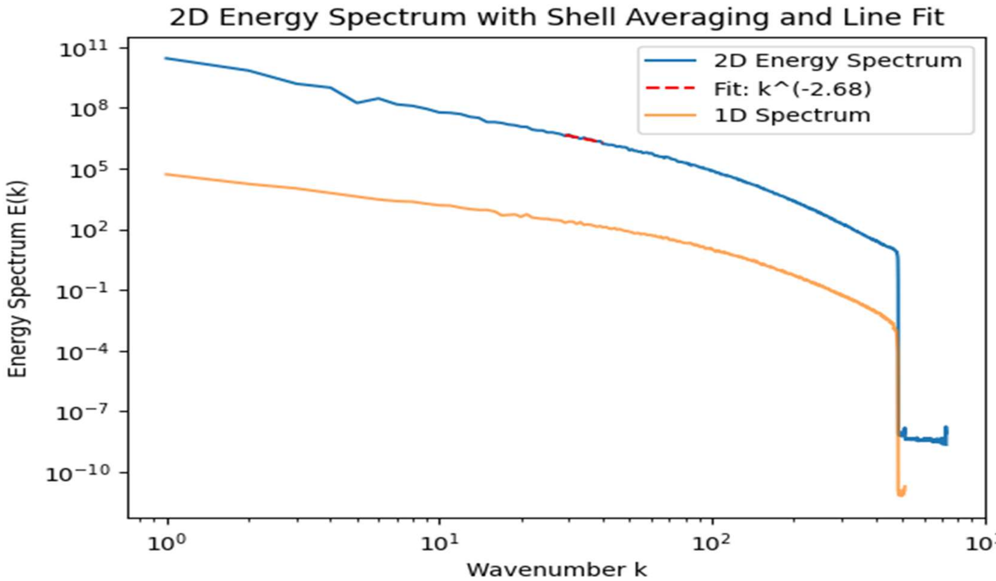
$$E(k) = \frac{1}{2}(|\hat{u}(k)|^2 + |\hat{v}(k)|^2)$$

Where k is the wavenumber, and $\hat{u}(k)$ and $\hat{v}(k)$ are the Fourier transform of the velocity fields



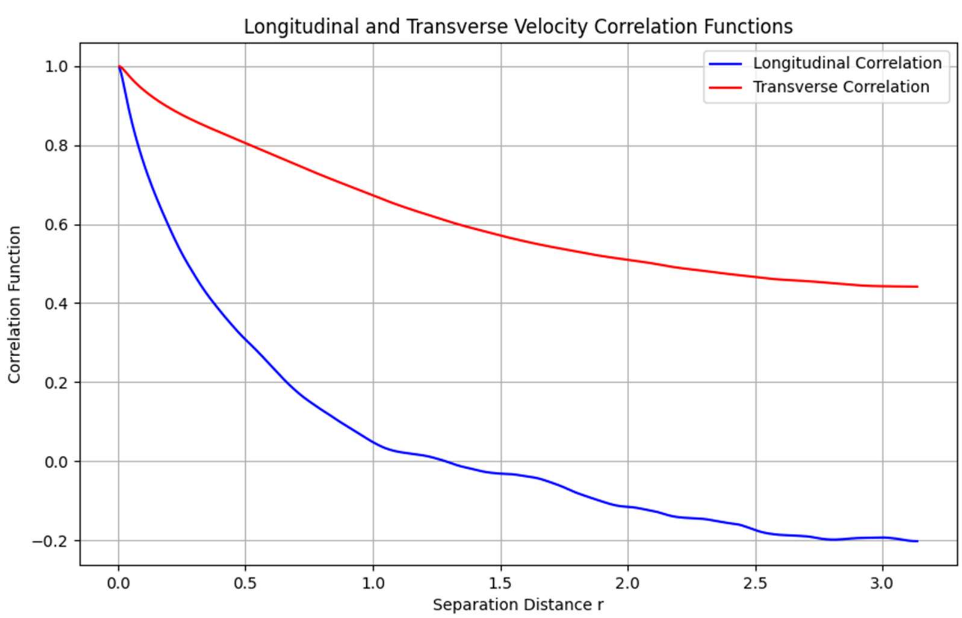
<u>The Energy Spectrum with the scaling exponent k^(−5/3) in the inertial range with a line-fit</u>

3.



<u>1D, 2D-energy spectrum vs wavenumber</u>

4.



<u>longitudinal and transverse velocity correlation functions in half domain</u>

The dependence between the two velocities at two points is measured by the time averages of the products of the quantities measured at two points. The correlation of the u' components of the turbulent velocity of these two points is defined as

$$\overline{u'(x)\, u'(x+r)}$$

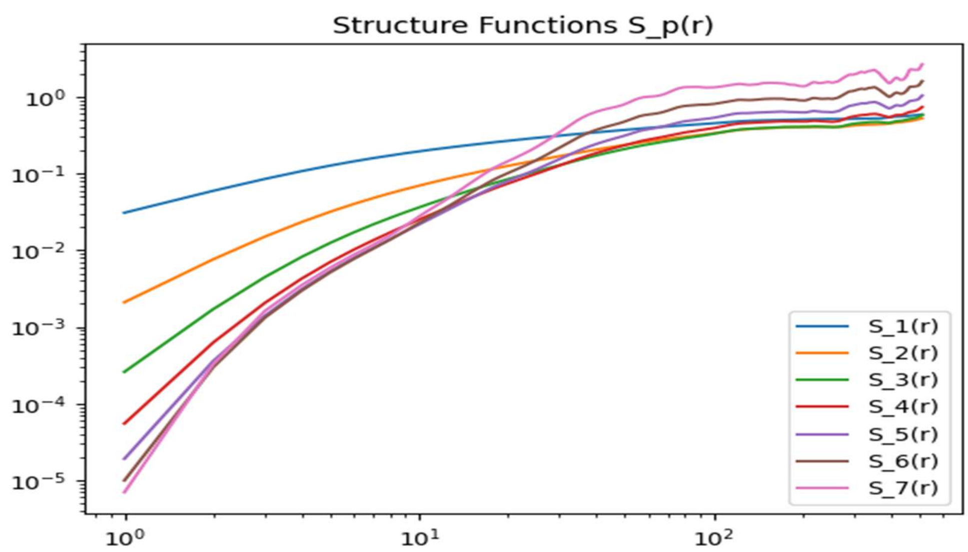
The non-dimensional form of the correlation

$$R(r) = \frac{\overline{u'(x)\, u'(x+r)}}{\left(\overline{u'^2}(x)\, \overline{u'^2}(x+r)\right)^{1/2}}$$

The value of $R\ (r)$ of unity signifies a perfect correlation of the two quantities involved and their motion is in phase. Negative value of the correlation function implies that the time averages of the velocities in the two correlated points have different signs

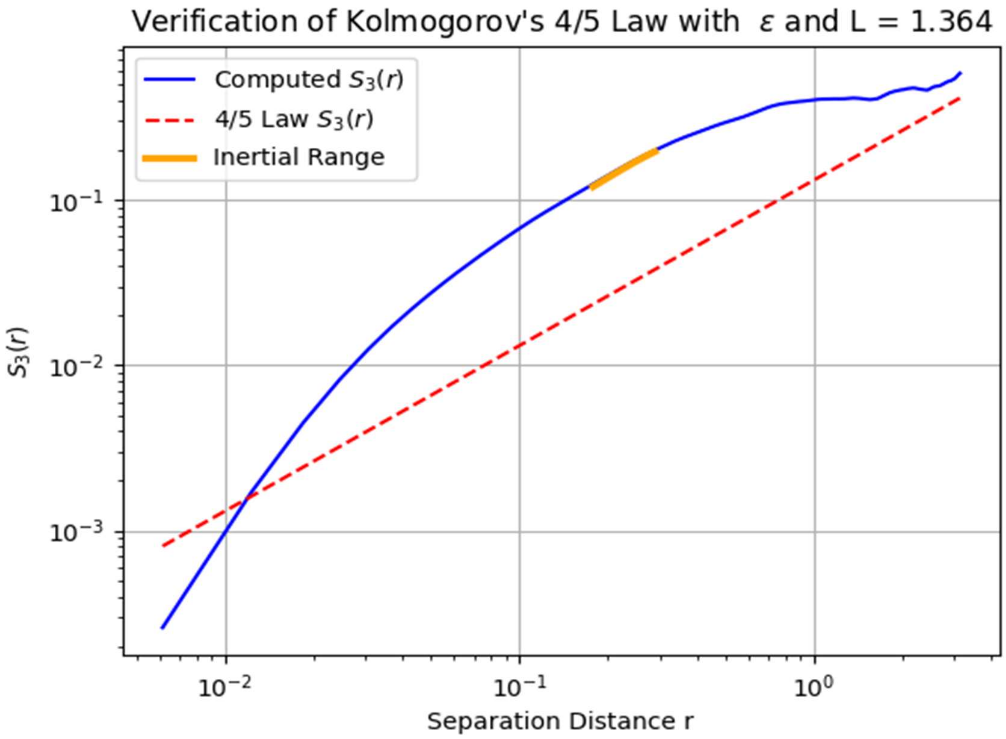
5.a

Structure functions are statistical tools used to quantify the differences in velocity at two points in a turbulent flow separated by a distance r. They describe the scaling behaviour of velocity fluctuations across different spatial scales, making them essential for understanding turbulence.



$S_p(r)$ v/s r

5.b

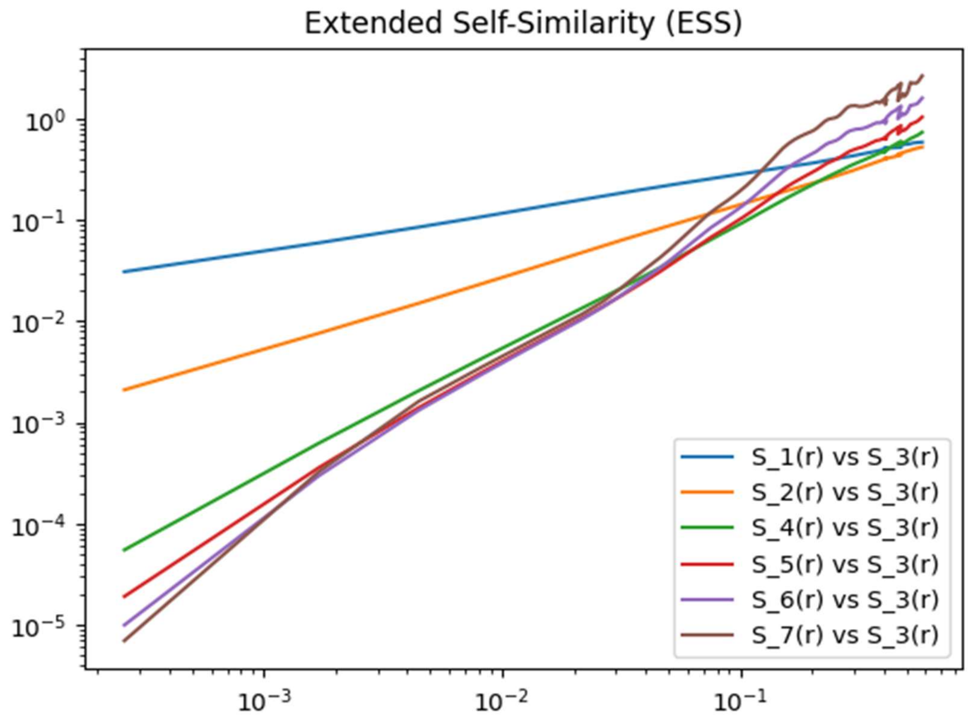


Kolmogorov's 4/5th law for S_3(r)

Kolmogorov's 4/5th law for S_3(r):

S[(u(t, x + r) − u(t, x)) · r |r|]^3 = − (4/5) ε |r|

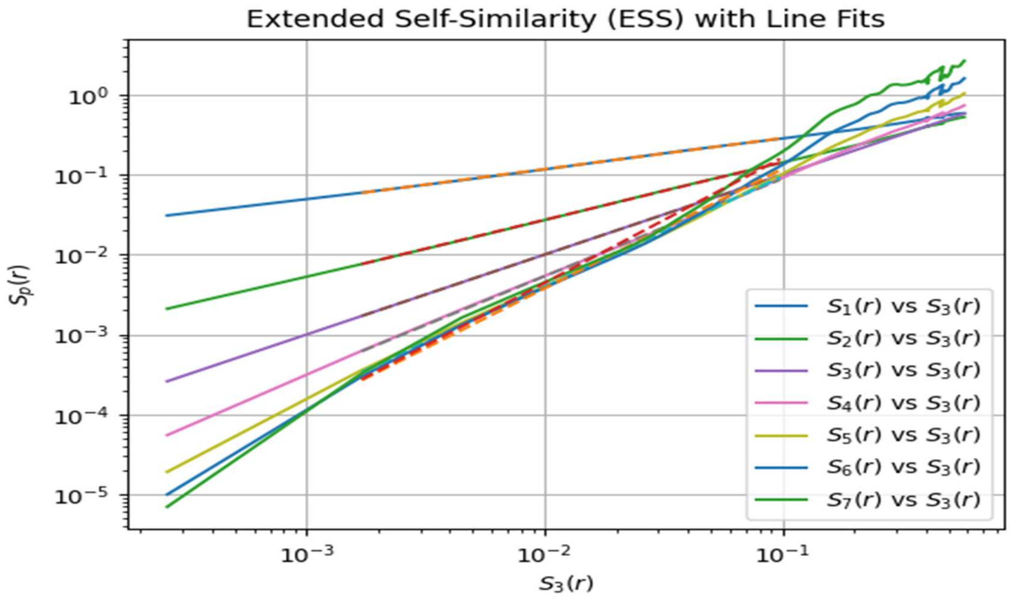
5.c



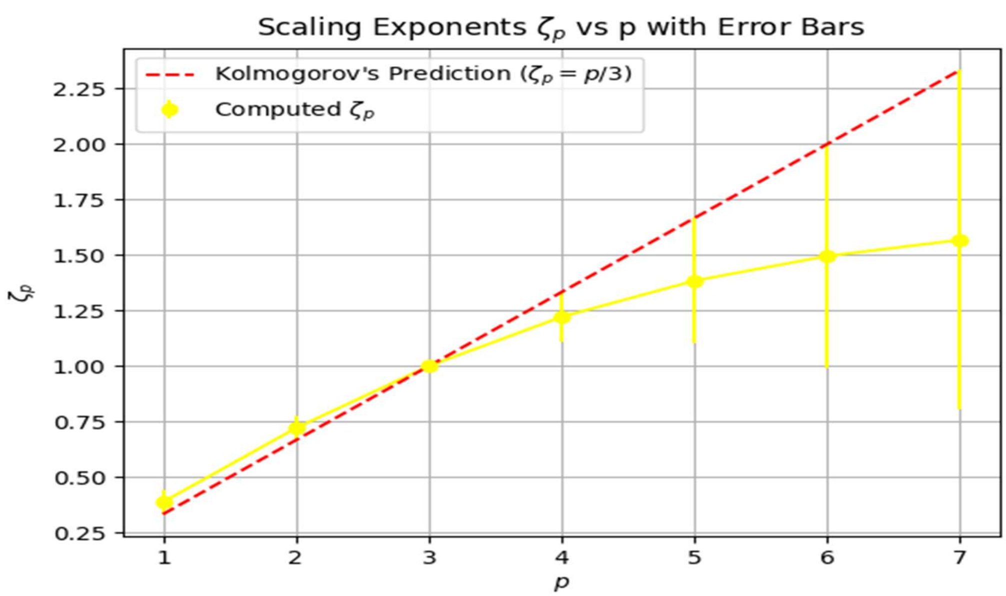
$S_p(r)$ versus $S_3(r)$

Extended Self-Similarity (ESS) is a technique that enhances the scaling range for structure functions. Instead of directly plotting against, ESS involves plotting against the third-order structure function

5.d



$S_p(r)$ versus $S_3(r)$ with line fits



scaling exponents, ζ_p and errorbars

- The source of anomalous scaling is the intermittent nature of turbulence, where energy dissipation is not uniform across scales.
- Large values of 'p' amplify this effect by giving more weight to extreme events, causing greater deviations from the expected p/3 scaling